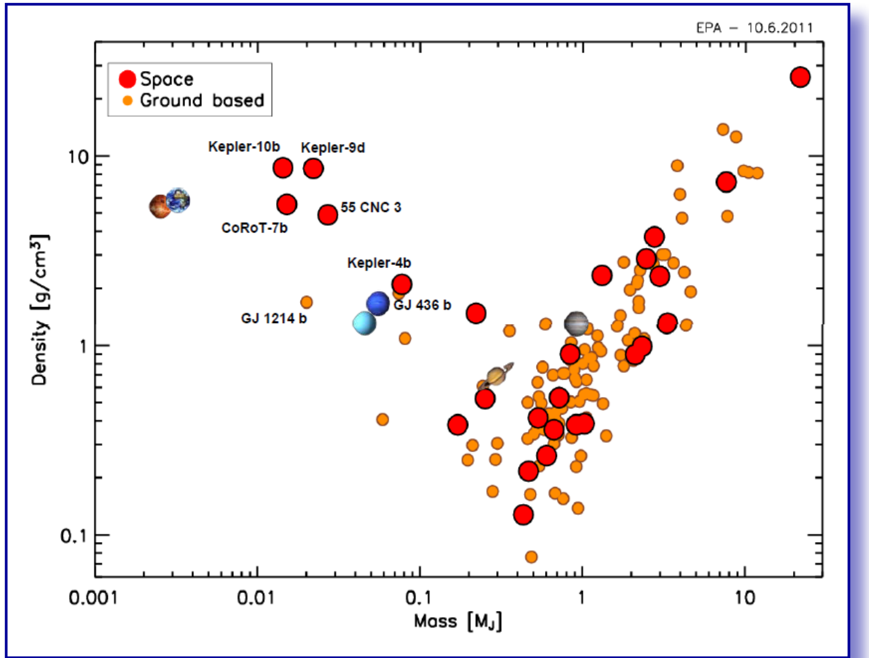
Astronomy from Space
Lecture 2
Exoplanets

Planet diversity



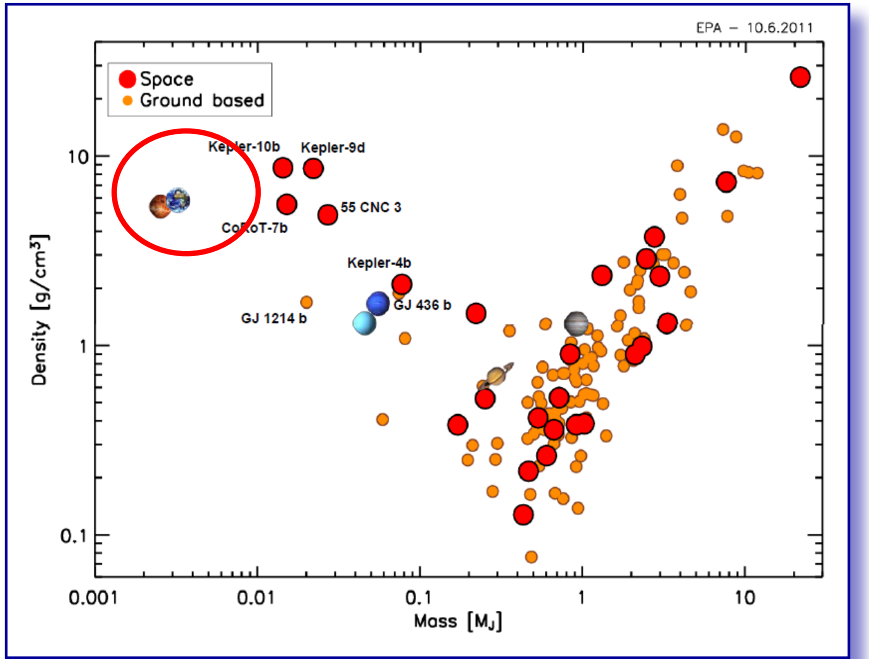
- Gas giant planets show a range of radii and mean densities for similar masses, including inflated planets

- A similar range is hinted for icy and rocky planets

Future:

- Detect a statistical sample of low-mass planets over a wide range of radii and masses

Planet diversety



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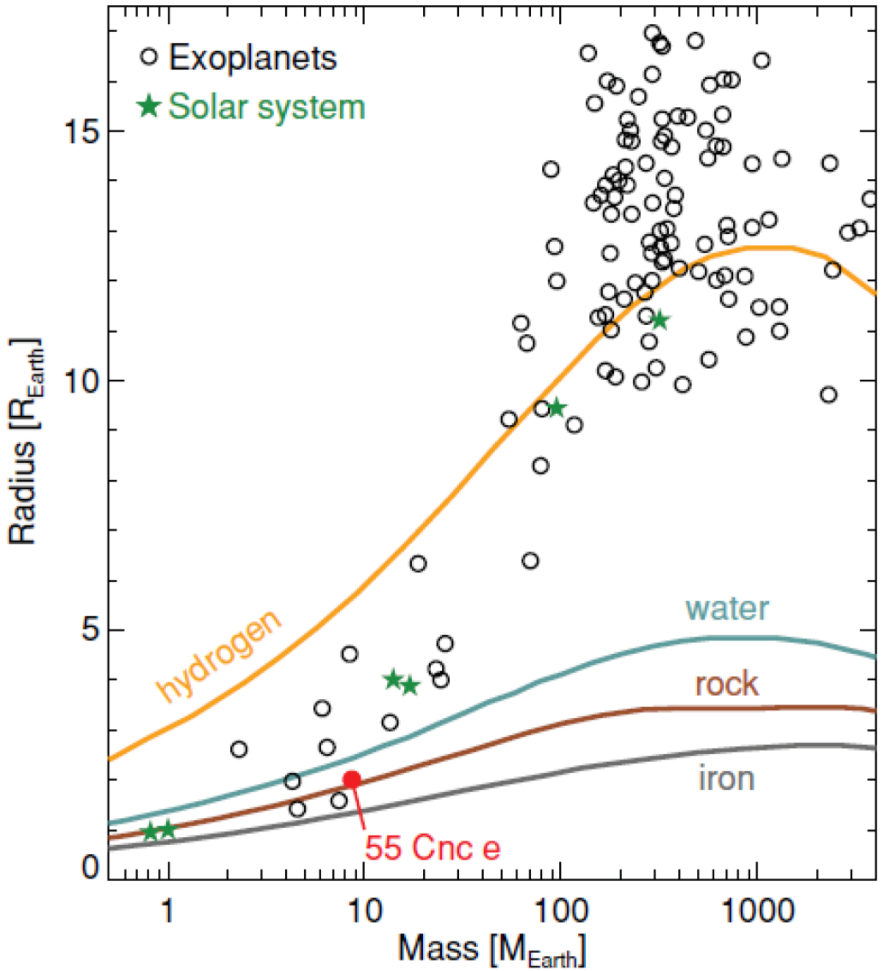
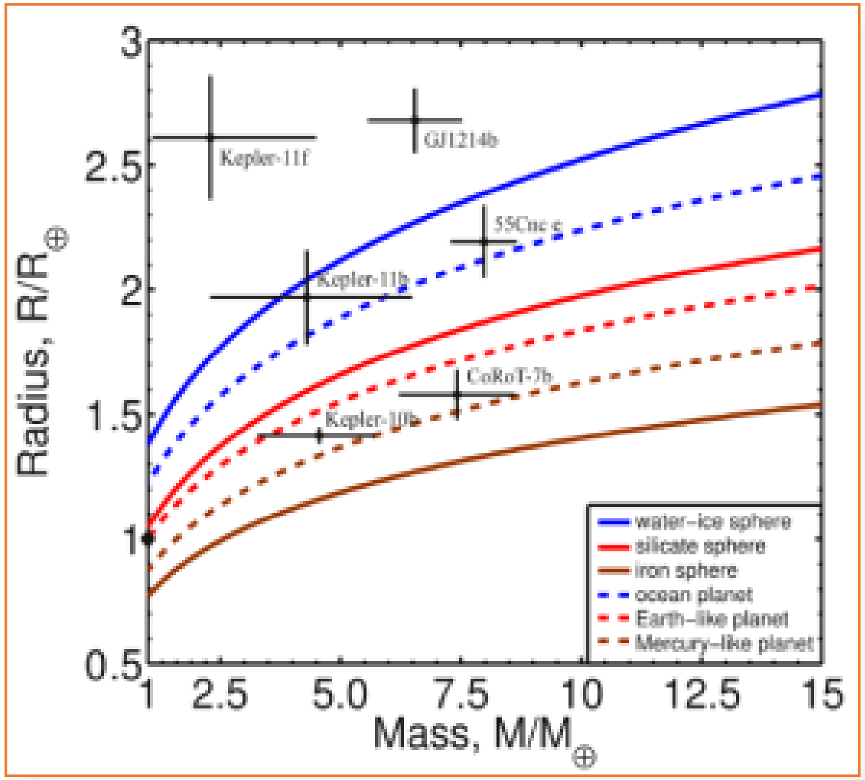
Why transits + RV data ?



