Universiteit Leiden

Shift-Symmetric Orbital Inflation

single field or multi-field?

based on *arXiv: 1901.03657*
with A. Achucarro, E. Copeland, O. Iarygina, G. Palma and Y. Welling;
and *arXiv: 1907.xxxxx*
with A. Achucarro, G. Palma and Y. Welling

Dong-Gang Wang

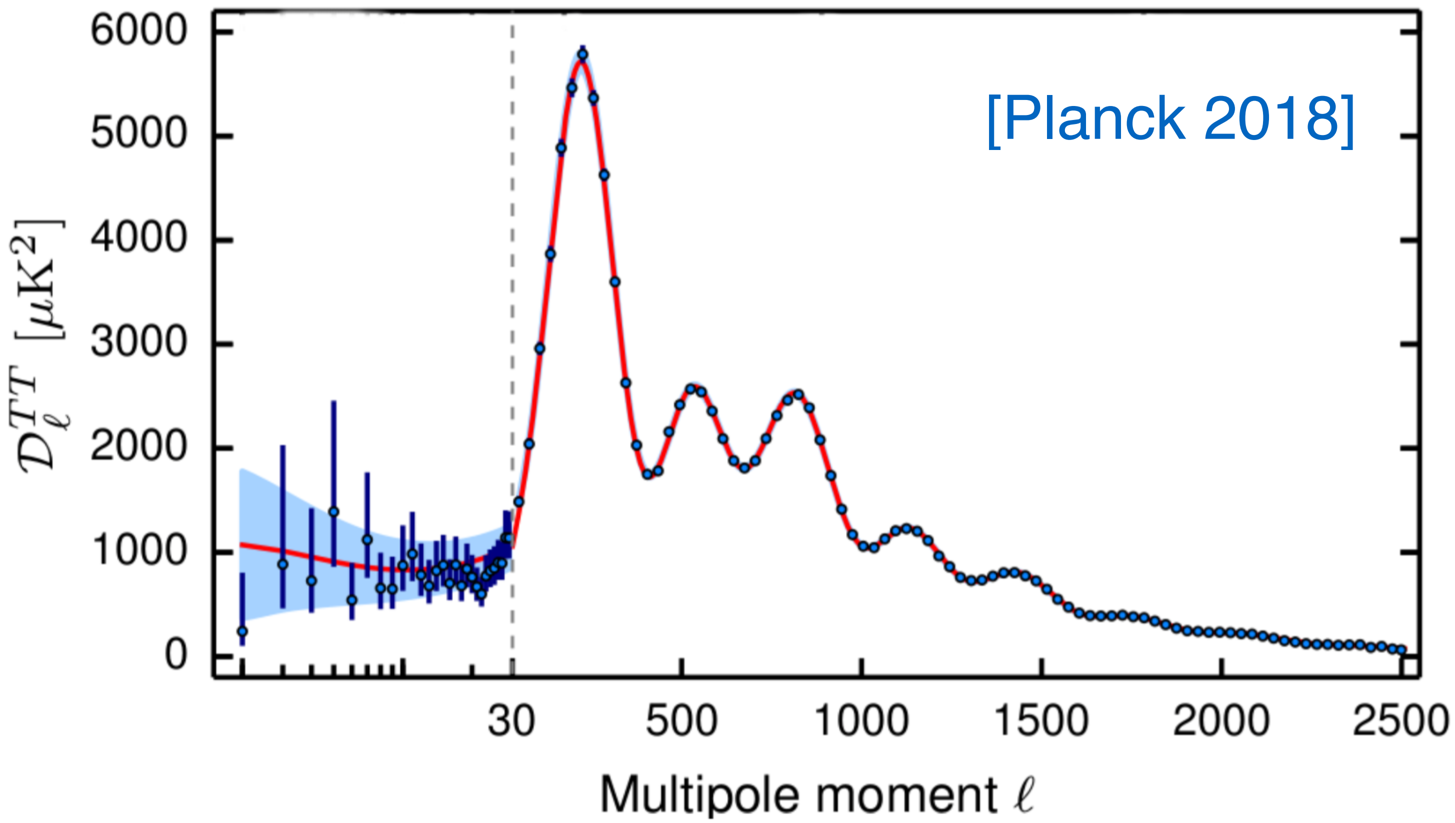
Leiden Observatory & Lorentz Institute

@Warsaw, July. 04, 2019

The success of single field slow-roll inflation

★ solved the horizon problem and flatness problem

★ quantum fluctuations of inflaton lead to **structure formation**



✦ Consistent with current CMB results

— nearly scale-invariant curvature perturbation

— small tensor-to-scalar ratio

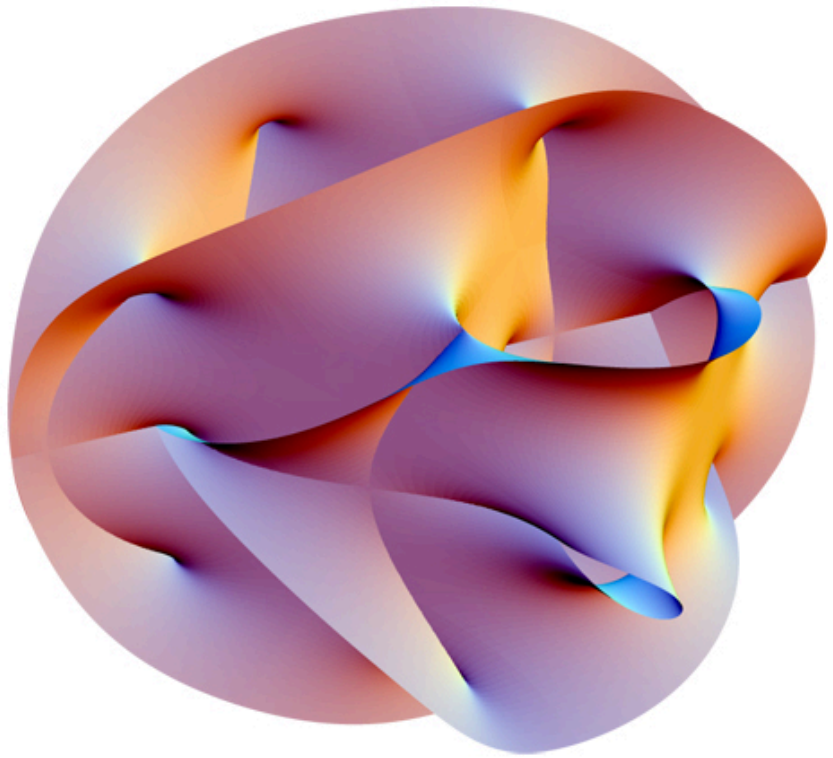
— small non-Gaussianity

— small isocurvature modes

Theoretical Challenges

— the realisation of inflation in fundamental theories

String inflation as an example



string theory in 10D spacetime

string compactification

KKLT
LVS
...

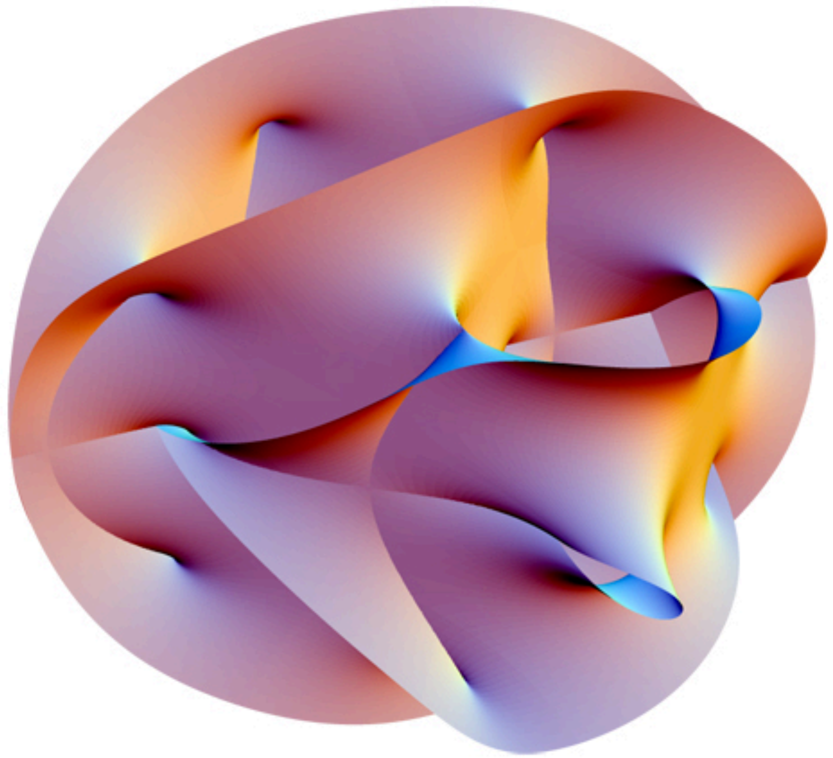
4D effective field theory
with many light fields

