

Young massive star clusters

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Abstract

Young massive clusters are dense aggregates of young stars that form the fundamental building blocks of galaxies. There are several specimens in the Milky Way Galaxy and the local group, but they are particularly abundant in starburst and interacting galaxies. The few young massive clusters that are close enough to resolve are of prime interest for studying the stellar mass function and the ecological interplay between stellar evolution and stellar dynamics. The distant unresolved clusters are effectively used to study the star-cluster mass function, and they provide excellent constraints on the formation mechanisms of young cluster populations. Young massive clusters are expected to be the nurseries for many unusual objects, including a wide range of exotic stars and binaries. So far only a few such objects have been found in young massive clusters, although their older siblings, the globular clusters, are unusually rich in stellar exotica.

In this review we limit ourselves to star clusters younger than 100 Myr and more massive than $10^4 M_{\odot}$, irrespective of the cluster size or environment. In particular we study clusters which are older than a current crossing time. We summarize the current knowledge of young massive star clusters, and discuss the state of the art in observations and dynamical modeling. We summarize the global properties of the currently known young massive star clusters in the local group and beyond, and discuss the nomenclature and range of numerical techniques utilized in simulations of young

massive star clusters. In order to make this review readable by observationally oriented astronomers as well as by theorists and computational astrophysicists, we also review the cross-disciplinary terminology.

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1 Introduction

Stars form in clusters (Lada & Lada 2003). In the Milky Way Galaxy, evidence for this statement comes from the global clustering of spectral O-type stars (Parker & Goodwin 2007), of which $\sim 70\%$ reside in young clusters (Gies 1987) and $\sim 50\%$ of the remaining field population are directly identified as runaways (de Wit et al. 2005). de Wit et al. (2005) found that only $\sim 4\%$ of O-type stars can be considered as having formed outside a cluster environment. However, after further analysis it was recently shown that a few of those may also be runaway stars (Gvaramadze & Bomans 2008, Schilbach & Röser 2008). Further evidence that clusters are the primary mode of star formation comes from the observed formation rate of stars in embedded clusters ($\sim 3 \times 10^3 M_{\odot} \text{ Myr}^{-1} \text{ kpc}^{-2}$; Lada & Lada 2003) which is comparable to the formation rate of field stars ($\sim 3\text{--}7 \times 10^3 M_{\odot} \text{ Myr}^{-1} \text{ kpc}^{-2}$; Miller & Scalo 1979). In addition, some 96% of the stars in the nearby Orion B star-forming region are clustered (Clarke, Bonnell & Hillenbrand 2000).

In nearby young starburst galaxies at least 20%, and possibly all, of the ultraviolet light appears to come from young star clusters (Meurer et al. 1995); this also seems to be the case for the observed $H\alpha$ and B-band luminosities in interacting galaxies, such as the Antennae (Fall, Chandar & Whitmore 2005) and NGC 3256 (Zepf et al. 1999). The fossil record of an early episode of star formation is evidenced by the present-day population of globular clusters, although the first (population III) stars seem not to have formed in clusters (Abel, Bryan & Norman 2002).

1.1 Scope of this review

According to the Oxford English Dictionary (2009) a globular star cluster (hereafter GC) is “*a roughly spherical cluster of stars, typically seen in galactic halos, containing large numbers of old, metal-poor stars.*” This definition succinctly defines the properties—shape, mass, spatial distribution, stellar content and metallicity—normally associated with globular clusters, although it is somewhat too restrictive about the metallicity. In these terms, a “young” globular cluster is a high-redshift ($z \sim 5$) object, probably undetectable with current technology against the background of its parent galaxy (Brodie & Strader 2006)^a. In current usage, however, the term “young globular cluster” has acquired a rather different meaning, referring instead to young, massive clusters that might some day come to exhibit properties comparable to the globular clusters observed today. Accordingly, for purposes of this review, our working definition of a young massive star cluster is “*a roughly spherical cluster of stars, found in both galactic disks and halos, containing large numbers of young stars*”. Any young cluster massive enough to survive for a Hubble time—regardless of its current location—meets our criteria for inclusion. Within the current context we cannot make a connection between cluster properties and environment, but from the discussion around Fig. 8 it is clear that environment has at least some influence on the global cluster characteristics (see §2.2).

Obviously, with this definition, young massive clusters cannot be the progenitors of the present-day GCs. We will refer to these alleged siblings as young massive clusters or YMCs throughout this review. In fact, it is unclear to what extent today’s YMCs will ever have

^aGlobular clusters at $z \sim 5$ are expected to be 2 magnitudes brighter than the detection limit of the James Webb Space Telescope.

properties similar to those of the (old) GCs. While we can reasonably expect that after (say) 10 Gyr they will be roughly spherical in shape and will have surface brightness distributions similar in character to those of the old GCs, other properties are not so easy to assess. For example, it is not known to what extent the future kinematic properties of these clusters, as a population, will resemble those of today’s GC systems. Young clusters, such as those in the Antennae (Fall, Chandar & Whitmore 2005), that formed in galaxy mergers may well come to populate the halo of the resultant galaxy (Kravtsov & Gnedin 2005), but it is less clear that the same is true of clusters currently forming in quiescent galactic disks.

The uncertainties increase when we consider in more detail the internal properties of clusters. Except for a few nearby cases, such as Westerlund 1 and the Arches cluster (see Tab. 2 in §2.3.1), observations of YMCs are limited to stellar masses $\gtrsim 1 M_{\odot}$, whereas the most massive stars observed in GCs are less than $1 M_{\odot}$. Thus there is no guarantee that the stellar mass function in YMCs below $\sim 1 M_{\odot}$ resembles the initial mass functions of known GCs, although we note that the stellar mass function in nearby open clusters is consistent with the distribution of low-mass stars in GCs (Chabrier 2003).

Finally, the initial binary fractions inferred for GCs are generally small, whereas YMCs appear to be binary-rich (see §6.1). However, the known binaries in massive clusters tend to be found among massive stars. The binary properties of low-mass stars are unknown. Again, the absence of any overlap in the observed stellar mass spectra makes it difficult to compare binary parameters in YMCs with those in GCS.

Our objective in this review is to summarize the current state of knowledge of young massive star clusters, the key physical processes governing their evolution and survival, and the extent to which we expect them to evolve into systems comparable to the old GCs observed in many galaxies today.

We will focus on observations of clusters younger than about 100 Myr and more massive than $10^4 M_{\odot}$. The age limit is somewhat arbitrary, but represents roughly the epoch at which a cluster can be said to have survived its birth and the subsequent early phase of vigorous stellar evolution (see §5), and to be entering the long-term evolutionary phase during which its lifetime is determined principally by stellar dynamical processes and external influences (see §3). The mass limit is such that lower mass clusters are unlikely to survive for more than a few gigayears, and therefore will never become part of an old “globular cluster” population. Based on the lifetimes presented in §5 (in particular in Eq. 18), we estimate that a cluster with an initial number of stars $N \simeq 10^5 M_{\odot}$ will survive for a typical GC age of ~ 10 Gyr. However, since young clusters more massive than $10^5 M_{\odot}$ are relatively rare, we relax our criterion in this review to include star clusters with masses as low as $10^4 M_{\odot}$.

We place no limits on cluster size, metallicity, or galactic location, for the practical reason that this would reduce still further our already small sample of young massive clusters, even though it may be clear that clusters like Arches and Quintuplet are unlikely to survive for more than a gigayear. The size distribution of YMCs, however, appears to be consistent with them evolving into GCs (Maíz-Apellániz 2002), so the absence of a limit has little material effect on our discussion.

In this review we will use the terms young, dense, and massive in relation to star clusters. Although not precise, these descriptions do have specific connotations. “Young” means star clusters that have survived the early, violent mass-loss phase during the first 100 Myr (see §5). “Dense” indicates that in some clusters the stars are packed together so closely that stellar

collisions start to play an important role (see §3.4.2). In terms of the Safronov number,^b young dense clusters have $\Theta \lesssim 10^2$. As a practical matter, we consider clusters to be dense if their relaxation time (Eq. 15) is less than $\sim 10^8$ years (Portegies Zwart, McMillan & Makino 2007). “Massive” indicates that we expect the cluster to survive for ~ 10 Gyr, into the “old globular cluster” regime. Tab. 1 summarizes the main parameters of the three different populations of star clusters.

cluster	age [Gyr]	m_{to} [M_{\odot}]	M [M_{\odot}]	r_{vir} [pc]	ρ_c [M_{\odot}/pc^3]	Z [Z_{\odot}]	location	t_{dyn} [Myr]	t_{rh} [Myr]
OC	$\lesssim 0.3$	$\lesssim 4$	$\lesssim 10^3$	1	$\lesssim 10^3$	~ 1	disk	~ 1	$\lesssim 100$
GC	$\gtrsim 10$	~ 0.8	$\gtrsim 10^5$	10	$\gtrsim 10^3$	< 1	halo	$\gtrsim 1$	$\gtrsim 1000$
YMC	$\lesssim 0.1$	$\gtrsim 5$	$\gtrsim 10^4$	1	$\gtrsim 10^3$	$\gtrsim 1$	galaxy	$\lesssim 1$	$\lesssim 100$

Table 1: Comparison of fundamental parameters for star cluster families relevant to this review: open cluster (OC), globular cluster (GC), and young massive cluster (YMC). The numbers in the columns are intended to be indicative of the population and are rounded-off, and should be used with care, but they provide some flavor of the various cluster types. The second column gives cluster age, followed by the turn-off mass (in M_{\odot}), the total cluster mass (in M_{\odot}), the virial radius r_{vir} , the core density, and the metallicity. The last three columns give the location in the Galaxy where these clusters are found, and the dynamical and relaxation time scales.

Figs. 1 and 2 compare the distributions of massive open star clusters, YMCs, and GCs in the Milky Way Galaxy. Their positions in the galaxy (Fig. 1) are quite distinct, but considering that YMCs are relatively massive compared to open clusters and are therefore observable to a larger distances, their locations do seem to resemble the open cluster distribution, rather than that of the GCs. However, in the mass-radius diagram (Fig. 2), YMCs seem more closely related to GC.

1.2 Properties of Cluster Systems

The Milky Way Galaxy contains some 150 GCs, with mass estimates ranging from $\sim 10^3$ (for AM4, a member of the Sgr dwarf spheroidal galaxy) to $2.2 \times 10^6 M_{\odot}$ (for NGC 5139, Omega Centauri)^c. If we assume constant $M/L = 2$, the current total mass in GCs is $\sim 3.5 \times 10^7 M_{\odot}$, or $\sim 0.07\%$ of the baryonic mass of the Galaxy and 0.005% of the total mass (including dark matter). The luminosity function of GCs in the Galaxy peaks at $M_V \approx -7.4$ mag,

^bThe Safronov (1969) number Θ is defined as the square of the ratio of the escape velocity from the stellar surface to the rms velocity in the cluster:

$$\Theta = \frac{1}{2} \left(\frac{v_{\star, \text{esc}}}{v_{\text{rms}}} \right)^2. \quad (1)$$

^cThese estimates are made assuming a constant mass-to-light ratio $M/L = 2$, with data from the (Harris 1996) catalog of Milky Way GCs <http://www.physics.mcmaster.ca/Globular.html>. Another useful catalog for Milky Way globular cluster data is available online <http://www.astro.caltech.edu/~george/glob/data.html> (Djorgovski & Meylan 1993)

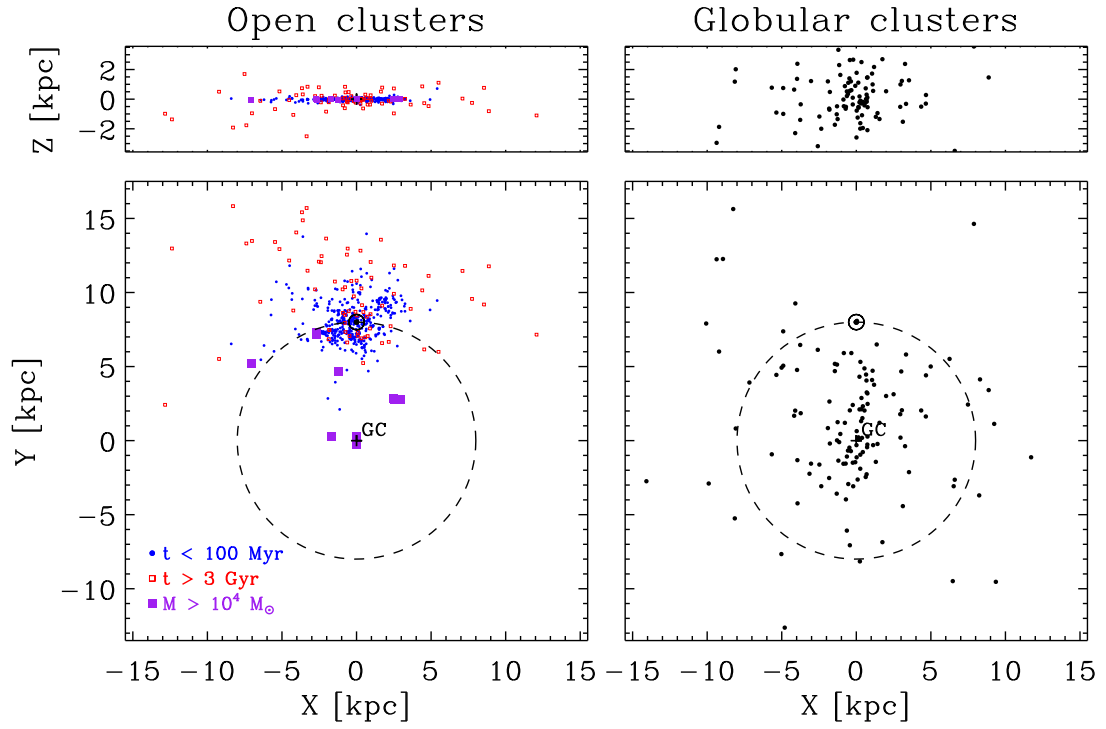


Figure 1: Left: Distribution of young (< 100 Myr) and old (> 3 Gyr) open clusters in the Galactic plane, based on the catalog of Dias et al. (2002). The old open clusters are found preferentially towards the Galactic anti-center and above the plane. The young massive clusters seem to be concentrated in the same quadrant as where the Sun resides, which probably is an observational selection effect. Right: Distribution of globular clusters, data from the catalog (Harris 1996).

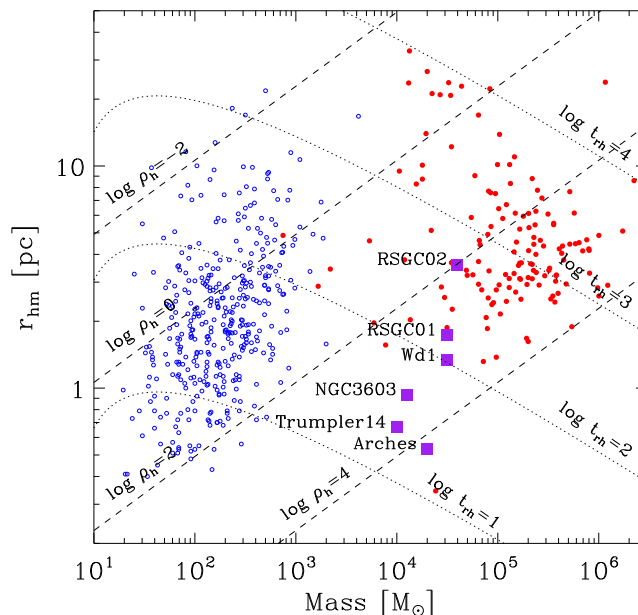


Figure 2: Radius-mass diagram of Milky Way open and globular clusters. Dashed and dotted lines represent constant half-mass density and half-mass relaxation time, respectively.

corresponding to a typical mass of $\sim 2 \times 10^5 M_\odot$, and has a (Gaussian) width of $\simeq 1.2$ mag (Harris 2001). The total initial mass for GCs is estimated to be $\sim 4\text{--}8 \times 10^8 M_\odot$ (Fall & Zhang 2001). Apparently, more than 90% of all globular star clusters have been disrupted during the last ~ 12 Gyr (Chernoff & Weinberg 1990), and the inferred total mass in disrupted clusters is comparable to the total mass of the Galactic stellar halo (Bell et al. 2008, see also § 5).

If all stars form in clusters, then all field stars must once have belonged to a cluster, and the observed old star clusters—the globular clusters and old open clusters—are the survivors. If GCs supplied the halo with stars, then the population of open star clusters must supply the disk. The open cluster databases of Kharchenko et al. (2005) and Piskunov et al. (2008), with 81 clusters, are probably complete to a distance of ~ 600 pc and have a mean mass of $\sim 500 M_\odot$. With an open cluster birth rate of $0.2\text{--}0.5 \text{ Myr}^{-1} \text{ kpc}^{-2}$ (Battinelli & Capuzzo-Dolcetta 1991, Piskunov et al. 2006), the implied total star formation rate in open clusters is $\sim 2 \times 10^2 M_\odot \text{ Myr}^{-1} \text{ kpc}^{-2}$, which is somewhat lower than independent estimates of the Galactic formation rate of stars in clusters Kennicutt (1998). (We note that, for an average cluster age of ~ 250 Myr these estimates imply a total of about 23,000–37,000 open star clusters currently in the Galaxy.) However, the formation rate of embedded clusters is considerably higher— $2\text{--}4 \text{ Myr}^{-1} \text{ kpc}^{-2}$ (Lada & Lada 2003)—and with a similar cluster mean mass the total star formation rate in embedded clusters is $\sim 3 \times 10^3 M_\odot \text{ Myr}^{-1} \text{ kpc}^{-2}$, comparable to the formation rate of field stars in the disk ($3\text{--}7 \times 10^3 M_\odot \text{ Myr}^{-1} \text{ kpc}^{-2}$; Miller & Scalo 1979). Although the uncertainties are large, this suggests that the majority of stars are formed in embedded clusters, but only a small fraction ($\sim 10\%$) survive the embedded phase to become open clusters.

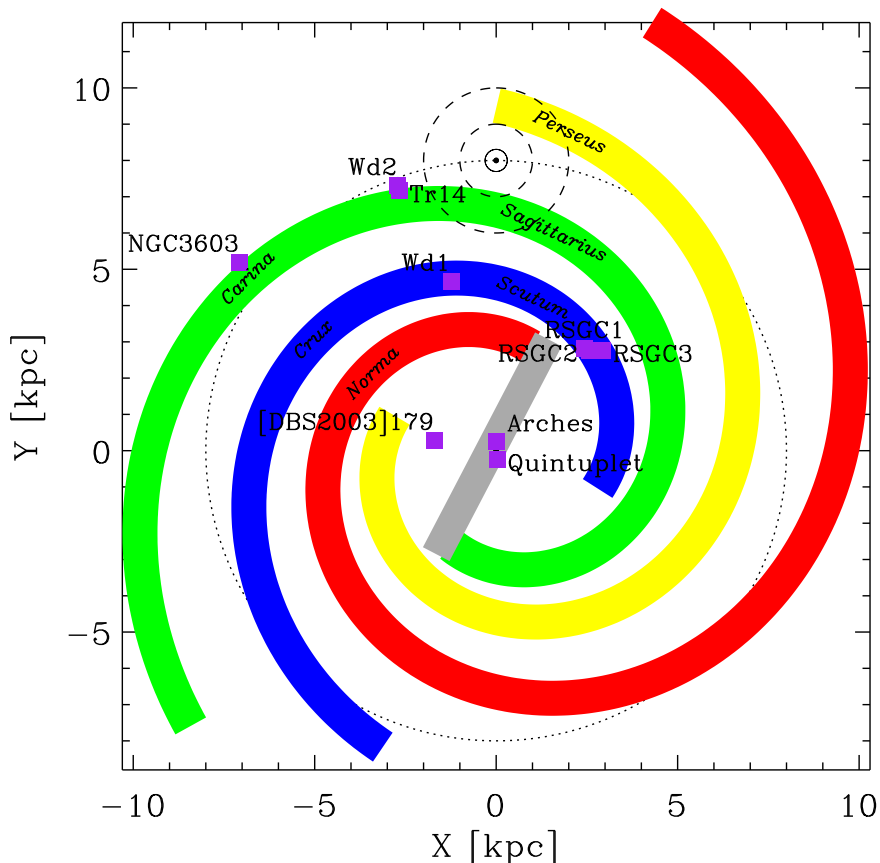


Figure 3: Top view of the Milky Way Galaxy, with the known spiral pattern (Vallée 2008) and young star clusters more massive than $\gtrsim 10^4 M_{\odot}$ identified. Basic cluster parameters are listed in Tab. 2. The location of the Sun is indicated by \odot , and its orbit by the dotted circle. Dashed lines indicate circles of 1 kpc and 2 kpc around the sun.

These estimates are sensitive to the underlying assumptions made about the star-formation history of the Galaxy and the duration of the embedded phase, as well as to observational selection effects. For example, the open cluster sample used by Battinelli & Capuzzo-Dolcetta (1991) is based on a luminosity limited sample of 100 clusters from the Lynga (1982) catalog, which claims to be complete to a distance of 2 kpc, but the mass of open clusters in this catalog between 600 pc and 2 kpc averages several $10^3 M_{\odot}$. The much higher mean mass of open clusters at large distance indicates that care has to be taken in using these catalogs, as there appear to be selection effects with respect to distance. Another problem arises from confining the analysis to a distance of 600 pc around the sun, since the cluster sample does not include any nearby spiral arms, where many young clusters form; Lada & Lada (2003) considered a sample of clusters within 2 kpc of the sun, which therefore includes many objects in the Perseus and Sagittarius arms (Fig. 3). These differences complicate direct comparison of cluster samples.

1.3 Terminology

The study of star clusters has traditionally been plagued by conflicting terminology used by theorists and observers. In this section we attempt to clarify some terms, with the goal of making this review readable by both observers and theorists.

1.3.1 Cluster center

Determining the center of a star cluster sounds like a trivial exercise, but in practice it is not easy. As with many other fundamental parameters in astrophysics, the center of a star cluster is not well defined observationally, although theorists have reached some consensus about its definition.

von Hoerner (1963) defined the center of a simulated (N -body) cluster as a density-weighted average of stellar positions:

$$x_{d,j} = \frac{\sum_i x_i \rho_i^{(j)}}{\sum_i \rho_i^{(j)}}. \quad (2)$$

Here $\rho_i^{(j)}$ is the density estimator of order j around star i , and x_i is the (3-dimensional) position vector of star i .

In direct N -body simulations (see §4.1.3), alternatives to Eq. 2 are preferred due to the computational expense of determining the local density $\rho_i^{(j)}$. The center of mass, often used in simple estimates, is generally *not* a good measure of the cluster center, as distant stars tend to dominate. This has led to approximate, but more efficient, estimators, such as the “modified” density center (Portegies Zwart et al. 2001), which iteratively determines the weighted mean of the positions of a specified Lagrangian fraction (typically $\sim 90\%$) of stars, relative to the modified density center. In general, it agrees well with the density center defined above.

Observationally it is considerably harder to define the cluster center, both because of the lack of full 3-dimensional stellar positions, and also because of observational selection effects, including low-luminosity stars, crowding and the broad range in luminosities of individual stars. Both number-averaged and luminosity-averaged estimators are found in the literature. In principle, the 2-dimensional equivalent of Eq. 2 could be used, but observers often prefer the point of maximal symmetry of the observed projected stellar distribution. An example is the technique adopted by McLaughlin et al. (2006) to determine the center of GC 47 Tuc.

1.3.2 Size scales

Massive star clusters tend to be approximately spherically symmetric in space, or at least circular on the sky, so the radius of a cluster is a meaningful measure of its size. Theorists often talk in terms of Lagrangian radii—distances from the center containing specific fractions of the total cluster mass. For observers, a similar definition can be formulated in terms of isophotes containing given fractions of the total luminosity. The **half-mass radius** (r_{hm} ; the 50% Lagrangian radius) is the distance from the cluster center containing half of the total mass. Observationally, the projected half-light radius, or effective radius r_{eff} , is often

used, although the total cluster light, which is obviously required to define the Lagrangian radii, can be hard to determine with confidence.

