Universiteit Leiden

Excursion in the curved field space

on the inflationary massive field with a curved manifold

based on arXiv: 1911.04459

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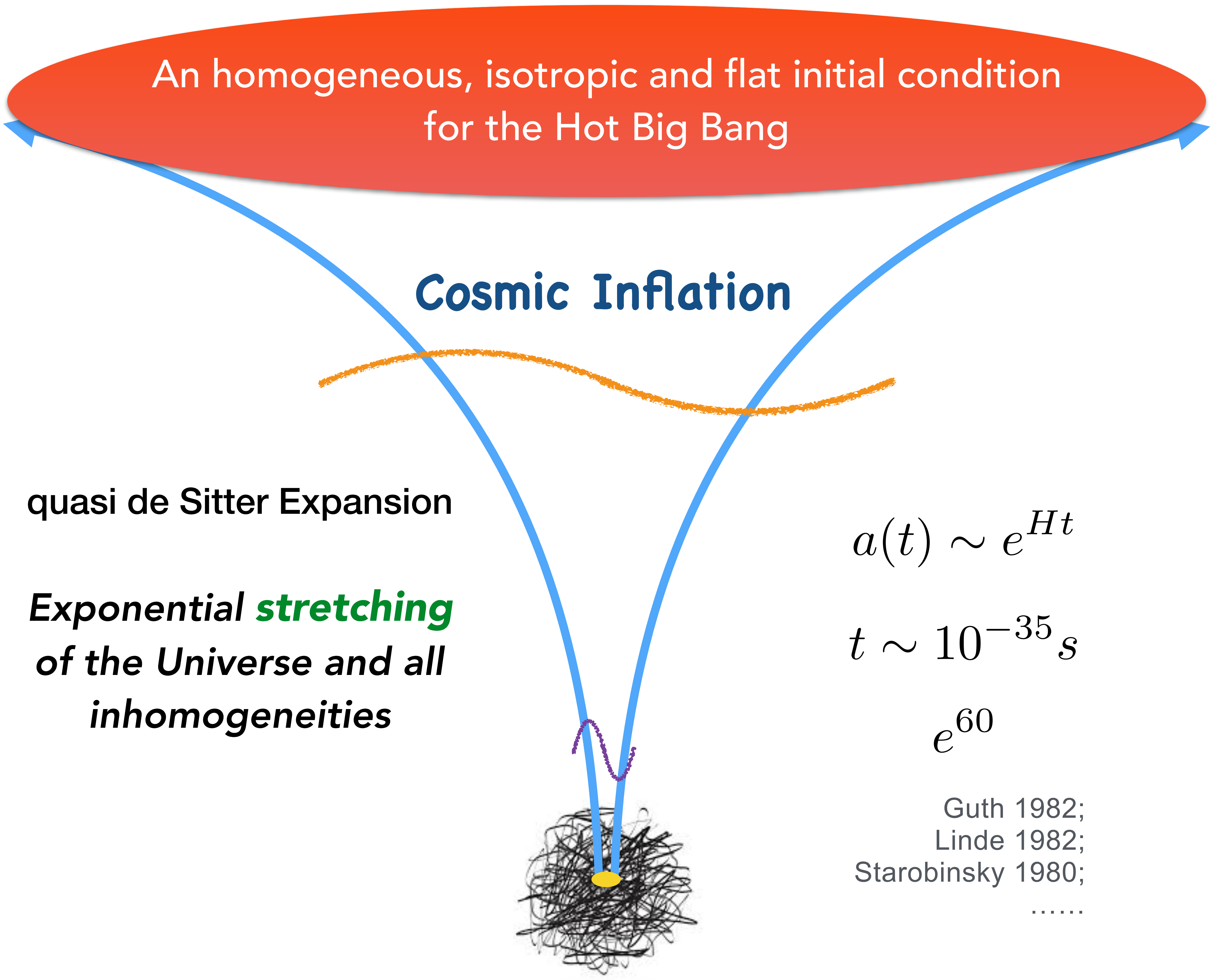
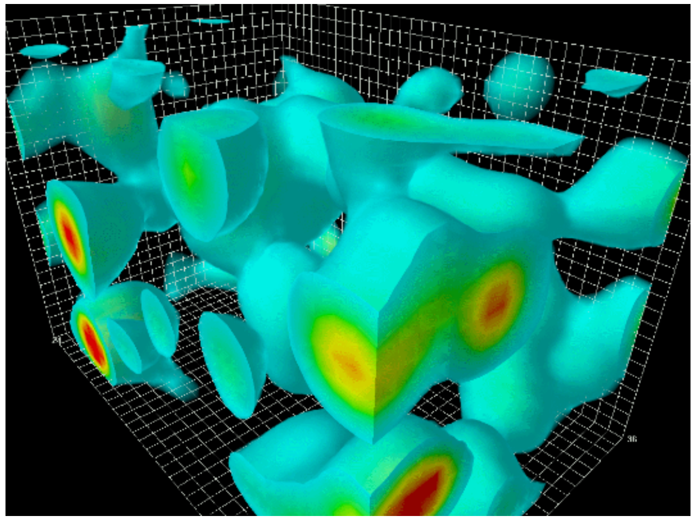
@Groningen, Dec. 12, 2019

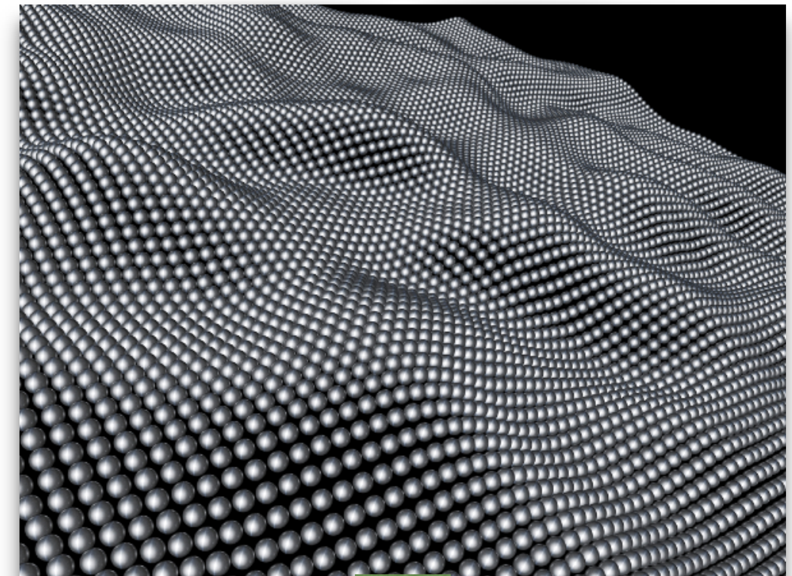

An homogeneous, isotropic and flat initial condition
for the Hot Big Bang
Cosmic Inflation
quasi de Sitter Expansion
Exponential ***stretching*** *of the Universe and all inhomogeneities*
$a(t) \sim e^{Ht}$
$t \sim 10^{-35} s$
e^{60}
Guth 1982;
Linde 1982;
Starobinsky 1980;
……