stabilize all
extra fields

single field inflation

eta-problen swampland conjectures

String inflation as an example



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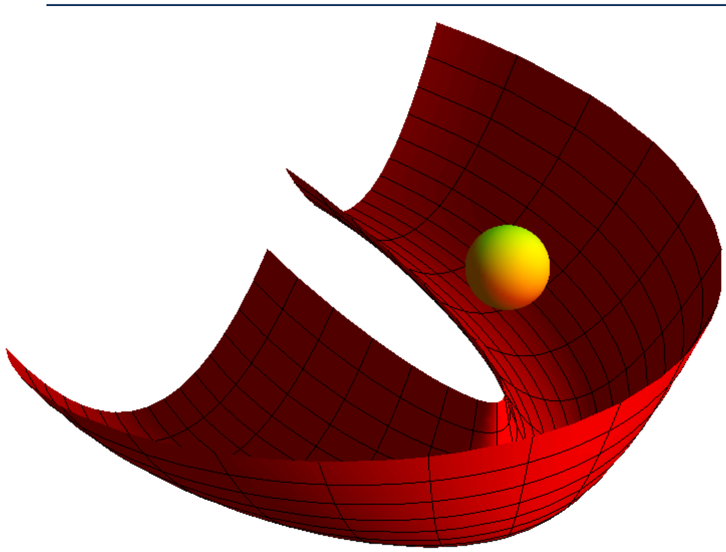
multi-field inflation

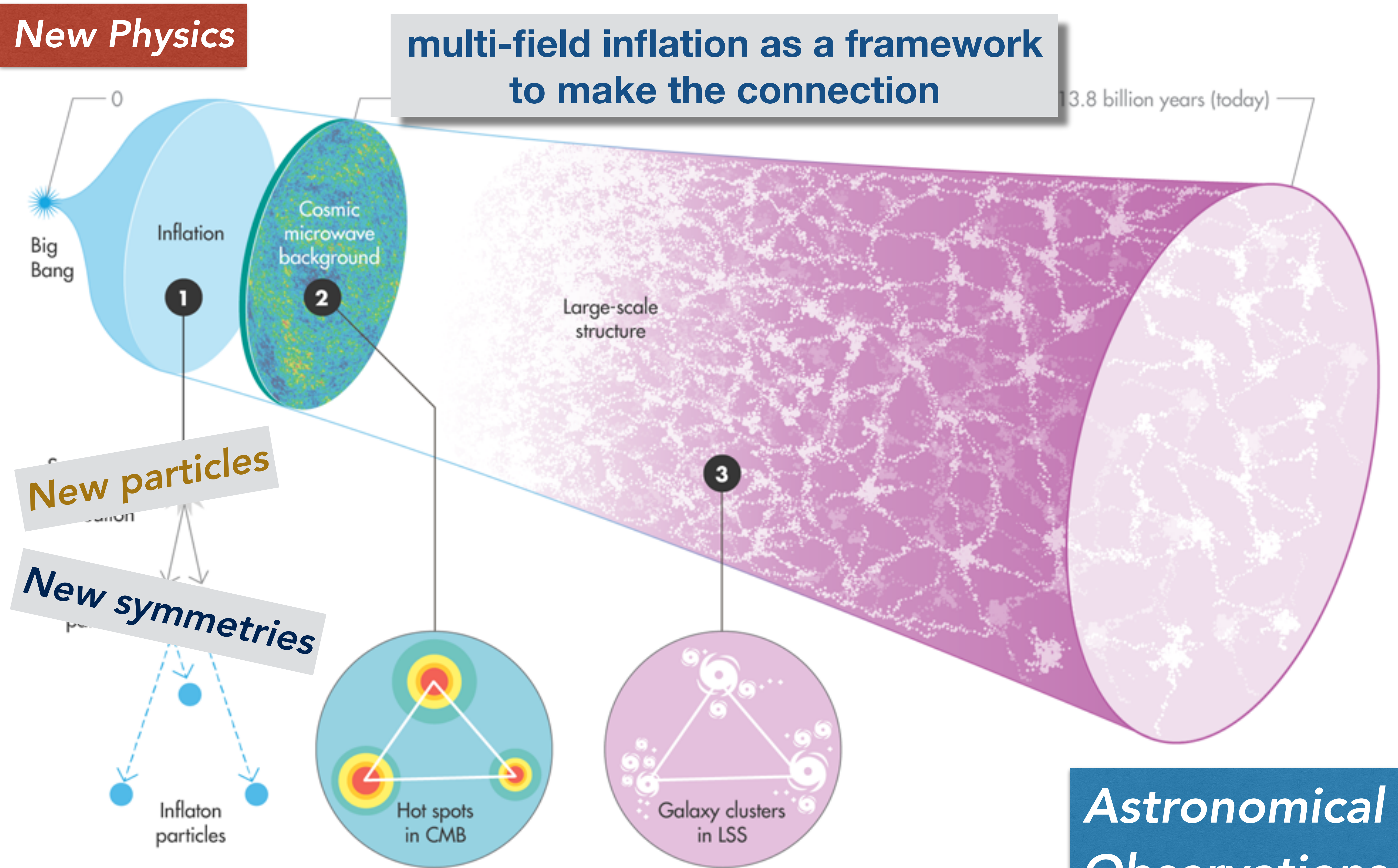


stabilize all extra fields

single field inflation

eta-problen swampland conjectures

From the phenomenological perspective…



[Lucy Reading-Ikkanda, Quantum Magzine]

Outline

- **Multi-field inflation in a nutshell**
  - — non-geodesic traj, isocurvature mass & self-interaction
- **Shift-symmetric orbital inflation**
  - — model, perturbations & pheno
- **Small non-Gaussianity in multi-field models**
  - — suppressed self-interaction & a scaling transformation
- **Outlook and discussion**
  - — theoretical implications

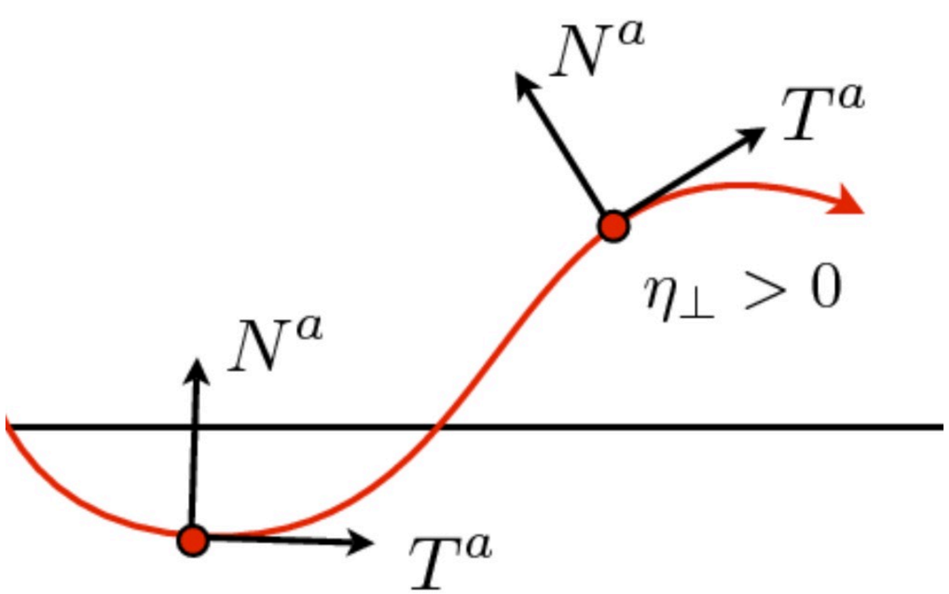
Multi-field inflation in a Nutshell

From the perspective of fundamental realization,
***ALL* the inflation models are essentially multi-field.**