Arguably more useful cluster scales are the virial radius r_{vir} , the core radius r_c , and the tidal radius r_t . The **virial radius** is defined as

$$r_{\text{vir}} \equiv \frac{GM^2}{2|U|}. \quad (3)$$

Here M is the total cluster mass, U is the total potential energy and G is the gravitational constant. This is clearly a theoretical definition, as neither the total mass nor the potential energy are actually observed. The potential energy may be obtained directly from the stellar masses and positions in an N -body simulation (see §4), or from a potential–density pair by $U = 2\pi \int \rho(r)\phi(r)r^2 dr$.

From an observational point of view, the parameter $\eta \equiv 6r_{\text{vir}}/r_{\text{eff}}$ is generally introduced to determine the dynamical mass of star clusters. In virial equilibrium ($U = -2T$, here T is the total kinetic energy of the cluster stars), $T/M = \frac{1}{2}v_{\text{rms}}^2 = \frac{3}{2}\sigma_{1D}^2$ for an isotropic system. The line-of-sight velocity dispersion σ_{1D} can be directly measured, yielding the cluster mass

$$M_{\text{vir}} = \eta \left(\frac{\sigma_{1D}^2 r_{\text{eff}}}{G} \right). \quad (4)$$

Fig. 4 presents the dependence of η on the parameters of some typical density profiles: the concentration $c \equiv \log(r_t/r_c)$ of a King (1966) model or γ in an Elson, Fall & Freeman (1987, hereafter EFF87) surface brightness profile,

$$\Sigma(r) = \Sigma_0 \left(1 + \frac{r^2}{a^2} \right)^{-\gamma/2}, \quad (5)$$

where a is a scale parameter and the 3-dimensional density profile has a logarithmic slope of $-\gamma_{3D} = -(\gamma + 1)$ for $r \gg a$, r_c and r_t are the core radius and tidal radius of the cluster, respectively (see below).

A Plummer (1911) density profile has $\gamma = 4$, $r_{\text{vir}}/r_{\text{eff}} = 16/3\pi$, and therefore $\eta \simeq 10$. The value $\eta = 9.75$, corresponding to $r_{\text{vir}} = 1.625r_{\text{eff}}$, is a reasonable and widely used choice for clusters with relatively shallow density profile ($\gamma \gtrsim 4$ or $W_0 \lesssim 8$). For $\gamma \leq 2$ the EFF87 profile has infinite mass, and the ratio $r_{\text{vir}}/r_{\text{eff}}$ drops sharply for $\gamma \lesssim 2.5$. The choice for $\eta = 9.75$ should be made cautiously, since many young clusters tend to have relatively shallow density profiles with $2 \lesssim \gamma \lesssim 3$, for which $\eta \lesssim 9$ (see Fig. 4 and also §2.3). In addition, mass segregation can have a severe effect on η , resulting in a variation of more than about a factor of 3 (Fleck et al. 2005).

Observers generally define the cluster **core radius**, r_c , as the distance from the cluster center at which the surface brightness drops by a factor of two from the central value. Unfortunately, theorists use at least two distinct definitions of r_c , depending on context. When the central density ρ_c and velocity dispersion $\langle v^2 \rangle$ are easily and stably defined, as is often the case for analytic, Fokker–Planck, and Monte-Carlo models (see §4.1), the definition

$$r_c = \sqrt{\frac{3\langle v^2 \rangle}{4\pi G\rho_c}} \quad (6)$$

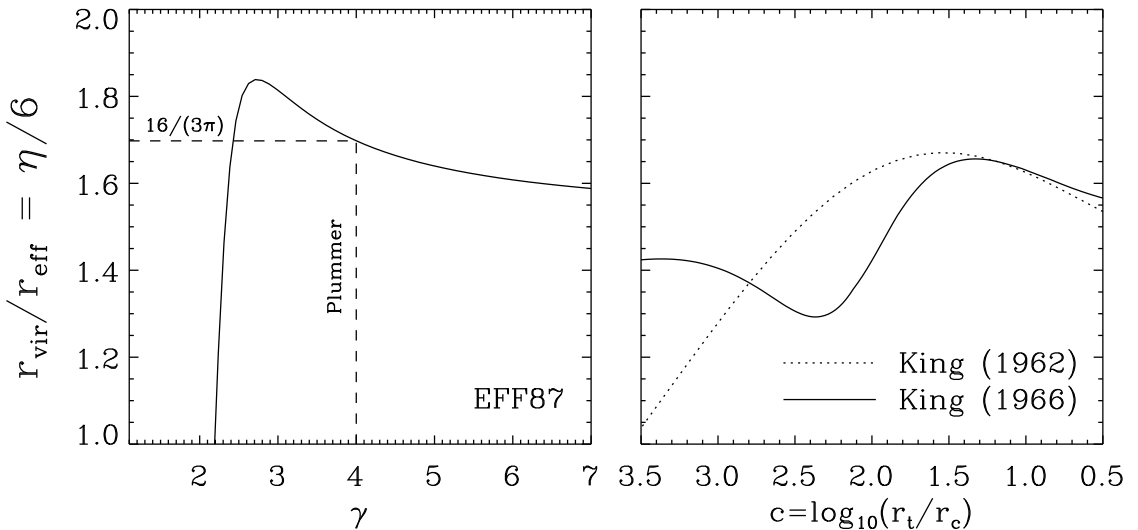


Figure 4: The ratio $r_{\text{vir}}/r_{\text{eff}}$ and the parameter η used to convert an observed 1-D velocity dispersion and half-light radius into a dynamical mass (Eq. 4) for Eq. 5 (left) and King (1962) and King (1966) models (right). The dashed line in the left panel indicates the analytical result for a Plummer (1911) model ($\gamma = 4$ in Eq. 5).

(King 1966) is often adopted. For typical cluster models this corresponds roughly to the radius at which the three-dimensional stellar density drops by a factor of 3, and the surface density by ~ 2 .

In N -body simulations, however, both ρ_c and $\langle v^2 \rangle$ are difficult to determine, as they are subject to substantial stochastic fluctuations. As a result, a density-weighted core radius is used instead. Specifically, for each star a local density ρ_i is defined using the star’s k nearest neighbors (Casertano & Hut 1985), where $k = 12$ is a common choice. A density center is then determined, either simply the location of the star having the greatest neighbor density, or as a mean stellar position, as in Eq. 2, except that the density estimator is ρ_i^2 . [The square is used rather than the first power, as originally suggested by Casertano & Hut (1985), to stabilize the algorithm and make it less sensitive to outliers.] The core radius then is the ρ_i^2 -weighted rms stellar distance from the density center:

$$r_c = \sqrt{\frac{\sum_i \rho_i^2 r_i^2}{\sum_i \rho_i^2}}. \quad (7)$$

Despite their rather different definitions, in practice the two “theoretical” core radii (Eqs. 6 and 7) behave quite comparably in simulations.

For simple models, the values of r_c and r_{vir} determine the density profile, which is generally assumed to be spherically symmetric. This is the case for the empirical King (1962) profiles and dynamical King (1966) models, both of which fit the observed surface brightness distribution of many Milky Way GCs. The dynamical King models are parameterized by a quantity W_0 representing the dimensionless depth of the cluster potential well.

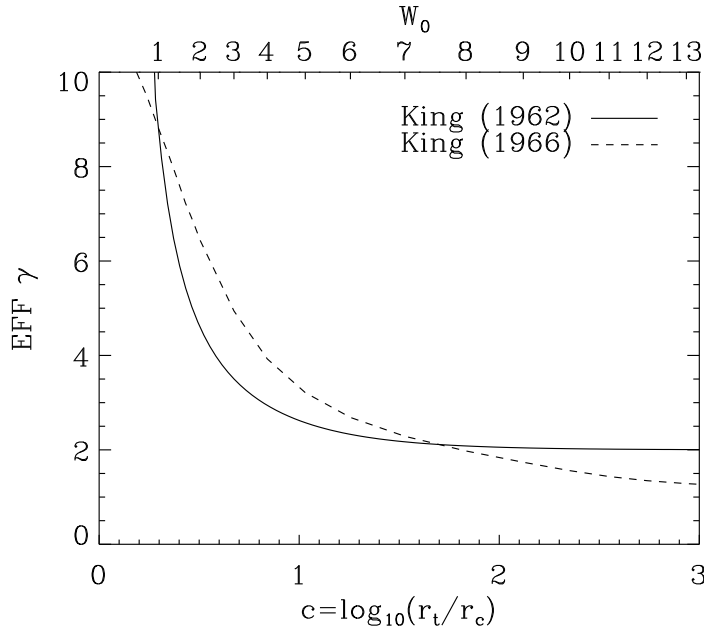


Figure 5: EFF87 model fits (with power-law index γ) to King surface brightness profiles (characterized by King concentration parameter $c = \log_{10} r_t/r_c$). The fits of Eq. 5 to the King profiles/models is done within the cluster half-mass radius.

Centrally concentrated clusters have $W_0 \gtrsim 8$, whereas shallow models have $W_0 \lesssim 4$. The empirical and dynamical King profiles are in good agreement for $W_0 \lesssim 7$.

Galactic GCs are well fit by King models, but the observed surface brightness profiles of young clusters in, for example, the LMC are not. They are much better represented by EFF87 profiles (Eq. 5), which have cores (different from the King-model cores) and power-law halos. For a King model with concentration $c \gtrsim 1$ ($W_0 \gtrsim 5$), the surface brightness drops to approximately half of its central value at $r = r_c$, as defined by Eq. 6, so the observed core radius is a good measure of the core radius of the underlying three-dimensional stellar density distribution. The core radius is often by observers adopted EFF87 surface brightness profile is

$$r_c = a (2^{2/\gamma} - 1)^{1/2}. \quad (8)$$

Here a is the scale parameter in the density profile. Thus, when an EFF87 surface brightness profile is fit to an observed cluster, Eq. 8 can be used to determine with good confidence the 3-dimensional core radius r_c of Eq. 6.

In Fig. 5 we compare the EFF87 profiles with the empirical King profiles and (projected) King models, by fitting Eq. 5 to each within the inner half-mass radius. For $c \rightarrow \infty$ the King (1962) surface brightness profile tends to a power-law with index -2 , which has (logarithmically) infinite mass. The King (1966) model in that case becomes an isothermal sphere ($\rho \propto r^{-2}$), also with (linearly) infinite mass, corresponding to $\gamma = 1$ in Eq. 5.

The **tidal radius** is the distance from the center of a star cluster where the gravitational pull from the cluster equals that of the parent Galaxy (von Hoerner 1957). It is a

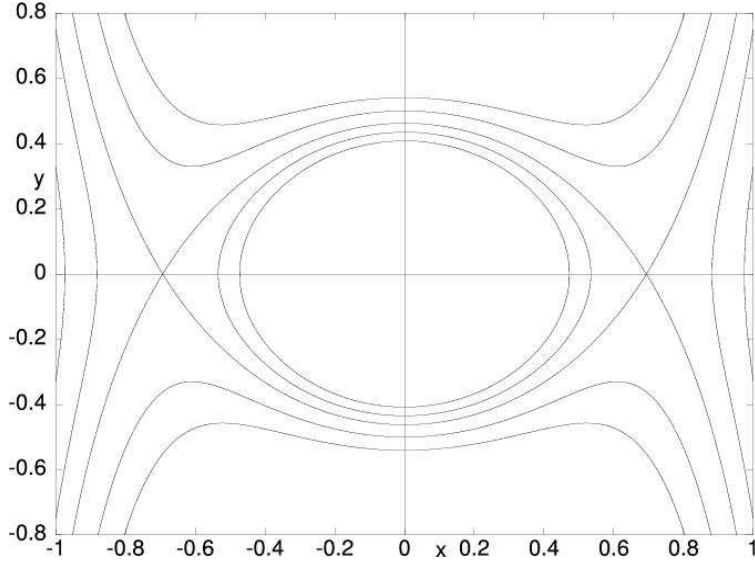


Figure 6: Equipotential surface for a star cluster in a galactic tidal field. The galactic center is to the right. The Jacobi surface is presented by the two crossing lines, to the right of which is the L_1 Lagrangian point and to the left is L_2 . (Image taken with courtesy from Fig. 3 of Heggie 2001).

1-dimensional concept which, when calculated in all directions results in the **Jacobi surface** (see Fig. 6), which is often expressed in a radius r_J from the cluster center. The Jacobi radius r_J is defined as the radius of a sphere with a volume identical to the enclosed volume of the Jacobi surface. For clarity and simplicity, the latter is often related to the **Roche radius**, as is used in relation to interacting star on a circular orbit. In practice all these radii indicate at what distance from the cluster center a star finds itself unbound from the collective (Read et al. 2006). Stars, however, do not usually escape from a cluster in a random direction, but along the Lagrangian radii nearest the star cluster, L_1 (in the direction towards the center of the host galaxy) and L_2 (away from the host Heggie 2001).

1.3.3 Time scales

The dynamics of any self-gravitating system is defined by two fundamental time scales, the dynamical timescale t_{dyn} , and the relaxation timescale t_{rl} .

The dynamical time scale is the time required for a typical star to cross the system; it is also the time scale on which the system (re)establishes dynamical equilibrium. A convenient formal definition in terms of conserved quantities is

$$t_{\text{dyn}} = \frac{GM^{5/2}}{(-4E)^{3/2}}, \quad (9)$$

where $E \equiv T + U$ is the total energy of the cluster. In virial equilibrium, $2T + U = 0$ and this expression assumes the more familiar form (Spitzer 1987)

$$t_{\text{dyn}} = \left(\frac{GM}{r_{\text{vir}}^3} \right)^{-1/2} \quad (10)$$

$$\sim 2 \times 10^4 \text{ yr} \left(\frac{M}{10^6 M_\odot} \right)^{-1/2} \left(\frac{r_{\text{vir}}}{1 \text{ pc}} \right)^{3/2}, \quad (11)$$

The relaxation time, t_{rl} , is typically much longer than t_{dyn} . It is the time scale on which two-body encounters transfer energy between individual stars and cause the system to establish thermal equilibrium. The local relaxation time is (Spitzer 1987):

$$t_{\text{rl}} = \frac{\langle v^2 \rangle^{3/2}}{15.4 G^2 m \rho \ln \Lambda}, \quad (12)$$

where m is the local mean stellar mass and ρ is the local density. The value of the parameter Λ ranges from $\Lambda \sim 0.11N$ for systems without a stellar mass function (Giersz & Heggie 1994) to $\Lambda = 0.4N$ for the theoretical case where all stars have the same mass and are distributed homogeneously with an isotropic velocity distribution (Spitzer 1987).

For a cluster in virial equilibrium we can replace all quantities by their cluster-wide averages, writing $\langle v^2 \rangle = GM/2r_{\text{vir}}$ and $\bar{\rho} \approx 3M/8\pi r_{\text{vir}}^3$, where we ignore the distinction between virial and half-mass quantities, so $r_{\text{hm}} \approx r_{\text{vir}}$. We thus obtain the “half-mass” two-body relaxation time (Spitzer 1987):

$$t_{\text{rh}} \simeq \frac{0.065 \langle v^2 \rangle^{3/2}}{G^2 \langle m \rangle \bar{\rho} \ln \Lambda} \quad (13)$$

$$= 0.14 \frac{N^{1/2} r_{\text{vir}}^{3/2}}{G^{1/2} \langle m \rangle^{1/2} \ln \Lambda} \quad (14)$$

$$\approx \frac{N}{7 \ln \Lambda} t_{\text{dyn}}, \quad (15)$$

where $\langle m \rangle \equiv N/M$ is the global mean stellar mass. If, for simplicity, we adopt $\ln \Lambda = 10$ as appropriate for the range in cluster masses of interest in this review, Eq. 13 becomes

$$t_{\text{rh}} \sim 2 \times 10^8 \text{ yr} \left(\frac{M}{10^6 M_\odot} \right)^{1/2} \left(\frac{r_{\text{vir}}}{1 \text{ pc}} \right)^{3/2} \left(\frac{\langle m \rangle}{M_\odot} \right)^{-1}. \quad (16)$$

Finally, we note that in real stellar systems the one-parameter simplicity of Eq. 15 is broken by the introduction of a third time scale independent of the dynamical properties of the cluster—the stellar evolution time scale $t_S \sim 10 \text{ Myr}$ for YMCs. This simple fact underlies almost all of the material presented in this review.

2 The properties of star clusters

The formation of clusters with masses comparable to present-day globular clusters is not restricted to the early universe. Even before the Hubble Space Telescope (HST) era, ground-based observations revealed numerous “bluish knots” and “super-star clusters” in M82 (van den Bergh 1971) and the relatively local ongoing galaxy merger NGC 7252 (Schweizer 1982) and in the starburst dwarf galaxy NGC 1569 (Arp & Sandage 1985). These and later well studied examples are promising candidates for the latest generation of “young globular

clusters” (Whitmore 2003, Larsen 2006). As discussed in §1 it is unclear whether, or to what extent, these young blue objects are actually related to the old globular clusters observed in the Milky Way Galaxy and many others. We review here some observational characteristics and attempt to assess the similarities and differences between the young and old populations. In § 2.1 we briefly visit some key results that were acquired from studies of young populations of clusters and in § 2.3 we present an overview of the properties of individual clusters from literature. Since there are too many clusters known that follow our definition of YMCs, we restrict ourselves to a sample of well studied clusters, for which at least the age, mass and radius have been determined. For more in depth reviews on observations of populations of YMCs we refer to Whitmore (2003) and Larsen (2006).

2.1 General characteristics

Traditionally, astronomers have drawn a clear distinction between the relatively young Milky Way open clusters associated with the Galactic disk, and the old globular clusters that reside mostly in the halo (see Fig. 1 and Tab. 1). This division does not hold outside the Milky Way. This is particularly evident in the Magellanic clouds, which host a wealth of YMCs that have received considerable attention since the 1960s (Hodge 1961). Their ages are comparable to many Milky Way open clusters (up to a few hundred megayears), but their masses and core densities exceed those of open clusters in the Milky Way, in some cases by several orders of magnitude (e.g. Elson & Fall 1985, see Tab. 1).

The discussion regarding the difference between open and globular clusters goes back a long while, and the complications regarding the identification of YMCs is well illustrated by R 136, a the central stellar conglomerate in the Tarantula nebula of the LMC. R 136 (see Fig. 7 and Tab. 3) was initially thought to be a single stellar object at least 2000 times more massive and about 10^8 times brighter than the Sun. In 1985, using speckle interferometry, Weigelt & Baier unambiguously resolved it into a group of stars. R 136 is now known to contain $\sim 10^5$ young stars, rather than one single extraordinary object (Massey & Hunter 1998, Andersen et al. 2009). It had already been noted that massive young clusters, such as R136, could be responsible for the giant HII regions found in other galaxies (Kennicutt & Chu 1988). The HST has later confirmed many extra-galactic YMCs, starting with the discovery in the interacting galaxy NGC 1275 (Holtzman et al. 1992).

With HST it is now possible to distinguish star clusters at distances of several tens of megaparsecs. One example is the cluster A1 in NGC 5194 (M51), shown in the right-hand panel of Figure 7. It may be viewed as a twin of R136, shown on the left. The two clusters have comparable ages of only a few megayears, masses of $\sim 10^5 M_{\odot}$, and very compact structures, with core radii of less than one parsec (Bastian et al. 2008). Since such clusters are small and their distances generally large, they are hard to find; the large number of objects already known suggests that there must be many clusters similar to R 136 waiting to be found.

YMCs are not only found in galaxies with violent star formation histories, but also in quiescent spirals (e.g. Larsen & Richtler 2000, Larsen 2004), and there are many similarities between the young cluster populations in these different environments. For example, the luminosity function (LF), defined as the number of clusters per unit luminosity (dN/dL) is well described by a power-law with index close to -2 (e.g. Whitmore & Schweizer 1995,

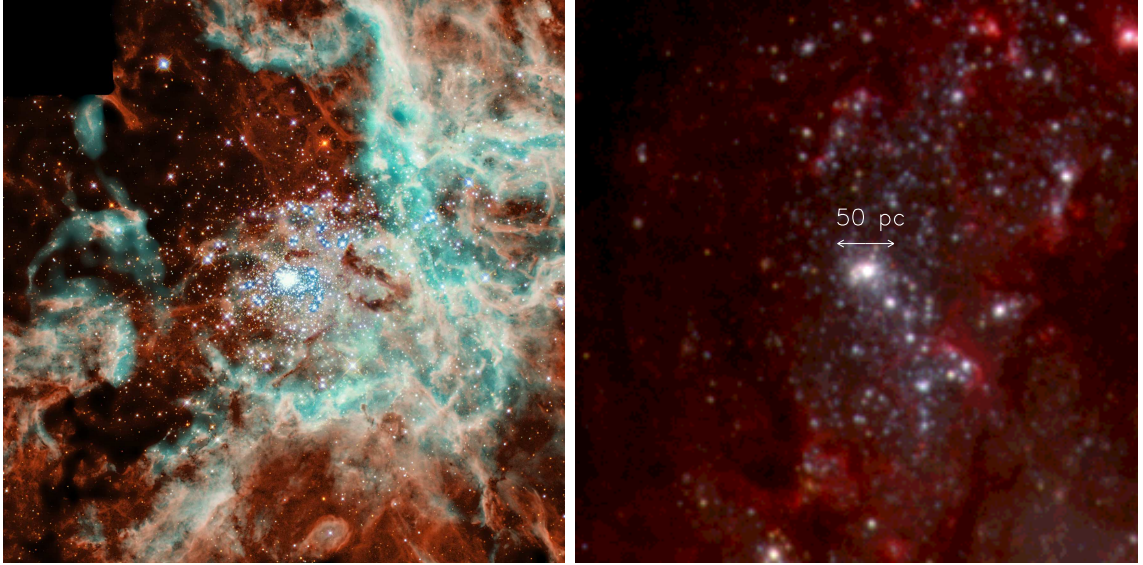


Figure 7: Images of two young (few megayears) massive ($\sim 10^5 M_\odot$) clusters. Left: A region of $50 \times 50 \text{ pc}^2$ around the cluster R136 in the 30 Doradus region of the LMC, at a distance of $\sim 50 \text{ kpc}$. Right: The star cluster A1 in the interacting galaxy M51 at a distance of 8.4 Mpc , with the scale of the R136 image indicated. Credit: NASA, ESA, and The Hubble Heritage Team (STScI/AURA).

Miller et al. 1997, Larsen 2002, de Grijs et al. 2003), and slightly steeper for the bright LFs (Whitmore et al. 1999, Larsen 2002, Gieles et al. 2006a).

A similar result is found from the relationship between the luminosity of the brightest cluster in a galaxy (L_{max}) and star formation rate (SFR) of the host galaxy, or the total number of clusters above a fixed detection limit in that galaxy (N_{cl}). Whitmore (2003) showed that L_{max} scales almost linearly with N_{cl} ; roughly as $L_{\text{max}} \propto N_{\text{cl}}^{0.75}$ (see also Larsen 2002, Weidner, Kroupa & Larsen 2004, Gieles et al. 2006a). This suggests that L_{max} is the result of the size of the sample, and therefore determined by statistics, rather than physics. That is, there is a larger probability to find bright clusters in galaxies with a large number of clusters, or a high a SFR. The relation between L_{max} and N_{cl} corresponds to a power-law index of -2.4 , supporting the finding that the bright end of the LF is steeper than -2 (since the L_{max} method traces the brightest clusters).

2.2 The cluster initial mass function

It is tempting to interpret the LF as the underlying cluster mass function. It is non-trivial, however, to make this translation, since it consists of clusters with different ages, and because clusters fade quickly during their first $\sim 1 \text{ Gyr}$ due to stellar evolution. Larsen (2009) showed that the L_{max} clusters have a large range of ages, with the brightest ones on average being younger than the fainter ones. This implies a dependence of M/L on L_{max} , in a way that the mass of the most *massive* cluster (M_{max}) increase much slower with N_{cl} and the SFR.