How diverse are planets?



- Mass-radius relationships for low-mass planets indicate their mean composition

Future:

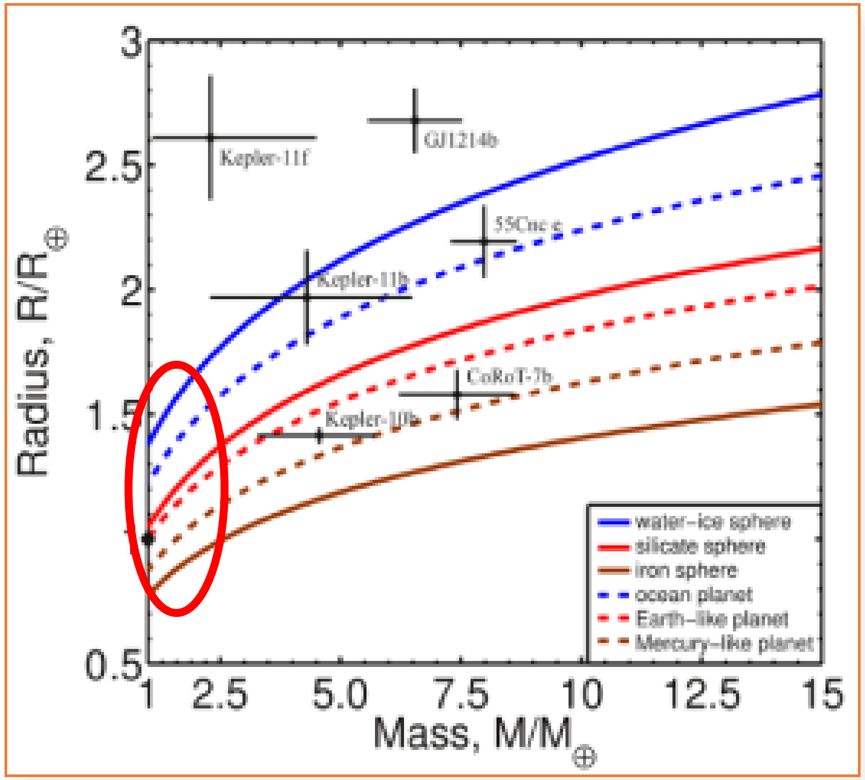
- Detect a statistical sample of low-mass planets

- Measure radii and masses to few % accuracy to constrain internal composition and structure

- Establish a classification of planets (e.g. Earth-like, Mercury-like, ice planets…)

- Determine planet gravity to investigate atmosphere loss processes and habitability

How diverse are planets?



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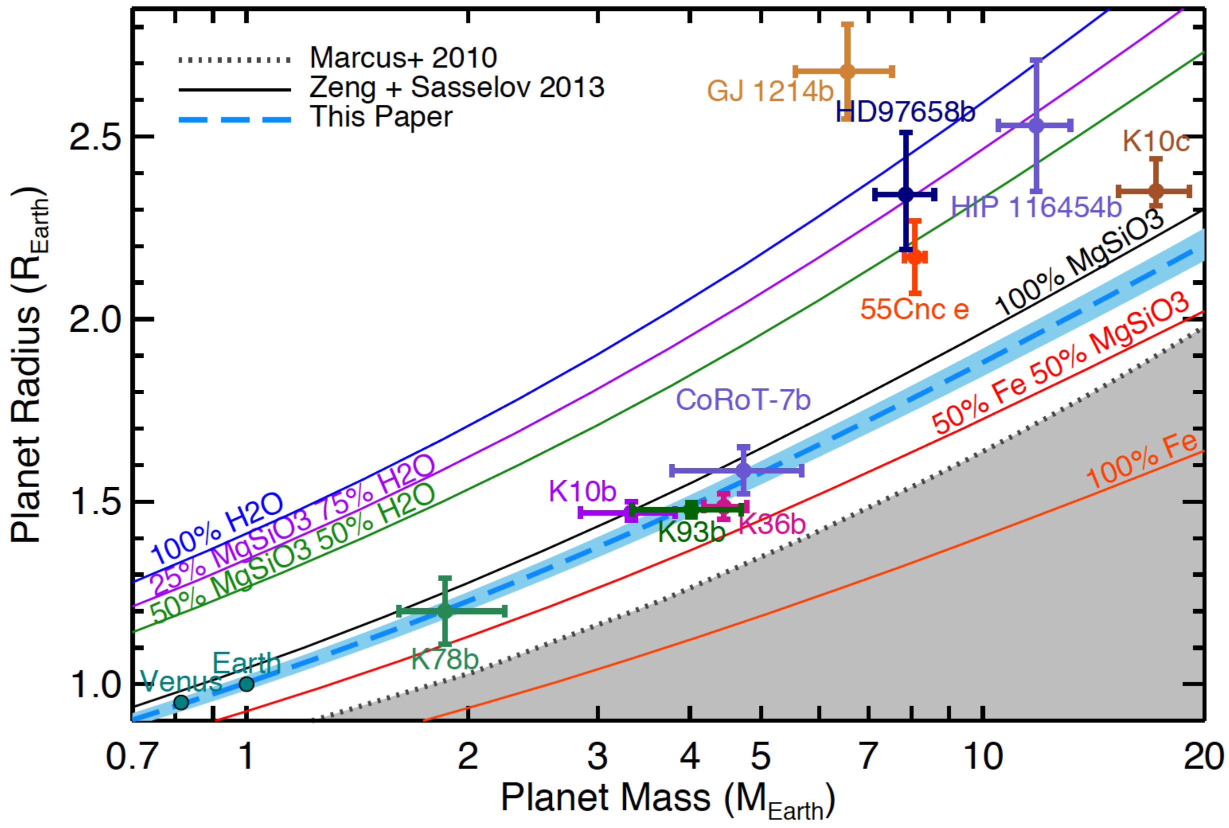