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}G_{ab}(\phi)g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

field space metric

potential



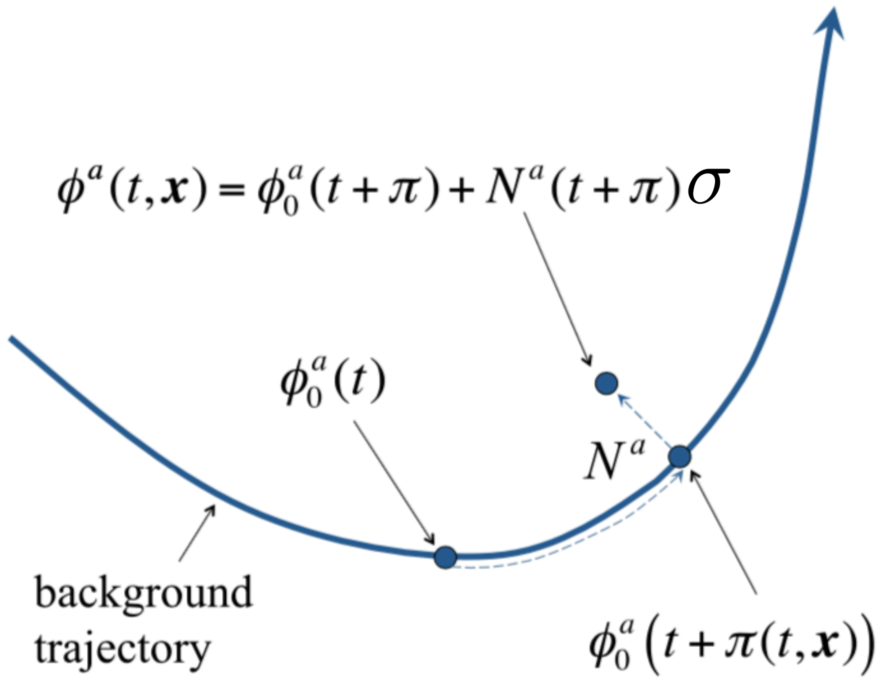
$$\Omega \equiv -N_a D_t T^a$$

$\Omega \neq 0$ ⟷ **Non-geodesic trajectory**

They can be effectively described by a single scalar field only if:

- 1) Inflaton rolls along the geodesics of the field space;
- 2) the extra fields are very heavy and can be integrated out

Perturbations in multi-field inflation

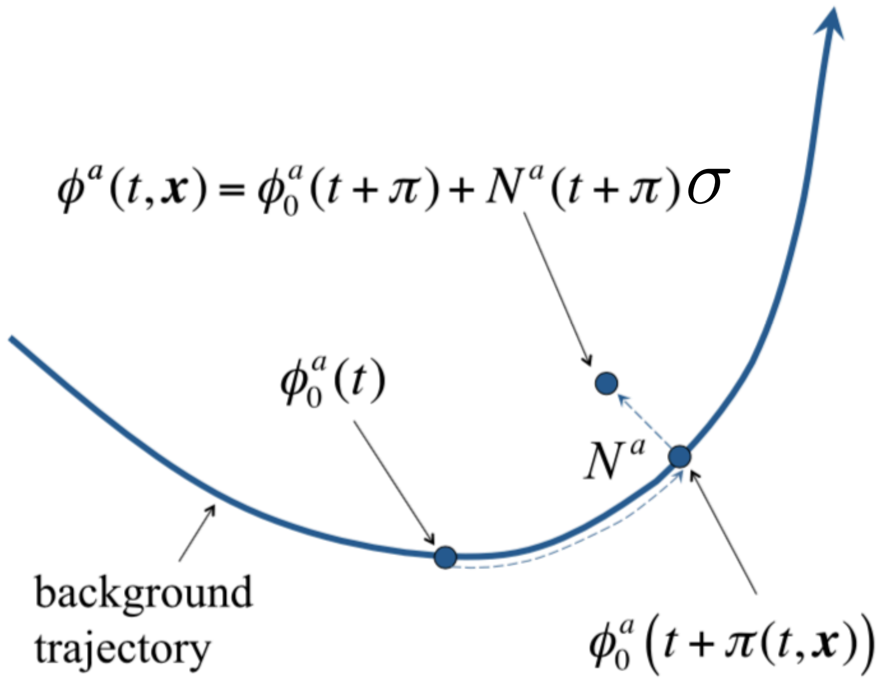


curvature modes $\mathcal{R} = H\pi$

isocurvature modes σ

$$S = \int d^4x\, a^3 \left[\epsilon(\dot{\mathcal{R}} - \alpha\sigma)^2 - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + \frac{1}{2}\left(\dot{\sigma}^2 - \frac{1}{a^2}(\nabla\sigma)^2\right) - \frac{1}{2}\mu^2\sigma^2\right]$$

Perturbations in multi-field inflation



- curvature modes $\mathcal{R} = H\pi$
- isocurvature modes σ

isocurvature mass $\mu^2 \equiv V_{NN} + \epsilon H^2 \mathbb{R} + 3\Omega^2$

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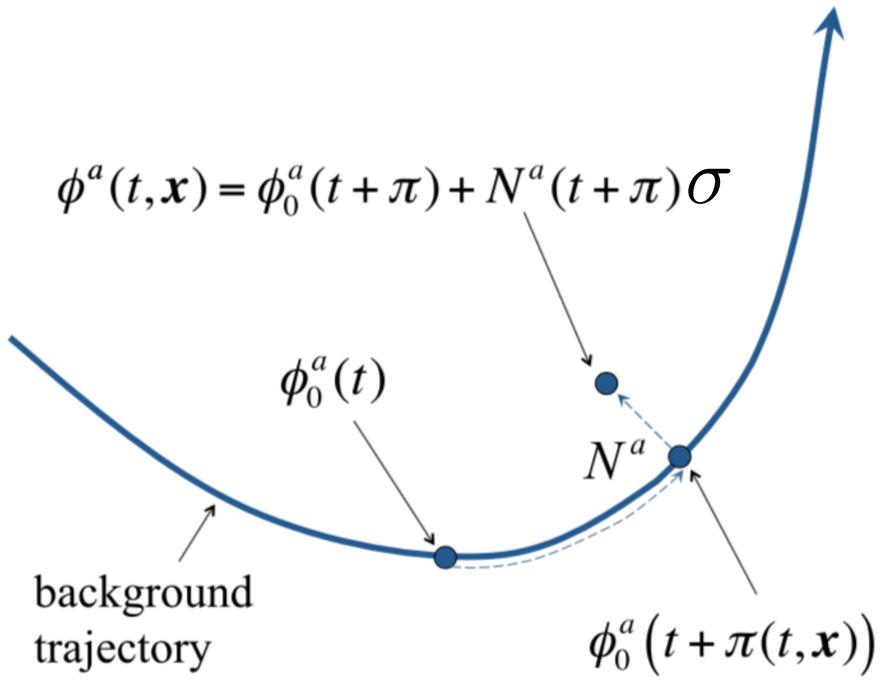
coupled evolution $\dot{\zeta} = \frac{2\Omega}{\sqrt{2\epsilon}}\sigma$

Perturbations in *multi-field inflation*



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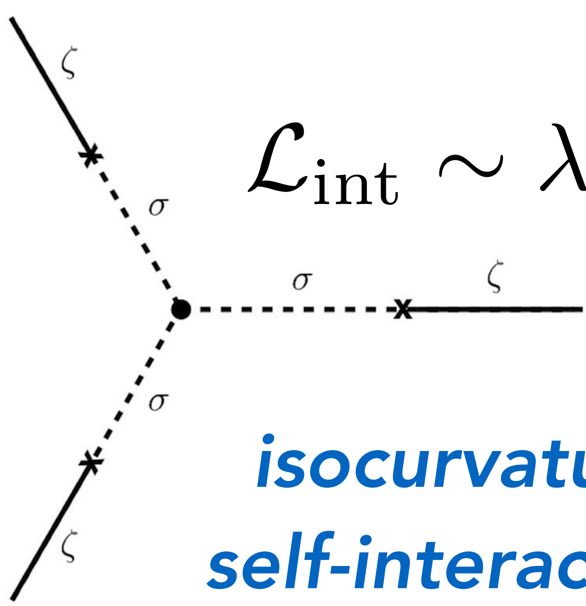
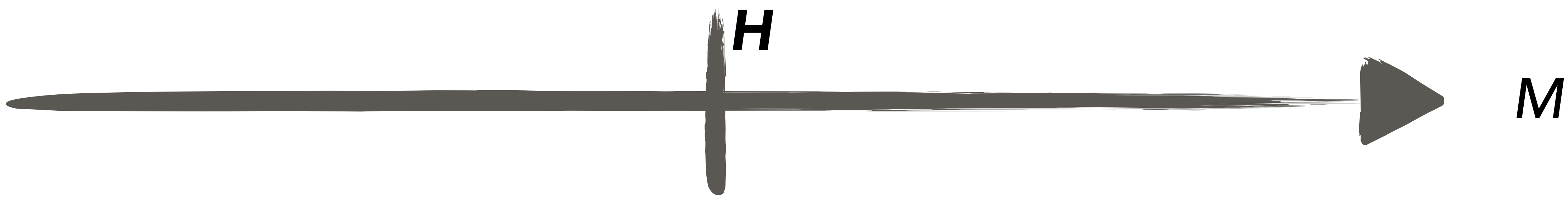