Determinations of cluster initial mass functions (CIMFs) are rare, since it is hard to

acquire the ages of the clusters, which are needed to select the youngest and to convert luminosity into mass. And even then, converting the present day mass to an initial mass requires detailed knowledge of the cluster's history. Several determinations of CIMFs have also found power-law functions with indices close to -2 (Zhang & Fall 1999, McCrady & Graham 2007, Bik et al. 2003), and other studies have found evidence for a truncation of this power-law at the high mass end (Gieles et al. 2006b, Bastian 2008, Larsen 2009). The functional form of the initial mass function for young star clusters is well represented by a Schechter (1976) distribution

$$\phi(M) \equiv \frac{dN}{dM} = A M^{-\beta} \exp(-M/M_*). \quad (17)$$

Here $\beta \simeq 2$ and the Schechter mass M_* is equivalent to the more familiar L_* for the luminosity function of galaxies. For Milky-Way type spiral galaxies $M_* \approx 2 \times 10^5 M_\odot$ (Gieles et al. 2006b, Larsen 2009). For interacting galaxies and luminous infrared galaxies Bastian (2008) obtains $M_* \gtrsim 10^6 M_\odot$. In the top panel of Fig. 8 we compare cluster mass functions for several galaxies to the Schechter function Eq. 17. The bottom panel compares the corresponding logarithmic slopes of the data with the Schechter function. Clearly the mass function of the Antennae clusters extends to higher masses, and this is not only due to the larger number of clusters, since the slope is also flatter at large masses. Thus the value of M_* depends on the local galactic environment; it is possible to form more massive clusters in starburst galaxies than in quiescent environments.

Indirect indications for the necessity of an upper mass also follow from statistical arguments. If we temporarily ignore the (exponential) truncation, i.e. we adopt a power law with $\beta = 2$ without the exponential part in Eq. 17, we can relate the cluster formation rate to the masses of the most massive clusters observed. Based on an overall star formation rate of $5000 M_\odot \text{ Myr}^{-1} \text{ kpc}^{-2}$ in the solar neighbourhood (Miller & Scalzo 1979), comparable to the average values found in external Milky Way type spirals (Kennicutt 1998), and assuming that $\sim 10\%$ of this mass ends up in bound star clusters, then the total mass formed in 10 Myr in clusters within 4 kpc of the Sun is $\sim 2 \times 10^5 M_\odot$. For our assumed power-law mass function, the most massive cluster contains $\sim 10\%$ of the total mass, so the mass of the most massive cluster is a few $\times 10^4 M_\odot$. Within a 4 kpc circle we find Westerlund 1, with a mass of $\sim 6 \times 10^4 M_\odot$ (see Tab. 2), in reasonable agreement with expectations.

Assuming the same star formation rate out to a distance of $R_G \sim 8 \text{ kpc}$, which is a conservative assumption since the star formation rate toward the Galactic center is probably higher, the expected most massive cluster becomes a factor of $8^2/4^2$ higher, or about $10^5 M_\odot$. Over a time span of 1 Gyr, clusters with masses of $\sim 10^7 M_\odot$ should have formed within that same distance, but if such a cluster existed it would most likely have been discovered already, unless it has been disrupted. For a quiescent environment like the Milky Way Galaxy, the maximum mass cluster that can form must be considerably smaller than in a starburst environment. Similar arguments hold for external galaxies, where the entire disk can be seen, and it is found that the high-mass end of the cluster mass function falls off more steeply than a power-law with exponent -2 (see Fig. 8).

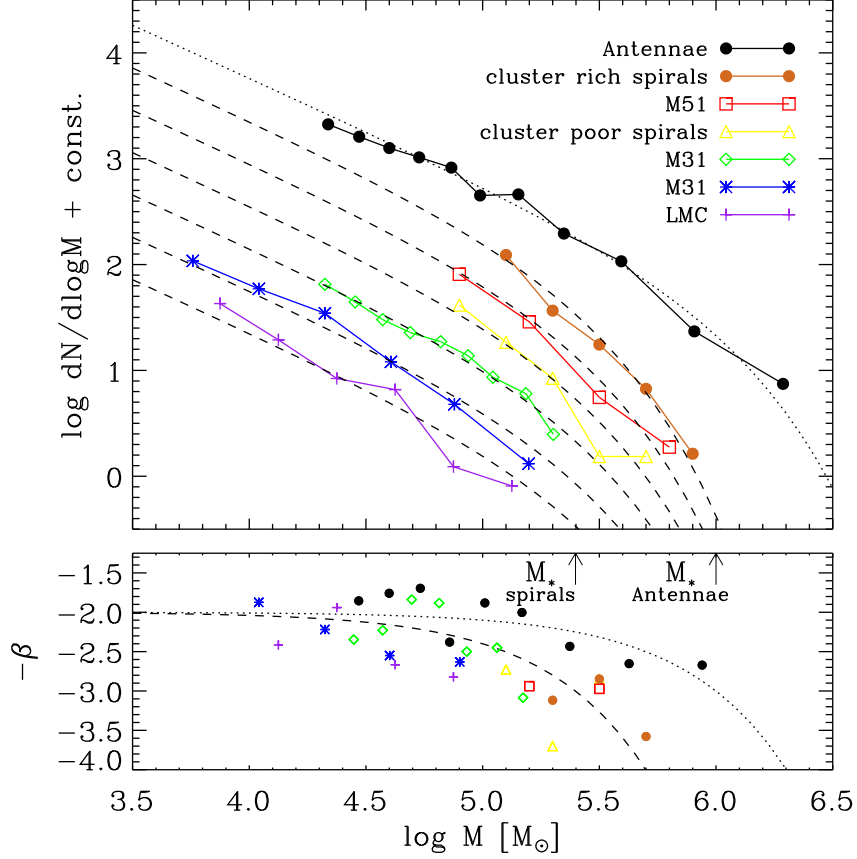


Figure 8: Top: Comparison of mass functions of clusters younger than ~ 1 Gyr in different galaxies. The results are taken from Larsen (2009) (LMC, cluster rich spirals and cluster poor spirals); Gieles (2009) (M51); Zhang & Fall (1999) (the Antennae galaxies) and Vansevičius et al. (2009) (two versions of the M31 cluster mass function). The cluster mass functions in spirals are compared to a Schechter function (Eq. 17) with $M_* = 2.5 \times 10^5 M_\odot$; for the Antennae, $M_* = 10^6 M_\odot$ is used. Bottom: the corresponding logarithmic slopes of the mass functions.

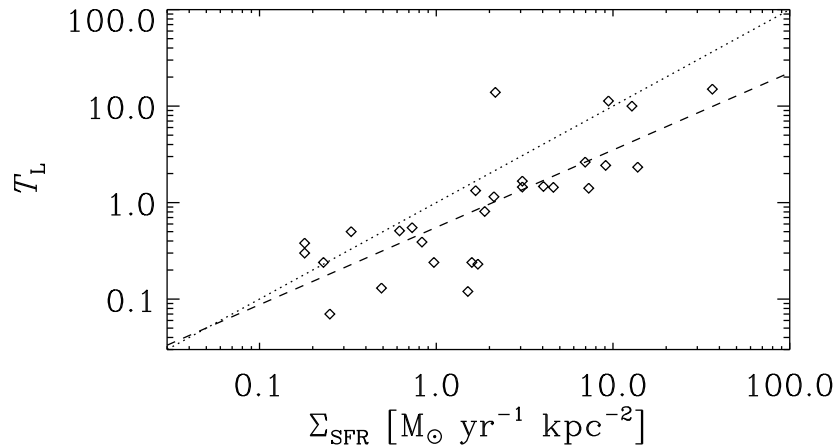


Figure 9: Specific U -band luminosity (T_L) for various cluster populations, as a function of Σ_{SFR} of the parent galaxy. The dashed line shows a power-law fit ($T_L \propto \Sigma_{\text{SFR}}^{0.8}$) and the dotted line is a linear relation. Data from Larsen & Richtler (2000).

2.2.1 The cluster formation efficiency

The number of globular clusters in a galaxy is often expressed in terms of the specific frequency, the number of globular clusters per unit luminosity of the host. For young clusters, this is probably not a very meaningful quantity, since these clusters form with a power-law mass function (or Schechter function, as in Eq. 17), and the luminosity of the host galaxy depends strongly on the age of the field star population. For this reason, Larsen & Richtler (2000) introduce the *specific luminosity*, $T_L = 100 L_{\text{clusters}}/L_{\text{galaxy}}$ for samples of cluster populations in different galaxies. They show that, in the U -band, T_L increases strongly with the star formation rate per unit area (Σ_{SFR} , see Figure 9). This suggests that galaxies with high Σ_{SFR} are more favorable for forming star clusters. A compelling property of this figure is that both axes are independent of distance.

2.3 Data on individual young and massive clusters

In this section we summarize observational results on individual YMCs (see also §6.4 for a number of well studied cases). We start with the Milky Way Galaxy star clusters (§2.3.1), followed by clusters within the Local Group (§2.3.2), and finally those beyond the Local Group (§2.3.3). We summarize the literature of clusters for which we found at least the cluster age, (photometric) mass and half-light (or effective) radius. For clusters of which more structural parameters were known, such as the density profile, we determine the virial radius using the relation presented in Fig. 4. For clusters of which only an estimate of r_{eff} was available we used the ratio $r_{\text{vir}}/r_{\text{eff}} = 1.7$ to determine r_{vir} . From r_{vir} and Eq. 11 we subsequently determine t_{dyn} . We clearly assumed here that the observed clusters are in virial equilibrium, which results in an overestimate for t_{dyn} for unbound (expanding) clusters.

2.3.1 In the Milky Way

According to our definition in §1, the Milky Way hosts several YMCs. Probably the best known are the Arches and Quintuplet clusters near the Galactic center (Figer et al. 2002, Figer, McLean & Morris 1999). The Arches cluster, with an age of only 1 or 2 Myr and a central density of $\sim 10^5 M_{\odot} \text{pc}^{-3}$, has been the topic of much debate. Because of its extreme stellar density and the galactic environment in which it resides, it may provide important clues about the universality of the stellar IMF. Some studies have claimed that the IMF is truncated below a few M_{\odot} (Stolte et al. 2005, Kim et al. 2006), and shows evidence for mass segregation (Stolte et al. 2002). Espinoza, Selman & Melnick (2009) suggest that the apparent mass segregation could simply be an observational bias, and that a Salpeter MF in the core cannot be ruled out (see also Harfst, Portegies Zwart & Stolte 2009, who reconstruct the cluster’s initial conditions by iterative N -body simulations.). The differential extinction across the cluster and the total visual extinction of $A_V \approx 30$ magnitudes make this a challenging cluster to study.

Currently the most massive young cluster known in our Galaxy is Westerlund 1 (Clark et al. 2005). Due to its relative proximity (~ 4 kpc), its lower extinction (although still a considerable $A_V \approx 10$ magnitudes), and slightly lower intrinsic stellar density, it represents a somewhat easier target than the Arches and Quintuplet for studies of the mass function. The IMF and structural parameters of Westerlund 1 and the somewhat less massive NGC 3603 have been investigated by several space-based instruments, and from the ground using adaptive optics. As in the case of the Arches cluster, as observations have improved the measured mass function in Westerlund 1 also appears to be consistent with a Salpeter distribution (e.g. Brandner et al. 2008, Harayama, Eisenhauer & Martins 2008)^d.

Data from individual clusters, supplemented by the recent overview by Pfalzner (2009), are presented in Table 2. As illustrated in Figure 2 of Pfalzner (2009), young star clusters appear to show two evolutionary sequences: a compact configuration starting from a radius of ~ 0.5 pc, to which we refer as dense clusters, and a second sequence with radii ~ 5 pc, which we call associations. A more quantitative distinction may be found in the ratio $\text{Age}/t_{\text{dyn}}$. Clusters older than their dynamical time scale ($\text{Age}/t_{\text{dyn}} > 1$) are associations, while the dense clusters (almost) all have $\text{Age}/t_{\text{dyn}} < 1$. More physical interpretations of these sequences are discussed in §5.

2.3.2 In the Local Group

An extensive overview of structural parameters, with age estimates, of clusters in the Magellanic clouds, was presented by Mackey & Gilmore (2003b,a). McLaughlin & van der Marel (2005) subsequently extended and improved this catalog. In Tab. 3 we summarize the properties of the 12 known YMCs in the Magellanic clouds. Comparison with Tab. 2 reveals the striking absence of Milky Way YMCs with ages between 10 and 100 Myr, whereas in the SMC and LMC, all YMCs except R136 have ages in this range. This may well be an observational effect: due to extinction in the Milky Way, it is hard to discover clusters without

^dWe here use the term “Salpeter” as a possibly hypothetical universal mass function, since from the observations of individual clusters it is not possible to make the distinction between the palette of flavors of mass functions adopted by theorists.

(1) Name	(2) Ref	(3) Age [Myr]	(4) $\log M_{\text{phot}}$	(5) $\log M_{\text{dyn}}$	(6) r_{c} [pc]	(7) r_{eff} [pc]	(8) γ	(9) r_{vir} [pc]	(10) t_{dyn} [Myr]	(11) Age/ t_{dyn}
Arches	1,2,3	2.00	4.30	—	0.20	0.40	—	0.67	0.06	34.81
CYgOB	4	2.50	4.40	—	—	5.20	—	8.67	2.40	1.04
DSB2003	4	3.50	3.80	—	—	1.20	—	2.00	0.53	6.59
NGC 3603	5	2.00	4.10	—	0.15	0.70	2.00	1.17	0.17	11.94
NGC 6231	6	2.00	4.00	—	—	3.00	—	5.00	1.67	1.20
NGC 6611	4	3.00	4.40	—	—	5.90	—	9.83	2.90	1.03
Quintuplet	7	4.00	4.00	—	1.00	2.00	—	3.33	0.91	4.41
RSGC 01	7	9.00	4.50	—	—	1.30	—	2.17	0.27	33.66
RSGC 02	8	17.00	4.60	—	—	2.70	—	4.50	0.71	23.83
RSGC 03	9	18.00	4.50	—	—	5.00	—	8.33	2.02	8.92
Trumpler 14	10,4	2.00	4.00	—	0.14	0.50	2.00	0.83	0.11	17.63
Wd 1	11,12	3.50	4.50	4.80	0.40	1.00	4.00	1.70	0.19	18.76
Wd 2	4	2.00	4.00	—	—	0.80	—	1.33	0.23	8.71
hPer	4	12.80	4.20	—	—	2.10	—	3.50	0.78	16.51
χ Per	4	12.80	4.10	—	—	2.50	—	4.17	1.13	11.33
IC 1805	4	2.00	4.20	—	—	12.50	—	20.83	11.26	0.18
I Lac 1	4	14.00	3.40	—	—	20.70	—	34.50	60.28	0.23
Lower Cen-Crux	4	11.50	3.30	—	—	15.00	—	25.00	41.72	0.28
NGC 2244	4	2.00	3.90	—	—	5.60	—	9.33	4.77	0.42
NGC 7380	4	2.00	3.80	—	—	6.50	—	10.83	6.69	0.30
ONC	13	1.00	3.65	—	0.20	2.00	2.00	3.33	1.36	0.74
Ori Ia	4	11.40	3.70	—	—	16.60	—	27.67	30.65	0.37
Ori Ib	4	1.70	3.60	—	—	6.30	—	10.50	8.04	0.21
Ori Ic	4	4.60	3.80	—	—	12.50	—	20.83	17.85	0.26
Upper Cen-Crux	4	14.50	3.60	—	—	22.10	—	36.83	52.82	0.27
U Sco	4	5.50	3.50	—	—	14.20	—	23.67	30.52	0.18

Table 2: Properties of YMCs (top) and associations (bottom) in the Milky Way, with the distinction based on age/ t_{dyn} . 1: Figer, McLean & Morris (1999); 2: Figer et al. (2002); 3: Stolte et al. (2002); 4: Pfalzner (2009); 5: Harayama, Eisenhauer & Martins (2008); 6: Sana et al. (2006); 7: Figer et al. (2006); 8: Davies et al. (2007); 9: Clark et al. (2009); 10: Ascenso et al. (2007); 11: Mengel & Tacconi-Garman (2007); 12: Brandner et al. (2008); 13: Hillenbrand & Hartmann (1998);

nebular emission.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Gal	Name	Ref	Age [Myr]	M_V [mag]	$\log M_{\text{phot}}$	$\log M_{\text{dyn}}$	r_c [pc]	r_{eff} [pc]	γ	r_{vir} [pc]	t_{dyn} [Myr]	Age/ t_{dyn}
LMC	NGC 1711	1,2	50.1	-9.05	4.33	-	2.13	4.18	2.78	7.68	2.17	23.10
LMC	NGC 1818	1,2	25.1	-9.62	4.42	-	2.45	4.73	2.76	8.69	2.35	10.67
LMC	NGC 1847	1,2	26.3	-10.95	4.94	-	2.05	4.08	2.05	1.41	0.08	312.52
LMC	NGC 1850	1,2	31.6	-12.65	5.72	6.00	2.55	5.67	2.18	5.27	0.25	127.06
LMC	NGC 2004	1,2	20.0	-9.70	4.41	-	1.57	3.75	2.53	6.69	1.61	12.39
LMC	NGC 2100	1,2	15.8	-10.35	4.59	-	1.22	3.30	2.44	5.61	1.00	15.78
LMC	NGC 2136	1,2	100.0	-8.70	4.34	-	1.99	3.09	3.79	5.32	1.24	80.76
LMC	NGC 2157	1,2	39.8	-9.17	4.34	4.98	2.35	3.85	3.45	6.77	1.78	22.41
LMC	NGC 2164	1,2	50.1	-8.67	4.19	5.16	1.91	3.63	2.96	6.62	2.04	24.59
LMC	NGC 2214	1,2	39.8	-8.67	4.14	5.42	2.14	5.23	2.26	6.62	2.16	18.43
LMC	R136	1,2	3.0	-13.62	5.52	-	0.32	0.69	2.43	1.16	0.03	92.91
M31	Vdb0	3	25.1	-10.03	4.85	-	1.40	7.40	-	12.33	2.43	10.35
M31	B015D	4	70.8	-9.94	4.85	-	0.46	23.44	-	39.07	13.68	5.17
M31	B040	4	79.4	-9.36	4.65	-	0.58	21.88	-	36.47	15.53	5.11
M31	B043	4	79.4	-8.96	4.49	-	0.76	5.13	-	8.55	2.12	37.46
M31	B066	4	70.8	-8.65	4.34	-	0.43	10.00	-	16.67	6.86	10.32
M31	B257D	4	79.4	-9.52	4.71	-	3.20	41.69	-	69.48	38.13	2.08
M31	B318	4	70.8	-8.92	4.45	-	0.81	8.32	-	13.87	4.59	15.44
M31	B327	4	50.1	-9.13	4.45	-	0.20	6.31	-	10.52	3.03	16.55
M31	B448	4	79.4	-9.47	4.70	-	0.59	25.12	-	41.87	18.04	4.40
SMC	NGC 330	5,2	25.1	-10.30	4.70	5.80	2.61	5.13	2.58	9.28	1.88	13.34

Table 3: Same as Table 2, but now for the Local Group. 1: Mackey & Gilmore (2003b); 2: McLaughlin & van der Marel (2005); 3: Perina et al. (2009); 4: Barmby et al. (2009); 5: Mackey & Gilmore (2003a);

2.3.3 Outside the Local Group

Young massive star clusters have been observed and identified well beyond the Local Group, providing exciting new opportunities for studies of star formation and population synthesis. Numerous population studies of extra-galactic star clusters have used broad-band photometry to study the LF, and hence derive age and mass distributions. Here we focus on individual YMCs which have been studied in detail, and for which structural parameters such as radii and density profiles have been determined. In Tab. 4 we present a compilation of the parameters of YMCs in galaxies in the Local Group and beyond.

2.4 Correlations

Based on Tables 2–4, Fig. 10 shows the evolution of core radius r_c and effective (half-light) radius r_{eff} with cluster age. The striking increase of r_c with time was reported by Mackey & Gilmore (2003b) for clusters in the LMC and by Bastian et al. (2008) for massive clusters in M51 and from a compilation of literature data. Brandner et al. (2008) found that r_{eff} increase with age for YMCs in the Milky Way.

There may be a rather strong selection bias toward dense objects in the studies of Mackey & Gilmore (2003b) and Bastian et al. (2008), i.e., there may be young clusters with large r_c

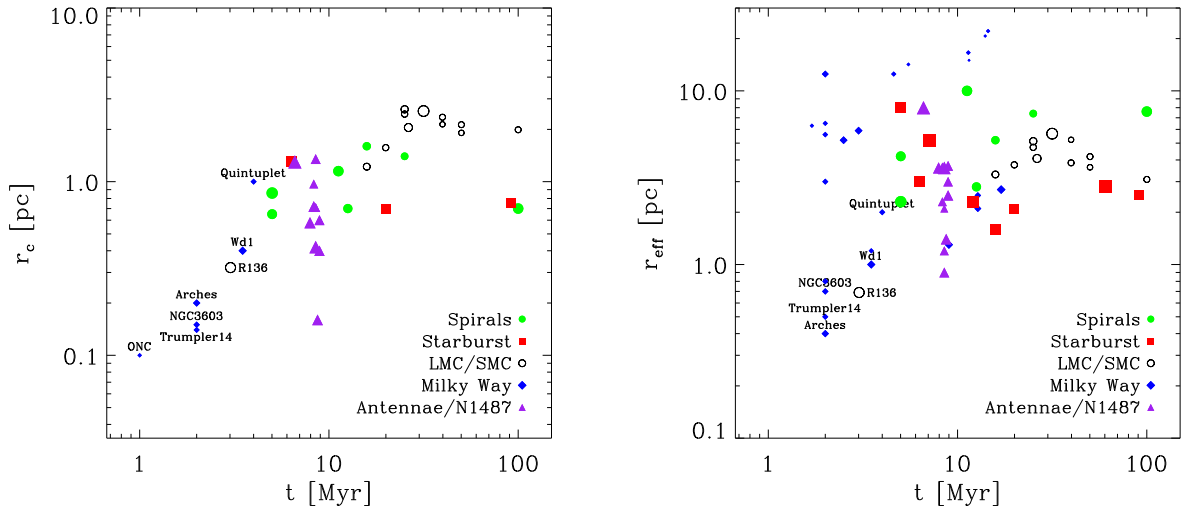


Figure 10: (r_{eff} , right) as a function of time for the clusters in Table 2, 3 & 4.

which simply have not been classified as star clusters. This becomes immediately apparent when we look at r_{eff} of young clusters from the study by Pfalzner (2009, see also Tab. 2). The value of r_c is not determined for these objects, resulting in a gap at the top left of Fig. 10. These low-density associations are probably not identified as genuine star clusters at large distances, and are therefore excluded from the sample.

The lack of dense ($r_c \lesssim 0.5$ pc and $r_{\text{eff}} \lesssim 1$ pc) clusters older than 10 megayear in Fig. 10 is probably real. In that case, the increase of a factor of 5 – 10 in both r_c and r_{eff} has dramatic implications for the evolution of these clusters, since it implies that in a very short time the densities drop by two or three orders of magnitudes. This may be explained in part by black-hole dynamics in the young star cluster (see §3.4.3 for further discussion). However, if very young clusters are mass segregated, the observed r_c (measured from the projected luminosity profile, see §1.3.2) could be considerably smaller than the real r_c (Fleck et al. 2006, Gaburov & Gieles 2008). This topic, and the various physical mechanisms that drive cluster expansion at these ages, are discussed in more detail in §3 and §5.