Dressing et al., 2015, Ap.J., 800, 135

Orbital Mechanics

Kepler's laws

1. The orbits of planets are elliptical, with the Sun at a focus

2. Radius vectors of planets sweep out equal areas per unit time

3. Squares of orbital periods are proportional to cubes of semimajor axes: P^2 (yr) = a_{pl}^3 (au)

Kepler derived these laws from the data of Tycho Brahe. Isaac Newton explained the laws with his more fundamental law of gravitation

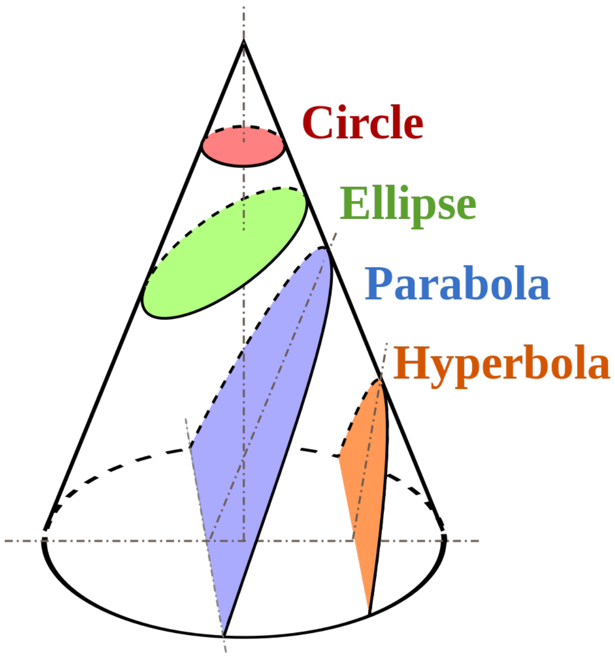
Keplerian orbits:

A mass always orbits a central body on the periphery of a conical section

Which type of section depends on the sign of the total energy of the body, $E_{tot} = E_{kin} - E_{pot}$:

$E_{tot} < 0$ ➔ Ellipse
$E_{tot} = 0$ ➔ Parabola
$E_{tot} > 0$ ➔ Hyperbola

In an elliptical orbit, the eccentricity, e, depends on the
angular momentum, **L**: Maximum **L** for a given energy ➔ a circular orbit



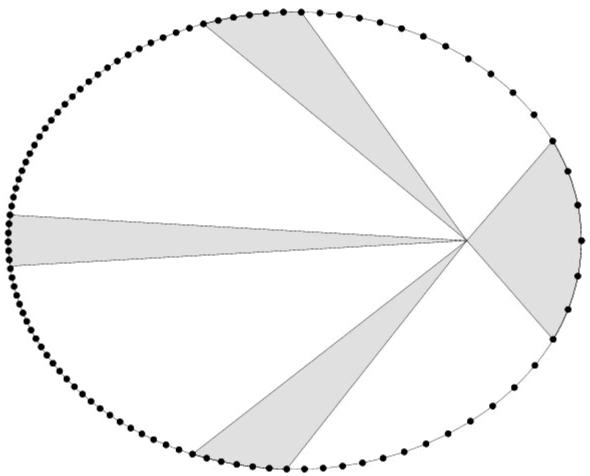
e = 0 ➔ Circle
e 0<e<1 ➔ Ellipse
e = 1.0 ➔ Parabola
e > 1.0 ➔ Hyperbola

Kepler's second law

A consequence of the conservation of angular momentum, **L**

Angular momentum**, L** = m_{pl} **V r** = Constant ➔ **V r** = Constant

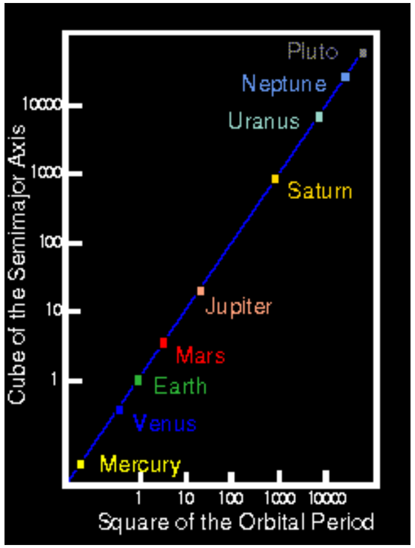
Element swept per unit time is dA = V r dt ⟵➔ dA/dt = **V r** = Constant



Kepler’s third law

$F_{grav} = F_{centripetal} = G\, M_{pl} M_{star}/(a_{pl}+a_{star})^2 \approx G\, M_{pl} M_{star}/a_{pl}^2$

$F_{centrifugal} = M_{pl}\, V_{pl}^{\ 2}/a_{pl} = 4\,\pi^2\, M_{pl} a_{pl}\, /\, P^2$;   since $(V_{pl} = 2\,\pi\, a_{pl}\, /\, P)$



$F_{centripetal} = F_{centrifugal} \rightarrow 4\,\pi^2\, a_{pl}^{\ 3} = G\, M_{star} P^2$

(Independant of M_{pl})

Another way:

$E_{kin} = M_{pl}\, V_{pl}^{\ 2}/2 = E_{pot} \approx G\, M_{pl} M_{star}/a_{pl} \rightarrow V_{pl} = 2\,\pi\, a_{pl}\, /\, P$