$\mathcal{L}_{\rm int} \sim \lambda\sigma^3$

isocurvature self-interaction

- isocurvature perturbations
- primordial non-Gaussianity

Different regimes of multi-field inflation



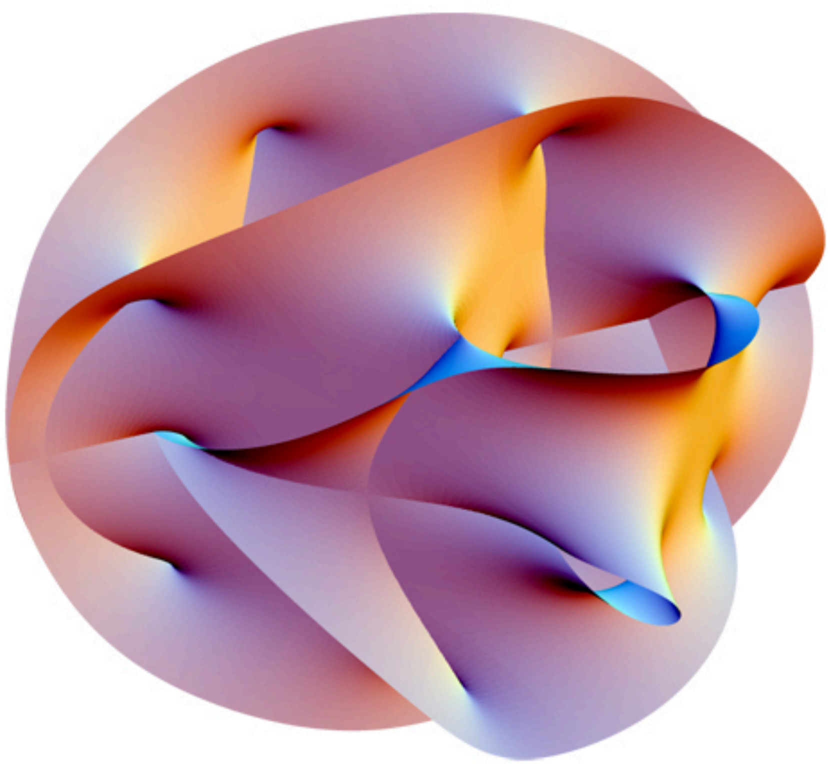
light $M<<H$	*medium* $M\sim H$	*heavy* $M>>H$
uncoupled; post-inflation effects	interact with inflaton	reduced c_s
O(1) local non-Gaussianity;	intermediate shapes of non-Gaussianity;	equilateral shape of non-Gaussianity;
Example: curvaton models	Example: quasi-single field inflation	Example: effective single field inflation
Enqvist & Sloth 2001, Lyth & Wands 2002, Moroi & Takahashi 2001	*Chen & Wang 2009; Arkani-Hamed & Maldacena 2015; …*	*Achucarro, Gong, Hardeman, Palma & Patil 2010; 2011*

[See Xingang and Yi's talks]

How about light fields m<<H **vigorously** coupled to inflaton?

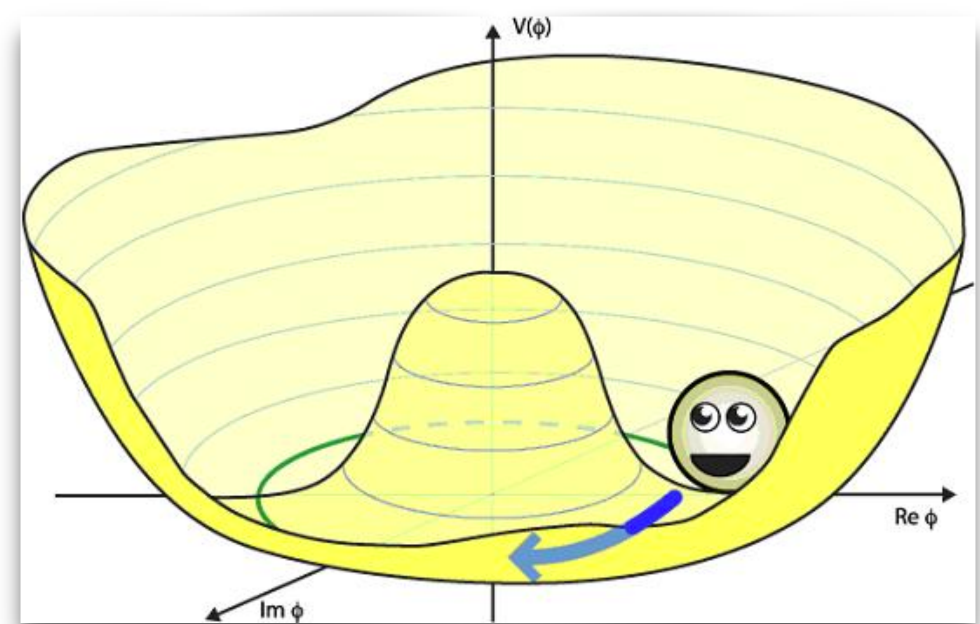
(which is more common in the fundamental perspective)

UV origin



moduli fields after string compactification without stabilisation

EFT point of view



pseudo-Goldstone fields living in the coset space *G/H* after spontaneous symmetry breaking

How about light fields m<<H **vigorously** coupled to inflaton?

(which is more common in the fundamental perspective)

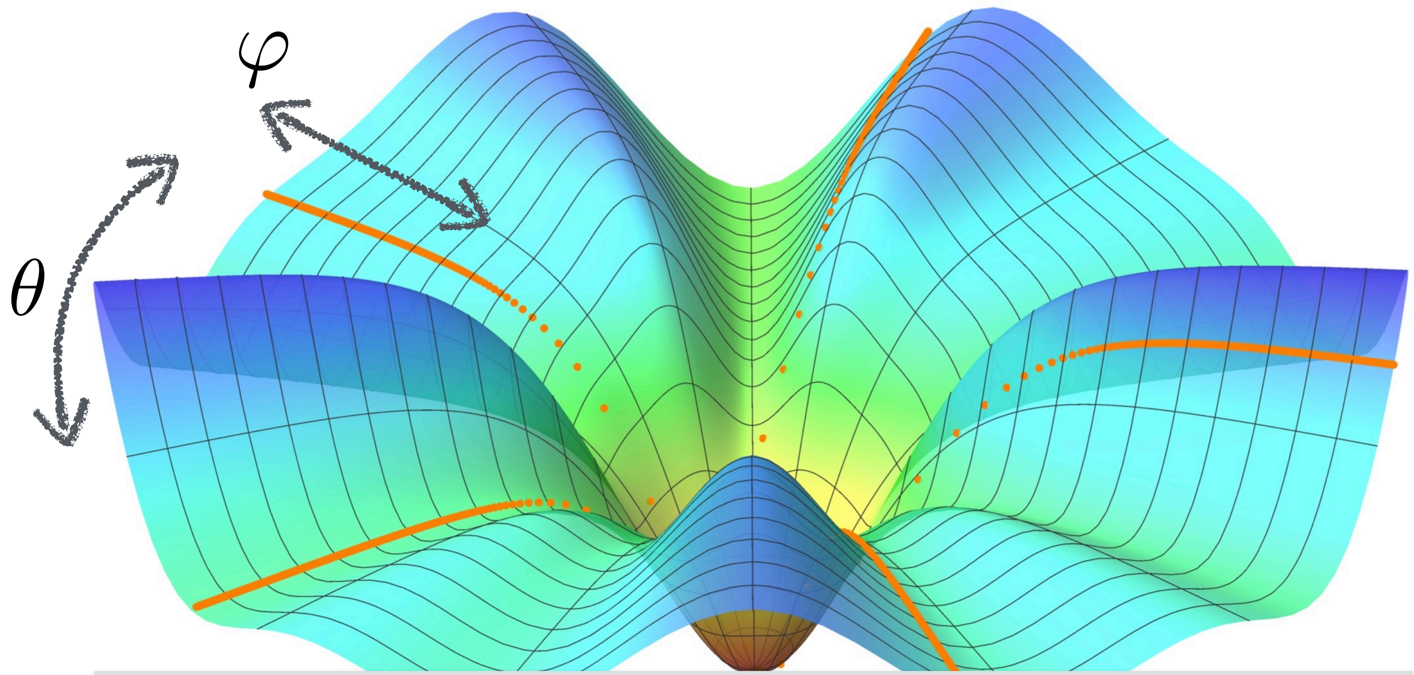
Do not go gentle into that "good" regime!

