Astronomical Observing Techniques 2019

Lecture 13: A summary

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Questions related to the topics emphasized on the slides

- What is the definition?
- What is the principle?
  - How is it applied? Give example(s).
- Why is this important/relevant?
- Equation?
  - Do I really understand it, also intuitively?
  - (If we derived it,) How is it derived?
  - Why is it important?
  - What is its application?

Preparation

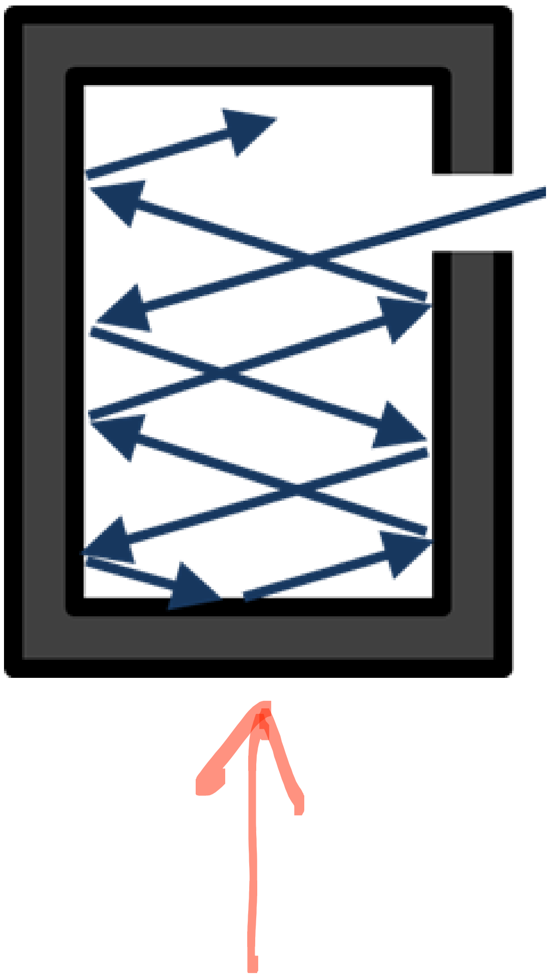
- Be able to answer all these questions
  - If in doubt, read up using the reading material
- Understand the questions and answers of the assignments and test exams

Also: bigger questions: ask them now and the TAs will come back to you fully prepared for tomorrows session

Lecture 1: Black Bodies in Space

1. Blackbody Radiation
    1. Planck Curve and Approximations
    2. Effective Temperature
    3. Brightness temperature
    4. Lambert
2. Photometry
    1. Flux and Intensity
    2. Magnitudes
    3. Photometric Systems
    4. Color Indices
    5. Surface Brightness

Blackbody Radiation



- Cavity at fixed T, thermal equilibrium
- Incoming radiation is continuously absorbed and re-emitted by cavity wall
- Small hole → escaping radiation will approximate black-body radiation independent of properties of cavity or hole

Kirchhoff’s Law

Gustav Kirchhoff stated in 1860 that “at thermal equilibrium, the power radiated by an object must be equal to the power absorbed.”

- Blackbody cavity in thermal equilibrium with completely opaque sides
- Opaque -> transmissivity $\tau = 0$
- emissivity ε = amount of emitted radiation
- Thermal equilibrium -> $\varepsilon + \rho = 1$

$$\left.\begin{aligned} \varepsilon &= 1-\rho \\ \alpha+\rho+\tau &= 1 \\ \tau &= 0 \end{aligned}\right\} \boxed{\alpha = \varepsilon}$$

Blackbody absorbs all radiation: $\alpha=\varepsilon=1$

Kirchhoff’s law applies to perfect black body at all wavelengths

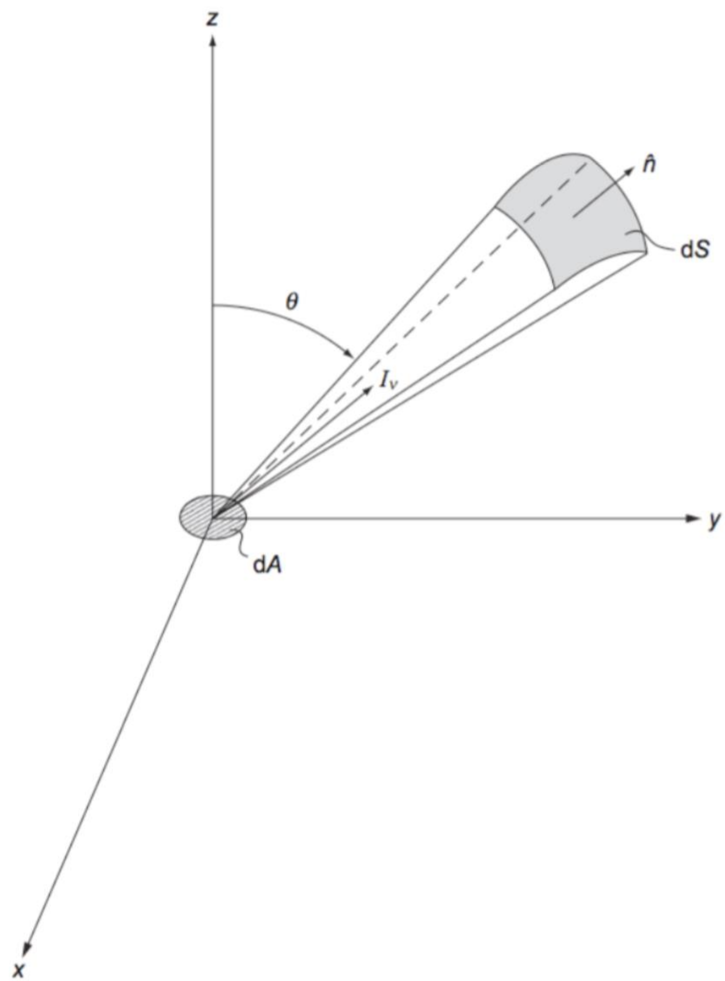


Figure 3.5 Illustration defining the specific intensity I_ν.

The specific intensity

the amount of energy received per unit area (at dA' =cosθdA, dA' is perpendicular to the line of sight) per unit time in the spectral range between ν and ν+dν, from solid angle dΩ (=dS/r^2)

$$I_\nu(\vec{r}, \hat{n}, t)$$

in units of [W m^{-2} sr^{-1} Hz^{-1}]

Independent of distance

Directly observed is:

Monochromatic radiative flux F$_\nu$

total energy from (part of) the star received per unit time per unit area per frequency range

Units: W Hz^{-1} m^{-2} 2

Planck Curve: Emission, Power & Temperature

Total radiated power per unit surface proportional to fourth power of temperature *T*:

$$\int_{\Omega}\int_{\nu} I_\nu(T)\,d\nu\,d\Omega = M = \sigma T^4$$

$\sigma = 5.67\cdot10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$ (Stefan-Boltzmann constant)

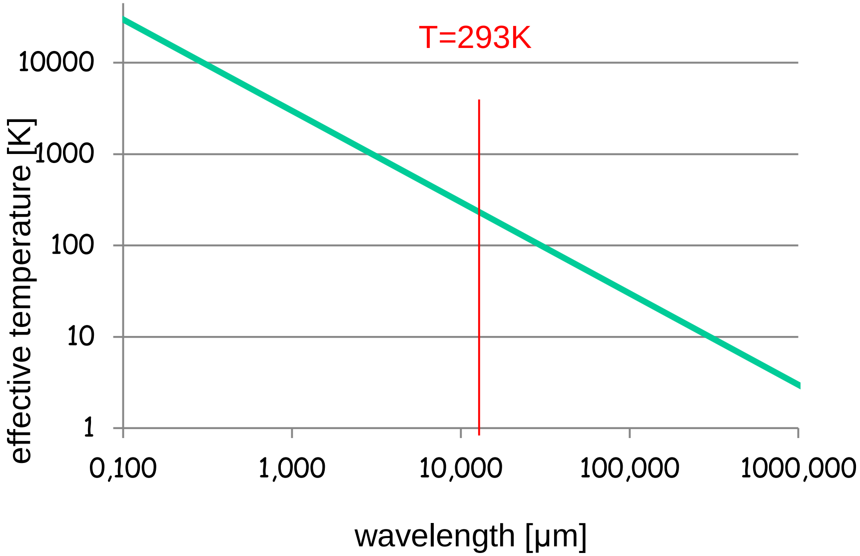
Assuming BB radiation, astronomers often specify the emission from objects via their effective temperature.

Wien's Law

Relation between temperature and frequency/wavelength at which the BB attains its maximum:

$$\frac{c}{\nu_{\max}} T = 5.096 \cdot 10^{-3} \text{ mK} \quad \text{or} \quad \lambda_{\max} T = 2.98 \cdot 10^{-3} \text{ mK}$$

Cooler BBs have peak emission at longer wavelengths and at lower intensities:



Planck Curve: Approximations

Planck:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

High frequencies (h*ν* >> kT) ➔ Wien approximation:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

Low frequencies (hv << kT) ➔ Rayleigh-Jeans approximation:

$$I_\nu(T) \approx \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2}$$

Effective Temperature

The effective temperature of a body such as a star or planet is the temperature of a black body that would emit the same total amount of electromagnetic radiation. Effective temperature is often used as an estimate of a body's surface temperature

Brightness Temperature: Definition

Brightness temperature: temperature of a perfect black body that reproduces the observed intensity of a grey body object at frequency *v.*

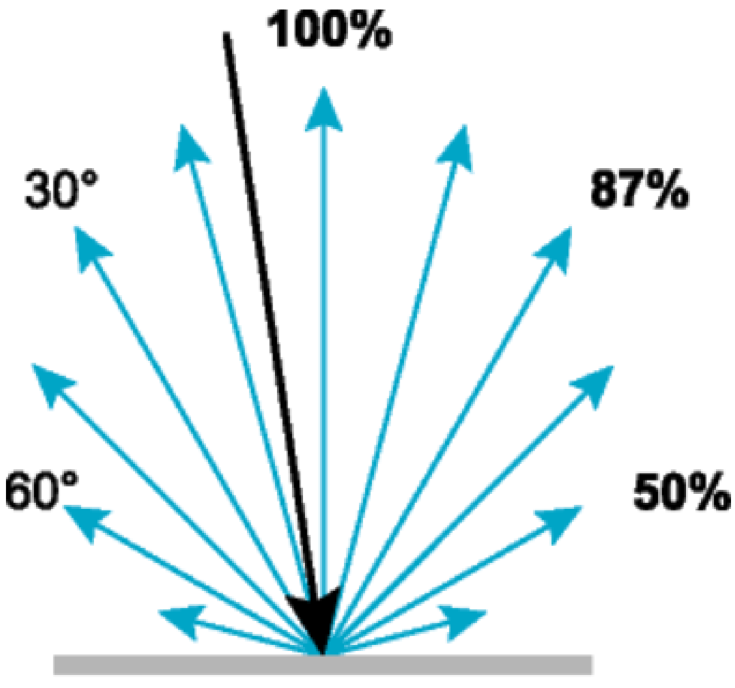
For low frequencies (h*v* << kT):

$$T_b = \varepsilon(\nu) \cdot T \underset{\text{Jeans}}{\overset{\text{Rayleigh-}}{=}} \varepsilon(\nu) \cdot \frac{c^2}{2k\nu^2} I_\nu$$

Only for perfect BBs is T_b the same for all frequencies.

Lambert: Cosine Law*

Lambert's cosine law: radiant intensity from an ideal, diffusively reflecting surface is directly proportional to the cosine of the angle θ between the surface normal and the observer.



Johann Heinrich Lambert
(1728 – 1777)

*Empirical law…. , mathematical derivation is complicated

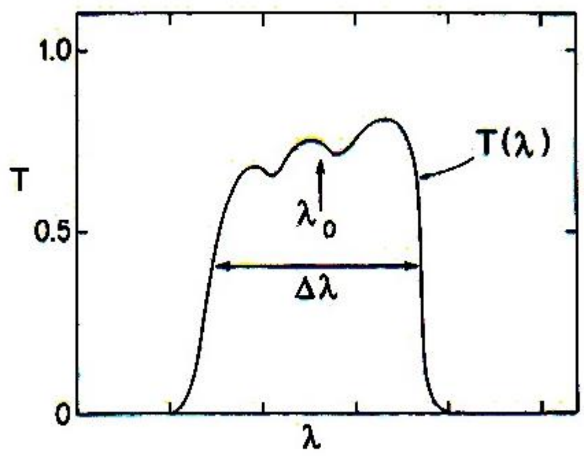
Magnitudes: Apparent Magnitude

Apparent magnitude is *relative* measure of monochromatic flux density F_λ of a source:

$$m_\lambda - M_0 = -2.5 \cdot \log\left(\frac{F_\lambda}{F_0}\right)$$

M_0 defines reference point (usually magnitude zero).

In practice, measurements through transmission filter $T(\lambda)$ that defines bandwidth:



$$m_\lambda - M = -2.5\log\int_0^\infty T(\lambda)F_\lambda d\lambda + 2.5\log\int_0^\infty T(\lambda)d\lambda$$

Magnitudes: Absolute Magnitude

- absolute magnitude = apparent magnitude of source if it were at distance D = 10 parsecs

$$M = m + 5 - 5\log D$$

- $m_\odot \approx -27$, $M_\odot = 4.83$ at visible wavelengths
- $M_{\text{Milky Way}} = -20.5 \rightarrow M_\odot - M_{\text{Milky Way}} = 25.3$
- Luminosity $L_{\text{Milky Way}} = 1.4 \times 10^{10}\, L_\odot$

Magnitudes: Bolometric Magnitude

Bolometric magnitude is luminosity expressed in magnitude units = integral of monochromatic flux over all wavelengths:

$$M_{bol} = -2.5 \cdot \log \frac{\int_0^\infty F(\lambda) d\lambda}{F_{bol}} \quad ; F_{bol} = 2.52 \cdot 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2}$$

If source radiates isotropically:

$$M_{bol} = -0.25 + 5 \cdot \log D - 2.5 \cdot \log \frac{L}{L_\Theta} \quad ; L_\Theta = 3.827 \cdot 10^{26}\ \mathrm{W}$$

Bolometric magnitude can also be derived from visual magnitude plus a bolometric correction BC: $M_{bol} = M_V + BC$

BC is large for stars that have a peak emission very different from the Sun's.

Photometric Systems: AB and STMAG

For given flux density F_ν, AB magnitude is defined as:

$$m(AB) = -2.5 \cdot \log F_\nu - 48.60$$

- object with constant flux per unit frequency interval has zero color
- zero point defined to match zero points of Johnson V-band
- used by SDSS and GALEX
- F_ν in units of [erg s^{-1} cm^2 Hz^{-1}]

STMAG system defined such that object with constant flux per unit wavelength interval has zero color. STMAGs are used by the HST photometry packages.

Color Index: Interstellar Extinction

- Absolute magnitude definition:

$$M = m + 5 - 5\log D$$

- Interstellar extinction *E* or absorption *A* affects the apparent magnitudes

$$E(B-V) = A(B) - A(V) = (B-V)_{\text{observed}} - (B-V)_{\text{intrinsic}}$$

- Need to include absorption to obtain correct absolute magnitude:

$$\boxed{M = m + 5 - 5\log D - A}$$

Photometric Systems: Conversions

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]	F_λ [W m^{-2} μm^{-1}]	F_ν [Jy]	
U	0.36	0.068	4.35×10^{-8}	1 880	Ultraviolet
B	0.44	0.098	7.20×10^{-8}	4 650	Blue
V	0.55	0.089	3.92×10^{-8}	3 950	Visible
R	0.70	0.22	1.76×10^{-8}	2 870	Red
I	0.90	0.24	8.3×10^{-9}	2 240	Infrared
J	1.25	0.30	3.4×10^{-9}	1 770	Infrared
H	1.65	0.35	7×10^{-10}	636	Infrared
K	2.20	0.40	3.9×10^{-10}	629	Infrared
L	3.40	0.55	8.1×10^{-11}	312	Infrared
M	5.0	0.3	2.2×10^{-11}	183	Infrared
N	10.2	5	1.23×10^{-12}	43	Infrared
Q	21.0	8	6.8×10^{-14}	10	Infrared

1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

Surface Brightness: Point & Extended Sources

Point sources = spatially unresolved

Brightness ~ 1 / distance^2

Size given by observing conditions

Extended sources = well resolved

Surface brightness ~ const(distance)

Brightness ~ $1/d^2$ and area size ~ $1/d^2$

Surface brightness [mag/arcsec^2] is constant with distance!

Lecture 2: Monsieur Fourier and his Elegant Transform

1. Introduction
   – Intuition, hearing, seeing
2. Fourier Series of Periodic Functions
3. Fourier Transforms
   – Properties
   – Examples
   – Numerical Fourier Transforms
4. Convolution, cross- and auto correlation
   – Parseval and Wiener-Khinchin Theorems
5. Sampling
   – Nyquist theorem
   – Aliasing

1. Introduction
Fourier Transformation

Functions *f(x)* and *F(s)* are Fourier pairs

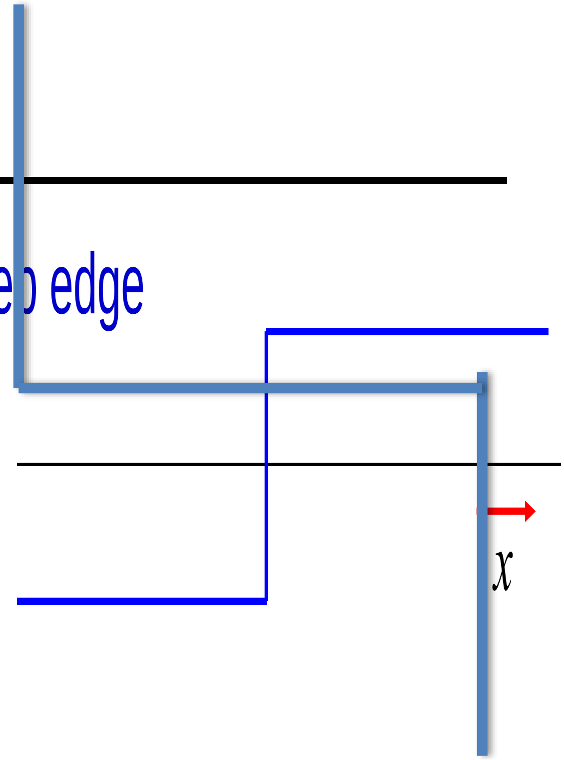
$$F(s) = \int_{-\infty}^{+\infty} f(x) \cdot e^{-i2\pi xs} dx$$

$$f(x) = \int_{-\infty}^{+\infty} F(s) \cdot e^{i2\pi xs} ds$$

inder: 1D Fourier Series

al frequency analysis of a step edge

$$= \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{otherwise} \end{cases}$$



A discrete sum of sines

lecomposition

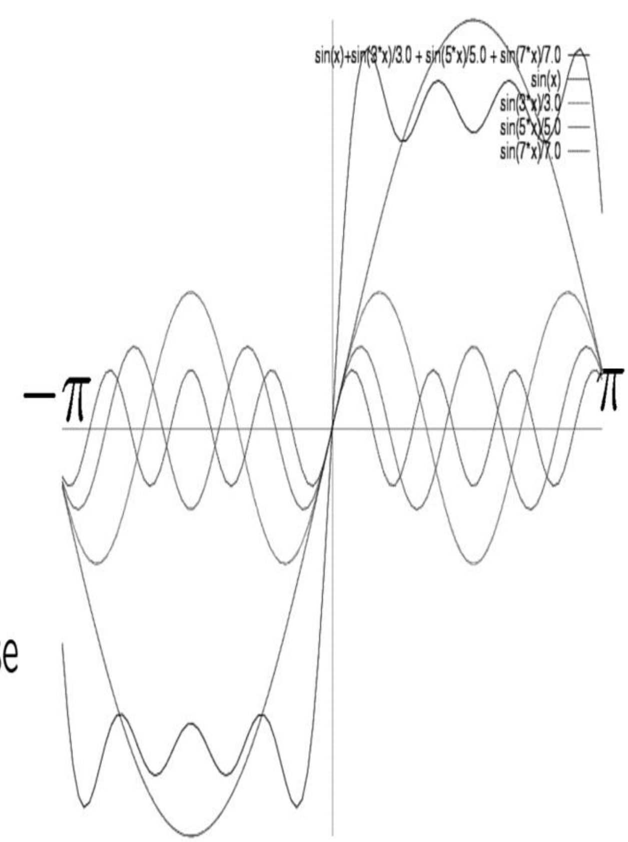
ies

$$\sum_n a_n \sin nx$$

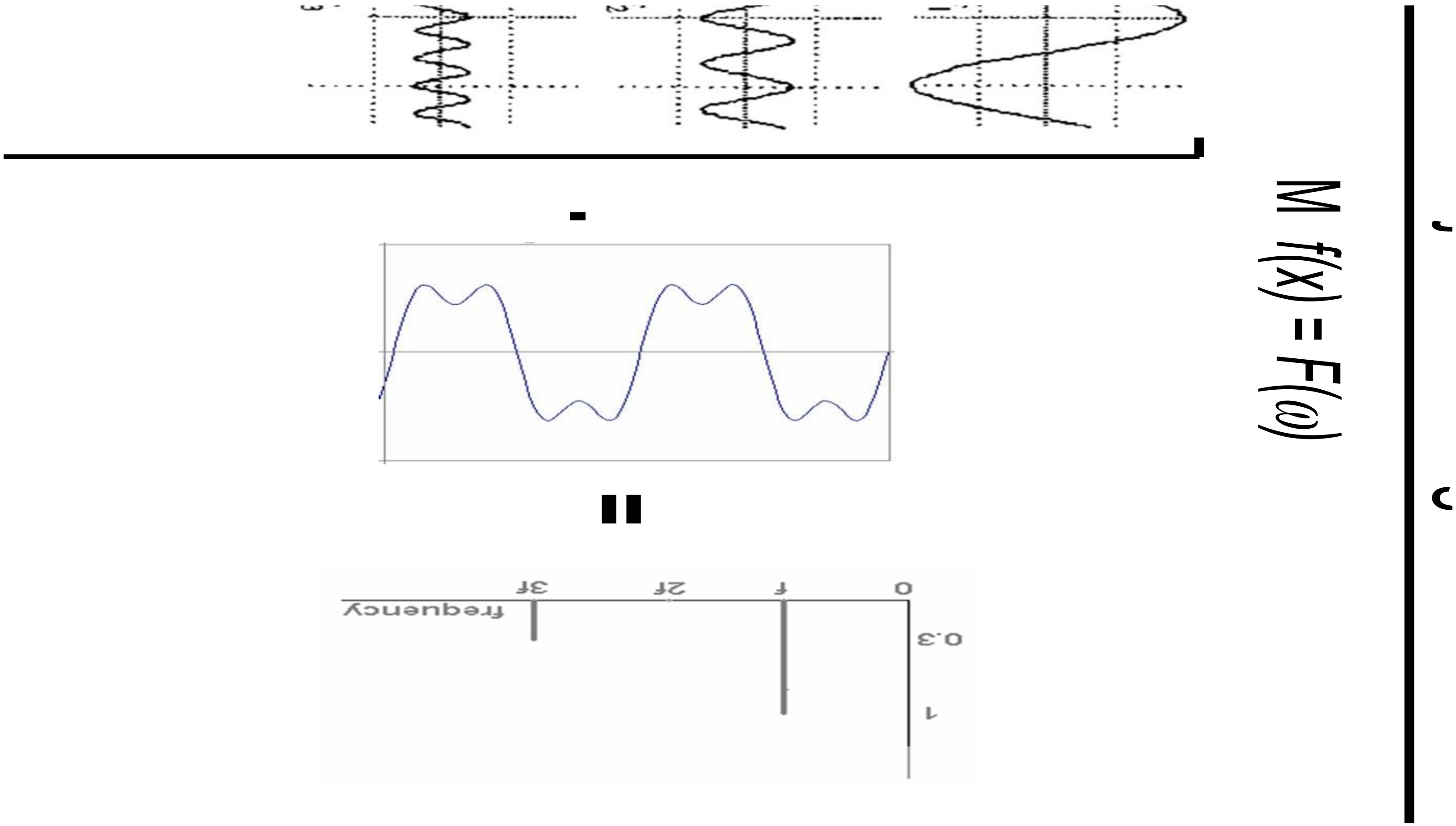
$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$\frac{2}{\pi}\int_{0}^{\pi} \sin nx \, dx = \begin{cases} \frac{4}{n\pi} & \text{if n odd} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=1} \frac{4}{(2n-1)\pi} \sin(2n-1)x$$



Fourier transform: just a change of basis



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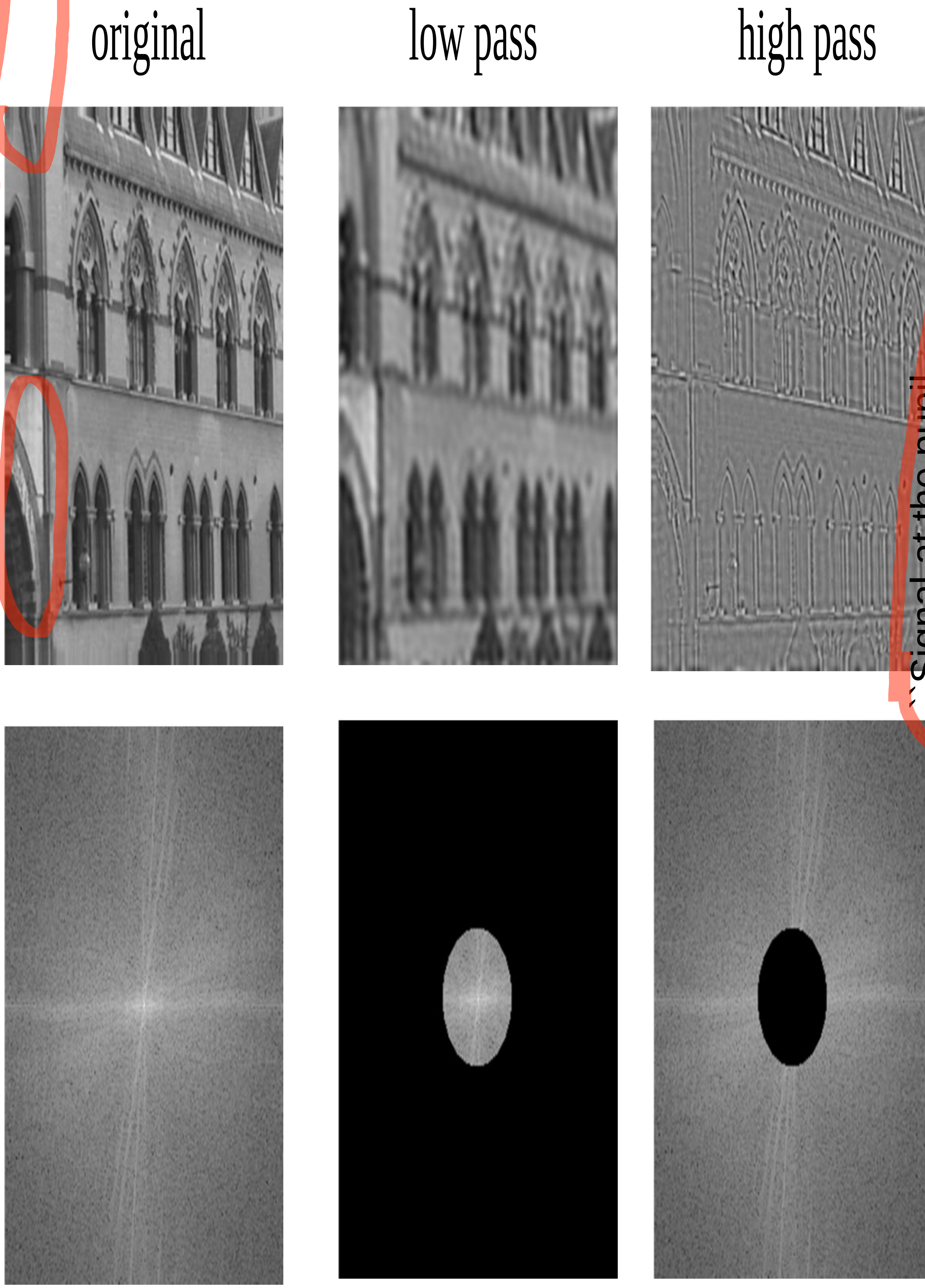


``Signal at the pupil
(‘mirror’) of a telescope”

2. Fourier Series of Periodic Functions

Decomposition using sines and cosines as orthonormal basis set

Periodic function: $f(x) = f(x+P)$

Fourier series: $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2pnx}{P}\right) + b_n \sin\left(\frac{2pnx}{P}\right)\right]$$

Fourier coefficients: $$a_n = \frac{2}{P}\int_{-P/2}^{P/2} f(x)\cos\left(\frac{2pnx}{P}\right)dx$$

$$b_n = \frac{2}{P}\int_{-P/2}^{P/2} f(x)\sin\left(\frac{2pnx}{P}\right)dx$$

Period: P

Frequency: $\nu = 1/P$

Angular frequency: $\omega = 2\pi/P$

Symmetries and Fourier Transforms

symmetry properties of Fourier transforms have many applications

- Define
  - even function: $f_{even}(-x) = f_{even}(x)$
  - odd function: $f_{odd}(-x) = -f_{odd}(x)$
- Hence
  - even part of f(x): $f_{even}(x) = ½[f(x)+f(-x)]$
  - odd part of f(x): $f_{odd}(x) = ½[f(x)-f(-x)]$
  - arbitrary function: $f(x) = f_{even}(x) + f_{odd}(x)$

Fourier Transform Symmetries

$$f(x) = f_{\text{even}}(x) + f_{\text{odd}}(x)$$

$$f_{\text{even}}(-x) = f_{\text{even}}(x) \qquad f_{\text{odd}}(-x) = -f_{\text{odd}}(x)$$

$$e^{-i2\pi xs} = \cos(2\pi xs) - i\sin(2\pi xs)$$

$$\Rightarrow \quad F(s) = 2\int_{0}^{+\infty} f_{\text{even}}(x)\cos(2\pi xs)\,dx$$

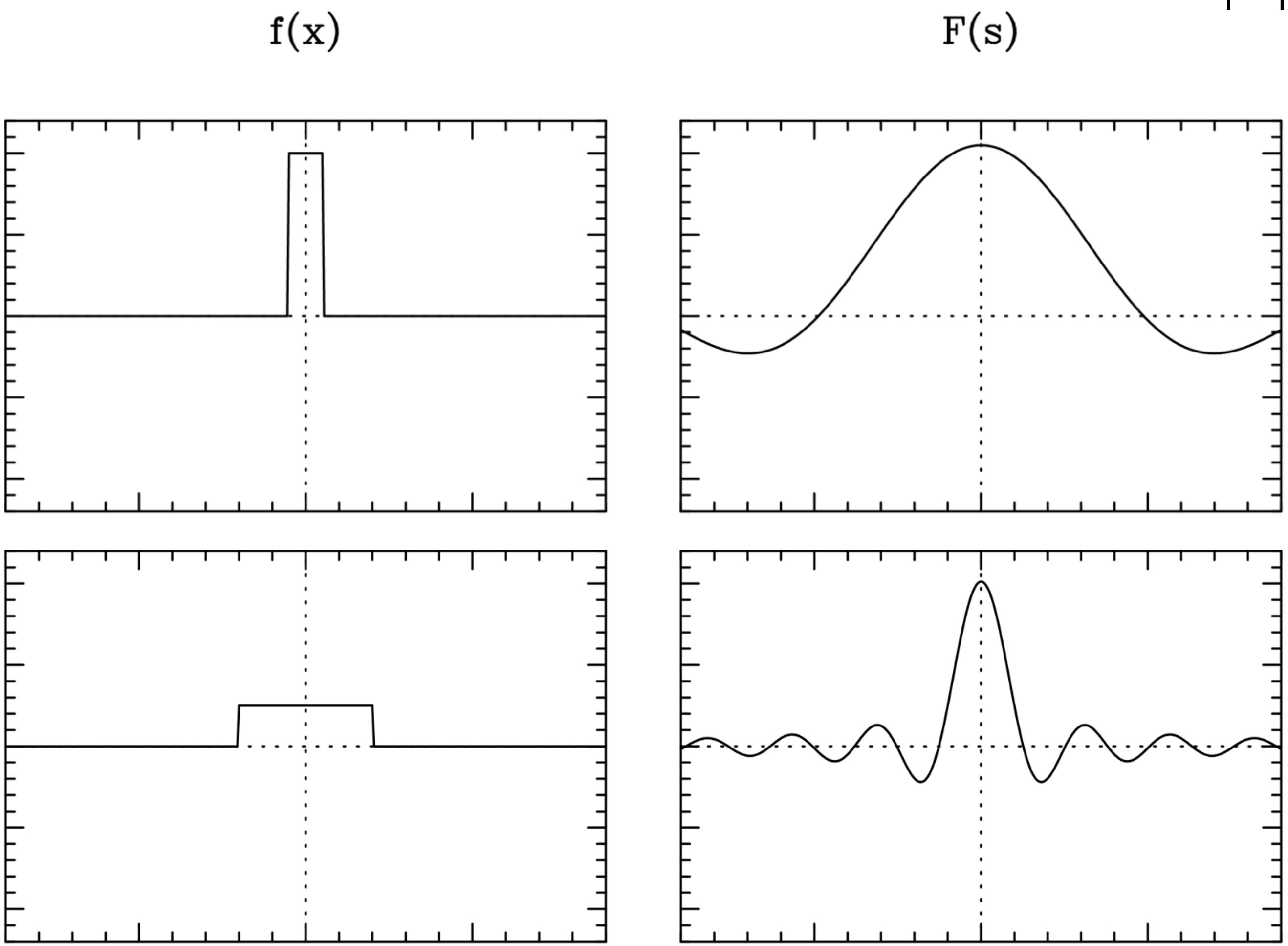
$$- i\, 2\int_{0}^{+\infty} f_{\text{odd}}(x)\sin(2\pi xs)\,dx$$

f(x) real: $f_{\text{even}}(x)$ transforms to (even) real part of F(s),

$f_{\text{odd}}(x)$ transforms to (odd) imaginary part of F(s).

Fourier Transform Similarity

Expansion of *f(x)* contracts F*(s)*: $f(x) \rightarrow f(ax) \Leftrightarrow \frac{1}{|a|} F\left(\frac{s}{a}\right)$



Fourier Transform Properties

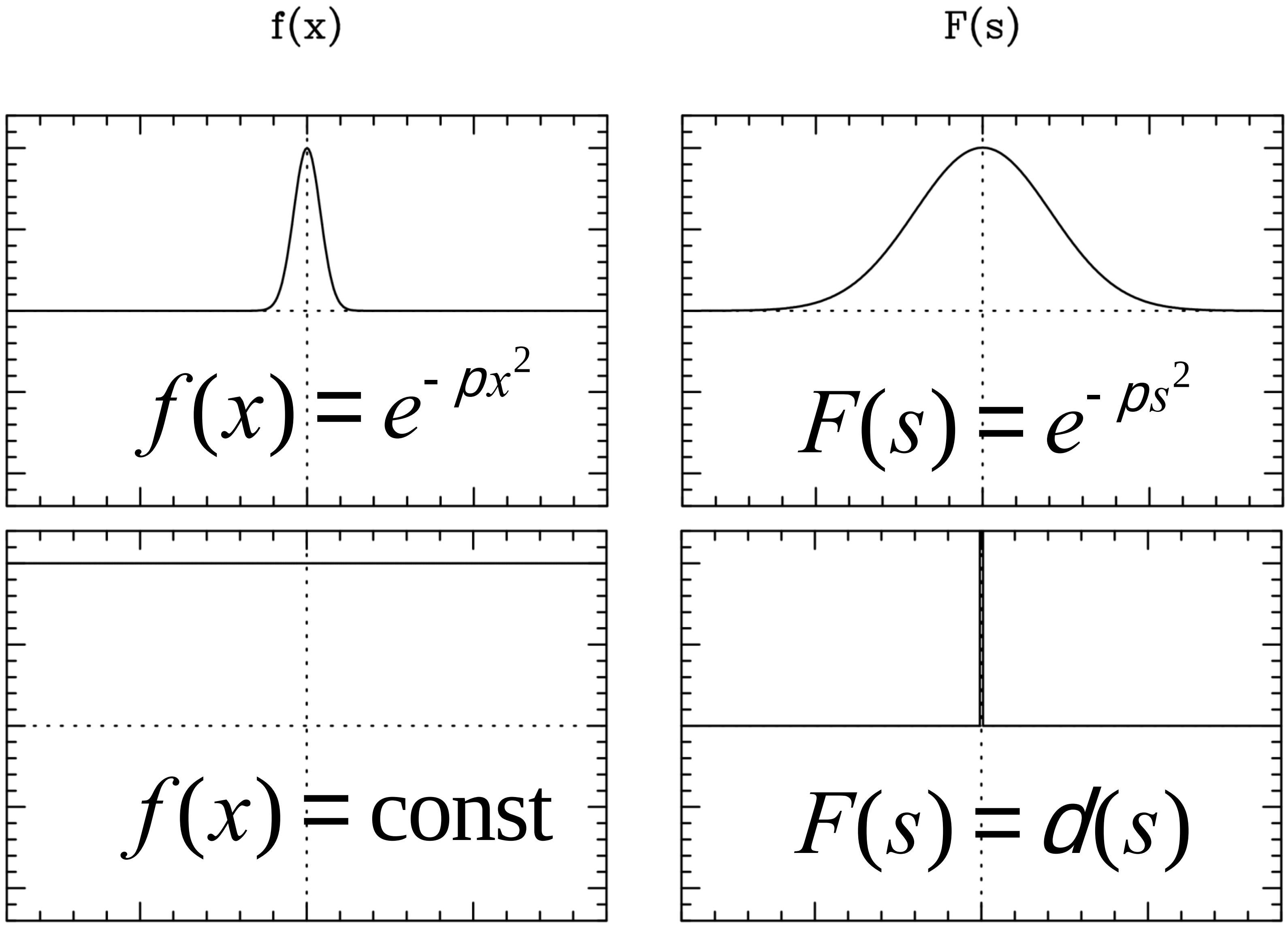
LINEARITY: $a \cdot f(x) + b \cdot g(x) \Leftrightarrow a \cdot F(s) + b \cdot G(s)$

TRANSLATION: $f(x-a) \Leftrightarrow e^{-i2\pi as} F(s)$

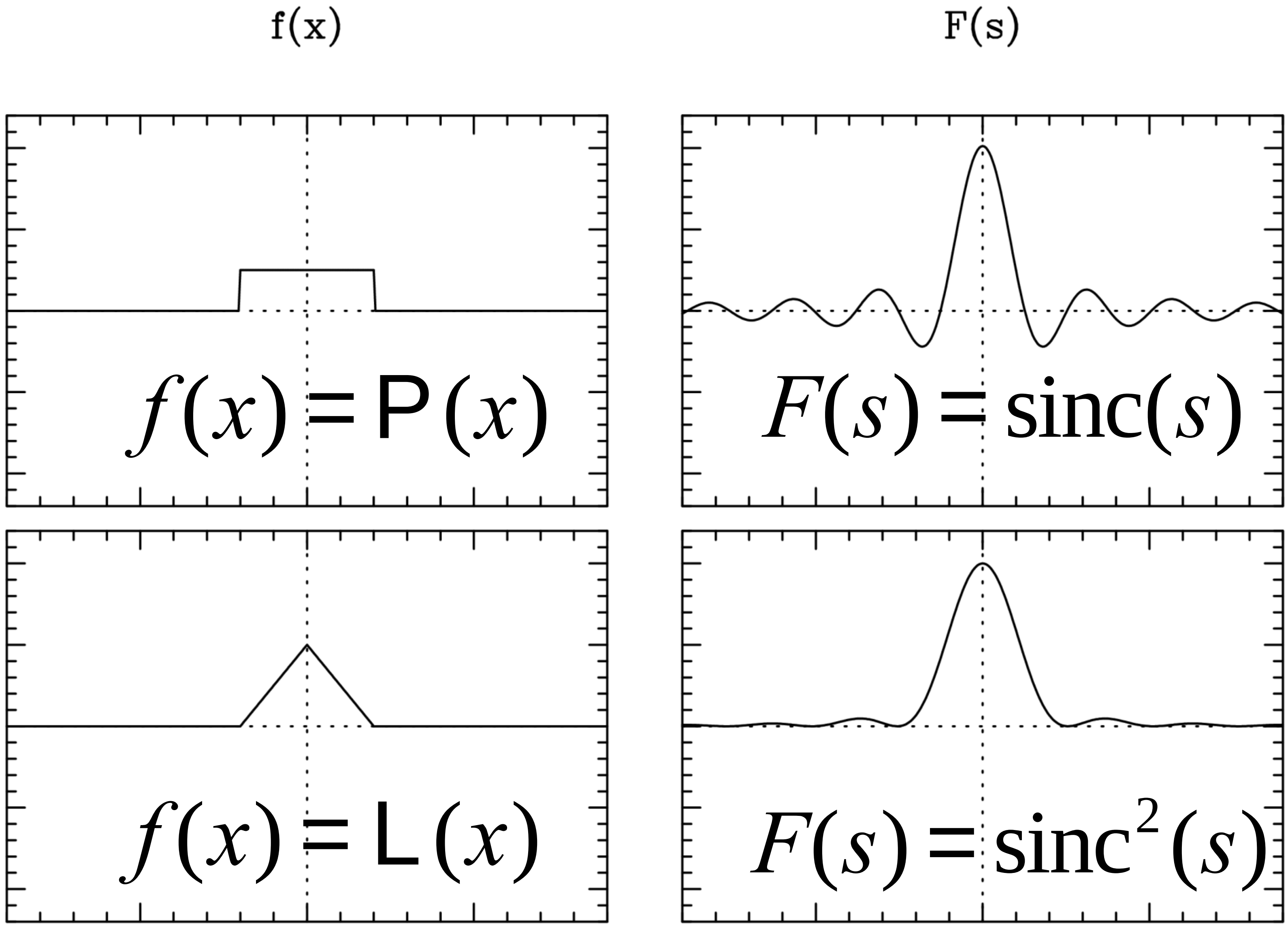
DERIVATIVE: $\dfrac{\partial^n f(x)}{\partial x^n} \Leftrightarrow (i2\pi s)^n F(s)$

INTEGRAL: $\int f(x) \partial x \Leftrightarrow (i2ps)^{-1} F(s) + cd(s)$

Important 1-D Fourier Pairs 1

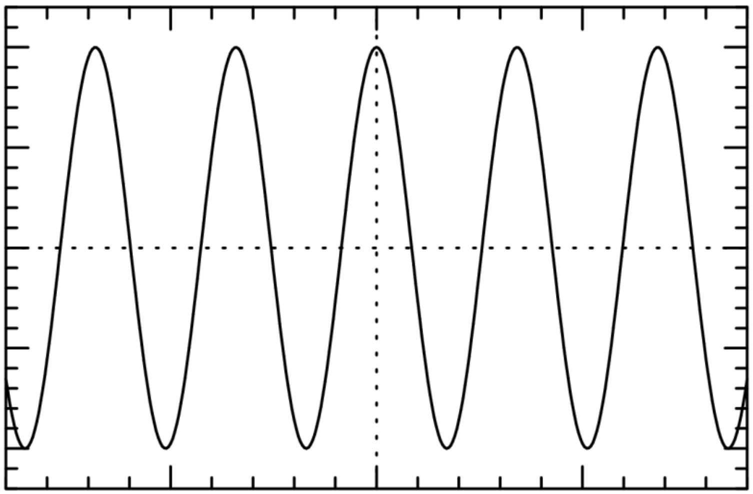


Important 1-D Fourier Pairs 2

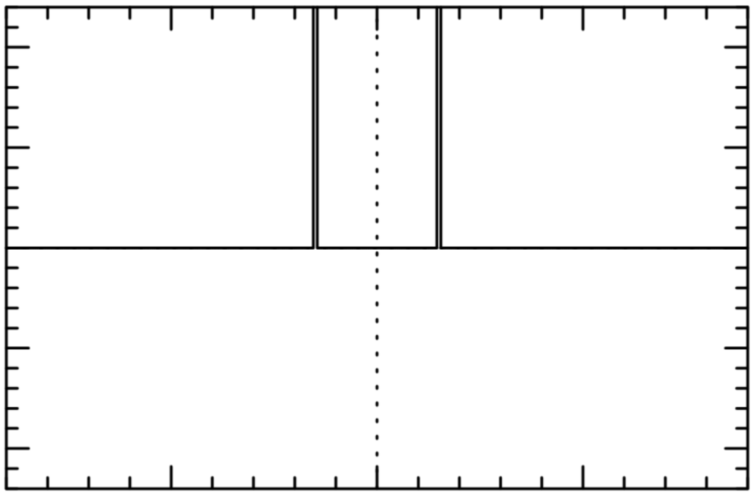


Important 1-D Fourier Pairs 3

f(x)



F(s)



$$f(x) = \cos(px) \qquad F(s) = d(s \pm \frac{1}{2})$$

Convolution

Convolution of two functions, $f*g$, is integral of product of functions after one is reversed and shifted:

$$h(x) = f(x) * g(x) = \int_{-\infty}^{+\infty} f(u) \cdot g(x-u)\,du$$

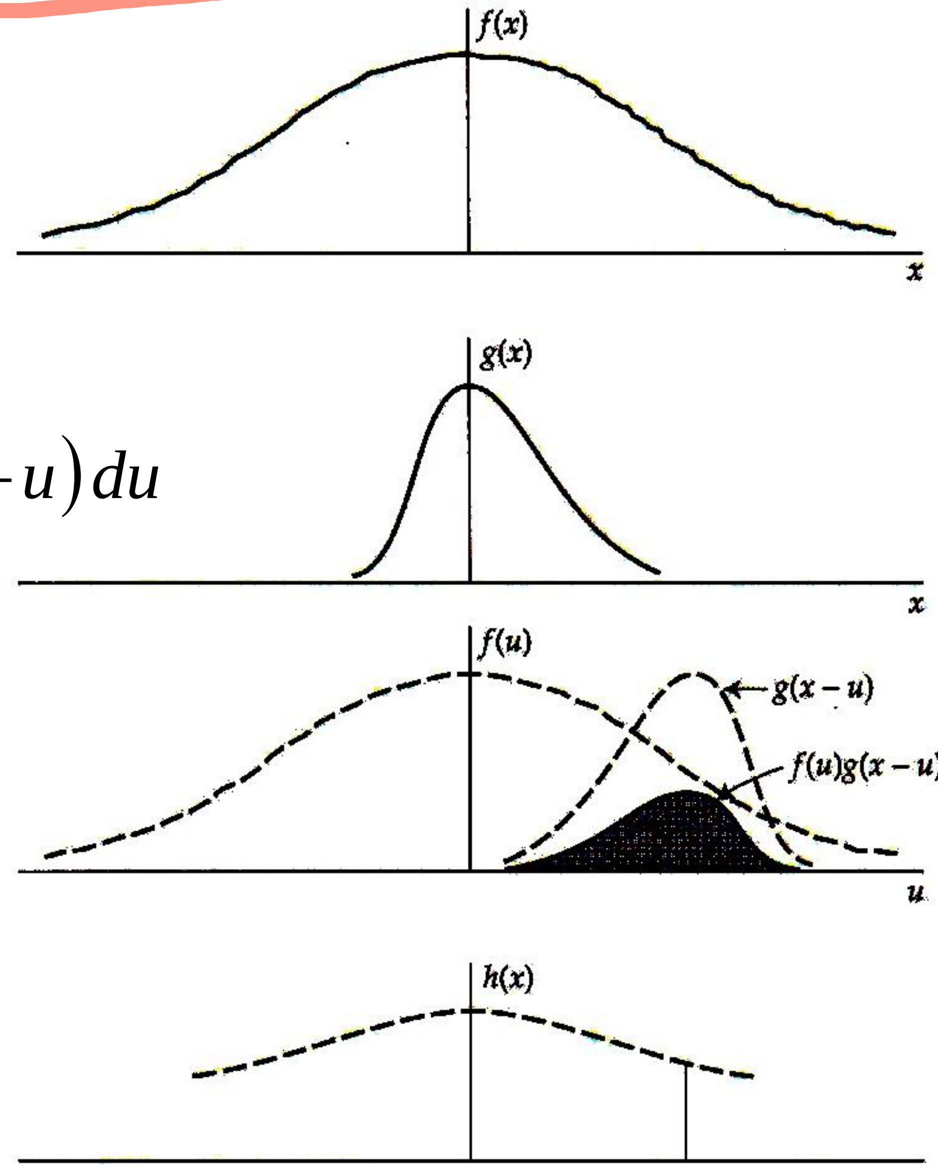
$$f(x) \Leftrightarrow F(s)$$

$$g(x) \Leftrightarrow G(s)$$

$$h(x) = f(x) * g(x)$$

$$\Leftrightarrow$$

$$F(s) \cdot G(s) = H(s)$$



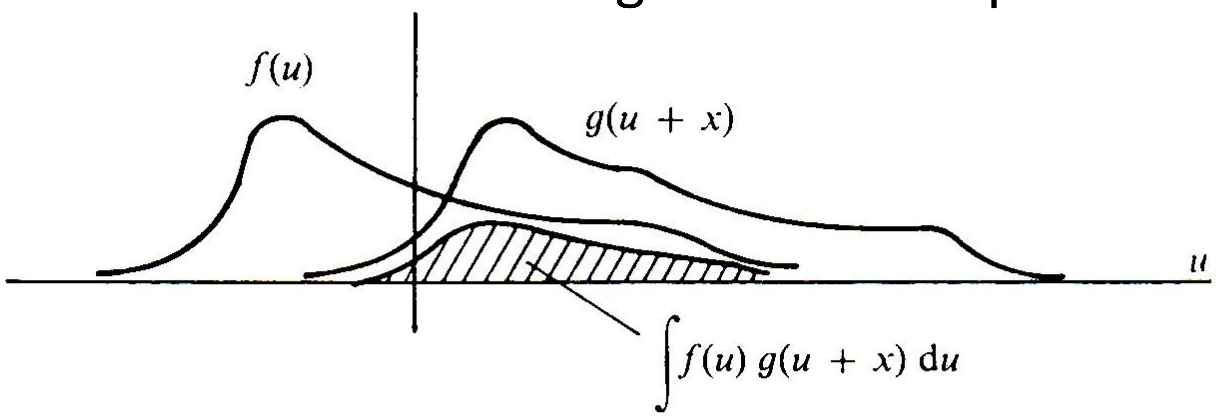
Cross-Correlation

Cross-correlation (or covariance) is measure of similarity of two waveforms as function of time-lag between them.

$$k(x) = f(x) \otimes g(x) = \int_{-\infty}^{+\infty} f(u) \cdot g(x+u)\, du$$

Difference between cross-correlation and convolution:

- Convolution reverses the signal ('-' sign)
- Cross-correlation shifts the signal and multiplies it with another



Interpretation: By how much (x) must *g(u)* be shifted to match *f(u)*? Answer given by maximum of *k(x)*

Cross-Correlation in Fourier Space

$$f(x) \Leftrightarrow F(s)$$

$$g(x) \Leftrightarrow G(s)$$

$$h(x) = f(x) \otimes g(x) \Leftrightarrow F(s) \cdot G^{*}(s) = H(s)$$

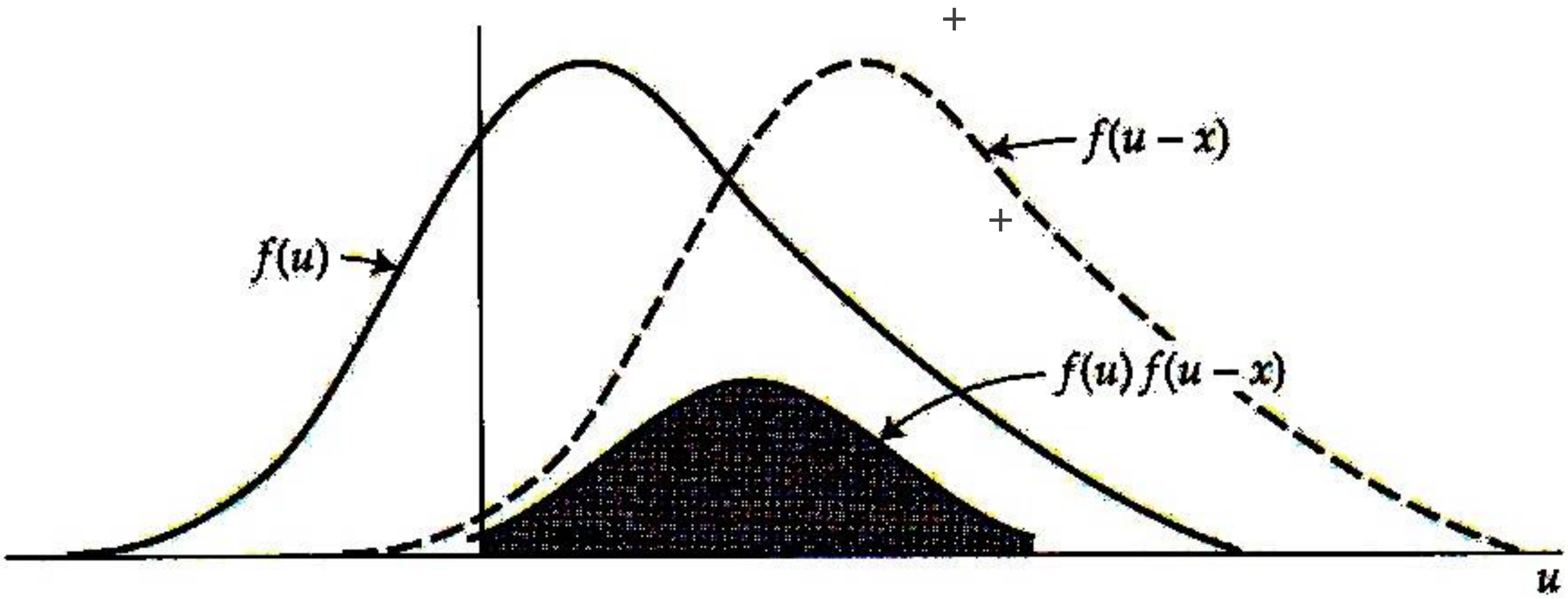
In contrast to convolution, in general

$$f \otimes g \neq g \otimes f$$

Auto-Correlation Theorem

Auto-correlation is cross-correlation of function with itself:

$$k(x) = f(x) \otimes f(x) = \int_{-\infty}^{+\infty} f(u) \cdot f(x+u)\,du$$



$$f(x) \otimes f(x) \Leftrightarrow F(s)F^*(s) = |F(s)|^2$$

Parseval's Theorem

Parseval's theorem (or Rayleigh's Energy Theorem):
Sum of square of a function is the same as sum of square of the Fourier transform:

$$\int_{-\infty}^{+\infty} |f(x)|^2 dx = \int_{-\infty}^{+\infty} |F(s)|^2 ds$$

<u>Interpretation:</u> Total energy contained in signal *f(x)*, summed over all x is equal to total energy of signal's Fourier transform *F(s)* summed over all frequencies *s*.