The three clusters with $r_c \simeq 0.7$ pc and $t \gtrsim 20$ Myr are NGC 1569-B, NGC 1569-30 and NGC 5236-502. NGC 1569 is a starburst dwarf galaxy, and cluster #502 in NGC5236 (M83) lies very close to the galactic nucleus. The YMCs in starburst dwarf galaxies (NGC 1705 and NGC 1569) are small (in terms of r_{eff}) even though they are older than 10 Myr (see Fig. 10, right panel). They are also massive, implying that they are among the densest survivors of the early evolutionary phases of gas removal and stellar evolution (see §3). The stellar populations in the dwarf starburst galaxies NGC 1705 and NGC 1569 are characterized by somewhat low metallicities (approximately half solar), but it is unlikely that this could explain the high densities of these objects. More likely the starburst dwarfs and the centers of galaxies are simply more efficient in forming dense massive clusters, as argued by Billett, Hunter & Elmegreen (2002) on the basis of a comparison of cluster populations in spirals and dwarf starbursts.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Gal	Name	Ref	Age [Myr]	M_V [mag]	$\log M_{\text{phot}}$	$\log M_{\text{dyn}}$	r_c [pc]	r_{eff} [pc]	γ	σ_{1D} [km s $^{-1}$]	r_{vir} [pc]	t_{dyn} [Myr]	Age/ t_{dyn}
ESO338-IG	23	1	7.08	-15.50	6.70	7.10	-	5.20	-	32.50	4.18	0.14	41.67
M51	3cl-a	2	15.85	-11.10	5.04	-	1.60	5.20	2.00	-	7.00	0.51	13.80
M51	3cl-b	2	5.01	-12.25	5.91	-	0.86	2.30	2.60	-	8.67	1.15	35.50
M51	a1	2	5.01	-12.15	5.47	-	0.65	4.20	1.90	-	5.45	0.23	9.86
M82	MGG 9	3,4,5	9.55	-13.42	5.92	6.36	-	2.60	-	15.90	4.33	0.15	64.76
M82	A1	6,5	6.31	-14.85	5.82	5.93	1.30	3.00	3.00	13.40	5.70	0.31	27.02
M82	F	7	60.26	-14.50	6.70	6.08	-	2.80	3.00	-	3.83	0.09	788.09
NGC 1140	1	8	5.01	-14.80	6.04	7.00	-	8.00	-	24.00	5.70	0.30	7.23
NGC 1487	2	9	8.51	-	5.20	5.30	0.71	1.20	-	11.10	5.09	0.39	80.35
NGC 1487	1	9	8.32	-	5.18	6.08	0.97	2.30	-	13.70	2.67	0.07	28.92
NGC 1487	3	9	8.51	-	4.88	5.78	0.71	2.10	-	14.30	5.09	0.08	24.01
NGC 1569	A	10,11,12	12.02	-14.10	6.20	5.52	-	2.30	-	15.70	3.70	0.14	135.27
NGC 1569	C	13	3.02	-	5.16	-	-	2.90	-	-	4.40	0.23	7.25
NGC 1569	B	14	19.95	-12.85	5.74	5.64	0.70	2.10	2.50	9.60	8.67	0.17	139.42
NGC 1569	30	11,12	91.20	-11.15	5.55	-	0.75	2.50	2.50	-	13.33	0.69	394.24
NGC 1705	1	7	15.85	-13.80	5.90	5.68	-	1.60	-	11.40	6.00	0.26	217.57
NGC 2403	II	13	4.47	-	5.35	-	-	11.80	-	-	10.00	0.88	1.63
NGC 4038	S2.1	9	8.91	-	5.47	5.95	0.60	3.70	-	11.50	13.33	0.45	21.21
NGC 4038	W99-1	15	8.13	-14.00	5.86	5.81	-	3.60	-	9.10	2.33	0.08	31.57
NGC 4038	W99-16	15	10.00	-12.70	5.46	6.51	-	6.00	-	15.80	6.00	0.26	11.39
NGC 4038	W99-2	9	6.61	-	6.42	6.48	1.30	8.00	-	14.10	6.00	0.31	14.76
NGC 4038	W99-15	9	8.71	-	5.70	6.00	0.16	1.40	-	20.20	1.50	0.05	116.04
NGC 4038	S1.1	9	7.94	-	5.85	6.00	0.58	3.60	-	12.50	6.00	0.17	30.50
NGC 4038	S1.2	9	8.32	-	5.70	5.90	0.72	3.60	-	11.50	6.17	0.42	26.87
NGC 4038	S1.5	9	8.51	-	5.48	5.60	1.35	0.90	-	12.00	4.17	0.20	170.77
NGC 4038	2000.1	9	8.51	-	6.23	6.38	0.42	3.60	-	20.00	5.00	0.34	50.62
NGC 4038	S2.2	9	8.91	-	5.60	5.60	0.40	2.50	-	9.50	3.83	0.29	44.35
NGC 4038	S2.3	9	8.91	-	5.38	5.40	0.60	3.00	-	7.00	2.00	0.11	26.19
NGC 4214	I-D	13	8.91	-	5.30	-	-	15.30	-	-	3.50	0.35	2.07
NGC 4449	N-1	13	10.96	-	6.57	-	-	16.90	-	-	4.83	0.42	9.48
NGC 4449	N-2	13	3.02	-	5.00	-	-	5.80	-	-	34.33	8.85	2.13
NGC 5236	805	16	12.59	-12.17	5.29	5.62	0.70	2.80	2.60	8.10	43.83	16.83	32.51
NGC 5236	502	16	100.00	-11.57	5.65	5.71	0.70	7.60	2.14	-	32.67	17.16	329.63
NGC 5253	I	13	11.48	-	5.38	-	-	4.00	-	-	19.67	2.75	21.91
NGC 5253	VI	13	10.96	-	4.93	-	-	3.10	-	-	50.00	15.38	18.27
NGC 6946	1447	16	11.22	-13.19	5.64	6.25	1.15	10.00	2.10	8.80	27.50	4.10	36.58
NGC 2403	I-B	13	6.03	-	4.82	-	-	26.30	-	-	55.00	12.13	0.36
NGC 2403	I-C	13	6.03	-	4.42	-	-	19.60	-	-	25.50	4.30	0.35
NGC 2403	I-A	13	6.03	-	5.06	-	-	20.60	-	-	36.17	12.05	0.68
NGC 2403	IV	13	4.47	-	5.07	-	-	30.00	-	-	139.83	33.64	0.29
NGC 4214	VI	13	10.96	-	4.93	-	-	35.90	-	-	59.83	23.65	0.46
NGC 4214	V	13	10.96	-	5.73	-	-	83.90	-	-	67.33	17.82	0.33
NGC 4214	VII	13	10.96	-	5.33	-	-	40.40	-	-	28.17	1.16	0.62
NGC 4214	I-A	13	3.47	-	5.44	-	-	16.50	-	-	9.67	1.42	0.85
NGC 4214	I-B	13	3.47	-	5.40	-	-	33.00	-	-	6.67	0.52	0.29
NGC 4214	II-C	13	2.00	-	4.86	-	-	21.70	-	-	23.00	7.18	0.17
NGC 5253	IV	13	3.47	-	4.72	-	-	13.80	-	-	5.17	0.60	0.48

Table 4: Same as Table 2, but now for objects outside the Local Group. 1: Östlin, Cumming & Bergvall (2007); 2: Bastian et al. (2008); 3: McCrady, Gilbert & Graham (2003); 4: Bastian et al. (2006); 5: McCrady & Graham (2007); 6: Smith et al. (2006); 7: Smith & Gallagher (2001); 8: Moll et al. (2007); 9: Mengel et al. (2008); 10: Ho & Filippenko (1996); 11: Hunter et al. (2000); 12: Anders et al. (2004); 13: Maíz-Apellániz (2001); 14: Larsen et al. (2008); 15: Mengel et al. (2002); 16: Larsen & Richtler (2004);

3 Dynamical processes in star clusters

Studies of the dynamical evolution of a young star cluster split naturally into two phases: (1) the first few megayears, during which stars are still forming and the cluster contains significant amounts of ambient gas, and (2) later times, when the cluster is largely gas-free. As a practical matter, we make a further distinction between early phase 2 and late phase 2, depending on the relative importance of stellar mass loss and internal dynamical evolution on the overall evolution of the cluster. An upper limit on the dividing line between phase 1 and phase 2 is the time of the first supernovae in the cluster, some 3 Myr after formation (Eggleton 2006), which expel any remaining gas not already ejected by winds and radiation from OB stars. The dividing line between early and late phase 2 may be placed anywhere between 100 Myr and 1 Gyr, depending on the initial mass, radius, and density profile of the cluster and the stellar mass function.

The evolution of the cluster during the first phase is a complex mix of gas dynamics, stellar dynamics, stellar evolution, and radiative transfer, and is currently incompletely understood (Elmegreen 2007, Price & Bate 2009). Unfortunately this leaves uncertain many basic (and critical) cluster properties, such as the duration and efficiency of the star-formation process, and hence the cluster survival probability and the stellar mass function at the beginning of phase 2 (see §5).

The second phase is the “ N -body simulation” stage familiar to many theorists. As discussed in more detail below, the processes driving the dynamical evolution here are well known and readily modeled, allowing significant inroads to be made into the task of interpreting cluster observations. However, since the outcome of phase 1 provides the initial conditions for phase 2, the proper starting configuration for these simulations remains largely a matter of conjecture. Theoretical studies generally consist of throughput experiments, mapping a set of assumed initial conditions into the subsequent observable state of the cluster at a later age.

Setting aside the many uncertainties surrounding the early (phase 1) evolution of the cluster, in this section we mainly describe the assumed state of the cluster at the start of phase 2 and the physical processes driving its subsequent evolution. For better or worse, N -body simulations generally assume quite idealized initial conditions (summarized in Tab. 5), with a spherically symmetric, gas-free cluster in virial equilibrium, with all stars already on the zero-age main-sequence.

3.1 Initial conditions

In the absence of a self-consistent understanding of cluster evolution during phase 1, assumptions must be made about the following key cluster properties before a phase 2 calculation can begin (Kroupa 2008). It must be noted that, in almost all cases, the choices are poorly constrained by observations.

- The *stellar mass function* $\phi(m) = dN/dm$ is typically taken to be a “standard” distribution derived from studies of the solar neighborhood (e.g. Miller & Scalo 1979, Salpeter 1955, Kroupa 2001) although it has been suggested that the mass functions of some YMCs may be deficient in low-mass stars and/or “top-heavy” (Smith & Gallagher

Table 5: Commonly adopted initial conditions for particle-based simulations of YMCs.

cluster property	parameter	min	max
number of stars	N	10^3	10^6
mass function equilibrium	single power law: $\phi(m) \propto m^{-2.35}$ $Q = -T/U$	$0.1M_{\odot}$	$100M_{\odot}$
density distribution	Plummer, King		
tidal field	r_t/r_J	0.25	1
concentration	W_0	1	16
Binary fraction	f_b	0	1
mass ratio	$\psi(q) = 1$	0	1
eccentricity	$\Xi(e) = 2e$	0	1
orbital period	$\Gamma(P) \propto 1/P$	RLOF	hard

2001, Stolte et al. 2002), in the sense that the slope $d \log N / d \log m$ of the mass function at the high-mass end is shallower (i.e. less negative) than the standard Salpeter value of -2.35 .

In addition to the mass function, in many cases minimum and maximum stellar masses are imposed. This is necessary for a pure power law, since the total mass would in general otherwise diverge, and often desirable for other distributions, for which convergence is not an issue. Often the minimum mass m_{\min} is chosen on the high side, $m_{\min} \gtrsim 1M_{\odot}$, to emulate a more massive cluster simply by ignoring the low-mass stars. This may suffice if one is interested in clusters younger than 100 Myr (early phase 2), but for older clusters the lower-mass stellar population is important, e.g. to reproduce the proper relaxation time. The maximum mass m_{\max} rarely poses a practical problem, although there may be some interesting correlations between the total cluster mass and the mass of the most massive star (Weidner & Kroupa 2004)

- *Mass Segregation.* Traditionally, dynamical simulations have begun without initial mass segregation—that is, the local stellar mass distribution does not vary systematically with location in the cluster. There is no good reason for this, other than simplicity. Evidence for initial mass segregation can be found in some young clusters (e.g. Hillenbrand & Hartmann 1998, Sabbi et al. 2008), simulations of star formation (e.g. Klessen 2001, Bonnell & Bate 2006), and dynamical evolution during phase 1 (McMillan, Vesperini & Portegies Zwart 2007; Allison et al. 2009). Several prescriptions have been used recently for initial mass segregation (Šubr, Kroupa & Baumgardt 2008; Baumgardt, De Marchi & Kroupa 2008; Vesperini, McMillan & Portegies Zwart 2009). They differ in detail, but lead to similar conclusions, namely that initial mass segregation may be critical to cluster survival (Gaburov & Gieles 2008; Vesperini, McMillan & Portegies Zwart 2009), since mass loss from centrally concentrated massive stars can be much more destructive than the same mass loss distributed throughout the body of the cluster (see §5).
- *Virial Ratio.* Simulations generally begin with a cluster in virial (dynamical) equi-

librium, with virial ratio $Q \equiv -T/U = 1/2$. As with most of the other simplifying assumptions described here, the principal reason for this choice is reduction of the dimensionality of the initial parameter space, but there is no compelling physical reason for it. The gas expulsion that marks the end of phase 1 is expected to leave the cluster significantly out of equilibrium, and quite possibly unbound (e.g. Hills 1980). The time scale for a cluster to return to virial equilibrium may be comparable to the time scale on which mass loss due to stellar evolution subsequently modifies the cluster structure (see §5.1).

- *Spatial Density and Velocity Distributions.* The initial density profiles of young star clusters are poorly constrained. Several standard models are used to model the stellar distribution: Plummer (1911) and truncated Maxwellian (King 1966) profiles are the most common. Other distributions, such as isothermal and homogeneous spheres, are also used (e.g. Scally, Clarke & McCaughrean 2005). King models provide good fits to many observed globulars, although their relation to YMCs is unclear (see §2). In the absence of strong observational constraints, the stellar velocity distribution is normally taken to be non-rotating and isotropic, with (for a Plummer model) the dispersion following the local potential with the assumed virial ratio (see §1.3.2).
- *Tidal Field.* Clusters do not exist in isolation, but rather are influenced by the local tidal field of their parent galaxy. In many cases (see §5.3.1), the field is modeled explicitly as an external potential or its quadrupole moment relative to the cluster center; however simple “stripping radius” prescriptions are also widely used. In practice incorporating a simple stripping radius instead of a self-consistent tidal field reduces the cluster lifetime with about a factor of two, at least if the stripping radius is taken identical to r_J . This effect can be compensated by adopting a larger cut-off radius (for example of $2r_J$). With few exceptions (e.g. Baumgardt & Makino 2003, Giersz & Heggie 2009), the parameters of the tidal field are held fixed in time, corresponding (for spherical or axisymmetric galactic potentials) to an orbit at fixed galactocentric radius. For a given orbit and cluster mass, the initial ratio of the cluster limiting radius (e.g. the “tidal” radius of a King 1966, model, see §1.3.2) to the Jacobi radius of the cluster in the local tidal field is a free parameter, often taken to be of order unity. In these cases the formal edge of the cluster in the King (1966) model (r_t) is then taken identical to the Jacobi radius (r_J). For a Plummer sphere, which extends to infinity, the tidal radius is often set as an artificial cut off at some relatively large distance.
- *Binary Fraction.* Binary stars are critical to cluster evolution during phase 2 (e.g. Hurley, Aarseth & Shara 2007; Portegies Zwart, McMillan & Makino 2007). It is not so clear how important they are during phase 1, when major structural changes are induced by mass loss and mass segregation (Clarke, Bonnell & Hillenbrand 2000; Bate, Bonnell & Bromm 2003). There are few, if any, observational constraints on the overall binary fraction in YMCs. Open clusters in the field typically have high binary fractions, approaching 100% in some cases (e.g. Mason et al. 2009; Bosch, Terlevich & Terlevich 2009). On the other hand, most recent studies of binaries in globular clusters suggest binary fractions of between $\sim 6\%$ and $\sim 15\%$ (Bellazzini et al. 2002, Sollima et al. 2007, Milone et al. 2008, Sommariva et al. 2009).

- *Binary Secondary Masses.* The mass of the secondary star in a binary is typically selected uniformly between some minimum mass and the mass of the primary (more massive) star (Duquennoy & Mayor 1991). With this choice, a binary tends to be more massive than the average cluster star, and this results in additional mass segregation of the binary population. This effect can be removed if desired by randomly selecting primary stars and splitting them into primary and secondary components, in which case adding binaries does not affect the mass function of cluster members (single stars and binaries), but it does introduce a deviation from the initial stellar mass function among binary components (Kroupa 1995).
- *Binary Orbital Elements.* In general, the choices made for binary elements tend to be defensive, given the lack of observational guidance. Apart from the introduction of a whole new set of initial parameters, the presence of primordial binaries also introduces new mass, length, and time scales to the problem, greatly complicating direct comparison between runs having different initial conditions.

The intrinsic initial distributions of binary periods are unknown, and are generally assumed to follow those observed in the solar neighborhood (Duquennoy & Mayor 1991). This appears to be consistent with a Gaussian in $\log P$ with a mean of $\log P = 4.8$ and a dispersion of $\sigma_{\log P} = 2.3$ in days. But flat in $\log P$ is also often adopted (Abt 1983). Observations of YMCs are of little help, as there are at most a handful of binaries with measured orbital parameters in YMCs, and those have very short orbital periods and high-mass components. The eccentricity distribution is usually taken to be thermal (Duquennoy & Mayor 1991). Binary orbital orientation and initial phase are chosen randomly.

- *Higher order multiples.* Primordial multiple stars are rarely included to phase-2 dynamical simulations. There are a few examples of calculations with primordial triples (van den Berk, Portegies Zwart & McMillan 2007) or hierarchical planetary systems (Spurzem et al. 2009). The complications of adding primordial multiples greatly increase the already significant challenges of including binary dynamics and evolution.

3.2 Multiple stellar populations

The discovery of multiple populations of main-sequence stars and giants in an increasing number of globular clusters (Piotto et al. 2005, Piotto 2008) and young (1–3Gyr) clusters in the LMC (Milone et al. 2009) has led to the realization that star clusters are not idealized entities with single well defined stellar populations. In some clusters, the observed stellar populations appear to be separated by less than $\sim 10^8$ years, well within our age range for young clusters. If similar processes are operating today, multiple populations should be expected in at least some observed young star clusters. For the unresolved extragalactic clusters, multiple populations will be hard to confirm, but for clusters in the local group this should be possible. At present, however, only one cluster, Sandage-96, exhibits a young (10–16 Myr) population together with a relatively old (32–100 Myr) population (Vinkó et al. 2009) (see also §6).

The discovery of a younger population indicates that a second epoch of star formation must have taken place early in the cluster’s lifetime (see §6.1.1). The differences in light-element abundances suggest that the second-generation (SG) stars formed out of gas containing matter processed through high-temperature CNO cycle reactions in first-generation (FG) stars.

The two main candidates currently suggested as possible sources of enriched gas for SG formation are rapidly rotating massive stars (Prantzos & Charbonnel 2006, Decressin et al. 2007) and massive (4–9 M_{\odot}) Asymptotic Giant Branch (AGB) stars (Ventura et al. 2001, Karakas & Lattanzio 2007). In the former case, in order to produce the large mass of SG stars suggested by observations (50% or more of the current mass of multiple-population clusters, Carretta et al. 2008), a highly anomalous FG mass function, with an unusually large fraction of massive stars, is required. In the latter scenario, the FG population has a normal IMF but was initially at least ten times more massive than is now observed. Baumgardt & Kroupa (2007) have studied the subsequent evolution and mixing of the two-component cluster in the first scenario. D’Ercole et al. (2008) have presented simulations of the second, in which the SG stars form deep in the potential well of a FG cluster destabilized by early mass loss. Most of the FG cluster dissolves, leaving a mixed FG/SG system after a few gigayears.

It is not known whether, or to what extent, this phenomenon occurs in observed YMCs (but see §6.4 and in particular the cluster Sandage-96). Obviously if it is, it significantly impacts the assumptions made for simulations of phase 2. Except where noted, we will not explicitly address the possibility of delayed SG star formation in this review.

3.3 Overview of cluster dynamical evolution

Most numerical studies start with initial conditions as described in §3.1. To the extent that stellar mass loss can be neglected, we can understand the dynamical evolution of a star cluster from the fundamental physics of self-gravitating systems, driven by relaxation.

3.3.1 Evaporation

The relaxation time (Eq. 15) is the time scale on which stars tend to establish a Maxwellian velocity distribution. A fraction ξ_e of the stars in the tail of that distribution have velocities larger than v_{esc} and consequently escape. Assuming that this high-velocity tail is refilled every t_{rh} , the dissolution time scale is $t_{\text{dis}} = t_{\text{rh}}/\xi_e$. For isolated clusters, $v_{\text{esc}} = 2v_{\text{rms}}$. For a Maxwellian velocity distribution, a fraction $\xi_e = 0.0074$ has $v > 2v_{\text{rms}}$, and hence $t_{\text{dis}} = 137t_{\text{rh}}$. For tidally limited cluster ξ_e is higher since v_{esc} is lower. For a typical cluster density profile $\xi_e \approx 0.033$, implying $t_{\text{dis}} \approx 30t_{\text{rh}}$ (Spitzer 1987).

The escape fraction ξ_e is often taken to be constant (e.g. Gnedin & Ostriker 1997), but it depends on r_{hm} (through v_{rms}) and also on the strength of the tidal field, or r_{J} (through v_{esc}). Effectively, ξ_e depends on the ratio $r_{\text{hm}}/r_{\text{J}}$ (e.g. Spitzer & Chevalier 1973, Wielen 1988). Gieles & Baumgardt (2008) show that $\xi_e \propto (r_{\text{hm}}/r_{\text{J}})^{3/2}$ for $r_{\text{hm}}/r_{\text{J}} \gtrsim 0.05$ (the so-called *tidal regime*). From Eq. 15 we then find, for clusters on circular orbits in the tidal regime, that $t_{\text{dis}} \propto N/\omega$, apart from the slowly varying Coulomb logarithm. Here $\omega \equiv V_G/R_G$ is the orbital angular frequency in the galaxy, where R_G and V_G are, respectively, the galactocentric distance and the velocity around the galaxy center. For a flat rotation curve, $t_{\text{dis}} \propto R_G$ for

a cluster of given mass (e.g. Chernoff & Weinberg 1990, Vesperini & Heggie 1997). This linear dependence of t_{dis} on R_G makes it difficult to explain the universality of the globular cluster mass function via dynamical evolution of a power-law initial cluster mass function (Vesperini et al. 2003), but this will not be discussed further here.

Baumgardt (2001) showed that t_{dyn} enters into the escape rate and found, for equal-mass stars, $t_{\text{dis}} \propto t_{\text{rh}}^{3/4} t_{\text{dyn}}^{1/4}$. Baumgardt & Makino (2003) found that this scaling also holds for models of clusters with a stellar mass spectrum, stellar evolution, and for different types of orbits in a logarithmic potential. Their result for t_{dis} can be summarized as

$$t_{\text{dis}} \approx 2 \text{ Myr} \left(\frac{N}{\ln \Lambda} \right)^{3/4} \frac{R_G}{\text{kpc}} \left(\frac{V_G}{220 \text{ km s}^{-1}} \right)^{-1} (1 - \varepsilon), \quad (18)$$

where ε is the eccentricity of the orbit. If the Coulomb logarithm is taken into account, the scaling is approximately $t_{\text{dis}} \propto N^{0.65}$ in the range of about 10^3 to $10^6 M_\odot$ (Lamers, Gieles & Portegies Zwart 2005).

3.3.2 Core collapse

Self-gravitating systems are inherently unstable, and no final equilibrium state exists for a star cluster. The evaporation of high-velocity stars and the internal effects of two-body relaxation, which transfers energy from the inner to the outer regions of the cluster, result in core collapse (Antonov 1962, Lynden-Bell & Wood 1968, Cohn 1980, Lynden-Bell & Eggleton 1980, Makino 1996). During this phase, the central portions of the cluster accelerate toward infinite density while the outer regions expand. The process is readily understood by recognizing that, according to the virial theorem, a self-gravitating system has negative specific heat—reducing its energy causes it to heat up. Hence, as relaxation transports energy from the (dynamically) warmer central core to the cooler outer regions, the core contracts and heats up as it loses energy. The time scale for the process to go to completion (i.e. a core of zero size and formally infinite density) is $t_{\text{cc}} \sim 15 t_{\text{rh}}$ for an initial Plummer sphere of identical masses. Starting with a more concentrated King (1966) distribution shortens the time of core collapse considerably (Quinlan 1996), as does a broad spectrum of masses (Inagaki & Saslaw 1985).