Quantum Fluctuations as the origin of structures

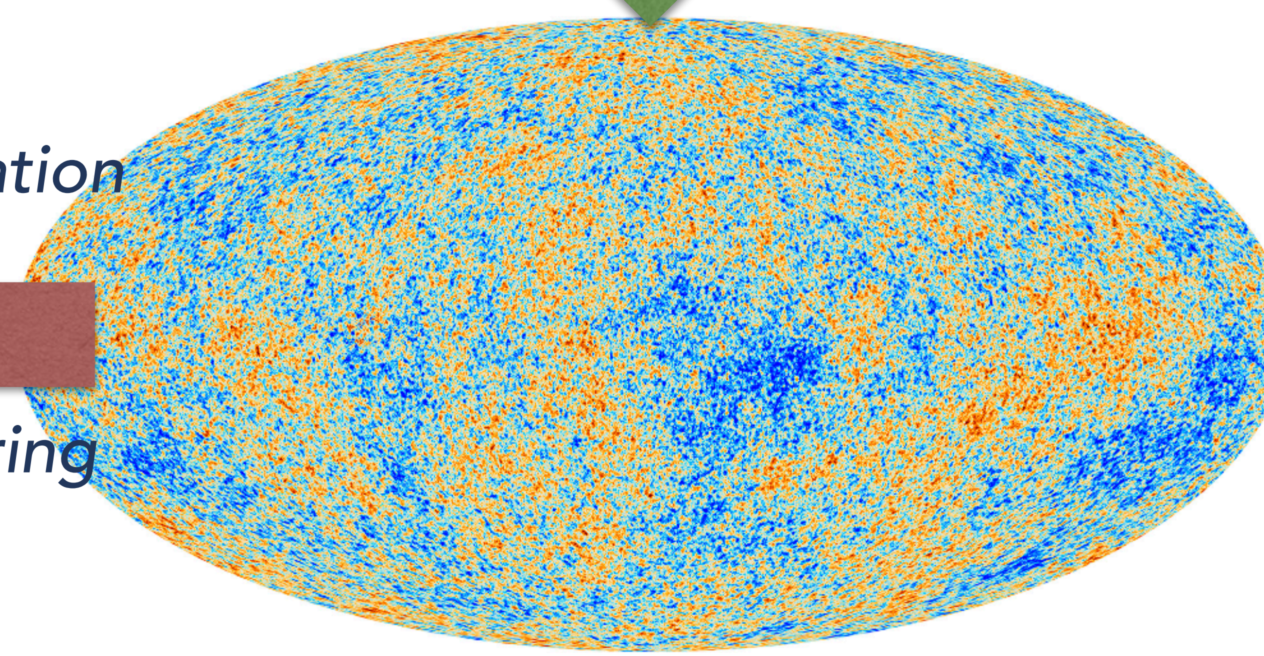


horizon-exit
stretch
decoherence

inflation ~ 10^{-35}s

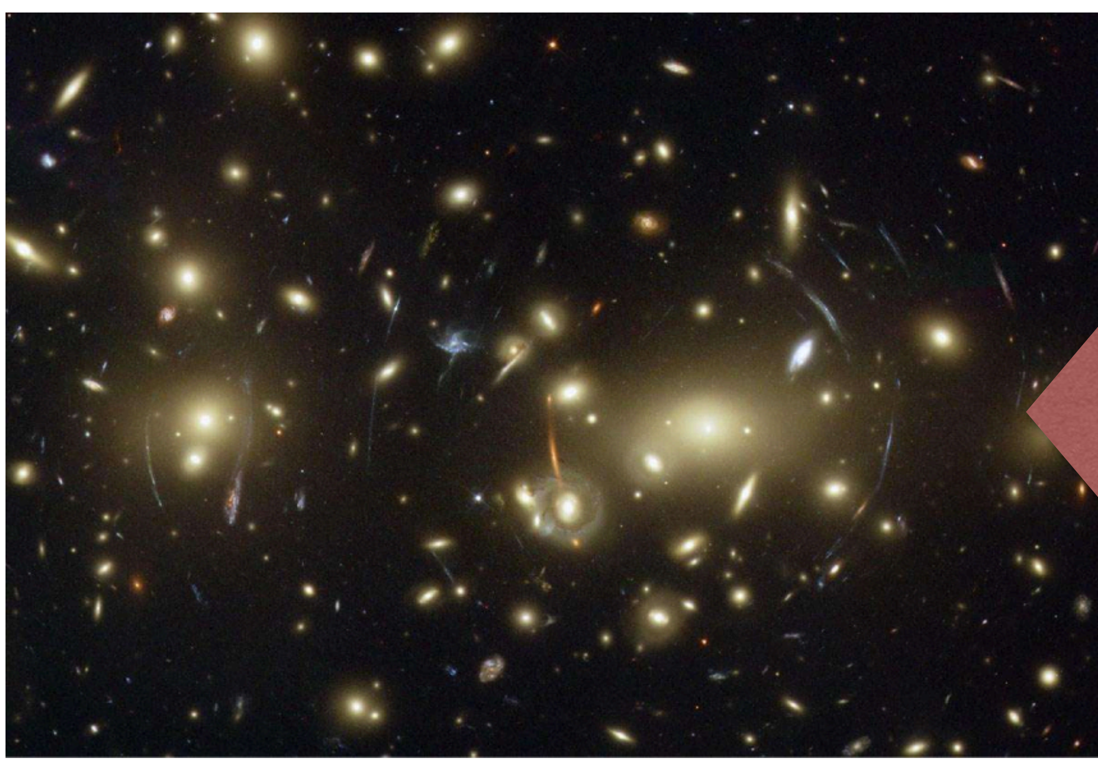


recombination



CMB ~ 380000 years

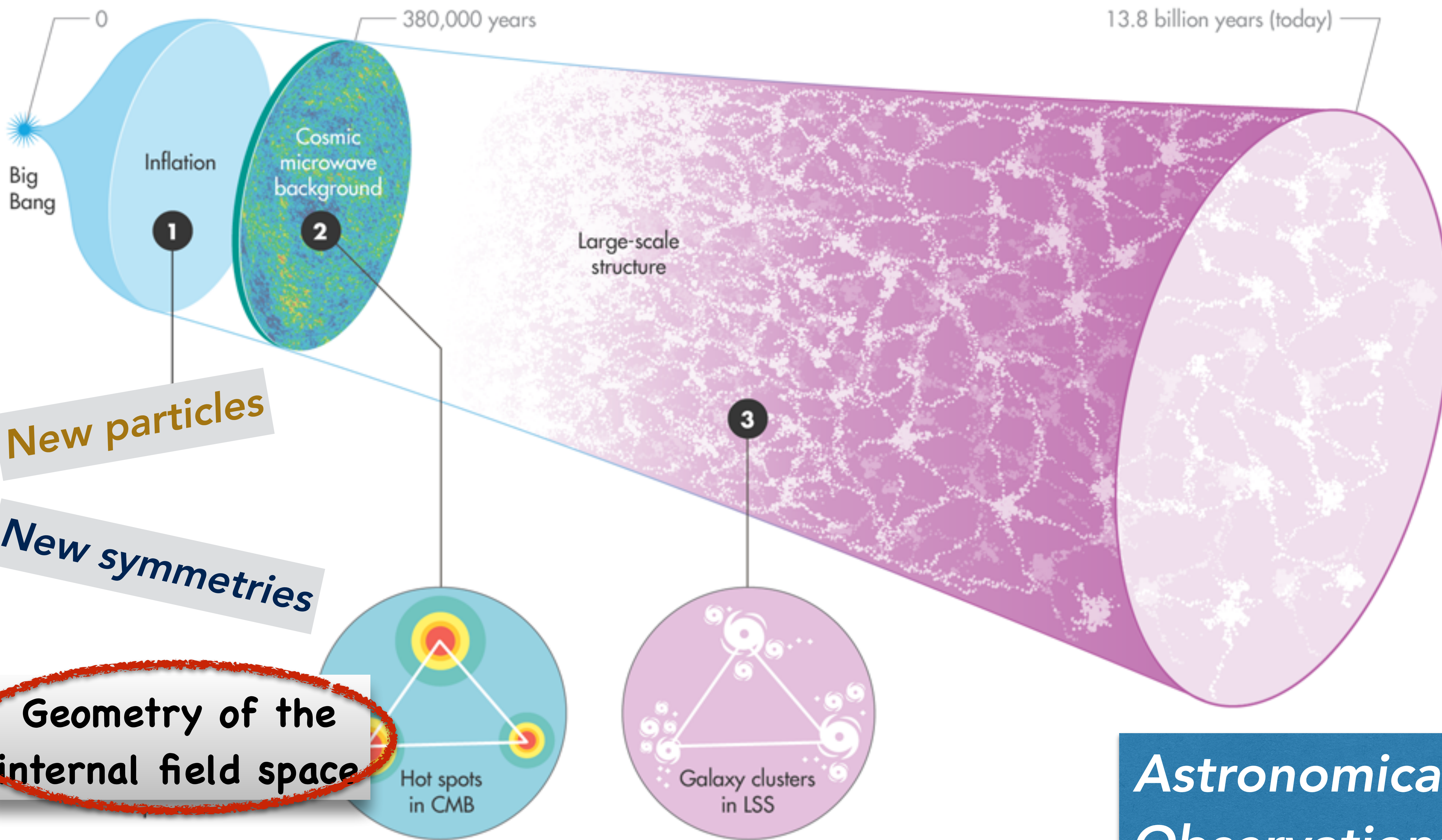
reionization
clustering



today ~ 13.7 billion years

Inflation as a natural laboratory at very high energy scales

New Physics $10^{14} GeV$



New particles

New symmetries

Geometry of the internal field space

Astronomical Observations

[Lucy Reading-Ikkanda, Quantum Magzine]

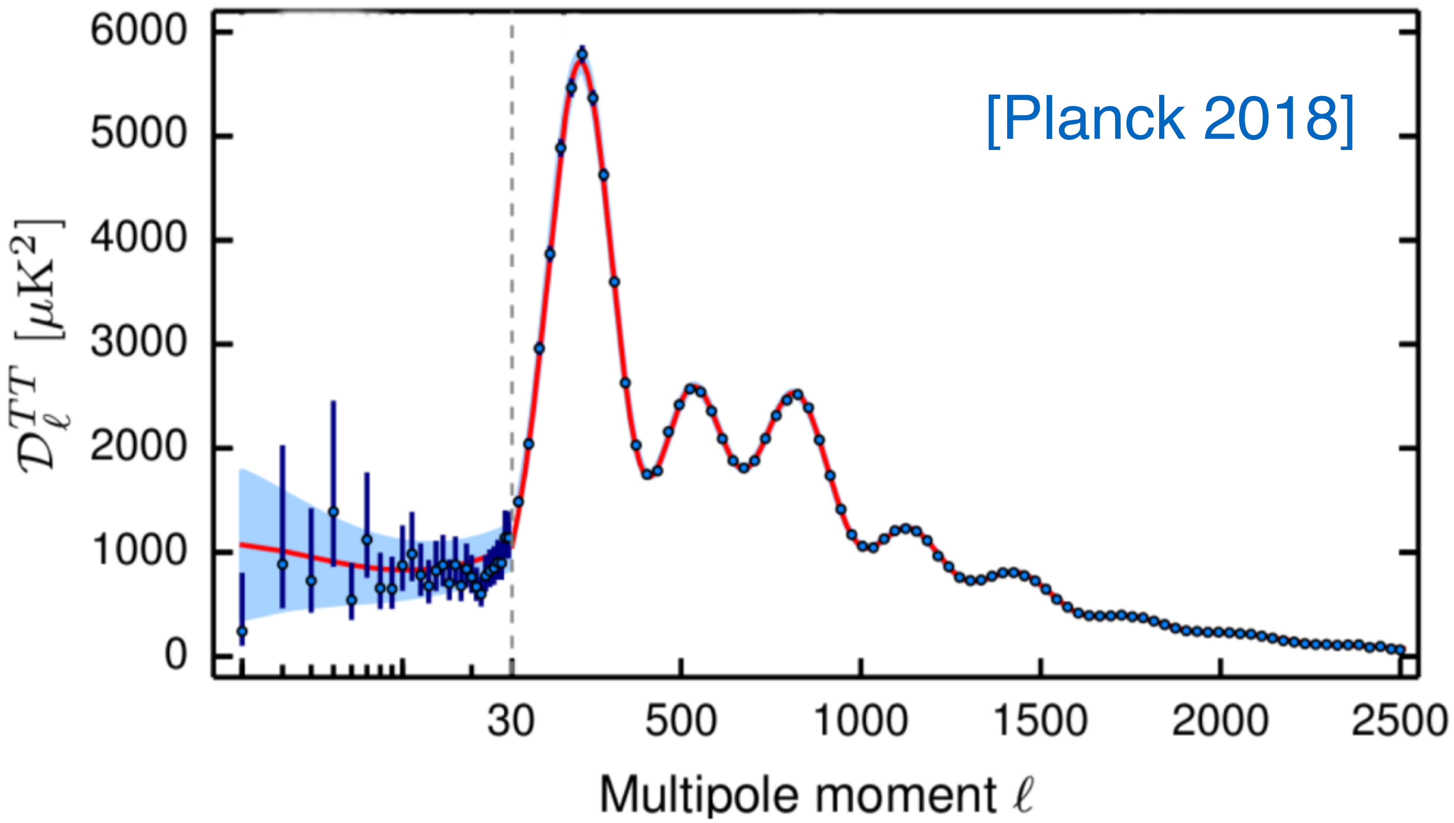
Outline

- Inflation with curved field space
  - — motivation, recent developments & examples
- Massive fields and non-Gaussianity
  - — quasi-single field inflation & the squeezed bispectrum
- When massive fields meet curved field space
  - — multi-field analysis & background EFT with dim-6 operator
- Phenomenology
  - — running isocurvature mass & modified collider signals
- Summary and outlook

The success of single field slow-roll inflation

★ solved the horizon problem and flatness problem

★ quantum fluctuations of inflaton lead to **structure formation**



✦ **Latest CMB results favor the phenomenology of single field inflation**

— nearly scale-invariant curvature perturbation

— small tensor-to-scalar ratio

— small non-Gaussianity

— small isocurvature modes

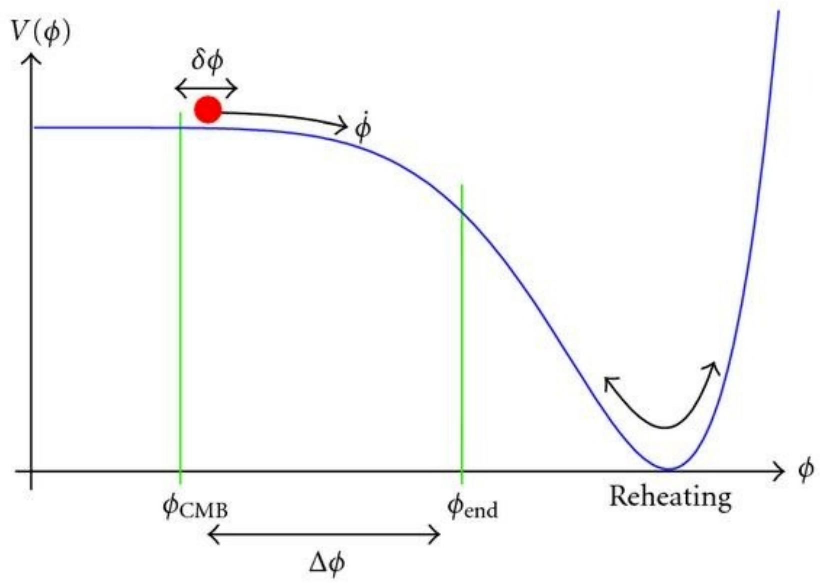
What is the inflaton?