Wiener-Khinchin Theorem

Wiener–Khinchin theorem states that the power spectral density S_f of a function *f(x)* is the Fourier transform of its auto-correlation function:

$$\left|F(s)\right|^2 = FT\left\{f(x) \otimes f(x)\right\}$$

$$\updownarrow$$

$$F(s) \cdot F^*(s)$$

Applications: E.g. in the analysis of linear time-invariant systems, when the inputs and outputs are not square integrable, i.e. their Fourier transforms do not exist.

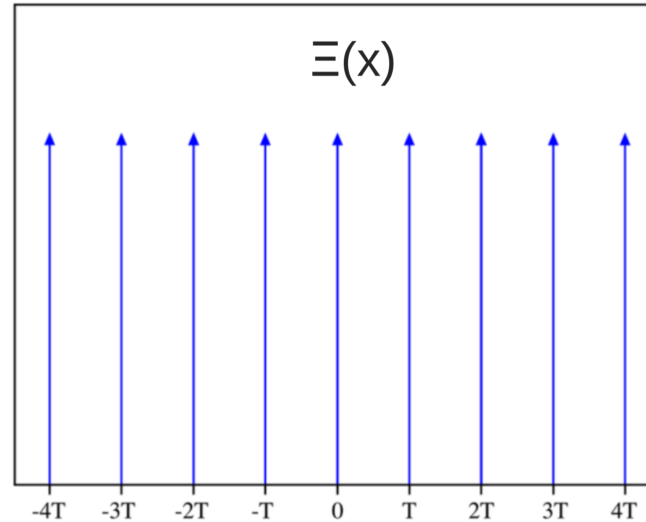
Equation Summary

Convolution	$h(x) = f(x) * g(x) = \int_{-\infty}^{+\infty} f(u) \cdot g(x-u)\, du$
Cross-correlation	$k(x) = f(x) \otimes g(x) = \int_{-\infty}^{+\infty} f(u) \cdot g(x+u)\, du$
Auto-correlation	$k(x) = f(x) \otimes f(x) = \int_{-\infty}^{+\infty} f(u) \cdot f(x+u)\, du$
Power spectrum	$S_f(s) = \lvert F(s) \rvert^2$
Parseval's theorem	$\int_{-\infty}^{+\infty} \lvert f(x) \rvert^2 dx = \int_{-\infty}^{+\infty} \lvert F(s) \rvert^2 ds$
Wiener-Khinchin theorem	$\lvert F(s) \rvert^2 = FT\{f(x)\,\text{Ä}\, f(x)\} = F(s) \times F^*(s)$

Dirac Comb

Dirac's delta "function":

$$f(x) = \delta(x) = \int_{-\infty}^{+\infty} e^{i2\pi sx}\, ds \;\rightarrow\; FT\{\delta(x)\} =$$



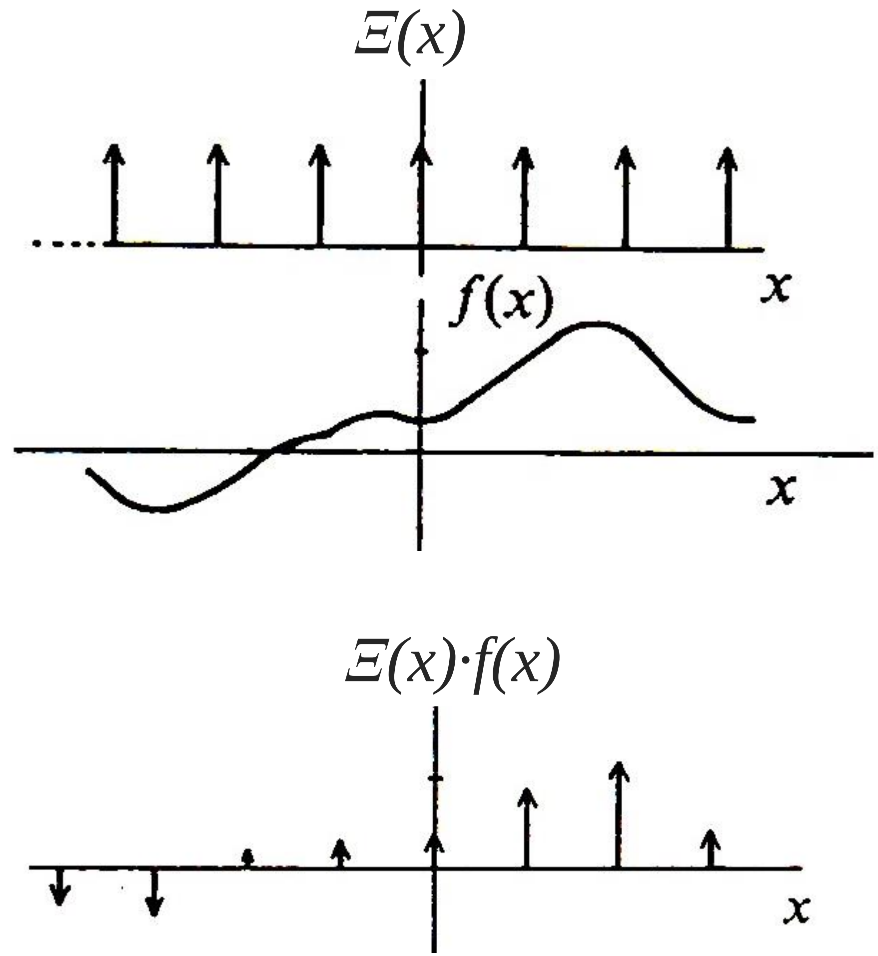
Dirac comb: infinite series of delta functions spaced at intervals of T:

$$\text{Ш}_T(x) = \sum_{k=-\infty}^{\infty} \delta(x - kT) \overset{Fourier}{\underset{series}{=}} \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{i2\pi nx/T}$$

- Fourier transform of Dirac comb is also a Dirac comb
- Dirac comb is also called impulse train or sampling function

5. Sampling

- Signal only sampled at discrete points in time
- Often constant sampling interval
- Sampling can be described as multiplication of true signal with Dirac comb
- Fourier transform of sampled signal is sampled Fourier transform of true signal



Nyquist Theorem

Sampling: signal at discrete values of x: $$f(x) \rightarrow f(x) \cdot Ш\left(\frac{x}{\Delta x}\right)$$

Interval between two successive readings is sampling rate

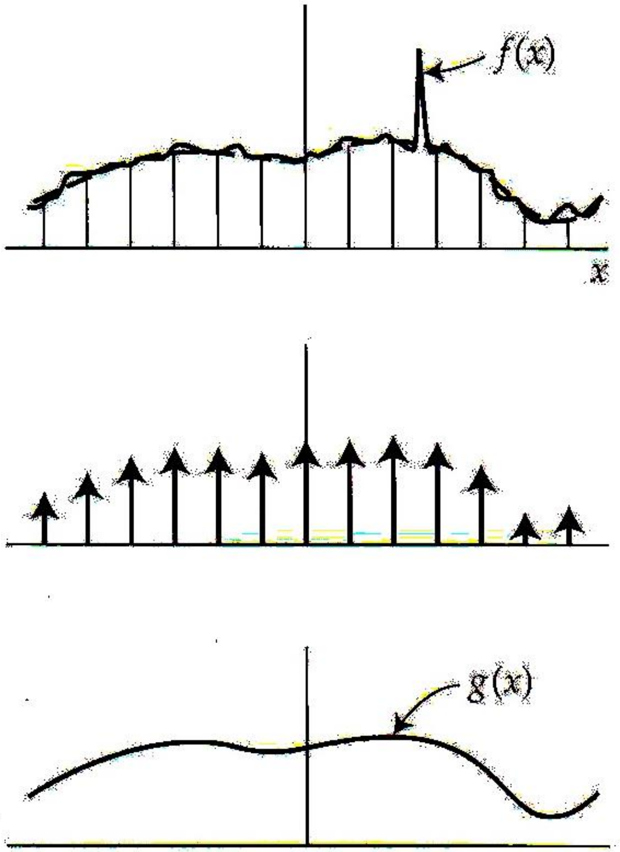
Critical sampling given by Nyquist theorem

Given f(x), its Fourier Transform F(s) defined on $[-s_{max}, s_{max}]$.

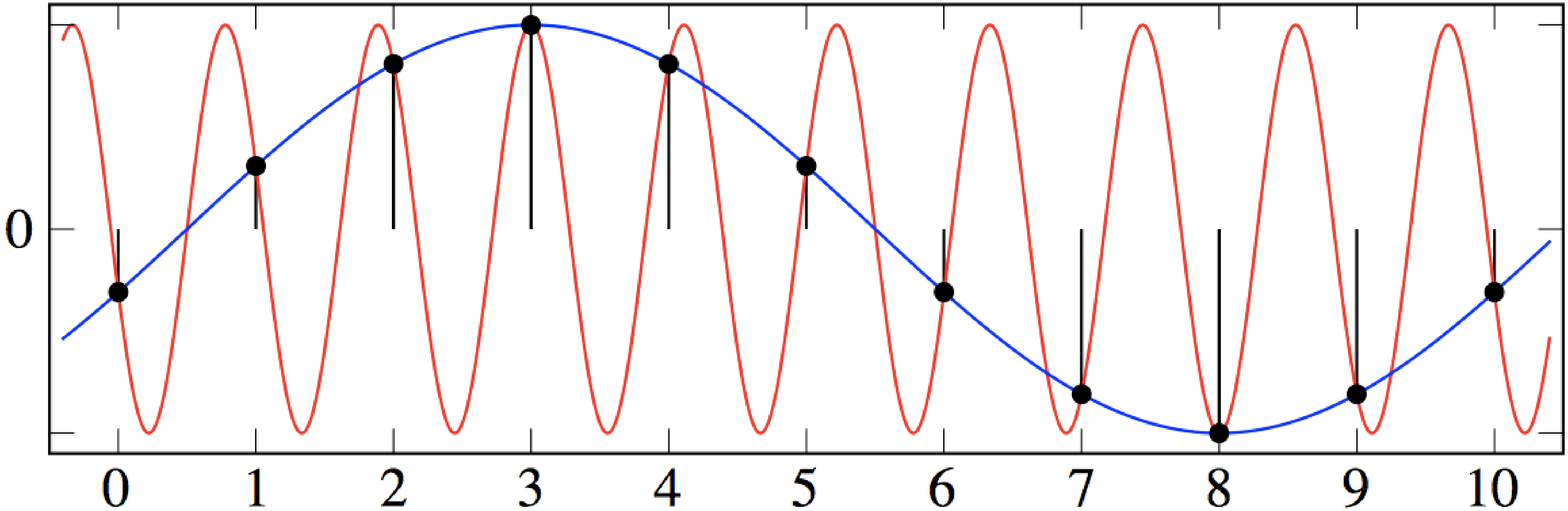
Sampled distribution of the form

$$g(x) = f(x) \cdot Ш\left(\frac{x}{\Delta x}\right)$$

with a sampling rate of $\Delta x = 1/(2s_{max})$ is enough to reconstruct *f(x)* for all *x*.



Aliasing



- undersampled, high frequencies look like well-sampled low frequencies
- create spurious components below Nyquist frequency
- may create major problems and uncertainties in determination of original signal

Lecture 3: Everything You Always Wanted to Know About Optics

1. Waves
2. Ideal Optics
3. Zernike-van Cittert Theorem
4. PSFs
5. Aberrations

Waves from Maxwell Equations

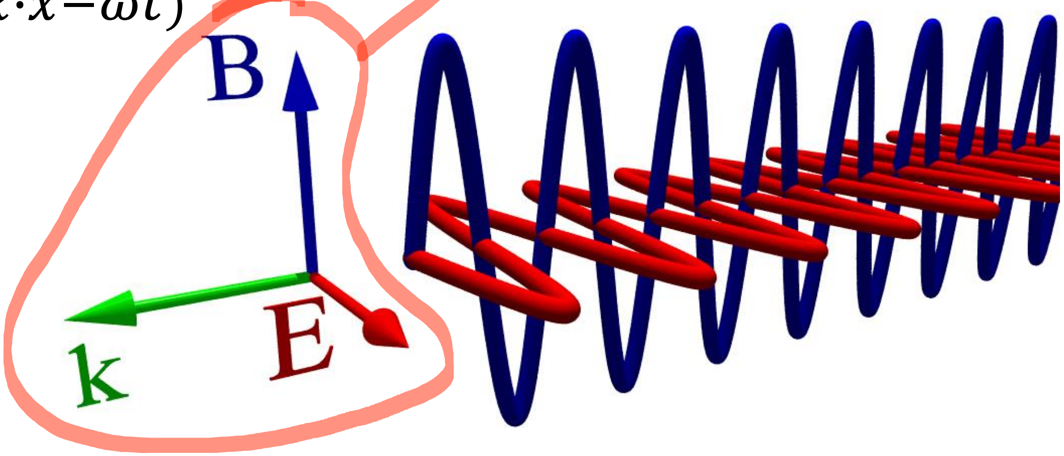
- Maxwell equations & linear material equations: differential equation for damped waves

$$\nabla^2 \vec{E} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{E}}{\partial t} = 0$$

- $\vec{E}$ electric field vector
- Material properties:
  - ε dielectric constant
  - μ magnetic permeability ($\mu = 1$ for most materials)
  - σ electrical conductivity (controls damping)
- c speed of light
- Same equation for magnetic field H

Plane Waves

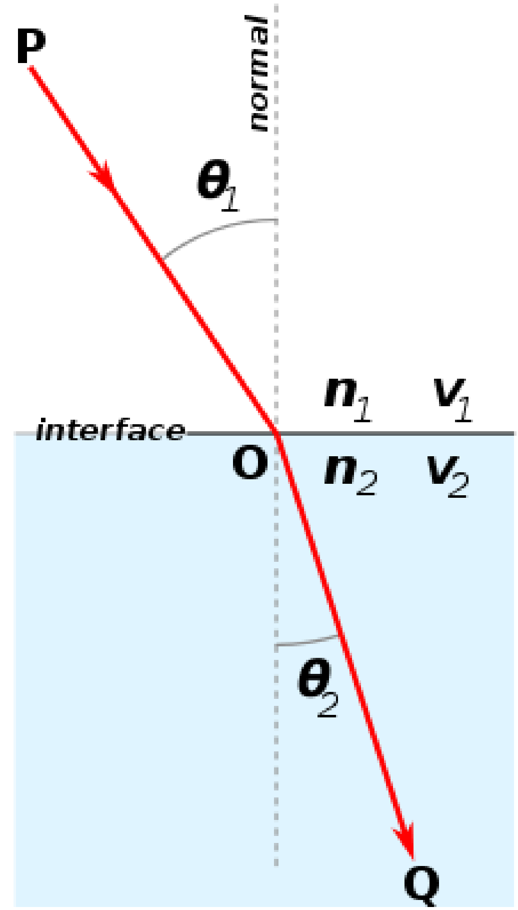
- Linear equation: sum of solutions is also solution
- Plane wave $\vec{E} = \overrightarrow{E_0} e^{i(\vec{k}\cdot\vec{x}-\omega t)}$
  - $\overrightarrow{E_0}$ complex constant vector (polarization)
  - $\vec{k}$ wave vector
  - $\vec{x}$ spatial location
  - ω angular frequency



- real electric field given by real part of E
- dispersion relation: $\vec{k} \cdot \vec{k} = \frac{\omega^2}{c^2}\tilde{n}^2, \tilde{n}^2 = \mu\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)$
- complex index of refraction $\tilde{n} = n + ik$
- $\vec{E}, \vec{H}, \vec{k}$ form right-handed triplet of orthogonal vectors

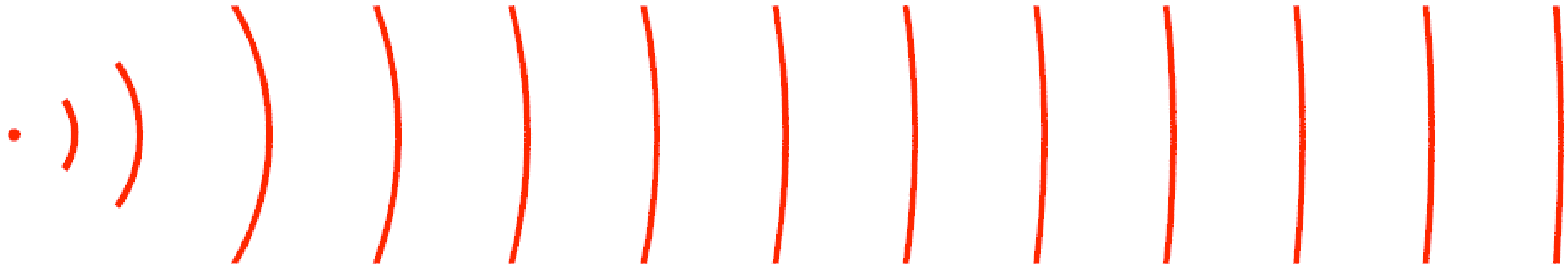
Snell's law

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$



Spherical and Plane Waves

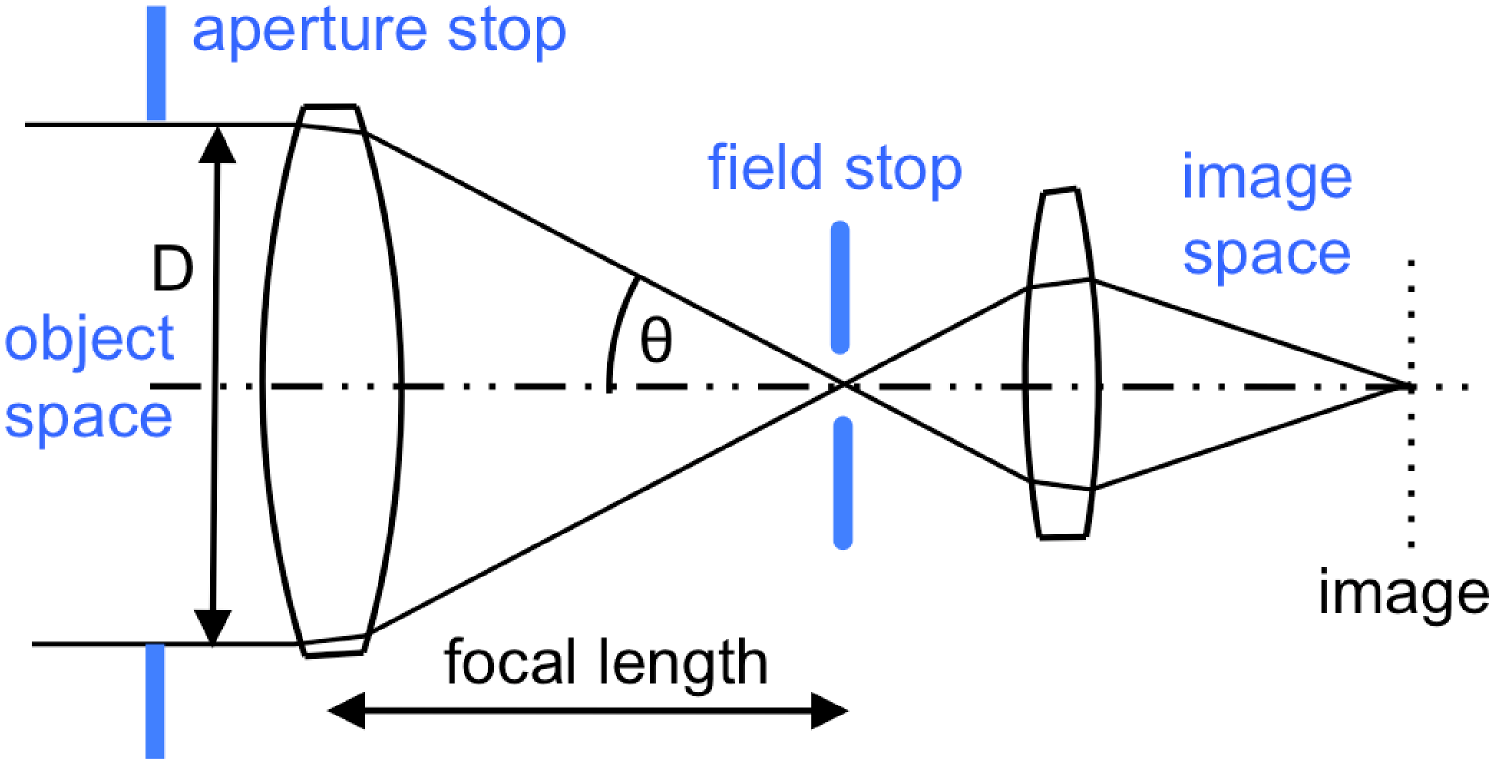
- light source: collection of sources of spherical waves
- astronomical sources: almost exclusively incoherent
- lasers, masers: coherent sources
- spherical wave originating at very large distance can be approximated by plane wave



Ideal Optical System

ideal optical system transforms plane wavefront into spherical, converging wavefront

Aperture and Field Stops

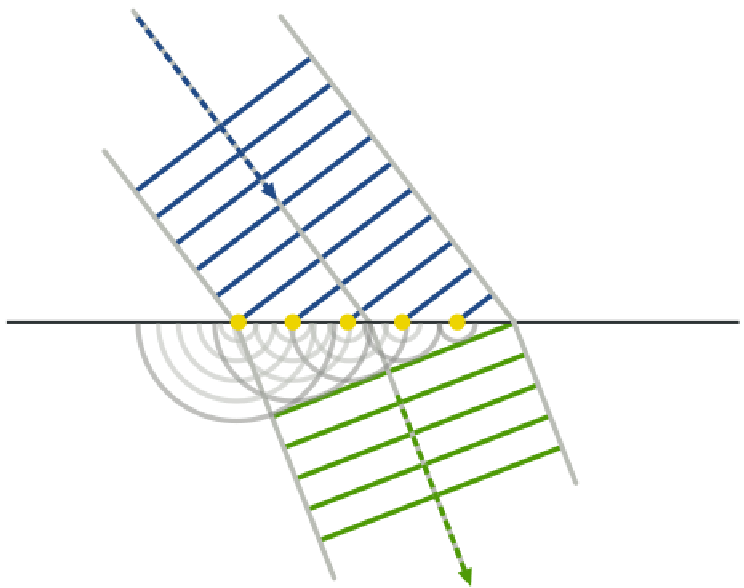


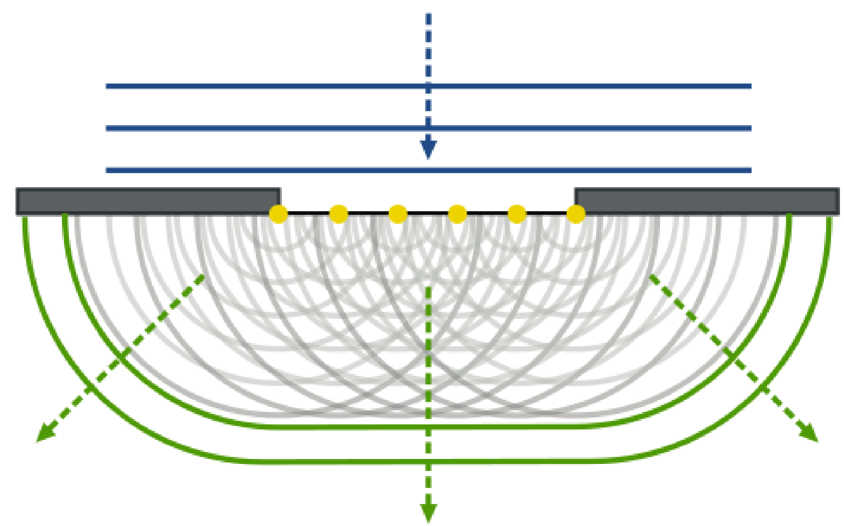
- Aperture stop: determines diameter of light cone from axial point on object.
- Field stop: determines the field of view of the system.

The Huygens-Fresnel Principle

Fermat's view: "A wavefront is a surface on which every point has the same OPD."

Huygens' view: "At a given time, each point on primary wavefront acts as a source of secondary spherical wavelets. These propagate with the same speed and frequency as the primary wave."





Reminder: Coherent Radiation

A light source may exhibit temporal and spatial coherence. The coherence function Γ_{12} between two points (1,2) is the cross-correlation between their complex amplitudes:

$$\Gamma_{12}(\tau) = \langle E_1(t+\tau) E_2^*(t) \rangle$$

The normalized representation is called the degree of coherence:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}}$$

which leads to an interference pattern* with an intensity distribution of:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}[\gamma_{12}(\tau)]$$

where

$$\begin{array}{ll} |\gamma_{12}| = 1 & \text{coherent} \\ |\gamma_{12}| = 0 & \text{incoherent} \\ 0 < |\gamma_{12}| < 1 & \text{partial coherence} \end{array}$$

and the visibility for $I_1 = I_2$.

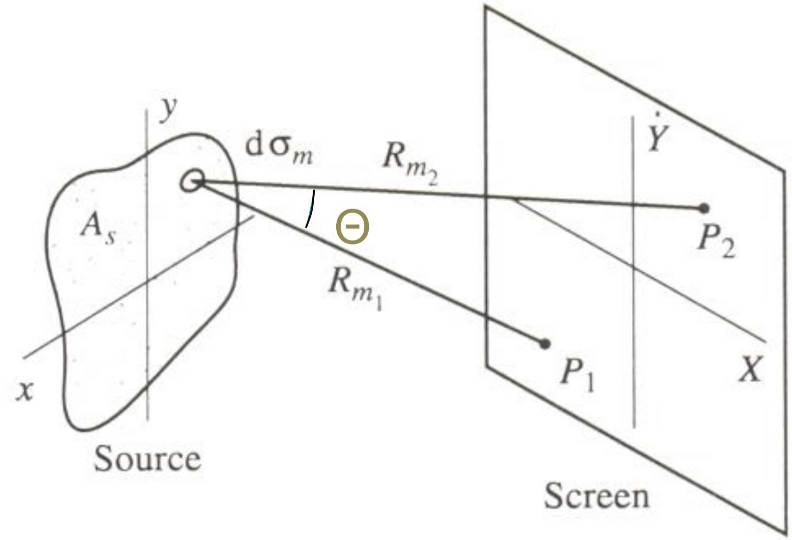
$$V = |\gamma_{12}(\tau)|$$

*e.g., from Young's double slit experiment

The Zernike-van Cittert Theorem (1)

Consider a monochromatic, extended, incoherent source A_s with intensity I(x,y).

Consider further a surface element dσ (dσ <<λ), which illuminates two points P_1 and P_2 at distances R_1 and R_2 on a screen.



The quantity measuring the correlation of the electric fields between P_1 and P_2 (for any surface element dσ at distance r) is:

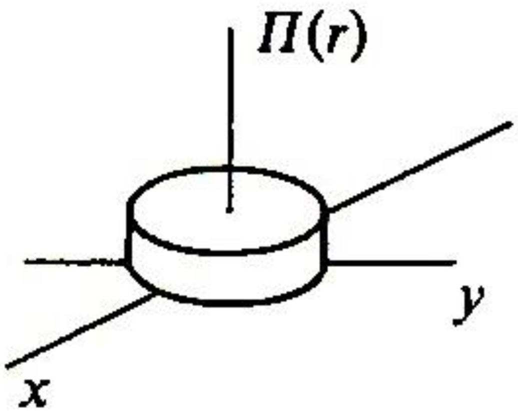
$$\langle V_1(t) V_2^*(t) \rangle = \int_{A_s} I(r) \frac{\exp[ik(R_1 - R_2)]}{R_1 R_2} dr$$

Fourier Pair in 2-D: Box Function

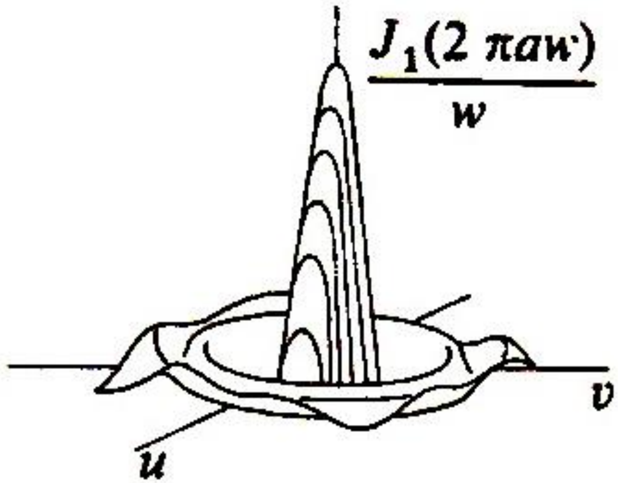
2-D box function with $r^2 = x^2 + y^2$: $$\Pi\left(\frac{r}{2}\right) = \begin{cases} 1 & \text{for } r < 1 \\ 0 & \text{for } r \geq 1 \end{cases}$$

Fourier Transform: $\Pi\left(\frac{r}{2}\right) \Leftrightarrow \frac{J_1(2\pi\omega)}{\omega}$ (1st order Bessel function J_1)

Electric Field in Telescope Aperture:

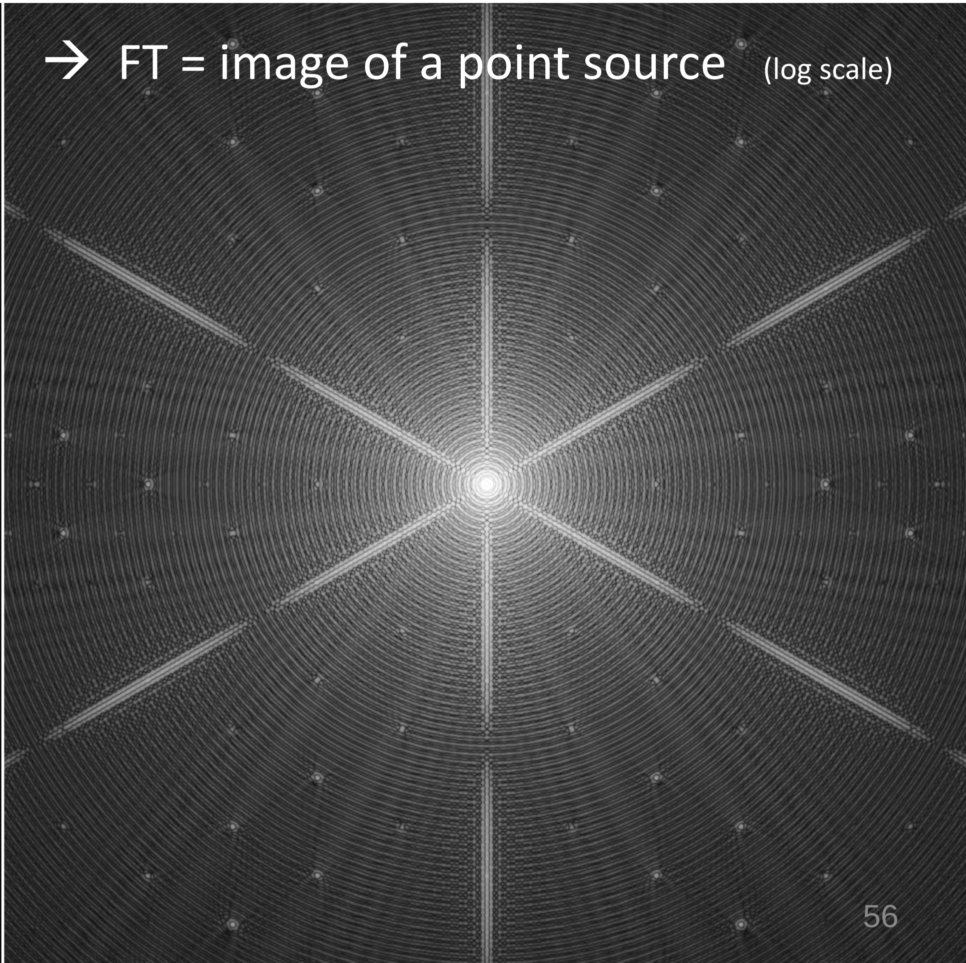


Electric Field in Focal plane:



Larger telescopes produce smaller Point Spread Functions (PSFs)!

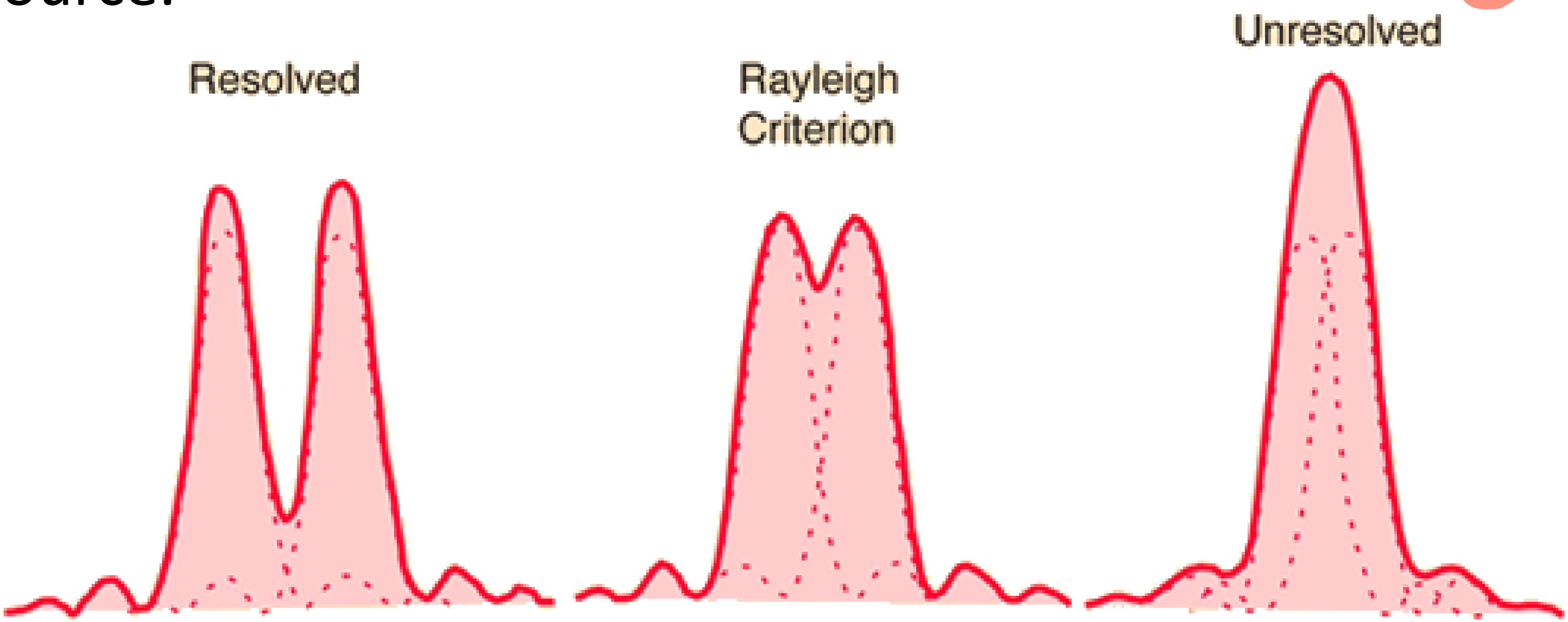
central obscuration,
monolithic mirror (pupil)
with 6 support-spiders



Angular Resolution: Rayleigh Criterion

Two sources can be resolved if the peak of the second source is no closer than the 1st dark Airy ring of the first source.

$$\sin\Theta = 1.22\frac{\lambda}{D}$$



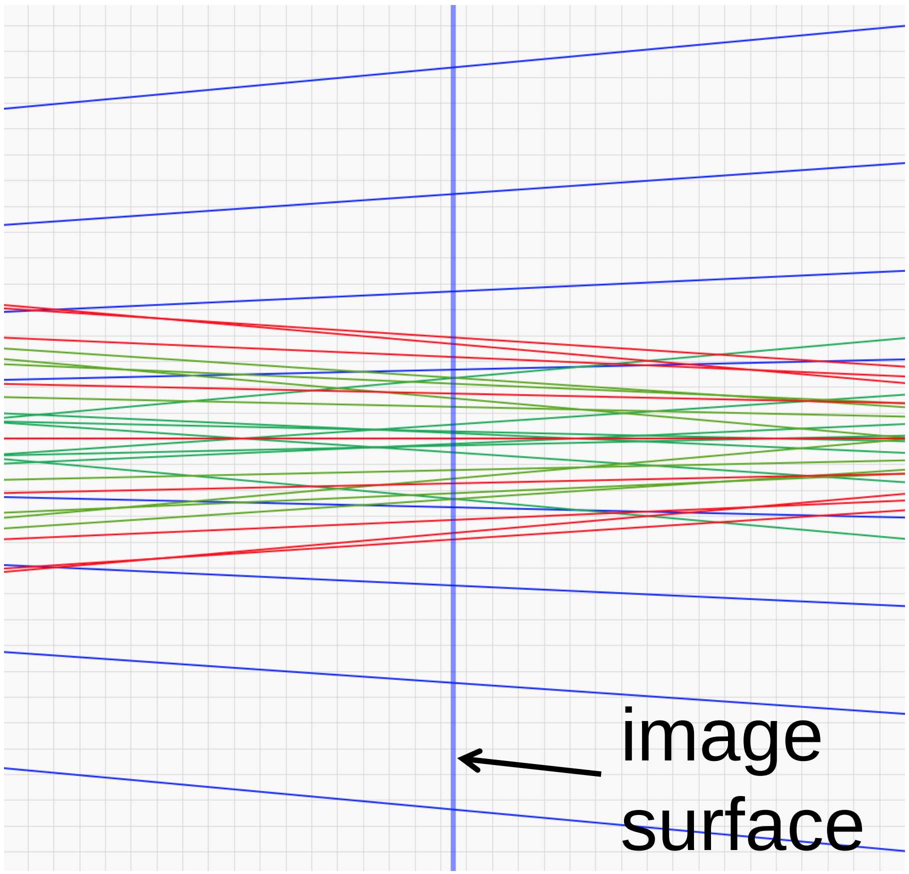
http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/Raylei.html

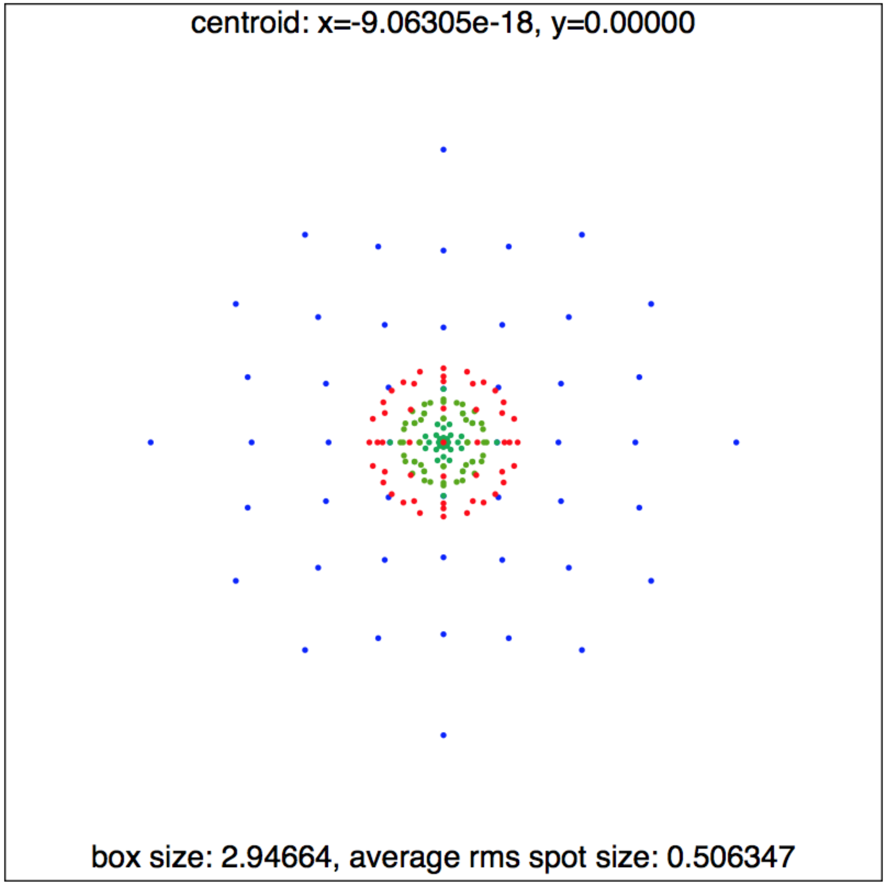
Aberrations

Aberrations are departures of the performance of an optical system from the ideal optical system.