large isocurvature perturbations

large local non-Gaussianity

One counter-example: *multi-field α-attractors*

Achucarro, Kallosh, Linde, DGW & Welling 2017



attractor trajectories in hyperbolic field space

"rolling on the ridge"

$$3H\dot{\varphi} \simeq -\frac{2\sqrt{2}}{\sqrt{3\alpha}} V_\rho e^{-\sqrt{\frac{2}{3\alpha}}\varphi}$$

$$\frac{\dot{\theta}}{H} \simeq -\frac{8}{3\alpha}\frac{V_\theta}{V} e^{-2\sqrt{\frac{2}{3\alpha}}\varphi}$$

non-geodesic motion, significant multi-field effects

Only the radial field contributions dominate in the final curvature perturbation

$$\zeta = \delta N = \frac{\partial N}{\partial \varphi}\delta\varphi + \frac{\partial N}{\partial \theta}\delta\theta + \frac{1}{2}\frac{\partial^2 N}{\partial \varphi^2}\delta\varphi^2 + \frac{1}{2}\frac{\partial^2 N}{\partial \theta^2}\delta\theta^2 + \frac{\partial^2 N}{\partial\theta\partial\varphi}\delta\theta\delta\varphi$$

$$n_s = 1 - \frac{2}{N} \quad \text{and} \quad r = \frac{12\alpha}{N^2}$$

$$f_{\rm NL} \simeq \frac{5}{6}\frac{\partial^2 N}{\partial\varphi^2} \Big/ \left(\frac{\partial N}{\partial \varphi}\right)^2 \simeq \frac{5}{6N}$$

[See more about α-attractors in Andrei's talk]

Shift-symmetric orbital inflation

inflaton trajectory along an ‘angular’ isometry direction with ***arbitrary*** ‘radius’.

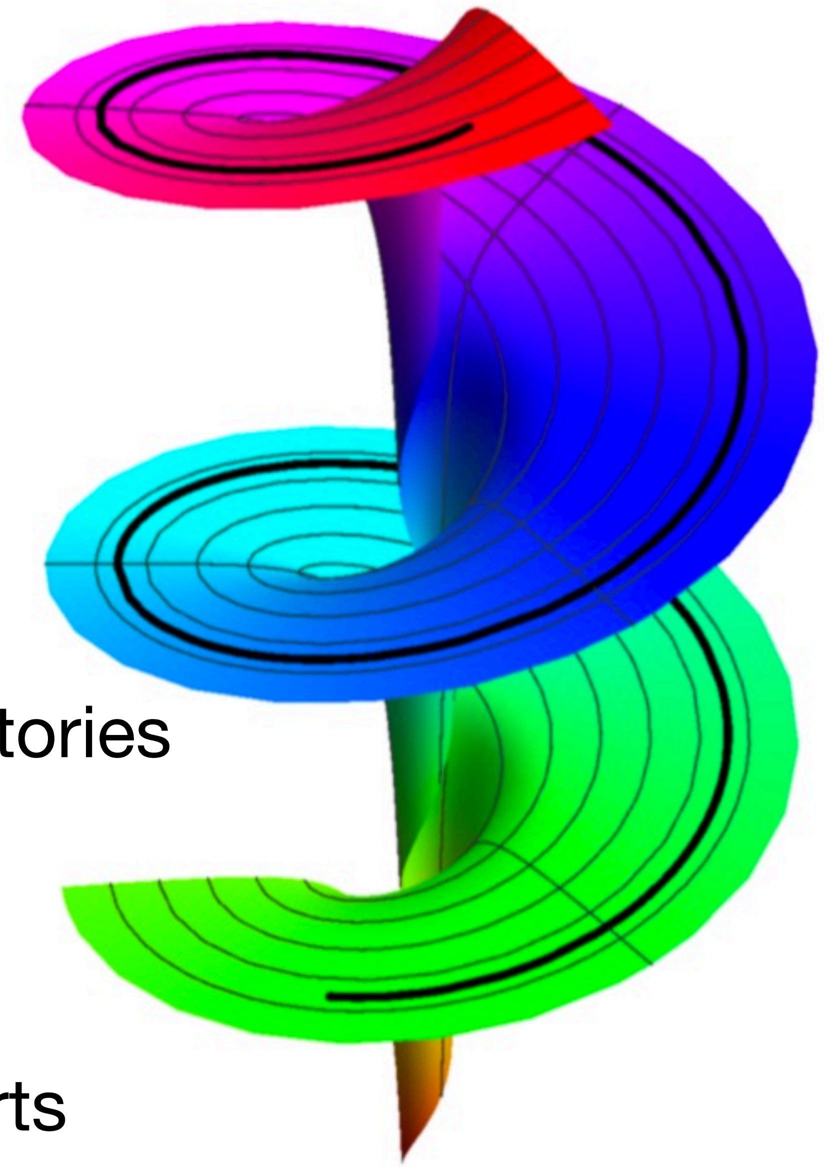
as another counter-example

Shift-symmetric orbital inflation — a toy model

Two-field Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[\rho^2(\partial\theta)^2 + (\partial\rho)^2\right] - \frac{1}{2}m^2\left(\theta^2 - \frac{2}{3\rho^2}\right)$$

flat field space   spiral-like potential



Exact and stable solutions for the trajectory

$$\rho = \rho_0, \quad \dot\theta = \pm\sqrt{\frac{2}{3}}\frac{m}{\rho_0^2};$$

The transformation which connects all the trajectories

$$\rho_c = \rho_0 + c, \quad \left(\theta_c^2\right)' = \frac{\left(\theta_0^2\right)'}{(1 + c/\kappa)^2}$$

The indicated **combined shift symmetry** for perts

$$\sigma \to \sigma + c, \quad \mathcal{R}' \to \mathcal{R}' + \frac{2}{\kappa}c$$

(massless)   (growing solution)

Shift-symmetric orbital inflation — basic setup

We begin with a two-field kinetic term with nontrivial field space metric

$$-\frac{1}{2}\left(f(\rho)\partial_\mu\theta\partial^\mu\theta + \partial_\mu\rho\partial^\mu\rho\right)$$

Next, we require the inflaton moves in the ***isometry*** direction, i.e.

$$\theta = \theta(t), \qquad \rho = \text{const.} \qquad\qquad \underline{\text{for } any \ \rho}$$

Then via Hamilton-Jacobi formalism, this orbital trajectory gives us

$$V = 3H^2 - 2\frac{H_\theta^2}{f(\rho)}$$

A class of exact background solutions $\dot{\theta} = -2\frac{H_\theta}{f}, \qquad \rho = \rho_0$

- **neutrally stable** *a new type of multi-field attractor!*
- constant turning with radius $\kappa = 2f/f_\rho$

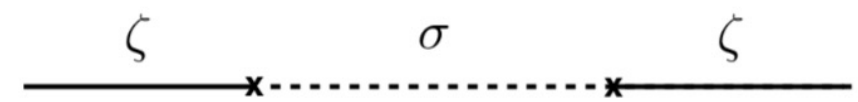
Shift-symmetric orbital inflation — perturbations