Theoretical Challenges

for the realisation of inflation in more fundamental theories

In spite of all the difficulties, we have learned…

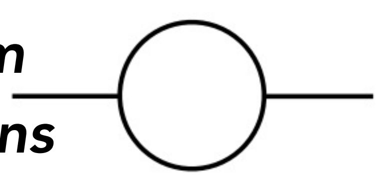
Lesson No.1: the flatness of the inflaton potential should be protected by **an approximate shift symmetry**



$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V_{\rm sr}(\phi)\right]$$

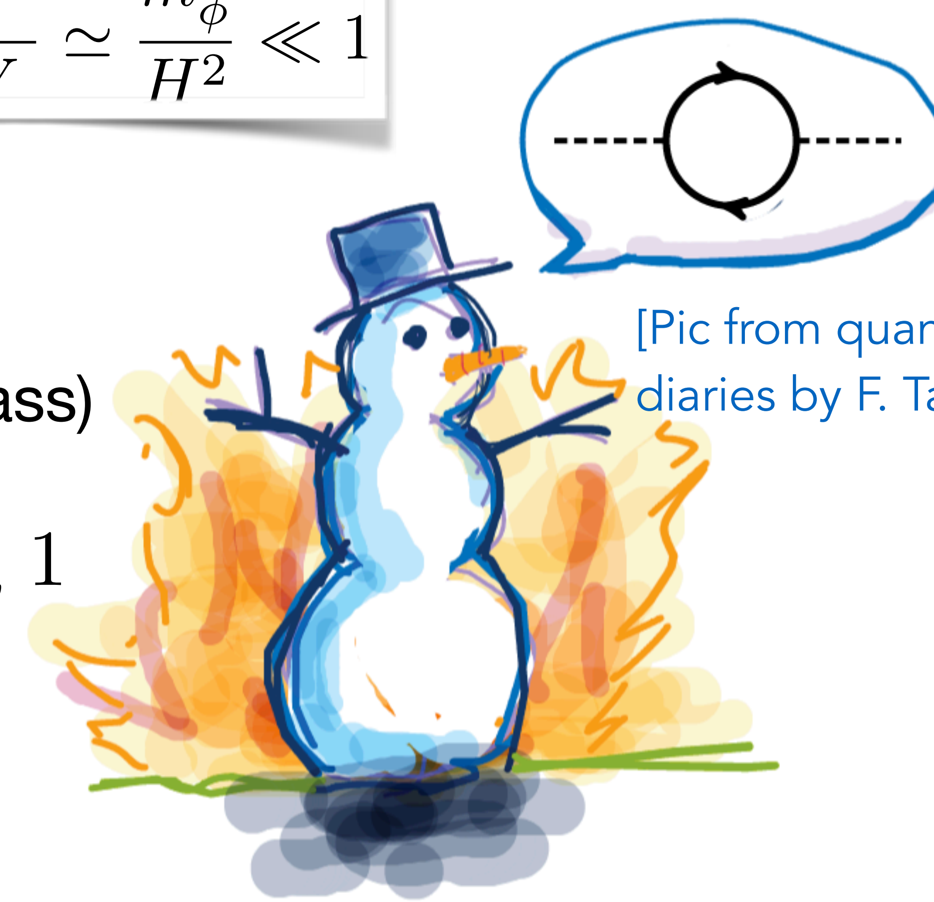
$$\eta_V \equiv M_{\rm pl}^2\frac{V''}{V} \simeq \frac{m_\phi^2}{H^2} \ll 1$$

η-problem the hierarchy problem of inflation (in analog of the one for Higgs mass)

quantum corrections  $\delta m_\phi^2 \gtrsim H^2$ → $\eta_V \gtrsim 1$

fine-tuning? or **global internal symmetries**

$$\phi \to \phi + \text{const.}$$



[Pic from quantum diaries by F. Tanedo]

A hierarchical system — snowball's chance in hell

In spite of all the difficulties, we have learned…

Lesson No.1: the flatness of the inflaton potential should be protected by **an approximate shift symmetry**

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V_{\rm sr}(\phi)\right]$$

$\eta_V \ll 1$ ***requires*** **shift symmetry** $\phi \to \phi + \text{const.}$

Lesson No.2: Besides the inflaton, other extra fields are also around and typically **they live in a curved field space**

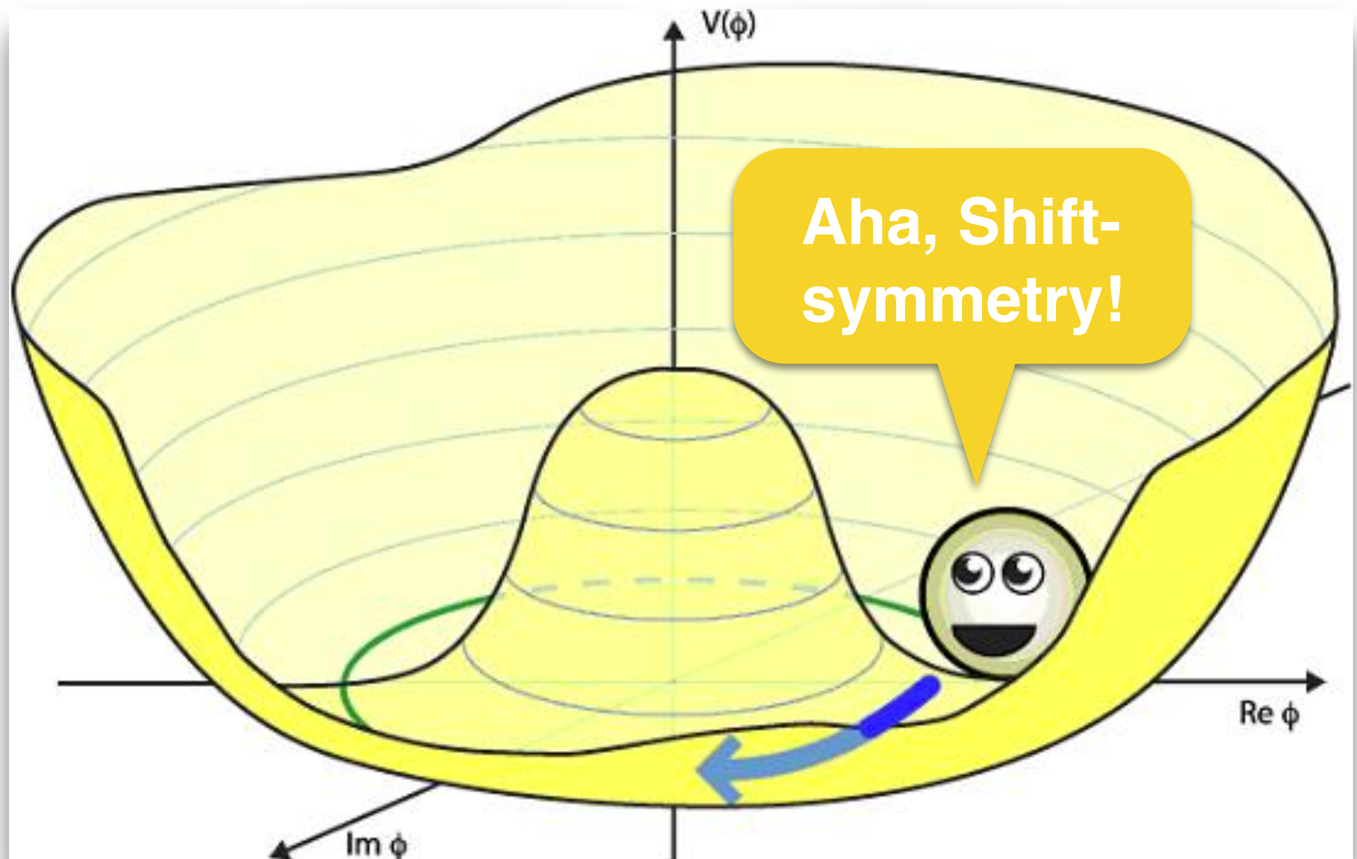
$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}G_{ab}(\phi)g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

Examples in UV constructions:

- **Kahler potential** in Supergravity *(e.g. alpha-attractors)*
- Different compatifications in String theory *(e.g. two-field axion-monodromy)*
- Coset Space **N**on-**L**inear **S**igma **M**odel as an **EFT**
- ……

The story of the *curved field space*

Inflation in coset space as an example



Burgess, Cicoli, Quevedo, Williams 2014; Klein, Roest, Stefanyszyn 2017;

The inflaton field is one of the pseudo-Nambu-Goldstone-Bosons (pNGB) living in the coset space *G/H* after some spontaneous symmetry breaking.

U(1)/∅	→	single pNGB	Natural Inflation
G/H	→	multiple pNGBs	Inflation with curved field space

The story of the curved field space

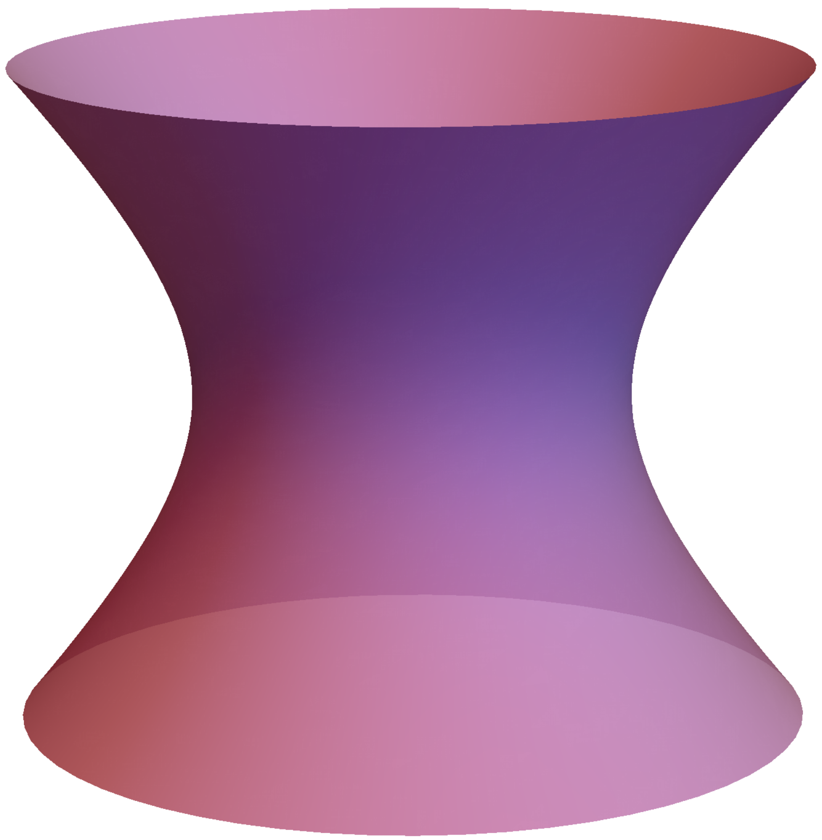
Inflation in coset space as an example

SO(3)/SO(2)

$$ds^2 = R^2 \left(d\varrho^2 + \cos^2 \varrho d\theta^2\right)$$

$$\mathbb{R} = 2/R^2$$

SO(2,1)/SO(2)



$$ds^2 = R^2 \left(d\varrho^2 + \cosh^2 \varrho d\theta^2\right)$$

$$\mathbb{R} = -2/R^2$$

The curvature radius *R* corresponds to the energy scale characterising the field space geometry.

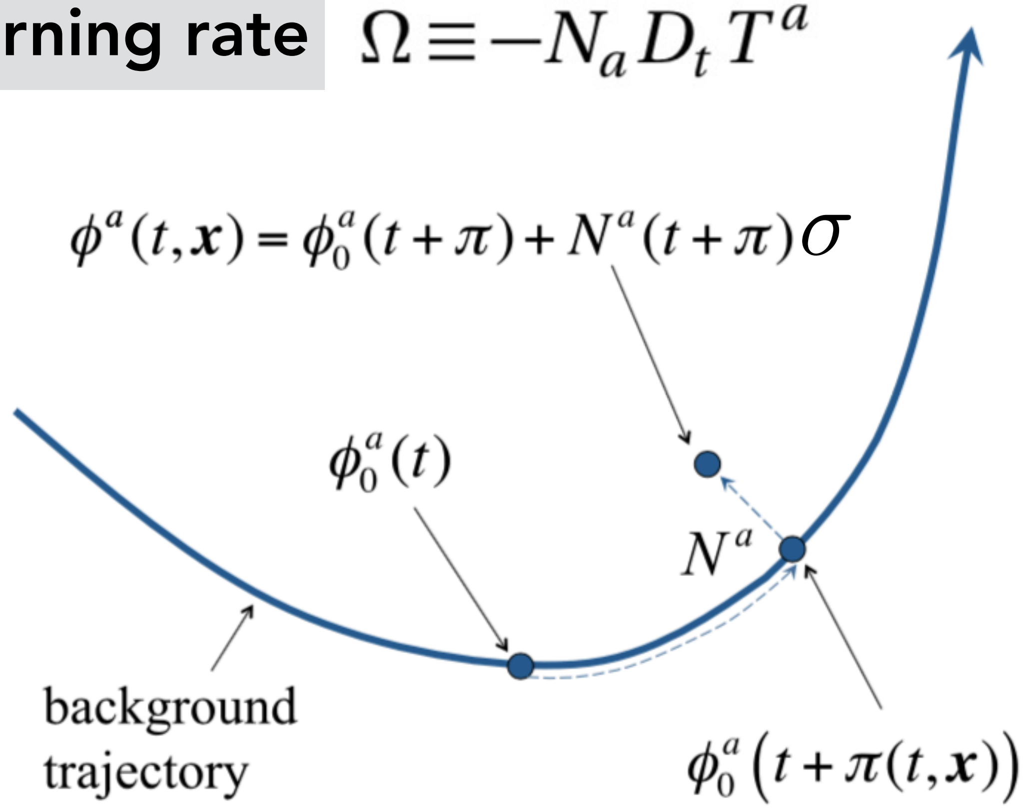
Multi-field inflation in a nutshell

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}G_{ab}(\phi)g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