1. On-axis aberrations: aberrations that can be seen everywhere in the image, also on the optical axis (center of the image)
2. Off-axis aberrations: aberrations that are absent on the optical axis (center of the image)
   a) Aberrations that degrade the image
   b) Aberrations that alter the image position

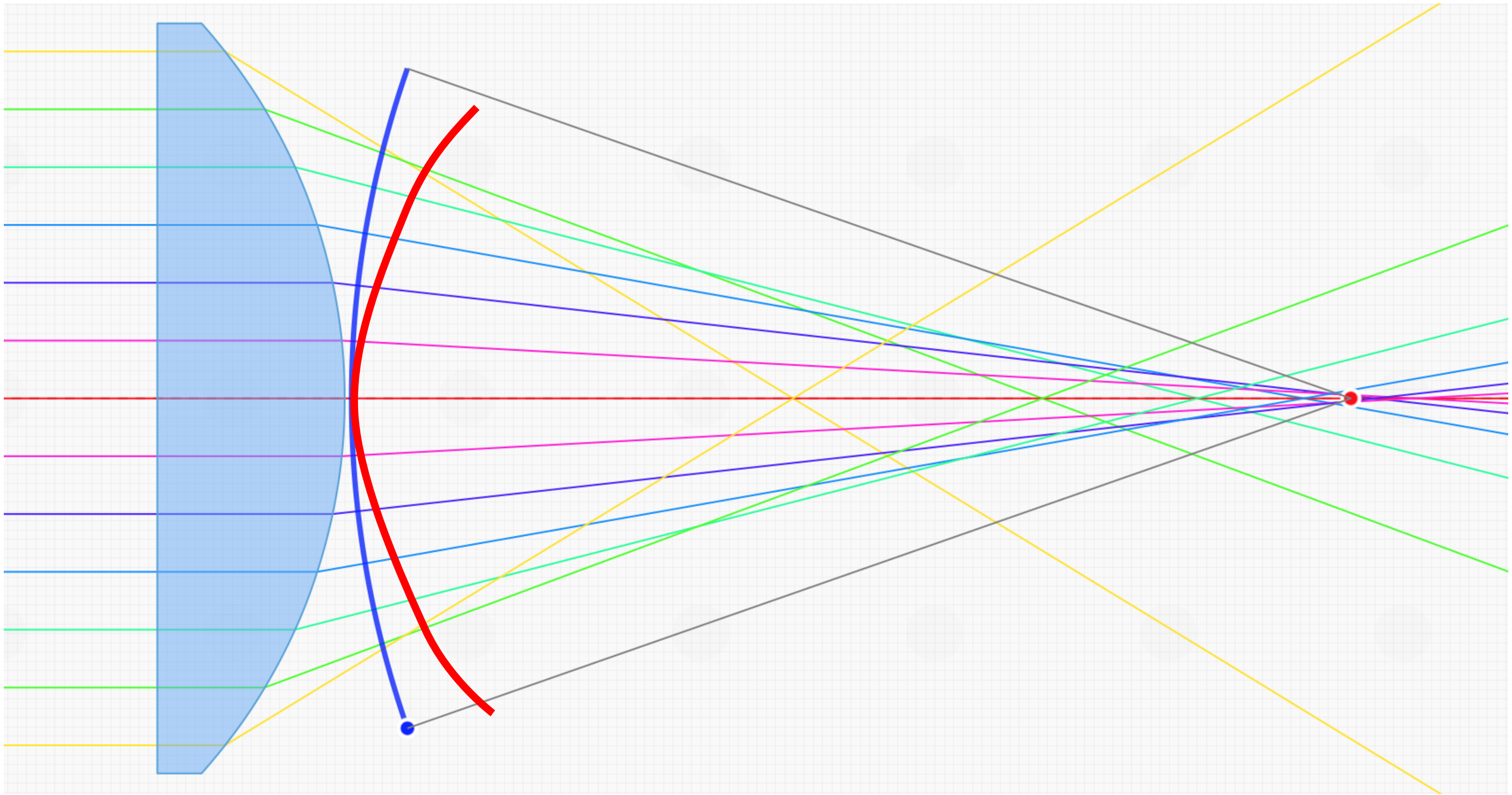
Spot Diagram





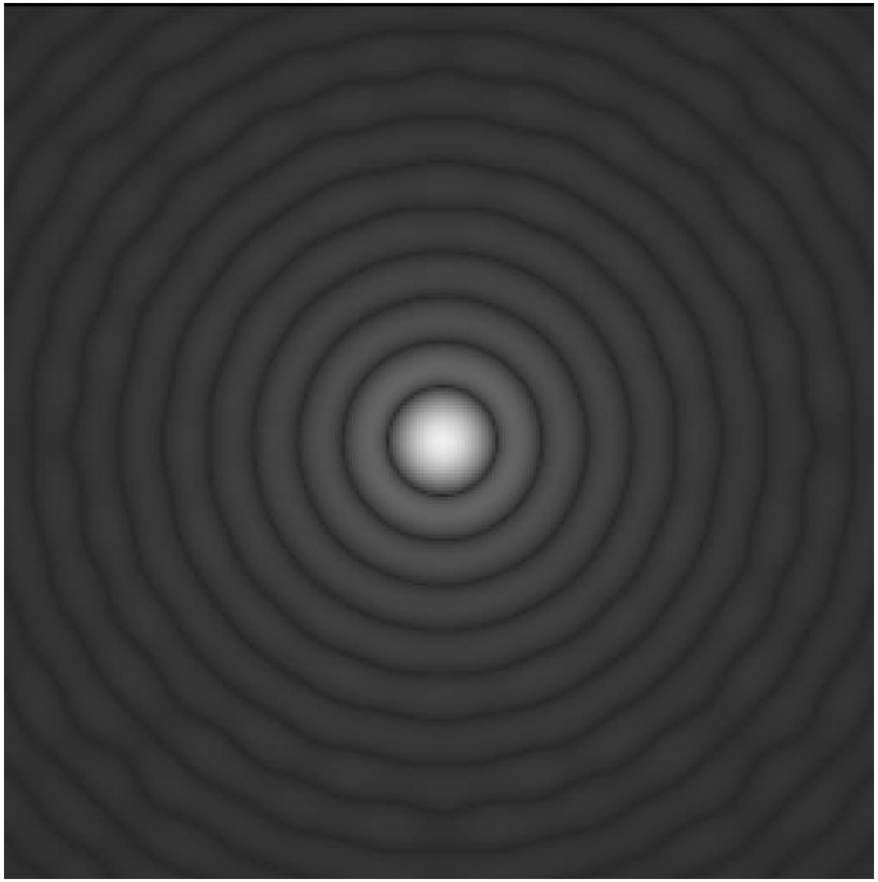
intersection of rays with image surface

Wavefront Error

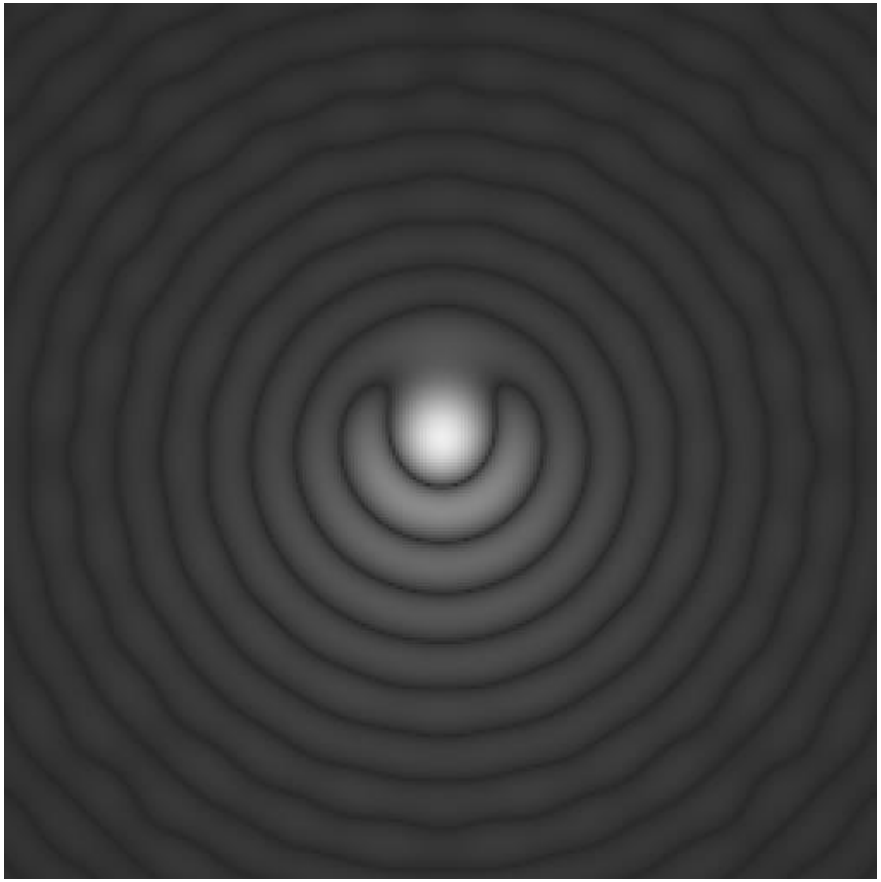


- deviation of surface that is normal to rays and spherical reference surface (distance between red & blue line)
- often shown as grayscale image or 3D surface

Aberrated Point-Spread Function



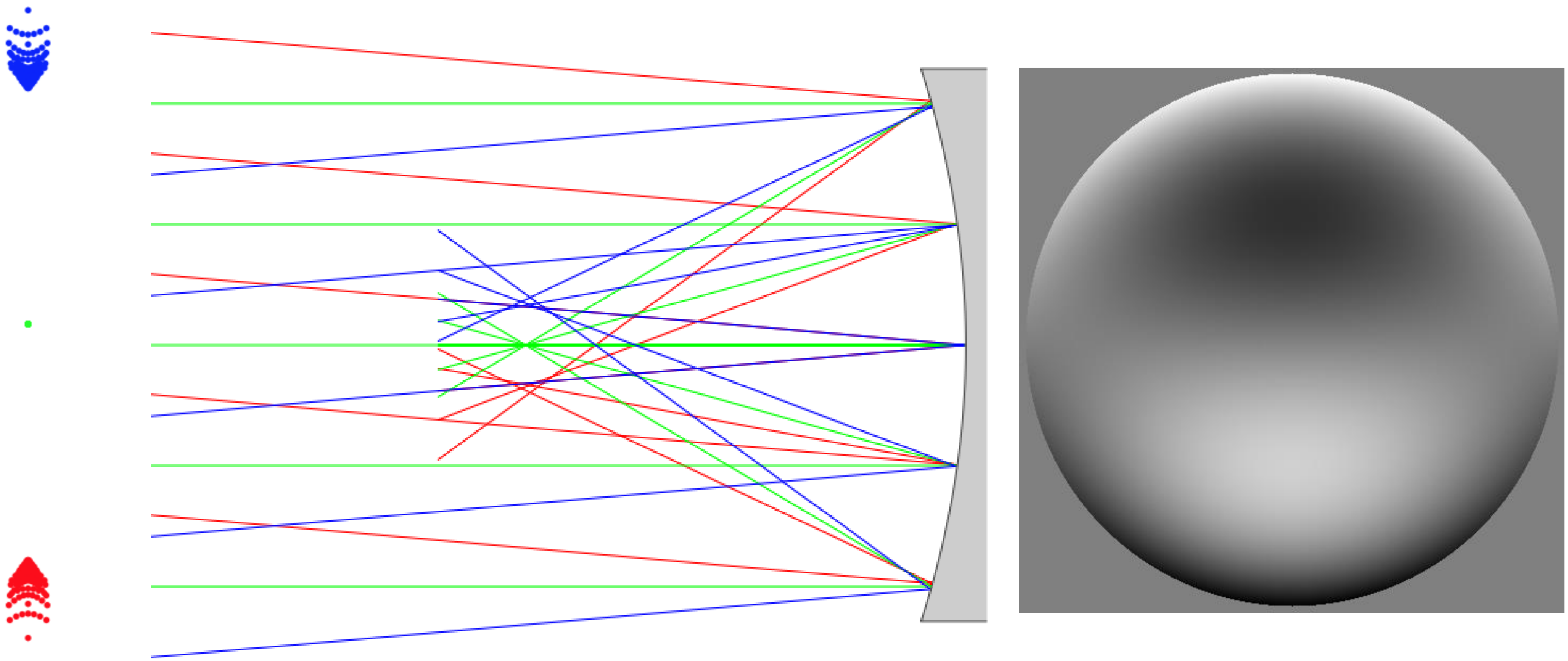
perfect



aberrated

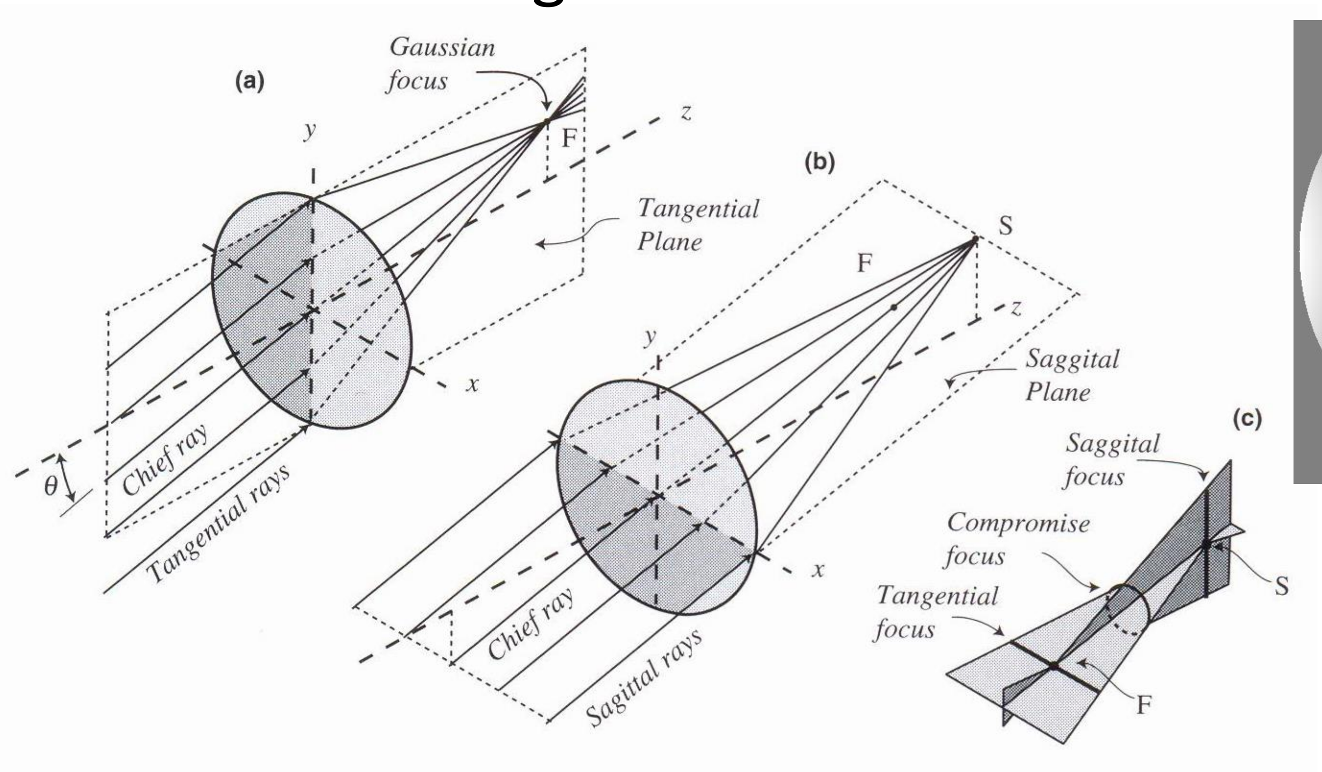
Coma

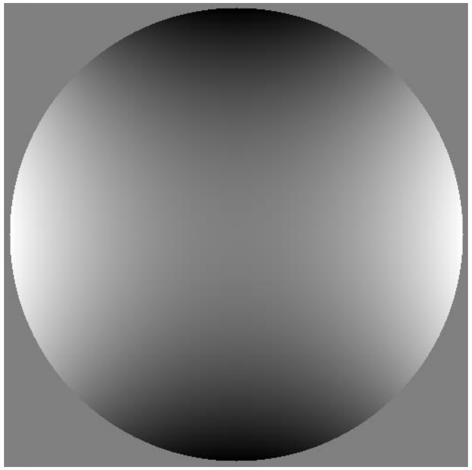
Variation of magnification across entrance pupil. Point sources will show a come-like tail. Coma is an inherent property of telescopes using parabolic mirrors



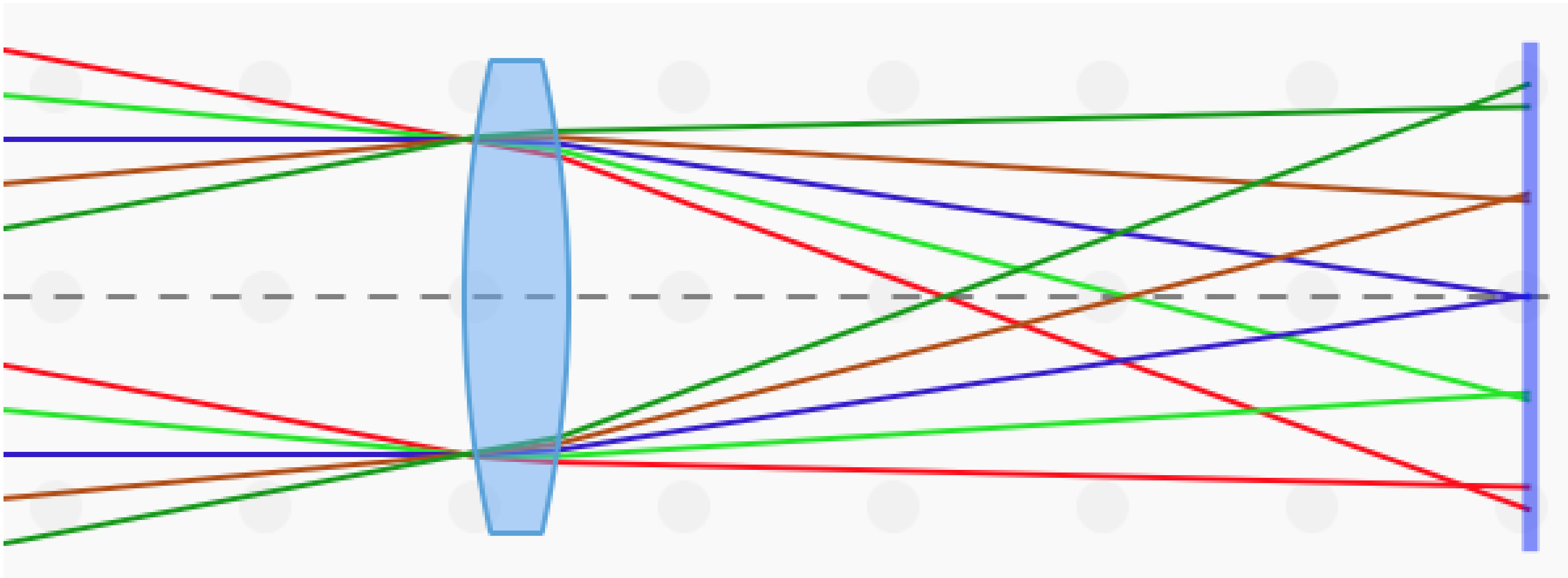
Astigmatism

From off-axis point A lens does not appear symmetrical but shortened in plane of incidence (tangential plane). Emergent wave will have a smaller radius of curvature for tangential plane than for plane normal to it (sagittal plane) and form an image closer to the lens.





Field Curvature

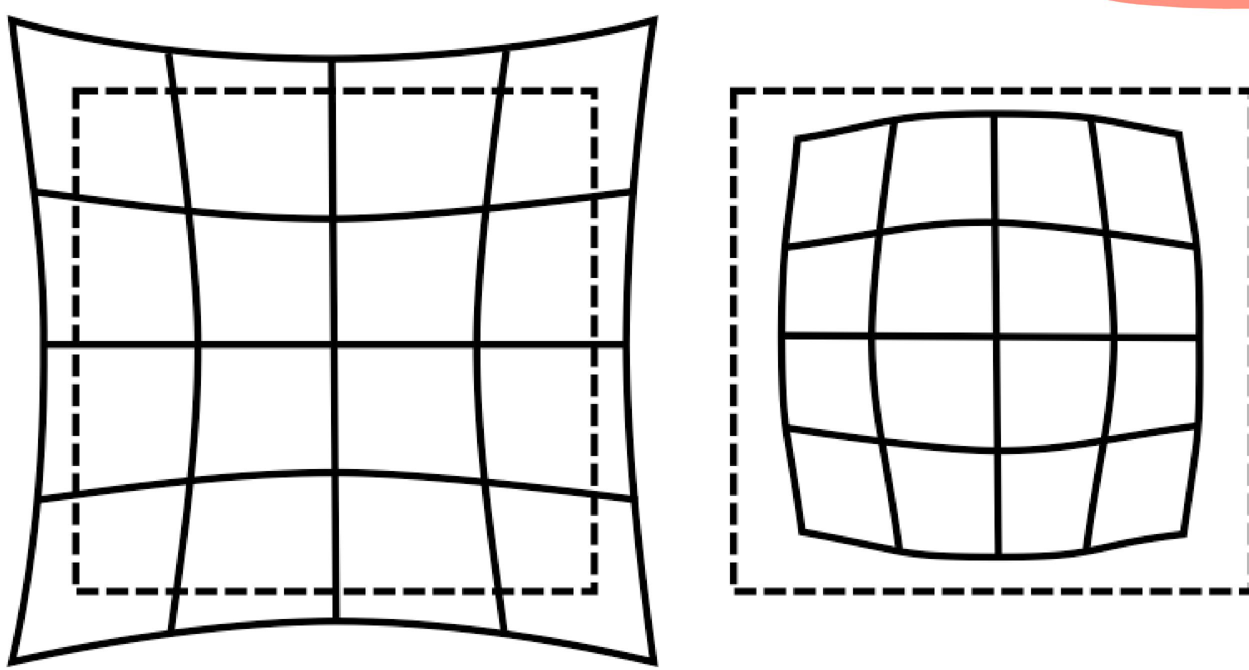


Only objects close to optical axis will be in focus on flat image plane. Off-axis objects will have different focal points.

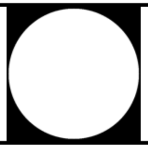
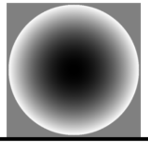
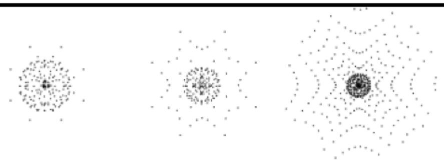
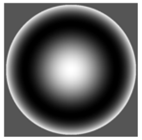
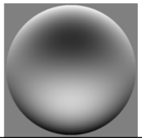
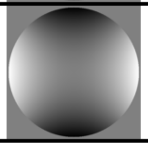
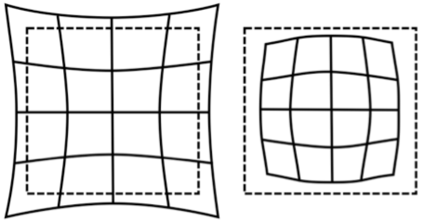
Distortion

Straight line on sky becomes curved line in focal plane because magnification depends on distance to optical axis.

1. Outer parts have larger magnification → pincushion
2. Outer parts have smaller magnification → barrel



Aberrations Summary

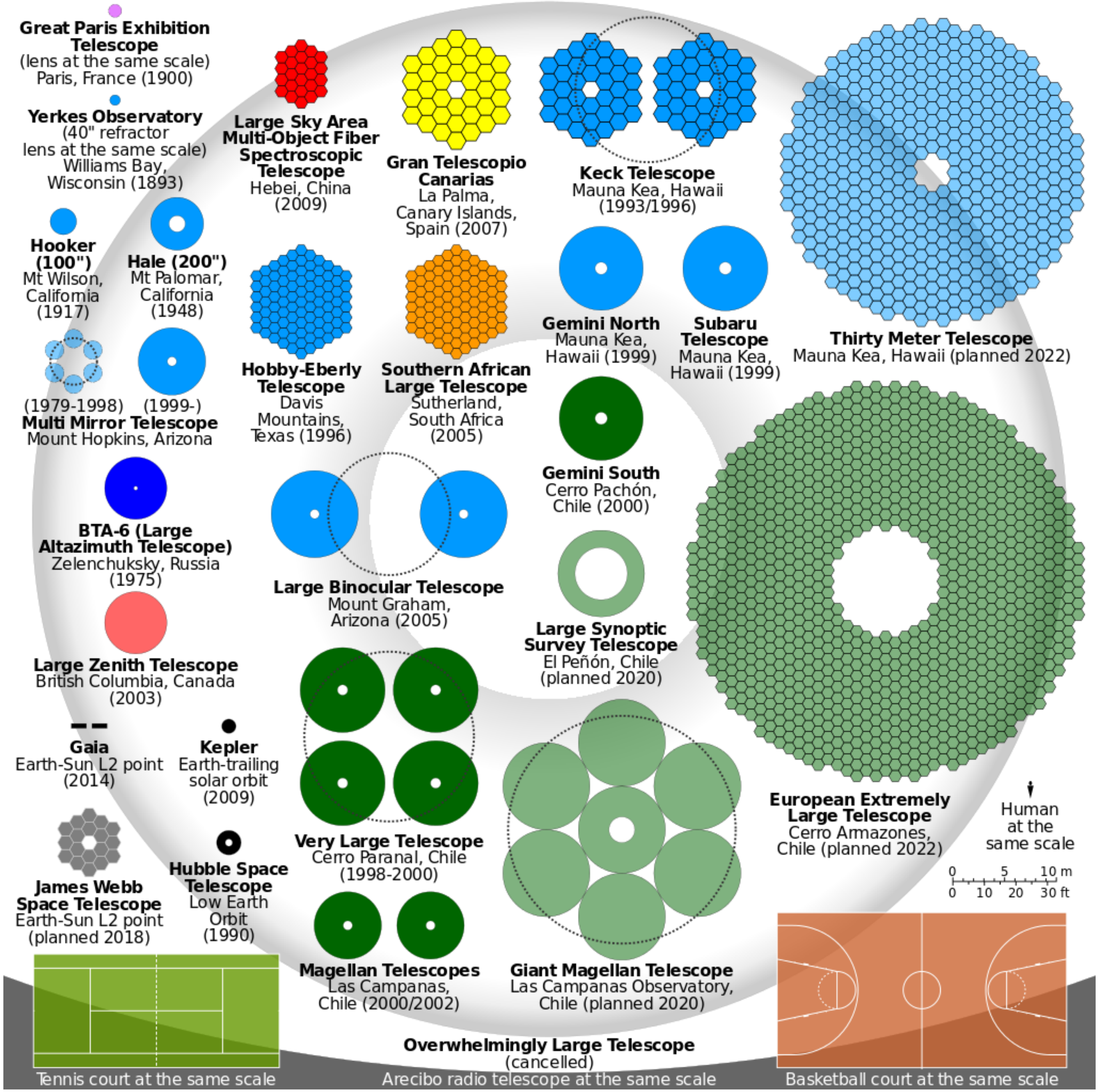
aberration	spot diagram / image	wavefront	scaling	
perfect			-	-
focus			$1/F^2$	-
spherical			$1/F^3$	-
coma			$1/F^2$	y
astigmatism			$1/F^2$	y^2
field curvature			$1/F^2$	y^2
distortion			-	y^3

Lecture 4: Eyes to the Skies

1. LOFAR
2. History
3. Optical telescopes
4. Space Telescope Orbits
5. Radio Telescopes
6. X-ray Telescopes

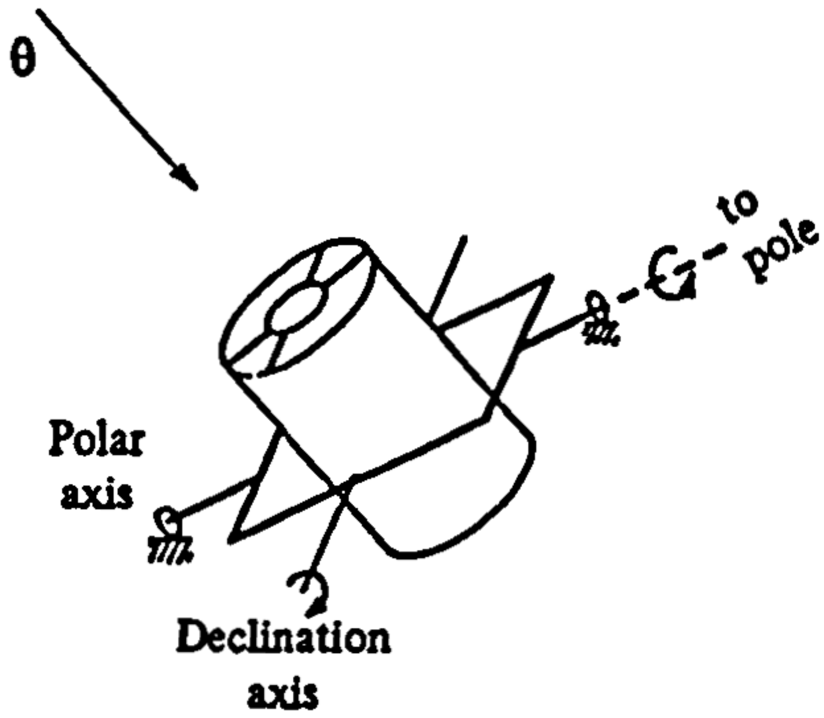
Telescope design considerations

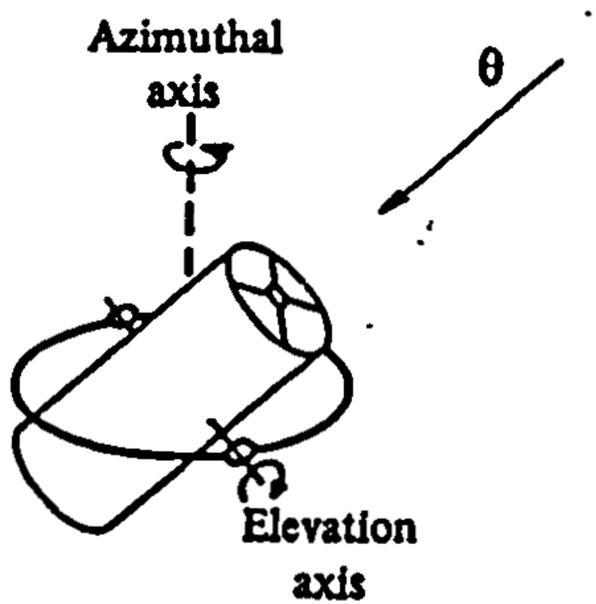
- F number/Plate scale/magnification/field of view
- Collecting area
- Focus configuration
- Mounts



commons.wikimedia.org/wiki/File:Comparison_optical_telescope_primary_mirrors.svg

Telescope mounts

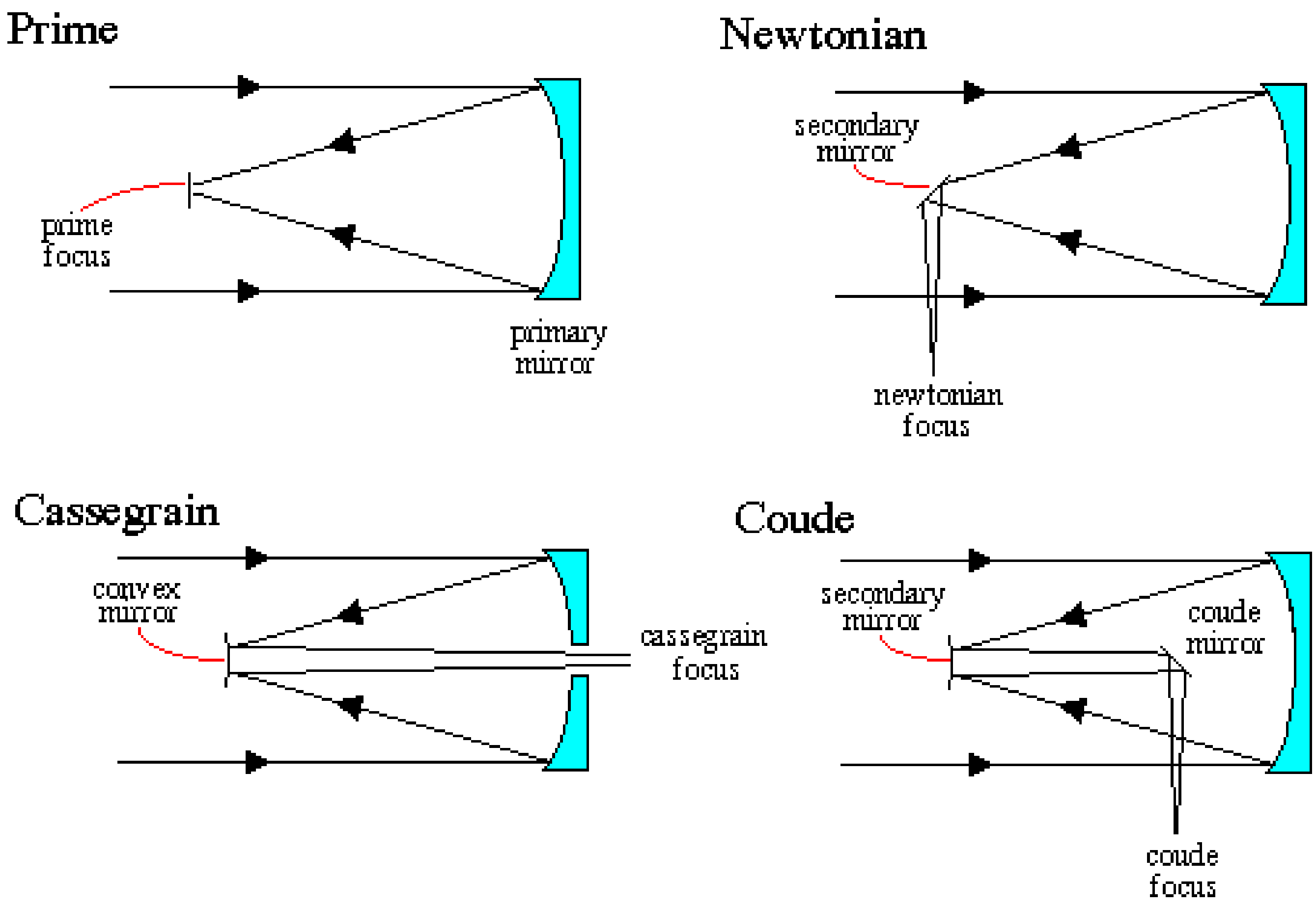




Equatorial

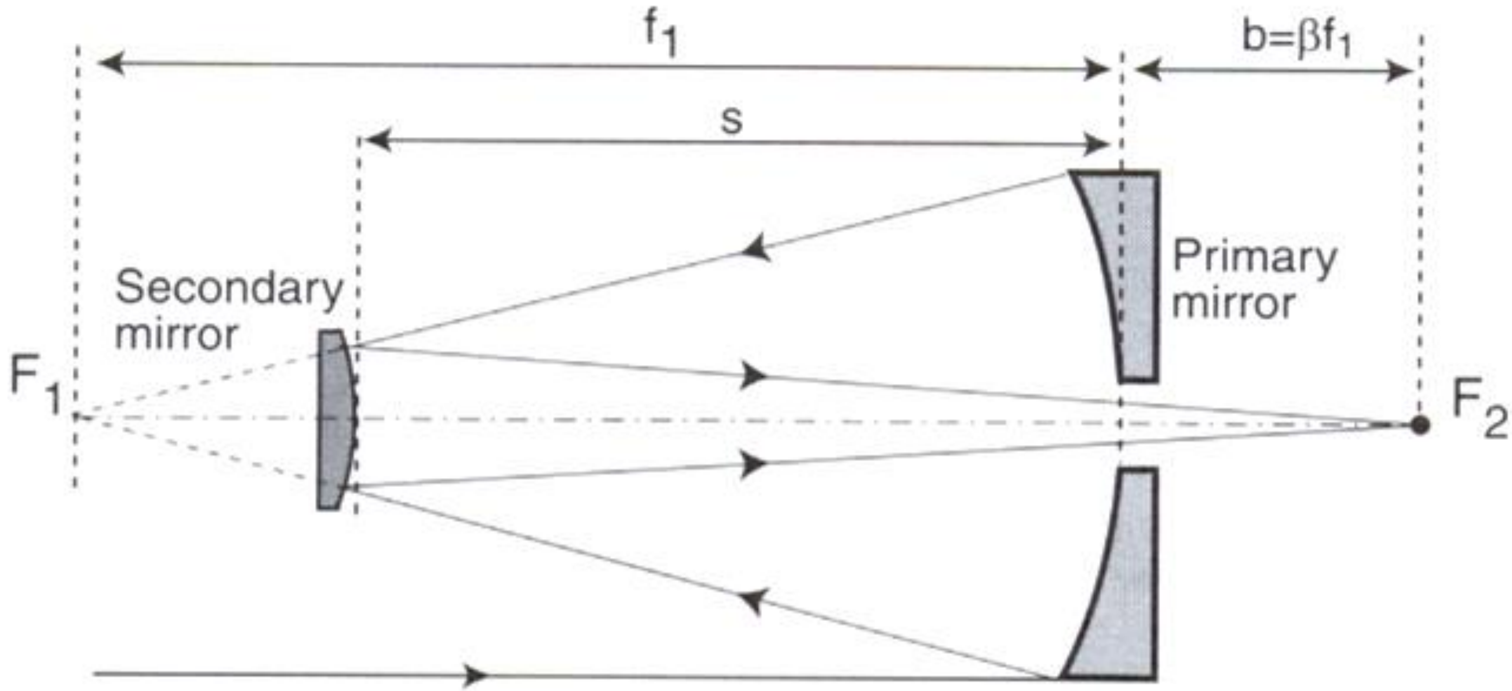
Altitude-Azimuth (altaz)

Telescope focus configurations



Ritchey-Chrétien Configuration

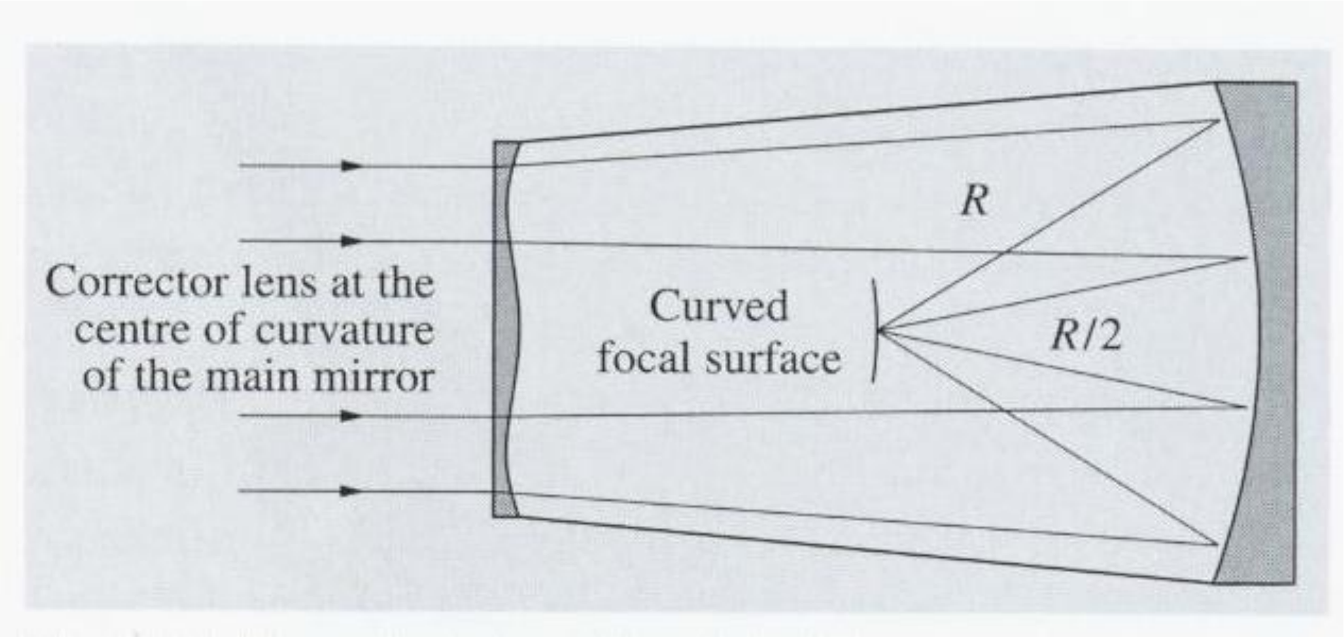
Modified Cassegrain with hyperbolic primary mirror (≈parabola) and hyperbolic secondary mirror eliminates spherical aberration and coma

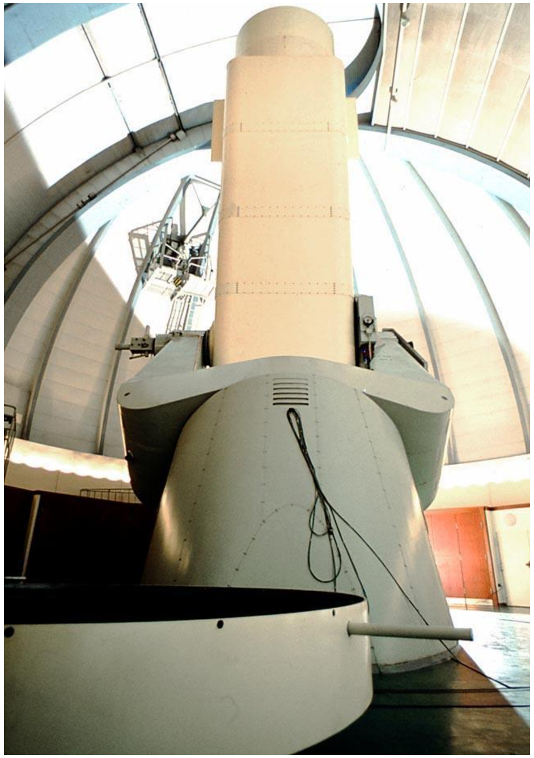


→large field of view, compact design, astigmatism

Schmidt Telescope

- Spherical primary mirror for maximum field of view (>5 deg) → no off-axis asymmetry *but* spherical aberrations
- correct spherical aberrations with corrector lens

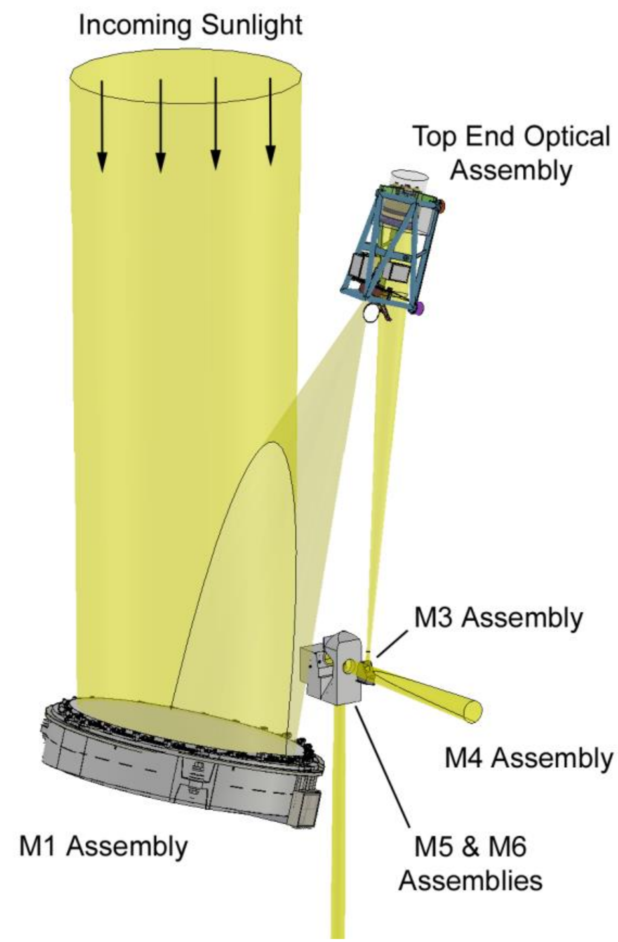




Two-meter Alfred-Jensch-Telescope in Tautenburg, the largest Schmidt camera in the world.

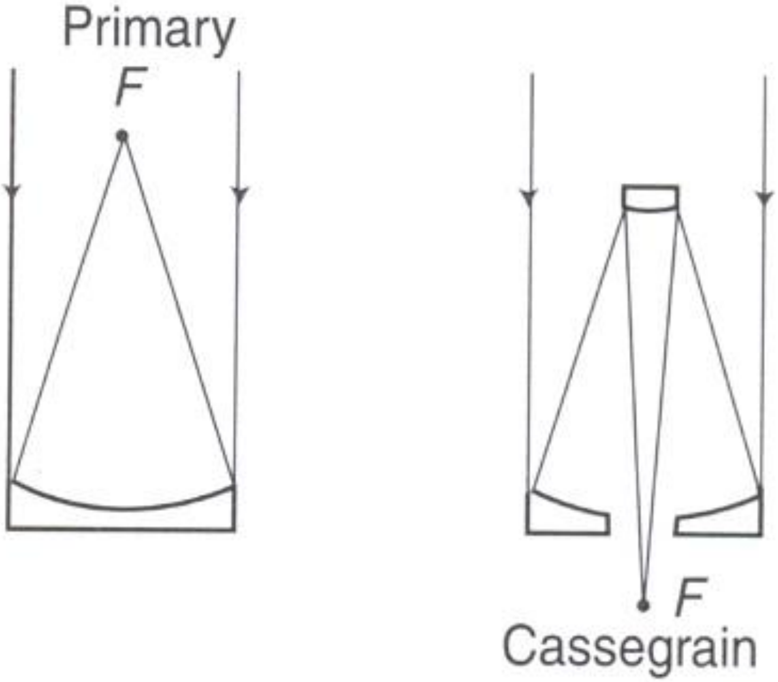
Gregorian Telescopes

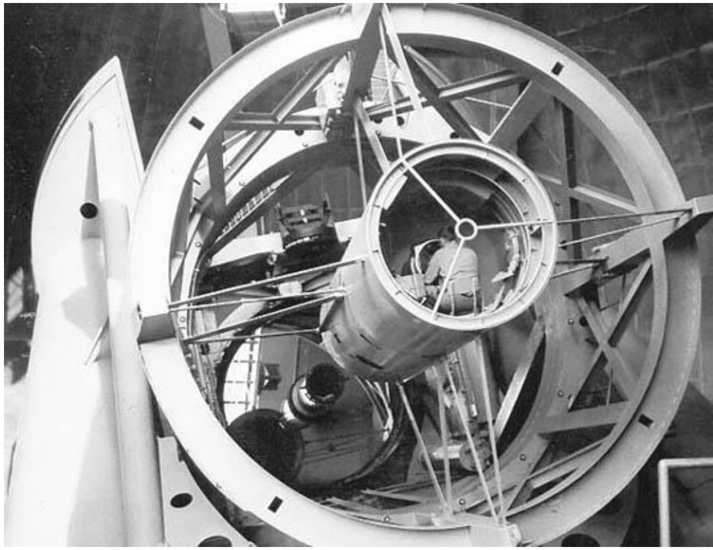
Large Binocular Telescope

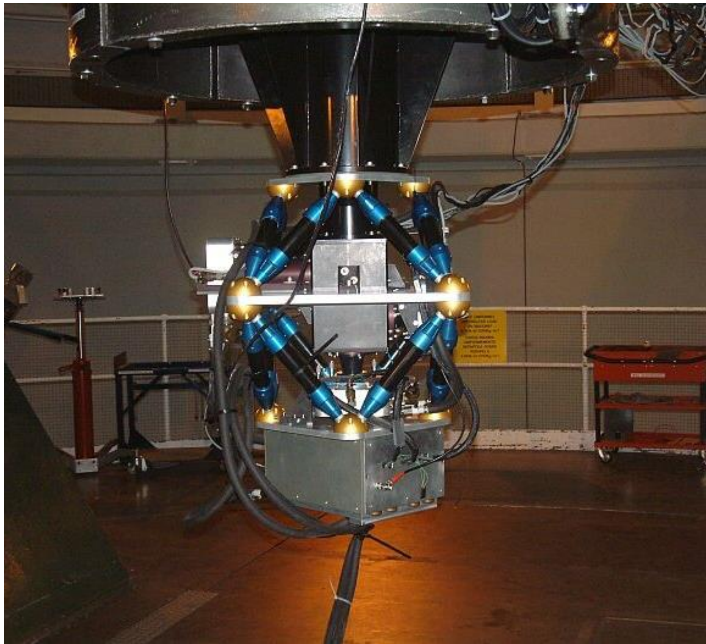


DKIST Solar Telescope

Telescope Foci – where to put the instruments



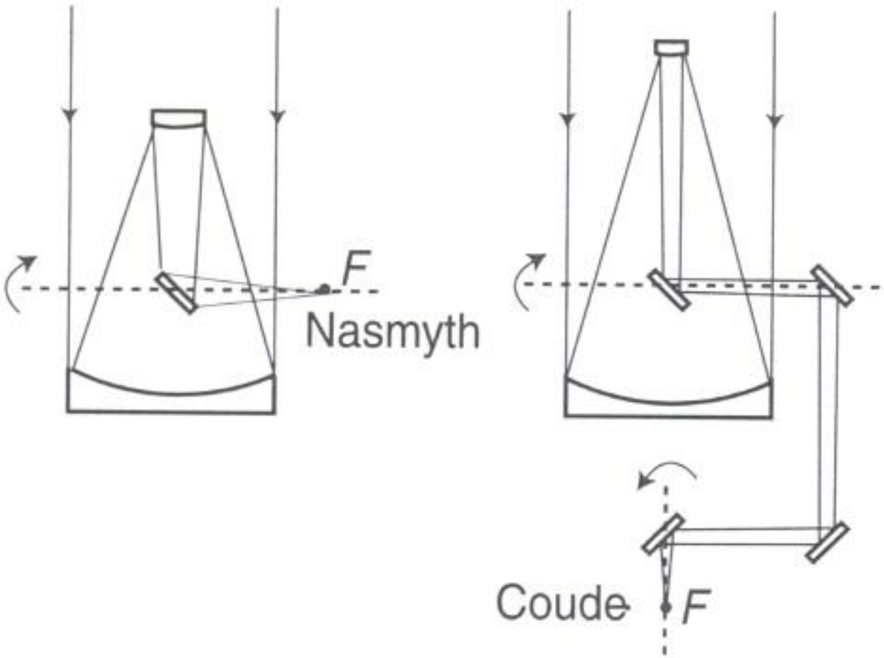




Prime focus – wide field, fast beam but difficult to access and not suitable for heavy instruments

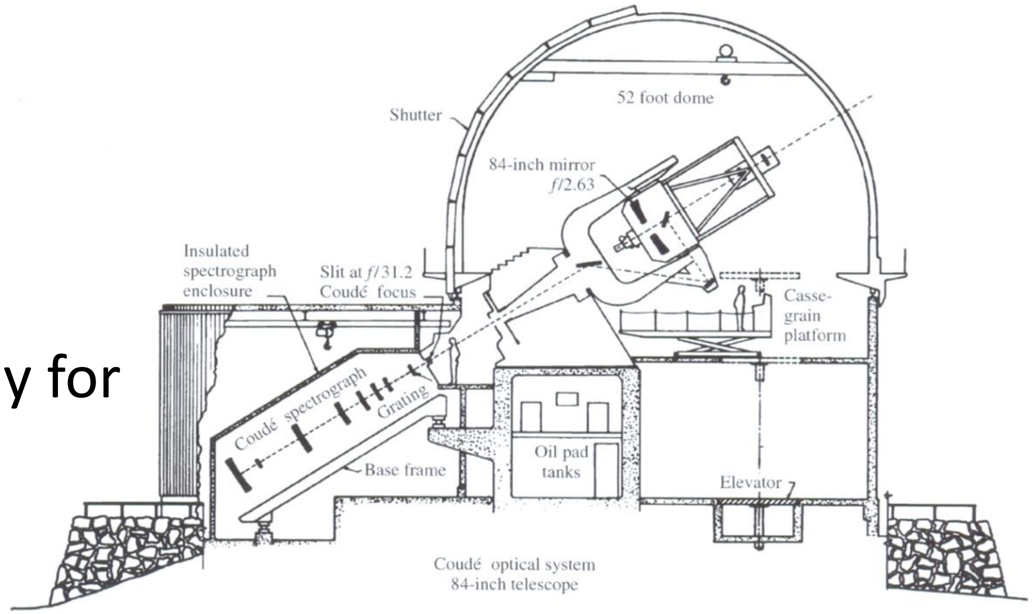
Cassegrain focus – moves with the telescope, no image rotation, but flexure may be a problem

Telescope Foci – where to put instruments (2)



Nasmyth – ideal for heavy instruments to put on a stable platform, but field rotates

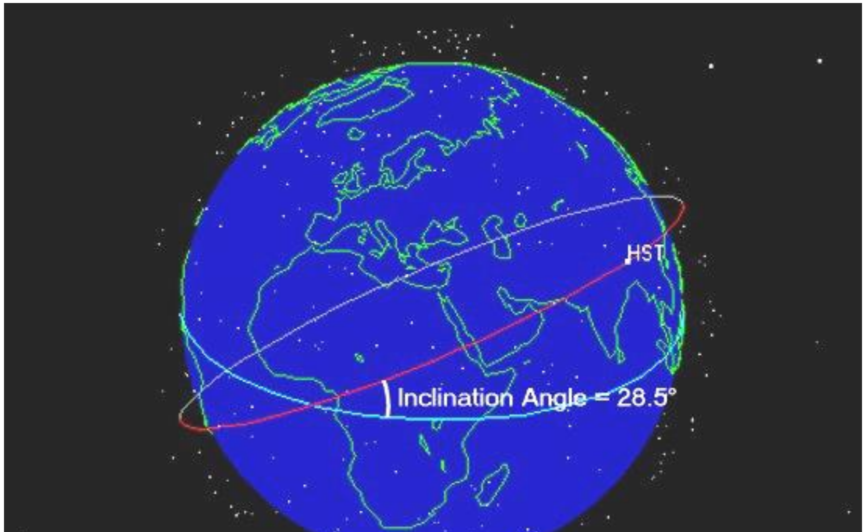
Coudé – very slow beam, usually for large spectrographs in the “basement”



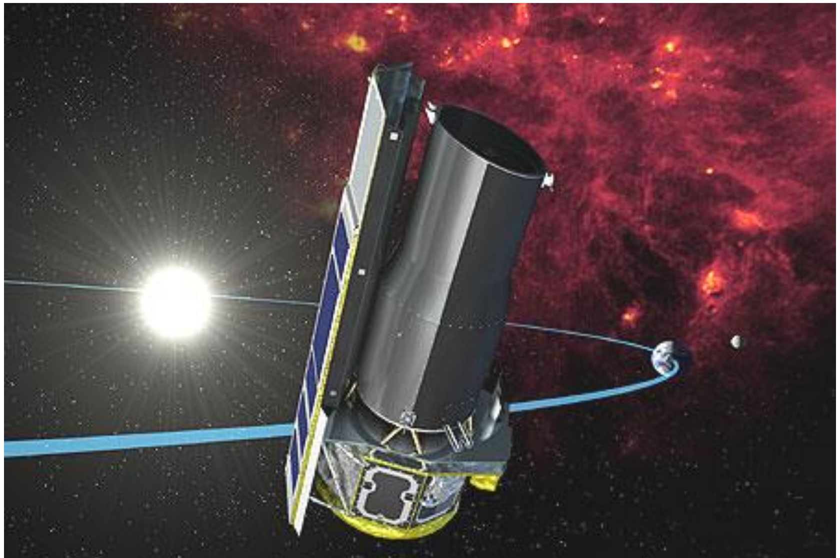
Space Telescope Orbits

Types of Orbits
- low Earth
- sun-synchronous
- geostationary
- Earth-trailing
- L2

Influence on Orbit Choice
- communications
- thermal background radiation
- space weather
- sky coverage
- access (servicing)



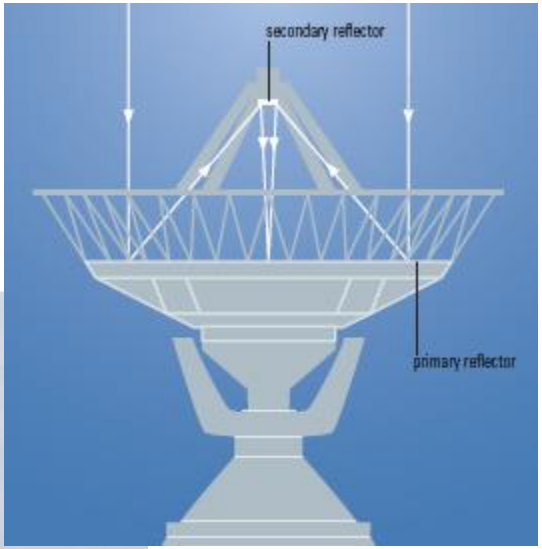
HST : low Earth orbit ~96 minutes



Spitzer: Earth-trailing solar orbit ~60 yr

Radio Telescopes

Dishes similar to optical telescopes but with much lower surface accuracy




Arecibo, Puerto Rico – 305m single-aperture telescope

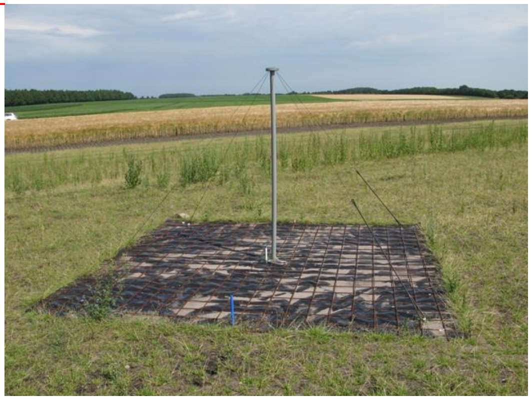

Effelsberg, Germany – 100m fully steerable telescope


Greenbank, USA – after structural collapse (now rebuilt)

Arrays and Interferometers

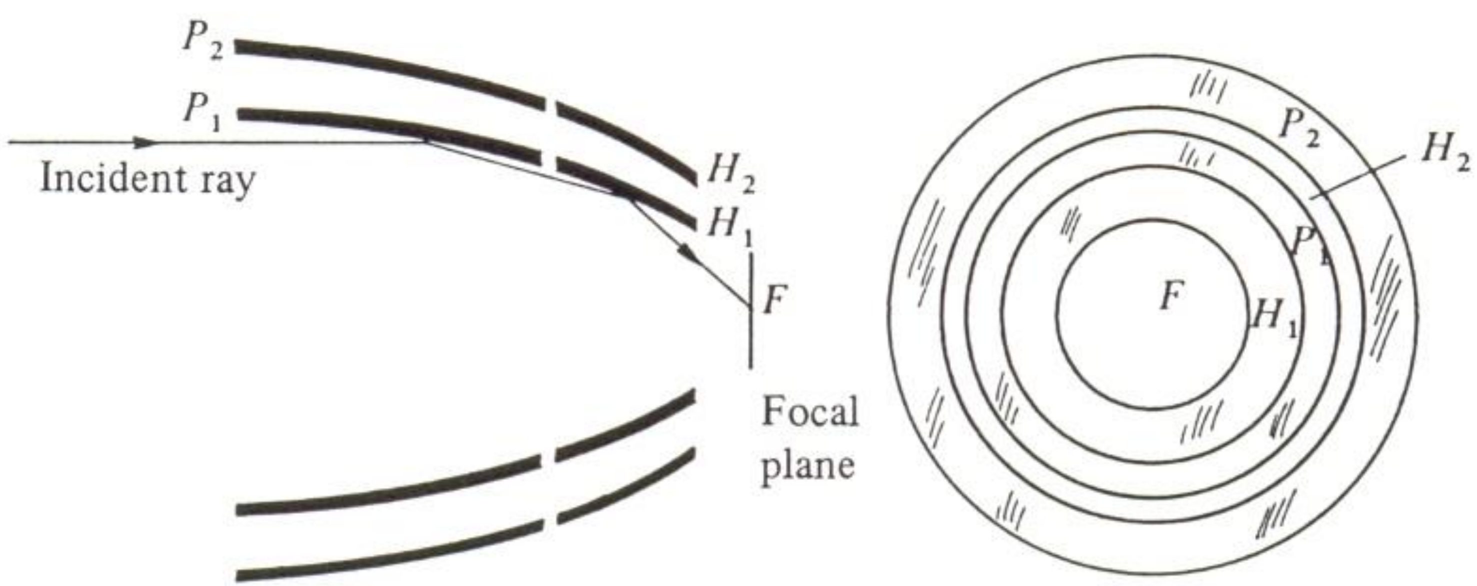
LOFAR

- LOw Frequency Array
- 2 types of low-cost antennae:
  - Low Band Antenna (LBA, 10-90 MHz)
  - High Band Antenna (HBA, 110-250 MHz)
- Antennae organized in 36 stations over 100 km
- Each station has 96 LBAs and 48 HBAs
- Baselines: 100m – 1500km



X-ray Telescopes

- X-rays close to normal incidence are largely absorbed, not reflected
- telescope optics based on glancing angle reflection
- typical reflecting materials for X-ray mirrors are gold and iridium (gold: critical reflection angle of 3.7 deg at 1 keV)



Lecture 5: Your Noise is My Signal

1. Signal and Noise
2. Examples of Noise in Astronomy
3. Noise in Astronomical Observations
4. Digitization Noise
5. Read Noise
6. Distribution and Density Functions
7. Normal Distribution
8. Poisson Distribution
9. Histogram
10. Mean, Variance and Standard Deviation
11. Noise Propagation
12. Periodic Signals
13. Gaussian Noise
14. Poisson Noise
15. Noise Measurements
16. Signal-to-Noise Ratio
17. Instrument Sensitivity

Signal and Noise

- Signal: data that is relevant for our science
- Noise: data that is irrelevant for our science
- Signal-to-noise ratio (SNR): ratio of relevant to irrelevant information

- Definition of signal and noise inherently depends on science objectives

Some Sources of Noise in Astronomical Data

Noise type	Signal	Background
Photon shot noise	X	X
Scintillation	X	
Cosmic rays		X
Image stability	X	
Read noise	X	X
Dark current noise	X	X
Charge transfer efficiencies (CCDs)	X	X
Flat fielding (non-linearity)	X	X
Digitization noise	X	X
Other calibration errors	X	X
Image subtraction	X	X

Three Probability Density Functions

- Binomial distribution
- Poisson distribution
- Gaussian/normal distribution

1.5.4 A rule of thumb

Lets see how useful this result is. When counting photons, if the expected number detected is n, the variance of the detected number is n: i.e. we expect typically to detect

$$n \pm \sqrt{n} \tag{28}$$

photons. Hence, just by detecting n counts, we can immediately say that the uncertainty on that measurement is $\pm\sqrt{n}$, without knowing anything else about the problem, and only assuming the counts are random. This is a very useful result, but beware: when stating an uncertainty like this we are assuming the underlying distribution is Gaussian (see later). Only for large n does the Poisson distribution look Gaussian (the Central Limit Theorem at work again), and we can assume the uncertainty $\sigma = \sqrt{n}$.