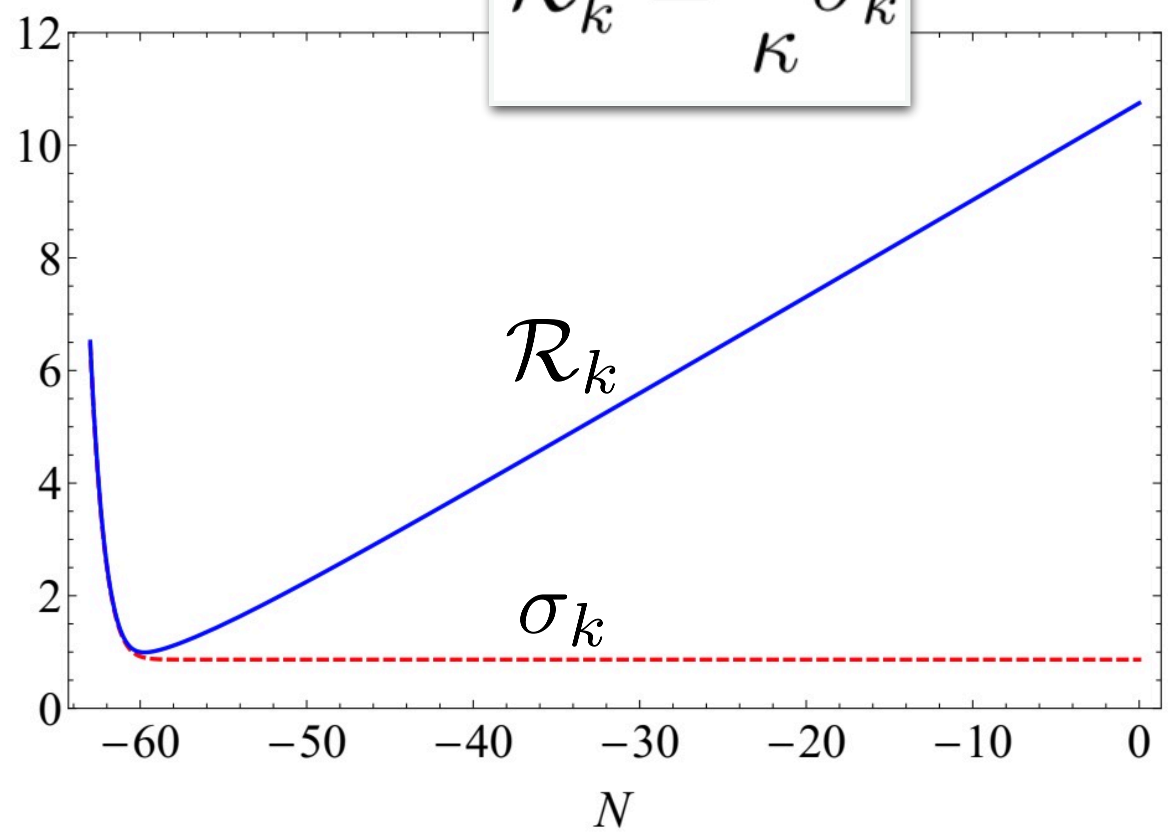
- Isocurvature modes become **exactly massless** and **freeze** on superhorizon scales

$$\mu^2 \equiv V_{NN} + \epsilon H^2 \left(\mathbb{R} + 6/\kappa^2\right) = 0 \qquad\qquad \sigma_k = \frac{H_*}{2\pi}$$

- Curvature perturbation is sourced by isocurvature and **grows** on super-horizon scales

$$\mathcal{R}_k' = \frac{2}{\kappa}\sigma_k$$





At the end of inflation

$$\mathcal{R}_k = \mathcal{R}_* + 2N_*\sigma_k/\kappa$$

"the ultra-light isocurvature scenario"

Achucarro, Atal, Germani, Palma 2016

Shift-symmetric orbital inflation — perturbations

Final power spectrum of curvature perturbation

$$P_{\mathcal{R}} = \frac{H_*^2}{8\pi^2\epsilon_*}(1+\mathcal{C})$$

$\mathcal{C} = 8\epsilon_* N_*^2/\kappa^2$ represents the isocurvature contributions

isocurvature power spectrum with $\mathcal{S} \equiv \sigma/\sqrt{2\epsilon}$ $\quad P_{\mathcal{S}} = \frac{H_*^2}{8\pi^2\epsilon_*}$

The interesting regime

$\mathcal{C} \gg 1$ ⟷ $8\epsilon_* \ll \kappa^2 \ll 8\epsilon_* N_*^2$

(small kappa / significant turning effects)

Only one DoF (isocurvature one) is responsible for the observed curvature perturbation

Isocurvature perturbations are dynamically suppressed

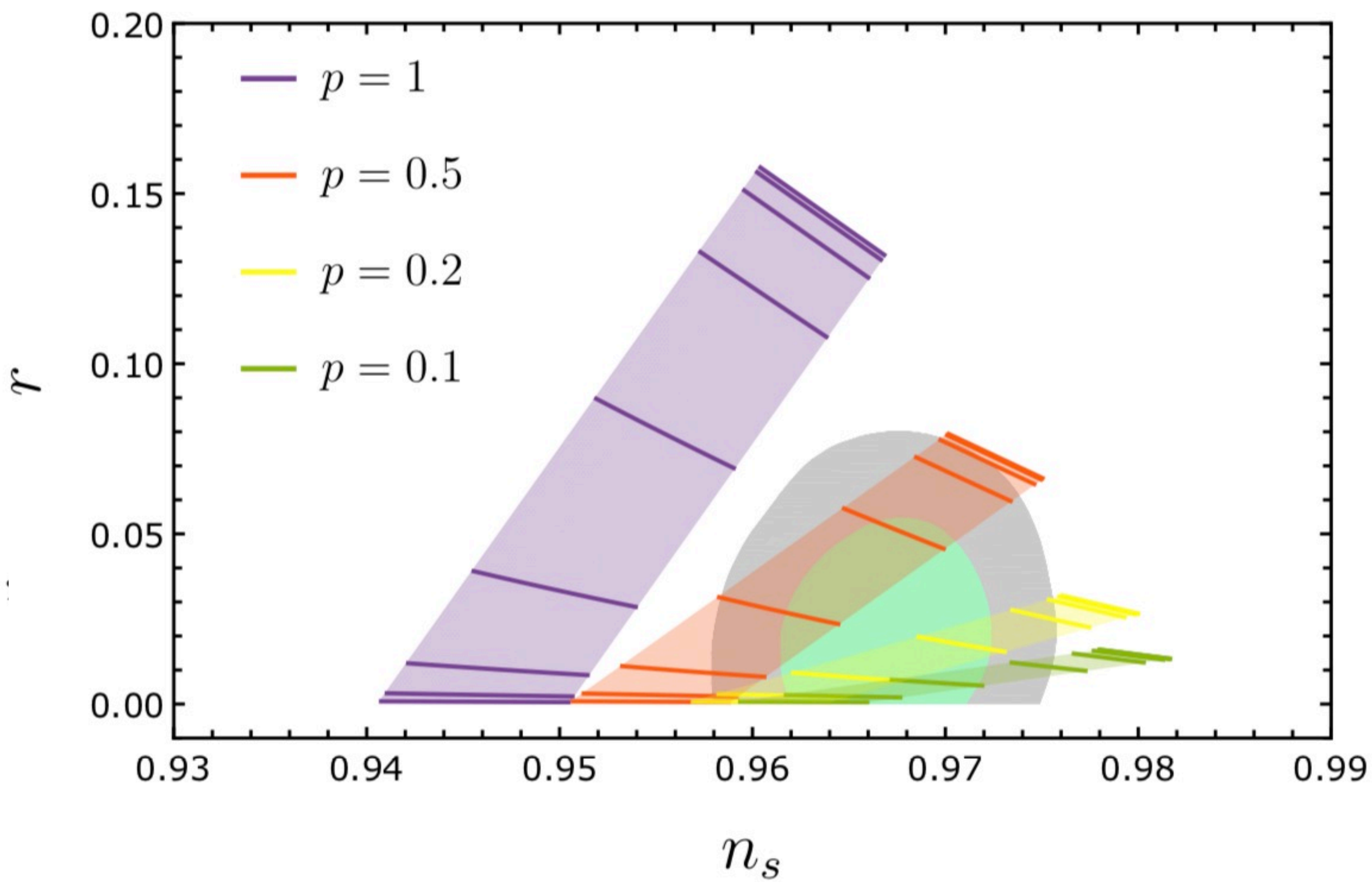
$$P_{\mathcal{S}}/P_{\mathcal{R}} \simeq 1/\mathcal{C} \ll 1$$