field space metric (G_{ab})

potential ($V(\phi)$)

turning rate $\Omega \equiv -N_a D_t T^a$



$\Omega \neq 0$ ⟺ **Non-geodesic trajectory**

- curvature modes $\mathcal{R} = H\pi$
- isocurvature modes σ

isocurvature mass $\mu^2 \equiv V_{NN} + \epsilon H^2\mathbb{R} + 3\Omega^2$

$$S = \int d^4x\, a^3\left[\epsilon(\dot{\mathcal{R}} - \alpha\sigma)^2 - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + \frac{1}{2}\left(\dot{\sigma}^2 - \frac{1}{a^2}(\nabla\sigma)^2\right) - \frac{1}{2}\mu^2\sigma^2\right]$$

coupled evolution $\dot{\zeta} = \frac{2\Omega}{\sqrt{2\epsilon}}\sigma$

The recent revival of interest in this direction

the axion-dilaton system:

$$\mathcal{L}_m = -\frac{1}{2}f(\rho)(\partial\theta)^2 - \frac{1}{2}(\partial\rho)^2 - V(\rho,\theta)$$

Alpha-attractors & multi-field extensions

— Kallosh, Linde, Roest 2013
Achucarro, Kallosh, Linde, **DGW**, Welling, 2017
Christodoulidis, Sfakianakis, Roest 2018

Geometrical destabilization

— Renaux-Petel, Turzynski 2015

Hyperinflation

— Brown 2017

Ultralight isocurvature / Orbital inflation

— Achucarro, Copeland, Iarygina, Palma, **DGW**, Welling, 2019
Achucarro, Welling, 2019
Achucarro, Palma, **DGW**, Welling, 2019

single-field like pheno

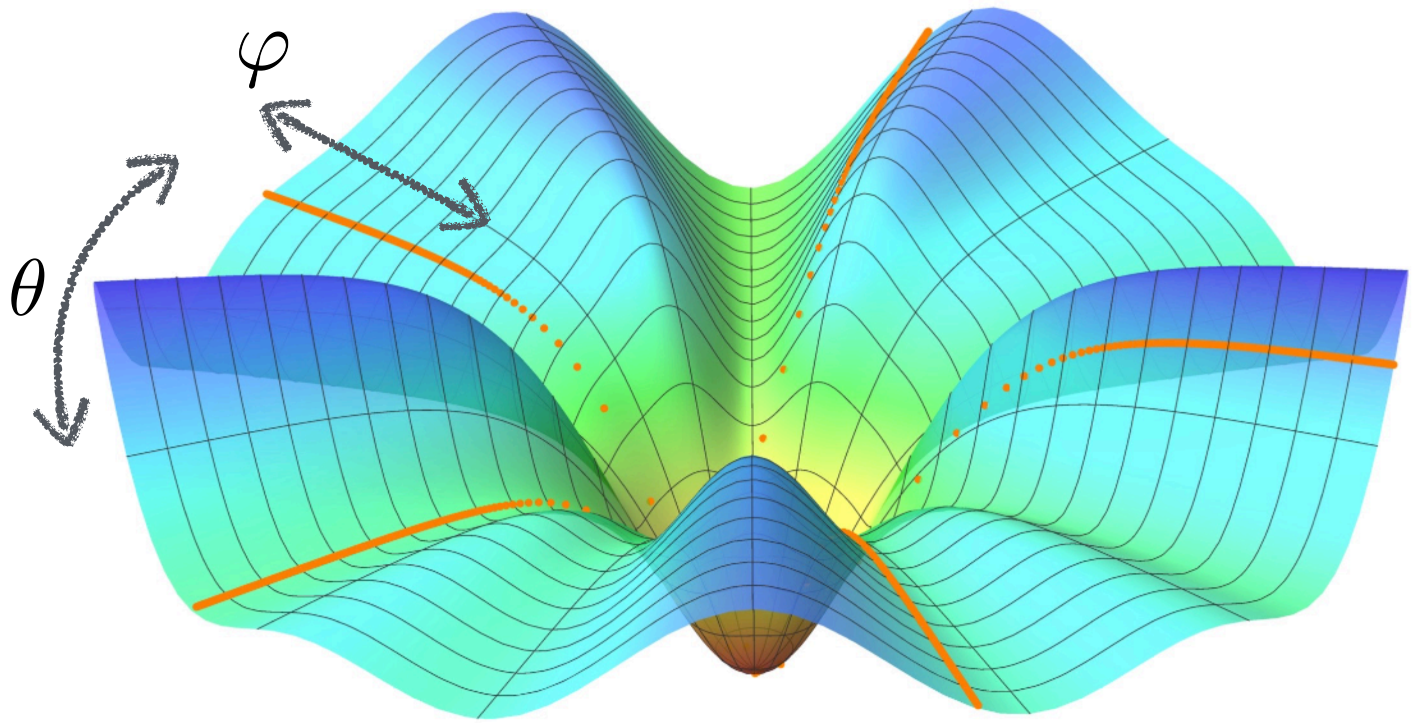
Analysis on the multi-field attractors

— Bjorkmo 2019
Christodoulidis, Sfakianakis, Roest 2019

……

Multi-field α-attractors: the magic of the hyperbolic space

Achucarro, Kallosh, Linde, DGW & Welling 2017



hyperbolic stretching on a multi-field potential

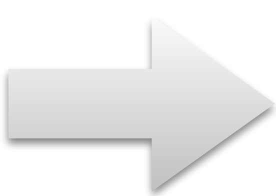
"rolling on the ridge"

$$3H\dot{\varphi} \simeq -\frac{2\sqrt{2}}{\sqrt{3\alpha}} V_\rho e^{-\sqrt{\frac{2}{3\alpha}}\varphi}$$

$$\frac{\dot{\theta}}{H} \simeq -\frac{8}{3\alpha}\frac{V_\theta}{V} e^{-2\sqrt{\frac{2}{3\alpha}}\varphi}$$

non-geodesic motion,
significant multi-field effects

Only the radial field contributions dominate in the final curvature perturbation



$$n_s = 1 - \frac{2}{N} \quad \text{and} \quad r = \frac{12\alpha}{N^2}$$

$$f_{\rm NL} \simeq \frac{5}{6}\frac{\partial^2 N}{\partial\varphi^2} \Big/ \left(\frac{\partial N}{\partial \varphi}\right)^2 \simeq \frac{5}{6N}$$

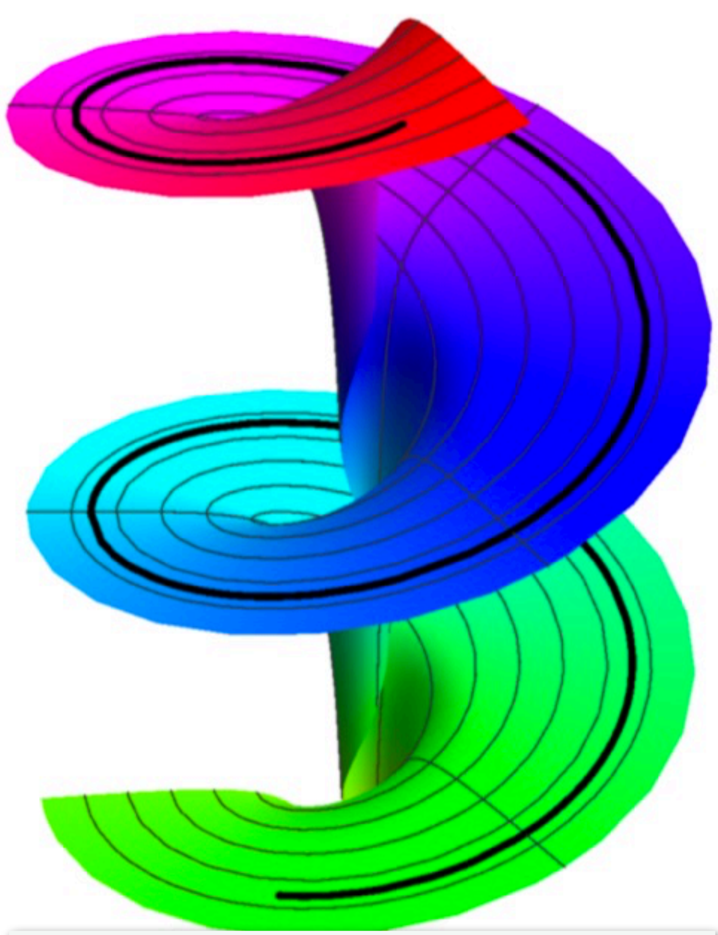
single field phenomenology

Shift-symmetric orbital inflation: ultra-light isocurvature

Achucarro, Copeland, Iarygina, Palma, **WDG** & Welling 2019
Achucarro, Palma, **WDG** & Welling 2019

$$-\frac{1}{2}\left(f(\rho)\partial_\mu\theta\partial^\mu\theta + \partial_\mu\rho\partial^\mu\rho\right)$$

The potential can be derived via Hamilton-Jacobi formalism:



$$\dot{\theta} = -2\frac{H_\theta}{f}, \qquad \rho = \rho_0$$

$$V = 3H^2 - 2\frac{H_\theta^2}{f(\rho)}$$

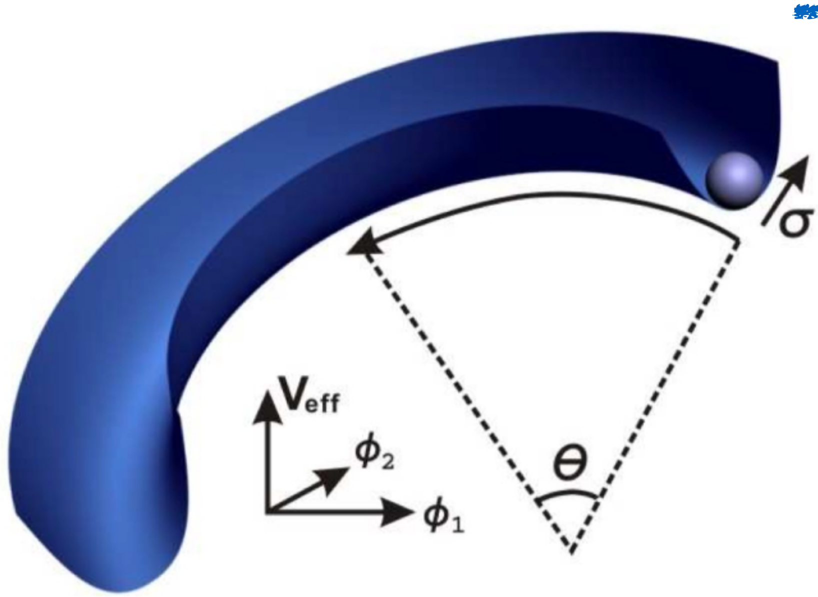
$$\mu^2 = 0 \qquad \sigma_k = \frac{H_*}{2\pi} \qquad \dot{\zeta} = \frac{2\Omega}{\sqrt{2\epsilon}}\sigma$$

- Only one degree of freedom (isocurvature) is responsible for observed curvature perts;
- The phenomenology mimics the one of single field inflation:
  - Small isocurvature perturbations
  - Small local non-Gaussianity

Is there any model-independent observational signatures of the inflationary curved field space?