Gaussian Noise

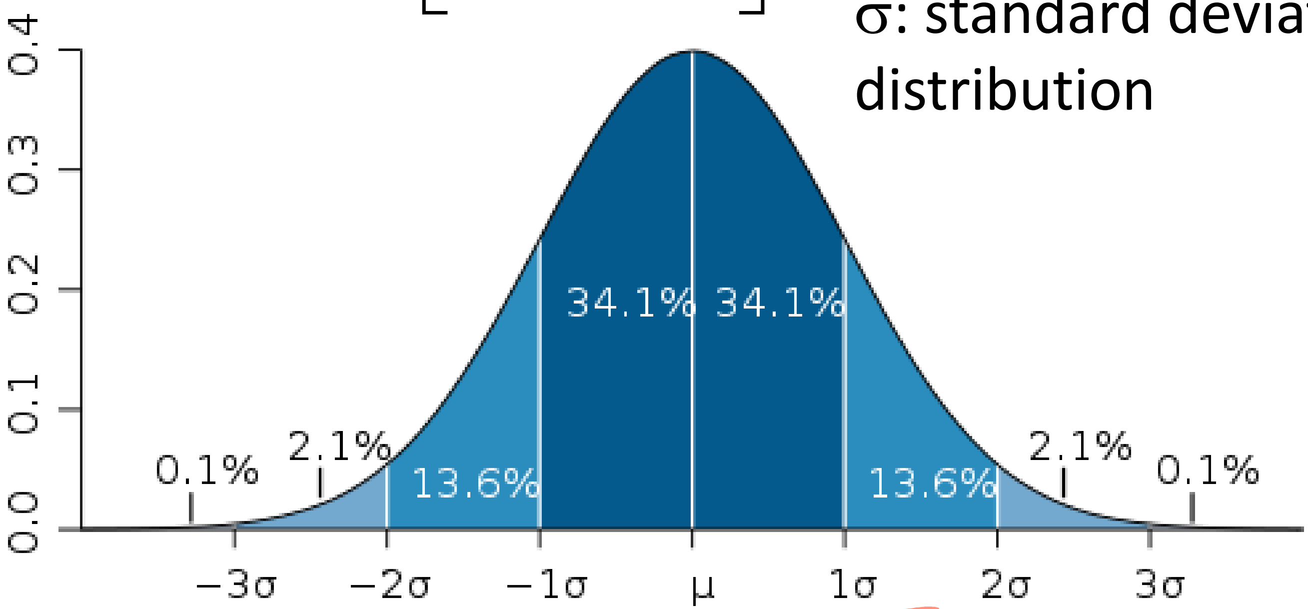
- Gaussian noise has Gaussian (normal) distribution
- Sometimes (incorrectly) called white noise (uncorrelated noise)

$$S = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2} \right]$$

x: actual value

μ: mean of distribution

σ: standard deviation of distribution



1-σ ~ 68%

2-σ ~ 95%

3-σ ~ 99.7%

Astronomers often consider S/N > 3σ or > 5σ as significant

Poisson Noise and Integration Time

- Integrate light from uniform, extended source on detector
- In finite time interval Δt, expect average of λ photons
- Statistical nature of photon arrival rate → some pixels will detect more, some less than λ photons.
- Noise of average signal λ (i.e., between pixels) is $\sqrt{\lambda}$
- Integrate for $2\times\Delta t$ → expect average of $2\times\lambda$ photons
- Noise of that signal is now $\sqrt{2\lambda}$, i.e., increased by $\sqrt{2}$
- With respect to integration time t, noise will only increase $\sim\sqrt{t}$ while signal increases $\sim t$

SNR and Integration Time

Assuming the signal suffers from Poisson shot noise. Let's calculate the dependence on integration time t_{int}:

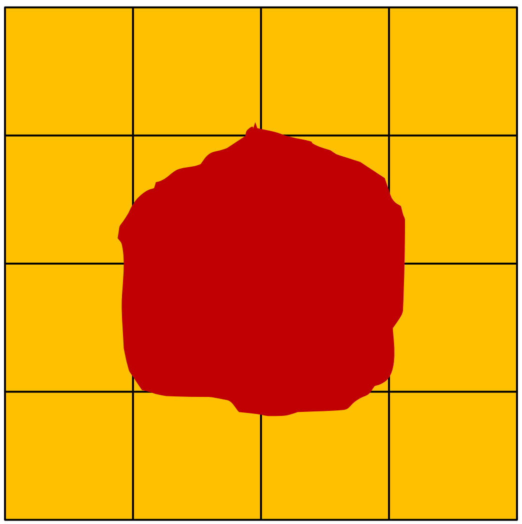
Integrating t_{int}: $$\sigma = \frac{S}{N}$$

Integrating $n \times t_{int}$: $$\sigma = \frac{n \cdot S}{\sqrt{n \cdot B}} \overset{N=\sqrt{B}}{=} \sqrt{n}\frac{S}{N} \quad \Rightarrow \quad \frac{S}{N} \propto \sqrt{t_{\text{int}}}$$

Need to integrate four times as long to double the SNR

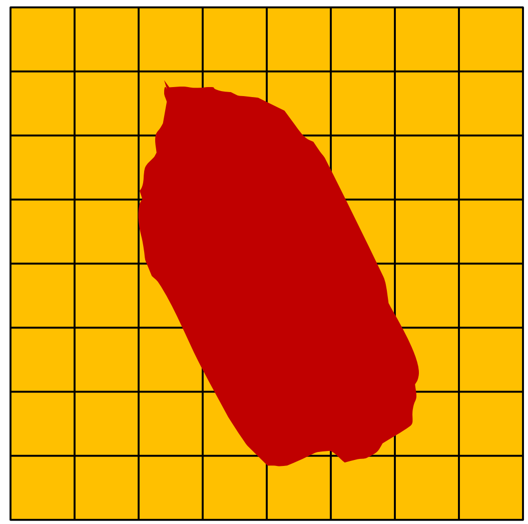
SNR Dependence on Source

Background (=noise) Target



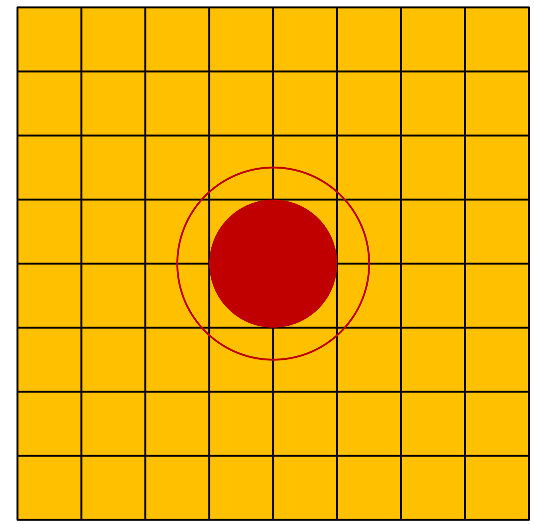
Seeing-limited point source

- pixel size ~ seeing
- PSF ≠ f(D)



Diffraction-limited, extended source

- pixel size ~ diff.lim
- PSF = f(D)
- target >> PSF



Diffraction-limited, point source

- pixel size ~ diff.lim
- PSF = f(D)
- target << PSF

(D: diameter telescope)

Instrument Sensitivity

Putting it all together, the total signal to noise ratio for a given experiment is:

$$\sigma = \frac{S_{el}}{N_{tot}} = \frac{S_{src} \cdot SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{\text{int}}}{N_{back}\sqrt{n_{pix}}}$$

$$\Rightarrow \; S_{src} = \frac{\sigma \cdot \sqrt{S_{back} \cdot t_{\text{int}} + I_d \cdot t_{\text{int}} + N_{read}^2 \cdot n} \cdot \sqrt{n_{pix}}}{SR \cdot \Delta\lambda \cdot A_{tel} \cdot \eta_D G \cdot \eta_{atm} \cdot \eta_{tot} \cdot t_{\text{int}}}$$

Noise Propagation

- same as error propagation
- function *f(u,v,...)* depends on variables *u,v, ...*
- estimate variance of *f* knowing variances $\sigma_u{}^2$, $\sigma_v{}^2$,... of variables *u,v, ...*

$$\sigma_f^2 = \frac{1}{N-1} \lim_{N\to\infty} \sum_{i=1}^{N} \left(f_i - \bar{f}\right)^2$$

- make assumption / approximation that average of *f* is well approximated by value of *f* for averages of variables: $\bar{f} = f(\bar{u}, \bar{v}, ¼)$

Lecture 6:
Your Favourite Radio Station

1. Brief history of Radio Astronomy
2. Emission mechanisms
3. Antennas
4. Telescopes
5. Beams
6. Frequency down conversion
7. Backends

Radio Emission Mechanisms

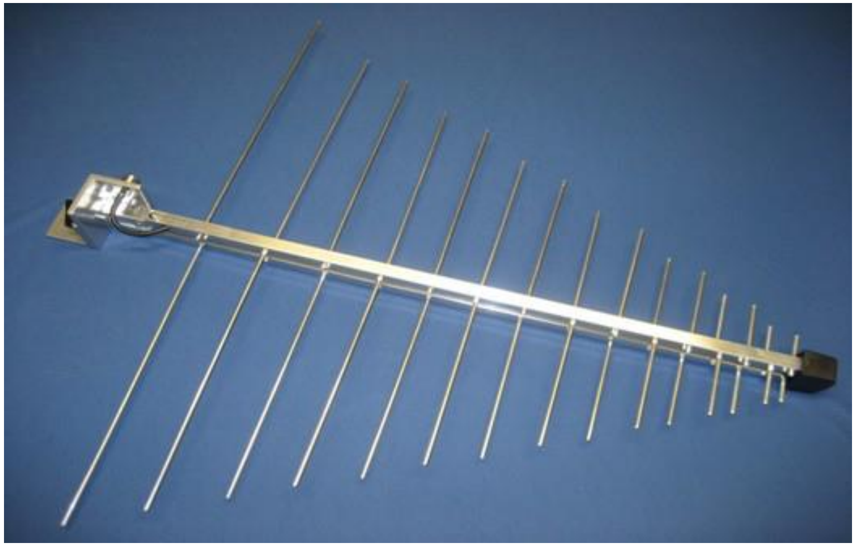
1. Synchrotron emission: electrons spiraling in magnetic fields
2. Free-free emission (thermal Bremsstrahlung): electrons scattering off ions
3. Thermal emission: blackbody radiation from gas and dust grains
4. Spectral lines: electrons bound to atom changing energy states

Spectral Lines

- Hydrogen 21-cm line
- Radio recombination lines: highly excited electrons transitioning between states with large orbital angular momentum states *n*
- Spectral lines due to molecules rotating and vibrating
- Masers

Receiving radiation using antennas

helical



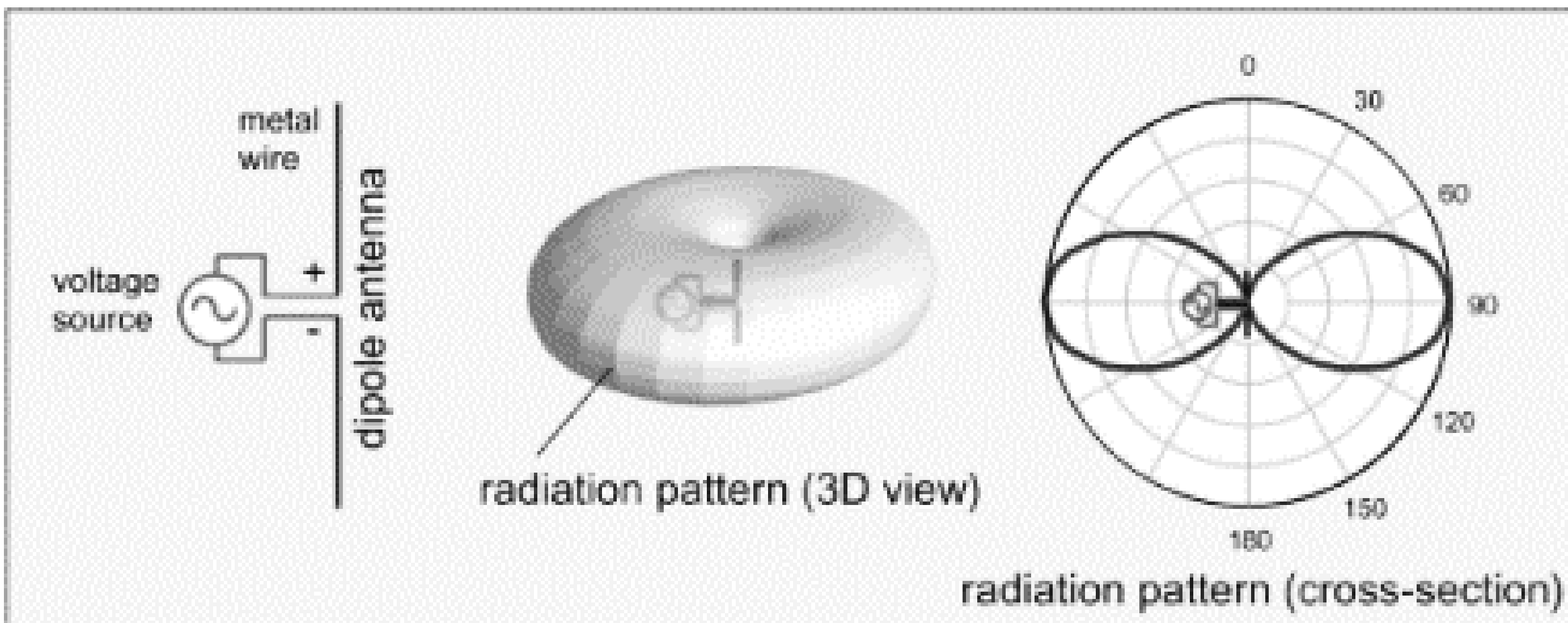
Log-periodic

URT2 – Ukraine

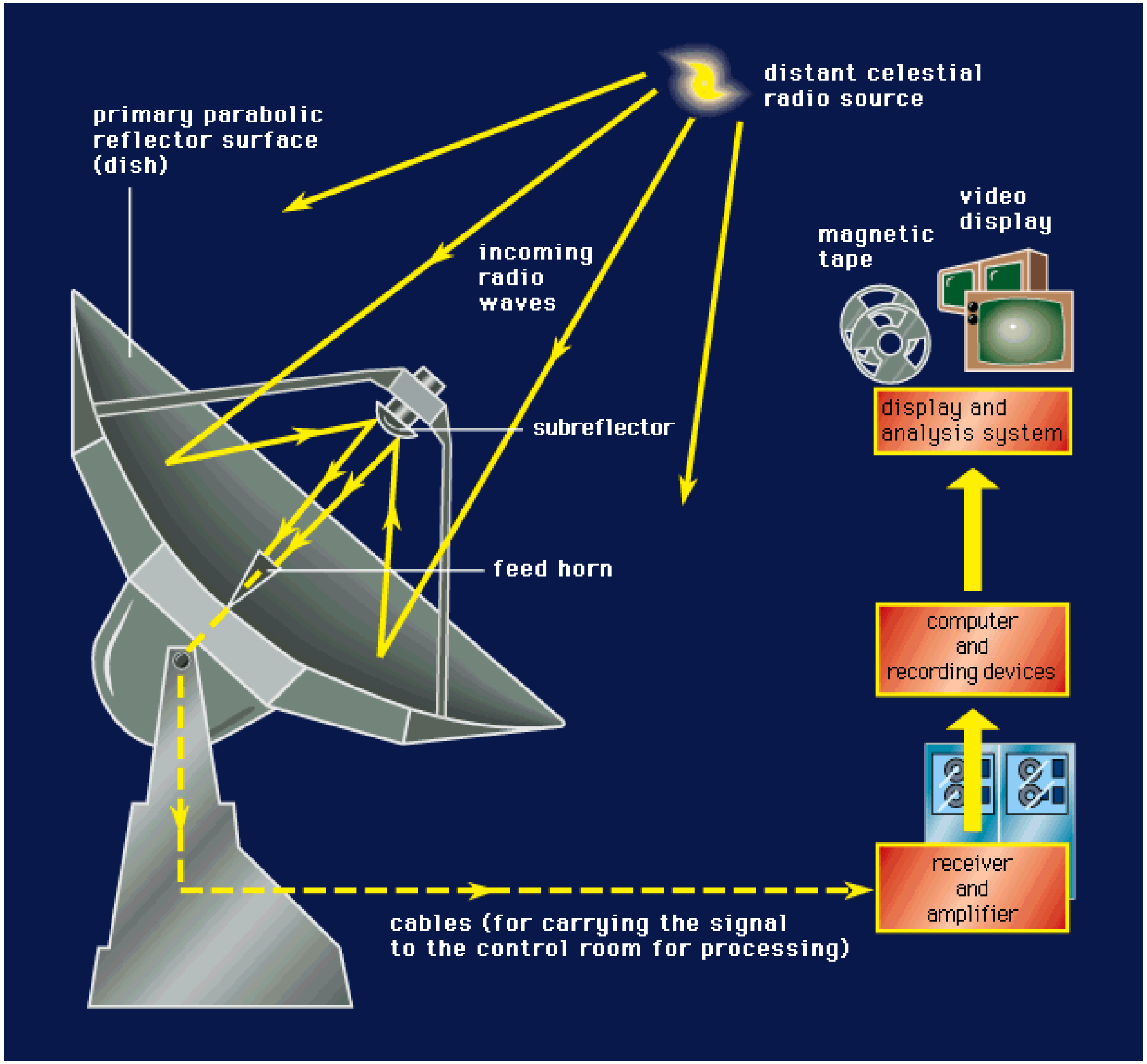
Concept SKA-low

Radiation from Hertz Dipole

- Radiation is linearly polarized
- Electric field lines along direction of dipole
- Radiation pattern has donut shape, defined by zone where phases match sufficiently well to combine constructively
- Along the direction of the dipole, the field is zero
- Best efficiency: size of dipole = 1/2 λ



Radio dishes



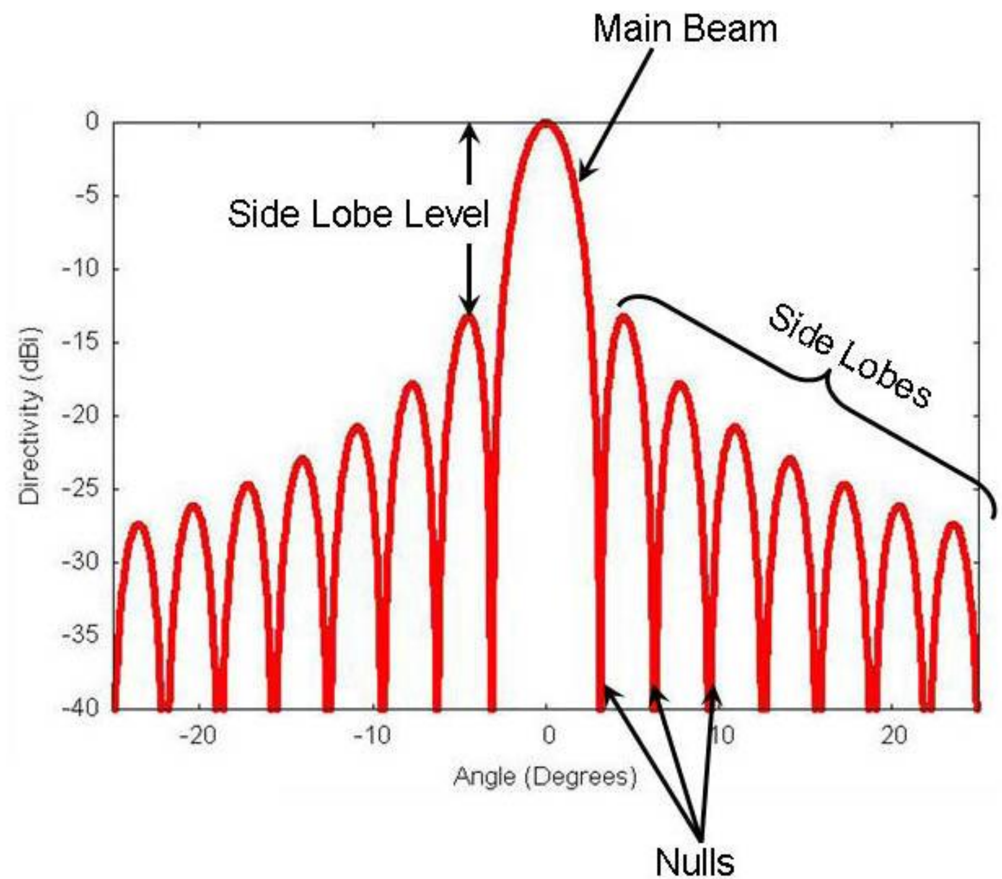
- Directly measure electric fields of electro-magnetic waves
- Electric fields excite currents in antenna
- Currents can be amplified and split electrically

Radio Beams

Similar to optical telescopes, angular resolution given by

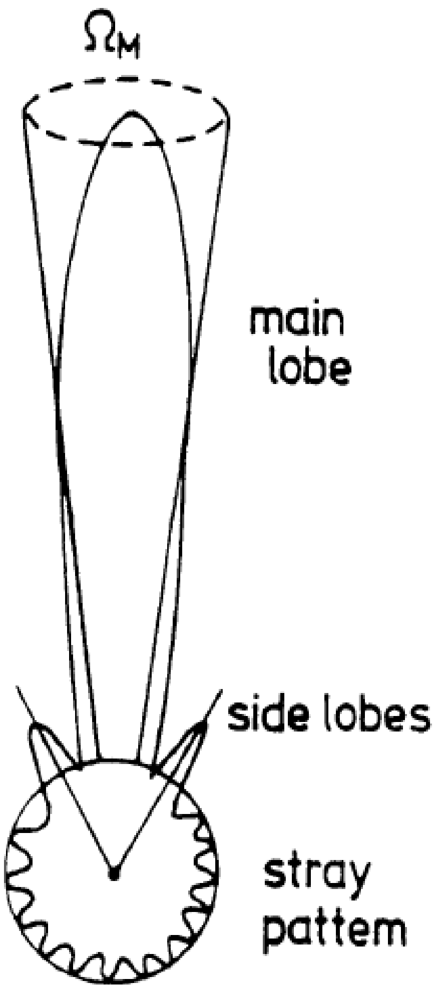
$$\theta = k\frac{\lambda}{D} \qquad \text{where } k \sim 1$$

- Radio beams also show Airy pattern
- lobes at various angles, directions where the radiated signal strength reaches a maximum
- nulls at angles where radiated signal strength falls to zero



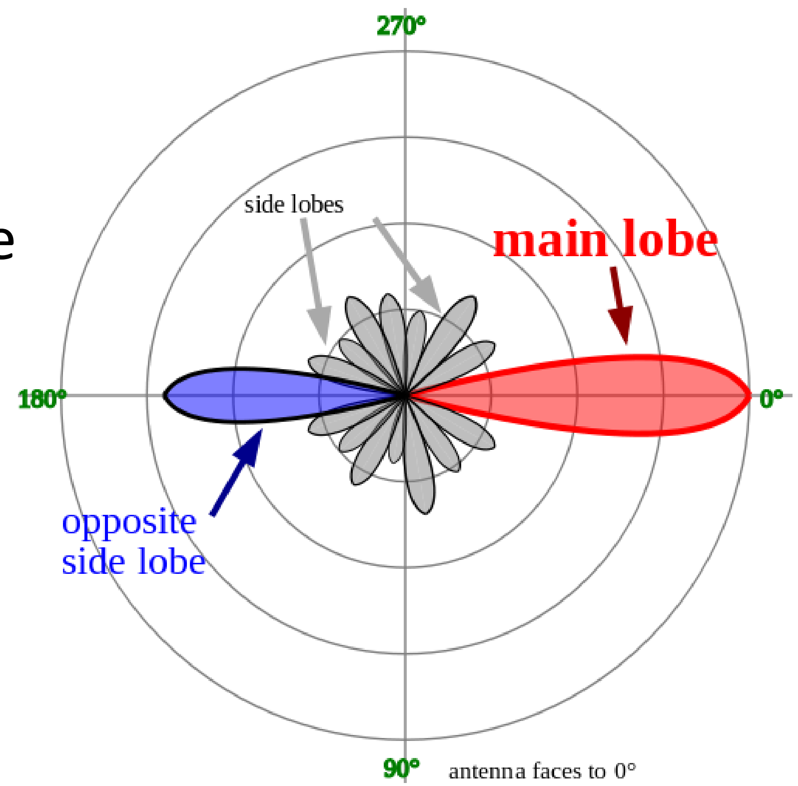
Main Beam and Sidelobes

Highest field strength in "main lobe", other lobes are called "sidelobes" (unwanted radiation in undesired directions)



Side lobes may pick up interfering signals, and increase the noise level in the receiver.

The side lobe in the opposite direction (180°) from the main lobe is called the "back lobe".



Main Beam Efficiency

The beam solid angle Ω_A in steradians of an antenna is given by:

$$\Omega_A = \iint_{4\pi} P_n(\vartheta,\varphi)\, d\Omega = \int_0^{2\pi}\int_0^{\pi} P_n(\vartheta,\varphi) \sin\vartheta \, d\vartheta \, d\varphi$$

With the normalized power pattern $$P_n(\vartheta,\varphi) = \frac{1}{P_{\max}} P(\vartheta,\varphi)$$

Main beam solid angle Ω_{MB} is:

$$\Omega_{MB} = \iint_{\text{main lobe}} P_n(\vartheta,\varphi)\, d\Omega$$

Main beam efficiency η_B, the fraction of the power concentrated in the main beam is $$\eta_B = \frac{\Omega_{MB}}{\Omega_A}$$

Main Beam Brightness Temperature

Relation between flux density S_ν and intensity I_ν:

$$S_\nu = \int_{\Omega_B} I_\nu(\theta,\varphi)\cos\theta \, \mathrm{d}\Omega$$

Definition of T_B:

$$T_B = \frac{c^2}{2k\nu^2} B_{RJ} = \frac{\lambda^2}{2k} B_{RJ}$$

For discrete sources, the source extent $\Delta\Omega$ is important. With $I_\nu = B_{RJ}$

$$S_\nu = \frac{2k\nu^2}{c^2} T_B \cdot \Delta\Omega$$

... or simplified for a source with a Gaussian shape:

$$\left[\frac{S_\nu}{\mathrm{Jy}}\right] = 0.0736 T_B \left[\frac{\theta}{\mathrm{arcsec}}\right]^2 \left[\frac{\lambda}{\mathrm{mm}}\right]^{-2}$$

Generally, for an antenna beam size $\vartheta_{\mathrm{beam}}$ the observed source size (for a Gaussian source) is:

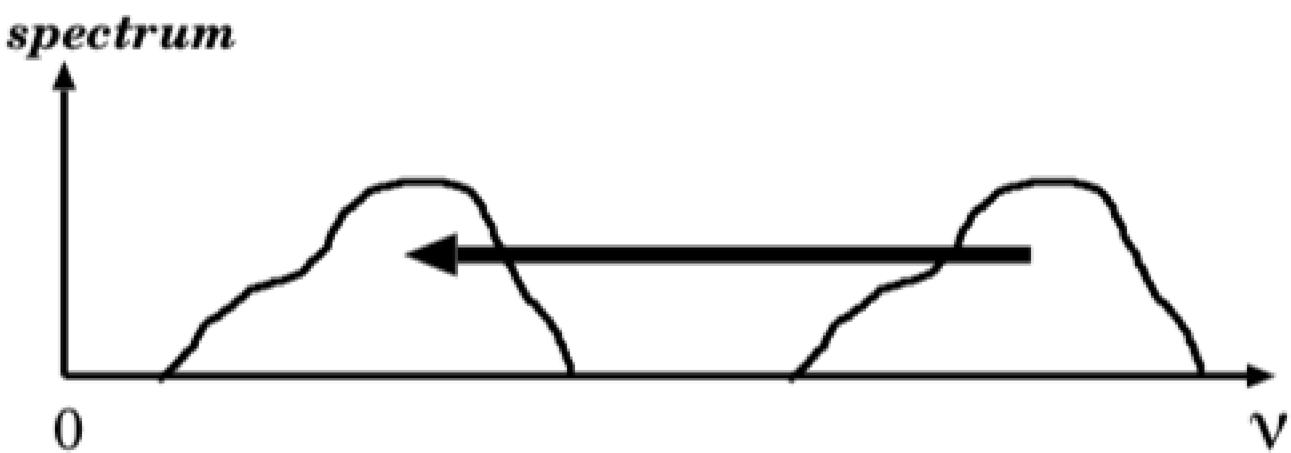
$$\theta^2_{observed} = \theta^2_{source} + \theta^2_{beam}$$

...which relates the true brightness temperature with the main beam brightness temperature:

$$T_{MB}\left(\theta^2_{source} + \theta^2_{beam}\right) = T_B \, \theta^2_{source}$$

Frequency down conversion

If a radio signal is band limited, that means it has zero spectral power outside a certain frequency range. We can shift the central frequency of such a signal (usually to the lower side) without losing any spectral information.



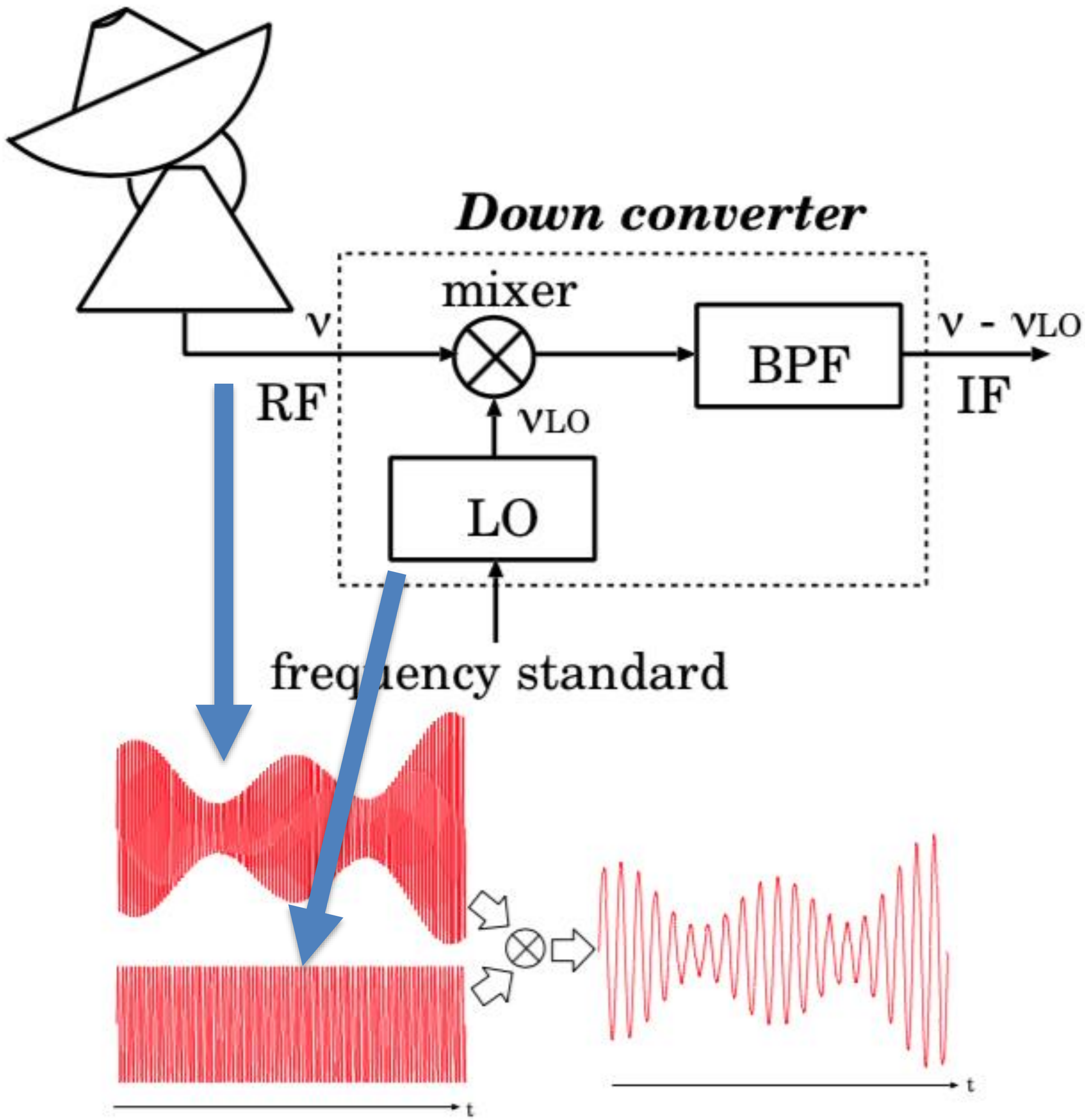


Figure 108: Elements and technical terms used in the frequency conversion.

- **radio frequency (RF)**: the high frequency involved in the band which is directly received by an antenna, sometimes called also as "sky frequency",

- **intermediate frequency (IF)**: the low frequency obtained by the frequency conversion,

- **local oscillator (LO)**: an oscillator which provides a sinusoidal reference signal with a specified frequency,

- **mixer**: a nonlinear device which multiplies the reference signal to the received signal,

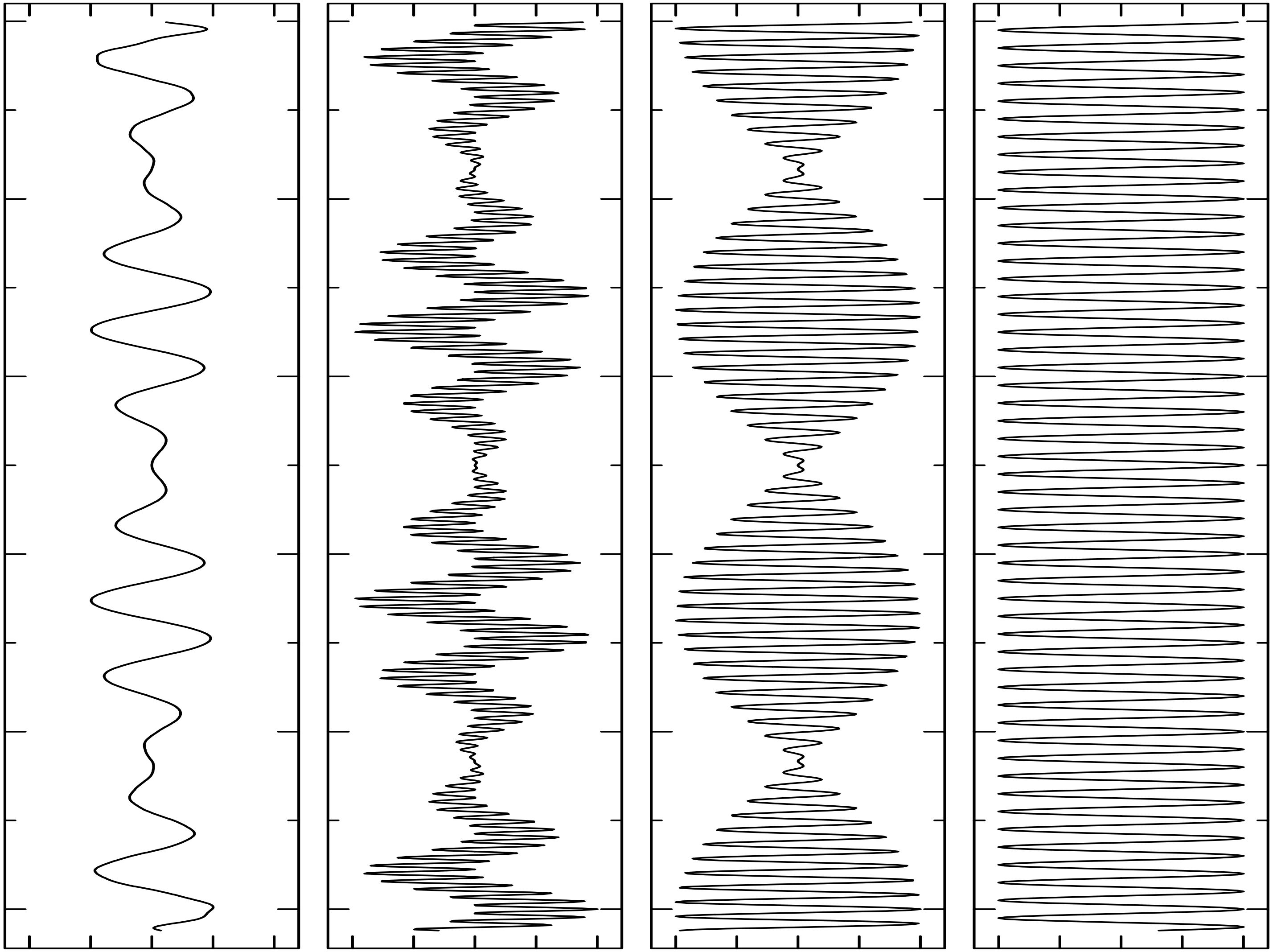
- **bandpass filter (BPF)**: a filter which passes only necessary band of the intermediate frequency and cuts off all other frequency components in the output from the mixer,

- **down converter**: a unit composed of the LO, mixer and BPF which converts RF ν to IF $\nu - \nu_{LO}$, where ν_{LO} is the frequency of the reference signal provided by the LO which is often called "LO frequency",

- **superheterodyne receiver**: a receiver based on the frequency conversion technique.

Figure 108 shows elements of the frequency conversion. For example, in a typical Mark III–type geodetic VLBI observation at 8 GHz, we can choose 8180 MHz $\sim$ 8600 MHz as the RF band and 8080 MHz as the LO frequency ν_{LO} to get 100 MHz $\sim$ 520 MHz as the IF band.

Mixing Example

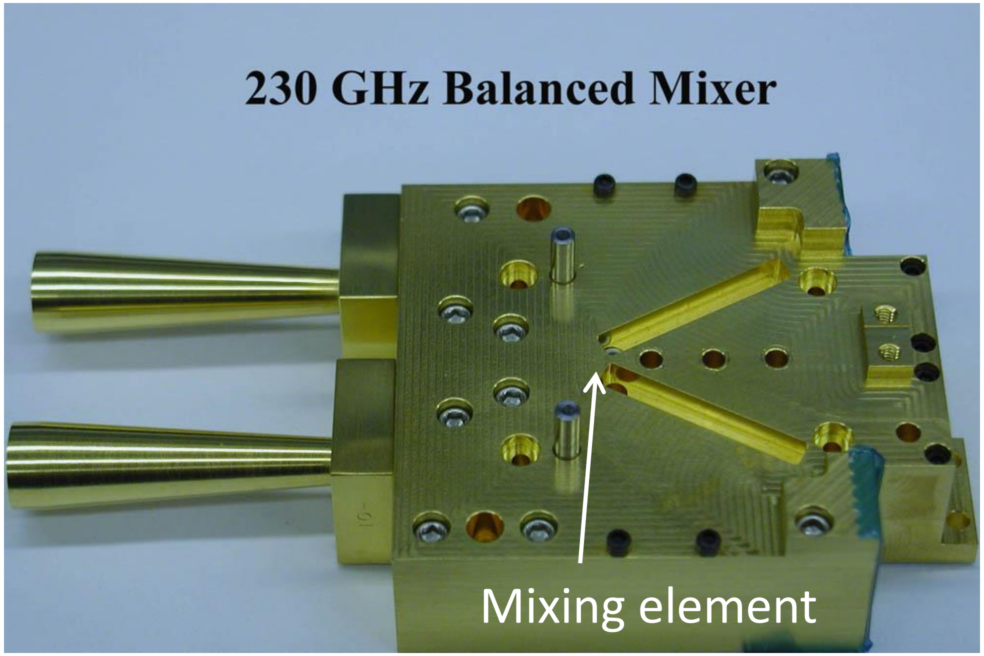


Mixer Technology

Example of a mixer:

Source signal →

Local oscillator signal →



Problem: good & fast "traditional" photo-conductors only exist for low frequencies

➔

- Schottky diodes
- SIS junctions
- Hot electron bolometers

Back End

The term “Back End” is used to specify the devices following the IF amplifiers. Many different back ends have been designed for specialized purposes such as continuum, spectral or polarization measurements.