In systems with a mass spectrum, two-body interactions accelerate the dynamical evolution by driving the system toward energy equipartition, in which the velocity dispersions of stars of different masses would have $m \langle v^2 \rangle \sim \text{constant}$. The result is mass segregation, where more massive stars slow down and sink toward the center of the cluster on a time scale (Spitzer 1969)

$$t_s \sim \frac{\langle m \rangle}{m} t_{\text{rl}}. \quad (19)$$

Portegies Zwart & McMillan (2002) and Gürkan, Freitag & Rasio (2004) find that, for a typical Kroupa (2001) mass function, the time scale for the most massive stars to reach the center and form a well defined high-density core is $\sim 0.2 t_{\text{rl}}$, where t_{rl} is the relaxation time of the region of interest (see Eq. 12), containing a significant number of massive stars—the core of a massive cluster, or the half-mass radius of a smaller one (in which case $t_{\text{rl}} = t_{\text{rh}}$, see Eq. 15). For dense clusters, t_s may be shorter than the time scale for stellar evolution, or for the first supernovae to occur (Portegies Zwart et al. 1999).

Thus, a collisional stellar system inevitably evolves toward a state in which the most massive objects become concentrated in the high-density central core (see §3.4.2). Dynamical evolution provides a natural and effective mechanism for concentrating astrophysically interesting objects in regions of high stellar density.

3.4 Internal Heating

On longer time scales, clusters that survive the early phases of mass loss enter a phase driven by the competition between relaxation and a variety of internal heating mechanisms (phase 2). High central densities lead to interactions among stars and binaries. Many of these interactions can act as energy sources to the cluster on larger scales, satisfying the relaxation-driven demands of the halo and temporarily stabilizing the core against collapse (Goodman & Hut 1989; Gao et al. 1991; McMillan, Hut & Makino 1990, 1991; Heggie & Aarseth 1992; Fregeau et al. 2003). On long time scales, these processes lead to a slow (relaxation time) overall expansion of the cluster, with $r_{\text{vir}} \propto t^{2/3}$, a result that follows from simple considerations of the energy flux through the half-mass radius (Hénon 1965).

While these processes are important to the long-term dynamical evolution, their relevance is somewhat different during the first 100 Myr (Portegies Zwart, McMillan & Makino 2007), which is largely dominated by stellar mass loss (early phase 2) and the segregation of the most massive stars. Often, their major effect is to enhance the rate of collisions and the formation of stellar exotica. We now consider in turn the following processes: (1) binary heating (§3.4.1), (2) stellar collisions (§3.4.2), and (3) black-hole heating (§3.4.3).

3.4.1 Binary Interactions

Irrespective of the way they form, binaries are often described by dynamicists as either “hard” or “soft.” A hard binary has binding energy greater than the mean stellar kinetic energy in the cluster (Heggie 1975): $|E_b| > \frac{1}{2}\langle mv^2 \rangle \approx \frac{1}{2}\langle m \rangle v_{\text{rms}}^2$, where $\langle m \rangle$ and v_{rms} are the local mean stellar mass and velocity dispersion. A binary with mass $m_b = m_1 + m_2$ and semi-major axis a_b has $E_b = -Gm_b/2a_b$, so hard binaries have $a_b < a_{\text{hard}}$, where

$$a_{\text{hard}} = \frac{Gm_b^2}{4\langle m \rangle v_{\text{rms}}^2} \approx 9.5 \times 10^4 R_{\odot} \left(\frac{m_b}{M_{\odot}} \right)^2 \left(\frac{v_{\text{rms}}}{\text{km s}^{-1}} \right)^{-2}. \quad (20)$$

The hard–soft distinction is helpful when discussing dynamical interactions between binaries and other cluster members. However, we note the definition of hardness depends on local cluster properties, so the nomenclature changes with environment, and even within the same cluster a binary that is hard in the halo could be soft in the core.

The dynamical significance of “hard” binaries (see Eq. 20) has been understood since the 1970s (Heggie 1975, Hills 1975, Hut & Bahcall 1983) When a hard binary interacts with another cluster star, the resultant binary (which may or may not have the same components as the original binary) tends, on average, to be harder than the original binary, making binary interactions a net heat source to the cluster. Soft binaries tend to be destroyed by encounters. For equal-mass systems, the mean energy liberated in a hard-binary encounter is proportional to E_b : $\langle \Delta E_b \rangle = \gamma E_b$, where $\gamma = 0.4$ for “resonant” interactions (Heggie 1975), and $\gamma \sim 0.2$ when wider “flybys” are taken into account (Spitzer 1987).

The liberated energy goes into the recoil of the binary and single star after the interaction. Adopting terminology commonly used in this field, we write the binary energy as $E_b = -hkT$, where $\frac{3}{2}kT = \langle \frac{1}{2}mv^2 \rangle$ and $h \gg 1$, so the total recoil energy, in the center of mass frame of the interaction, is γhkT . For the general case $\frac{m_b}{m_b+m}$ of this energy goes to the single star (with mass m) and $\frac{m}{m_b+m}$ to the binary, which for an equal-mass stars reduces to $\frac{2}{3}$ for the single star and $\frac{1}{3}$ for the binary. Neglecting the thermal motion of the center of mass frame, we identify three regimes:

1. If $\frac{2}{3}\gamma hkT < \frac{1}{2}mv_{\text{esc}}^2 = 2mv_{\text{rms}}^2 = 6kT$, i.e. $h < 45$, neither the binary nor the single star acquires enough energy to escape the cluster. Binaries in this stage are bounced from the core to fall back by dynamical friction in a process that we call “binary convection”.
2. If $\frac{2}{3}\gamma hkT > 6kT$ but $\frac{1}{3}\gamma hkT < 4mv_{\text{rms}}^2 = 12kT$, i.e. $45 < h < 180$, the single star escapes, but the binary is retained. We refer to such a binary as a “bully.”
3. If $h > 36/\gamma = 180$, both the binary and the single star are ejected. Such a binary is a “self-ejecter.”

These numbers are only illustrative, and for a binary with more components more massive than average, as is often the case, the threshold for bullying behavior drops, while that for self-ejection increases.

Tab. 6 places these considerations in a more physical context. Note that, since the closest approach between particles in a resonant interaction may be as little as a few percent of the binary semi-major axis (Hut & Inagaki 1985, McMillan 1986b), the hardest binaries may well experience physical stellar collisions rather than hardening to the point of ejection; and a collision tends to soften the surviving binary. Alternatively, before their next interaction, they may enter the regime in which internal processes, such as tidal circularization and/or Roche-lobe overflow, become important. The future of such a binary may be determined by the internal evolution of its component stars, rather than by further encounters.

Table 6: Terminology (first column) and characterization (second and third columns) for the various stages of a binary (see text). The subsequent columns give the orbital separation a of a binary (in units of AU) with a total mass $m_1 + m_2 \equiv m_b = 10\langle m \rangle$ or $m_b = 100\langle m \rangle$, in a cluster with a mass of $M = 10^5 M_\odot$ and virial radius $r_{\text{vir}} = 1 \text{ pc}$ and $r_{\text{vir}} = 10 \text{ pc}$.

Binary	relation	E_b [kT]	$M = 10^5 M_\odot$				Unit
			$r_{\text{vir}} = 1 \text{ pc}$		$r_{\text{vir}} = 10 \text{ pc}$		
			$m_b = 10$	$m_b = 100$	$m_b = 10$	$m_b = 100$	
hard	$E_b > \frac{3}{2}kT$	1	7.2×10^4	7.2×10^6	7.2×10^5	7.2×10^7	AU
bully	$v_{\text{rec}} > v_{\text{esc}}m/m_b$	10	1.7×10^3	1.7×10^6	1.7×10^4	1.7×10^7	AU
tenured	$t_{\text{enc}} > t_{\text{rh}}$	100	52	58	53	5.8	AU
self-eject	$v_{\text{rec}} > v_{\text{esc}}m_b/m$	100	0.016	1.6	33	3.3×10^3	AU

The binary encounter time scale is $t_{\text{enc}} = (n\sigma v_{\text{rms}})^{-1}$, where n is the local stellar density and σ is the encounter cross section (see Eq. 23). If we arbitrarily compute the binary

interaction cross section as that for a flyby within 3 binary semi-major axes, consistent with the encounters contributing to the Spitzer (1987) value $\gamma = 0.2$, and again assume equal masses ($m_b = 2m$), we find

$$t_{\text{enc}} \sim 8ht_{\text{rl}}, \quad (21)$$

where we have used Eq. 12 and taken $\ln \Lambda = 10$. Thus the net local heating rate per binary during the 100% efficient phase (#1 above), when the recoil energy remains in the cluster due to “binary convection” is

$$\mathcal{E}_{\text{bin}} = \gamma h k T t_{\text{enc}}^{-1} \sim 0.1 k T / t_{\text{rl}}, \quad (22)$$

that is, *on average*, each binary heats the cluster at a roughly constant rate. During the “bully” phase, the heating rate drops almost to one-third of this value, the true value of one-third is not reached since the ejected single stars still give energy via indirect heating. For “self-ejecting” binaries, the heating rate is zero.

Binary–binary interactions also heat the cluster, although the extra degrees of freedom complicate somewhat the above discussion. If the binaries differ widely in semi-major axes, the interaction can be handled in the three-body approximation, with the harder binary considered a point mass. If the semi-major axes are more comparable, as a rule of thumb the harder binary tends to disrupt the wider one (Bacon, Sigurdsson & Davies 1996).

Numerical experiments over the past three decades have unambiguously shown how initial binaries segregate to the cluster core, interact, and support the core against further collapse (Heggie & Aarseth 1992; McMillan, Hut & Makino 1990). The respite is only temporary, however. Sufficiently hard binaries are ejected from the cluster by the recoil from their last interaction (self-ejection, see Tab. 6), and binaries may be destroyed, either by interactions with harder binaries, or when two or more stars collide during the interaction. For large initial binary fractions, this binary supported phase may exceed the age of the universe or the lifetime of the cluster against tidal dissolution. However, for low initial binary fractions, as now appears to have been the case for the globular clusters observed today (Heggie & Giersz 2008, Giersz & Heggie 2009), the binaries can be depleted before the cluster dissolves, and core collapse resumes (Fregeau et al. 2003).

Thus binary dynamics drives the evolution of the cluster while, simultaneously, the combination of cluster dynamics and internal stellar processes determine the internal evolution of each binary. This interplay between stellar evolution and stellar dynamics is sometimes referred to as *star-cluster ecology* (Heggie 1992) or the *binary zoo* (Davies et al. 2006) or *stellar promiscuity* (Hurley & Shara 2002).

3.4.2 Stellar Collisions

In systems without significant binary fractions—either initially or following the depletion of core binaries—core collapse may continue to densities at which actual stellar collisions occur. In young clusters, the density increase may be enhanced by rapid segregation of the most massive stars in the system to the cluster core. Since the stellar escape speed greatly exceeds the rms speed of cluster stars ($\theta < 100$ in Eq. 1), collisions lead to mergers of the stars involved, with only small fractional mass loss (Benz & Hills 1987, Freitag & Benz 2001). If the merger products did not evolve, the effect of collisions would be to dissipate

kinetic energy, and hence cool the system, accelerating core collapse (Portegies Zwart et al. 1999). However, when accelerated stellar evolution is taken into account, the (time averaged) enhanced mass loss can result in a net heating effect (Chatterjee, Fregeau & Rasio 2008).

The cross section for an encounter between two objects of masses m_1 and m_2 and radii r_1 and r_2 , respectively, is

$$\sigma = \pi r^2 \left[1 + \frac{2G(m_1 + m_2)}{rv^2} \right] \quad (23)$$

(Hills & Day 1976), where v is the relative velocity between the two objects, and $r = r_1 + r_2$. For $r \ll G(m_1 + m_2)/v^2$, as is usually the case for the star clusters discussed in this review, the encounter is dominated by the second term (gravitational focusing), and Eq. 23 reduces to

$$\sigma \approx 2\pi r \frac{Gm}{v^2}, \quad (24)$$

which is nearly independent of the properties of the other stars.

Encounters (collisions) between single stars are unlikely, unless one (or both) of the stars is very large and/or very massive, or the local density is very high. The presence of primordial binaries increases the number of stars in a cluster, and therefore increases the chance of a traffic accident. Hard binaries (see §3.4.1) are also in the gravitational focusing regime, so the earlier binary interaction cross section is obtained by setting $r = a$ in Eq. 24. As just discussed, such an encounter can lead to the hardening of the binary and ejection of the single star and possibly also the binary, but it may also lead to a hydrodynamical encounter, i.e. a physical collision between two of the stars. Due to the large extent of the hydrodynamical mess which typically results from the collision between two stars, it is quite likely that the third star is engulfed also to participate in the colliding (Fregeau et al. 2004). Since binaries generally have semi-major axes much greater than the radii of the component stars, such *binary-mediated collisions* play important roles in determining the stellar collision rate in YMCs (Portegies Zwart & McMillan 2002). The rate of increase of the star’s mass due to collisions then is

$$\begin{aligned} \frac{dM}{dt} &\approx \rho_c \sigma v \approx 2\pi G M r_* \rho_c / v \\ &= 0.6 \left(\frac{m}{100M_\odot} \right) \left(\frac{a}{100R_\odot} \right) \left(\frac{\rho_c}{10^6 M_\odot / \text{pc}^3} \right) \left(\frac{10 \text{ km s}^{-1}}{v} \right) M_\odot / \text{Myr}. \end{aligned} \quad (25)$$

Thus a massive star ($m \sim r_* \sim 1$) in a dense stellar core ($\rho_c \sim 10^6 M_\odot / \text{pc}^3$) will experience numerous collisions during its $\sim 3 - 5$ Myr lifetime. Binary encounters can substantially increase this rate (Portegies Zwart & McMillan 2002), leading to significant numbers of mergers in lower-density, binary rich environments. Massive binaries in young dense clusters tend to be collision targets rather than heat sources (Gürkan, Freitag & Rasio 2004).

In a sufficiently dense system, repeated stellar collisions can lead to a so-called “collision runaway” (Portegies Zwart et al. 1999), in which a massive star or collision product suffers repeated mergers and grows enormously in mass before exploding as a supernova (Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004; Gürkan, Freitag & Rasio 2004). This has frequently been cited as a possible mechanism for producing intermediate-mass black holes (IMBHs) in star clusters. However, while the dynamics is simple, numerous uncertainties in

the stellar evolution and mass loss of the resultant merger product have been pointed out in the recent literature, suggesting that the net growth rate, and hence the final mass of the resulting IMBH, may be much lower than suggested by purely dynamical simulations—perhaps as little as a few hundred solar masses (Yungelson et al. 2008, Glebbeek et al. 2009, Vanbeveren et al. 2009). An alternative formation mechanism for more massive IMBHs, involving gas accretion onto a seed $\sim 100 M_\odot$ black hole during a second round of star formation early in the cluster’s lifetime (Vesperini et al. 2009), and by repeated collisions between stellar-mass black holes during the late phase 2 of the evolution of the star cluster (Miller & Hamilton 2002).

3.4.3 Black Hole Heating

An IMBH in a star cluster can be an efficient source of energy to the stellar system. Stars diffuse by two-body relaxation deeper and deeper into the IMBH’s potential well, and eventually are tidally disrupted and consumed by the black hole (Bahcall & Wolf 1976). The energy lost during the process heats the system. The heating rate for an IMBH of mass M_{BH} in a cluster core of density ρ_c and velocity dispersion v_c is

$$\mathcal{E}_{bh} \sim \frac{G^5 \langle m \rangle \rho_c^2 M_{BH}^3 \ln \Lambda}{v_c^7}. \quad (26)$$

Although cores are promising environments for the formation of IMBHs, they may not be the best place to look for evidence of massive black holes today. Dynamical heating by even a modest IMBH is likely to lead to a cluster containing a fairly extended core (Baumgardt, Makino & Hut 2005). Comparing the outward energy flux from stars relaxing inward in the (Bahcall–Wolf) cusp surrounding the IMBH to the outward flux implied by two-body relaxation at the cluster half-mass radius, Heggie et al. (2007) estimate the equilibrium ratio of the half-mass (r_{hm}) to the core (r_c) radius in a cluster of mass M . Calibrating to simulations, they conclude that for systems with equal mass, except of course the black holes

$$\frac{r_{hm}}{r_c} \sim 0.23 \left(\frac{M}{M_{BH}} \right)^{3/4}. \quad (27)$$

Trenti et al. (2007) has suggested that the imprint of this process can be seen in his “isolated and relaxed” sample of simulated open clusters having relaxation times less than 1 Gyr, a half-mass to tidal radius ratio $r_{hm}/r_t < 0.1$, and an orbital ellipticity of less than 0.1. Roughly half of the clusters in this sample have core radii substantially larger than would be expected on the basis of simple stellar dynamics and binary heating. However, Hurley (2007) has argued that such anomalously large core to half-mass ratios may also be explained by the presence of a stellar-mass BH binaries heating the cores of these clusters (see also Merritt et al. 2004, Mackey et al. 2007, 2008). Many of the above discussed results and much of our physical understanding of the dynamical evolution of star clusters has been developed and calibrated by means of simulations.

4 Simulating star clusters

The evolution of a young star cluster is a complex problem combining stellar dynamics, gas dynamics, and stellar evolution, all contributing in important ways to the cluster’s appearance and long-term survival probability (see also §3 and §5). Over the past decade, significant progress has been made in modeling many of these processes simultaneously in numerical simulations of clusters during phase 2 (see §3). A striking omission is the self-consistent treatment of the interaction between stars and gas during phase 1. We focus here on simulations of phase-2 clusters, first describing treatments of stellar dynamics (§4.1), then turning to the inclusion of other physical processes into the mix (§4.2).

4.1 Dynamical algorithms

A broad spectrum of numerical methodologies is available for simulating the dynamical evolution of young star clusters. In approximate order of increasing algorithmic and physical complexity, but not necessarily in increasing numerical complexity, the various methods may be summarized as follows.

- *Static Models* are self-consistent potential–density pairs for specific choices of phase-space distribution functions (Plummer 1911, King 1966, Binney & Tremaine 2008). They have been instrumental in furthering our understanding of cluster structure, and provide a framework for semi-analytical treatments of cluster dynamics. However, they do not lend themselves to detailed study of star cluster evolution, and we will not discuss them further here, instead referring the reader to the discussion in §1.3.2, or to (Spitzer 1987).
- *“Continuum” Models* treat the cluster as a quasi-static continuous fluid whose phase-space distribution function evolves under the influence of two-body relaxation and other energy sources (such as binary heating) that operate on relaxation time scales (see Eq. 13).
- *Monte–Carlo Models* treat some or all components of the cluster as pseudo-particles whose statistical properties represent the continuum properties of the system, and whose randomly chosen interactions model relaxation and other processes driving the long-term evolution.
- *Direct N-body Models* follow the individual orbits of all stars in the system, automatically including dynamical and relaxation processes, and modeling other physical processes on a star-by-star basis.

We now consider the last three categories in more detail.

4.1.1 Continuum methods

The two leading classes of continuum models are gas-sphere (Lynden-Bell & Eggleton 1980, Bettwieser & Sugimoto 1984, Deiters & Spurzem 2001) and Fokker–Planck (Cohn 1979; Shapiro 1985; Chernoff & Weinberg 1990; Drukier, Fahlman & Richer 1992; Takahashi 1996,

1997; Takahashi & Portegies Zwart 1998) methods. They have mainly been applied to spherically symmetric systems, although axisymmetric extensions to rotating systems have also been implemented (Einsel & Spurzem 1999; Kim et al. 2002; Kim, Lee & Spurzem 2004), and some limited experiments with rudimentary binary treatments have also been performed (Gao et al. 1991).

Both approaches start with the collisional Boltzmann equation as the basic description for a stellar system, then simplify it by averaging the distribution function $f(\mathbf{x}, \mathbf{v})$ in different ways. Gas-sphere methods proceed in a manner closely analogous to the derivation of the equations of fluid motion, taking velocity averages to construct the moments of the distribution: $\rho = \int d^3v f(\mathbf{x}, \mathbf{v})$, $\mathbf{u} = \int d^3v \mathbf{v} f(\mathbf{x}, \mathbf{v})$, $\sigma^2 = \frac{1}{3} \int d^3v v^2 f(\mathbf{x}, \mathbf{v})$, etc. Application of a closure condition leads to a set of equations identical to those of a classical conducting fluid, in which the conductivity depends inversely on the local relaxation time. Fokker–Planck methods transform the Boltzmann equation by orbit-averaging all quantities and recasting the equation as a diffusion equation in $E - J$ space, where E is stellar energy and J is angular momentum. Since both E and J are conserved orbital quantities in a static, spherically symmetric system, two-body relaxation enters into the problem via the diffusion coefficients.

These methods have been of enormous value in developing and refining theoretical insights into the fundamental physical processes driving the dynamical evolution of stellar systems (Bettwieser & Sugimoto 1984). However, as the degree of realism demanded of the simulation increases—adding a mass spectrum, stellar evolution, binaries, etc.—the algorithms rapidly become cumbersome, inefficient, and of questionable validity (Portegies Zwart & Takahashi 1999). As a result, they are generally not applied to the young stellar systems of interest here. The major approaches currently used for simulating young massive clusters are particle-based Monte–Carlo or direct N -body codes.

4.1.2 Monte-Carlo methods

Depending on one’s point of view, Monte–Carlo methods can be regarded as particle algorithms for solving the partial differential equations arising from the continuum models, or approximating the long-term average gravitational interactions of a large collection of particles. The early techniques developed in the 1970s and 1980s (Spitzer & Hart 1971, Henon 1973, Spitzer 1975, Stodolkiewicz 1982, 1986) fall into the former category, but recent studies, in particular (Giersz 1998; Joshi, Rasio & Portegies Zwart 2000; Freitag & Benz 2001; Giersz 2001; Fregeau et al. 2003; Giersz 2006; Fregeau & Rasio 2007), adopt the latter view. The hybrid Monte–Carlo scheme of (Giersz 1998, 2001, Giersz & Spurzem 2003) combines a gas-sphere treatment of the “background” stellar population with a Monte–Carlo realization of the orbits and interactions of binaries and other objects of interest. With their hybrid method they have performed the first simulations of an entire globular cluster, from a very early (although gas depleted) phase to complete dissolution (Heggie & Giersz 2008, Giersz & Heggie 2009).

Monte-Carlo methods are designed for efficient computation of relaxation effects in collisional stellar systems, a task which they accomplish by reducing stellar orbits to their orbital elements—energy and angular momentum—effectively orbit averaging the motion of each star. Relaxation is modeled by randomly selecting pairs of stars and applying interactions between them in such a way that, on average, the correct rate is obtained. This may

be implemented in a number of ways, but interactions are generally realized on time scales comparable to the orbit-averaged relaxation time. As a result, Monte-Carlo schemes can be orders of magnitude faster than direct N -body codes. For example, Joshi, Rasio & Portegies Zwart (2000) report a CPU time scaling for their Monte-Carlo scheme of $O(N^{1.4})$ for core-collapse problems, compared to N^3 for N -body methods, as discussed below. To achieve these speeds, however, the geometry of the system must be simple enough that the orbital integrals can be computed from a star's instantaneous energy and angular momentum. In practice, this limits the approach to spherically symmetric systems in virial equilibrium, and global dynamical processes occurring on relaxation (or longer) time scales.

4.1.3 N -body methods

N -body codes incorporate detailed descriptions of stellar dynamics at all levels, using direct integration of the individual (Newtonian) stellar equations of motion for all stars (Aarseth 2003, Heggie & Hut 2003). Their major attraction is that they are assumption-free, in the sense that all stellar interactions are automatically included to all orders, without the need for any simplifying approximations or the inclusion of additional reaction rates to model particular physical processes of interest. Thus, problems inherent to Monte-Carlo methods (see §4.1.2), related to departures from virial equilibrium, spherical symmetry, statistical fluctuations, the form of (and indeed the existence of) phase space distribution functions, and the possibility of interactions not explicitly coded in advance, simply do not arise, and therefore do not require fine-tuning as in the Monte-Carlo models. In addition, including the properties of individual stars, such as stellar evolution, is relatively straightforward.