Shift-symmetric orbital inflation — phenomenology

Consider a concrete potential with $H \sim \theta^p$

$$r \simeq \frac{8p\kappa^2}{N_*\kappa^2 + 4pN_*^2}$$

$$n_s - 1 \simeq -\frac{p+1}{N_*} - \frac{4p}{\kappa^2 + 4pN_*}$$



- When $\kappa \to \infty$, we go back to chaotic inflation with $V \propto \phi^{2p}$

$$r \simeq \frac{8p}{N_*} \qquad n_s - 1 \simeq -\frac{p+1}{N_*}$$

- For the interesting regime with significant turning (small kappa) $\mathcal{C} \gg 1$

$$r = 2\kappa^2/N_*^2 = 16\epsilon_*/\mathcal{C} \qquad n_s - 1 = -(p+2)/N_*$$

Shift-symmetric orbital inflation — non-Gaussianity

Solving the exact models in terms of e-folds $N = f(\rho)\theta^2/4p - p/2$

δN formalism

$$f_{\rm NL}^{\delta N} = \frac{5}{6}\left(\frac{N_\theta^2 N_{\theta\theta}}{f(\rho)^2} + N_\rho^2 N_{\rho\rho} + \frac{N_\rho N_\theta N_{\rho\theta}}{f(\rho)}\right) / \left(\frac{N_\theta^2}{f(\rho)} + N_\rho^2\right)^2$$

$$\Rightarrow \quad f_{\rm NL}^{\rm loc} = \frac{5}{12}\eta_*\left[1 - \frac{\mathcal{C}^2}{(1+\mathcal{C})^2}\frac{\kappa^2\mathbb{R}}{2}\right] \qquad \mathcal{C} = 8\epsilon_* N_*^2/\kappa^2$$

- Single field regime $\mathcal{C} = 2p^2/(\epsilon_*\kappa^2) \to 0$ $\Rightarrow$ $f_{\rm NL}^{\rm loc} = \frac{5}{12}\eta_*$
- When **turning is significant** $\mathcal{C} \gg 1$

$$f_{\rm NL}^{\rm loc} \simeq \frac{5}{6}\frac{N_{\rho\rho}}{N_\rho^2} = \frac{5}{12}\eta_*\left(1 - \frac{\kappa^2\mathbb{R}}{2}\right)$$

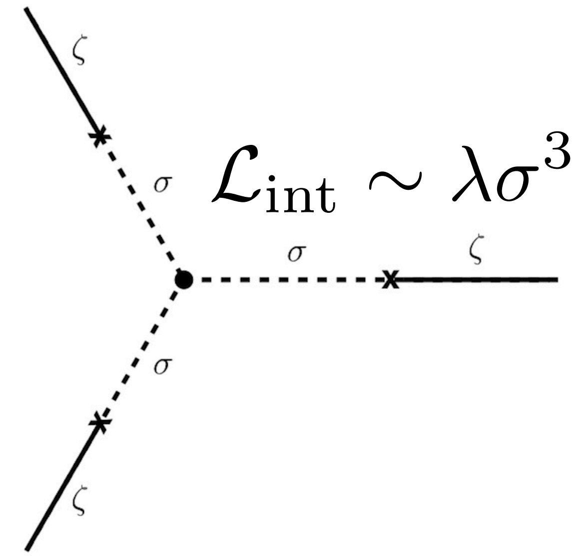
Still slow-roll suppressed!

- For the intermediate regime $\mathcal{C} \sim \mathcal{O}(1)$ $\Rightarrow$ $f_{\rm NL}^{\rm loc} \sim -5p\mathbb{R}/12$

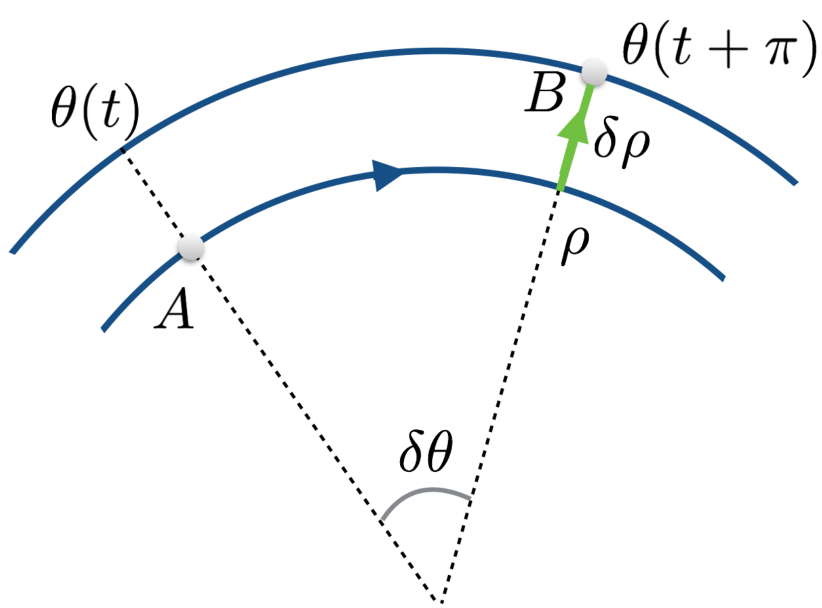
Small non-Gaussianity in multi-field inflation

In general, can we neglect the isocurvature self-interaction?

small isocurvature mass	?	***small isocurvature self-interaction***
$\mu^2 \equiv V_{NN} + \epsilon H^2 \mathbb{R} + 3\Omega^2$		$\lambda = V_{NNN} + \dots$



Scaling transformation *as the origin of the ultra-light field*



- Background:

$$Y(x) \to Y'(x') = Y(x) + \Lambda c,$$
$$X(x) \to X'(x') = e^{-c\Lambda/R_0} X(x), \quad S \to S' = e^{2c} S$$
$$x^\mu \to x'^\mu = e^c x^\mu.$$

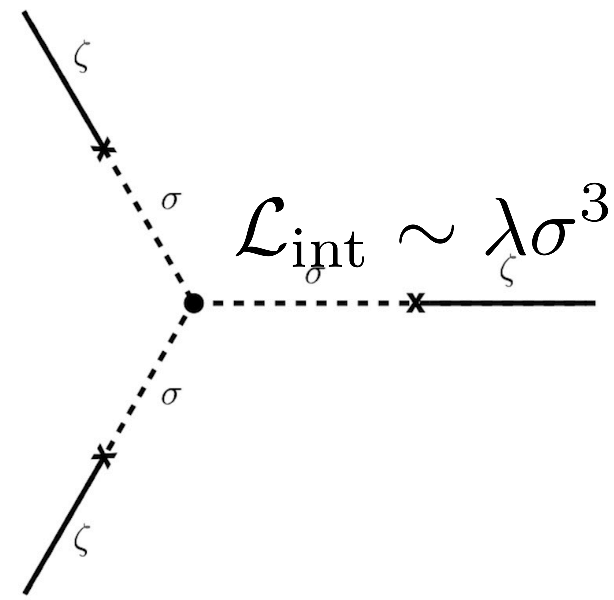
- Perturbations:

$$Y(x) = \bar{Y}\left(t + \pi(x)\right) + \mathcal{F}(x)$$
$$X(x) = e^{-\mathcal{F}(x)/R_0} \bar{X}\left(t + \pi(x)\right)$$

the ultra-light field

Small non-Gaussianity in multi-field inflation

Consequence of the scaling transformation



$$\mathcal{F}_S(x) = \mathcal{F}'(e^{\mathcal{F}_L} x).$$

$$\langle \mathcal{F}_S(\mathbf{k}_1)\mathcal{F}_S(\mathbf{k}_2)\mathcal{F}_L(\mathbf{k}_3)\rangle_{\text{sq}} = (2\pi)^3\delta^{(3)}(\sum_i \mathbf{k}_i)(1-n_{\mathcal{F}})P_{\mathcal{F}}(k_L)P_{\mathcal{F}}(k_s).$$