Lecture 7: Fringes, Damned Fringes and Interferometry

1. Introduction
2. Young's slit experiment
3. Van Cittert – Zernike relation
   - UV plane, sampling, tapering, deconvolution, self-calibration

- Optical interferometers
- Radio interferometers

Very Large Array VLA

- Y-shaped array, 27 telescopes on railroad tracks
- 25-m diameter telescopes, New Mexico, USA
- Configurations spanning 1.0, 3.4, 11, and 36 km

Cygnus A at 6 cm with VLA

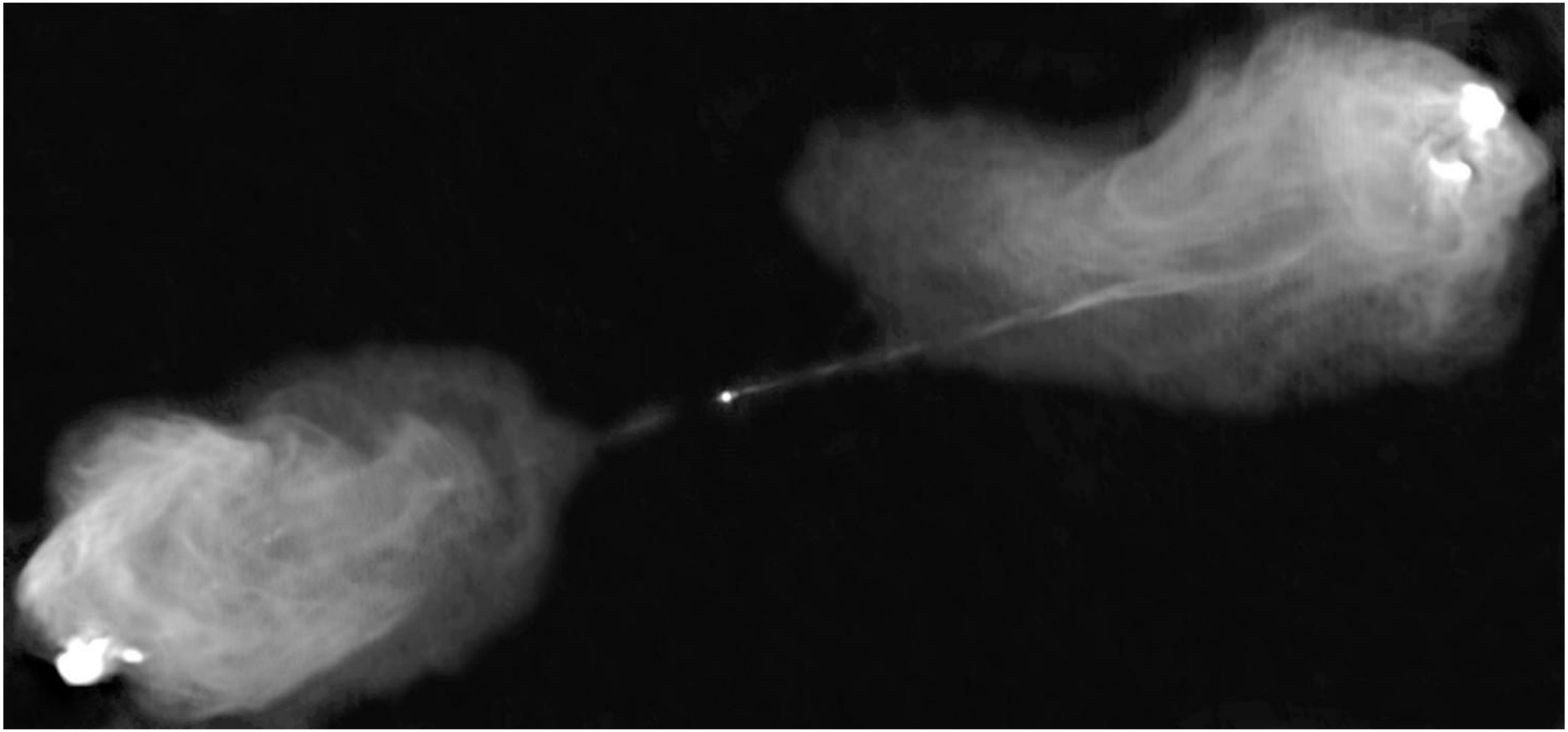
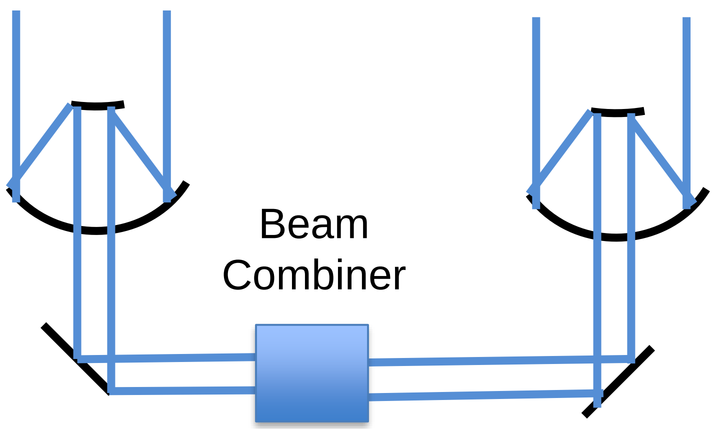


Image courtesy of NRAO/AUI

Interferometers

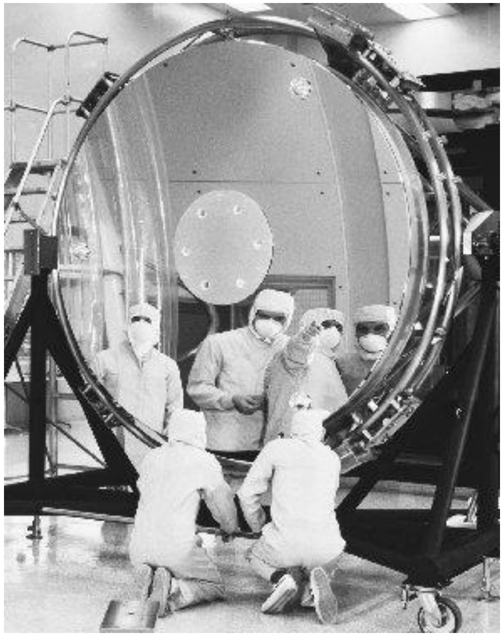
- Goal: increase resolution
- General principle: Coherently combine ≥2 beams
- Requires accuracies of optics $\ll \lambda$



Increase Angular Resolution λ/D

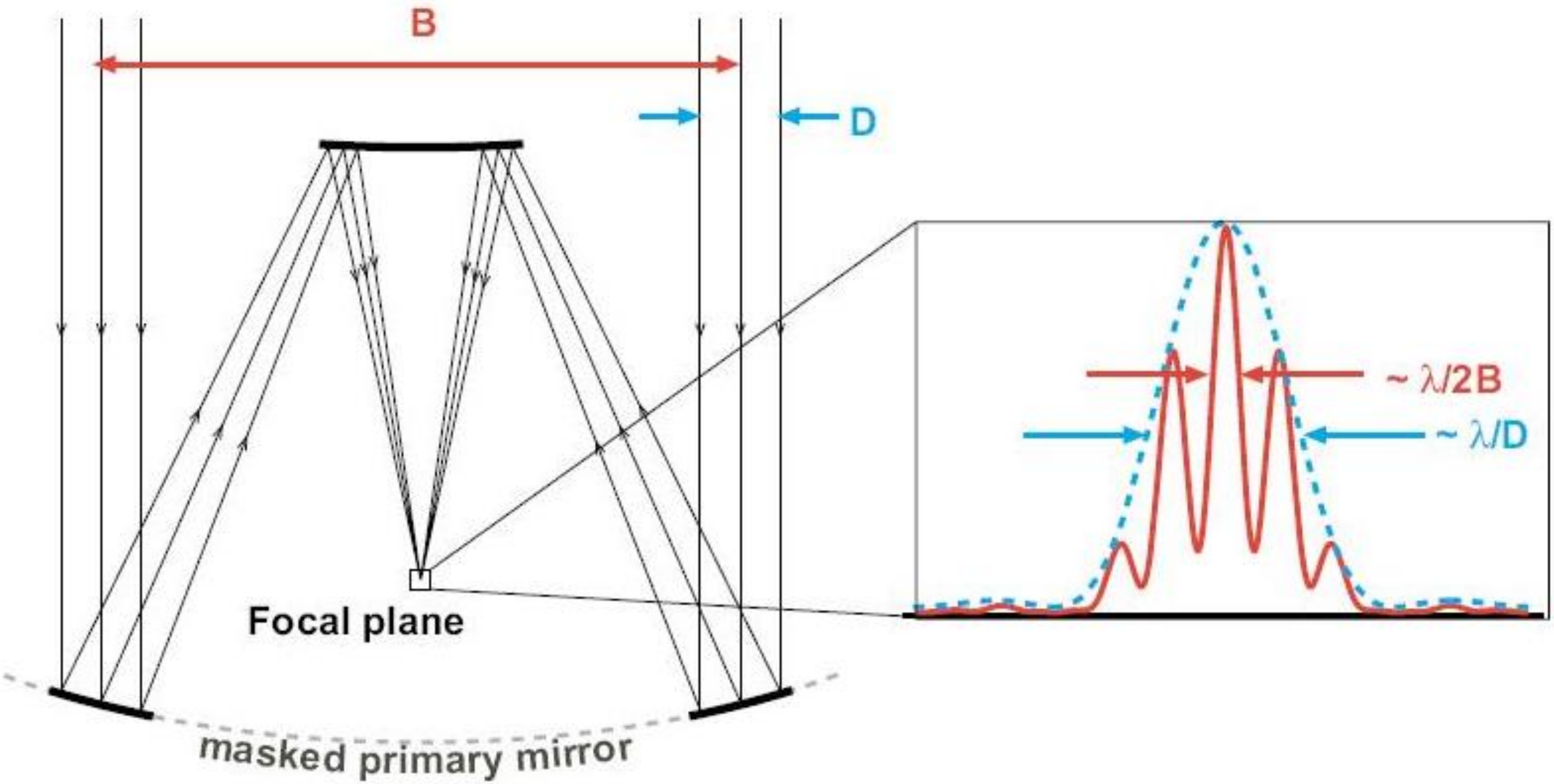
$D = D_{telescope}$

$D = B_{baseline}$

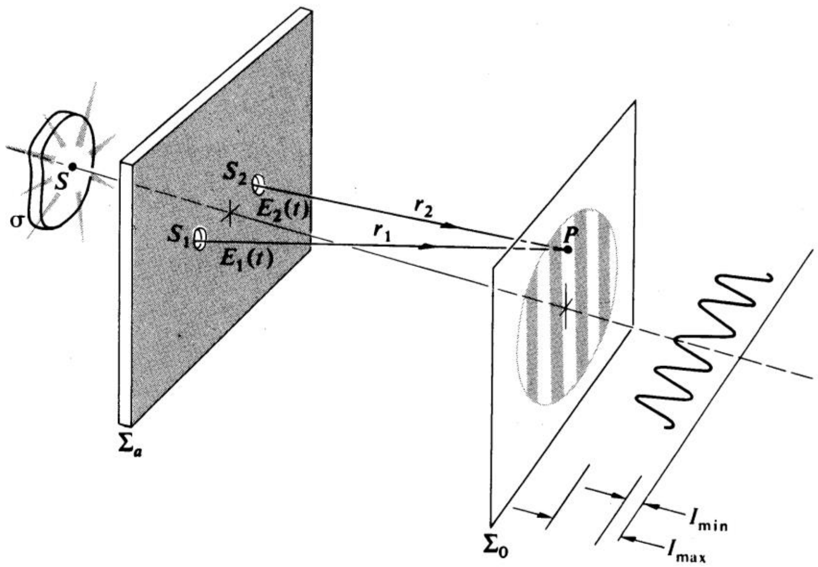


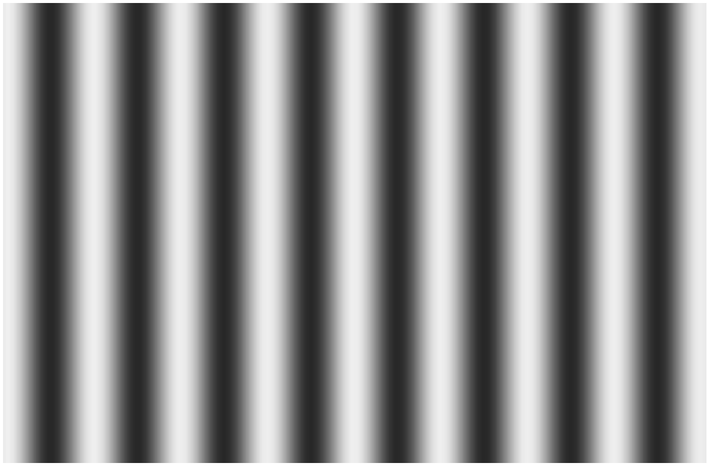
PSF of Masked Aperture

Interferometry is like masking a giant telescope:



Young's Double Slit Experiment



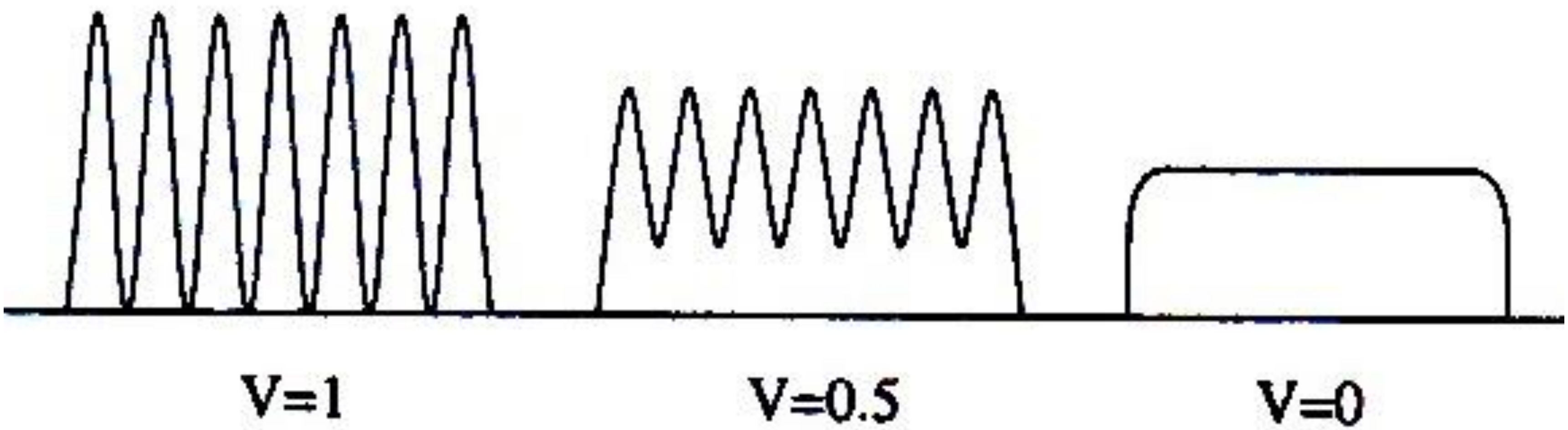


- monochromatic wave
- infinitely small pinholes
- source S generates fields
  - $E(r_1,t) = E_1(t)$ at S_1
  - $E(r_2,t) = E_2(t)$ at S_2
- two spherical waves from pinholes interfere on screen
- electrical field at P is sum of electrical field originating from S_1 and S_2
- Maxima if path lengths differences are k*wavelength

Visibility

- Fringe visibility

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$



- Fringe pattern of extended object is fringe pattern of point-source (PSF) convolved with object
- Still a fringe pattern but with reduced visibility

Complex visibility

- Basic measurement is the cross-correlation of the electric field at two locations, $\mathbf{r_1}$ and $\mathbf{r_2}$:

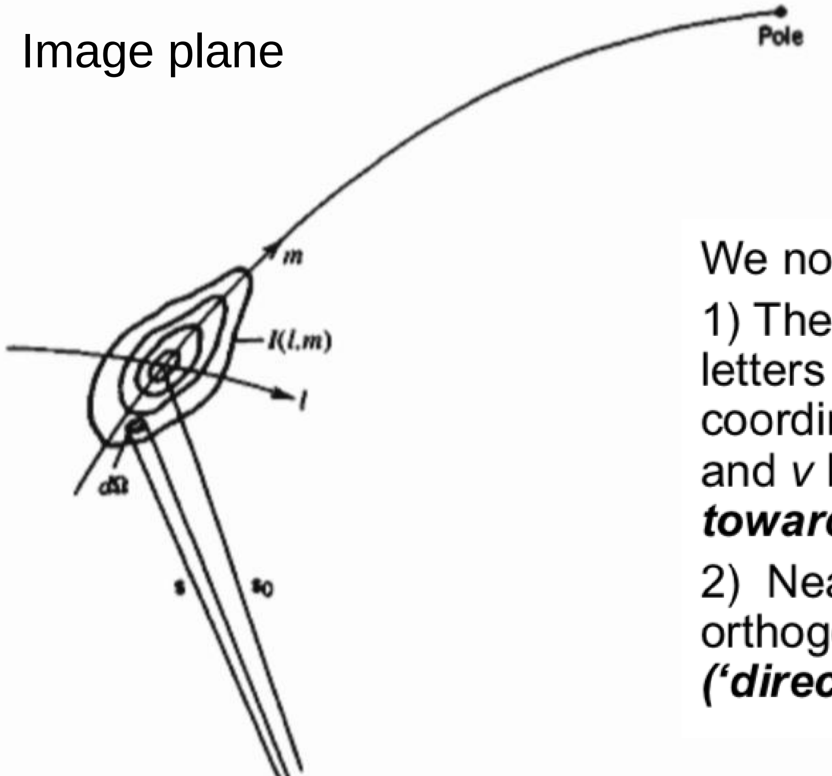
$$V_\nu(\mathbf{r_1}, \mathbf{r_2}) =< E_\nu(\mathbf{r_1}) E_\nu^*(\mathbf{r_2}) >$$

- Using the wavelength observed as the unit, define (u,v,w) coordinate system

$$(u, v, w) = (\mathbf{r_1} - \mathbf{r_2})/\lambda$$

- Hence the complex visibility for an interferometer in a plane: $V_\nu(u, v)$

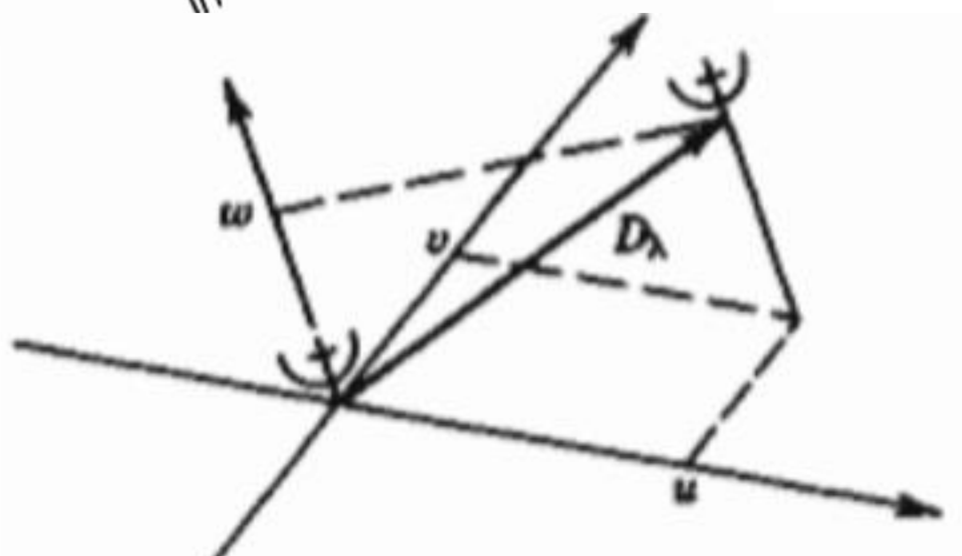
Image plane



We now ***introduce two coordinate systems***:

1) The ***array coordinates*** are designated with the letters ***u,v,w*** and represent a righthanded Cartesian coordinate system where w is towards the source and *u* and *v* lie in a plane orthogonal to the *w*-direction: ***u towards the east and v towards north.***

2) Near the target (in direction ***s***) we define a plane with orthogonal coordinates ***l,m which are angles ('direction cosines') in the directions of u and v***.



UV or pupil plan

The van Cittert - Zernike relation (II)

In the ***small angle approximation*** the visibility equation then takes on the simple 2-D form:

$$V(u,v) = \int\int I(l,m) e^{-2\pi i(ul+vm)} dl dm$$

This equation is called the ***van Cittert - Zernike relation*** who first derived it in an optical context. The Fourier inversion of this equation leads to:

$$I(l,m) = \int\int V(u,v) e^{2\pi i(ul+vm)} du dv$$

Hence by measuring the visibility functio *V(u,v)* , we can, through a Fourier transform, derive the brightness distribution I(l,m). Therefore, the more u,v samples we measure the more complete our knowledge of the source structure.

Each point in the u-v plane samples one component of the Fourier transform of the brightness distribution. The points at ***small u,v record large scale structure and vice-versa.***

4.3. Effect of discrete sampling

In practice the spatial coherence function V is not known everywhere but is sampled at particular places on the u-v plane. The sampling can be described by a *sampling function* $S(u, v)$, which is zero where no data have been taken. One can then calculate a function

$$I_\nu^D(l, m) = \iint V_\nu(u, v) S(u, v) e^{2\pi i(ul+vm)}\, du\, dv\,. \tag{1–10}$$

Radio astronomers often refer to $I_\nu^D(l, m)$ as the *dirty image*; its relation to the desired intensity distribution $I_\nu(l, m)$ is (using the convolution theorem for Fourier transforms):

$$I_\nu^D = I_\nu * B\,, \tag{1–11}$$

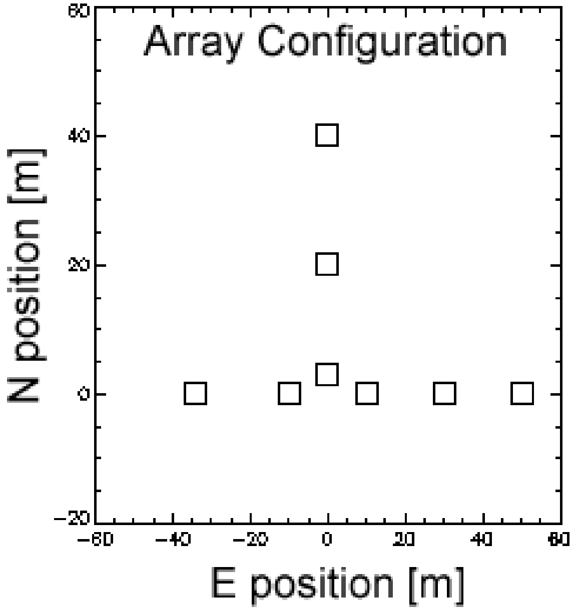
where the in-line asterisk denotes convolution and

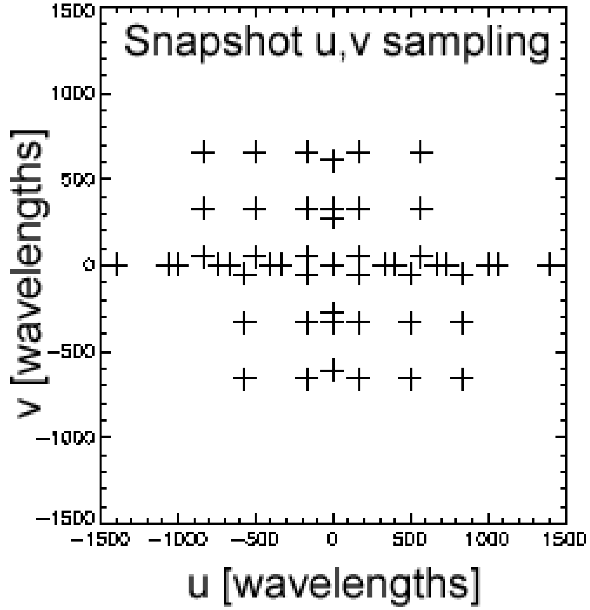
$$B(l, m) = \iint S(u, v) e^{2\pi i(ul+vm)}\, du\, dv \tag{1–12}$$

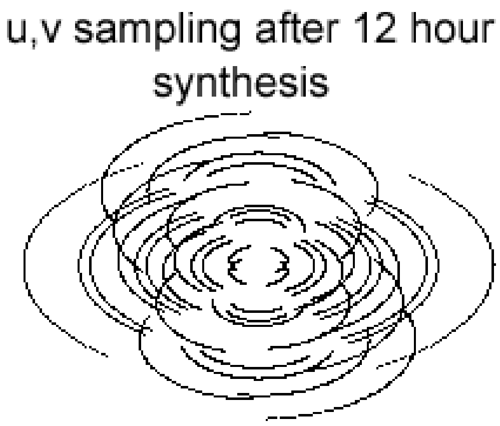
is the *synthesized beam* or *point spread function* corresponding to the sampling function $S(u, v)$. Equation 1–11 says that I^D is the true intensity distribution I convolved with the synthesized beam B.

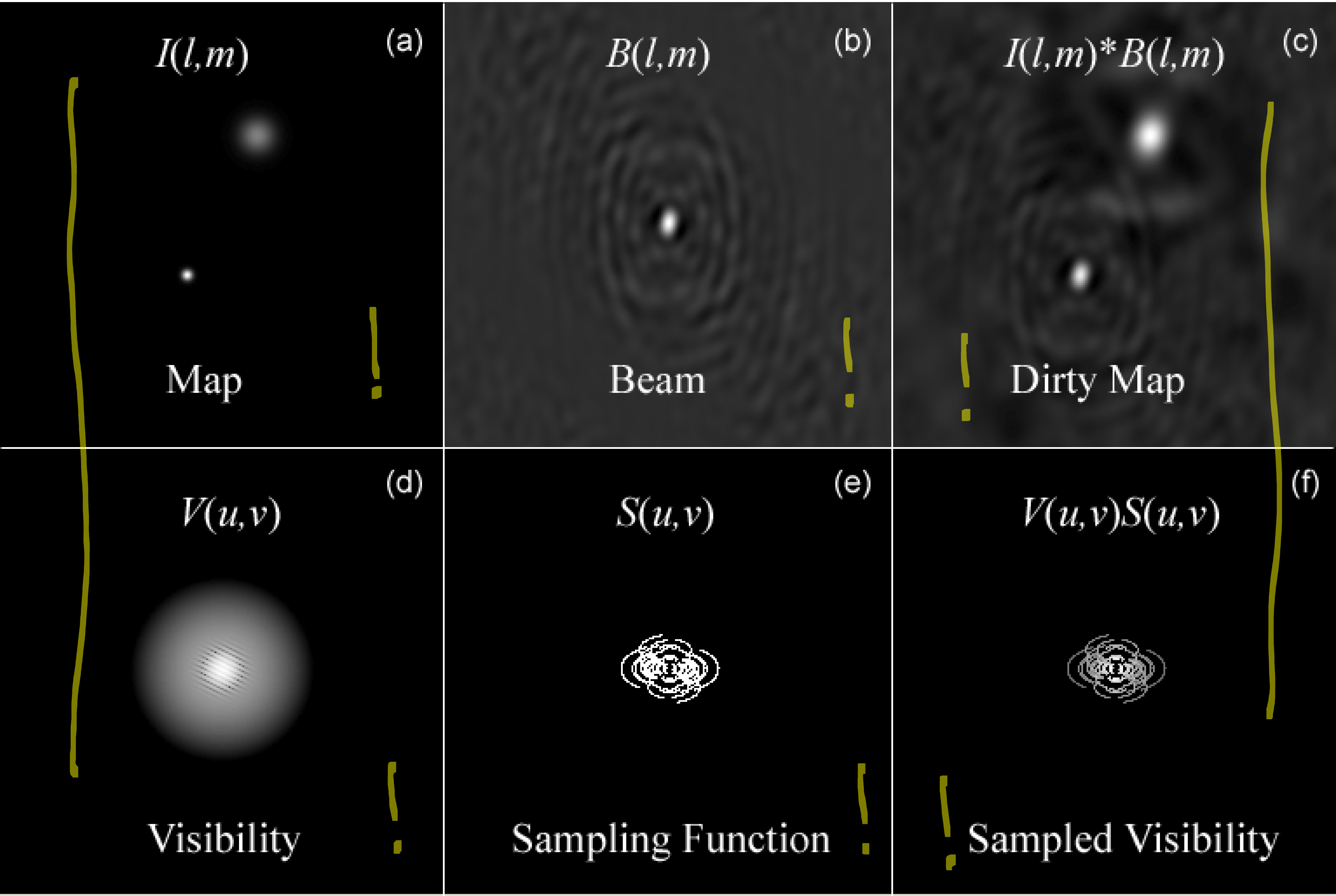
Tapering

- Weighing the visibilities as a function of baseline length: $T(u,v) = T(\sqrt{u^2+v^2})$
- Often a Gaussian is used: Gaussian tapering.
- Tapering with a Gaussian with FWHM of w, gives a PSF with a FWHM of 1/w


Array Configuration
N position [m]
E position [m]
60
40
20
0
−20
−60
−40
−20
0
20
40
60

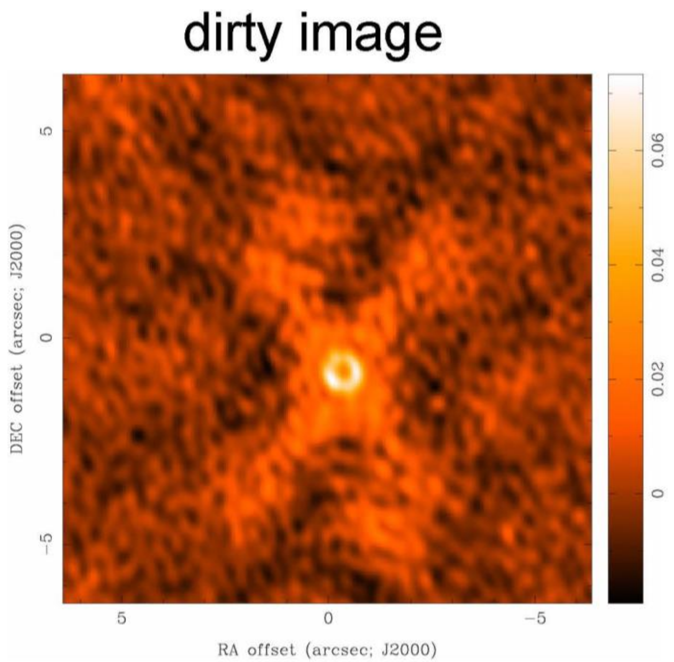

Snapshot u,v sampling
v [wavelengths]
u [wavelengths]
1500
1000
500
0
−500
−1000
−1500
−1500
−1000
−500
0
500
1000
1500

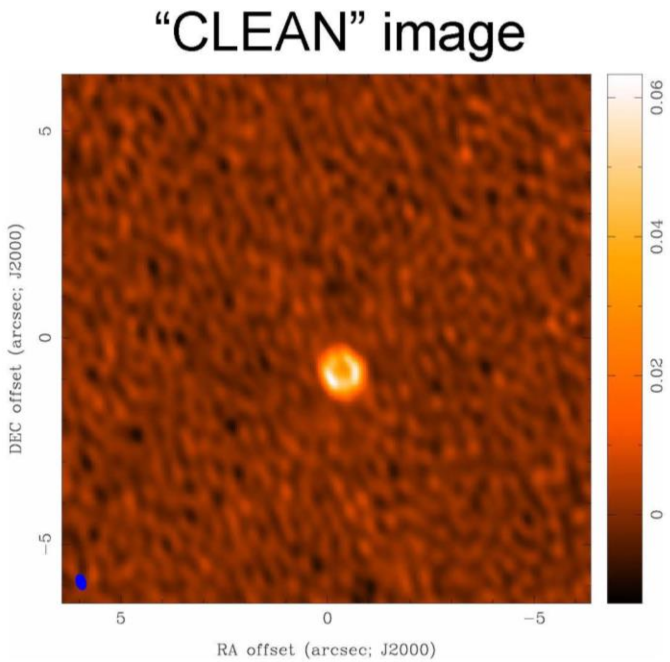

u,v sampling after 12 hour synthesis


(a)
$I(l,m)$
Map
(b)
$B(l,m)$
Beam
(c)
$I(l,m)*B(l,m)$
Dirty Map
(d)
$V(u,v)$
Visibility
(e)
$S(u,v)$
Sampling Function
(f)
$V(u,v)S(u,v)$
Sampled Visibility

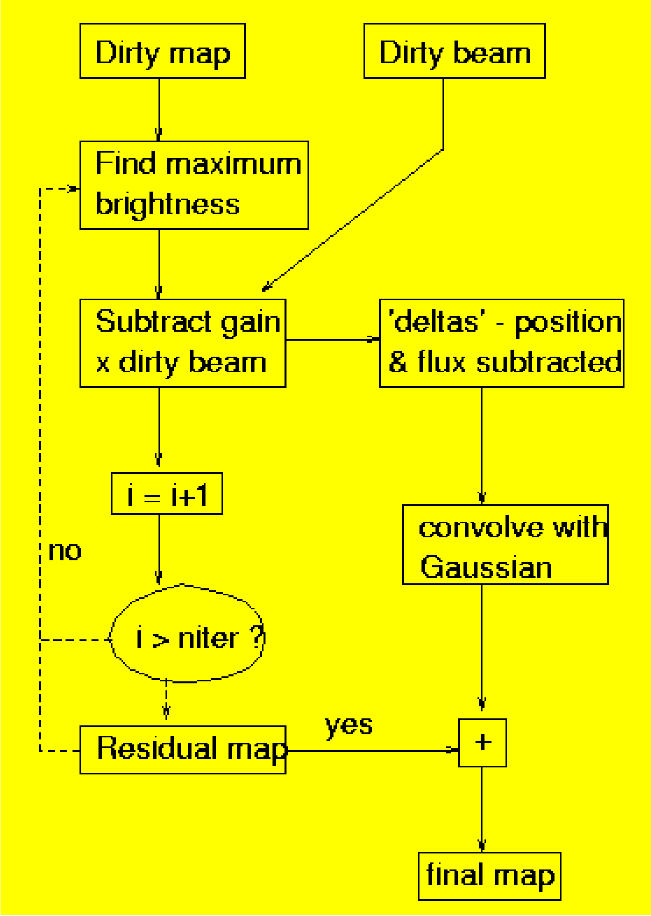
Deconvolution

- difficult to do science with dirty image
- deconvolve b(x,y) from $T^D(x,y)$ to recover T(x,y)
- information is missing and noise is present -> difficult problem





Hogbom CLEAN deconvolution



Brute-force iterative deconvolution using the dirty beam

Effectively reconstructs information in unsampled parts of the u-v plane by assuming sky is sum of point sources

Self calibration

Calibrating a synthesis array is one of the most difficult aspects of its operation and, in many cases, is the most important factor in determining the quality of the final deconvolved image. Small quasi-random errors in the amplitude and phase calibration of the visibility data scatter power and so produce an increased level of "rumble" in the weaker regions of the image, and other systematic errors can lead to a variety of artifacts in the image.

The ordinary calibration procedure (see Lecture 5) relies on frequent observations of radio sources of known structure, strength and position in order to determine empirical corrections for time-variable instrumental and environmental factors that cannot be measured, or monitored, directly. The relationship between the visibility $\widetilde{V}_{ij}$ observed at time t on the i-j baseline and the true visibility $V_{ij}(t)$ can be written very generally as

$$\widetilde{V}_{ij}(t) = g_i(t)g_j^*(t)G_{ij}(t)V_{ij}(t) + \varepsilon_{ij}(t) \tag{10–1}$$

The multiplicative factors $g_i(t)$ and $g_j(t)$ represent the effects of the complex gains of the array elements i and j; $G_{ij}(t)$ is the non-factorable part of the gain on the i-j baseline; $\varepsilon_{ij}(t)$ is an additive offset term; and $\epsilon_{ij}(t)$ is a pure, zero-mean, noise term representing the thermal noise. The effects of $G_{ij}(t)$ and $\varepsilon_{ij}(t)$, which cannot be split into antenna-dependent parts, can usually be reduced to a satisfactory degree by clever design (see Lecture 4), so we will ignore them during this lecture. Equation 10–1 then simplifies to

$$\widetilde{V}_{ij}(t) = g_i(t)g_j^*(t)V_{ij}(t) + \epsilon_{ij}(t)\,. \tag{10–2}$$

The *element gain* (usually called the *antenna gain* in radio astronomy) really describes the properties of the elements relative to some reference (usually one array element for phase and a "mean" array element for amplitude). Although this use of the word "gain" may seem confusing, it is quite helpful in lumping all element-based properties together. The gain for any one array element has two contributing components: firstly, a slowly varying instrumental part and secondly, a more rapidly varying part due to the atmosphere (troposphere and ionosphere) above the element. Variations in the phase part of the atmospheric component nearly always dominate the overall variation of the element gains (see Sec. 4 of Lecture 5).

Selfcalibration

1. Start with a decent image of the sky
2. Calculate $V_{ij}(t)$ from a Fourier transform of this sky image
3. Solve eq. 10-2 for gains $g_i(t)$
   - note that this can been done since N telescopes (with N gains $g_i(t)$) give N(N-1)/2 ~ N^2 visibilities
4. Produce improved image using better calibrated visibilities
5. Repeat 2-4 with improved image, until no further improvement

Interferometer Components

Optical Interferometer

- Telescopes
- Delay Lines
- Beam Combiner
- Fringe Tracker
- Detector

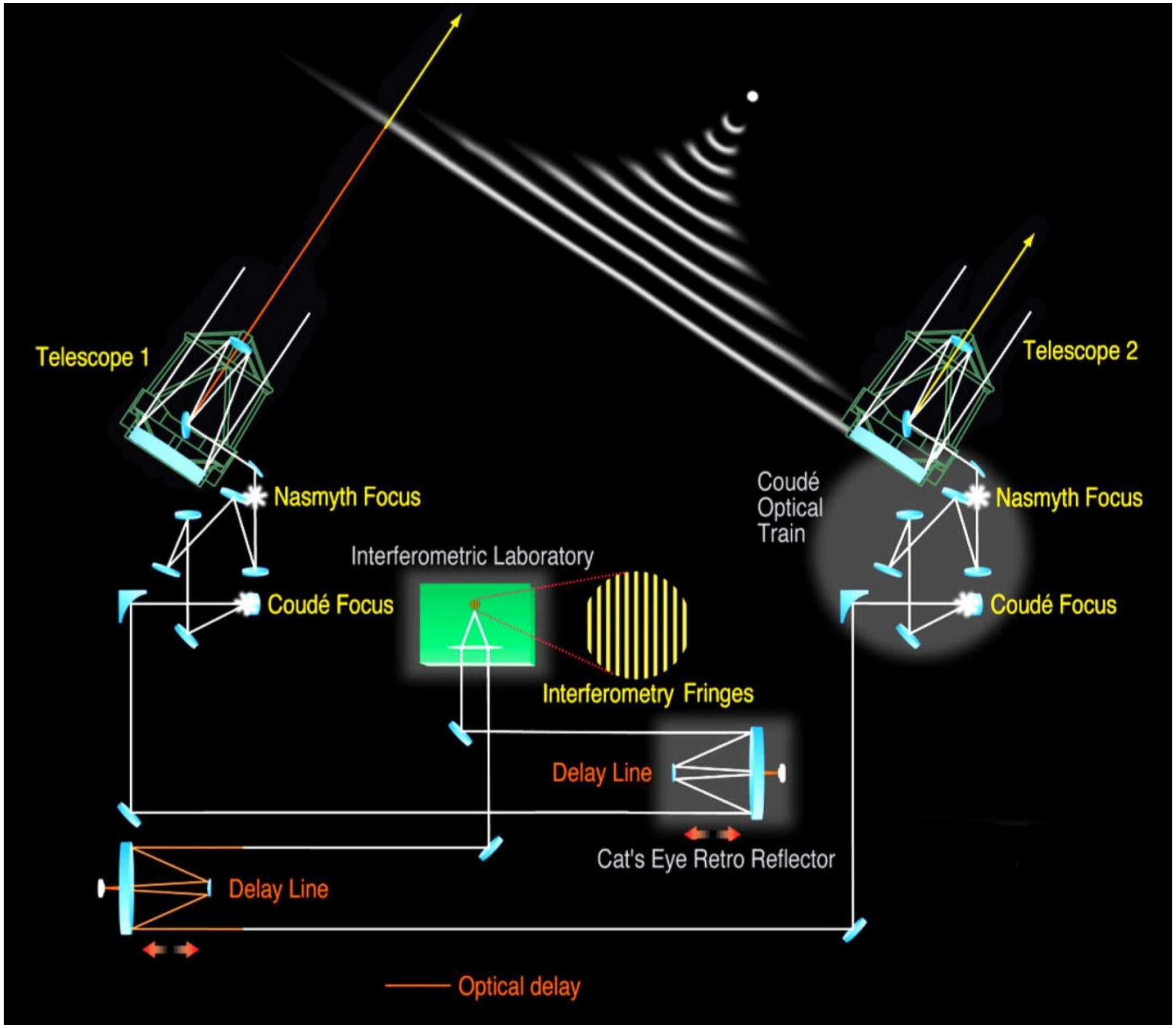
(Normally) measures only amplitude of visibilities

Radio Interferometer

- Telescopes
- Receiver
- Signal Delay
- Correlator

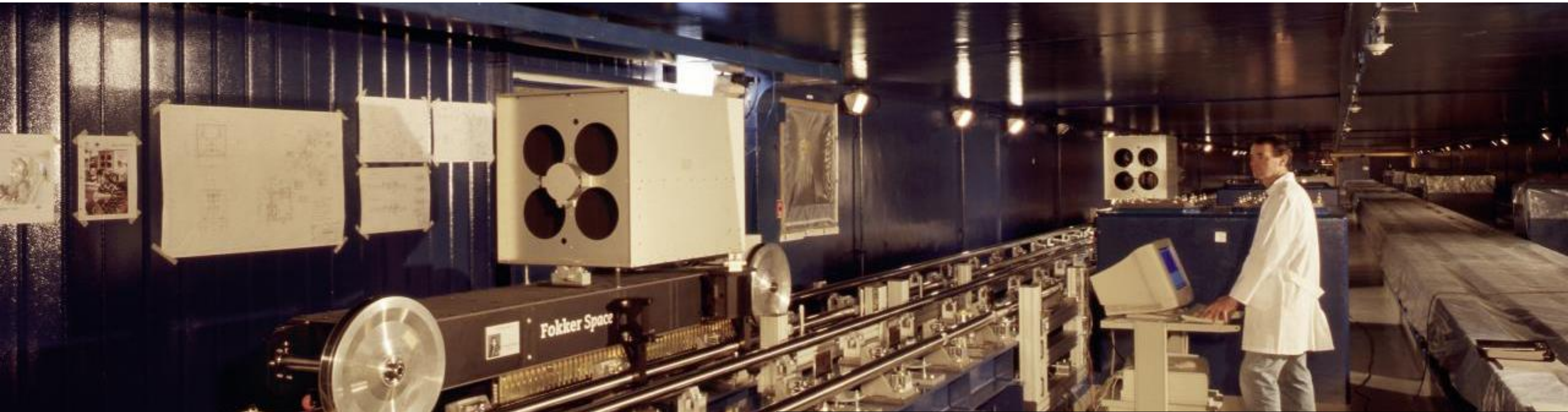
Measures complex visibilities

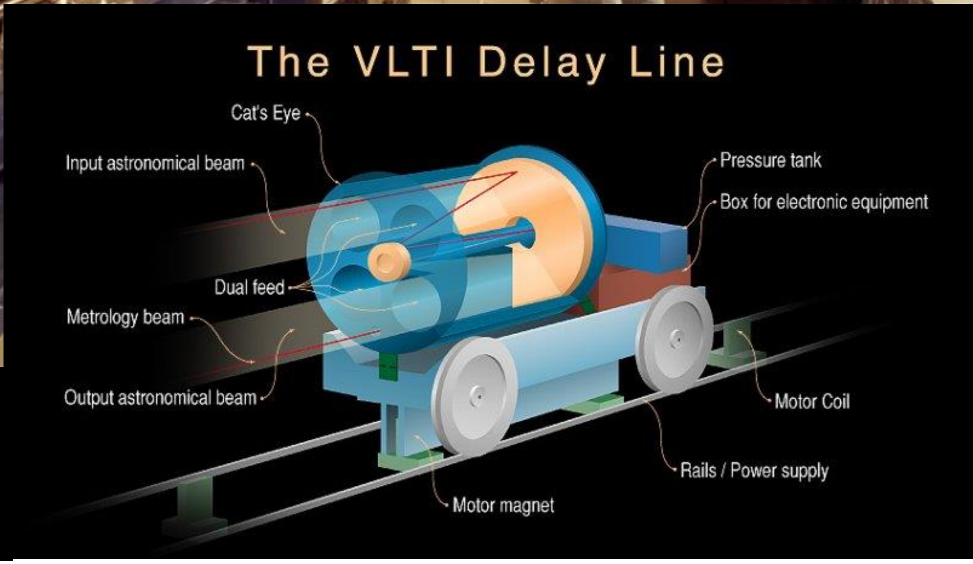
Optical Interferometry



Delay Lines

Compensate optical path difference between telescopes

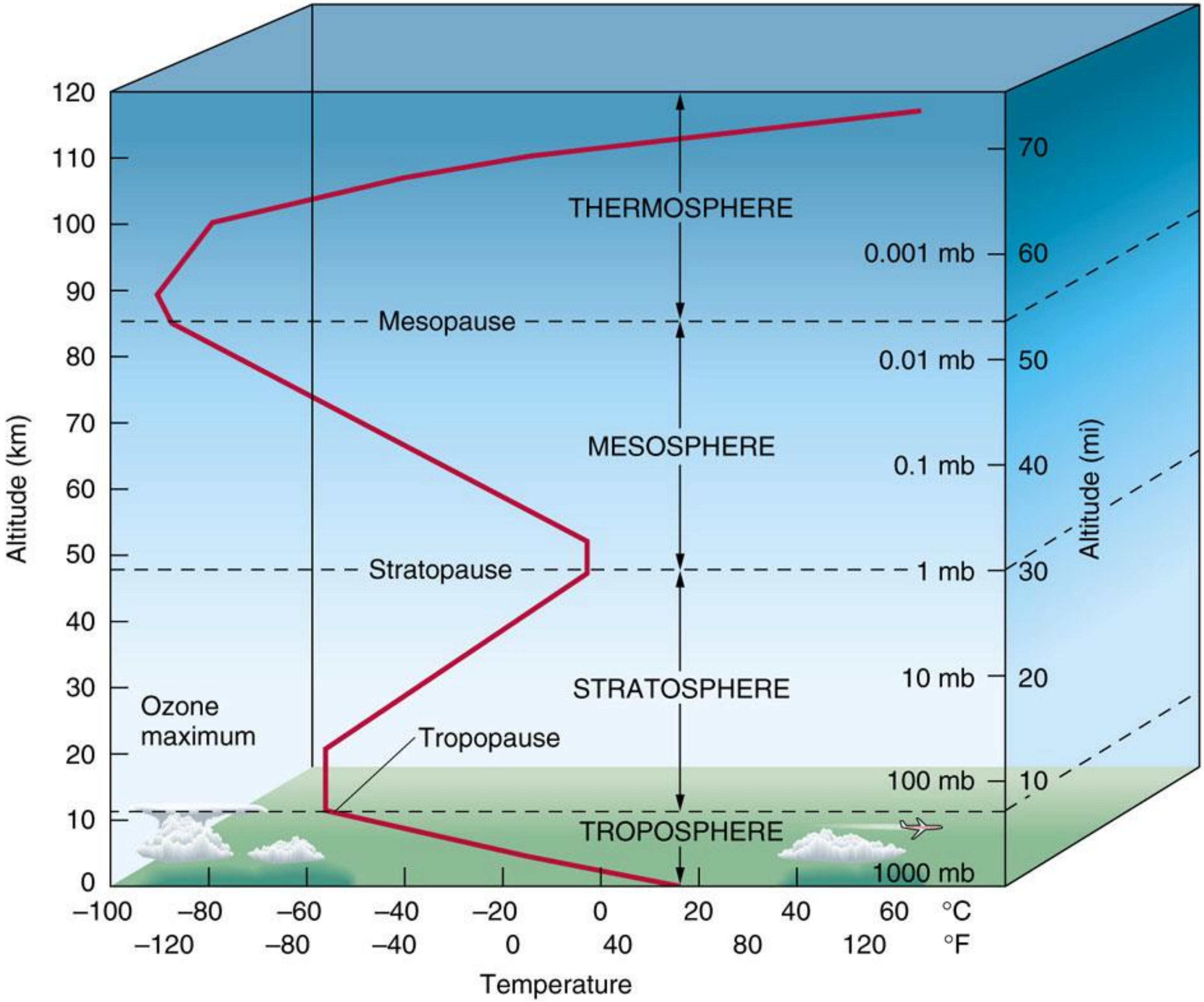




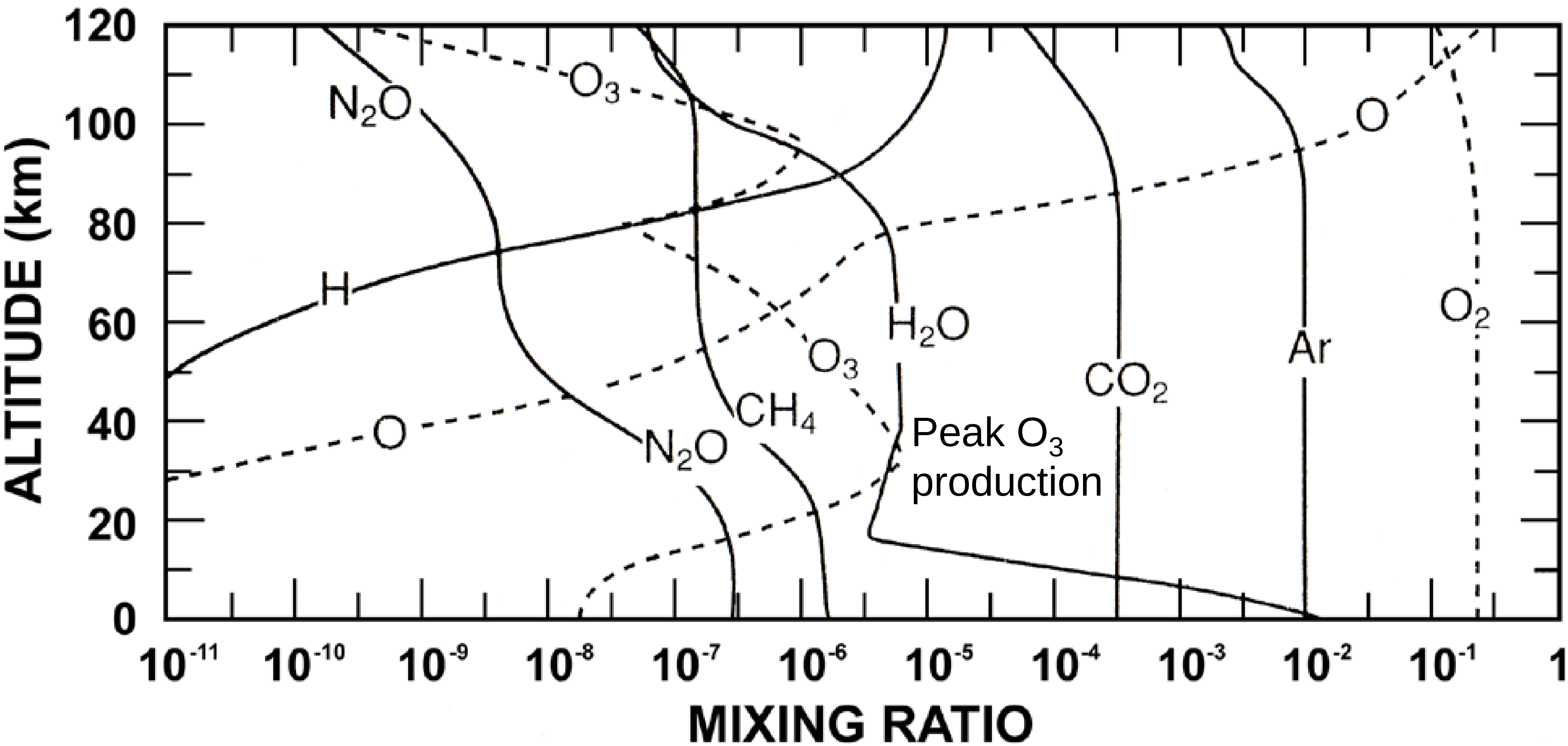
Challenge: travel over tens of meters, positioning to fractions of micrometers ➔ dynamic range of > 10^9 !