$\rightarrow 4\,\pi^2\, a_{pl}^{\ 3} = G\, M_{star} P^2$

A consequence of the conservation of Energy, E

Most important equation: Kepler's third law

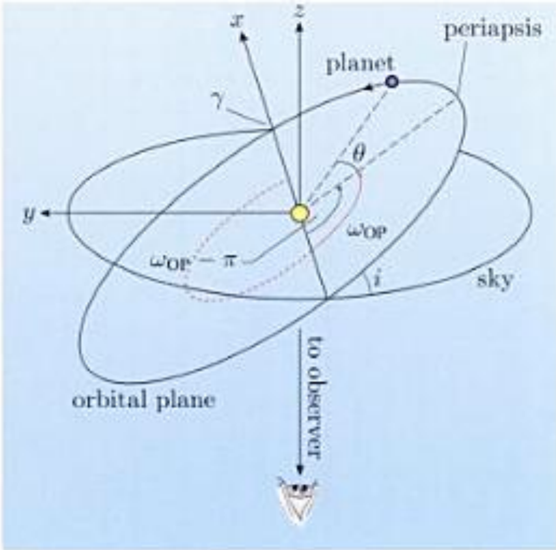
$$\frac{a^3}{P^2} = \frac{G\left(M_* + M_P\right)}{4\pi^2}$$

Where a = $a_P + a_*$

$$a \cong a_p \cong \left(\frac{G}{4\pi^2}\right)^{1/3} m_*^{1/3} P^{2/3}$$

Not true for new class of planets with $a_p \sim R_*$

Astrometry of the situation: The motion of a planet in the astrocentric frame



Pericentre = periapsis = periastron = closest point to star of planet

Inclination, i = angle between sky and orbital plane

X-axis is intersection between orbit and plane-of-sky

γ is the intersection of the orbit with the positive x-axis

θ, the `true anomaly' is the angle between the direction of periapsis(= periastron) and the current position of the body, as seen from the main focus of the ellipse (position of star)

ω_{OP} is the orientation of the periapsis w.r.t. γ

x, y, are coordinates in plane of sky, z-axis is ←→ line-of-sight from us to star

$$v_x = -\frac{2\pi a}{P\sqrt{1-e^2}}\left(\sin\left(\theta+\omega_{OP}\right)+e\sin\omega_{OP}\right)$$

This is the velocity component of the planet in the rest frame of the host star – **the astrocentric frame**

$$v_y = -\frac{2\pi a\cos i}{P\sqrt{1-e^2}}\left(\cos\left(\theta+\omega_{OP}\right)+e\cos\omega_{OP}\right)$$

$$v_z = -\frac{2\pi a\sin i}{P\sqrt{1-e^2}}\left(\cos\left(\theta+\omega_{OP}\right)+e\cos\omega_{OP}\right)$$

Both the star and the planet are, however, really moving around **the barycentre = common centre of mass** – each in an ellipse. The barycentre is one of the foci of each ellipse

$$v_* = -\frac{M_P}{M_P+M_*}v$$

Where v is the astrocentric orbital velocity of the planet

We can only observe, spectroscopically, the stellar velocity component along the line of sight

Then the observed z component of the stellar velocity will be:

$$V(t) = V_{0,z} + \frac{2\pi a M_P \sin i}{(M_P + M_*)P\sqrt{1-e^2}}\left(\cos\left(\theta(t) + \omega_{OP}\right) + e\cos\omega_{OP}\right)$$

Where $V_{0,z}$ is the motion of the star due to its galactic motion,
Index z is the component along the line-of-sight and $\theta(t)$ is the true anomaly changing with time .
$V_{0,z}$ is thus the Radial Velocity, (index *RV*)