The price of all these advantages is computational expense. Each of the N particles must interact with every other particle a few hundred times over the course of every orbit, each interaction requires $O(N)$ force calculations, and a typical (relaxation time) run spans $O(N)$ orbits (see Eq. 15). The resulting $O(N^3)$ scaling of the total CPU time means that, even with the best time-step algorithms, integrating even a fairly small system of, say, $N \sim 10^5$ stars requires sustained teraflops speeds for several months (Hut, Makino & McMillan 1988). Radically improved performance can be achieved by writing better software, or by building faster computers (or both). In fact, the remarkable speed-up of N -body codes over the last four decades has mainly been due to advances in hardware, and in a lesser extend due to software.

Substantial performance improvements were realized by adopting better (individual) time stepping schemes (as opposed to earlier shared time step schemes), in which particles advance using steps appropriate to their individual orbits, rather than a single step for all. Further gains were made by utilizing neighbor schemes (Ahmad & Cohen 1973), which divide the force on every particle into irregular (rapidly varying) and regular (slowly varying) parts, due (loosely speaking) to nearby and more distant bodies. By recomputing the regular force at every particle step, but extrapolating the more expensive $O(N)$ regular force for most time steps, and recomputing it only on longer time scales, significant improvements in efficiency have been realized. A multi-level generalization of this approach by Dorband, Hemsendorf & Merritt (2003) is incorporated into the collisional NBODY6++ (Spurzem 1999).

Another important algorithmic improvement was introduced in the mid-1980s with the development of tree codes (Barnes & Hut 1986), which reduce the force calculation com-

plexity from $O(N)$ to $O(\log N)$. Despite their algorithmic efficiency, tree codes have not been widely used in modeling collisional systems. This seems principally to be because of lingering technical concerns about their long-term accuracy in systems dominated by relaxation processes and their performance in clusters with large dynamic ranges in densities and time scales, even though these objections may no longer be well founded (Moore et al. 1999, Dehnen 2000). Very promising direct–treecode methods have recently been developed to model the dynamical interaction between a cluster and the surrounding galactic population (McMillan & Aarseth 1993, Fujii et al. 2007, Portegies Zwart et al. 2009).

4.1.4 Parallelization

Individual time step schemes are generally hard to optimize on parallel machines. For those architectures, block time step schemes (McMillan 1986a, Makino et al. 2006) offer substantially better performance. By rounding each star’s “natural” step down to the nearest negative integer power of two, such a scheme effectively discretizes the time variable, allowing the possibility that large blocks of stars will be “next” on the time step list, and so can be efficiently integrated in parallel.

The two most important parallel integration techniques are the *ring* and *copy* algorithms. Both have advantages and disadvantages, but the execution times for each, on computers with p processors, scale as N/p , while the communication times scale as p (Harfst et al. 2007). Both algorithms are implemented in a range of N -body codes, including NBODY6++ (Dorband, Hemsendorf & Merritt 2003) and the *kira* integrator in *Starlab* (Portegies Zwart et al. 2008). The two-dimensional lattice parallelization for direct N -body kernels has comparable CPU time scaling, but the communication has a weaker scaling ($\propto 1/\sqrt{p}$), enabling the code to maintain satisfactory performance even on computers with $p \gtrsim 10^3$ processors (Makino 2002, Bisseling 2004). So far, however, this scheme has not been implemented in a production N -body code.

An interesting further step is to use a widely distributed grid of computers (Foster & Kesselman 2004). In this extreme form of parallel computing the computational bottleneck often shifts from the $O(N^2)$ force calculation (see §4.1.3) to communication (latency and bandwidth). However, even in the worst-case scenario the communication costs scale $\propto N$, so, for a sufficiently large number of stars even intercontinental grid computing can be practical (Hoekstra et al. 2008). In addition, if (as seems likely—see §4.2) future simulation environments will combine a range of codes in addition to pure stellar dynamics to address the evolution of YMCs in detail, grid computing may provide the solution to the problem of limited supply of local computer resources. This is particularly relevant if the desired algorithms for solving stellar dynamics, stellar evolution, hydrodynamics, etc, require a diversity in computer architectures that may not be locally available.

4.1.5 Hardware acceleration

A quantum leap in gravitational N -body simulation speed come from the introduction of special purpose computers. All N -body codes, including neighbor schemes and treecodes, suffer from the cost of computing inter-particle forces at every step along the orbit. A technological solution in widespread use is the “GRAPE” (short for “GRAVity PipE”) series of

machines developed by Sugimoto and co-workers at Tokyo University (Ebisuzaki et al. 1993). Abandoning algorithmic sophistication in favor of simplicity and raw computing power, these machines achieved high performance by mating a fourth-order Hermite integration scheme (Makino & Aarseth 1992) with special-purpose hardware in the form of highly parallel, pipelined “Newtonian force accelerators” implementing the computation of all inter-particle forces entirely in hardware. Operationally, the hardware is simple to program, as it merely replaces the function that computes the (regular) force on a particle by a call to the hardware interface libraries; the remainder of the user’s N -body code is unchanged. The effect of GRAPE on simulations of stellar systems has been nothing short of revolutionary. Today, GRAPE-enabled code lies at the heart of almost all detailed N -body simulations of star clusters and dense stellar systems.

Recently, Graphics Processing Units (GPUs) have achieved speeds and price/performance levels previously attainable only by GRAPE systems (see Portegies Zwart, Belleman & Geldof 2007; Hamada & Iitaka 2007; Belleman, Bédorf & Portegies Zwart 2008; Gaburov, Harfst & Portegies Zwart 2009) for recent GPU implementations of the GRAPE interface). In addition, the programming model for GPUs (as well as the GRAPE-DR, Makino 2005), means that many other kinds of algorithms can (in principle) be accelerated, although, in practice, it currently seems that CPU-intensive operations such as direct N -body force summation show substantially better acceleration than, say, treecodes running on the same hardware. It appears that commodity components may be poised to outpace special-purpose computers in this specialized area of computational science, just as they have already done in general-purpose computing.

4.2 The kitchen sink

Consistent with our growing understanding of the role of stellar and binary interactions in collisional stellar systems, the leading programs in this field are “kitchen sink” packages that combine treatments of dynamics, stellar and binary evolution, and stellar hydrodynamics within a single simulation. Of these, the most widely used are the N -body codes NBODY (Hurley et al. 2001, Aarseth 2003) and `kira` which is part of the `starlab` package (e.g. Portegies Zwart et al. 2001), and the Monte-Carlo codes developed by Mirek Giersz (Giersz 1998, Heggie & Giersz 2008, Giersz & Heggie 2009), Mark Freitag (Fregeau et al. 2003; Freitag, Rasio & Baumgardt 2006) and John Fregeau (Fregeau & Rasio 2007).

Despite the differences in their handling of the large-scale dynamics, as just outlined, these codes all employ conceptually similar approaches to stellar and binary evolution and collisions. All use approximate descriptions of stellar evolution, generally derived from look-up tables based on the detailed evolutionary models of Eggleton, Fitchett & Tout (1989) and Hurley, Pols & Tout (2000). They also rely on semi-analytic or heuristic rule-based treatments of binary evolution (Portegies Zwart & Verbunt 1996; Hurley, Tout & Pols 2002), conceptually similar from code to code, but significantly different in detail.

In most cases, collisions are implemented in the simple “sticky-sphere” approximation, where stars are taken to collide (and merge) if they approach within the sum of their effective radii. The effective radii may be calibrated using hydrodynamical simulations, and mass loss may be included in some approximate way. Freitag’s Monte-Carlo code, geared mainly to studies of galactic nuclei, uses a more sophisticated approach, interpolating encounter

outcomes from a pre-computed grid of smoothed particles hydrodynamics (SPH) simulations (Freitag & Benz 2005). An interesting alternative, though currently only operational in AMUSE (see § 4.3), is the “Make Me A Star” package (MMAS; Lombardi et al. 2003)^e and its extension “Make Me a Massive Star” (MMAMS; Gaburov, Lombardi & Portegies Zwart 2008)^f. MMA(M)S constructs a merged stellar model by sorting the fluid elements of the original stars by entropy or density, then recomputing their equilibrium configuration, using mass loss and shock heating data derived from SPH calculations.

Small-scale dynamics of multiple stellar encounters, such as binary and higher-order encounters, are often handled by look-up from pre-computed cross sections or—more commonly—by direct integration, either in isolation or as part of a larger N -body calculation. Codes employing direct integration may also include post-Newtonian terms in the interactions between compact objects (Kupi, Amaro-Seoane & Spurzem 2006).

4.3 Future prospects

The very comprehensiveness of kitchen-sink codes gives them the great advantage of applicability to complex stellar systems, but also the significant disadvantage of inflexibility. By selecting one of these codes, one is implicitly choosing a particular hard-coded combination of dynamical integrator, stellar and binary evolution schemes, collision prescription, and treatment of multiple dynamics. The structure of these codes is such that implementing a different algorithm within the larger framework is difficult at best, and practically impossible.

However, studies of dense stellar systems force interactions between programs that were never intended to interact with other programs, and by extension require new communication channels between the programmers responsible for them. Closely related to this effort is the “MUSE” (MULTiscale, MULTiphysics Software Environment) project^g (Portegies Zwart et al. 2009), and its successor AMUSE (Astrophysical MULTipurpose Software Environment), two ambitious open-source efforts in code integration. (A)MUSE aims at the self-consistent integration of dynamics, collisions, stellar evolution, and other relevant physical processes, thereby realizing one vision of the MODEST^h community (Hut et al. 2003, Sills et al. 2003, Davies et al. 2006). The long-term goal is a comprehensive environment for modeling dense stellar systems, including multiphysics/legacy codes and flexible interfaces to integrate existing software (written in many languages) within this unifying environment.

Apart from the future developments regarding the flexibility of ‘lego’ simulation environment, the builders of the GRAPE are aiming at optimizing hardware by ‘returning’ to machine-dependend implementations of the force operations in N -body codes. Particularly promising are the recent optimizations of the inner force-calculation operations using the IA-32 Streaming SIMD Extensions 2 (SSE2) as are described by Nitadori, Makino & Hut (2006). In addition, (Nitadori & Makino 2008) have developed extensions of the standard fourth-order Hermite scheme to higher (sixth and eighth) orders. Other intriguing future prospects come from the hybridization of direct methods with hierarchical tree and particle-

^eSee <http://webpub.allegheeny.edu/employee/j/jalobar/mmas/>

^fSee <http://castle.strw.leidenuniv.nl/>

^g<http://muse.li>

^hMODEST stands for MODEling DENSE STellar systems, and can be found at <http://www.manybody.org/modest>.

mesh algorithms, an approach which is currently being developed by Nitadori and Makino (2009, private communication), but which is not yet operational.

Although we could easily write many more pages on the various numerical issues related to simulating young massive star clusters, we have to be realistic and return to more practical matters. In the last theoretical SS we discuss the interplay between theory, observations and numerical modeling of YMCs, and the extent to which they provide a basis for consistency or contradiction.

5 The survival of star clusters

The realization that the majority of star formation occurs in embedded clusters, whereas only a small fraction of stars in the Galactic disk currently reside in clusters (see §1), indicates that most clusters and associations are relatively short lived; they dissolve on time scales comparable to the median age of open clusters in the solar neighborhood (Kharchenko et al. 2005), which is about 250 Myr (see §1.2).

Historically, studies of the lifetimes of star clusters have focused on open clusters in the Milky Way. The scarcity of open clusters with ages $\gtrsim 1$ Gyr was reported independently in several studies (von Hoerner 1958; van den Bergh 1957; Oort, Kerr & Westerhout 1958), and has been attributed to their short median lifetimes (about 250 Myr; Wielen 1971), rather than, say, a variation in the formation history or a detection bias toward young objects. These short cluster lifetimes have been explained as due to the destructive effects of encounters with giant molecular clouds (GMCs) (Spitzer 1958). A typical Galactic cluster with a density of $\sim 1 M_{\odot} \text{pc}^{-3}$ can survive the heating due to passing GMCs for about 250 Myr. The remarkable agreement between the inferred mean lifetime and the expected survival time in the Galactic disk implicated GMCs as responsible for the destruction of open clusters (see §5.3). The argument is further supported by the radial offset of the old (few Gyrs) open clusters toward the anticenter of the Galactic disk and away from the plane of the disk, where the density of GMCs is low (van den Bergh & McClure 1980), as is illustrated in Fig. 1. The the galactic bulge and spiral structure also contribute, though in lesser extent, to the destruction of open clusters (Weinberg 1994).

By comparing the age distributions of clusters in the Magellanic clouds with those in the Milky Way Galaxy, the former population is found to be on average older and also more massive than the local population (Elson & Fall 1985, Hodge 1987). The higher average cluster mass in the sample of Magellanic cloud clusters is a consequence of the difficulty in detecting low mass clusters. The apparent longer lifetimes of the Magellanic clusters could imply that more massive clusters tend to live longer, although the longer lifetimes could also be explained by the lower density of GMCs, the absence of bulges and spiral structures and the overall weaker tidal fields in the Magellanic clouds. However, in §5.3.2 we argue that GMCs are unlikely to play an important role in the lifetime of a YMC. The mechanism leading to the destruction of star clusters is therefore of major importance for understanding the evolution of star clusters from youth to old age.

5.1 Early violent gas expulsion

Possibly the greatest discrepancy between star cluster simulations and observations lies in the first few million years of the evolution (phase 1 in §3). Real star clusters are formed in a complicated interaction between gas and gravity, which is imperfectly understood. Once a primordial gas cloud starts to condense into stars dynamical evolution also begins (Bate, Bonnell & Bromm 2003). At the end of the star formation process, probably brought about by the developing winds of the most massive stars or the first supernovae, the residual gas is ejected from the protostellar cluster. The gas expulsion phase is expected to be short—on the order of several dynamical times (Eq. 11)—and places the remaining stellar population in a super-virial state, making the young cluster vulnerable to dissolution. The sharp decrease in the number of young and embedded star clusters at an age of a few megayears is thought to be a consequence of this early process, and is often referred to as “infant mortality” (Lada & Lada 2003).

5.1.1 Theoretical considerations

The total mass $M = M_g + M_*$ of the primordial cluster contains contributions from stars M_* and gas M_g . In virial equilibrium, the rms velocity of the cluster is

$$v_{\text{rms}}^2 = \frac{GM}{2r_{\text{vir}}}. \quad (28)$$

which in observable quantities becomes

$$\sigma_{1D}^2 = \frac{GM}{\eta r_{\text{eff}}}, \quad (29)$$

with $\eta \approx 10$ (see §1.3.2).

The response of the cluster to the loss of the residual gas depends on the gas expulsion time scale t_{exp} relative to the dynamical time scale t_{dyn} of the cluster (see Eq. 9). For many young clusters $t_{\text{exp}} \ll t_{\text{dyn}}$, in which case removing the gas shocks the cluster. In the extreme case, the positions and velocities of the stars remains fixed during the gas expulsion phase, and the response of the cluster to losing a fraction of gas, $f_e \equiv M_*/M$, can be calculated under the assumption that the mass loss is instantaneous. The rms velocity of the stars immediately before and after gas loss is then given by Eq. 28; the stellar positions are also unchanged. As a consequence, the cluster expands and re-establishes equilibrium at a new virial radius given by (Hills 1980)

$$\frac{r_{\text{vir}}}{r_{\text{vir}}(t=0)} = \frac{f_e}{2f_e - 1}. \quad (30)$$

For high star formation efficiency ($f_e \gtrsim 0.5$) the arguments leading to Eq. 30 may be reasonable, as in this case a relatively small fraction of the stars is lost after the gas is expelled. However, the energy argument is too simple to determine the survival probability if $f_e \lesssim 0.5$. Losing more than half of the total mass ($f_e \leq 0.5$) by explosive gas expulsion is devastating for the cluster, leading to its complete dissolution in a few dynamical time scales. A low

star formation efficiency may explain the majority of disrupted young and embedded clusters, but even for $f_e \lesssim 0.5$ some small portion of the cluster can in practice remain bound. There are several independent arguments for the survival of embedded clusters even for a star formation efficiency as low as $f_e \lesssim 0.1$.

The most important argument against the above simple analysis (Eq. 30) is the fact that the time scale for gas expulsion, t_{exp} , in practice is short but not instantaneous. This time scale should not be compared to the (global) half-mass crossing time, but rather to the local dynamical time, which depends strongly on the distance to the cluster center. For example, the dynamical time scale in the cluster core has $t_{\text{core}}/t_{\text{dyn}} = (\rho_{\text{hm}}/\rho_c)^{1/2}$ and this fraction ranges from $\lesssim 0.01$ for a concentrated cluster ($W_0 \gtrsim 12$, or $c > 2.7$) to ~ 0.2 for a shallow potential ($c < 1$ or $W_0 \lesssim 5$). For concentrated clusters ($c \gtrsim 2$), the gas expulsion occurs more or less instantaneously for stars in the outskirts, but stars in the core respond adiabatically to the loss of gas. If $t_{\text{exp}} > t_{\text{dyn}}$, the cluster is likely to survive; in particular, the core may respond with just a slight expansion (of at most a factor of 2), even in extreme cases, $f_e \lesssim 0.1$ (Geyer & Burkert 2001).

Further complications arise from the clumping of the gas and stellar distributions in the embedded cluster (Fellhauer, Wilkinson & Kroupa 2009). It is likely that the radial dependence of the star formation process causes the central part of the cluster to be depleted in gas, whereas the outskirts are relatively gas rich (Bonnell, Bate & Vine 2003). This radial variation of the star formation efficiency, combined with the process of competitive star formation (Bonnell, Bate & Vine 2003), may actually render the cluster sub-virial after the gas is ejected, for example if stars formed in the collapsing cloud are dynamically cold (Lada, Margulis & Dearborn 1984). In that case, Eq. 30 depends on the fractional deviation from virial equilibrium q_{vir} , to become $r_{\text{vir}}/r_{\text{vir}}(t=0) = f_e/(2f_e - q_{\text{vir}}^2)$, i.e. the condition for complete disruption becomes $f_e = q_{\text{vir}}^2/2$ (Proszkow et al. 2009, Goodwin 2009). The cluster survival probability then depends on the entire star formation process, not just on its overall efficiency (Goodwin & Bastian 2006). A further deviation from the simple formulation comes from the effect of high-velocity escapers, which can carry away a considerable fraction of the cluster’s binding energy, leaving the remaining stars more strongly bound (Baumgardt & Kroupa 2007).

Thus the survival probability of an embedded cluster cannot be determined by a single parameter, such as the star formation efficiency, as the complete formation process of stars, clumps of stars, and the entire cluster comes into play. The process whereby stars form in massive star clusters is still poorly explored terrain within astrophysics, and the formation of sub-clumps and clusters is even less well charted.

5.1.2 Observational constraints

What sounds convincing from a theoretical standpoint is often very hard to support with observations. There are a number of interesting observational indications of infant mortality, and of the associated dissolution time scales, but since the embedded phase is short ($\sim 1 - 2$ Myr), many parameters are poorly constrained by observations, and the interpretations of models tend to be sensitive to assumptions made about the initial conditions.

Many observed young clusters, particularly the extragalactic population in Tab. 4, appear to be super-virial, which naively would lead to the early termination of the cluster’s existence.

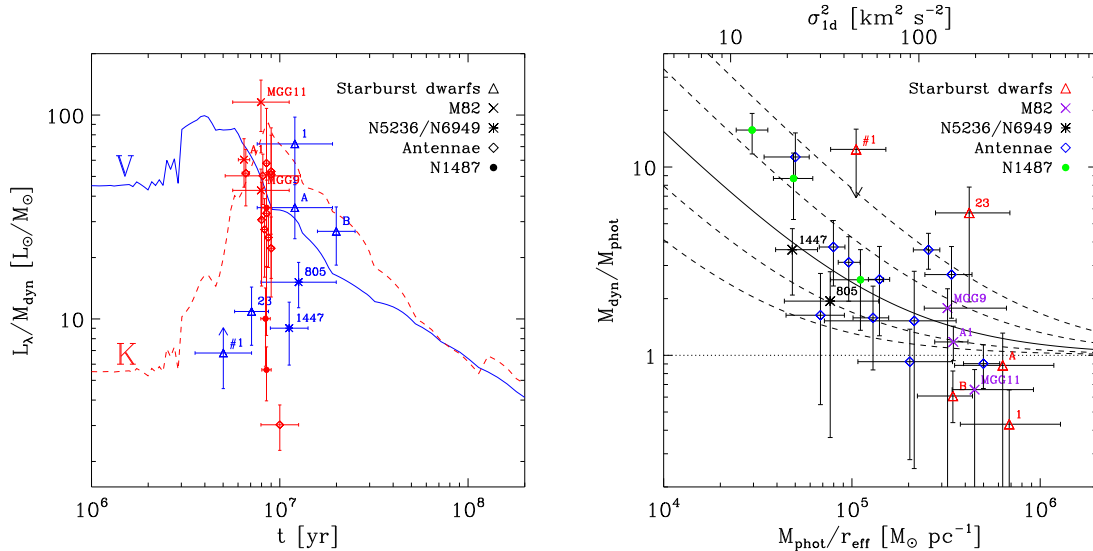


Figure 11: *Left*: Light to dynamical mass ratio for 24 clusters found in the literature and also given in Table 4. The blue and red points refer, respectively, to studies done in the optical and near infrared. Different symbols correspond to different parent galaxy types. Photometric evolution from the Bruzual & Charlot (2003) single stellar population models, using a Chabrier IMF, in the V and K bands is indicated by the full (blue) dashed (red) lines, respectively. *Right*: Dynamical mass over photometric mass for the same clusters, shown as a function of $M_{\text{phot}}/r_{\text{eff}}$, which is a proxy of σ_{1D} (Eq. 4). The full line is a prediction of the effect of binaries on M_{dyn} , with 1- σ and 2- σ variations due to stochastic fluctuations shown as dashed lines (Reproduced from Fig. 3 of Gieles, Sana & Portegies Zwart 2009).

This is most easily seen in the higher value of their dynamical mass M_{dyn} compared to the photometric mass M_{phot} . The former is derived from measurements of the velocity dispersion and the radius of the cluster and the use of Eq. 4. The latter is derived from the total luminosity, calibrated to single stellar population models (see §2). Bastian et al. (2006) determine M_{phot} and M_{dyn} from a compilation of 19 clusters and find that both independent mass estimates are consistent for the somewhat older ($\gtrsim 50 - 100$ Myr) clusters, but that for young (~ 10 Myr) star clusters $M_{\text{dyn}} > M_{\text{phot}}$. Goodwin & Bastian (2006) explain this as a signature of the primordial gas expulsion, and hence of the process of infant mortality.

In Fig. 11 we present an updated version of Fig. 5 of Bastian et al. (2006), showing the ratio of light to dynamical mass of 24 clusters, taken from Table 4, each of which is about 10 megayear old (see §2). The age range is quite narrow because the red supergiant phase, which starts around 10 Myr, makes these clusters extremely bright (especially in the near infrared), whereas younger clusters are still heavily obscured.