Lecture 8: Trouble is in the Air

1. Layers in the atmosphere
2. Atmospheric absorption
3. Emission from the atmosphere and space
4. Scattering and dispersion in the atmosphere
5. Turbulence in the atmosphere and seeing
6. Low frequency radio astronomy and the ionosphere

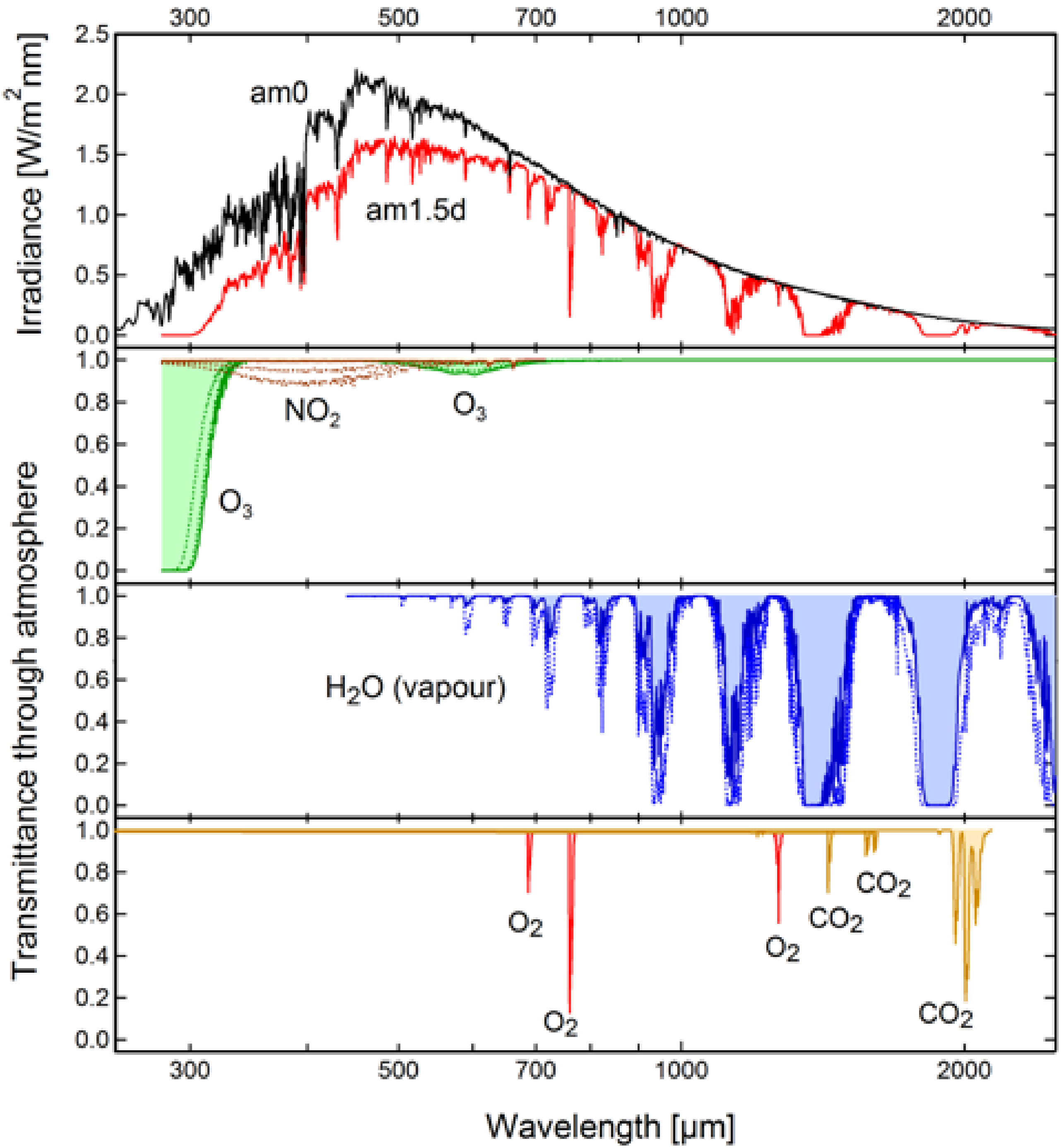

THERMOSPHERE
Mesopause
MESOSPHERE
Stratopause
STRATOSPHERE
Ozone maximum
Tropopause
TROPOSPHERE
0.001 mb
0.01 mb
0.1 mb
1 mb
10 mb
100 mb
1000 mb
Altitude (km)
Altitude (mi)
Temperature
°C
°F

Atmospheric Composition: Mixing Ratios



The mixing ratio is the number density of the gas under consideration divided by number density of all gases in dry air. Not depicted are N_2 and the rare noble gases, whose mixing ratios are nearly constant up to 100 km because of their chemical stability. The N_2 mixing ratio curve would parallel the O_2 curve and lie to its right, starting at a sea-level value of 0.78.

Schatter 2009: Atmospheric Composition and Vertical Structure

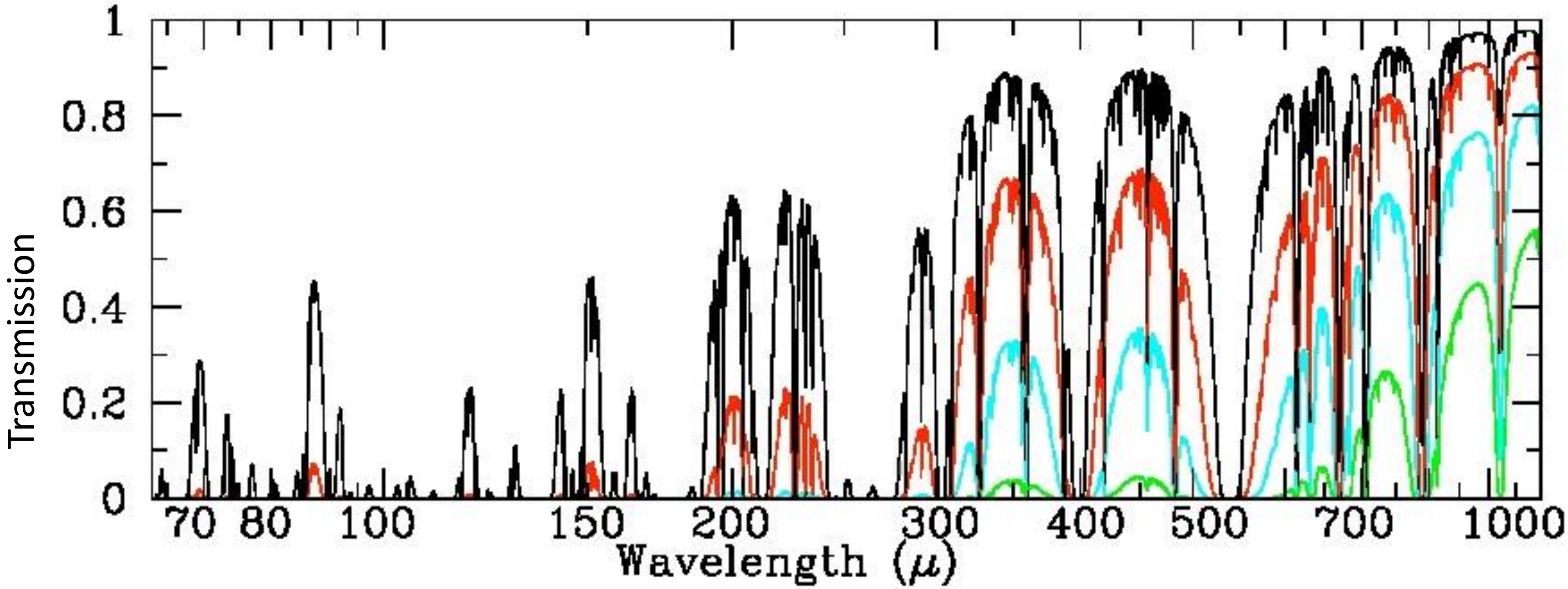


300
500
700
1000
2000
Irradiance [W/m^2 nm]
am0
am1.5d
Transmittance through atmosphere
NO2
O3
O3
H2O (vapour)
O2
O2
O2
CO2
CO2
CO2
Wavelength [µm]

Absorption by Water Vapour

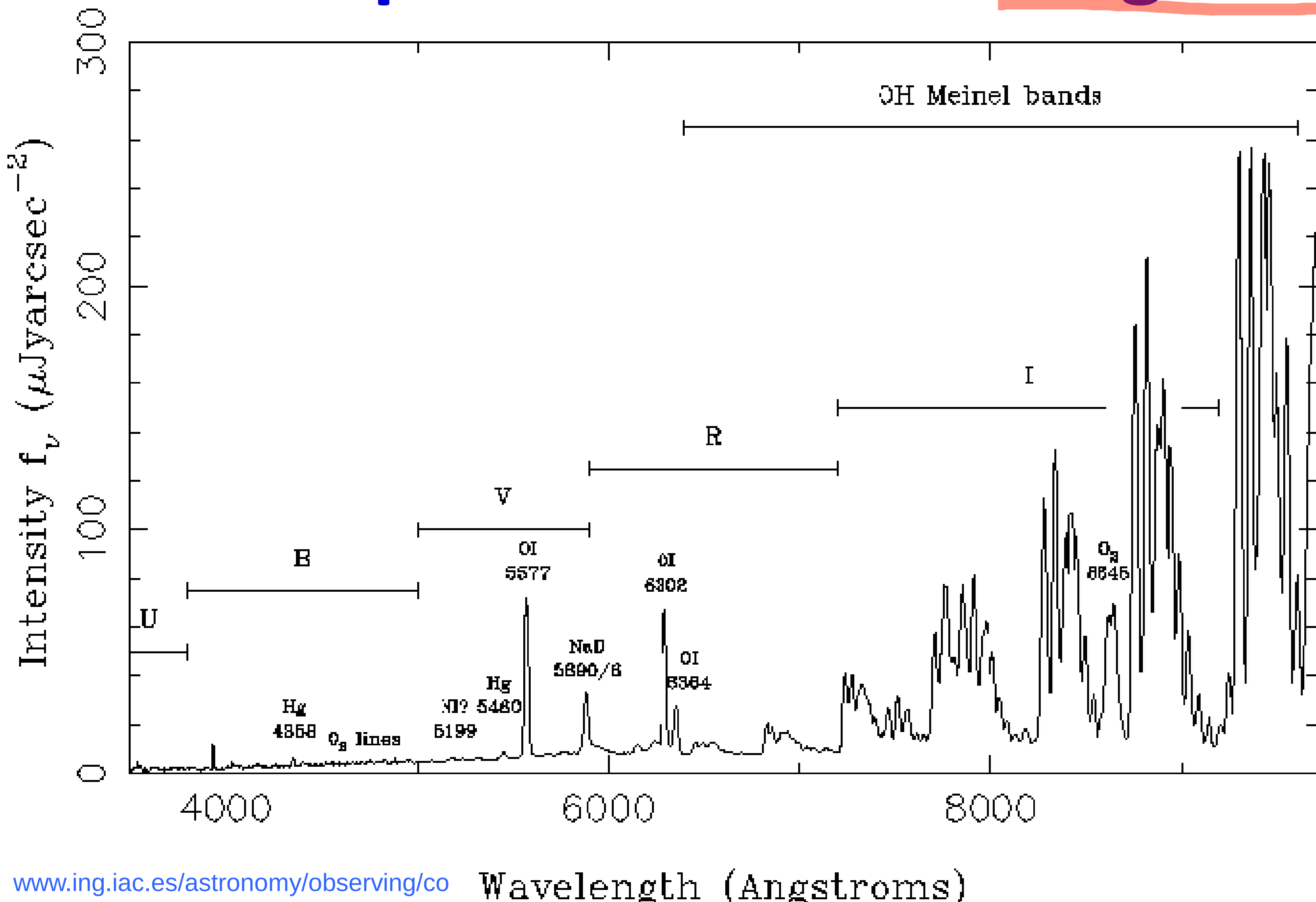
Scale height for PWV is only ~3 km

→ observatories at high altitudes



0.1 mm PWV (Hawaii/ALMA), 0.4 mm PWV, 1.0 mm PWV, 3.0 mm PWV

Atmospheric Emission: Airglow



www.ing.iac.es/astronomy/observing/conditions/skybr/skybr.html

Thermal Atmospheric Emission

- Atmosphere in local thermodynamic equilibrium (LTE) <60 km, i.e. excitation levels are thermally populated
- Full radiative transfer calculation needed
- For τ << 1 use approximation:

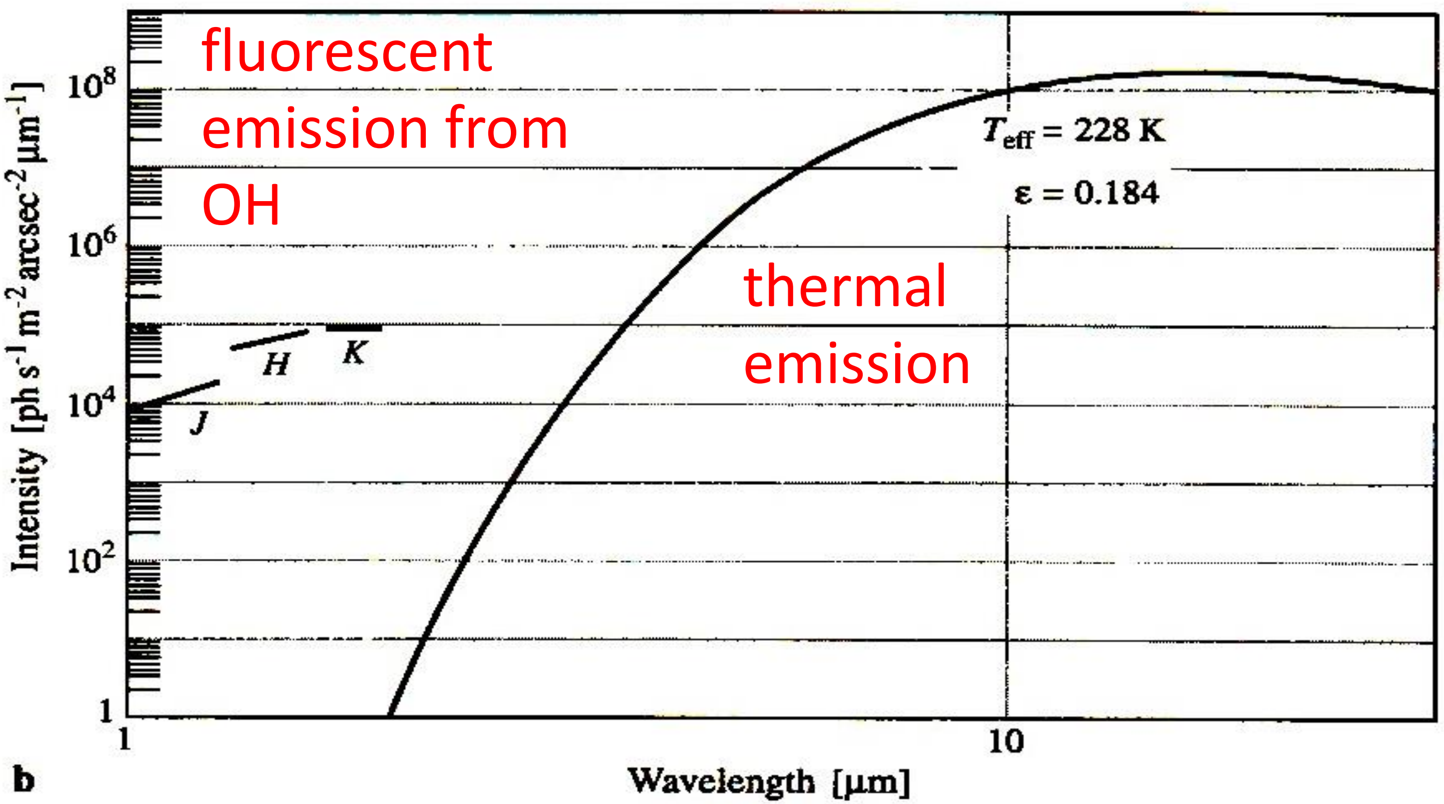
$$I_\lambda(z) = \frac{\tau_\lambda B_\lambda(T_{\mathrm{mean}})}{\cos\theta}$$

$B_\lambda(T_{\mathrm{mean}})$: Planck function at mean temperature T_{mean} of atmosphere

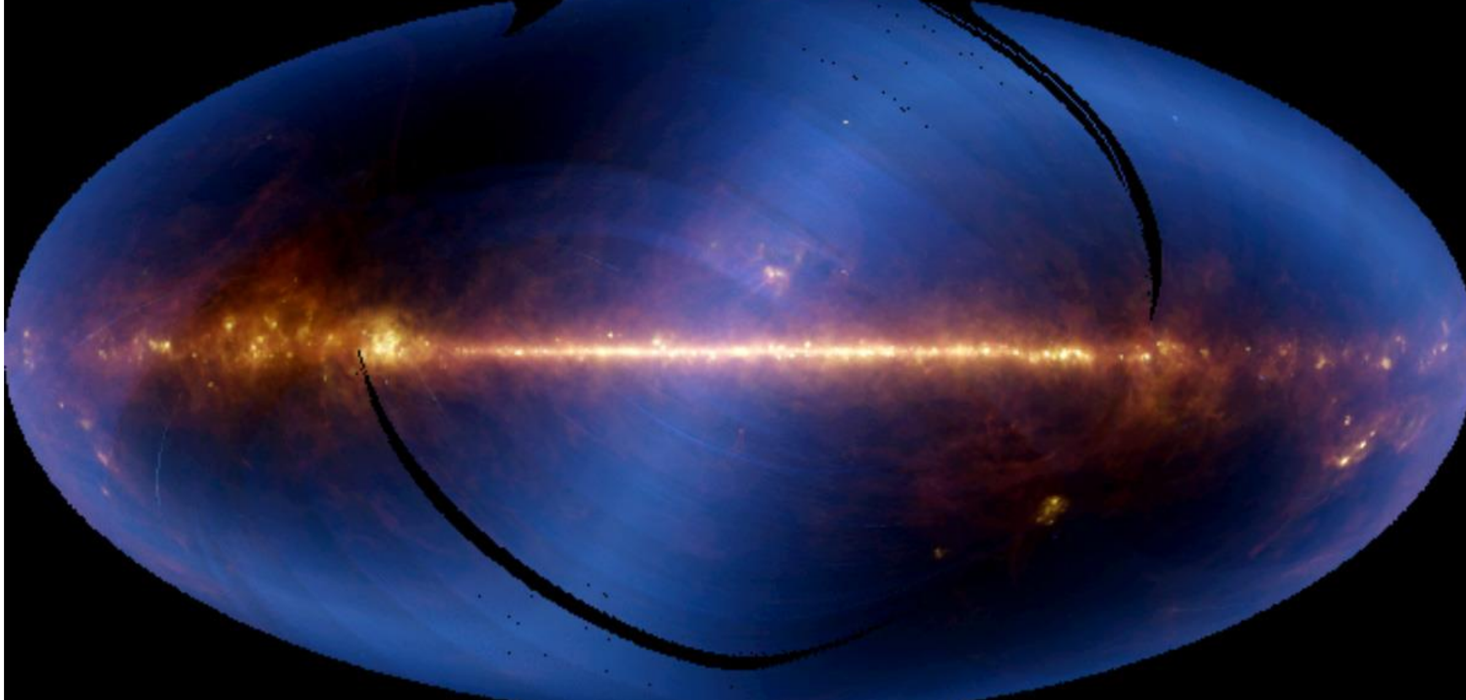
For T = 250 K and θ = 0

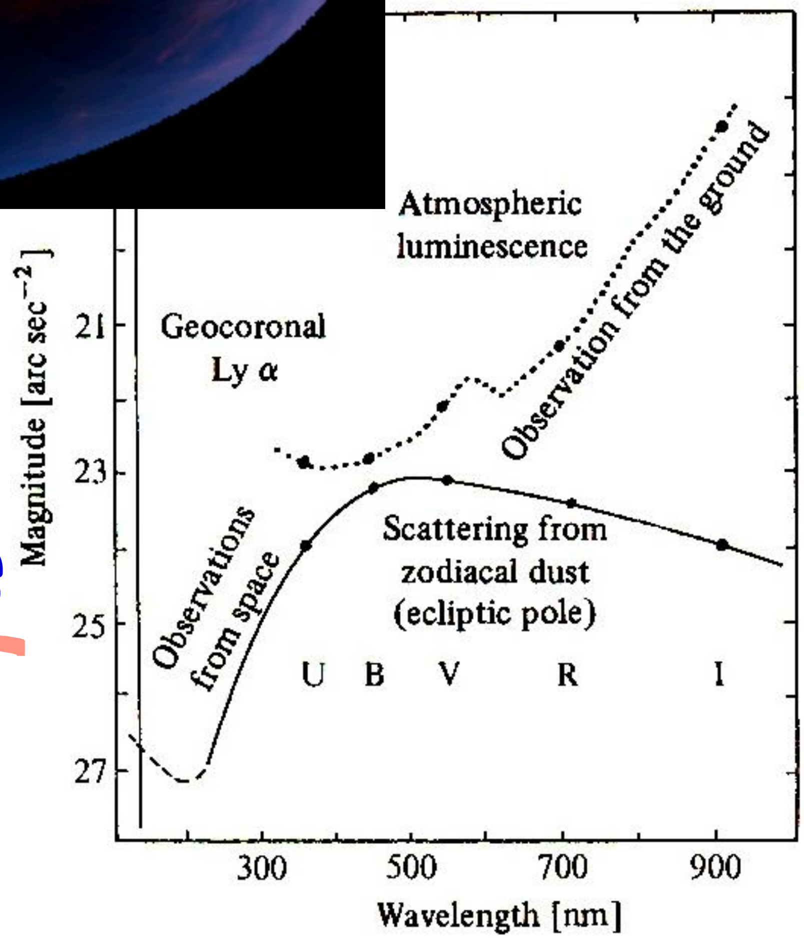
Spectral band (cf. Sect. 3.3)	L	M	N	Q
Mean wavelength [μm]	3.4	5.0	10.2	21.0
Mean optical depth τ	0.15	0.3	0.08	0.3
Magnitude [arcsec^{-2}]	8.1	2.0	−2.1	−5.8
Intensity [Jy arcsec^{-2}][a]	0.16	22.5	250	2 100

Fluorescent and Thermal Emission



Sky surface brightness is important as even an unresolved point source has a finite angular diameter when viewed through a telescope.





Emission from Space

Molecular Scattering

- Molecular scattering in visible and NIR is Rayleigh scattering; scattering cross-section given by:

$$\sigma_R(\lambda) = \frac{8\pi^3}{3}\frac{\left(n^2-1\right)^2}{N^2\lambda^4}$$

  where *N* is the number of molecules per unit volume and *n* is the refractive index of air ($n-1 \sim 8\cdot10^{-5}\, P/T$).
- Rayleigh scattering is not isotropic:

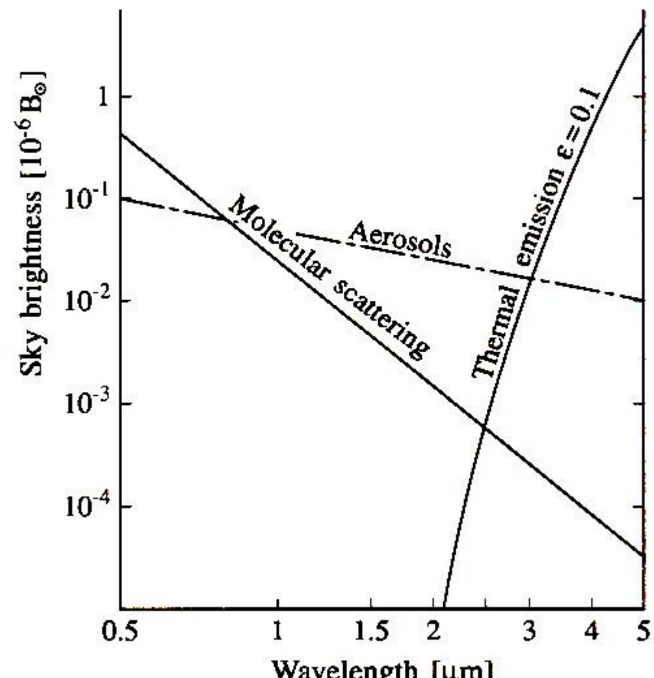
$$I_{scattered} = I_0 \frac{3}{16\pi}\sigma_R\left(1+\cos^2\theta\right)d\omega$$

Aerosol Scattering

- Aerosols (sea salt, hydrocarbons, volcanic dust) much larger diameter *d* than air molecules → NOT Rayleigh scattering
- Aerosol scattering described by Mie theory (classical electrodynamics, "scattering efficiency factor" Q):

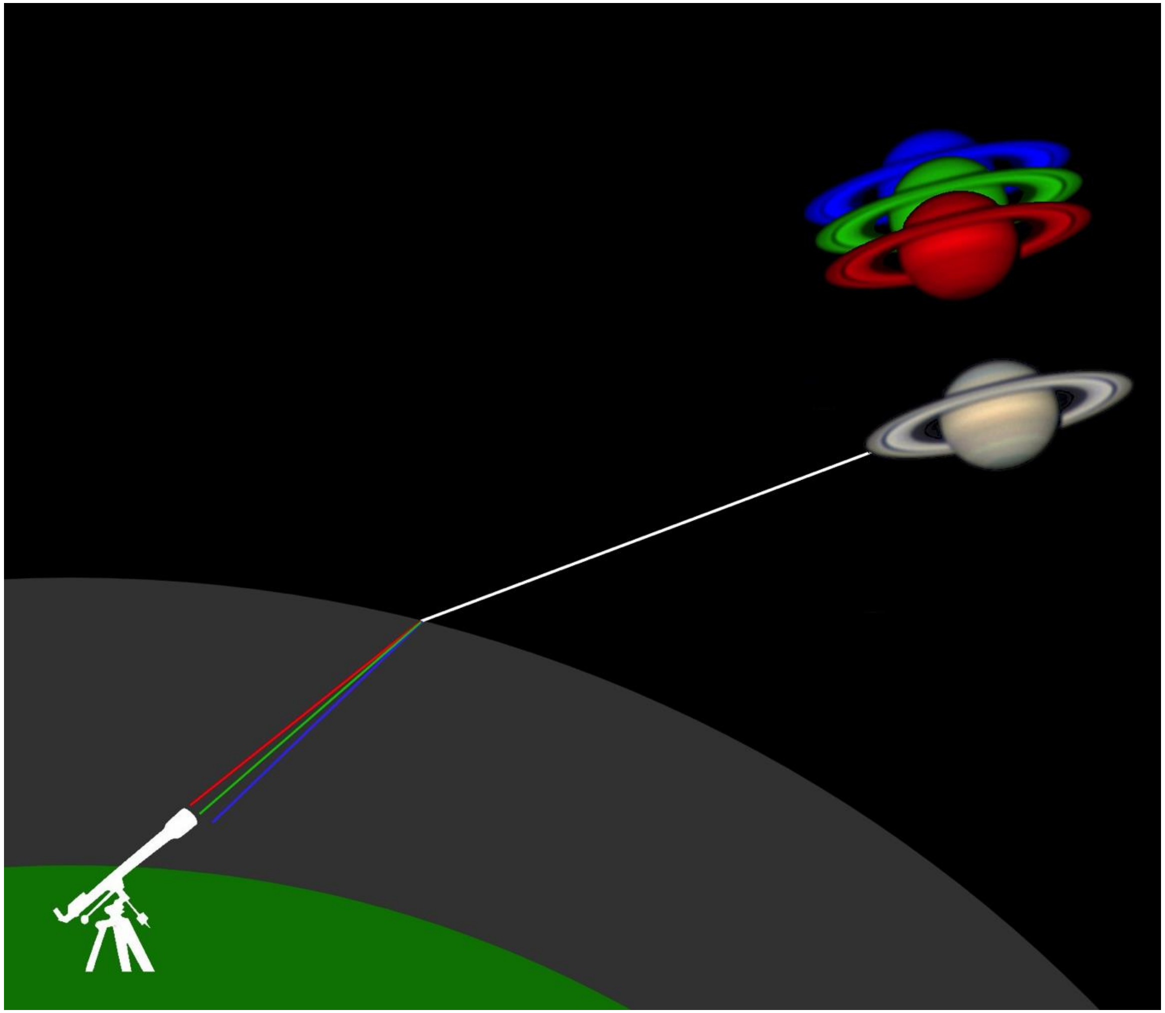
$$Q_{\text{scattering}} = \frac{\sigma_M}{\pi a^2} = \frac{\text{scattering cross section}}{\text{geometrical cross section}}$$

- *d* >> *λ*: $Q_{\text{scattering}} \sim Q_{\text{absorption}}$
  - scattered power equal to absorbed power
  - effective cross section is twice the geometrical size
- *d* ~ *λ*: $Q_s \sim 1/\lambda$ (for dielectric spheres):
  - scattered intensity goes with 1/λ



Atmospheric Refraction

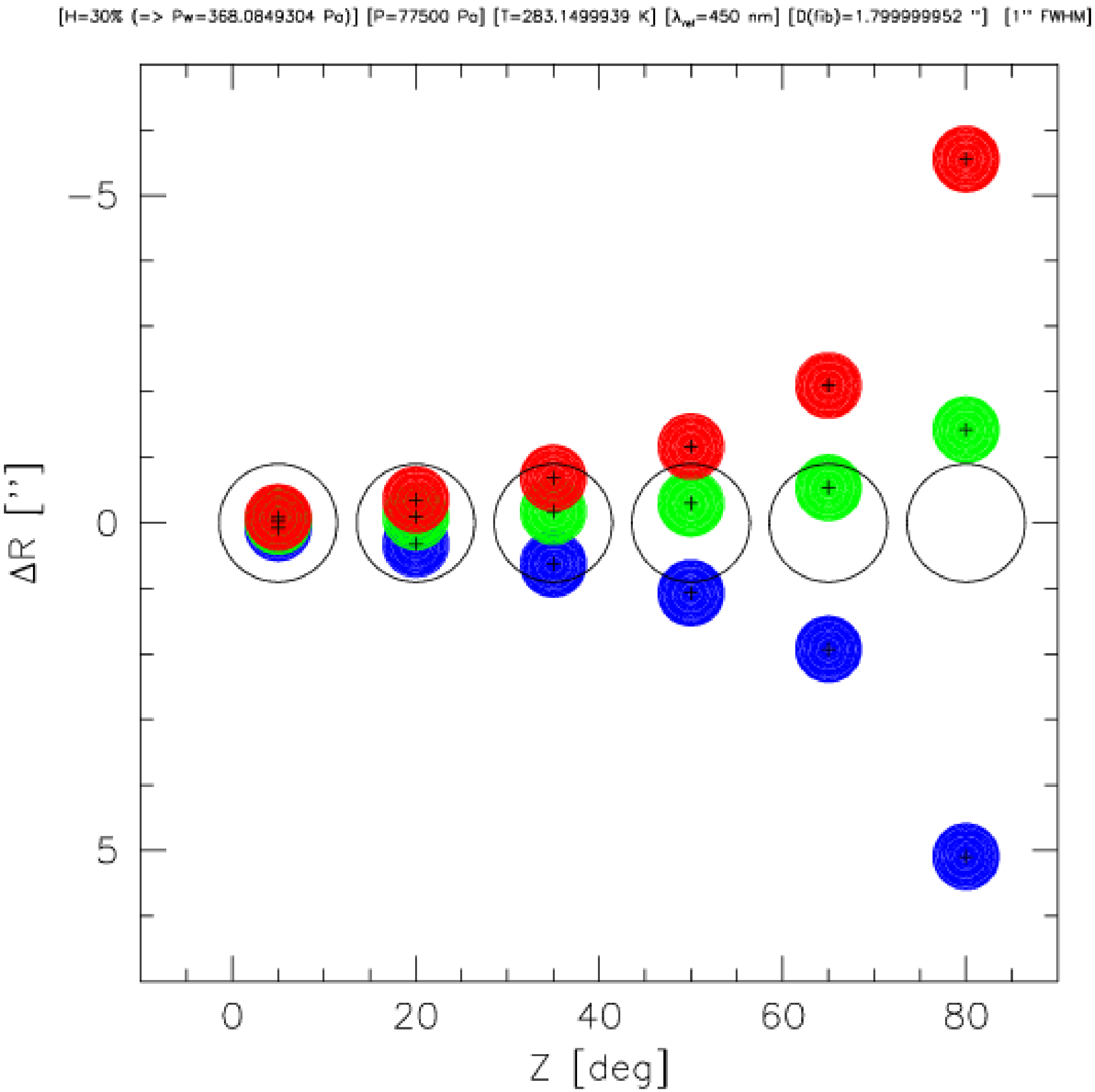
Atmospheric Dispersion



www.skyinspector.co.uk/Atm-Dispersion-Corrector-ADC(2587060).htm

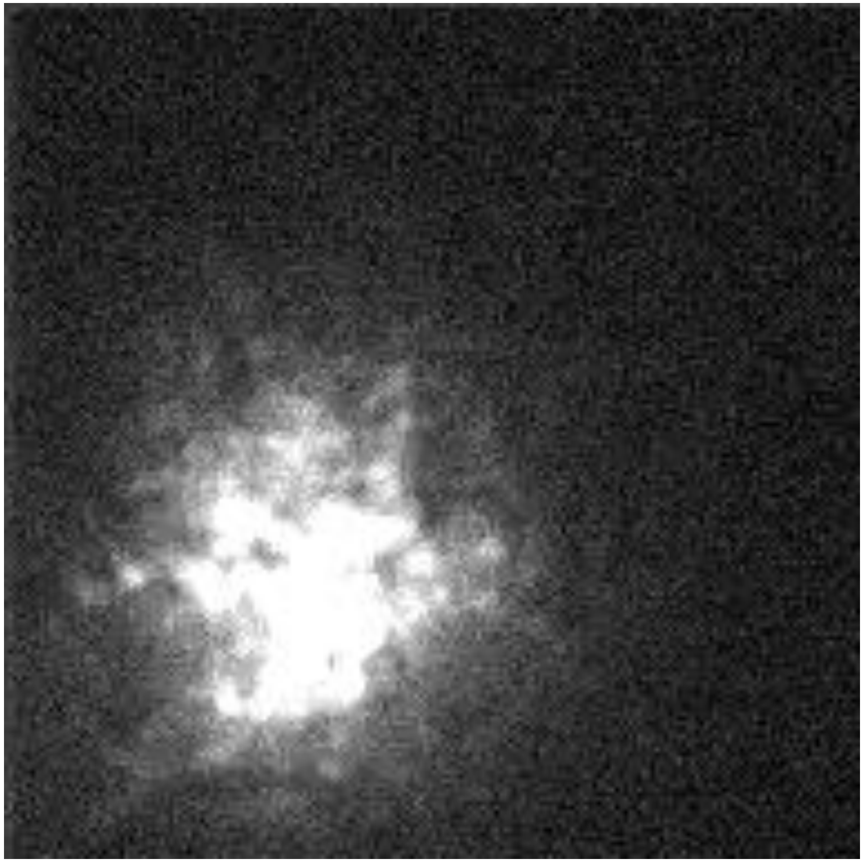
- Dispersion: Elongation of points in broadband filters due to n(λ) → "rainbow"
- Dispersion depends strongly on airmass and wavelength
- No problem if dispersion < λ/D ← o.k. for small or seeing limited telescopes, but big problem for large, diffraction limited telescopes (e.g. E-ELT)

Atmospheric Dispersion

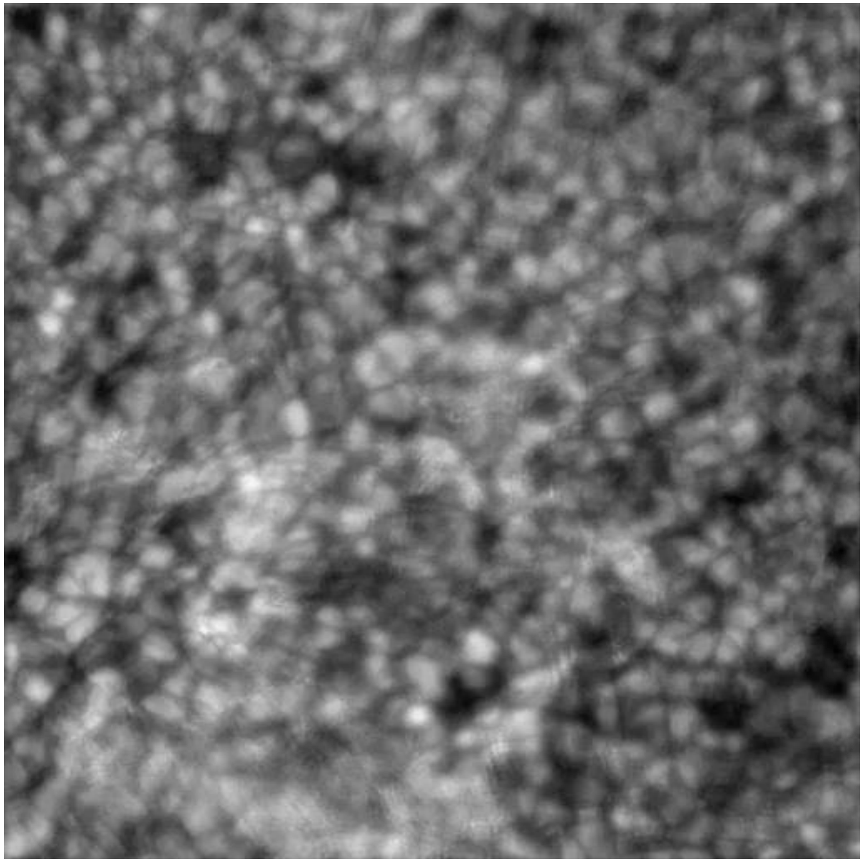


Solution: Atmospheric Dispersion Corrector: a set of prisms that correct for this

Atmospheric Turbulence: Seeing

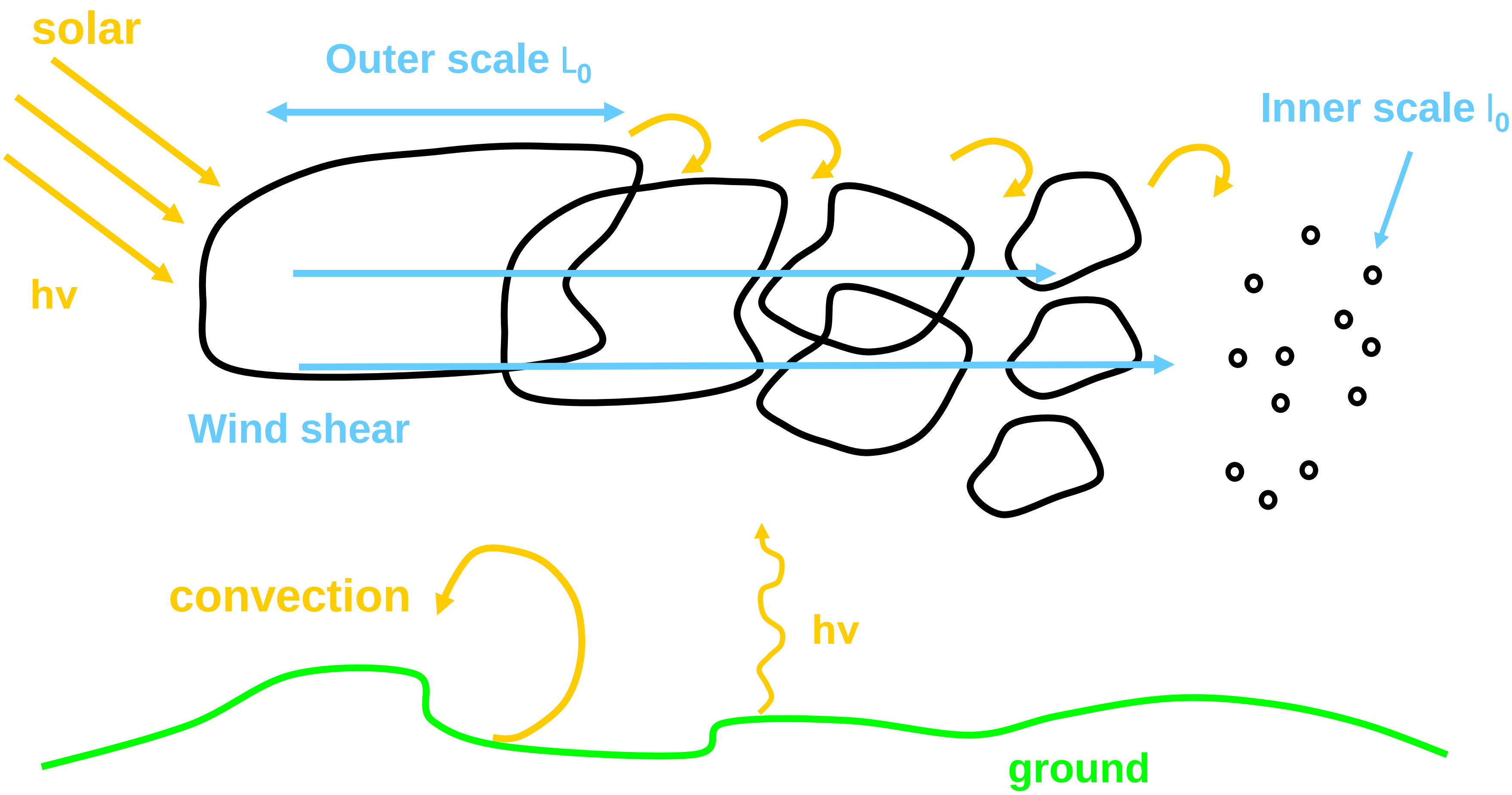


α Ori



solar photosphere

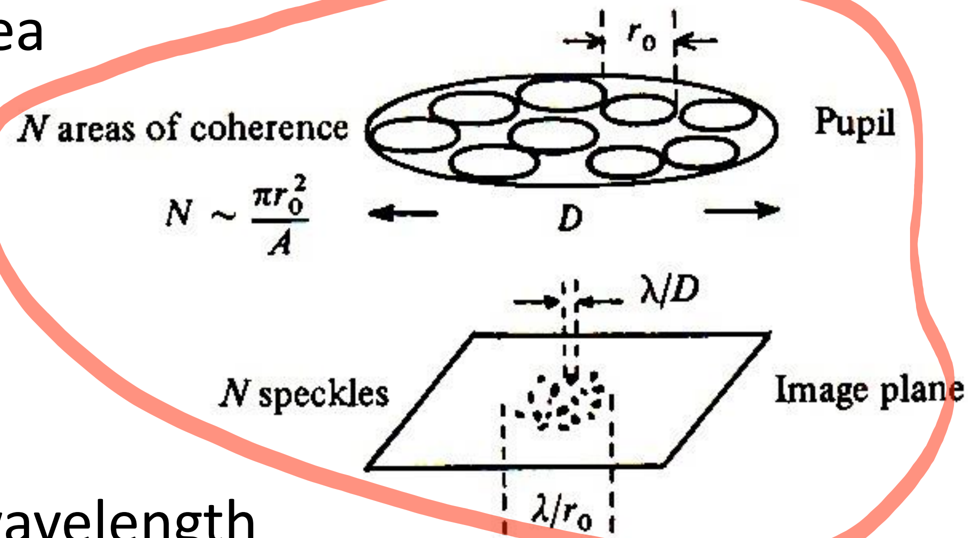
Atmospheric Turbulence: Origin



Fried Parameter r_0

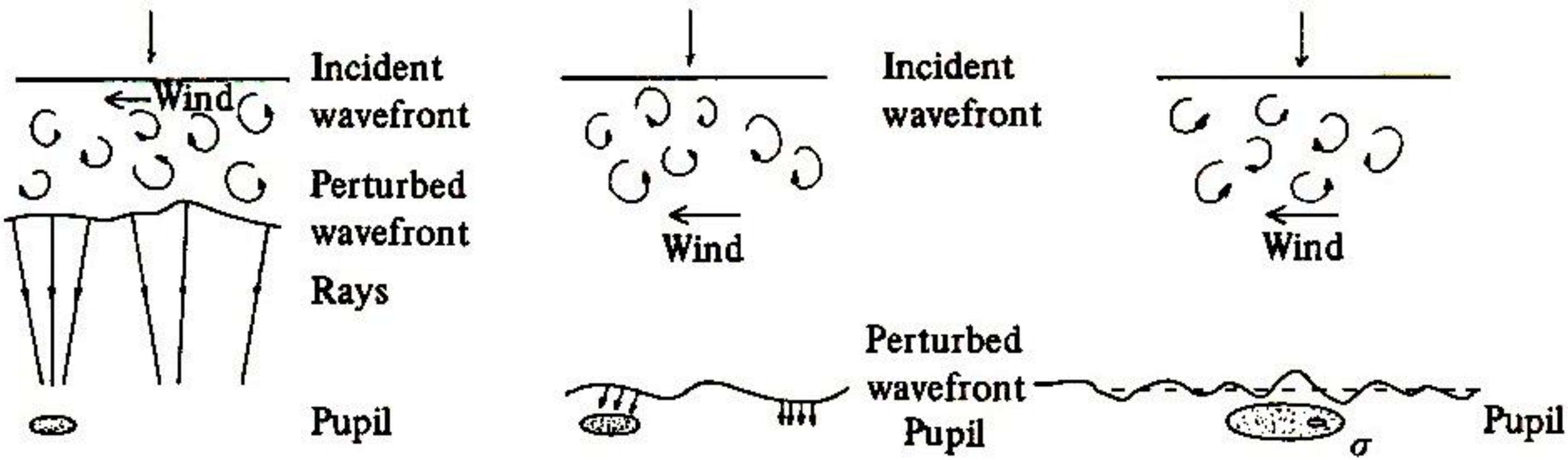
- radius of spatial coherence area (of wavefront) given by Fried parameter r_0 :

$$r_0(\lambda) = 0.185\lambda^{6/5}\left[\int_0^\infty C_n^2(z)dz\right]^{-3/5}$$



- r_0 increases as 6/5 power of wavelength
- r_0 is average scale over which rms optical phase distortion is 1 rad
- angle $\Delta\theta=\lambda/r_0$ is the seeing in arcsec
- r_0 at good sites: 10 - 30 cm (at 0.5µm)
- seeing is roughly equivalent to FWHM of long-exposure image of a point source (Point Spread Function)

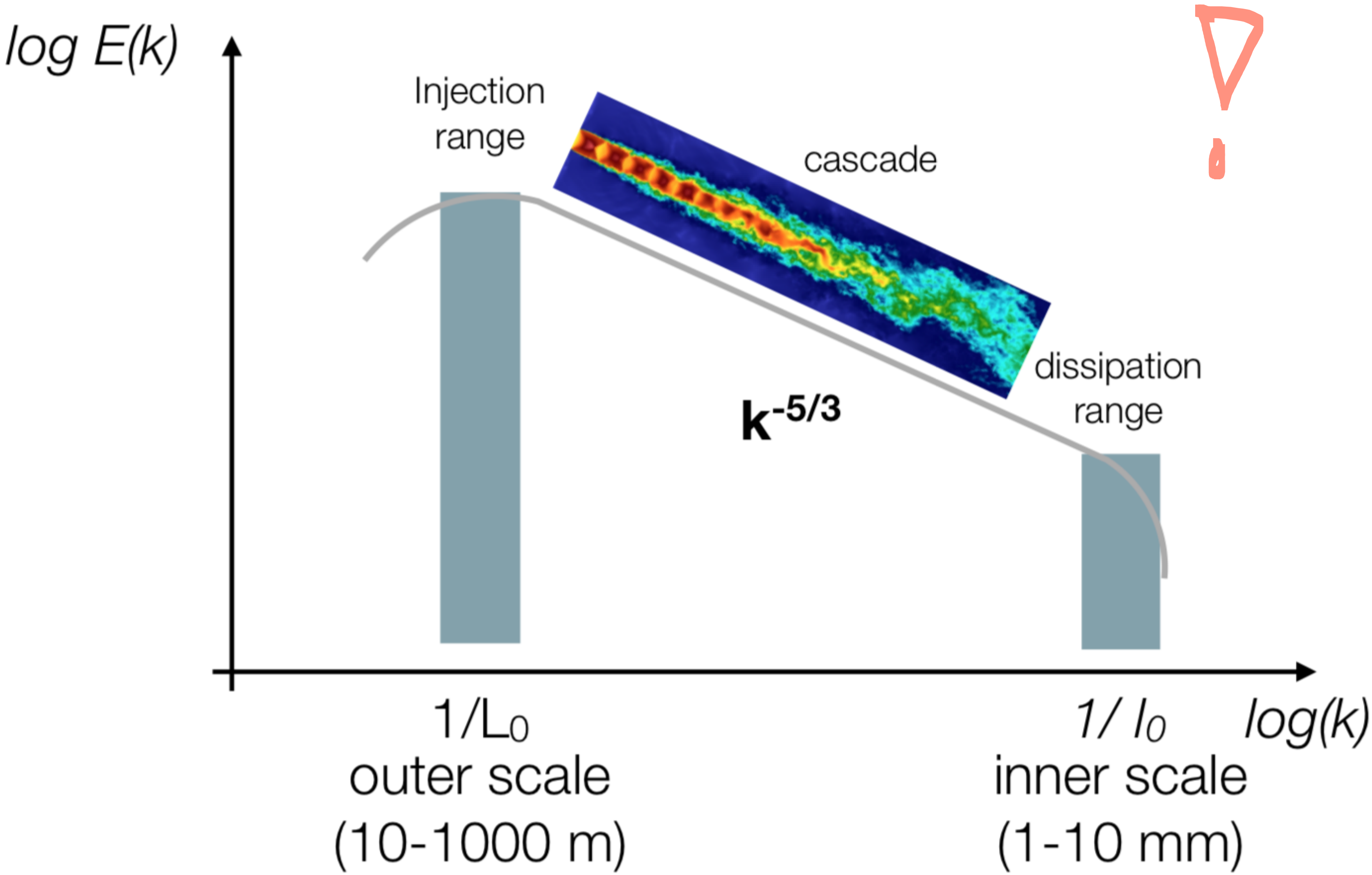
Aspects of Image Degradation



Scintillation
energy received by pupil varies in time (stars flicker)

Image Motion
average slope of wavefront at pupil varies ("tip-tilt", stars move around)

Image Blurring
wavefront is not flat ("seeing")



http://www.astro.caltech.edu/~dmawet/teaching/ay122a_atmosphericphysics-3.pdf

Air Refractive Index Fluctuations

- winds mix layers of different temperature → fluctuations of temperature T → fluctuations of density ρ → fluctuations of refractive index n
- 1K temperature difference changes n by 1×10^{-6}
- variation of 0.01K along path of 10km:
  10^4 m × 10^{-8} = 10^{-4} m = 100 waves at 1μm
- refractive index of water vapour is less than that of air → moist air has smaller refractive index

Long Exposure through Turbulence

- atmospheric coherence time:
  maximum time delay for RMS wavefront error to be less than 1 rad (v is mean wind velocity)

$$\tau_0 = 0.314 \frac{r_0}{\bar{v}}$$

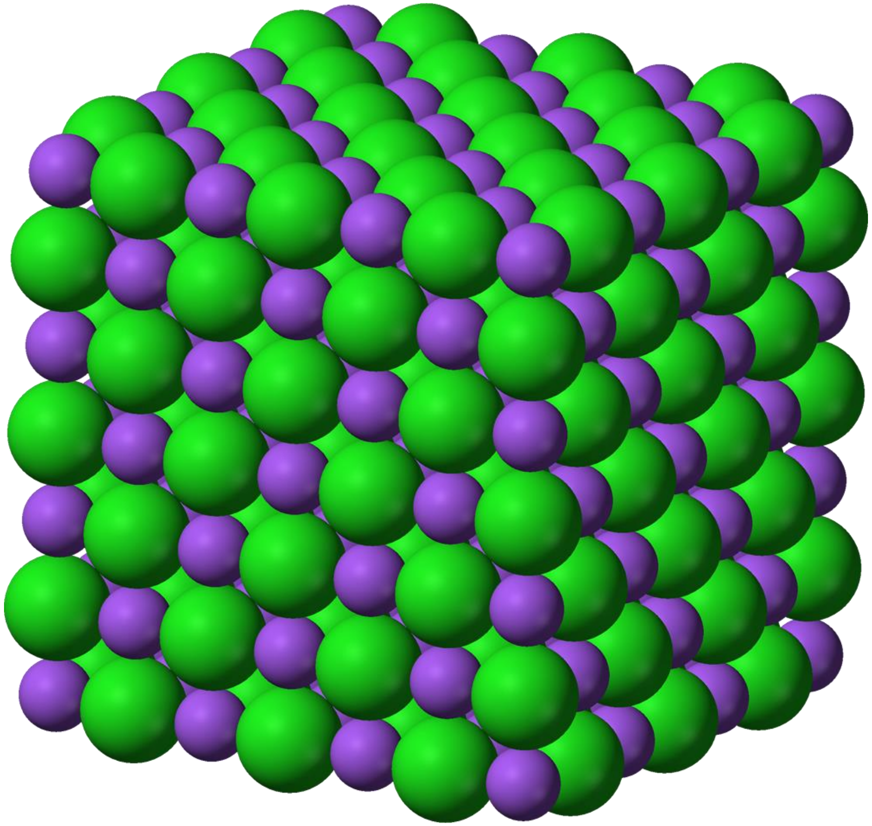
- integration time $t_{int} >> \tau_0$
  → image is mean of many instantaneous images
- angular resolution ~λ/r_0 instead of ~λ/D
- for D > r_0, bigger telescopes will *not* provide sharper images!