$$A_{RV} = \frac{2\pi a M_P \sin i}{(M_P + M_*)P\sqrt{1-e^2}}$$

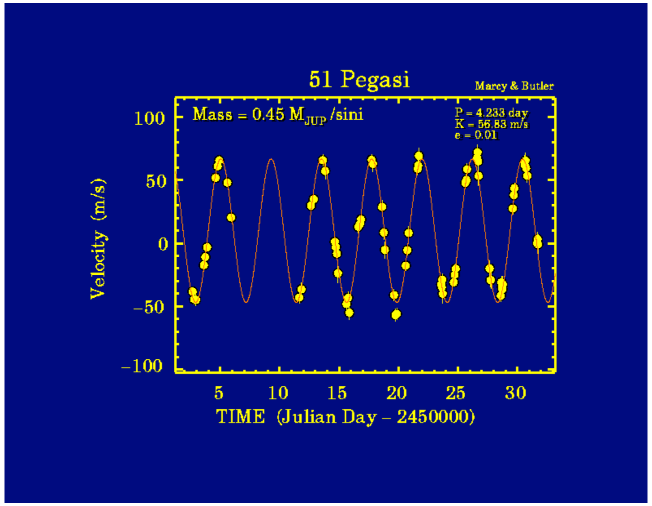
Is the amplitude of the radial velocity

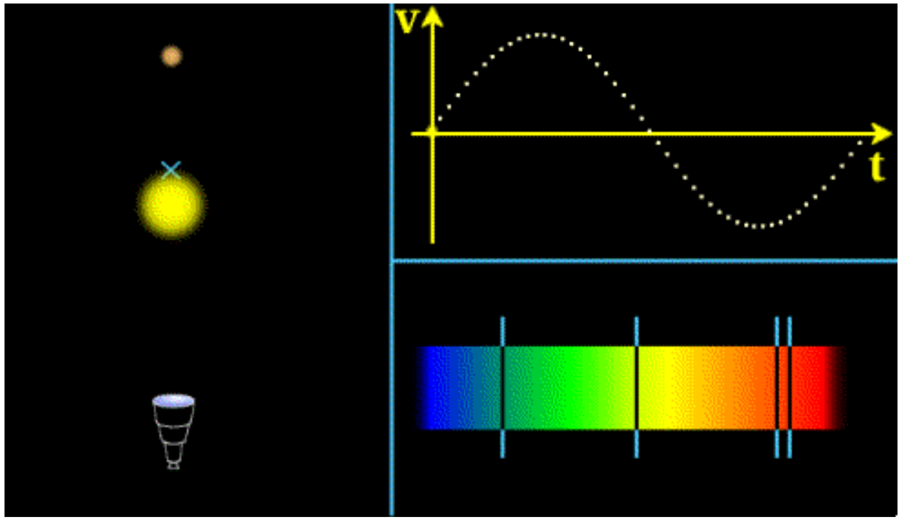
Radial Velocity observations

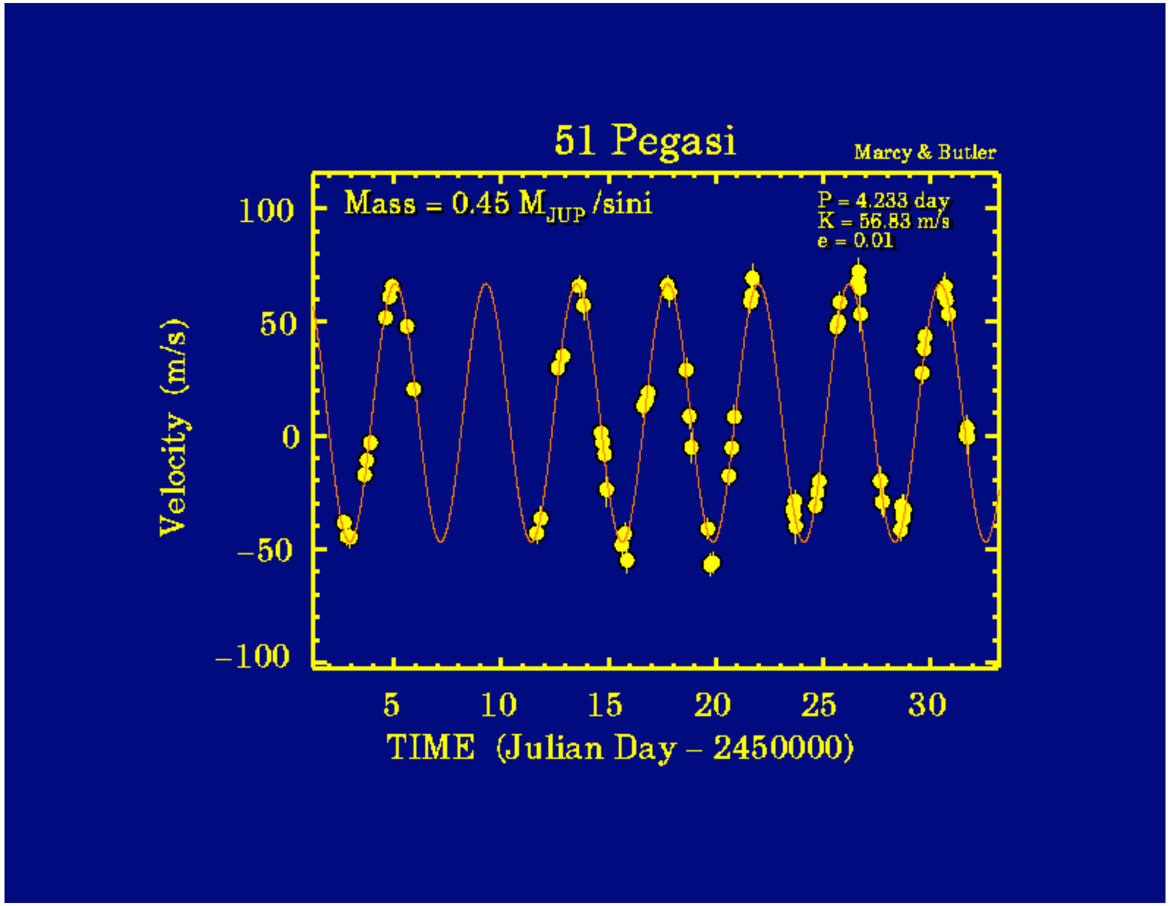
Radial Velocity method

Radial Velocity method (i.e. the Doppler effect!)
- Analysing spectrum ➔ Mass of planet if orbital inclination known

Otherwise M_p/ sin i !!!

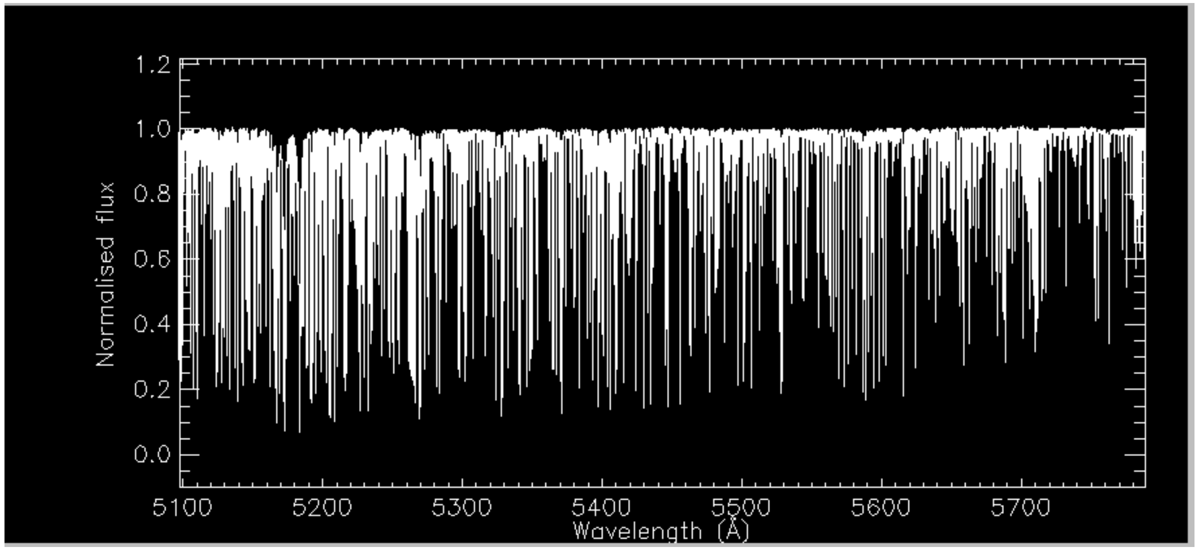






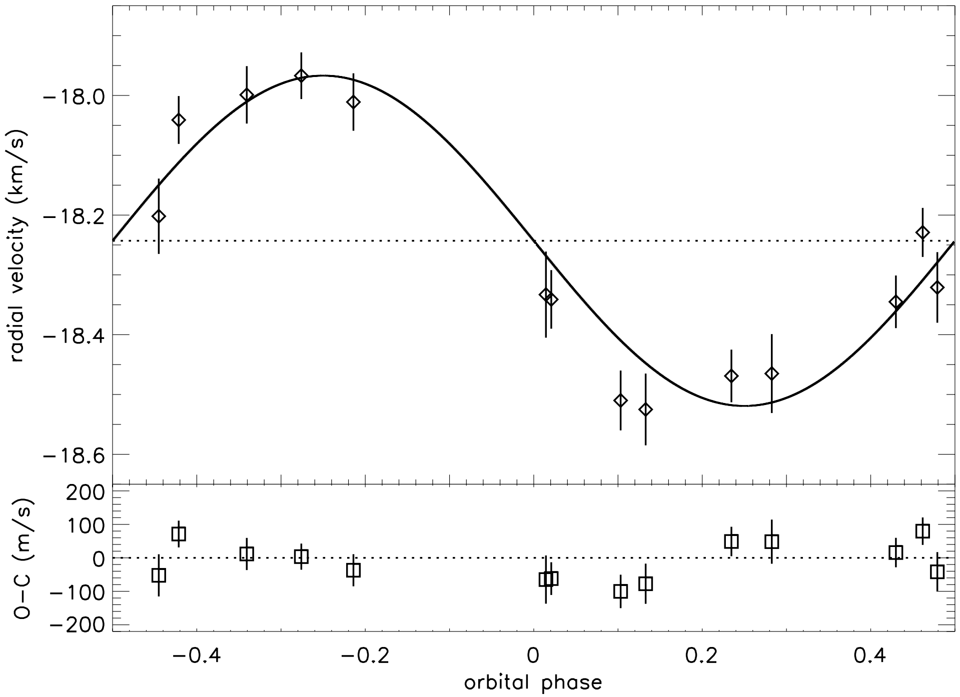
51 Pegasi
Marcy & Butler
Mass = 0.45 M_JUP /sini
P = 4.233 day
K = 56.83 m/s
e = 0.01
Velocity (m/s)
100
50
0
−50
−100
5
10
15
20
25
30
TIME (Julian Day − 2450000)