Quasi-single field inflation (QSFI) and massive fields

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\right]$$



flat field space with polar coordinate

massive isocurvature field

$$m \sim \mathcal{O}(H)$$

Chen & Wang 2010;
Baumann & Green 2011;
……

Background EFT of inflation approach:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}M^2\sigma^2 + \frac{\sigma(\partial_\mu\phi)^2}{\tilde{\Lambda}}$$

the inflaton sector	the massive field sector	mixing by a dim-5 operator
(slow-roll)	$m \sim \mathcal{O}(H)$	***leading correction***
shift symmetry	no shift symmetry	

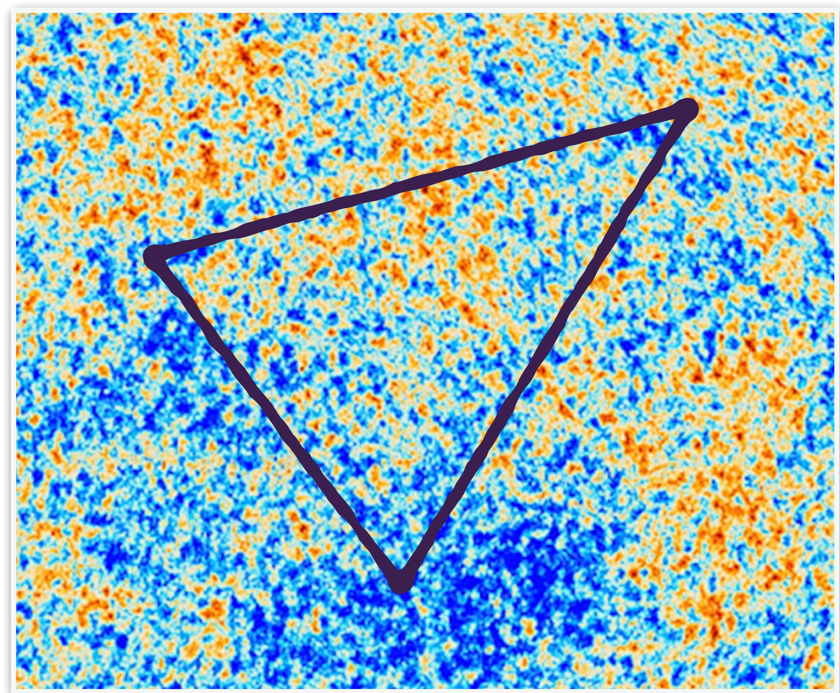
Weinberg 2008;
Assassi, Baumann, Green, McAllister 2013;
……

Triangles in the sky

Primordial non-Gaussianity

beyond the Gaussian statistics, beyond the power spectrum…

3-point correlation function in the primordial perturbations

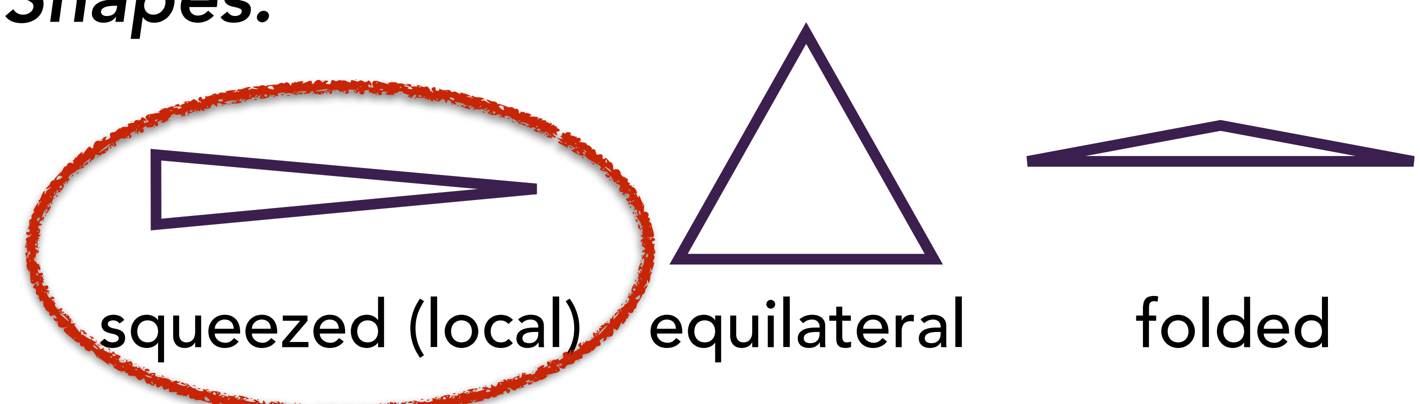


a lot of information but too small to be detected now

Theorists are needed to do template searches!

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B_\zeta(k_1, k_2, k_3)$$

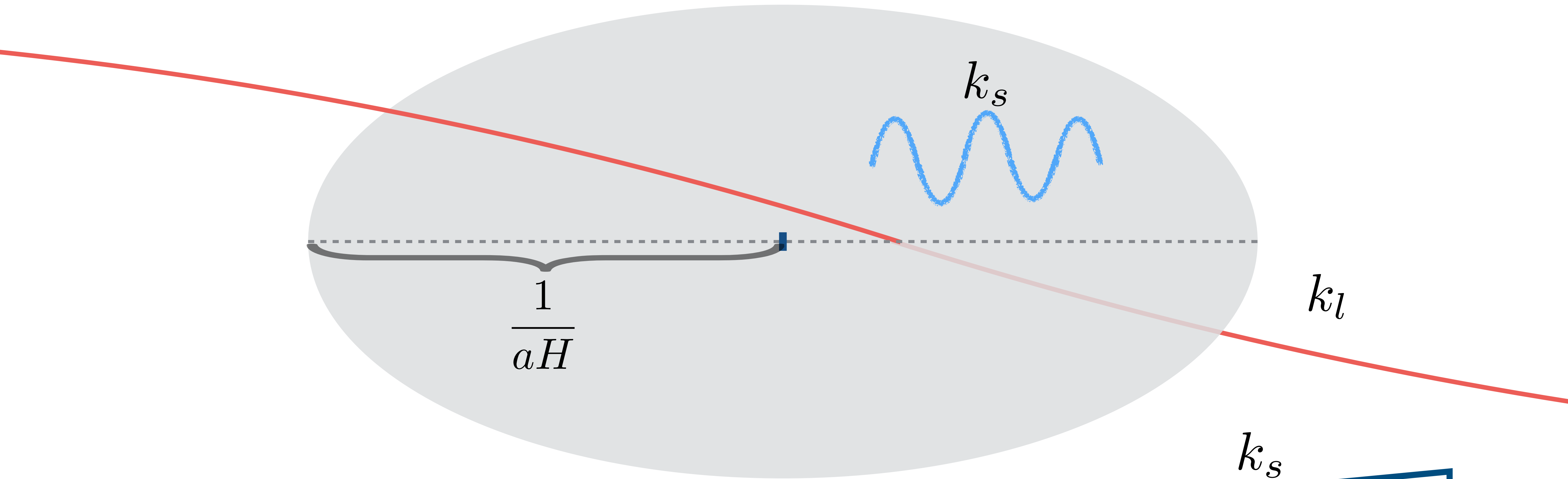
- ***Shapes:***

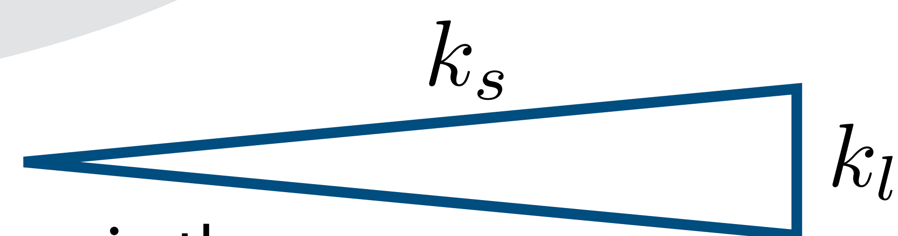


- ***Amplitudes:***

$$f_{\rm NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{10}{3} \frac{\mathcal{A}_{\rm tot}(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{\sum_i k_i^3}$$

The squeezed limit of the scalar bispectrum





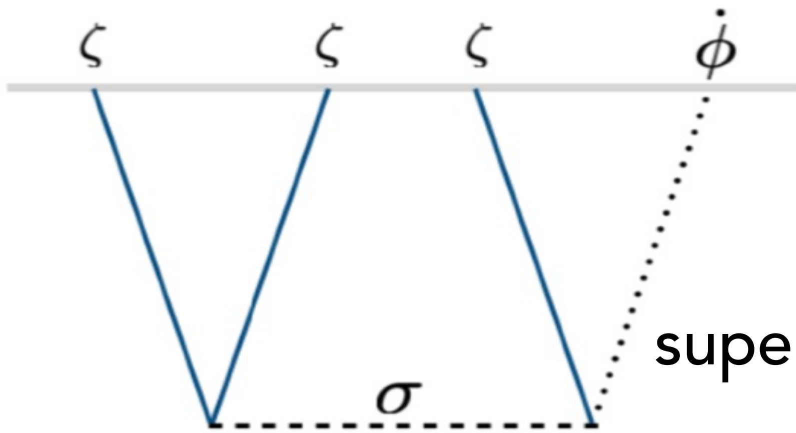
long wavelength mode modulation on the short wavelength ones → bispectrum in the squeezed triangle

Example: consistency relation in single field inflation

$$\lim_{k_3\to 0}\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = -(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,P_\zeta(k_1)P_\zeta(k_3)\,\frac{d\ln k_3^3 P_\zeta(k_3)}{d\ln k_3}$$
$$= (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,(1-n_s)\,P_\zeta(k_1)P_\zeta(k_3)\,.$$

Maldacena 2002

Massive fields and the squeezed limit



Chen & Wang 2010;
Baumann & Green 2011;
Noumi, Yamaguchi, Yokoyama 2012
……

the modulation of the
superhorizon isocurvatue modes

Superhorizon decay of the massive field: $\sigma_k'' + \cancel{\frac{k^2}{a^2H^2}\sigma_k} + 3\sigma_k' + \frac{\mu^2}{H^2}\sigma_k = 0$

<u>a damped oscillator:</u>

$$\sigma_k(N) \propto e^{-(3/2\pm\nu)(N-N_k)}$$

$\mu < 3H/2$ underdamped decay

$$\nu = \sqrt{\frac{9}{4} - \frac{\mu^2}{H^2}}$$

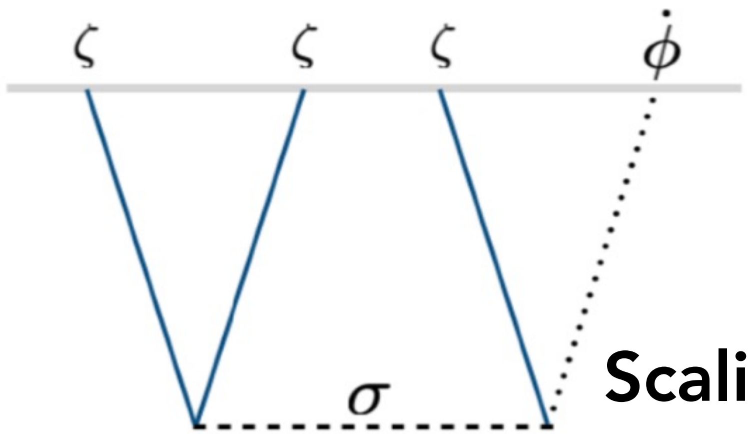
$\mu > 3H/2$ overdamped oscillation

Scaling behaviour in the squeezed bispectrum:

power-law $\lim_{k_l \ll k_s} B_\zeta \propto \frac{1}{k_l^3 k_s^3}\left(\frac{k_l}{k_s}\right)^{3/2-\nu}$ for $\mu < 3H/2$,

oscillatory $\lim_{k_l \ll k_s} B_\zeta \propto \frac{1}{k_l^3 k_s^3}\left(\frac{k_l}{k_s}\right)^{3/2} \cos\left[i\nu \ln\left(\frac{k_l}{k_s}\right) + \delta_\nu\right]$ for $\mu > 3H/2$.