Lecture 9: Silicon Eyes 1

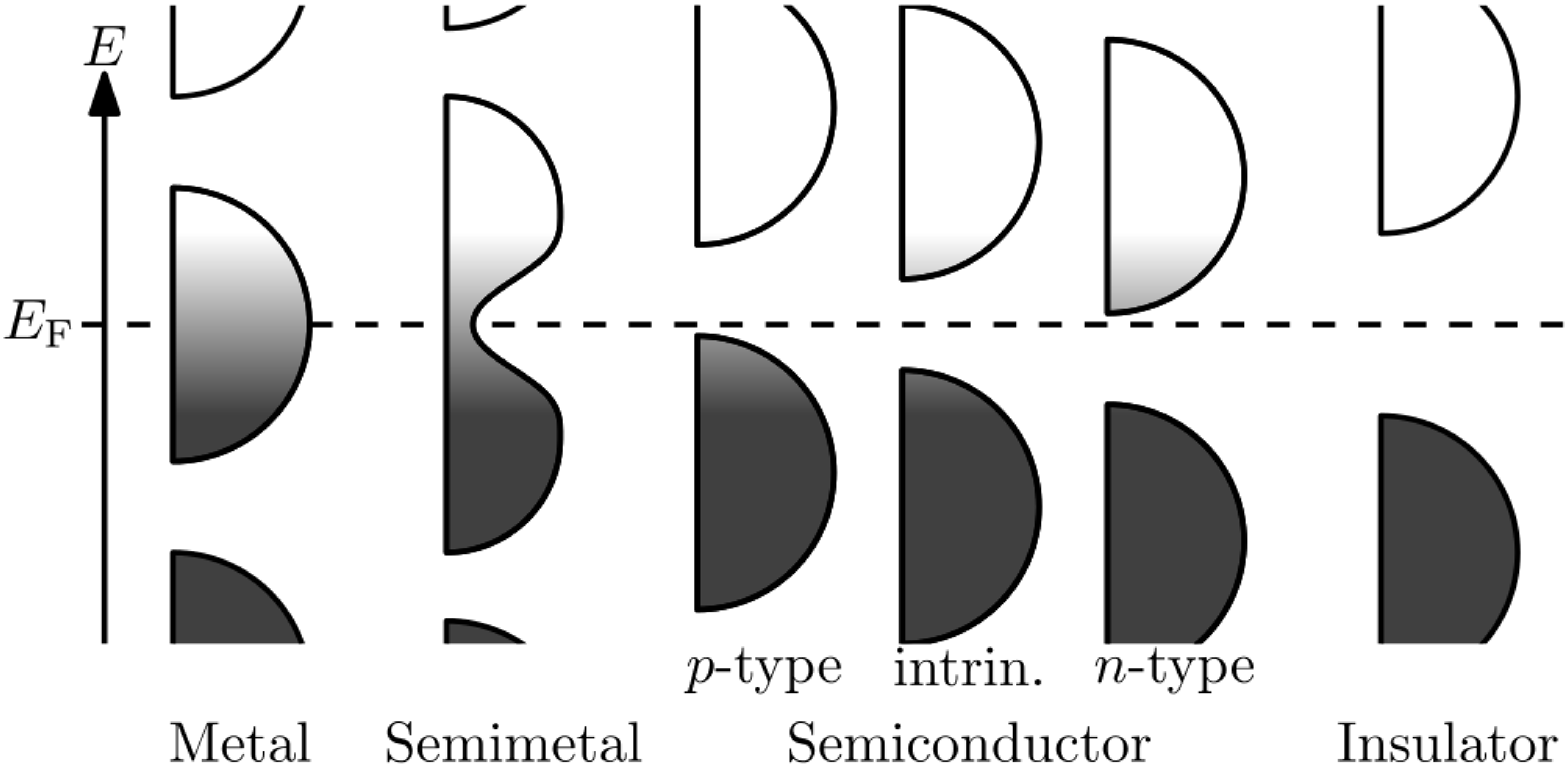
1. Physical principles
   – Periodic table, Covalent Bond, Crystal lattices, Electronic Bands, Fermi Energy and Fermi Function, Electric Conductivity, Band Gap and Conduction Band
2. Intrinsic Photoconductors
   – Photo-Current
3. Extrinsic Photoconductors
   – Depletion Zone
4. Photodiodes
5. Charge Coupled Devices

Crystal Lattice

- crystals: periodic arrangement of atoms, ions or molecules
- smallest group of atoms that repeats is unit cell
- unit cells repeat at lattice points
- crystal structure and symmetry determine many physical properties

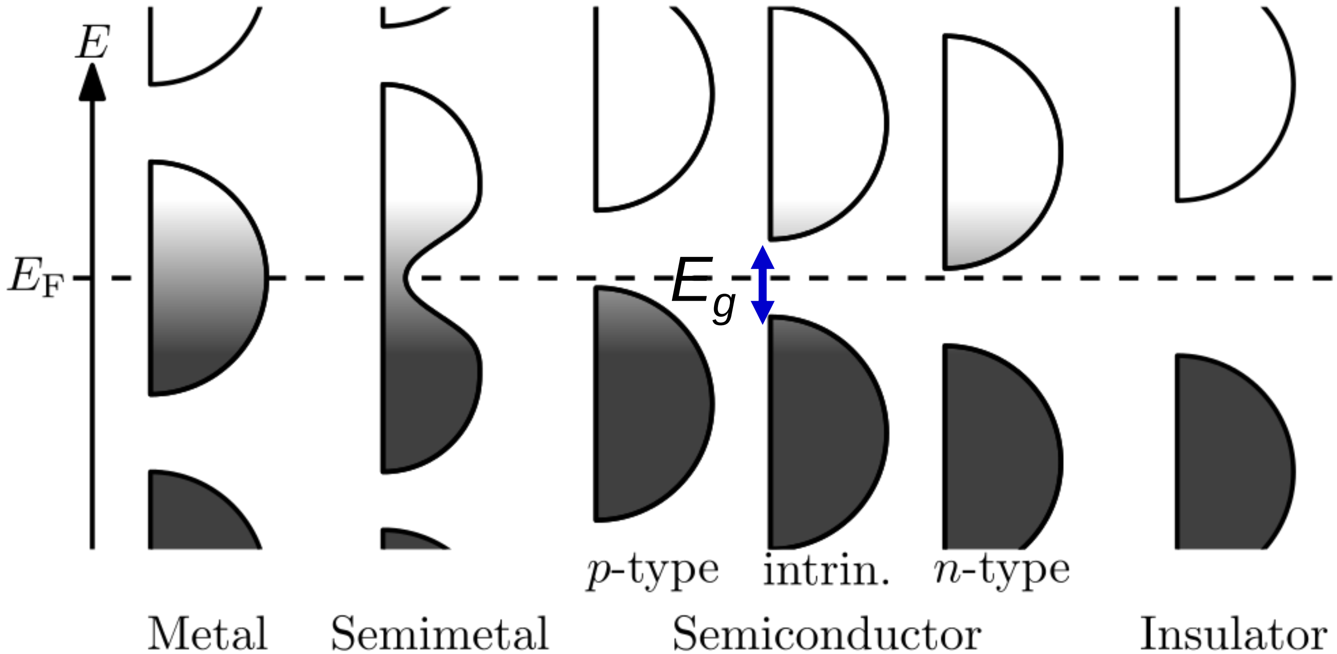


purple: Na^+, green: Cl^-



Filling of the electronic states in various types of materials at equilibrium. Here, height is energy while width is the density of available states for a certain energy in the material listed. The shade follows the Fermi–Dirac distribution (**black** = all states filled, **white** = no state filled). In metals and semimetals the Fermi level E_F lies inside at least one band. In insulators and semiconductors the Fermi level is inside a band gap; however, in semiconductors the bands are near enough to the Fermi level to be thermally populated with electrons or holes.

Overcoming the Bandgap



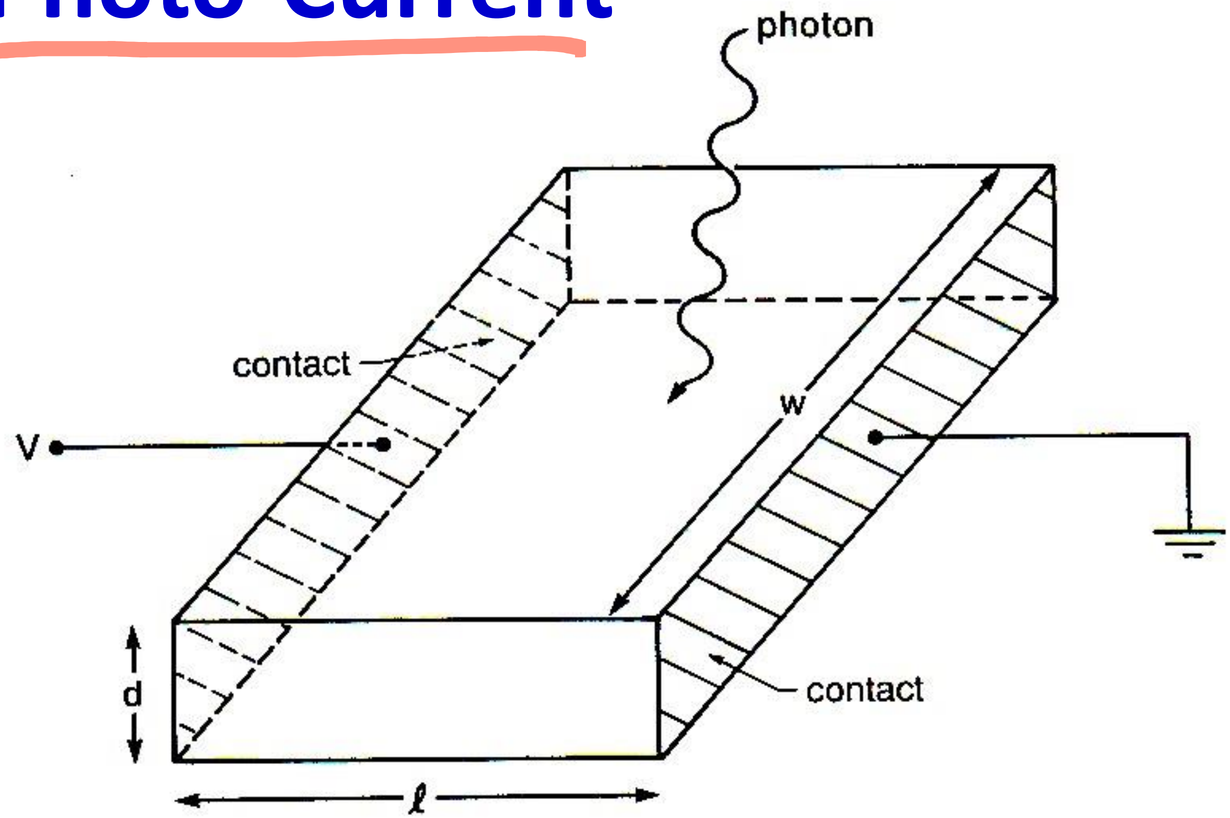
Overcome bandgap E_g by lifting e^- into conduction band:

1. external excitation, e.g. via a photon ←photon detector
2. thermal excitation
3. impurities

Photo-Current

$$\sigma = \frac{1}{R_d}\frac{l}{wd} = qn_0\mu_n$$

$$n_0 = \frac{j\,ht}{wdl}$$



where:

q = elementary charge

n_0 = number density of charge carriers

φ = photon flux

η = quantum efficiency

τ = mean lifetime before recombination

μ_n = electron mobility; drift velocity $v=\mu_n E$

Important Quantities and Definitions

Quantum efficiency η = $\dfrac{\text{\# absorbed photons}}{\text{\# incoming photons}}$

Responsivity S = $\dfrac{\text{electrical output signal}}{\text{input photon power}}$

Wavelength cutoff: $\lambda_c = \dfrac{hc}{E_g} = \dfrac{1.24\mu m}{E_g[eV]}$ E_g:Energy bandgap

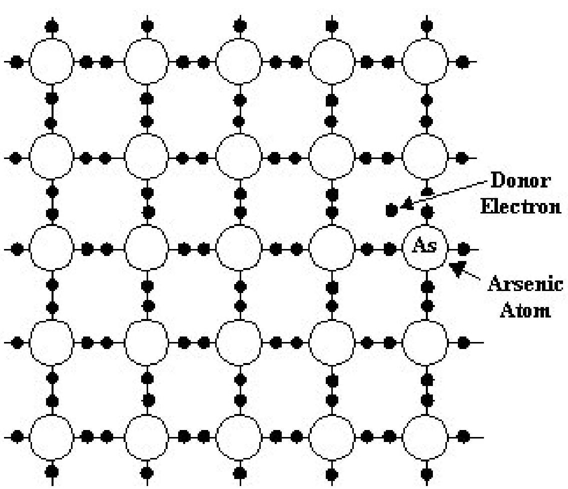
Photo-current: $I_{ph} = q\varphi\eta G$

Photoconductive gain G: $G = \dfrac{I_{ph}}{q\varphi\eta} = \dfrac{\tau}{\tau_t} = \dfrac{\text{carrier lifetime}}{\text{transit time}}$

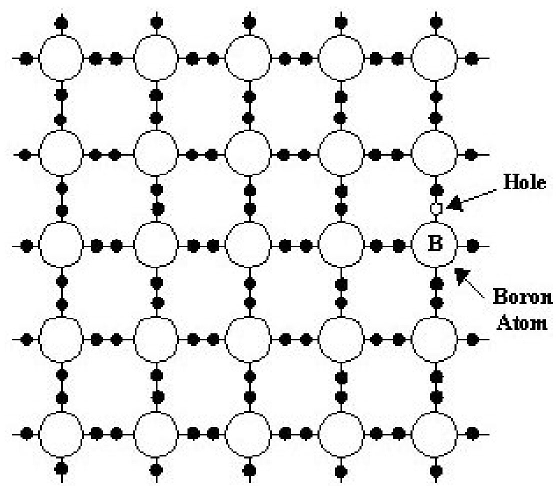
The product ηG describes the probability that an incoming photon will produce an electric charge that will reach an electrode

Extrinsic Semiconductors

- extrinsic semiconductors:
  charge carriers = electrons (n-type) or holes (p-type)
- addition of impurities at low concentration to provide excess electrons or holes
- much reduced bandgap -> longer wavelength cutoff



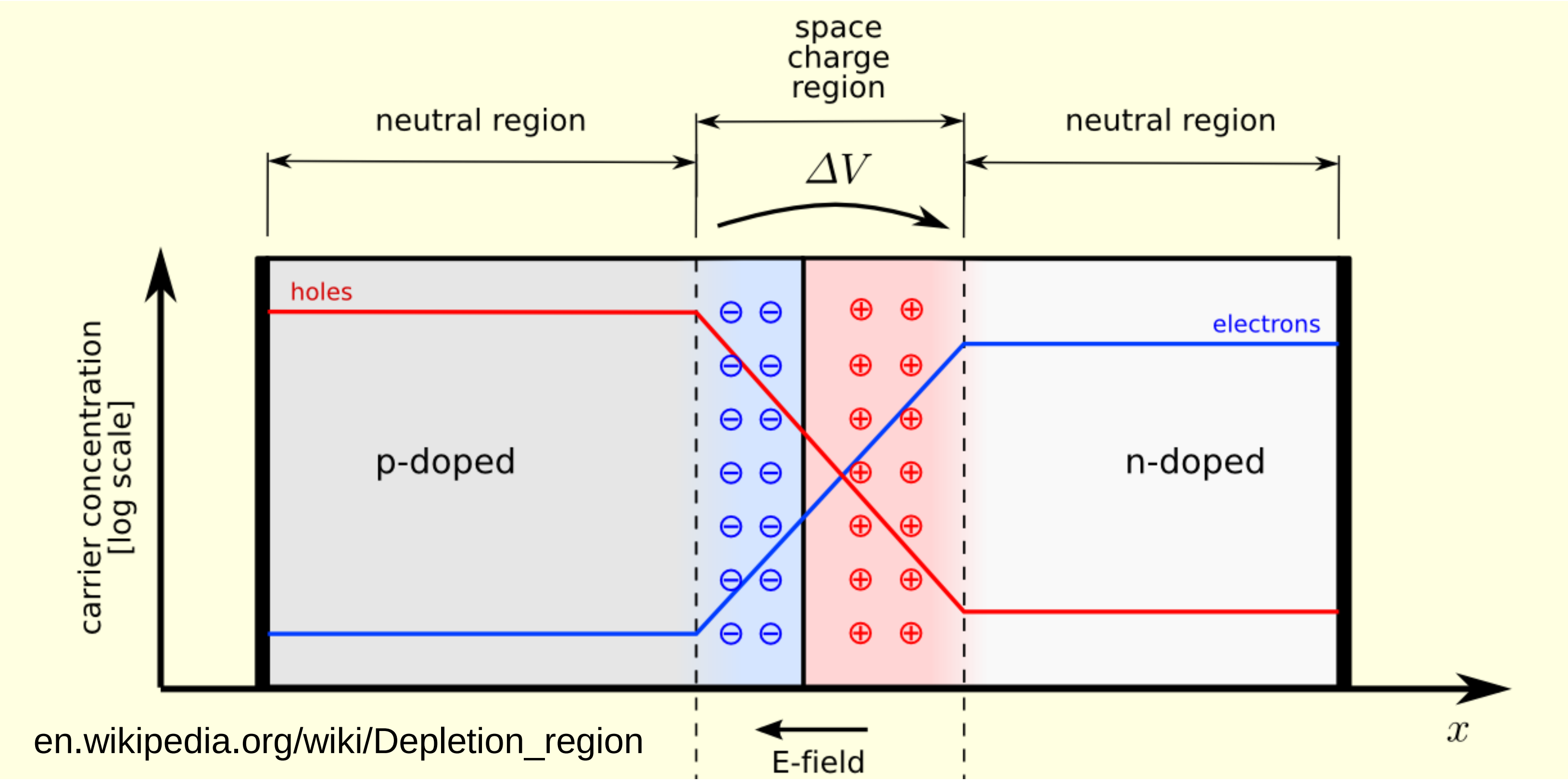
N-type silicon



P-type silicon

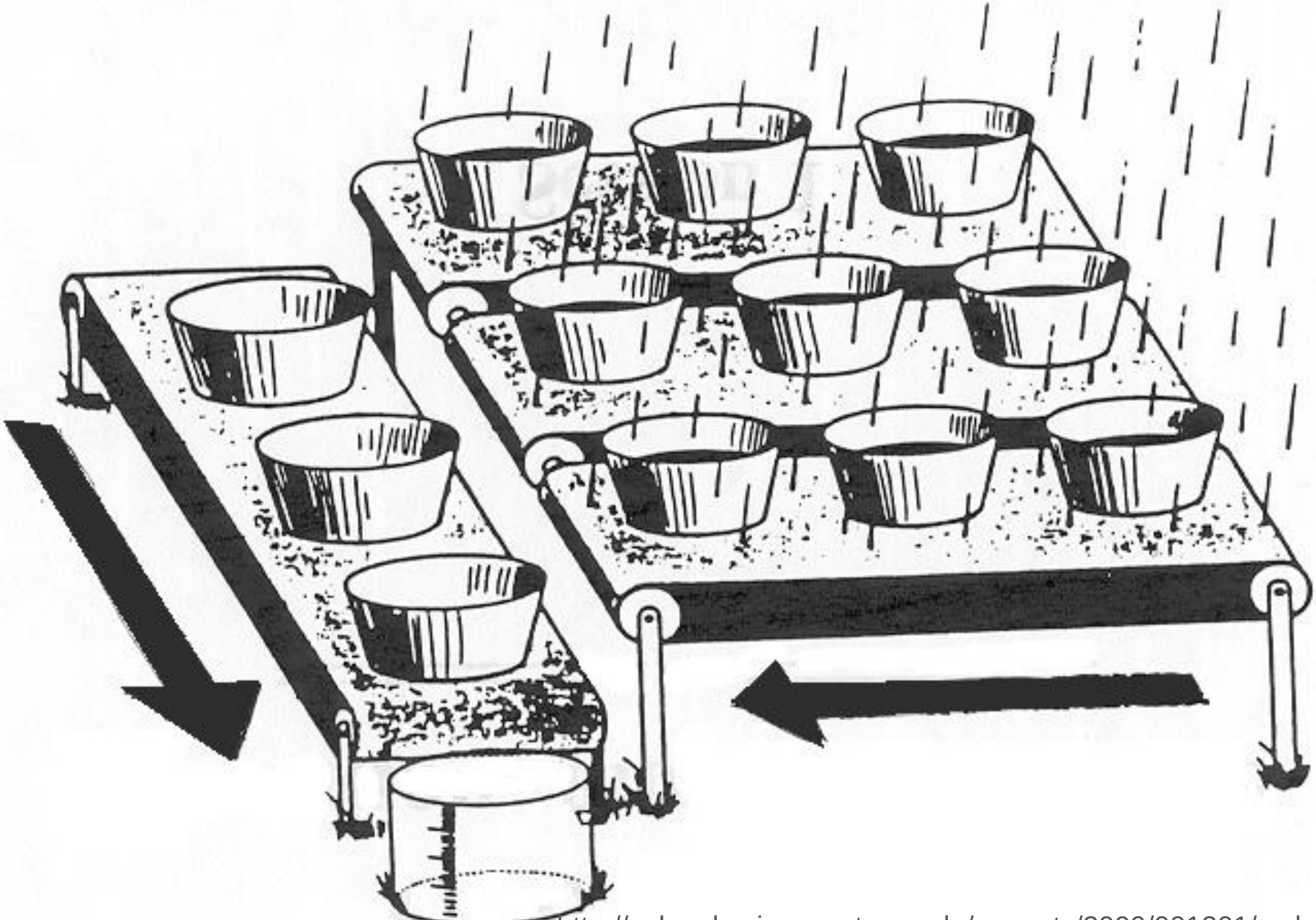
Example: addition of boron to silicon in the ratio 1:100,000 increases its conductivity by a factor of 1000!

PN Junction + Depletion Zone



en.wikipedia.org/wiki/Depletion_region

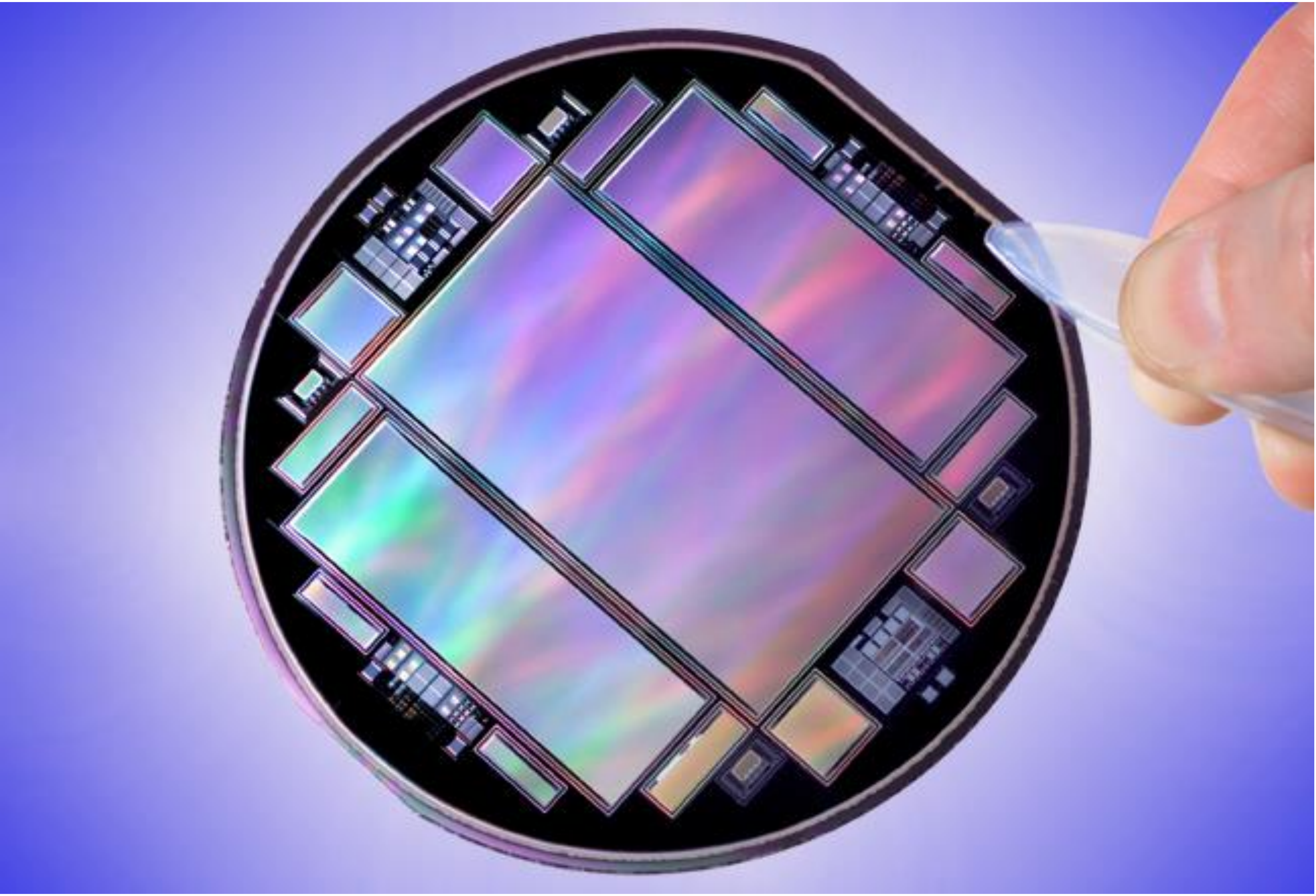
- junction between p- and n-doped Si (both are electrically neutral)
- e^- migrate to P-side, holes migrate to N-side
- e^- can only flow over large distances in n-type material, holes can only flow in p-type material



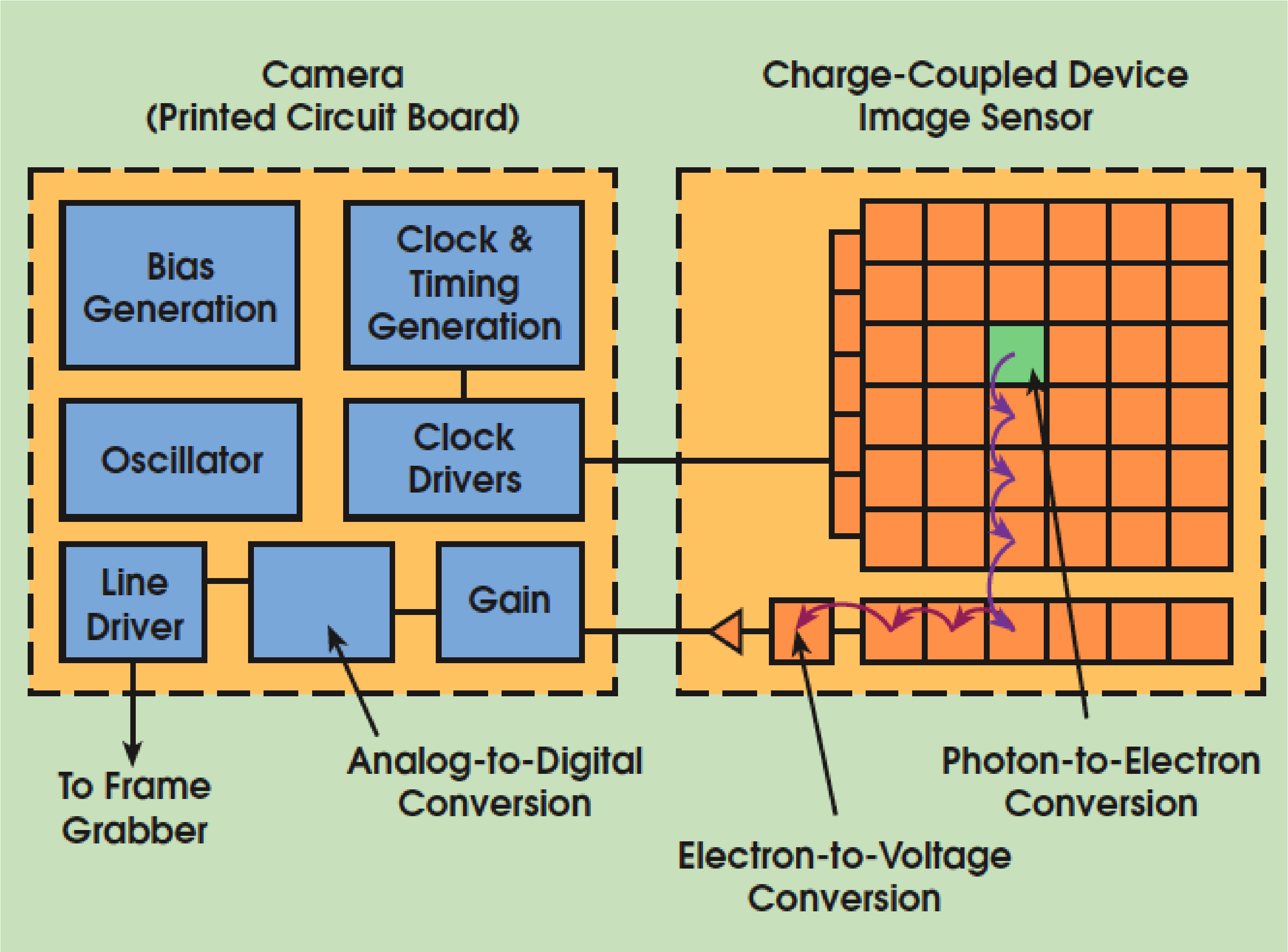
http://solar.physics.montana.edu/nuggets/2000/001201/ccd.png

Lecture 10: Silicon Eyes 2

1. CCDs
2. CCD Data Reduction
3. CMOS
4. Infrared Arrays
5. Chopping and Nodding
6. Detector Artefacts
7. Bolometers



CCD Camera



Camera
(Printed Circuit Board)
Charge-Coupled Device
Image Sensor
Bias Generation
Clock & Timing Generation
Oscillator
Clock Drivers
Line Driver
Gain
To Frame Grabber
Analog-to-Digital Conversion
Electron-to-Voltage Conversion
Photon-to-Electron Conversion

- Dynamic range: ratio of maximum to minimum measurable signal
  - maximum number of events in a CCD pixel is determined by photoelectron "full well" capacity or digitization maximum (typically 2 bytes);
  - minimum is determined by dark current/readout noise

  Applies to a single exposure; effective dynamic range can be increased with multiple exposures

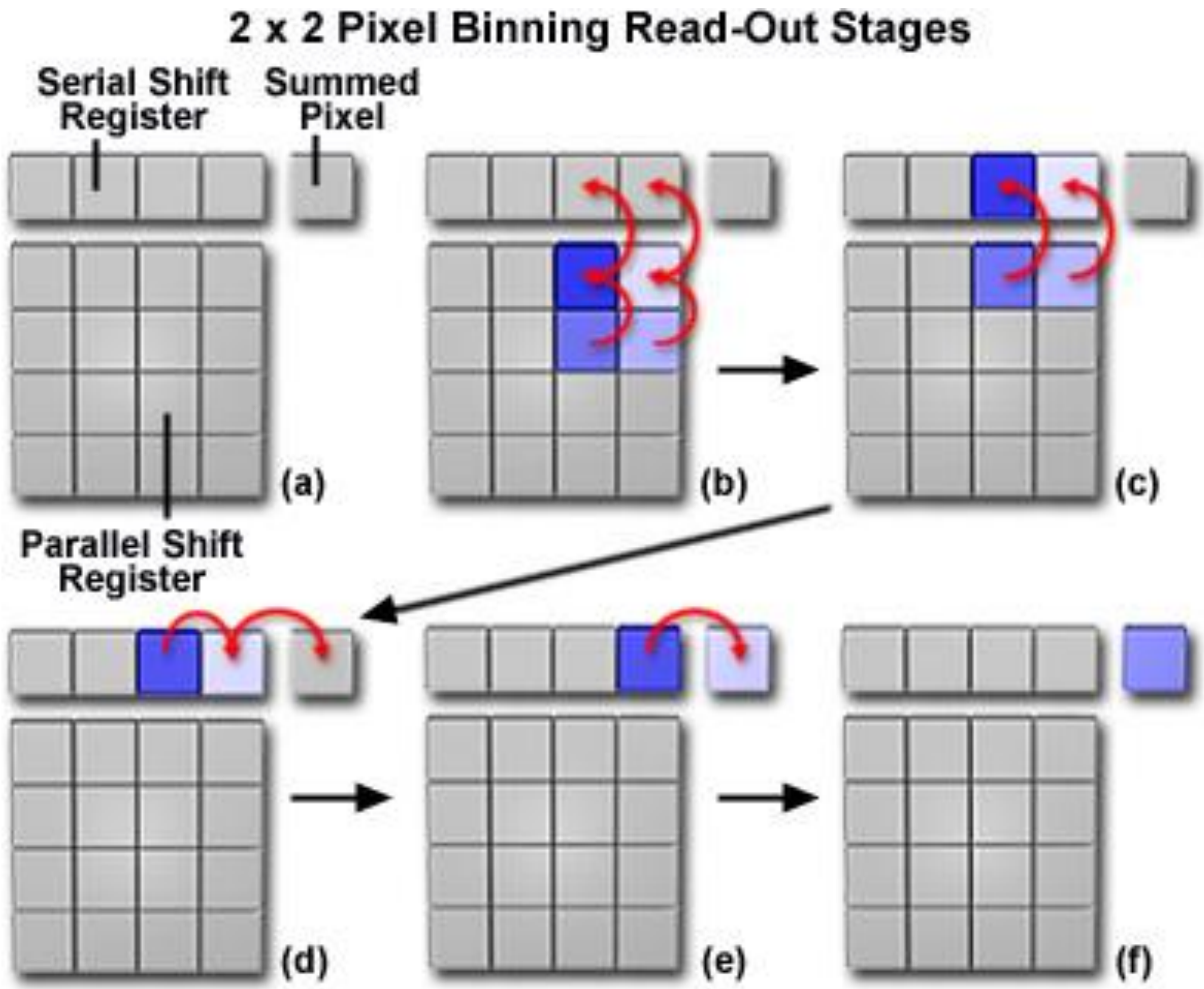
- Linear Range: range of signals for which [Output] = k x [Input], where k is a constant. Generally smaller than the calibratable range

- Threshold: minimum measurable signal determined by sky or detector noise

- Saturation point: level where detector response ceases to increase with the signal

- Readout noise: main origins:
  - on-chip amplifier translating charge (electrons) to analog voltage
  - wires between on-chip amplifier and analog-to-digital converter acting as antennae

Binning



hamamatsu.magnet.fsu.edu/articles/images/pixelbinningfigure1.jpg

From Photons to Digital Units

- Photons S create electrons according to individual pixel sensitivity (flat field f)
- Finite temperature of detector adds thermally generated electrons (dark current d)
- Voltage offset on Analog-Digital Converter (bias b)
- Hence measured signal:

$$b + d + f \cdot S$$

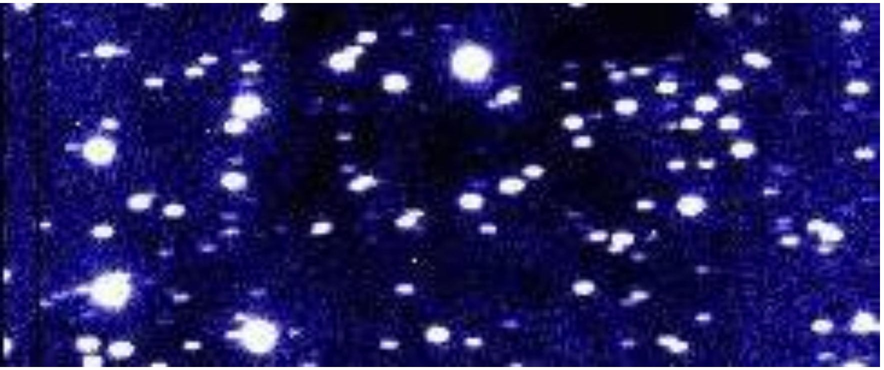
Common Flatfield Methods

1. Dome flats: illuminate a white screen within the dome (can be done during the day, but may introduce spectral artefacts)
2. Twilight flats: observe the twilight sky during sunrise and/or sunset (high S/N but time is often too short to get flatfields for all instrument configurations)
3. Self-calibration: use the observations themselves (e.g. average all data)

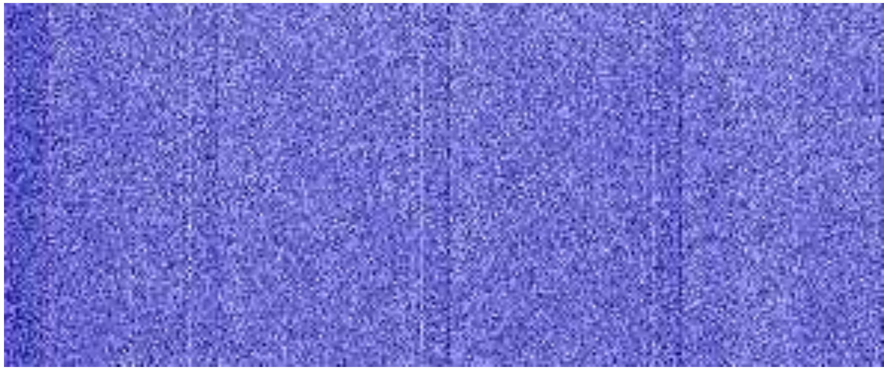
In practice: methods can be combined

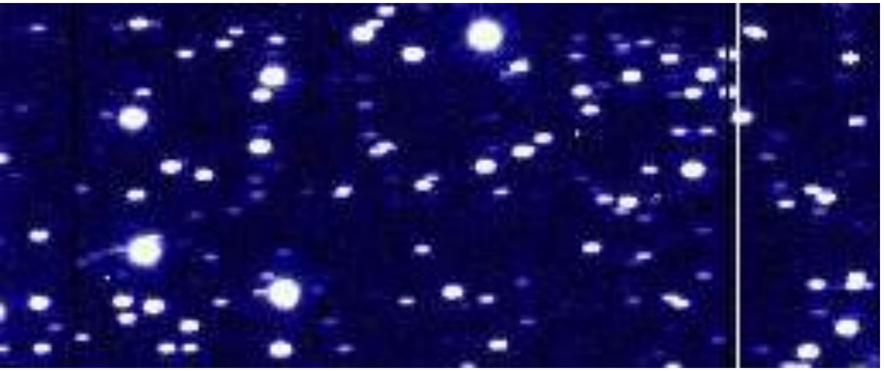
CCD Data Reduction

Raw



Bias + Dark Current



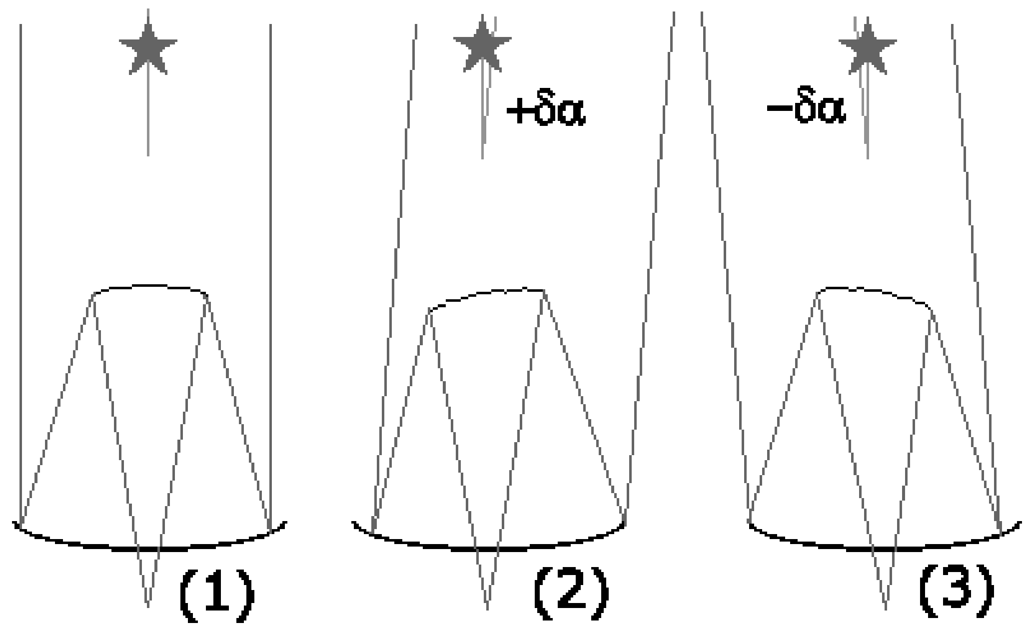


Reduced

Flatfield

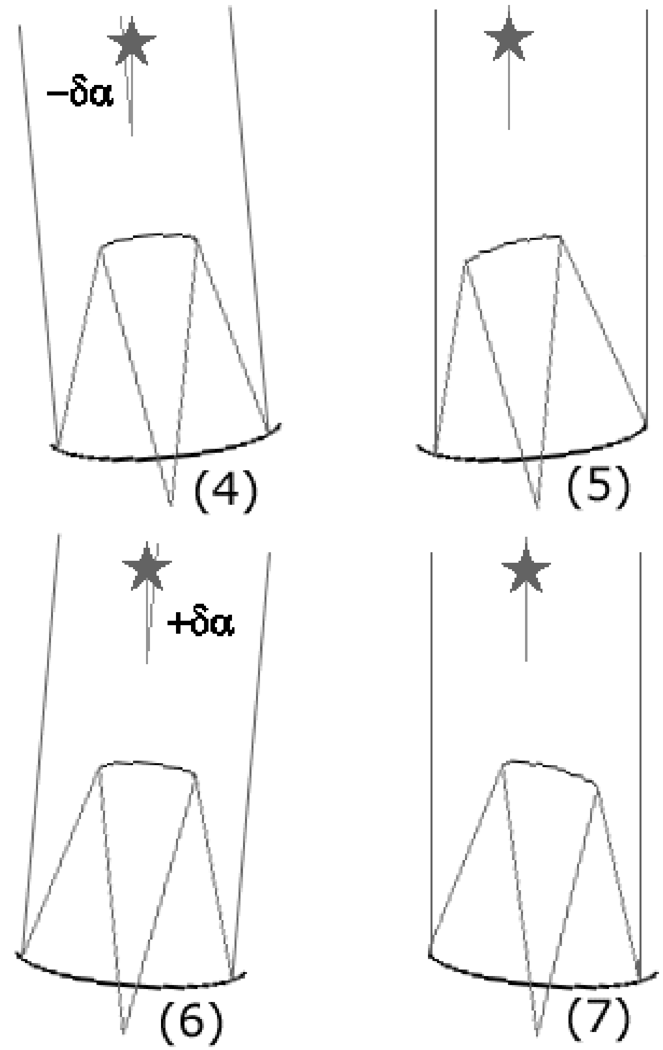
Chopping

- IR images dominated by background signal
- Background subtraction must be very precise
- Move secondary mirror **quickly** to slightly change pointing
- Assumes that background is relatively flat

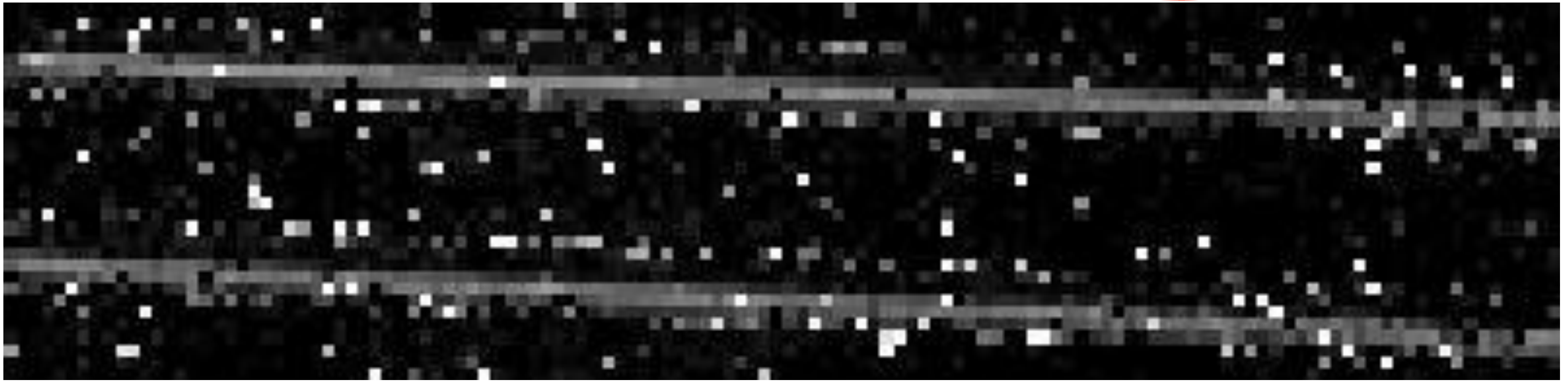


Nodding

- Moving telescope keeps optical path the same
- Nodding is **slow**
- Best of both: combine fast chopping with slow nodding



Detector Artefacts: Bad Pixels

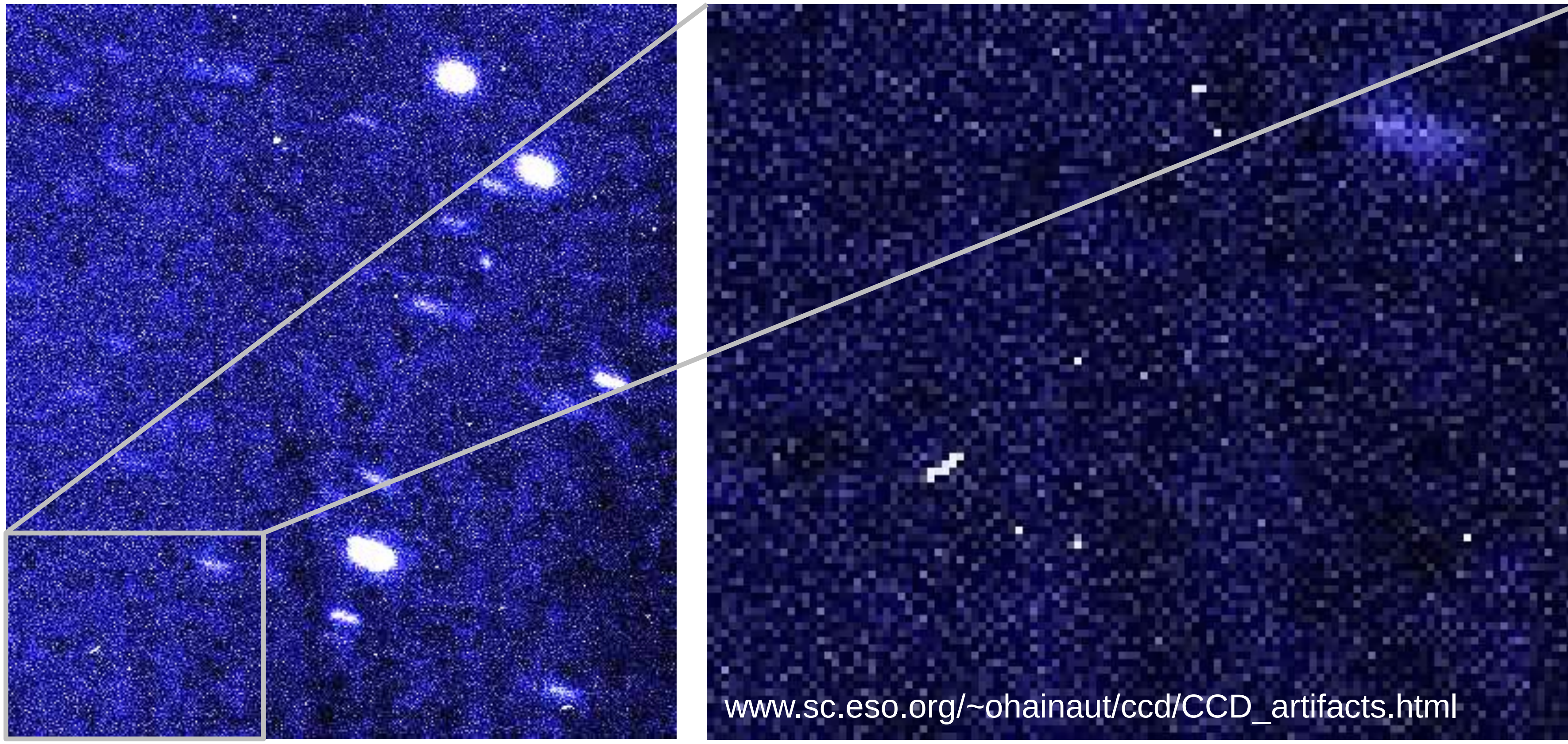


- dead, hot and rogue pixels, rows, columns
- bias and dark correction help somewhat
- can often us “dead-pixel map” and replace with median of surrounding pixels

Detector Artefacts: Latent Images

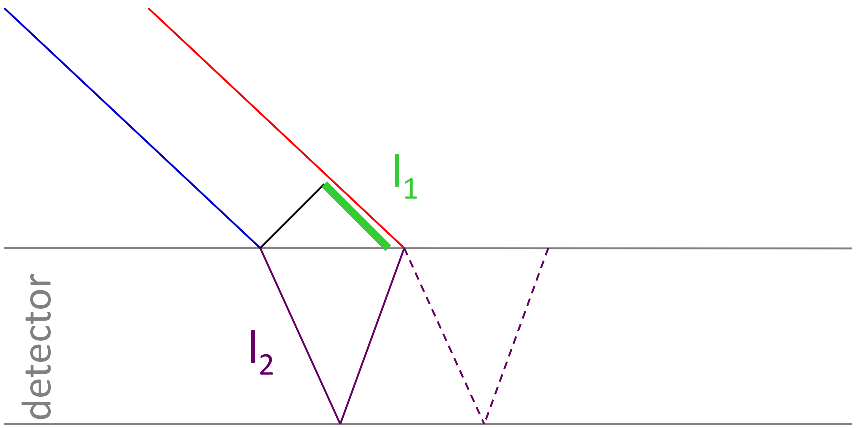
- mostly seen in hybrid (IR) arrays
- Mitigation:
  - avoid overexposure
  - wait
  - additional resets

Detector Artifacts: Cosmic Rays

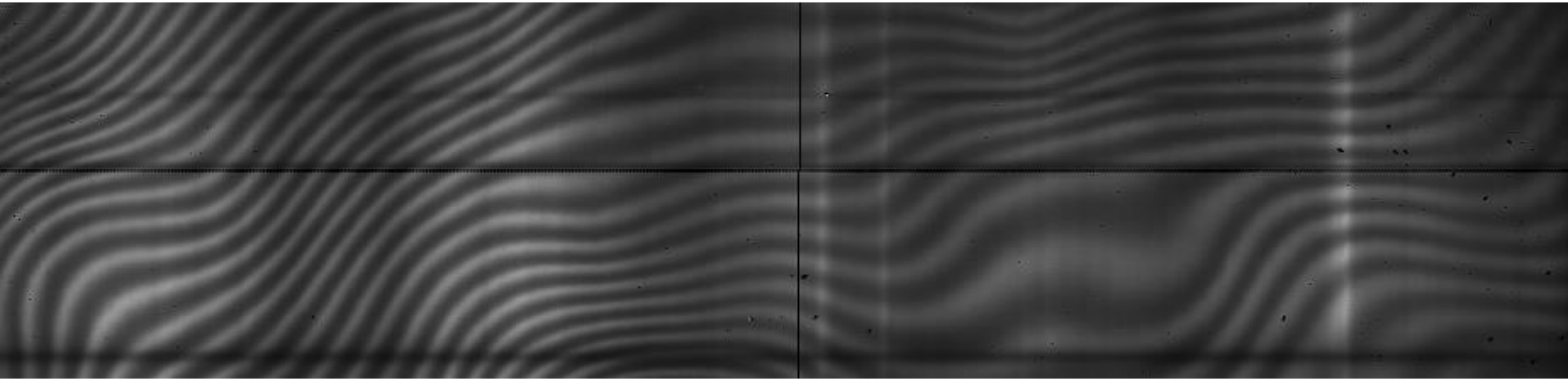


- cosmic ray particles free electrons in detector
- remove with median filtering of several exposures

Detector Artefacts: Fringing

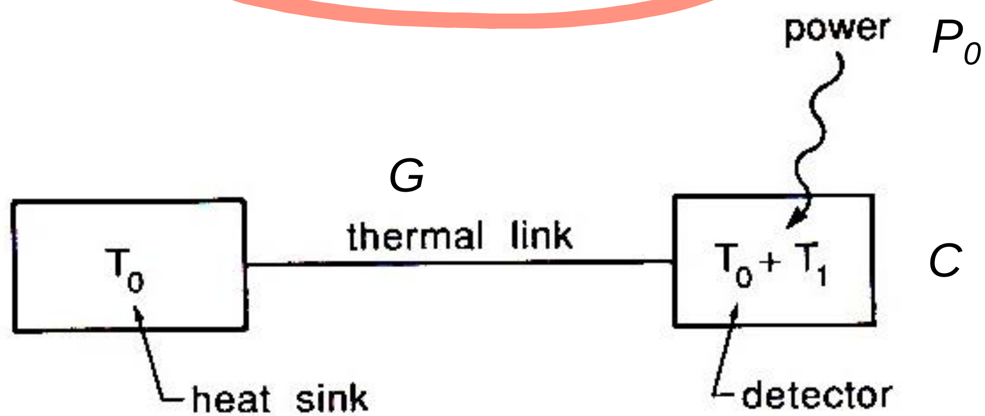


If the phase difference between l_1 and $n \bullet l_2$ is an even multiple of π constructive interference occurs. If an odd multiple destructive interference occurs → fringes = wave pattern.