Doppler spectroscopy:



1. Star & planet orbit their common barycentre according to Newton's laws of gravitation & motion
2. The Doppler effect provide a measure of the relative velocity between source and observer
3. The visible portion of F, G, K and M star have a very large number of absorption lines which make it easier to measure wavelength shifts with the Doppler effect
4. The measured wavelength is measured w.r.t a reference spectrum – gas cell (I) or lamp (ThAr)
5. Or cross correlated to Radial Velocity standard spectrum

We measure the radial velocity of the star during the orbit at different phases



And the we can calculate m_p from the radial velocity curve:

One often put the expression for the semi major axis in the formula for A_{RV} :

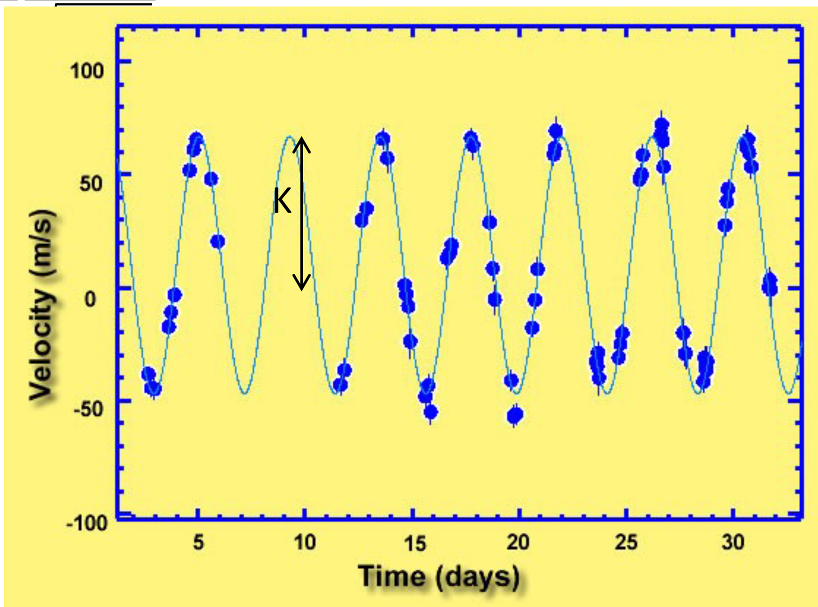
$$a \cong a_p \cong \left(\frac{G}{4\pi^2}\right)^{1/3} m_*^{1/3} P^{2/3}$$

$$A_{RV} = \frac{2\pi a M_P \sin i}{\left(M_P + M_*\right) P \sqrt{1-e^2}}$$

$$A_{RV} = \frac{2\pi \left(\frac{G}{4\pi^2}\right)^{\frac{1}{3}} m_*^{1/3} P^{2/3} m_p \sin i}{\left(m_p + m_*\right) P} \frac{1}{\sqrt{1-e^2}}$$

$$= \left(\frac{2\pi G}{P}\right)^{1/3} \frac{m_p \sin i}{m_*^{2/3}} \frac{1}{\sqrt{1-e^2}}$$

And call A_{RV} = K



$$V_{rad}(t) = V_{0,z} + K\left[\cos\left(\theta(t) + \omega_{OP}\right) + e\cos\left(\omega_{OP}\right)\right]$$

K is related to the masses of the 2 bodies through the so-called `mass-function'. Some rearranging: ($M_P << M_*$)

$$\frac{(M_p \sin i)^3}{(M_* + M_p)^2} = \frac{P}{2\pi G} K^3 \left(1-e^2\right)^{3/2}$$

Which gives the planet minimum mass accordingly:

$$M_p \sin i \cong \left(\frac{P}{2\pi G}\right)^{1/3} K M_*^{2/3} \left(1-e^2\right)^{1/2}$$

Assuming that $m_p << m_*$.
Introducing the same approximation into Kepler's third law yields for the semi-major axis of the system:

$$a \cong a_p \cong \left(\frac{G}{4\pi^2}\right)^{1/3} M_*^{1/3} P^{2/3}$$

Fitting radial velocity data with a Keplerian ➔ thus provides 4 of the 6 orbital elements

Next determine stellar mass from another source:
- spectral modeling
- light curve analysis

Expected radial velocity curve semi-amplitude

$$K_* = \left(\frac{2\pi G}{P}\right)^{\frac{1}{3}} \frac{m_p \sin i}{m_*^{2/3}} \frac{1}{\sqrt{1-e^2}}$$

- Let's assume:

$M_* = 1\ M_{Sun}$ $e = 0$
$M_p = 1\ M_{Jup}$ $i = 90°$

a	K
AU	m/s
0.02	201
1	28
5.2	12

$M_* = 1\ M_{Sun}$ $e = 0$
$M_p = 5\ M_{Earth}$ $i = 90°$

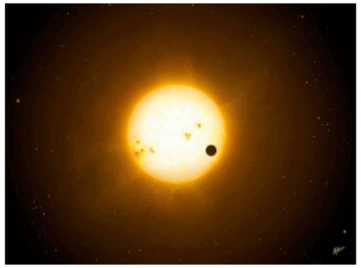
a	K
AU	m/s
0.02	3
1	0.45
5.2	0.19

$M_* = 1\ M_{Sun}$ $e = 0$
$M_p = 1\ M_{Earth}$ $i = 90°$

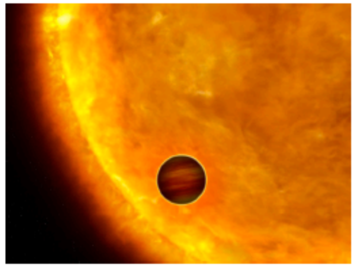
a	K
AU	m/s
0.02	0.63
1	0.09
5.2	0.04

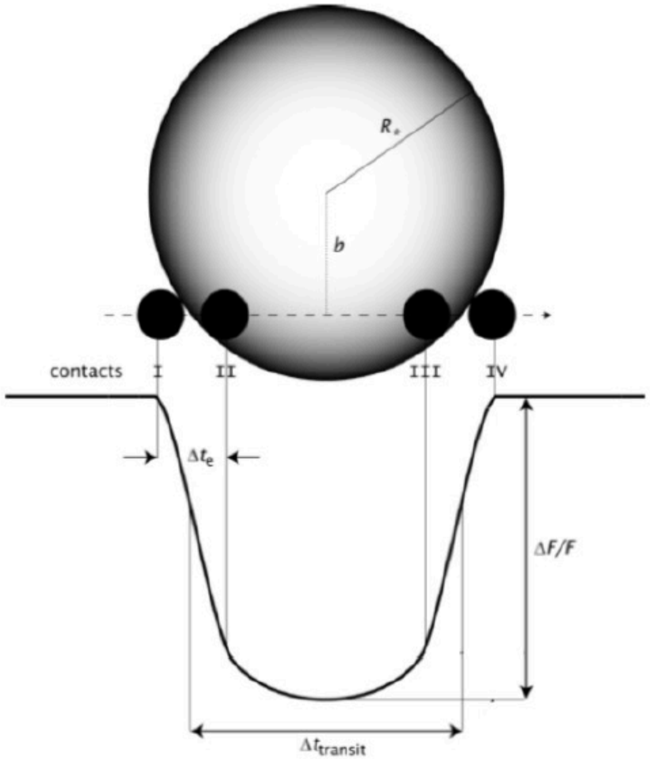
With HARPS@ESO-3.6m and HIRES@Keck we can reach a precision of *only* 1 m/s!

Exoplanet transit Observations



Exoplanet detection methods
Transit photometry





The inclination *i* of the orbit with respect to the line-of-sight can be measured

$$\frac{\Delta F}{F} = \left(\frac{R_p}{R_*}\right)^2$$

R_p can be derived !

So what is the chance of a planetary transit?

The disc of the planet need to pass across the disc of the star

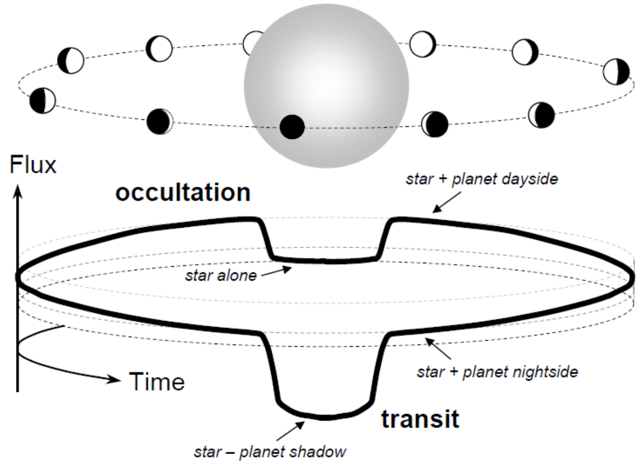
The closest approach of the planet to the center of the stellar disc happen at the "inferior conjunction". This is defining the phase ϕ = 0.0

$d(\phi = 0.0) = a \cos i$ is the distance between the centres of the two discs

i must satisfy: $a \cos i \leq R_* + R_p$

Transit probability = $(R_* + R_p)$ / a $\approx R_* / a$

Transit photometry



Advantages:

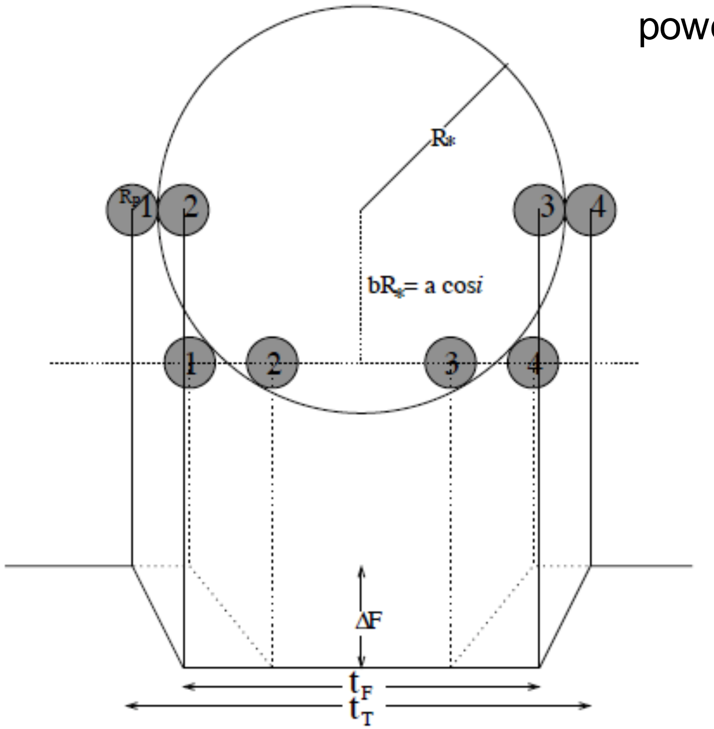
- It allows us to measure the planetary radius
- A wealth of precious information can be gained:

- Transmission spectroscopy → Atmosphere composition
- Planet occultation → Albedo, atmosphere composition
- "Changing phase" → Albedo, atmosphere composition
- Rossiter McLaughlin effect → Star/planet system obliquity

Drawbacks:

- Low probability of transit detection → Large number of observed stars
- It is most sensitive to close-in planets (complementary to direct imaging and micro-lensing)
- High-precision, time-series, continuous photometry is needed
- It produces many false-positives

Transits are in principle a very powerful method:



b is impact parameter!!