Massive fields and the squeezed limit



$$\nu = \sqrt{\frac{9}{4} - \frac{\mu^2}{H^2}}$$

Chen & Wang 2010;
Baumann & Green 2011;
Noumi, Yamaguchi, Yokoyama 2012
……

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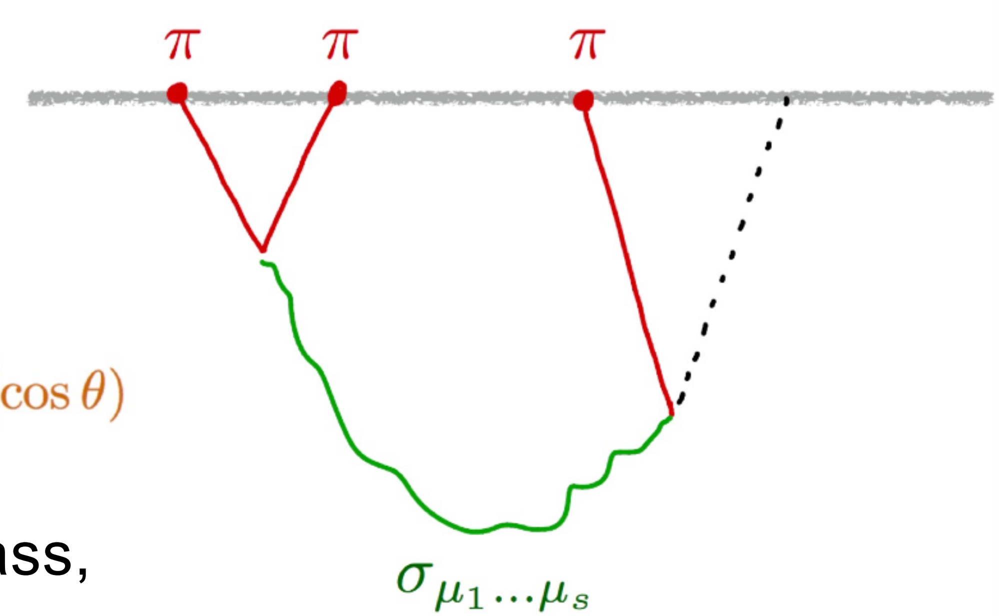
Cosmological Collider Physics

Arkani-Hamed and Maldacena 2015
Lee, Baumann and Pimentel 2016

$$\lim_{k_L \to 0} \langle \zeta_{\vec{k}_S} \zeta_{\vec{k}_S} \zeta_{\vec{k}_L} \rangle \propto \left(\frac{k_L}{k_S}\right)^{3/2} \cos\left[\frac{M}{H} \ln\left(\frac{k_L}{k_S}\right) + \delta\right] P_S(\cos\theta)$$

In the squeezed limit, oscillation measures mass,
angular dependence measures spin



Massive field with curved field space

A natural extension of QSFI

$$\mathcal{L}_m = -\frac{1}{2}f(\rho)(\partial\theta)^2 - \frac{1}{2}(\partial\rho)^2 - V(\rho) - V_{sr}(\theta)$$

metric function examples

$$f(\rho) = \rho^2$$ flat field space — the original model

$$f(\rho) = R^2\cos^2(\rho/R)$$ spherical field space — positively curved

$$f(\rho) = R^2\cosh^2(\rho/R)$$ hyperbolic field space — negatively curved

stabilized radial field: $V''(\rho_0) \simeq \mathcal{O}(H^2)$

field space curvature:

$$\mathbb{R} = \frac{f'(\rho)^2}{2f(\rho)^2} - \frac{f''(\rho)}{f(\rho)}$$

angular turning motion: $\theta = \theta(t), \qquad \rho = \text{const.}$

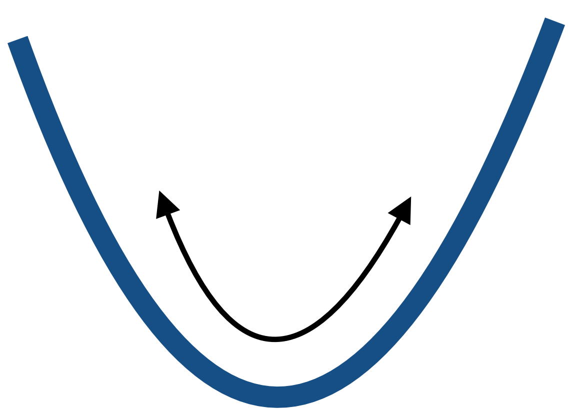
with turning rate: $\Omega \equiv -N_a D_t T^a = \frac{f'(\rho_0)}{2\sqrt{f(\rho_0)}}\dot{\theta}$

Mass scales in multi-field inflation

isocurvature mass $\mu^2 \equiv V_{NN} + \epsilon H^2 \mathbb{R} + 3\Omega^2$

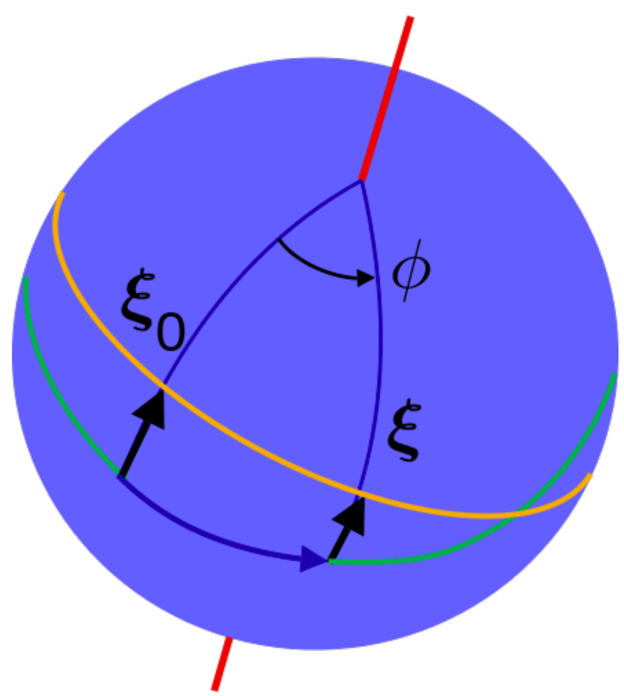
Hessian of the potential

$V_{NN} \equiv N^a N^b \nabla_a \nabla_b V$



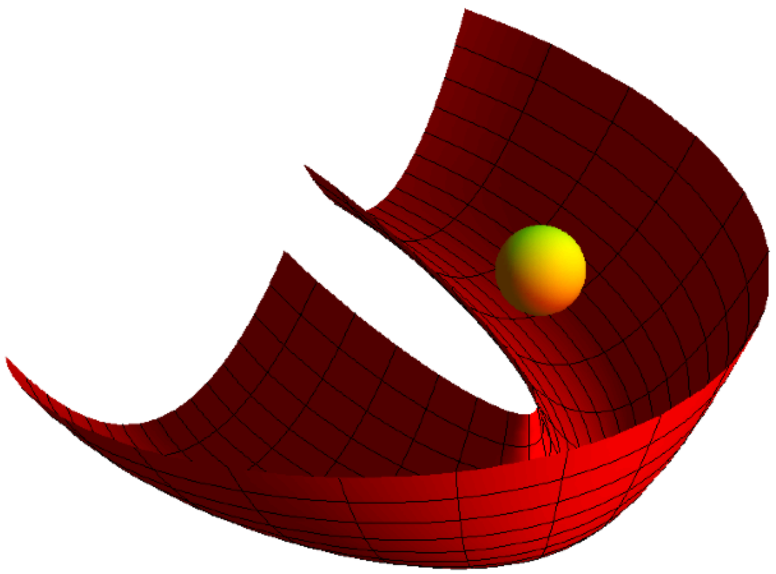
the “bare” mass

curvature of the field space



analogy of the geodesic deviation in GR

turning rate



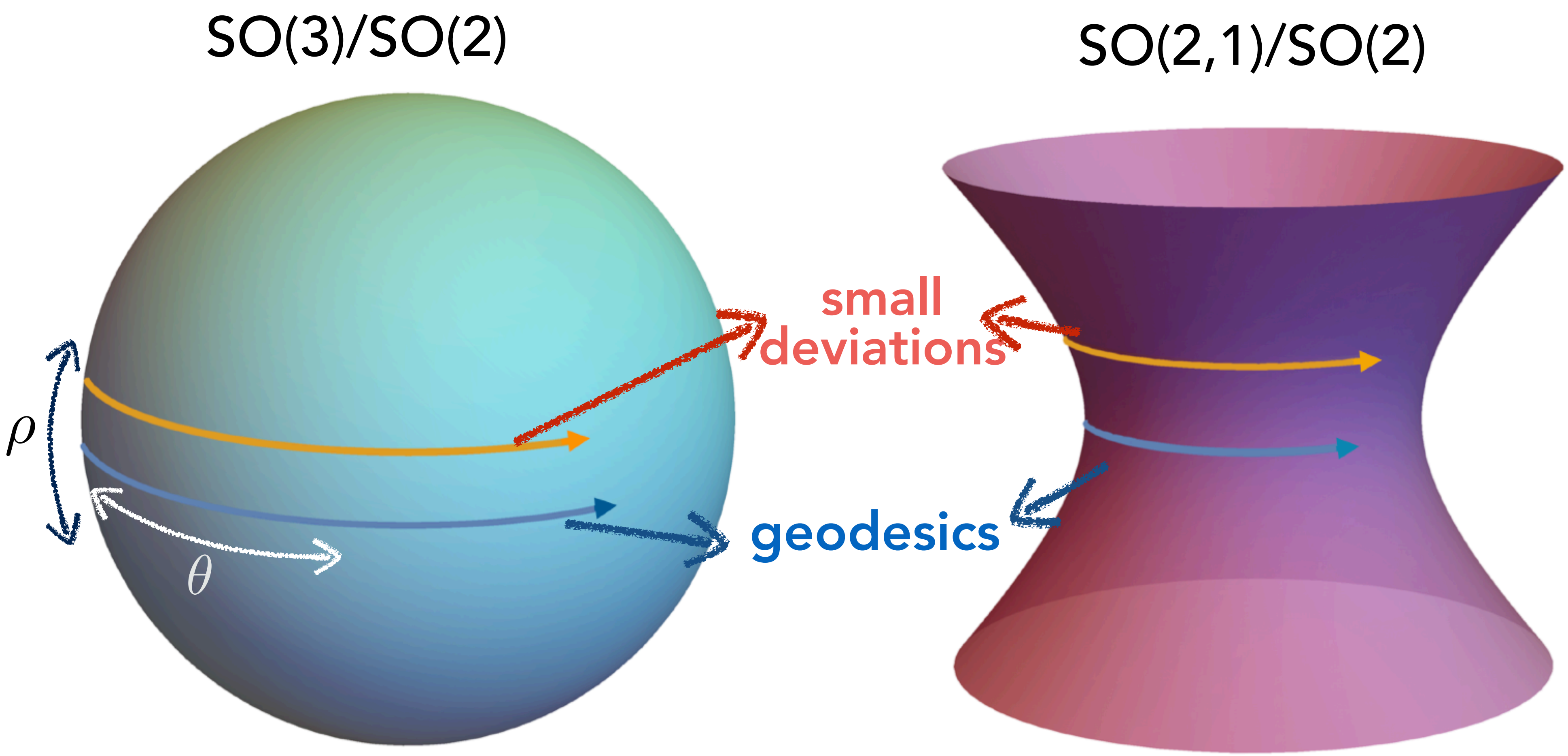
weakly coupled regime

$\Omega \ll H$

The effect of the field space curvature

Inflation in coset space as an example

spontaneous symmetry probing (SSP)
Nicolas, Piazza 2011



$$K = -\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}R^2\cos^2\left(\frac{\rho}{R}\right)(\partial\theta)^2 \qquad K = -\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}R^2\cosh^2\left(\frac{\rho}{R}\right)(\partial\theta)^2$$

mass correction for the not-rolling Goldstone

$$\delta m_\rho^2 = \dot\theta^2 = \frac{\dot\phi^2}{R^2} \qquad \delta m_\rho^2 = -\dot\theta^2 = -\frac{\dot\phi^2}{R^2}$$