Many of the clusters in Fig. 11 and Tab.4 appear to have dynamical masses too high for their luminosities, and the intuitive explanation is that these clusters are expanding and possibly even unbound (Goodwin & Bastian 2006). The time needed for a cluster to completely dissolve, or to find a new virial equilibrium after gas expulsion, is about $\sim 20 - 30 t_{\text{dyn}}$ (see for example Fig. 8 in Baumgardt & Kroupa 2007), where t_{dyn} is the initial

dynamical time, when the gas and the stars are still bound. Hence, to ‘catch’ an unbound or expanding cluster at 10 Myr, the initial t_{dyn} should be $\gtrsim 0.5$ Myr. This corresponds to an half-mass density of $\rho_{\text{hm}} \lesssim 300 M_{\odot} \text{pc}^{-3}$ (Eq. 11 and $\rho_{\text{hm}} \equiv 3M/8\pi r_{\text{hm}}^3$). Clusters with shorter initial t_{dyn} (higher density, see Eq. 11) have expanded into the field, or found a new equilibrium a few megayears after gas expulsion, and are not observable as super-virial clusters at 10 Myr.ⁱ

The initial density of the clusters in Fig. 11 is unknown, but is likely to have been higher in the past than it is today. In addition we may attempt to estimate their initial densities from their current densities. In Fig. 12 we show the radii and masses of the clusters under discussion, together with lines of constant ρ_{hm} . The present day densities are $\rho_{\text{hm}} \approx 10^{3\pm 1} M_{\odot} \text{pc}^{-3}$. The densities in the embedded phase were $O(1/f_e^4)$ higher—the additional factor of $1/f_e^3$ comes from the reduction in cluster mass (by a factor $1/f_e$), the adiabatic expansion (contributing another factor of $1/f_e$, see Eq. 32) and the non-linear response, for a total of roughly $1/f_e^3$. The initial dynamical times were therefore a factor of $1/f_e^2$ shorter than t_{dyn} after phase 1. Based on their physical ages of ~ 10 megayears and the fact that the gas ejection phase does not last beyond the moment of the first supernova (within ~ 3 Myr, see §4), these clusters have evolved for at least $10/f_e^2$ to $100/f_e^2$ initial dynamical times, and hence must be bound (McCradly, Gilbert & Graham 2003; McCradly & Graham 2007), and the observed discrepancy between M_{phot} and M_{dyn} cannot originate from the overall expansion of the cluster.

We are still left with the question why the observed velocity dispersions in some of these clusters are higher than would be expected from the virial theorem. Several independent and implicit assumptions enter the derivation of M_{dyn} and M_{phot} , and each of them could be wrong. The stellar mass function, for example, could be bottom-heavy, i.e. steeper than Salpeter or with an excess of low-mass stars. Such a mass function would result in a velocity dispersion in virial equilibrium higher than that of a cluster with a Salpeter IMF, with little effect on M_{phot} . However, to explain the observed discrepancy, the cluster mass function must deviate substantially from the canonical mass functions. We do not favor this conjecture, since the only star cluster for which the mass function was once anticipated to be deficient in low-mass stars, the Arches (Stolte et al. 2005), turns out to have a rather normal mass function at least down to $1 M_{\odot}$ (Kim et al. 2006).

Another interesting possibility is provided by the binarity of red supergiants, which dominate the observed luminosity. A relatively high fraction of hard binaries (see §3.4.1) leads to an overestimate of the cluster velocity dispersion due to the contribution from their internal orbital motion. This leads to an overestimate of σ_{1D} , and therefore of the mass of the cluster (Kouwenhoven & de Grijs 2008). For typical open clusters, with $\sigma_{\text{1D}} \approx 1 \text{ km s}^{-1}$, this can only account for a factor of ~ 2 increase of M_{dyn} (Kouwenhoven & de Grijs 2008). However, young star clusters are dominated by $\sim 13 - 22 M_{\odot}$ red supergiants, and a binary fraction of $\sim 25\%$ among these stars could explain an apparent dynamical mass of up to an order of magnitude more than the photometric mass (Gieles, Sana & Portegies Zwart 2009). As a consequence the discrepancy between M_{dyn} and M_{phot} is larger for clusters with high stellar

ⁱThe models of Goodwin & Bastian (2006) start with a density in the embedded phase of $\sim 60 M_{\odot} \text{pc}^{-3}$ ($t_{\text{dyn}} \approx 1$ Myr), which according to their analysis results in a gas expulsion timescale of at least 25 Myr.

5.2.1 Theoretical considerations

The time scale for mass loss depends on the mode in which it is achieved; supernova explosions, Wolf-Rayet winds and AGB expulsion result in high mass loss rates, whereas the general mass loss for an older stellar population is relatively slow. When the time scales for mass loss by stellar evolution is considerably longer than t_{dyn} , the cluster responds isothermally, expanding through a series of virial equilibria. For small f_e , Eq. 30 reduces to

$$\frac{\delta r_{\text{vir}}}{r_{\text{vir}}} = \frac{\delta M}{M}, \quad (31)$$

and therefore

$$\frac{r_{\text{vir}}}{r_{\text{vir}}(t=0)} = \frac{M(t=0)}{M}. \quad (32)$$

Even losing half the mass by slow stellar evolution (which for the canonical IMF would not occur within a Hubble time), the cluster would expand by only a factor of two. Note that in the instantaneous approximation (§5.1 and Eq. 30), such mass loss would lead to the dissolution of the cluster.

In reality, the situation is more complicated, in particular because of the connection between dynamical evolution and stellar mass loss. For real clusters the expansion due to stellar mass loss is considerably more severe than suggested above, and can even result in complete disruption if the cluster is mass segregated before the bulk of the stellar evolution takes place (Vesperini, McMillan & Portegies Zwart 2009). Even an initially unsegregated cluster can still undergo mass segregation during the period when the residual gas is being ejected, and certainly during the early evolution of its stars (Applegate 1986), which can also lead to enhanced expansion at later times. The expansion of a mass segregated cluster, however, will not be homologous, as the massive (segregated) core stellar population tends to lose relatively more mass than the lower-mass halo stars. The result is a more dramatic expansion of the cluster core, with less severe effects farther out.

This effect is illustrated in Fig. 13, which presents the results of an isolated $N = 128k$ body simulation, run on a GRAPE-4 (Makino & Taiji 1998) using the `starlab` software environment (Portegies Zwart et al. 2001). The figure shows the evolution of the core radius with and without stellar evolution. Without stellar evolution the core tends to shrink, and eventually reaches core collapse (Bettwieser & Sugimoto 1984), whereas with stellar mass loss the core expands (Portegies Zwart, McMillan & Makino 2007).

The combined effects of mass loss by stellar evolution and dynamical evolution in the tidal field of a host galaxy was studied extensively by Takahashi & Portegies Zwart (2000), Baumgardt & Makino (2003), Fukushige & Heggie (1995). They showed that when clusters expand to a radius of $\sim 0.5 r_J$ they lose equilibrium and most of their stars overflow r_J (see §1.3.2) in a few crossing times.

We conclude that in early phase 2 the overall evolution of the cluster is completely dominated by expansion due to stellar mass loss. However, since most of the mass loss comes from massive stars in the core, the core expansion is considerably larger than expected for the global mass function. The early phase 2 lasts until the response of the cluster to stellar mass loss diminishes and from that moment the cluster can continue to expand until it completely dissolves or until the cluster core starts to contract again due to internal dynamical effects, at which point late phase 2 begins.

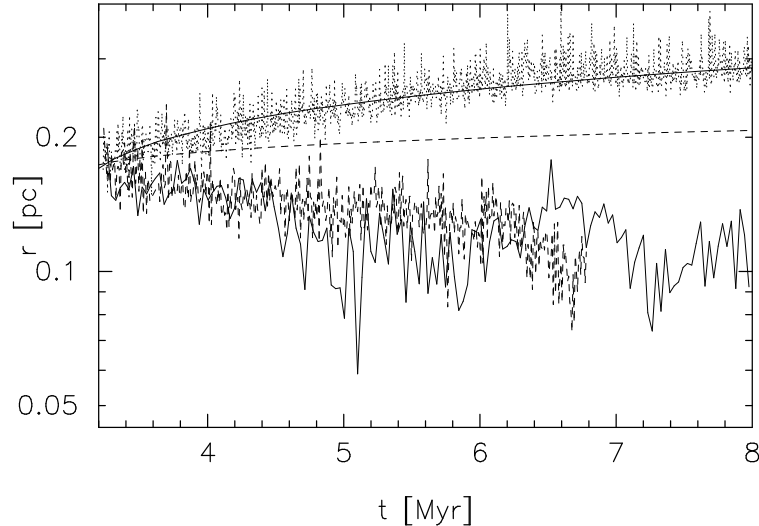


Figure 13: Evolution of the core radius during phase 2 of an N -body simulation ($N = 128k$, $r_{\text{vir}} = 3.2 \text{ pc}$, King $W_0 = 12$, Mass function is Salpeter between $1 M_{\odot}$ and $100 M_{\odot}$ Portegies Zwart, McMillan & Makino 2007). After about 3 Myr stellar mass loss dominates the evolution of the cluster core radius. The dotted curve (top curve) is the result of the full simulation, with both stellar evolution and binary dynamics included; the irregular dashed curve is calculated with the same initial realization but without stellar evolution. The wiggly solid curve (bottom) is calculated without stellar mass loss and without binary dynamics by collapsing the binaries in single objects. The smooth dashed line shows the expected expansion of the core, assuming adiabatic mass loss for a Salpeter initial mass. The smooth solid curve is computed assuming mass loss by stellar evolution from a Salpeter mass function with a lower limit of $15 M_{\odot}$, rather than the $0.1 M_{\odot}$ used in the simulation.

5.2.2 Observational constraints

As we have just seen, rapid dissolution due to stellar evolution mass loss occurs when $r_{\text{hm}}/r_{\text{J}} \gtrsim 0.5$. All the clusters in Fig. 12 have, within a factor of two, $r_{\text{hm}}/r_{\text{J}} \approx 0.03$, suggesting that they are probably all stable against stellar mass loss. Pfalzner (2009) recognizes two evolutionary sequences in young Galactic star clusters, from which she draws a similar conclusion. The first sequence of dense “starburst clusters”, containing the Arches cluster, NGC 3603 and Trumpler 14, starts at a density of $\sim 10^5 M_{\odot} \text{ pc}^{-3}$ at an age of a few megayears. These clusters appear to expand at constant mass, up to an age of 10–20 Myr, where we find the the red supergiant clusters RSGC01 and RSGC02. At that age the cluster density has dropped to $\sim 10^3 M_{\odot} \text{ pc}^{-3}$. The second sequence of “leaky clusters” (which is identical to our definition of associations, see § 2.1) starts at the same age but with a much lower densities of $\sim 10 M_{\odot} \text{ pc}^{-3}$, and expand while $M \propto 1/r_{\text{hm}}$ to densities comparable to the field star density. The clusters in Fig. 12 may be compared with the red supergiant clusters in the Milky Way, i.e. the end point of the dense cluster sequence. The associations discussed by Pfalzner (2009, she refers to them as “leaky” clusters) and listed in Table 2 have dynamical times that exceed the cluster age, and are expected to be unstable against

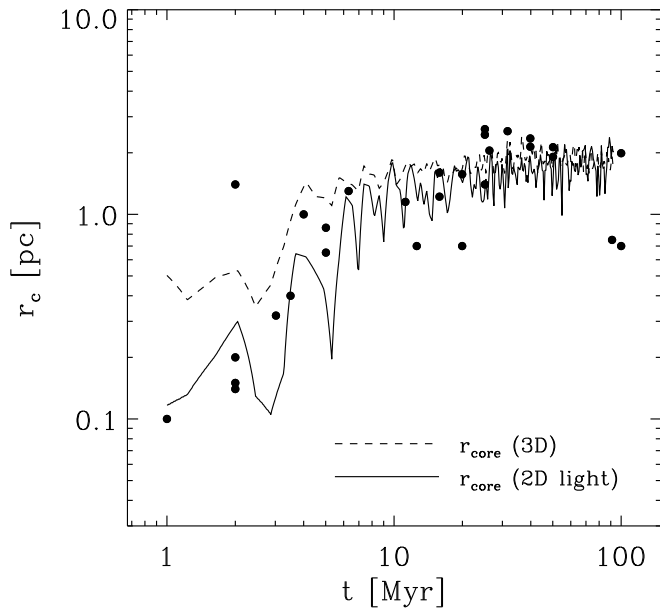


Figure 14: Evolution of observed core radii values of clusters, compared to the results of an N -body simulation including mass loss due to stellar evolution. For the simulation, the projected surface density profile was constructed and r_c was determined using Eq. 5 (full line). The dashed line shows the 3D core radius discussed in §1.3.2.

mass loss by stellar evolution.

In §2 we discussed the remarkable increase in the observed core radii of clusters with ages 1–50 Myr (Mackey & Gilmore 2003b, Bastian et al. 2008). Earlier studies argued that this effect was the result of early gas expulsion (phase 1), but as discussed in §2 the time scale for gas expulsion is too short to affect the growth of r_c over such a long period. Mass loss from the young stellar population (early phase 2) can contribute to some extent to the observed trend, but its effect is probably too weak to explain it completely (Mackey et al. 2007). A more likely solution is dynamical heating from relatively massive objects, such as massive stars and stellar mass black holes, sinking to the cluster center (Merritt et al. 2004).

An additional effect not yet discussed is the difference between the observed r_c , usually resulting from a fit to the surface brightness profile (Eq. 5), and the 3D dynamical core radius described in §1.3.2. If a young star cluster is mass segregated, possibly already from its formation process, then the light is dominated by the few massive stars in the core, which can lead to an underestimate of r_c (Fleck et al. 2005, Gaburov & Gieles 2008). Mackey et al. (2007, 2008) showed that when taking this into account, a remarkable increase of r_c is “observed,” while the “real” core radius changes only very little.

We illustrate this in Fig. 14. The same data points as in Fig. 10 are shown, together with the results of an N -body simulation (lines), where the 3D core radius, as defined in §1 and the r_c resulting from a fit of the EFF87 profile (Eq. 5) to the projected light in the simulated cluster. The cluster consists of $N = 32k$ single stars initially distributed according

to a (King 1966) profile with $W_0 = 8$ and $r_{\text{vir}} = 2.5 \text{ pc}$. Before stellar evolution was turned on, we evolved the cluster for $0.20t_{\text{rh}}$ to mimic some degree of primordial mass segregation, in which stars with masses $\gtrsim 5 M_{\odot}$ are more centrally concentrated than the less massive stars. Since the massive stars dominate the light, the observed r_c is almost a factor of five smaller than the 3D version at $t = 0$. The observed r_c increase by nearly a factor of ten in a few tens of megayears, while the 3D core radius expands only by a factor of two. After $\sim 30 \text{ Myr}$ the two quantities roughly agree.

Primordial mass segregation has a profound effect on the evolution of a star cluster, but possibly much more relevant for this review is its consequences for cluster observations. The assumption that a young ($\lesssim 10 \text{ Myr}$) cluster is not mass segregated, when in reality it is, can dramatically alter observationally derived quantities, such as the cluster size, velocity dispersion, density profile, and central density.

5.3 External perturbations and evaporation

5.3.1 Theoretical considerations

One important external disruptive factor, first considered by Spitzer (1958, see §5.1), is encounters between clusters and giant molecular clouds. Since GMCs are typically more massive than clusters, the cluster is more affected by an encounter than the cloud (Theuns 1991). The cluster lifetime due to heating by passing clouds is inversely proportional to the volume density of molecular gas, ρ_{gas} , and proportional to the density of the cluster:

$$t_{\text{dis}}^{\text{GMC}} \approx 1 \text{ Gyr} \left(\frac{0.03 M_{\odot} \text{ pc}^{-3}}{\rho_{\text{gas}}} \right) \left(\frac{\rho_{\text{hm}}}{10 M_{\odot} \text{ pc}^{-3}} \right). \quad (33)$$

This result is typical of disruption by external tidal perturbations operating on short time scales ($\lesssim t_{\text{dyn}}$), also known as tidal shocks (e.g. Ostriker, Spitzer & Chevalier 1972). Here $0.03 M_{\odot} \text{ pc}^{-3}$ is the molecular gas density in the solar neighborhood and the constant is taken from Gieles et al. (2006c), which is an update from the seminal result by Spitzer (1958). The dependence of t_{dis} on ρ_{gas} indicates that the lifetimes of star clusters scales roughly inversely with the observable surface density of molecular gas, Σ_{gas} .

This result enables us to make order of magnitude estimates of the lifetimes of clusters in other galaxies. In spiral galaxies, YMC disruption by GMC encounters is especially important during the early stages of cluster evolution, since clusters form in the thin gaseous disk where ρ_{gas} is high. Older clusters are typically more associated with the thick disk, where ρ_{gas} is low and GMC encounters are less frequent. Since young ($\lesssim 1 \text{ Gyr}$) clusters in spiral galaxies have only a small range in radii (e.g. Larsen 2004), more massive clusters tend to have higher densities, making them less vulnerable to encounters with GMCs, which explains their longer lifetimes compared to their lower-mass counterparts (Gieles et al. 2006c). It is not clear whether the lack of a mass-radius relation is a universal property imprinted at cluster formation, or the result of evolution.

5.3.2 Observational constraints

Several studies (Zepf et al. 1999, Larsen 2004) discuss the lack of any clear correlation between the size of a cluster and its mass or luminosity. For the 24 clusters in Tab. 4 we

tentatively overplot lines of constant density in Fig. 12, which could indicate some sort of a trend with $\rho_{\text{hm}} \approx 10^{3\pm 1} \text{ M}_{\odot}$.

In Fig. 12 we limited ourselves to a narrow range of cluster ages, whereas most literature studies did not enforce this age limit and considered clusters with a large age spread. The existence of a mass-radius relation would be important for understanding cluster disruption, since $t_{\text{dis}}^{\text{GMC}} \propto \rho_{\text{hm}}$ (§5.3.1). If clusters form with a constant density, i.e. a mass-radius relation of the form $r_{\text{hm}} \propto M^{1/3}$, then their destruction time due to GMC encounters is independent of cluster mass. Additional complications arise from the time dependence of a mass-radius relation. Clusters older than the ones shown in Fig. 12 seem to have a near constant radius, i.e. $r_{\text{hm}} = \text{constant}$, which could be the consequence of evolutionary effects, such as mass segregation and stellar mass loss. This would lead to a destruction time scale by GMCs ($t_{\text{dis}}^{\text{GMC}}$) that is dependent of the cluster mass: $t_{\text{dis}}^{\text{GMC}} \propto M/r_{\text{hm}}^3$. In any case, the YMCs listed in Tab. 4 are unlikely to be destroyed by passing GMCs in a short time-scale, as $t_{\text{dis}}^{\text{GMC}}$ exceeds a Hubble time due to their high densities.

While considering mass loss from star clusters it is convenient to distinguish between two fundamental processes: evaporation and tidal stripping. Evaporation is the steady loss of stars from the cluster driven by the continuous repopulation by relaxation of the high-velocity tail of the Maxwellian velocity distribution (see § 3.3.1, Eq. 13 and, e.g. Ambartsumian 1938, Spitzer 1940). This process has been the subject of numerous comprehensive numerical studies (Spitzer 1987, Aarseth 2003, Heggie & Hut 2003, Baumgardt & Makino 2003).

Tidal stripping is the prompt removal of stars that find themselves outside the cluster Jacobi radius (r_{J} , see 1.3.2) due to internal processes such as stellar mass loss or a change in the external tidal field, for example as the cluster approaches pericenter in its orbit around its parent galaxy. On a $\sim 100 \text{ Myr}$ time scale, and for clusters with masses $\gtrsim 10^4 \text{ M}_{\odot}$, relaxation is unlikely to be important, and so tidal stripping dominates the cluster mass loss.

Until the 1990s, the main targets of cluster disruption studies were the open clusters in the Milky Way and the YMCs in the Magellanic clouds. Since then, HST observations have established the properties of large populations containing more massive clusters in quiescent spiral galaxies (e.g. Larsen & Richtler 2000), interacting galaxies (e.g. Whitmore et al. 1999, Bastian et al. 2005), and merger products (e.g. Miller et al. 1997). The primary tool used in studies of cluster disruption is the cluster age distribution. Different groups give different weights to the various factors described in §5.1 in interpreting the results, but empirical cluster disruption studies follow one of two basic models, in which the disruption is either *externally* or *internally* driven.

In the externally driven model, the dissolution time follows a simple scaling relation with cluster mass and environment, following the age and mass distributions of luminosity-limited cluster samples in different galaxies (Boutloukos & Lamers 2003). Variations in dissolution time scales are explained by differences in the tidal field strength (Lamers, Gieles & Portegies Zwart 2005) and the GMC density (Gieles et al. 2006c).

The internally driven model assumes that internal processes dominate the cluster disruption and that roughly 80–90% of all remaining clusters are destroyed during each decade in age, resulting in a (mass limited) age distribution that declines inversely with time ($\propto t^{-1}$). The main assumptions are that the majority (80 – 90%) of clusters dissolve within a few hundred megayears and that the disruption rate does not depend on mass. This model

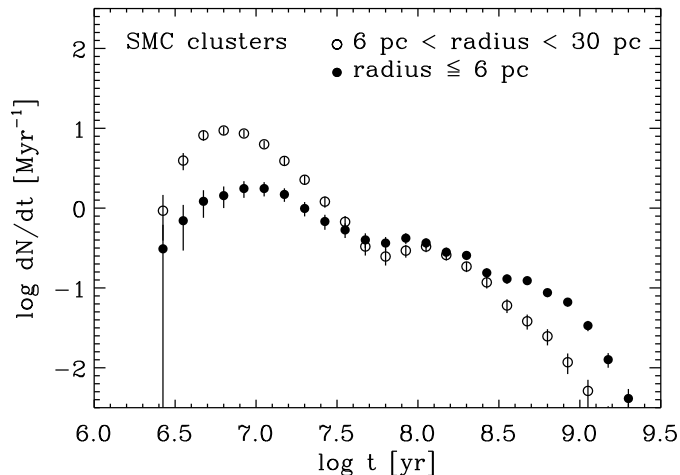


Figure 15: Age distribution of SMC clusters based on the catalog of Chiosi et al. (2006). The sample is split into small and large clusters/associations, with the boundary at a radius of 6 pc. The histograms are made using a 0.5 dex bin width with different starting values (boxcar averaging).

however, is calibrated against the cluster population in the Antennae galaxies, as discussed in §5.1 (Fall, Chandar & Whitmore 2005; Whitmore, Chandar & Fall 2007), and while it is quite consistent with these observations, it is not clear how applicable it is to other galactic environments.

The various cluster dissolution models have led to some controversy, resulting in a number of spirited discussions at conferences and in the literature. Chandar, Fall & Whitmore (2006) demonstrated that the age distribution of SMC clusters declines $\propto t^{-0.85}$, which is consistent with their results for the Antennae. Gieles, Lamers & Portegies Zwart (2007) are able to reproduce these results only if they impose an incompleteness in the detection of clusters, which is consistent with the arguments of de Grijs & Goodwin (2008). As a consequence Gieles, Lamers & Portegies Zwart (2007) conclude, using the same data sample used by Chandar, Fall & Whitmore (2006), that the age distribution of massive ($\gtrsim 10^{3.5} M_{\odot}$) clusters in the LMC younger than a few hundred megayears is not affected by internal processes, contradicting the findings of Chandar, Fall & Whitmore (2006).

The conclusions of Gieles, Lamers & Portegies Zwart (2007) are supported by Boutloukos & Lamers (2003), who show that for a constant formation rate of clusters and without disruption, the age distribution of a luminosity-limited cluster sample declines $\propto t^{-\zeta(\beta-1)}$, where $-\beta$ is the index of the cluster initial mass function. The parameter ζ describes how the cluster fades with age, due to stellar evolution: $F_{\lambda}(t) \propto t^{-\zeta}$, where F_{λ} is the flux at wavelength λ of a cluster with constant mass. For the U and V-bands $\zeta \approx 1.0$ and 0.7 , respectively, which for $\beta = 2$ results in $t^{-\eta}$, with $0.7 \lesssim \eta \lesssim 1.0$, due to fading alone. If the distribution of cluster masses is described by a Schechter function (see Eq. 17 in §2.2) the age distribution of a luminosity-limited sample is not affected by disruption and is as steep as $0.9 \lesssim \eta \lesssim 1.4$.

The internal disruption model for the age distribution relies on the assumption that the cluster formation history is constant within the age range considered (a few hundred megayears). Bastian et al. (2009) demonstrate that in the Antennae a variable cluster formation rate can (at least) partly explain the decline in the number of clusters with age, without the need to invoke rapid cluster disruption.

