1 Problem 1: Splitting a black hole

As mass falls into a black hole, $M$ increases, and therefore the area of the horizon must also increase. This is also true for non spherically symmetric infall. This is a manifestation of the area increase theorem for black holes, which states that the horizon area can never decrease. Show that this implies that a black hole can never split into two black holes preserving total mass.

2 Problem 2: Brightness of an evaporating black hole

In class we discussed the evaporation of black holes by Hawking radiation. Suppose a primordial black hole (i.e., a black hole formed in the first fraction of a second following the Big Bang) evaporates now. Suppose also that there is an astronomer surveying the sky for such events by covering every position for exactly 1 sec before moving on to the next position. Suppose finally that this astronomer catches the final second of this black hole, which appears in his data as a transient source as bright as the Sun placed at a distance of 10 pc (i.e., the brightness of a typical star). Give an order-of-magnitude estimate of the distance of the evaporating black hole. Make any reasonable approximation that you like as long as it is order-of-magnitude correct.