Very simple: Two spheres that pass in front of each other!
A geometric interpretation of effects is possible

Adding some physical relationships and we can determine some of the parameters of a star-planet system using the lightcurve alone

4 Assumptions + 2 Conditions

- The planet orbit is circular
- $M_p << M_*$ and the companion is dark compared to central star
- The stellar mass-radius relation is known
- The light comes from a single star (no blends)

  C1 The eclipses have flat bottoms ⇔ the companion is fully superimposed on the disk

  C2 The period can be derived from the light curve (e.g. observed two consecutive eclipses)

3 geometrical equations:

The transit depth ΔF:

$$\Delta F = \frac{F_{no-transit} - F_{transit}}{F_{no-transit}} = \left(\frac{R_P}{R_*}\right)^2$$

The transit shape as the ratio of the duration of the flat part (t_F) to the total transit duration (t_T)

$$\frac{\sin(t_F \pi / P)}{\sin(t_T \pi / P)} = \frac{\left\{\left[1-(R_P/R_*)\right]^2 - \left[(a/R_*)\cos i\right]^2\right\}^{1/2}}{\left\{\left[1+(R_P/R_*)\right]^2 - \left[(a/R_*)\cos i\right]^2\right\}^{1/2}}$$

And the total transit duration t_T

$$t_T = \frac{P}{\pi} \arcsin\left(\frac{R_*}{a}\left\{\frac{\left[1+(R_P/R_*)\right]^2 - \left[(a/R_*)\cos i\right]^2}{1-\cos^2 i}\right\}^{1/2}\right)$$

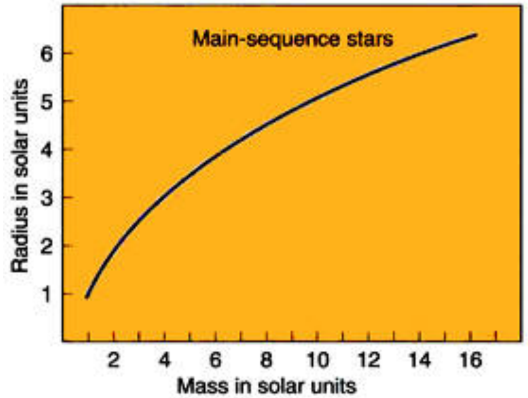
Two physical relations will break the degeneracy:

$$P^2 = \frac{4\pi^2 a^3}{G\left(M_* + M_p\right)}$$

Kepler's third law

$$R_* = kM_*^x$$

+ a stellar mass radius relation
R = $M^{0.8}$ assumed for main sequence stars



But....Limbdarkening

And data must be of very high quality!!!!

Solve for M_*, R_*, i, a and R_p

Remember the assumptions!!!

Excellent papers: Seager & Mallen-Ornelas, 2003, Ap.J.585, 1038
Giménez, A. 2006, A&A, 450, 1231

The analytical solution

An analytical solution, provides physical information about the system

- Stellar mass M_*
- Stellar radius R_*
- Planetary radius R_p
- Orbital semimajor axis a
- Orbital inclination i

....And note – The planetary mass is missing!!

The analytical solution – 4 parameters derived from the equations

$R_p/R_* = (\Delta F)^{1/2}$

$$b \equiv \frac{a}{R_\star}\cos i = \left[\frac{(1-\sqrt[2]{\Delta F})^2 - [\sin^2(t_F\pi/P)/\sin^2(t_T\pi/P)](1+\sqrt[2]{\Delta F})^2}{1-[\sin^2(t_F\pi/P)/\sin^2(t_T\pi/P)}\right]^{1/2}$$

$$\frac{a}{R_\star} = \left[\frac{(1-\sqrt{\Delta F})^2 - b^2[1-\sin^2(t_T\pi/P)]}{\sin^2(t_T\pi/P)}\right]^{1/2}$$

With Kepler's third law and $M_p << M_*$ ➔

$$\rho_\star \equiv \frac{M_\star}{R_\star^3} = \left[\frac{(1-\sqrt{\Delta F})^2 - b^2[1-\sin^2(t_T\pi/P)]}{\sin^2(t_T\pi/P)}\right]^{3/2}$$

Analytic solution - simplifications

We can simplify the analytical solution (see Seager and M-O)

$$b = \left[\frac{(1-\sqrt{\Delta F})^2 - (t_F/t_T)(1+\sqrt{\Delta F})^2}{1-(t_F/t_T)^2} \right]^{1/2}$$

$$\frac{a}{R_\star} = \frac{2P}{\pi} \frac{\Delta F^{1/4}}{(t_T^2 - t_F^2)^{1/2}}$$

$$\rho_\star = \frac{32}{G\pi} P \frac{\Delta F^{3/4}}{(t_T^2 - t_F^2)^{3/2}}$$

With the expression for the radius the same

$R_p/R_* = (\Delta F)^{1/2}$

Where do the error bars come from?

$$\frac{(M_p \sin i)^3}{(M_* + M_p)^2} = \frac{P}{2\pi G} K^3 \left(1 - e^2\right)^{3/2}$$

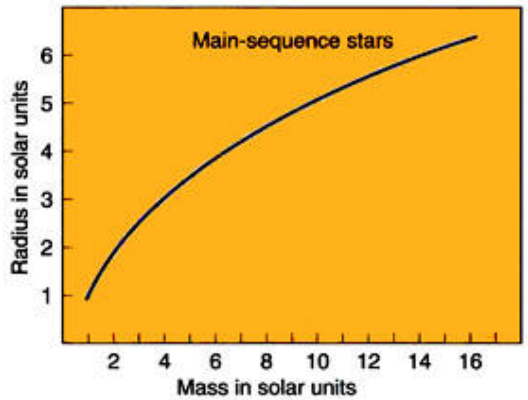
$$\Delta F = \frac{F_{no-transit} - F_{transit}}{F_{no-transit}} = \left(\frac{R_P}{R_*}\right)^2$$

So if we want to know the planetary mass, radii, with a precision better than ~ few % we must know the parameters of the star to the same precision

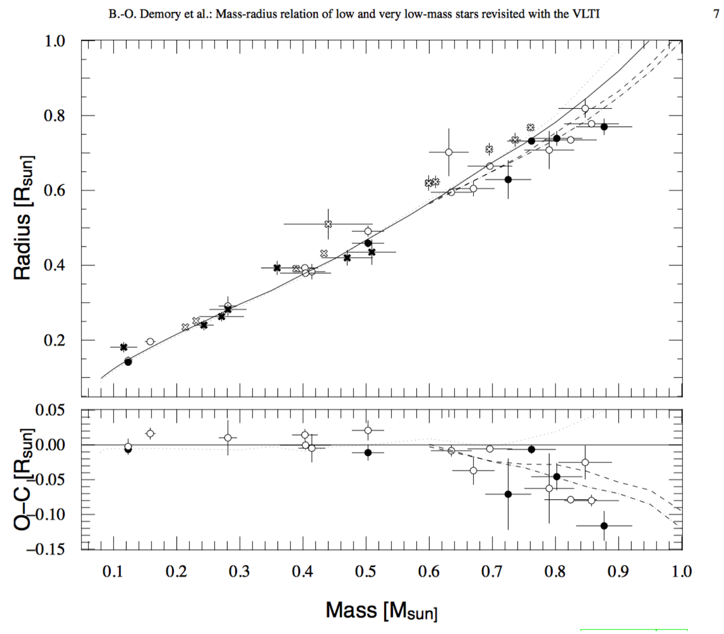
And there is a way of doing this – and it will be required to go to space to do it....

Stellar Masses and radii

We have a stellar mass radius relation
$R = M^{0.8}$ assumed for main sequence stars.
This has been derived using binary stars and modeling ➔



But in reality ➔



➔ Errors > 20% and in the mass range where our interest will be

End of lecture 2