Squeezed bispectrum of curvature perts:

isocurvature self-interaction *superhorizon effects*

$$\lim_{k_l/k_s \to 0} \langle \mathcal{R}(k_l)\mathcal{R}(k_s)\mathcal{R}(k_s)\rangle = (1-n_{\mathcal{F}})\left(\frac{n}{\epsilon_* R_0}\right)^3 P_{\mathcal{F}}(k_l)P_{\mathcal{F}}(k_s) + \left(\frac{n}{\epsilon_* R_0}\right)^4 \left(\frac{2\epsilon_*}{n} + \eta_*\right) P_{\mathcal{F}}(k_l)P_{\mathcal{F}}(k_s)$$

$$\approx \left(\frac{2\epsilon_*}{n} + \eta_*\right) P_{\mathcal{R}}(k_l)P_{\mathcal{R}}(k_s) \ .$$

To appear soon, Achucarro, Palma, DGW, Welling

Small non-Gaussianity in multi-field inflation

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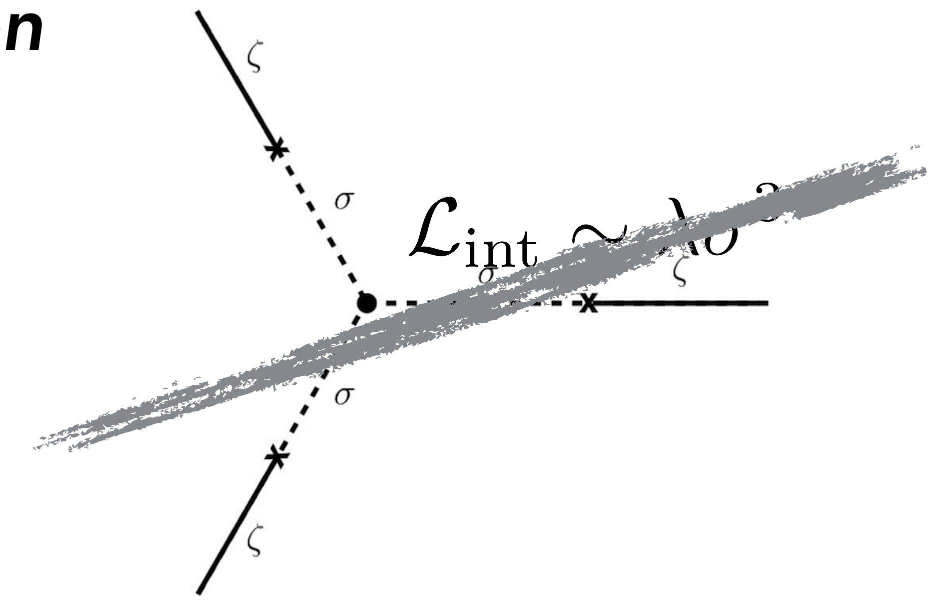

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$$(2\pi)^3\delta^{(3)}\left(\sum_i \mathbf{k}_i\right)(1-n_{\mathcal{F}})P_{\mathcal{F}}(k_L)P_{\mathcal{F}}(k_s).$$

Squeezed bispectrum of curvature perts:

isocurvature self-interaction — **superhorizon effects**

$$\lim_{k_l/k_s\to 0}\langle \mathcal{R}(k_l)\mathcal{R}(k_s)\mathcal{R}(k_s)\rangle = (1-n_{\mathcal{F}})\left(\frac{n}{\epsilon_* R_0}\right)^3 P_{\mathcal{F}}(k_l)P_{\mathcal{F}}(k_s) + \left(\frac{n}{\epsilon_* R_0}\right)^4\left(\frac{2\epsilon_*}{n}+\eta_*\right)P_{\mathcal{F}}(k_l)P_{\mathcal{F}}(k_s)$$
$$\approx \left(\frac{2\epsilon_*}{n}+\eta_*\right)P_{\mathcal{R}}(k_l)P_{\mathcal{R}}(k_s)\ .$$

To appear soon, Achucarro, Palma, DGW, Welling

One generic lesson: local non-G is negligible if

- 1) *only one* DoF is responsible for the final curvature perts;
- 2) the self-interaction of this DoF is small.

Final Remarks on *shift-symmetric orbital inflation*

$\mathcal{C} \gg 1$

- ***Only one degree of freedom*** (the one with isocurvature origin) is responsible for observational predictions;

- Its phenomenology mimics the one of single field inflation

  - ***Non-Gaussianity* is negligible**, even though the interaction between curvature and isocurvature perts is big (small isocurvature self-interaction)

- Future observations on isocurvature modes may help us to distinguish single-field and our model, but it relies on the details of reheating;

- Generalization: exact solvable multi-field system without shift-symmetry;

  Orbital inflation [Achucarro, Welling 2019]

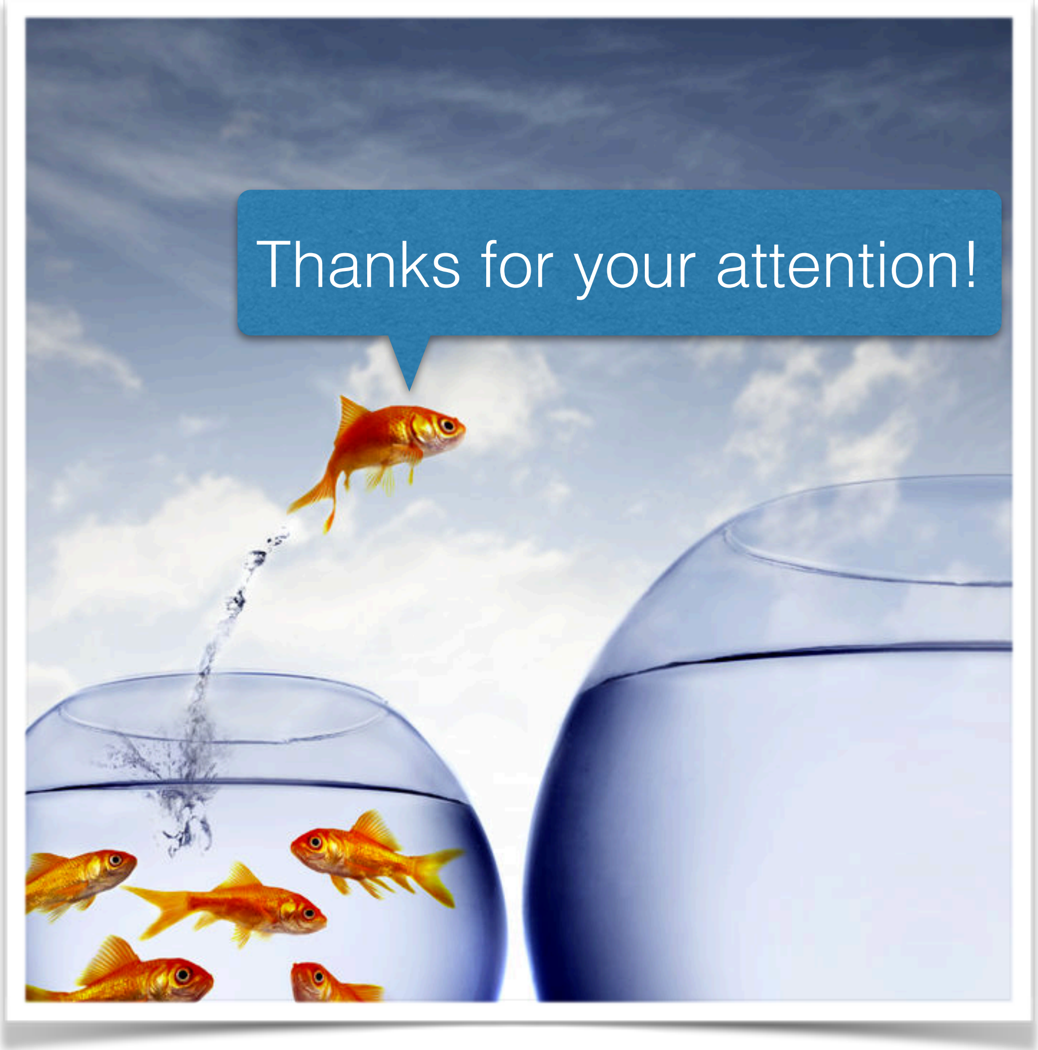
- Possible connection with other multi-field attractors, swampland conjectures?

  [Christodoulidis, Sfakianakis, Roest 2019], [Aragam, Paban, Rosati 2019]

- UV origin of the combined shift symmetry / ultra-light fields?

Take home message

- This new regime of multi-field attractors tells us ...



we should have the courage to dive into

the UV completion of inflation with unstablized light fields

:-)