1 Problem 1: Is it a black hole?

Consider the spacetime defined by the line element
\[ ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \left(1 - \frac{M}{r}\right)^{-2} dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right). \] (1.1)

Except for \( r = M \), the coordinate \( t \) is always timelike and \( r \) is always spacelike.

(a) Find a transformation to new coordinates \((v, r, \theta, \phi)\) (analogous to Eq. (12.1) in Hartle) that has \( g_{rr} = 0 \) and shows that the geometry is not singular at \( r = M \). \textit{Hint:} Start by writing \( t = v - F(r) \) and define \( F' = dF/dr \).

(b) Sketch a \((\tilde{t}, r)\) spacetime diagram (analogous to Fig. 12.2 in Hartle) showing the world lines of incoming and outgoing radial light rays and the light cones.

(c) Is this the geometry of a black hole?