The discussion of the distribution of the number of clusters with age is complicated by the distinction between associations and dense clusters (see §2.3.1 and §5.1.2), which may be hard to distinguish at large distances. This is illustrated in Fig. 12, which shows the age distribution of clusters and associations in the central region of the SMC (Chiosi et al. 2006). The sample is divided into two subsamples of ~ 200 clusters each, based on size. The age distribution of the large ($r_{\text{eff}} > 6\text{pc}$) clusters falls off much more rapidly than that of the more compact clusters. The median radii of the two samples are 4.5 pc and 9 pc, respectively, but at a distance of 20 Mpc it would be very difficult to tell these two groups apart. We suggest that by relaxing the cluster size in the observationally selected sample one includes more short-lived associations, which if resolved should probably not be considered genuine star clusters.

6 Lusus Naturæ

Globular clusters are of interest because of their old age, their assumed relatively homogeneous populations, their relative isolation in their parent galaxies, and because of their abundance of unusual objects, such as blue stragglers, x-ray binaries, radio pulsars, etc., often referred to collectively as *stellar exotica*, or *lusus naturæ*[‡]. In the disk of the Milky Way, such objects form through internal evolutionary processes in individual stars or close binary systems. In star clusters, these processes are augmented by stellar interactions, mediated by the high encounter frequency in dense cluster cores.

Stellar encounters generate new channels for the formation of exotic objects, but can also catalyze existing channels. For example, a binary encounter may lead directly to a collision and the formation of a blue straggler, or its effect may be indirect, perhaps resulting in an exchange that eventually (billions of years later) leads to the formation of a low-mass x-ray binary. Repeated encounters can transform binaries and multiple stellar systems, multiplying the channels for the production of exotica (Davies 1995, Hurley & Shara 2002, Davies et al. 2006). A clear understanding of the formation and evolution of these objects can provide insight into the past dynamical evolution of the cluster (Davies 2009).

Many of the stellar exotica observed in globular clusters today are the results of processes that began when the cluster was young. In some cases, they are the products of the interplay between dynamical and evolutionary processes involving stars and binaries during the first ~ 100 Myr of the cluster lifetime. The primordial seeds for many *lusus naturæ* were planted during this period (Portegies Zwart, McMillan & Makino 2007). Later phases of dynamical evolution, such as core collapse, may produce additional generations of exotica. Thus, even though we remain uncertain as to whether or not observed YMCs will ever become “true” globular clusters (see §1), they nevertheless provide a convenient testbed for the study of the progenitors of stellar exotica.

[‡]Lusus Naturæ is Latin for the freaks of nature, mutants or monsters.

The progenitors of observed *lusus naturæ* in globular clusters are not necessarily easy to identify in the young cluster population, although in some cases the evolutionary link is well established (Glebbeek, Pols & Hurley 2008). There may well be entire populations of peculiar objects in young star clusters that do not lead to observable interesting objects at later stages, and some objects destined for peculiarity may look perfectly ordinary at early times. An example of the latter is the dormant blue straggler population consisting of stars that were rejuvenated by mass transfer or collisions while still on the main sequence, and now lurk among their fellow main-sequence stars until they remain behind after the others traverse the Hertzsprung gap (Portegies Zwart, Hut & Verbunt 1997).

6.1 Binary Stars

Exotic objects in star clusters are closely related to binaries, as they often form via internal binary evolution or during dynamical interactions between binaries and other stars. Examples are the formation of blue stragglers (§6.1.1), colliding wind binaries (§6.1.2), and anomalous x-ray pulsars (§6.2.1), all of which require the presence of binary stars in the system. In some cases, such as the slow evolution of an accreting X-ray pulsar that leads from a low-mass x-ray binary to a binary millisecond pulsar, the evolutionary track is readily established (Bhattacharya & van den Heuvel 1991). In others, however, the punctuated equilibrium through which these objects evolve makes it virtually impossible to catch the key transitions as they occur. Examples are collisions between stars, or the common-envelope phase in the Darwin–Riemann instability of a contact binary.

We will distinguish two fundamental types of binaries in star clusters: (1) “primordial” binaries, which formed contemporaneously with the stars in the cluster as a crucial part of the star-formation process (Goodman & Hut 1989), and (2) “dynamical” binaries, which formed later via stellar interactions, often long after the component stars reached the main sequence. One can wonder to what extent this limited terminology is still usable for binaries that experiences one or more exchange interactions. It is for example, quite possible that two stars that were initially single end-up in a binary after two exchanges. As a practical matter we would still consider such a binary primordial. The second group of dynamical binaries may be further divided into two sub-categories—binaries formed by conservative three- and four-body stellar dynamical interactions, and binaries formed by dissipative two-body tidal capture. The latter process was introduced by Fabian, Pringle & Rees (1975) to explain the relatively high specific frequency of low-mass X-ray binaries in globular clusters. It has fallen somewhat out of favor since the late 1980s, but it may be entering a revival of sorts Ogilvie & Lin (2004). Many of the curious objects discussed in this § are related to binarity, either primordial or tidal, although we tentatively will use this distinction in the formation process.

6.1.1 Blue Stragglers

A blue straggler is a star which exceeds the cluster main-sequence turnoff in both temperature and luminosity, but which is not on the horizontal branch. Blue stragglers populate the region blueward of the turnoff, as if they lagged behind on the cluster main sequence while the other stars aged. The first (34) blue stragglers were discovered in the globular cluster M3

by Sandage (1953). At least 8 plausible explanations have been proposed for the formation of blue stragglers (see Leonard 1989). Of these, two are currently in favor:

- direct merger between two stars (Hills & Day 1976),
- mass transfer in a close semi-detached binary star (McCrea 1964).

The latter scenario is supported by the discovery of two blue stragglers, in the young open star clusters NGC 663 and NGC 6649, which have been found to be the donors in Be/X-ray binary systems (Marco, Negueruela & Motch 2007). The discovery of a blue straggler in the old open cluster M67, which appears to be about 2.5 times more massive than the turn-off mass, favors the former view (Leonard 1996).

Both favored mechanisms for blue straggler formation appear plausible in YMCs. However, no blue stragglers have yet been identified in any observed YMC, although this may be explained by the absence of a clear turn-off in the resolved clusters, which makes the identification (and the definition) of a blue straggler impractical. There are, however a number of “odd” stars in YMCs that might possibly evolve to resemble blue stragglers in the future. Objects consistent with this broadened definition include four O3 If/WN6-A stars in the star cluster R136 in the 30 Doradus region of the LMC (Campbell et al. 1992, Brandl et al. 1996).

Several interesting correlations exist between the numbers of blue stragglers in globular clusters and the numbers of red giants (Ferraro, Fusi Pecci & Bellazzini 1995), and also with the binary fraction (Sollima et al. 2008; Knigge, Leigh & Sills 2009). In addition, Davies, Piotto & de Angeli (2004) find that the number of blue stragglers is independent of integrated absolute magnitude M_V of the cluster, and use this fact to argue that both production mechanisms are relevant.

6.1.2 Colliding wind binaries

Binaries containing two massive stars, such as Wolf-Rayet stars, with strong stellar winds often exhibit intense radio and/or x-ray emission. Since this process requires copious stellar mass loss in a fast wind, these sources do not occur in globular clusters, but YMCs appear to be excellent hosts for such systems. Several young and dense star clusters exhibit x-ray and radio emission from colliding wind binaries. In some relatively nearby cases, R136 (Portegies Zwart, Pooley & Lewin 2002), Wd1 (Clark et al. 2005, Crowther et al. 2006), and the Arches and Quintuplet clusters (Lang et al. 2005), the counterparts of the radio and x-ray sources have been identified.

6.2 Compact objects

A number of YMCs have been observed in the radio and x-ray, revealing a wealth of sources, even richer than found in globular clusters (Clark et al. 2008). Among the x-ray sources is a large population of accreting neutron stars, stellar-mass black holes and possibly intermediate-mass black holes. Because of crowding in the central regions of these clusters, where most of the x-ray sources are found, very few sources have optical counterparts.

With the adopted age limit of 100 Myr, only relatively few white dwarfs have formed—about as many as neutron stars—and cataclysmic variables are not expected. The youth of these clusters seem to make it unlikely that any low-mass x-ray binaries or millisecond pulsars will be found.

6.2.1 Magnetars

Shortly after a supernova (within $\sim 10^5$ years), a newly formed neutron star may become observable as a magnetar, which can have a magnetic field strength exceeding $\sim 10^{15}$ gauss (Kouveliotou et al. 1999). The population of magnetars is subdivided into two classes: soft gamma-ray repeaters (SGRs) and anomalous x-ray pulsars (AXPs)^k. As the products of supernovae, one might naively expect these objects to reside mainly in YMCs, and indeed about half (3 of 8) of the known SGRs and one-tenth (1 of 10) of the AXPs are known to reside in such systems. This is a remarkably high fraction, given that only 0.05% of the stellar mass of the Galaxy resides in star clusters (see §1).

The single cluster AXP is CXOU J164710.2-455216 (Muno et al. 2006) in the Galactic young star cluster Westerlund 1. It exhibits a 20-ms burst with energy $\sim 10^{37}$ erg (15-150 keV), and spins down at a rate of $P/\dot{P} \simeq -10^{-4}$ (Muno et al. 2007), quite typical of a magnetar.

Several interesting sources are hosted by young star clusters in the LMC. These include the microquasar LS I +61° 303, which may have been ejected from the cluster IC 1805 (Mirabel, Rodrigues & Liu 2004). The relatively low-density young star cluster SL 463, too small for inclusion in this review, seems to be associated with SGR 0526-66 (Klose et al. 2004) at a projected distance of ~ 30 pc. Two other SGRs associated with relatively low-density young star clusters are SGR 1806-20, at a projected distance of ~ 0.4 pc from the core of its parent cluster (Mirabel, Fuchs & Chaty 2000; Corbel & Eikenberry 2004), and SGR 1900+14, at ~ 0.8 pc (Mirabel, Fuchs & Chaty 2000; Vrba et al. 2000). This latter SGR has a measured proper motion of 70 mas/yr away from the cluster, suggesting that it indeed was ejected from the parent cluster (DeLuca et al. 2009), which is relatively old (14 ± 1 Myr) compared to the usual SGR-producing stars (Davies et al. 2009). The actual association between cluster and SGR is hard to establish in this case since the distances to both objects are ill constrained.

6.2.2 Ultra-luminous X-ray sources

Globular clusters are known to host an enormous excess of low-mass x-ray binaries compared to the rest of the Galaxy. Much of this excess is attributed to the dynamical environment in dense cluster cores (Fabian, Pringle & Rees 1975; Pooley et al. 2003). Young star clusters are sites of intense dynamical activity, so it is not surprising that YMCs also host many x-ray sources. The majority of x-ray point sources in external galaxies appear to be associated with young star clusters, as is the case for example in the Antennae system (NGC4038/39) Zezas et al. (2002). The nature of most of these x-ray sources is unknown, and we can only guess at their origin.

^ksee <http://www.physics.mcgill.ca/~pulsar/magnetar/main.html>.

We limit ourselves here to the most striking x-ray sources, the subclass of ultra-luminous x-ray sources (ULXs), which are characterized by x-ray luminosity $L_x \gtrsim 1.3 \times 10^{39}$ erg/s, the maximum isotropic luminosity that can be produced by a $10 M_\odot$ black hole accreting pure hydrogen (King et al. 2001). For practical reasons we round the threshold up to $L_x \gtrsim 10^{40}$ erg/s, mainly to ensure that such luminosities are unlikely to be produced by stellar-mass black holes accreting at the Eddington rate from a main-sequence companion star. These ULXs are responsible for the brightest stellar x-ray sources in the sky.

Several models attempt to explain the high x-ray luminosity of the ULXs, but at present there is no consensus in the community on the source of the x-rays. The current leading models are:

- anisotropic (collimated or beamed) emission from an accreting stellar-mass black hole (although porosity, turbulence, and bubbles also provide interesting alternatives, King et al. 2001, King 2002),
- accretion from an evolved star onto a stellar-mass black hole, which can, in principle, lead to an accretion rate higher than from a main-sequence star, and therefore a higher x-ray luminosity (Madhusudhan et al. 2006),
- accretion from a companion star onto an “intermediate-mass black hole” (IMBH), with mass $\gtrsim 100 M_\odot$ (Portegies Zwart et al. 2004).

ULXs tend to be hosted by starburst and spiral galaxies (Makishima et al. 2000). Some of the brightest are associated with YMCs; a leading example is the ULX in the star cluster MGG 11 in the starburst galaxy M82 (Kaaret et al. 2001). The association with YMCs argues in favor of an accreting black hole of $\sim 1000 M_\odot$ (Portegies Zwart et al. 2004). The object in MGG 11 is particularly interesting, as it shows a strong quasi-periodic oscillation in the 50-100 mHz frequency range (Strohmayer & Mushotzky 2003), providing a strong argument against beamed emission, and supporting the hypothesis that the x-ray luminosity comes from an accreting black hole of $200\text{--}5000 M_\odot$. Further support is provided by the detected periodic variation of ~ 62 days, which can be explained if the black hole is orbited by a $22\text{--}25 M_\odot$ Roche-lobe filling donor star (Patruno et al. 2006).

ULXs have been associated with YMCs in NGC 5204 (Liu, Bregman & Seitzer 2004), the starburst galaxies M82 (Kaaret et al. 2001) and NGC1313 (Grisé et al. 2008), the edge-on spiral NGC 4565 (Wu et al. 2002), the interacting galaxies M51 (Liu et al. 2002), NGC4038/39 (Antennae Fabbiano, Zezas & Murray 2001) and ESO 350-40 (Cartwheel Gao et al. 2003), and the type 1.5 Seyfert galaxy NGC 1275 (González-Martín, Fabian & Sanders 2006). The higher abundance of ULXs in active, starburst, and interacting galaxies may be related to the empirical fact that YMCs tend to form in these environments; 60% of ULXs are associated with active star-forming regions (Swartz, Tennant & Soria 2009, although the definition of a ULX used here is somewhat faint).

If the counterpart of a ULX hosts an IMBHs, it is likely to be the the acceptor from a windy or Roche-lobe overflowing massive star, as seems to be the case in the $10^{39}\text{--}10^{41}$ erg/s ULX in NGC 5204, where the donor is identified as a B0 Ib supergiant with a 10-day orbital period (Liu, Bregman & Seitzer 2004), and in ULX M51 X-7, which has has an even shorter orbital period of only 2.1 hr (Liu et al. 2002), although no stellar companion has

been identified. The black holes that may be responsible for the ULXs NGC 1313 X-1 and X-2 (0.2-10.0 keV) may have masses in the range 100–1000 M_{\odot} (Miller et al. 2003), and in these cases YMCs have been identified as optical counterparts.

6.3 Explosive events

6.3.1 Supernovae

Supernova are relatively rare events, occurring about once every 100 years in a galaxy like the Milky Way (Cappellaro, Evans & Turatto 1999). Although type I supernovae are unlikely to occur in star clusters younger than 100 Myr (Pfahl, Scannapieco & Bildsten 2009), at these ages the seeds may be planted for a rich future of type I events (Shara & Hurley 2002).

Since most massive stars tend to reside in clusters, as discussed in §1 and §2, it is probable that the majority of type Ib/c and type II supernovae occur in star clusters. However, since most supernovae occur in distant galaxies it is hard to find optical counterparts, and very few associations of supernovae with young star clusters have been reported. Based on the peculiar metallicity of SN 1987A, Efremov (1991) argue that the star originated in the young LMC cluster MKM90. SN 2006gy (Foley et al. 2006), which is a candidate for the formation of an IMBH, may have been the result of a collision runaway (§3.4.2) in a YMC (Portegies Zwart & van den Heuvel 2007). However, perhaps the strongest case is the peculiar type IIp supernova SN 2004dj (probably produced by a 12–20 M_{\odot} star) in the spiral galaxy NGC 2403; as it faded, the star cluster Sandage-96 reappeared (Wang et al. 2005).

6.3.2 Gamma-ray bursts

Any word written about gamma-ray bursts is likely to trigger its own burst of e-mail, but we cannot resist the temptation to devote a few lines to this fascinating transient phenomenon. Gamma-ray bursts come in two types, of short ($\lesssim 2$ s) and long duration, respectively Mészáros (2002). Several theories exist which point to either globular or young star clusters as possible hosts for gamma-ray bursts. Colliding compact objects are often cited as sources for the short bursts (Narayan, Piran & Shemi 1991). Long bursts are thought to be hosted by massive star forming regions (Paczynski 1998). Of particular interest is the elusive relation between the long bursts and YMCs (Efremov 2000). The models for long-duration gamma-ray bursts should be particularly applicable to YMCs, as they require rapidly rotating high-mass stars (Heger et al. 2003), which could be achieved quite naturally by stellar collisions in a YMC (see §3.4.2).

6.4 Summary of exotica

We have discussed several examples of exotica in YMCs, but other curiosities remain. Many of these exotic objects are well studied in globular clusters, but similar scrutiny is so far lacking in their younger siblings. Rather than providing a detailed description of each of the oddities found, we simply mention a few developments and recent discoveries of what today we call exotic objects, which one day we may consider “normal.” The following list summarizes a number of peculiar clusters, noteworthy because of a unique source or object.

The list is far from complete, but at the very least it indicates the diversity of objects found in YMCs.

- **R136** Contains some 13 colliding wind binaries (Portegies Zwart, Pooley & Lewin 2002), and possibly 3 blue stragglers (even though no clear turn-off can be distinguished). In addition, the star cluster shows an OH (1720 MHz) Maser, which is probably related to the surrounding nebula rather than the star cluster itself (Roberts & Yusef-Zadeh 2005).
- The **Arches cluster** contains 10 radio point sources (Lang et al. 2005) and several colliding wind binaries.
- **MGG11** is a YMC in M82 which may contain a ULX (Kaaret et al. 2001), although the Chandra error box is slightly offset.
- The **Quintuplet cluster** contains the Pistol star (Figer et al. 1998), a candidate for the most massive star in the Galaxy, at a projected distance of about 1 pc, as well as 9 radio point sources (Lang et al. 2005).
- **Westerlund 1** hosts the anomalous x-ray pulsar CXOU J164710.2-455216 (Muno et al. 2006), and a wealth of x-ray sources (Clark et al. 2008).
- **Westerlund 2** hosts the massive Wolf-Rayet binary WR 20a, containing two WN6ha stars, at a distance of 1.1 pc from its center (Rauw et al. 2005).
- **NGC 663 and NGC 6649** contain blue stragglers which found to be the donors of Be/X-ray binary systems (Marco, Negueruela & Motch 2007).
- **Sandage-96** is a $\sim 96,000 M_{\odot}$ star cluster in NGC 2403 in which a type IIp supernova was detected (Wang et al. 2005). This star cluster also exhibits multiple stellar populations (Vinkó et al. 2009).

6.4.1 Planetary nebulae and supernova remnants

The nuclear evolution of a sufficiently massive star ($\gtrsim 8 M_{\odot}$) is associated with a supernova explosion (see §6.3.1), while lower mass stars fizzle into the background after a short, bright post-AGB phase. But following these lower mass events a roughly spherical gas shell—a planetary nebula—remains visible for a much longer time than either the supernova or the post-AGB star, illuminated by the central stellar remnant and shocks as the out-flowing gas encounters the interstellar medium.

Every star experiences either a supernova or a post AGB phase and a cluster of 5×10^4 stars will experience some 350 supernova, leading to an equal number of remnants, while during the first 100 Myr a similar number of planetary nebulae will form. The formation rate of planetary nebulae and supernova remnants is thus $\sim 7/\text{Myr}$. With an observable lifetime for a nebula of about 10^4 yr, we naively expect to see one nebula for every ~ 14 star clusters. Among 80 YMCs Larsen & Richtler (2006) found 6 with a planetary nebula, consistent with our naive estimate. No planetary nebulae or supernova remnants have so far been found in any of the YMCs in the Local Group.

6.4.2 Brown dwarfs and planets

We will say little here about brown dwarfs and planets in YMCs since the observations are sparse and the general topic deserves its own review. However, a few words are in order.

Young dense star clusters are generally too distant for planets to be detectable using current methods, and no planets have been found to date (Udry & Santos 2007). Globular clusters seem to be deficient in planets (e.g. 47 Tuc, Gilliland et al. 2000), possibly because of their low metallicities (Weldrake et al. 2005). However, the recent HARPS discovery of a planet around a metal poor star (Mayor et al. 2009) makes this argument less convincing. The planet found orbiting the 11 ms pulsar B1620-26 in the metal poor environment of the globular cluster M4 (Backer 1993; Thorsett, Arzoumanian & Taylor 1993), was formed by a different mechanism than planets around solar-type stars, and we do not (yet) expect such planets in YMCs. Star clusters in high-metallicity environments, such as Westerlund 1 and NGC 3603, could be brimming with planets, but none have yet been found.

We see no reason why YMCs should be deficient in planets. However, the high interaction rate in a dense cluster could make a planetary system short-lived. Disruption of a planetary system may leave the planet separated from its parent star (Spurzem et al. 2009). Several such free-floating objects have been found in the Orion Trapezium cluster (Lucas & Roche 2000). Once dislocated from its parent star, a planet will easily escape the cluster, though.

6.4.3 YMCs in a galactic context

Due to supernovae and winds from massive stars, YMCs are the sources of cluster winds (Silich, Tenorio-Tagle & Rodríguez-González 2004), which may trigger supplanting winds and chimneys, as in the Perseus arm of the Milky Way Galaxy (Normandeau, Taylor & Dewdney 1996). Although little studied from the point of view of cluster evolution, this is an important topic that provides a possible and rather natural link between the evolution of a YMC and that of its parent galaxy.

7 Concluding Remarks: Young Globular Clusters?

The discovery of large numbers of young star clusters, particularly in other galaxies, over the last decade has led to the realization that such clusters are responsible for a significant fraction of all current star formation in the local universe. The study of these systems, and especially their lifetimes against various stellar evolutionary and dynamical processes, is therefore of critical importance to several branches of stellar and galactic astrophysics.

Star clusters appear to form with a cluster mass function described by a power-law with index -2 . This mass function seems to be the same for both open and globular clusters, and does not depend significantly on the local galactic environment or on the specific characteristics of the giant molecular clouds from which the clusters formed.

Young massive star clusters evolve and eventually dissolve due to the combined effects of a number of physical processes, the most important (for clusters that survive the early expulsion of their natal gas) being mass loss due to stellar evolution. The most massive clusters, such as those found in the Antennae system, have expected lifetimes comparable to the age of the universe, and we could well imagine that the Antennae will someday be

a medium-sized elliptical galaxy with an extended population of intermediate-age clusters having overall properties quite comparable to the old globular clusters seen in other ellipticals.

The seeds of the exotica observed in many present-day globular clusters were sown during the infancy of those systems, in the strongly coupled mix of stellar dynamics and stellar evolution that characterized their early evolution. Young massive clusters destined to survive to ages comparable to the globular clusters appear to contain much richer populations of stellar exotica (per unit mass) than are found in the field, and may provide important testbeds for this unique period in cluster evolution. Our limiting cluster age of 100 Myr is chosen in part to include the period when this ecological interplay is strongest.

From an observational point of view, little is known of the formation and evolution of stellar exotica in young massive star clusters, mainly because such clusters are relatively rare. The Milky Way contains only half a dozen, and the closest lies ~ 4 kpc away, too distant for detailed study of the individual stars in its central region. A number of studies have drawn connections between young massive clusters and exotic objects, such as unusual supernovae, magnetars, x-ray binaries, and ultraluminous x-ray sources. This renders YMCs in the same league as the old GC's, which are brimming with curios objects.

In the end, despite the connections, we are unconvinced that “young globular cluster” is an appropriate term for the young massive clusters discussed in this review. For this reason we have omitted the term completely from our discussion, to concentrate on the massive $\gtrsim 10^4 M_\odot$ star clusters with ages $\lesssim 100$ Myr. An important parameter for a young bound cluster appears to be its age relative to its current dynamical time scale t_{dyn} . For unbound stellar agglomerates or associations, t_{dyn} exceeds the system age, indicating that the cluster is either extremely young or expanding into the tidal field of its parent galaxy. For the typical bound star clusters in this review, t_{dyn} is smaller than the current age, whereas for an association it is the other way around.

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