Problem 1: Displacement due to a passing gravitational wave

A plane gravitational wave described by the small metric disturbance $h_{\alpha\beta}$ propagates in a flat background spacetime. The wave is propagating in the positive $z$ direction. We also have 2 test masses, which, in Cartesian coordinates, are located at $(0,0,0)$ and at $(X,Y,Z)$. Let $L_*$ be the unperturbed distance between the 2 test masses. We will denote the straight line path between the two test masses with the symbol $\gamma$ and define a spatial unit vector $\vec{n}$ with coordinates $n^i$, which is a tangent vector to $\gamma$. Show that the time-dependent change in distance between the masses, produced by the passing gravitational wave, is then given by

$$\delta L(t) = \frac{1}{2} \int_0^{L_*} d\lambda h_{ij}(t - n^z \lambda)n^i n^j. \quad (1.1)$$

Hint: begin by writing down a parametric form for the path $\gamma$, involving the quantities defined above, using a parameter $\lambda$ that varies along the path $\gamma$, i.e., write down an expression for $x^i_\gamma(\lambda)$ where $x^i_\gamma$ are the spatial coordinates along the path $\gamma$. 