Massive field in the background EFT of inflation

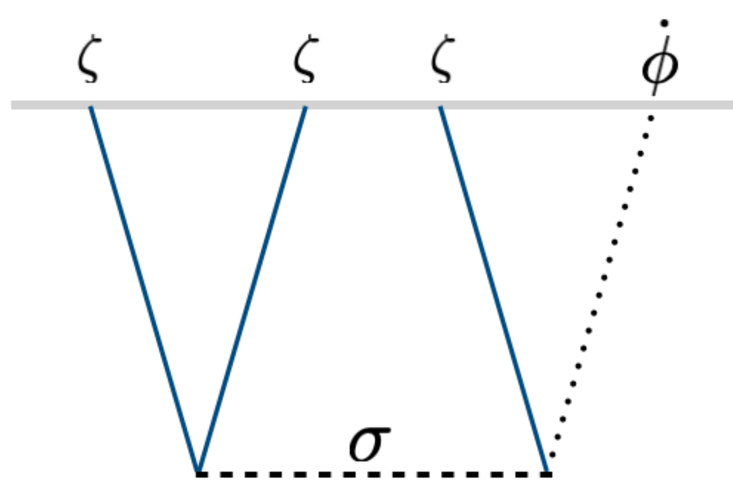
$\rho/\Lambda \ll 1$

dim-5 operator

$$\mathcal{L}^5_{\rm int} = -\frac{1}{2\Lambda_1}(\partial\varphi)^2\rho$$

leading order interaction term:

- leads to the mixing between curvature and isocurvature modes
- contributes to the bispetrum via the following Feynman diagram



a lot of discussions in the literature

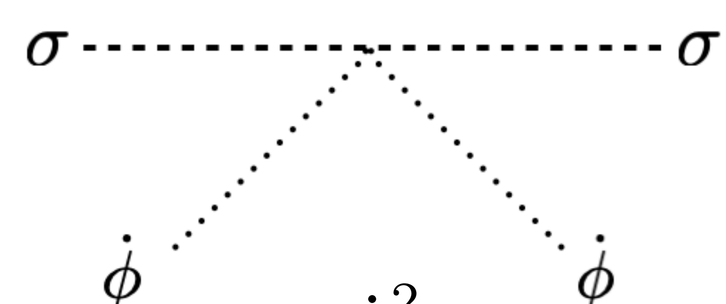
dim-6 operator

$$\mathcal{L}^6_{\rm int} = \pm\frac{1}{2\Lambda_2^2}(\partial\varphi)^2\rho^2$$

next-to-leading order term:

- no linear mixing between curvature and isocurvature modes
- contributes to the bispetrum only through loop diagrams (small)

but a new mass correction!



$$\pm\frac{\dot\varphi^2}{2\Lambda_2^2}\sigma^2$$

first noticed in geometrical destabilization

Revisiting the background EFT of inflation

bridging the gap with the curved field space

$$\mathcal{L}_m = -\frac{1}{2}\left(1 + c_1\frac{\rho}{\Lambda} + c_2\frac{\rho^2}{\Lambda^2}\right)(\partial\varphi)^2 - \frac{1}{2}(\partial\rho)^2 - \frac{1}{2}m^2\rho^2 - V_{\rm sr}(\varphi)$$

$$\rho/\Lambda \ll 1$$

metric function

$$f(\rho) = 1 + c_1\frac{\rho}{\Lambda} + c_2\frac{\rho^2}{\Lambda^2}$$

field space curvature

$$\mathbb{R} \simeq -\frac{2c_2 - c_1^2/2}{\Lambda^2} + \mathcal{O}\left(\frac{\rho}{\Lambda}\right)$$

Consider an approximate Z2 symmetry: $|c_1| \ll |c_2| = 1$

equivalent to the trajectory with small deviation from geodesics in coset space

$$R^2\cos^2\left(\frac{\rho}{R}\right) = R^2\left[1 - 2\frac{\rho_b}{R}\frac{\rho - \rho_b}{R} - \frac{(\rho-\rho_b)^2}{R^2}\right] + \dots$$

$$c_1 = -2\rho_b/R$$
$$c_2 = -1$$

Λ = R

the cutoff scale Λ of the dim-6 operator plays the role of the curvature scale of the field space

The running isocurvature mass

multi-field analysis:

$$\mu^2 \simeq V''(\rho_0) + \frac{\dot\phi^2}{2}\mathbb{R} = V''(\rho_0) \pm \frac{\dot\phi^2}{R^2}$$

EFT approach:

$$\mu^2 \simeq m^2 - c_2\left(\frac{\dot\varphi}{\Lambda}\right)^2 = m^2 - 2c_2\epsilon H^2\left(\frac{M_{\rm pl}}{\Lambda}\right)^2$$

the size of the mass correction:

$R = \Lambda \sim M_{\rm Pl}$ ⇒ $\delta\mu^2_{\mathbb{R}} \sim \epsilon H^2$

$R = \Lambda \sim 3000H$ ⇒ $\delta\mu^2_{\mathbb{R}} \sim H^2$

$\dot\phi^2 \simeq 10^7 H^4$

This mass correction is time-dependent in nature

The time-dependence in the slow-roll inflaton

$$\epsilon(N) \simeq \epsilon(N_l) + \epsilon'(N_l)(N - N_l) = \epsilon_l\left[1 + \eta_l(N - N_l)\right]$$

The isocurvature mass is running!

⇒

$$\mu^2(N) = \mu_l^2 + \lambda(N - N_l)H^2$$

$$\lambda = \eta_l\frac{\dot\phi^2}{2H^2}\mathbb{R} = -\eta_l\frac{c_2}{H^2}\frac{\dot\varphi^2}{\Lambda^2}$$

Superhorizon decay of the isocurvature perturbation

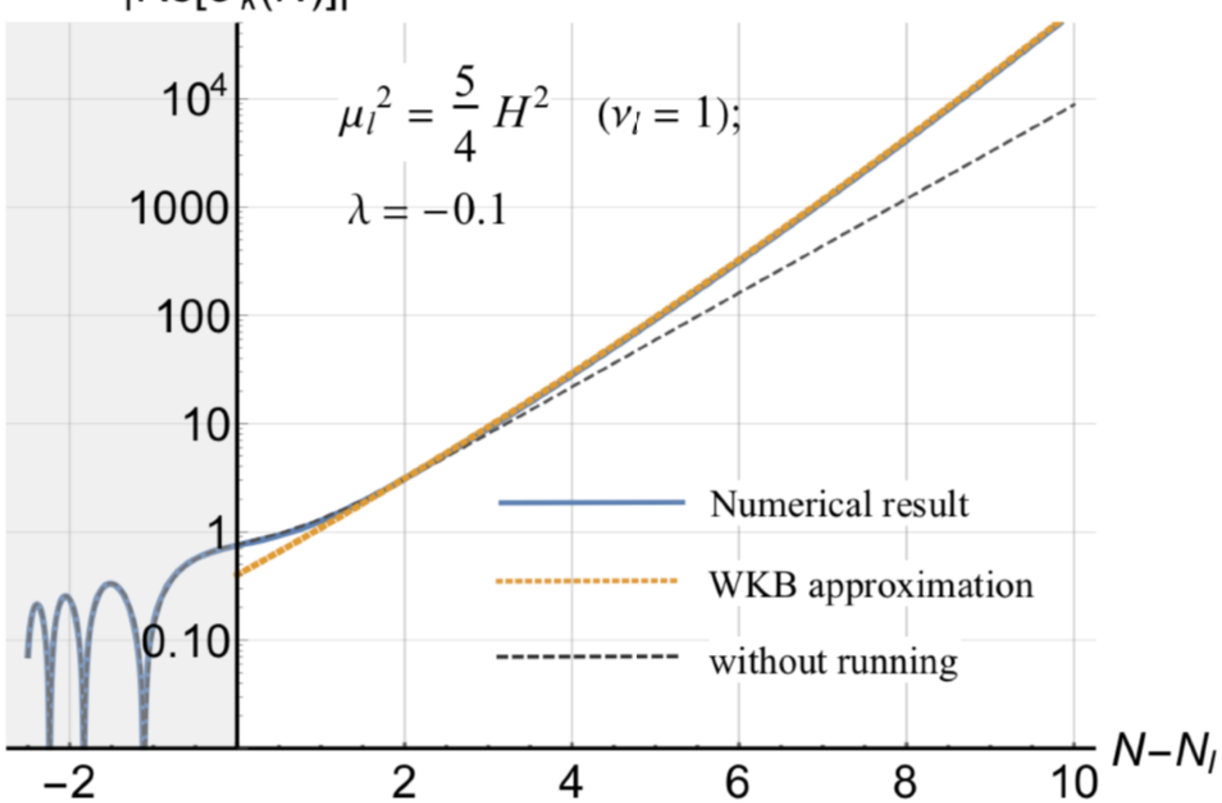
with running mass $\mu^2(N) = \mu_l^2 + \lambda(N - N_l)H^2$

a rescaled mode function $\widetilde{\sigma}_k = e^{3N/2}\sigma_k$

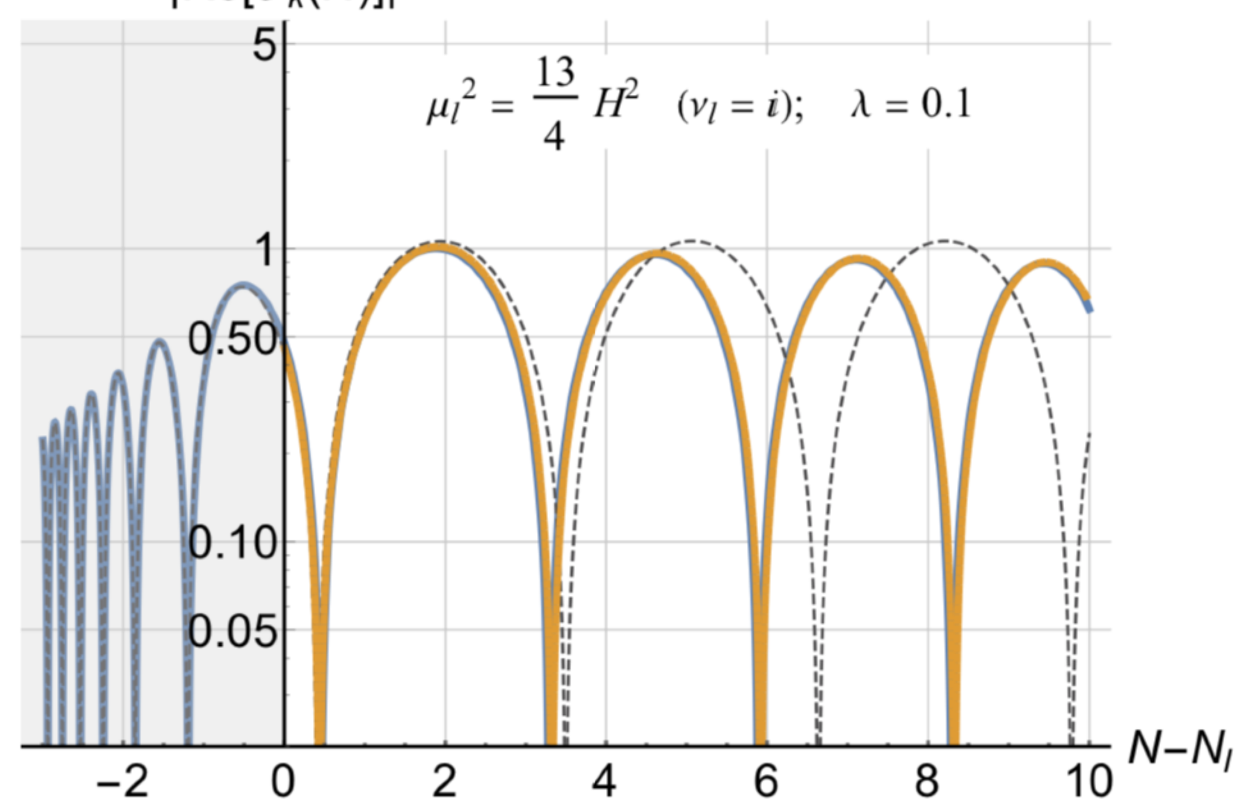
the superhorizon EoM $$\widetilde{\sigma}''_{k_l} - \left[\nu_l^2 - \lambda(N - N_l)\right]\widetilde{\sigma}_{k_l} = 0$$ $\nu_l^2 = 9/4 - \mu_l^2/H^2$

Numerical results / Exact solutions with Airy functions / WKB approximations:

negative-running mass in the μ < 3H/2 regime



positive-running mass in the μ > 3H/2 regime



Superhorizon decay of the isocurvature perturbation

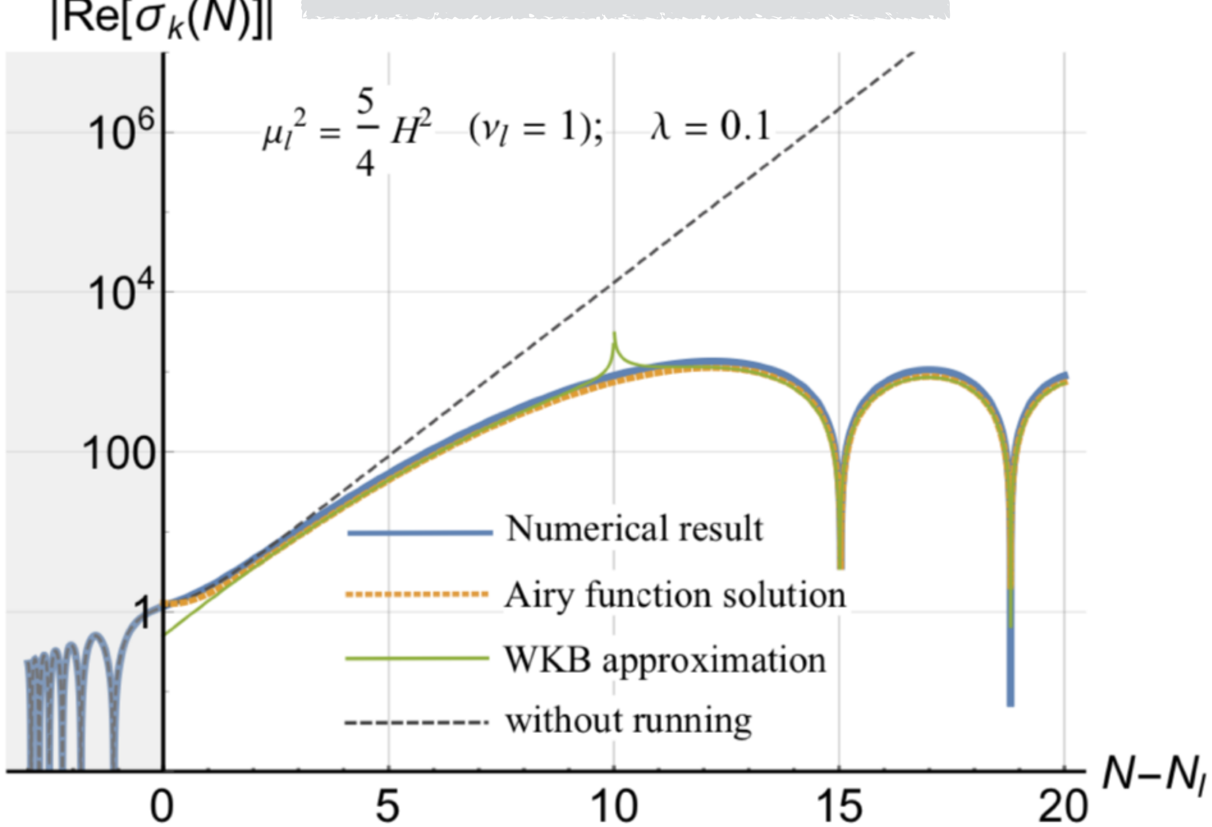
with running mass $\mu^2(N) = \mu_l^2 + \lambda(N - N_l)H^2$

a rescaled mode function $\widetilde{\sigma}_k = e^{3N/2}\sigma_k$

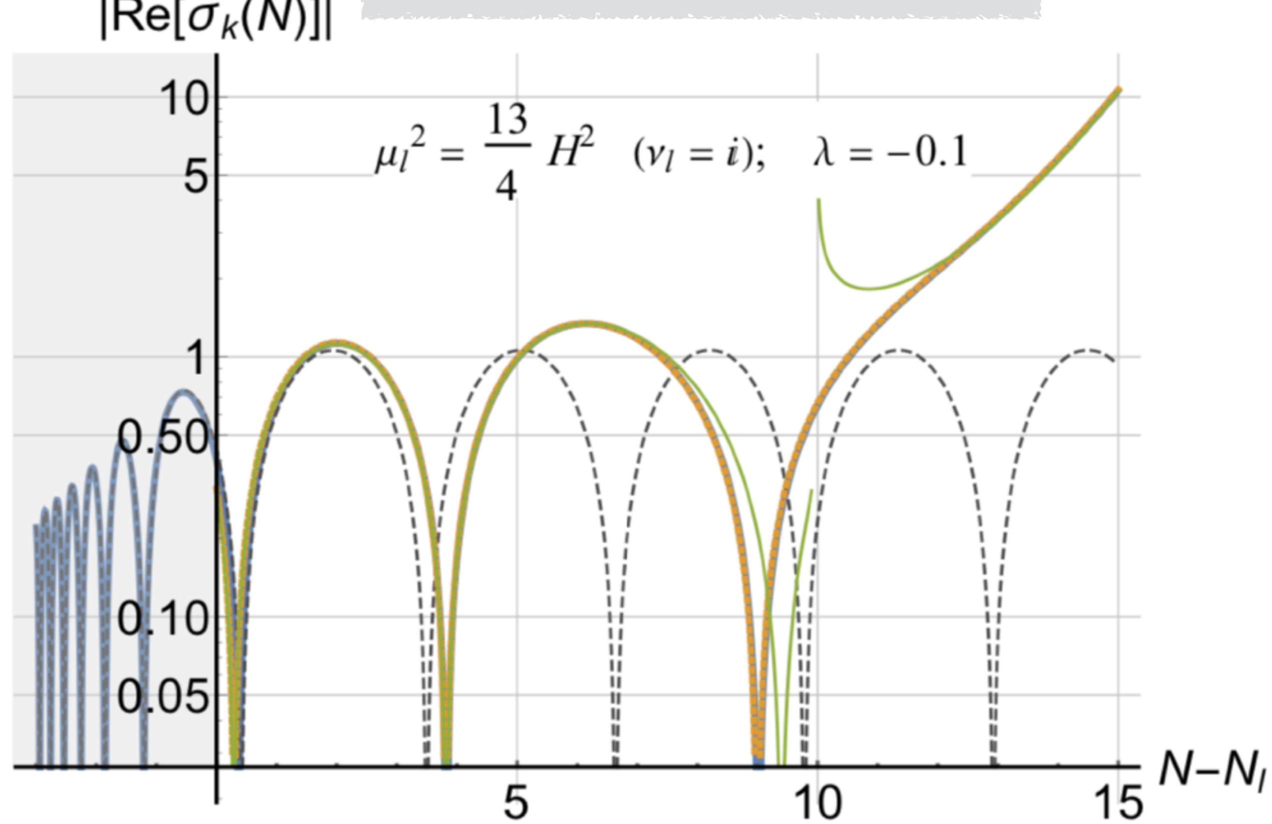
the superhorizon EoM $\widetilde{\sigma}''_{k_l} - \left[\nu_l^2 - \lambda(N - N_l)\right]\widetilde{\sigma}_{k_l} = 0$ $\qquad \nu_l^2 = 9/4 - \mu_l^2/H^2$

Numerical results / Exact solutions with Airy functions / WKB approximations:

positive-running mass from μ < 3H/2 regime



negative-running mass from μ > 3H/2 regime



Phenomenology in primordial non-Gaussianity

- **Running in the μ < 3H/2 regime** — via WKB method

$$\lim_{k_l \ll k_s} B_\zeta \propto \frac{1}{k_l^3 k_s^3}\left(\frac{k_l}{k_s}\right)^{3/2-\nu_l+\alpha_\nu \ln(k_s/k_l)}, \quad \text{with } \alpha_\nu \equiv \frac{\lambda}{4\nu_l} = \frac{1}{4\nu_l}\epsilon_l M_{\rm p}^2 \mathbb{R} \cdot \eta_l .$$

- **Running in the μ > 3H/2 regime** — via WKB method

$$\lim_{k_l \ll k_s} B_\zeta \propto \frac{1}{k_l^3 k_s^3}\left(\frac{k_l}{k_s}\right)^{3/2} \cos\left[i\nu_l \ln\left(\frac{k_l}{k_s}\right) - i\alpha_\nu \ln^2\left(\frac{k_l}{k_s}\right) + \delta_l\right], \quad \text{with } \alpha_\nu \equiv \frac{\lambda}{4\nu_l}$$

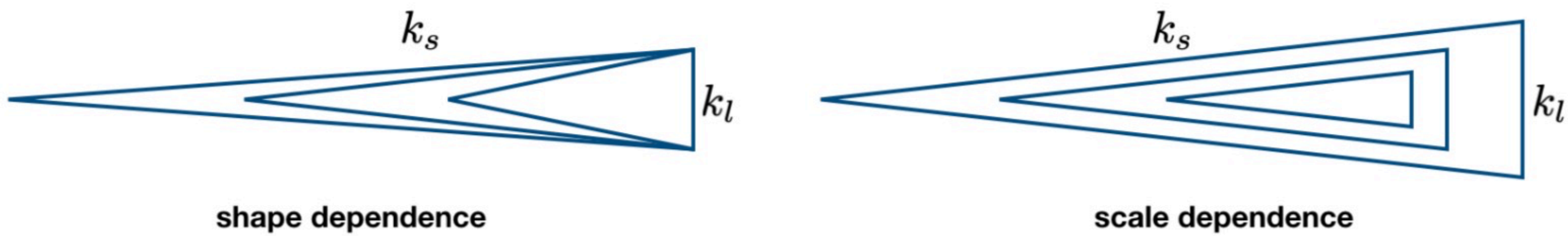
- **Running through μ = 3H/2** — WKB breaks down

large deviation from the QSFI predictions

- ***positive-running*** transition signal from power-law to oscillatory scalings
- ***negative-running*** transition signal from oscillatory to power-law scalings

Final Comments:

shape dependence vs. scale dependence



scale-dependent halo bias

the galaxy power spectrum
with non-Gaussian initial condition

$$P_{\rm hh}(k) = (b_1 + \Delta b_1^{\rm NG}(k))^2 P_{\rm mm}(k)$$

sensitive to the squeezed limit of the scalar bispectrum

the standard QSFI scaling:

$$b^{\rm NG}(k) \propto k^{-1/2-\nu}$$

$$b^{\rm NG}(k) \propto k^{-1/2}\cos(i\nu \ln k)$$

the new scaling with running:

$$b^{\rm NG}(k) \propto k^{-1/2-\nu_l-\alpha_\nu \ln k}$$

$$b^{\rm NG}(k) \propto k^{-1/2}\cos(i\nu_l \ln k - i\alpha_\nu \ln^2 k)$$

Summary and Discussions

- ***Curved field space*** is a typical consequence in the fundamental realizations of inflation, which has drawn a lot of attention recently;
- **The background EFT** provides an equivalent description, where a *dim-6 operator* plays an important role for the field space curvatue.
- **Massive field** provides an illuminating channel for observational signatures of inflationary curved field space in the squeezed bispectrum:
  - *running behaviour of the scaling* measures the field space curvature;
  - transition signals between power-law and oscillatory scalings.
- Possible degeneracy with other physical effects?
- **More *generic* and *model-independent*** signatures of curved field space?
- More systematical understanding of its implications on *cosmological colliders*

More adventures ahead in the curved field space!