Detector Artefacts: Blooming

Bolometer

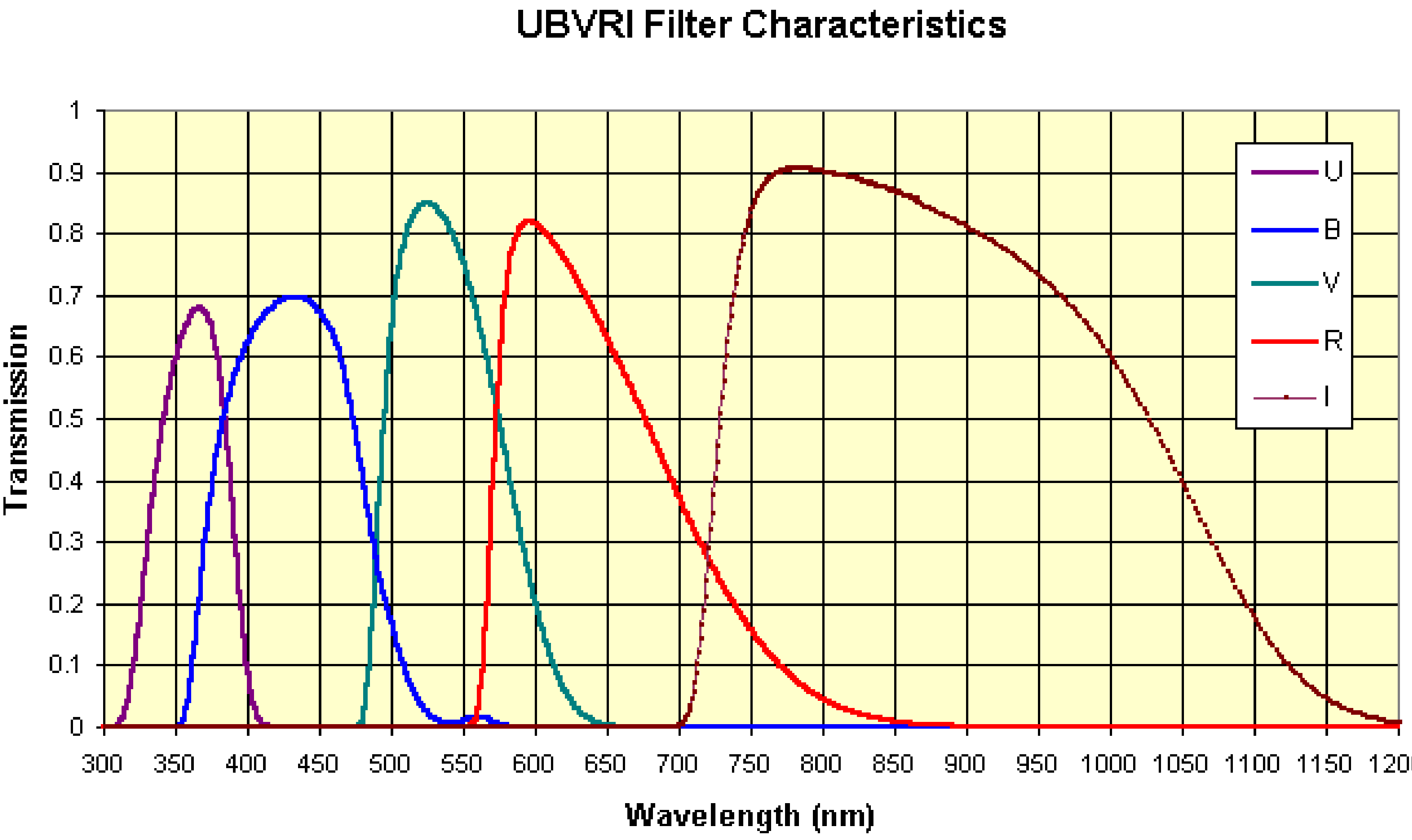


- photon flux power P_0 on detector
- incoming photons increase temperature of detector by T_1
- weak thermal link to heat sink at T_0
- thermal link conductance $G=P_0/T_1$
- steady state energy transfer to heat sink is $P_0=GT_1$

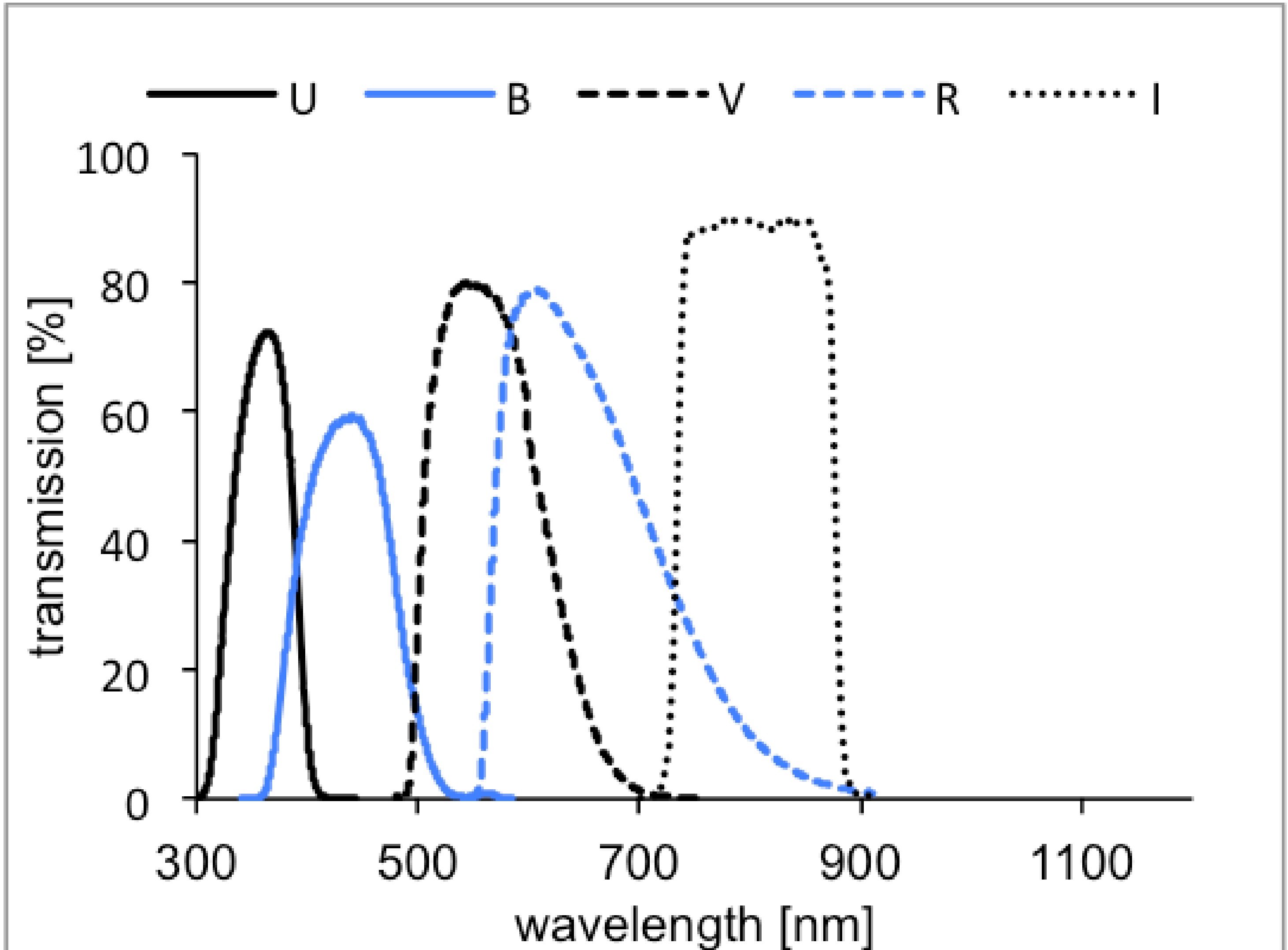
Lecture 11-II:
Pink and other Coloured Glasses

1. Imaging Overview
2. UBVRI Photometry and other filter systems
3. Fabry-Perot Tuneable Filter
4. Interference Filters
5. Photometry
6. Astrometry

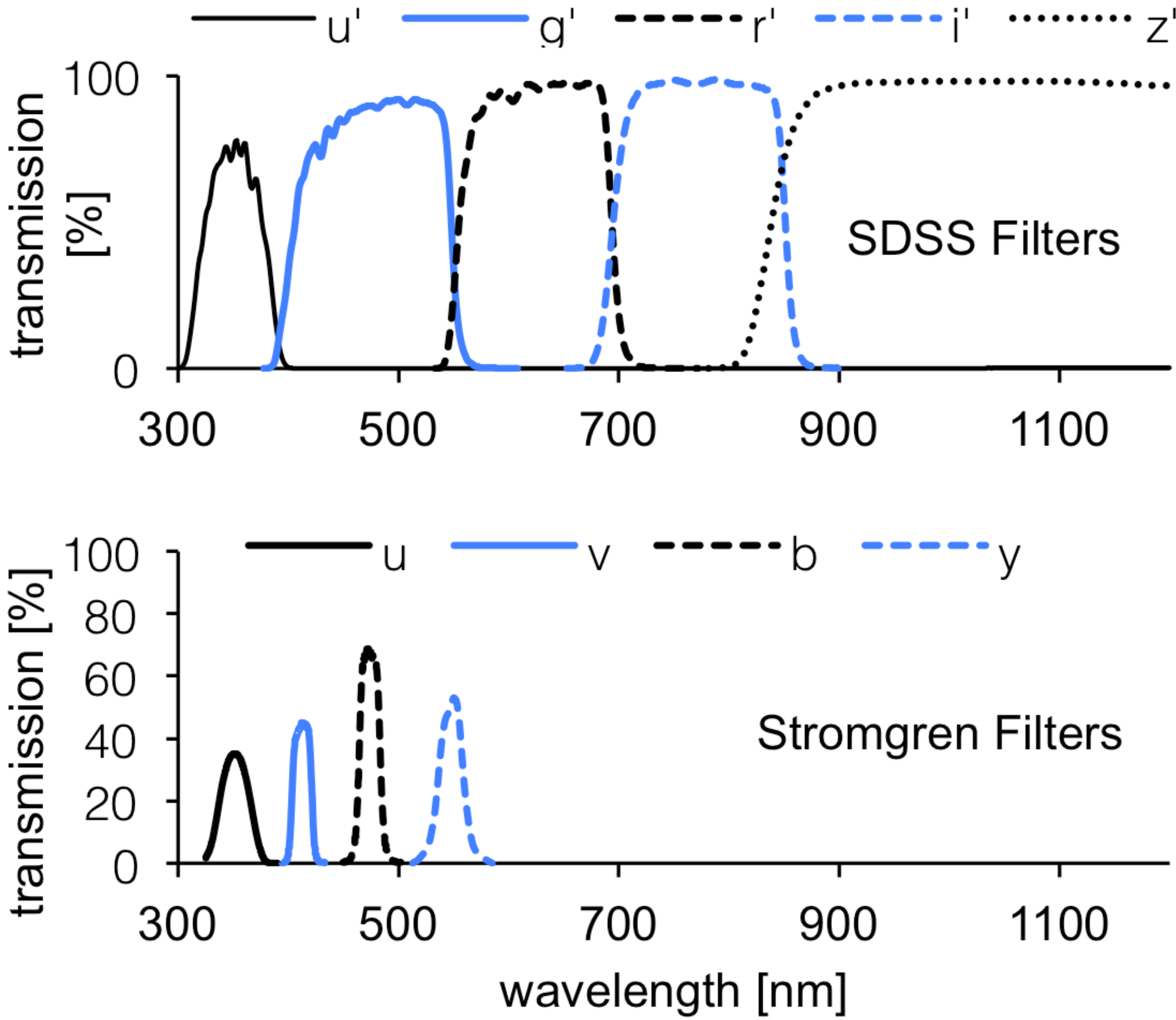
(Old) UBVRI


UBVRI Filter Characteristics
Transmission
Wavelength (nm)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
300
350
400
450
500
550
600
650
700
750
800
850
900
950
1000
1050
1100
1150
1200
U
B
V
R
I

Modern UBVRI Filter Profiles



Other Filter Systems

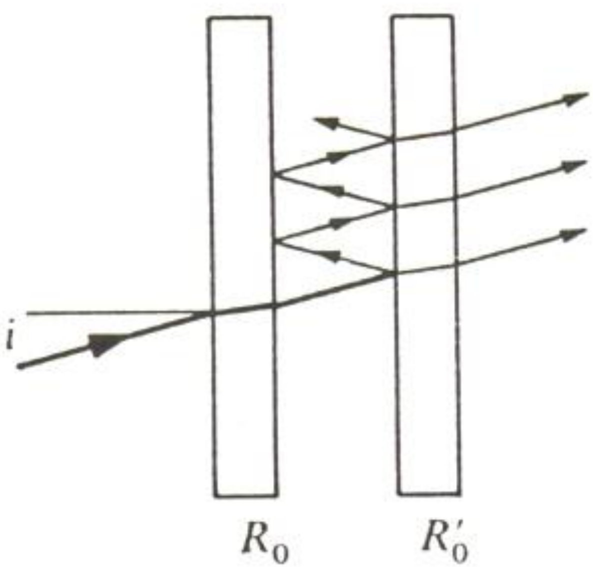


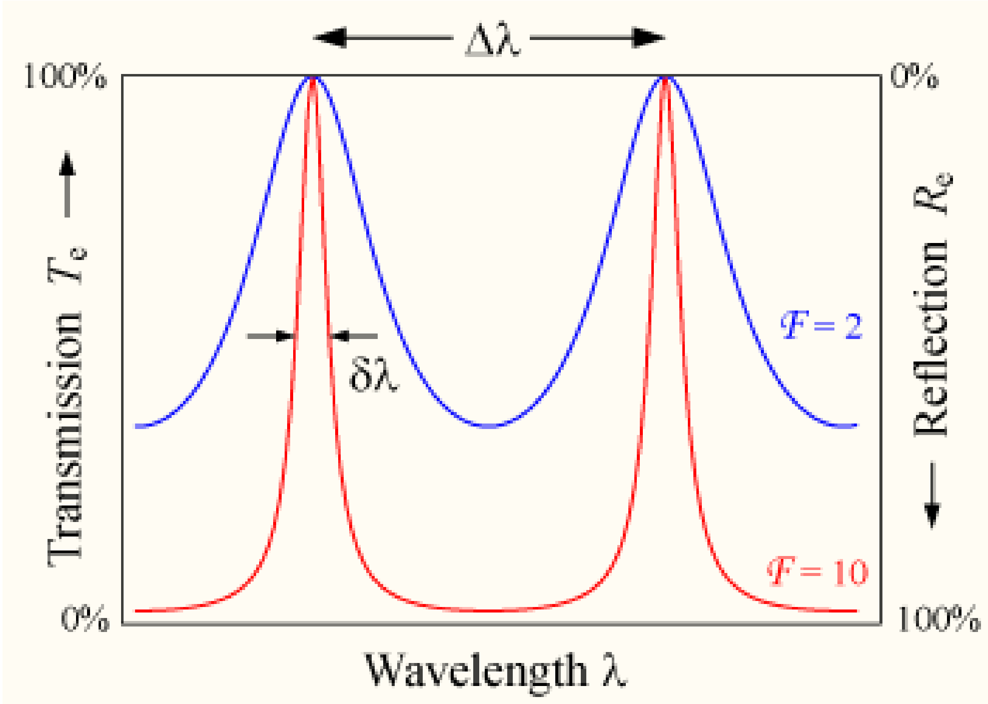
- Sloan Digital Sky Survey (SDSS) for faint galaxy classification

- Stromgren for better sensitivity to stellar properties (metallicity, temperature, surface gravity)

Fabry-Perot Etalon

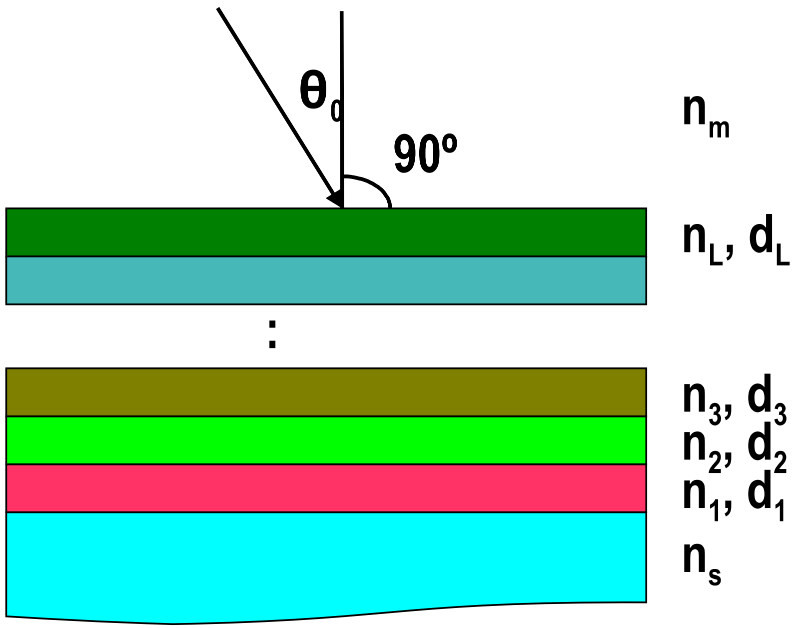
- 2 parallel plates with adjustable plate separation
- highly reflective coatings on inside surfaces
- periodic transmission peaks due to constructive interference of successively reflected beams
- transmission peak location changes linearly with plate separation





Interference Filters

- stack of thin films
- can be tailored to almost any specifications
- sensitive to temperature, humidity, angle of incidence
- tune in wavelength with temperature, angle of incidence
- spectral resolution R ~ 3 – 1000

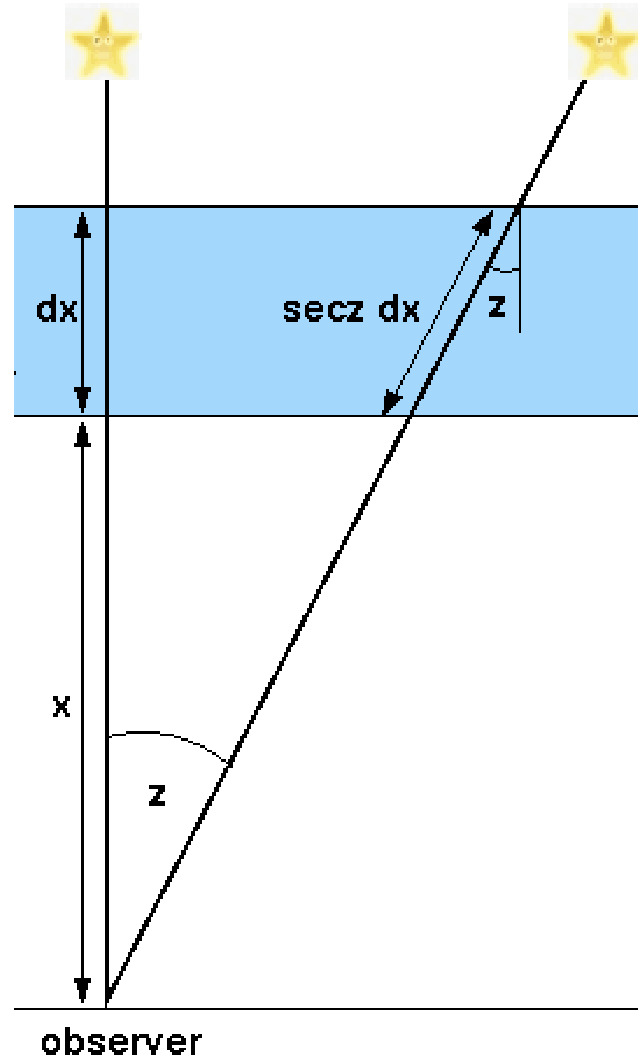


Photometry

- goal: determine flux of an astronomical object in well-defined wavelength range
- problems:
  - seeing
  - extinction
  - sky background
  - telescope, instrument, detector
- calibration with (standard) stars
- all-sky photometry: compare objects all over the sky
- differential photometry: compare objects on same CCD exposure

Air Mass

- extinction due to atmosphere along line-of-sight
- *airmass* = amount of air one looks through
- at zenith ($z=0$): airmass X=1.0
- at zenith distance z, airmass $X \approx \sec z = 1/\cos z$
- $X(z=60°) \approx 2$



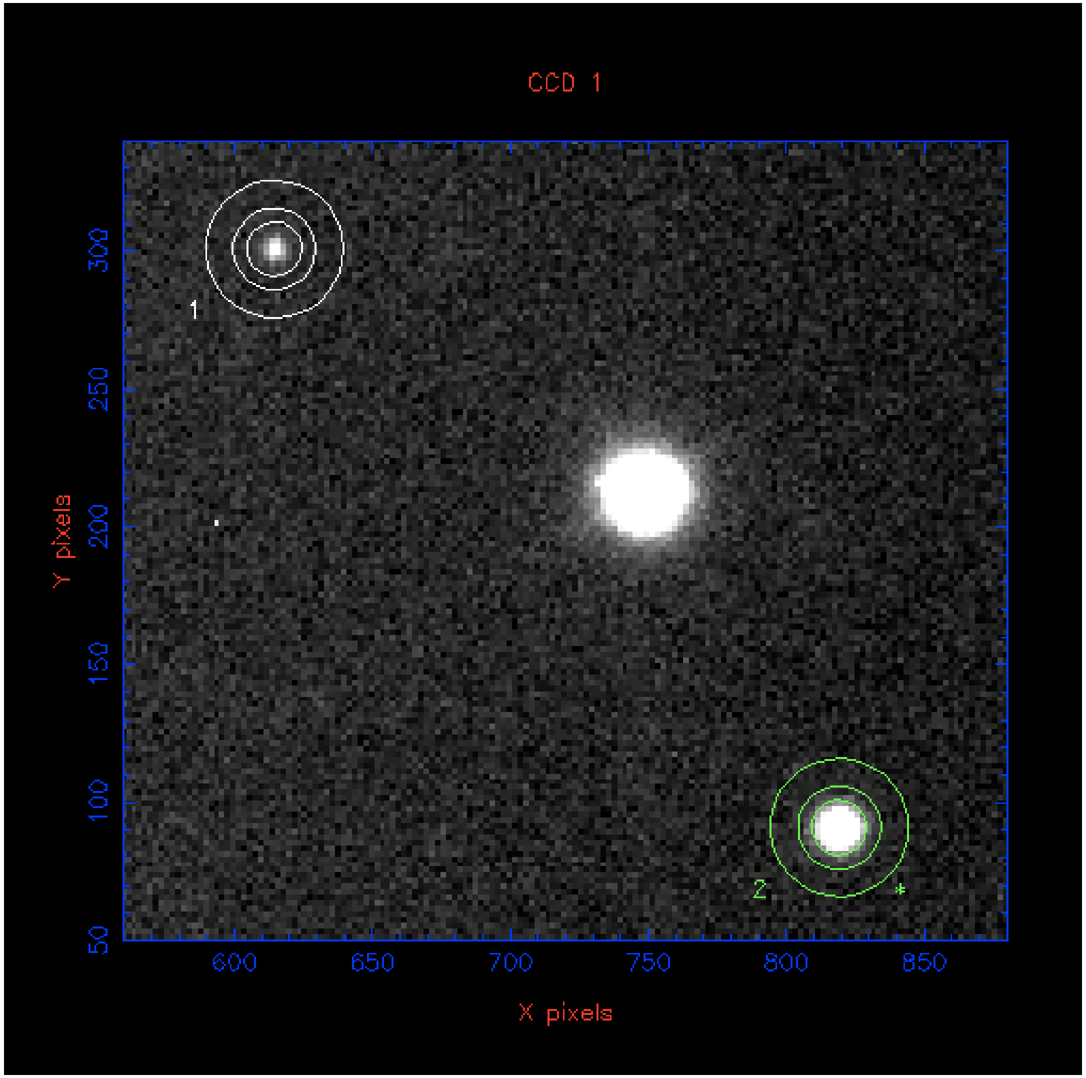
Photometric Observations (1)

- observe object and standard stars with different colors
- observe standard stars at low and high airmass to determine their (color-dependent) extinction coefficients
- reduce images with bias, dark, flat field
- measure fluxes with aperture photometry
- calculate instrumental magnitudes:
  $m_{inst} = -2.5\log(f_i/t_{exp})$

Photometric Observations (2)

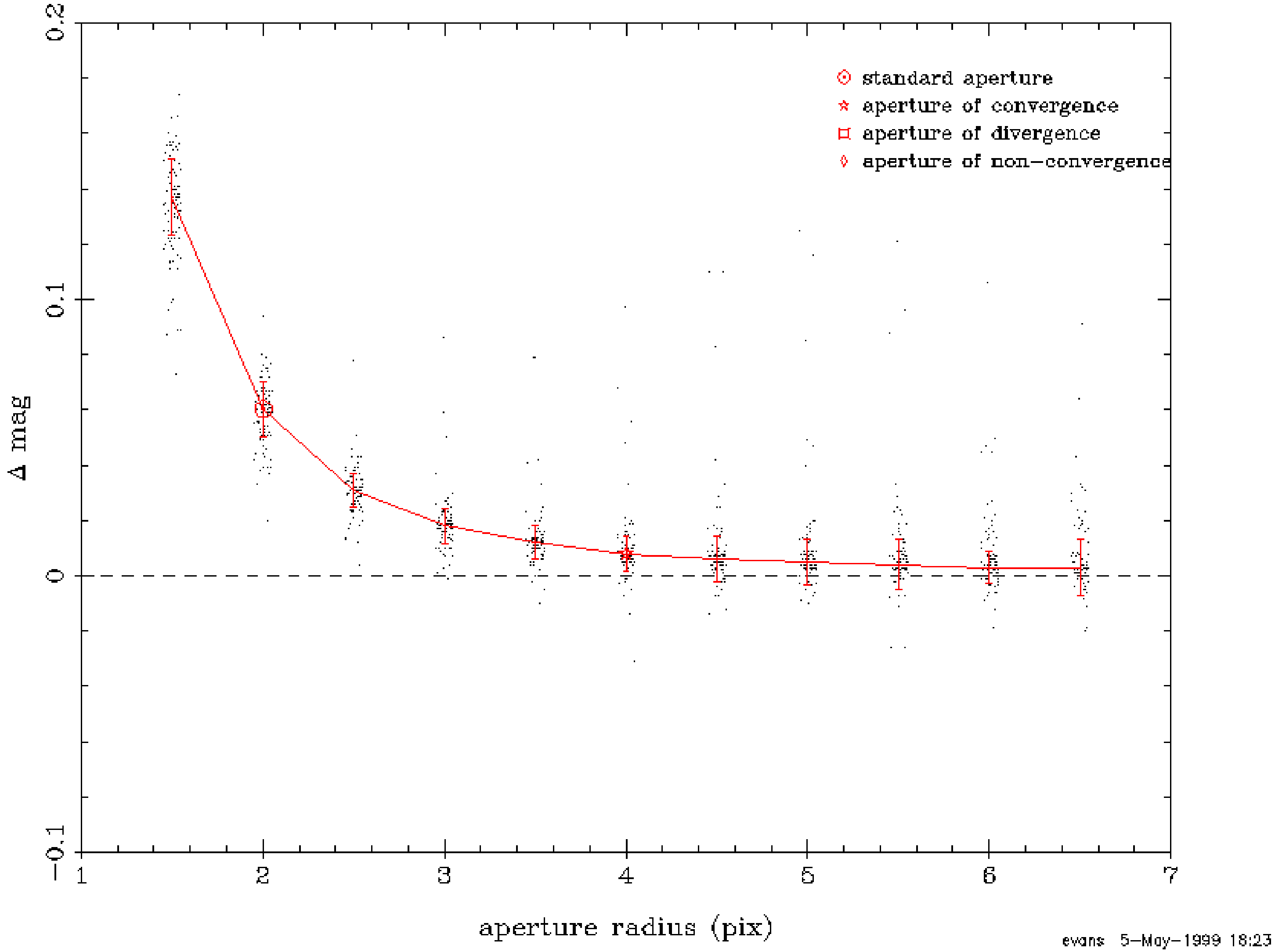
- calibrate instrumental magnitude for zero airmass: $m_0 = m_{inst} - K \sec z$
- zero point from standard stars: $m_{zp} = m_{std} - m_{std,inst}$
- remove zero point: $m = m_{zp} + m_{inst}$
- absolute photometry: determine actual magnitudes with calibrations derived from standard stars
- differential photometry: does not require calibrations

Aperture Photometry



- determine location of star by fitting 2-D Gaussian or Moffat PSF model
- determine and remove sky background

Aperture Photometry Growth Curve



www.ipac.caltech.edu/2mass/releases/allsky/doc/figures/seciv4cf2.gif

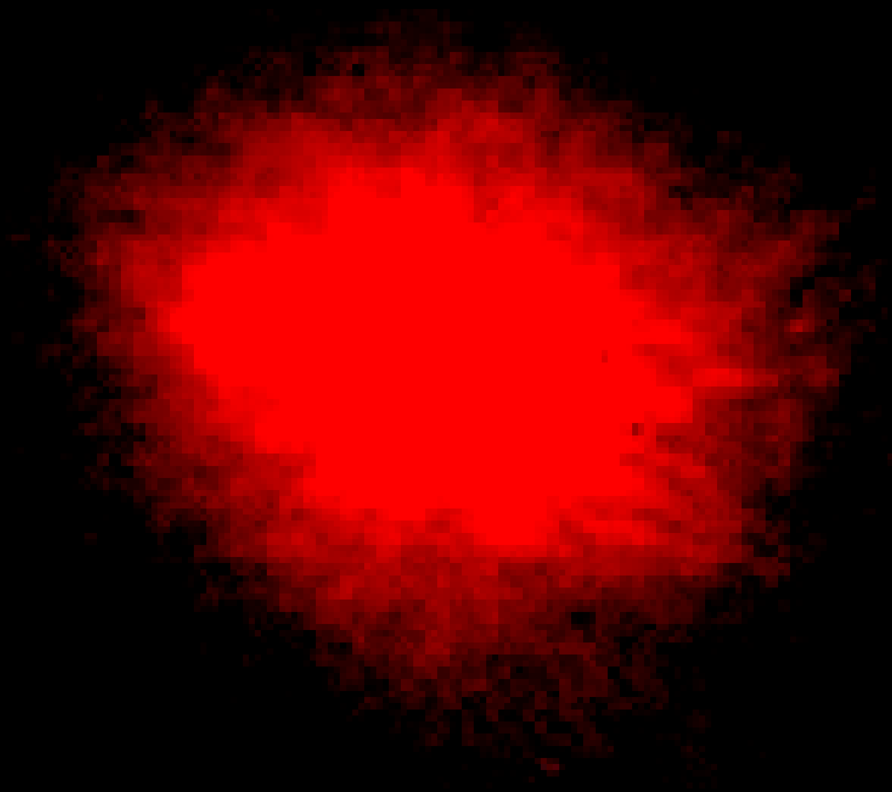
Astrometry

- Determine precise location (and motion of objects)
- Parallax as distance measurement
- Asteroid and comet orbit determination
- Binary masses
- Stellar motions and associations
- Stars around central black hole in Milky Way

Lecture 13:
Twinkle, twinkle little star ... No more!

1. The Power of Adaptive Optics
2. Seeing
3. Resolution & Sensitivity Improvements
4. Adaptive Optics Principles
5. Key Components
6. Wavefront Description
7. Shack Hartmann Wavefront Sensor
8. Other Wavefront Sensors
9. Deformable Mirrors
10. Adaptive Secondary Mirrors
11. Influence Matrix
12. Controls
13. Error Terms
14. Anisoplanatism
15. Laser Guide Stars
16. Adaptive Optics Operations Modes
17. Multi-Conjugate Adaptive Optics
18. Extreme Adaptive Optics

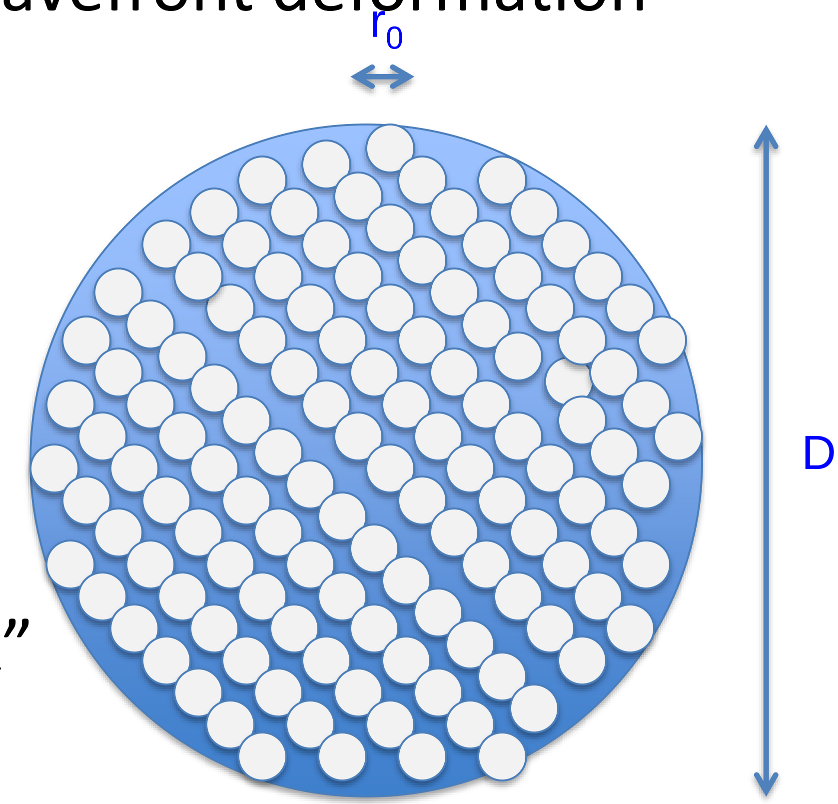
The Power of Adaptive Optics



oldweb.lbto.org/AO/AOpressrelease.htm

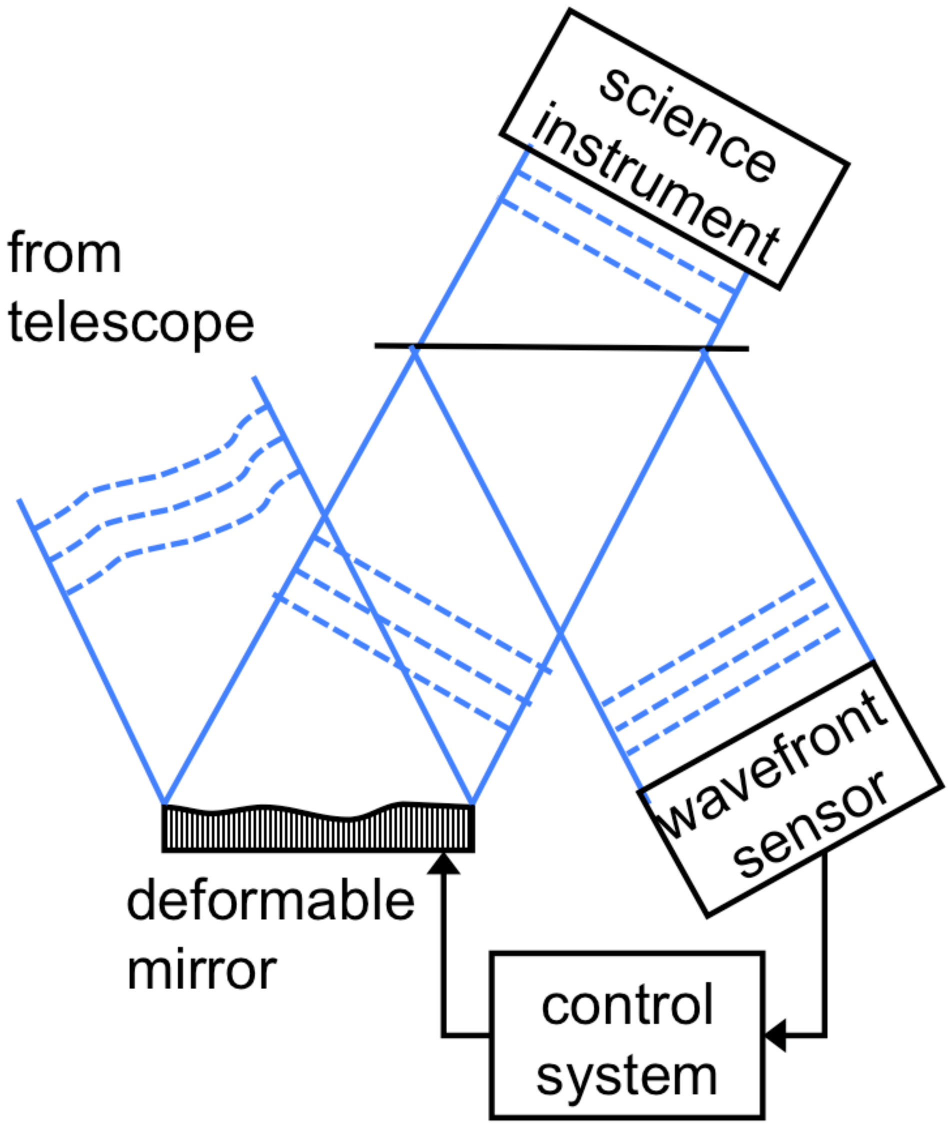
Adaptive Optics Principle

- Maximum scale of tolerated wavefront deformation is r_0 → subdivide telescope into apertures with diameter r_0
- Measure wavefront deformation
- Correct wavefront deformation by "bending back" patches of size r_0



- Number of subapertures is $(D/r_0)^2$ at observing wavelength → requires hundreds to thousands of actuators for large telescopes

Adaptive Optics Scheme



Wavefront Description: Zernike Polynomials

They are a sequence of polynomials $Z_n^m(\rho,\phi)$ that are orthogonal on the unit disk: $\sum a_{m,n} Z_n^m(\rho,\phi)$

There are even and odd Zernike polynomials. The even ones are defined as

$$Z_n^m(\rho,\varphi) = R_n^m(\rho)\,\cos(m\,\varphi)$$

and the odd ones as

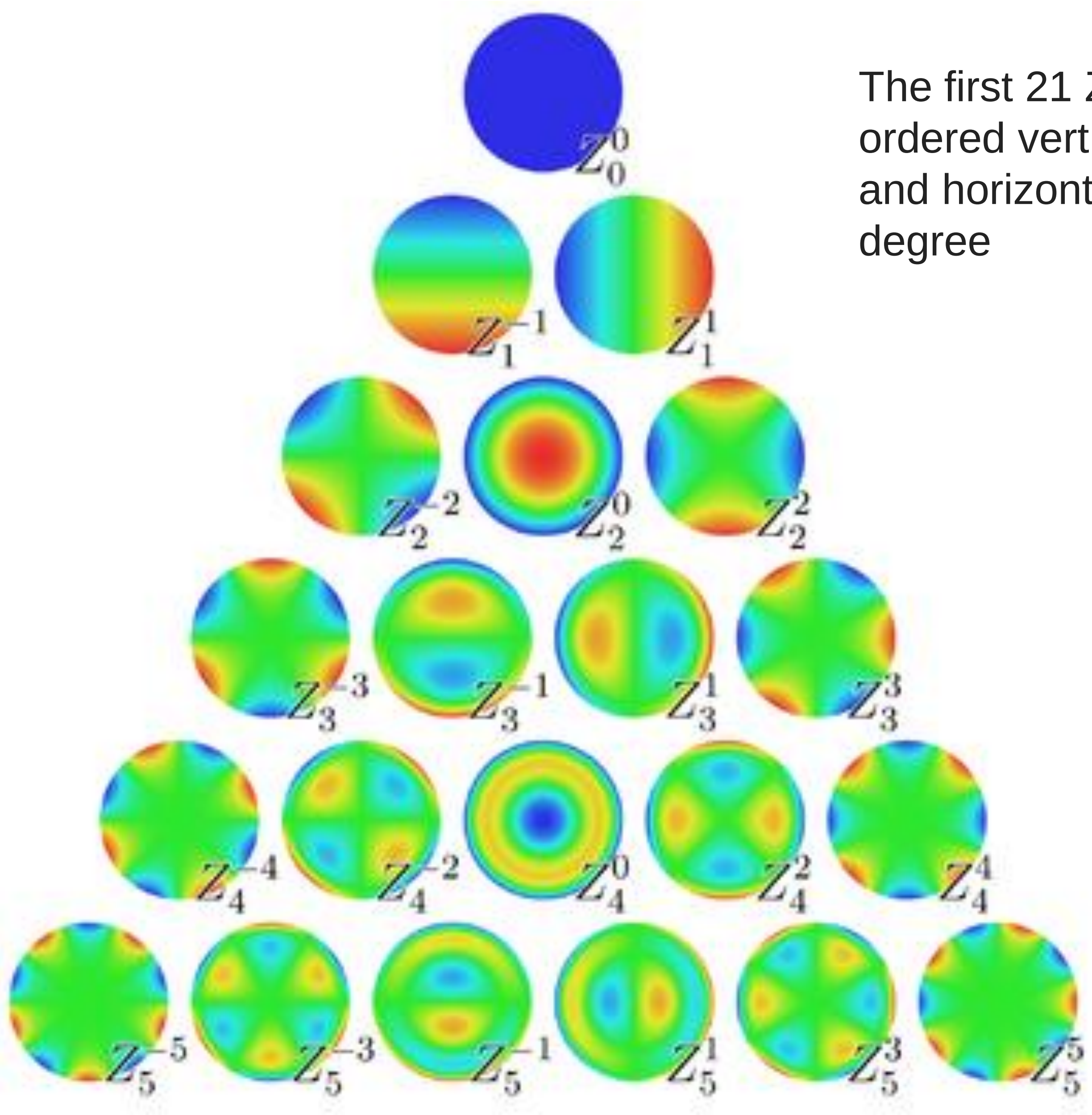
$$Z_n^{-m}(\rho,\varphi) = R_n^m(\rho)\,\sin(m\,\varphi),$$

where *m* and *n* are nonnegative integers with $n \geq m$, ϕ is the azimuthal angle, ρ is the radial distance $0 \leq \rho \leq 1$, and $R^m{}_n$ are the radial polynomials defined below. Zernike polynomials have the property of being limited to a range of −1 to +1, i.e. $|Z_n^m(\rho,\varphi)| \leq 1$. The radial polynomials $R^m{}_n$ are defined as

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k\,(n-k)!}{k!\,\left(\frac{n+m}{2}-k\right)!\,\left(\frac{n-m}{2}-k\right)!}\;\rho^{n-2\,k}$$

for $n - m$ even, and are identically 0 for $n - m$ odd.

https://en.wikipedia.org/wiki/Zernike_polynomials



The first 21 Zernike polynomials, ordered vertically by radial degree and horizontally by azimuthal degree

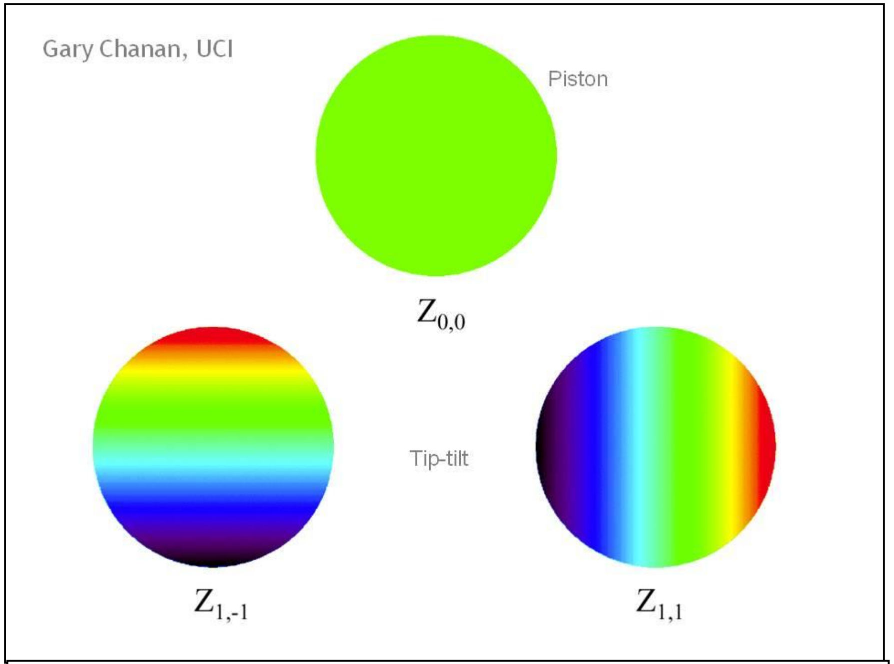
$$Z_{0,0} = 1 \qquad \text{piston}$$

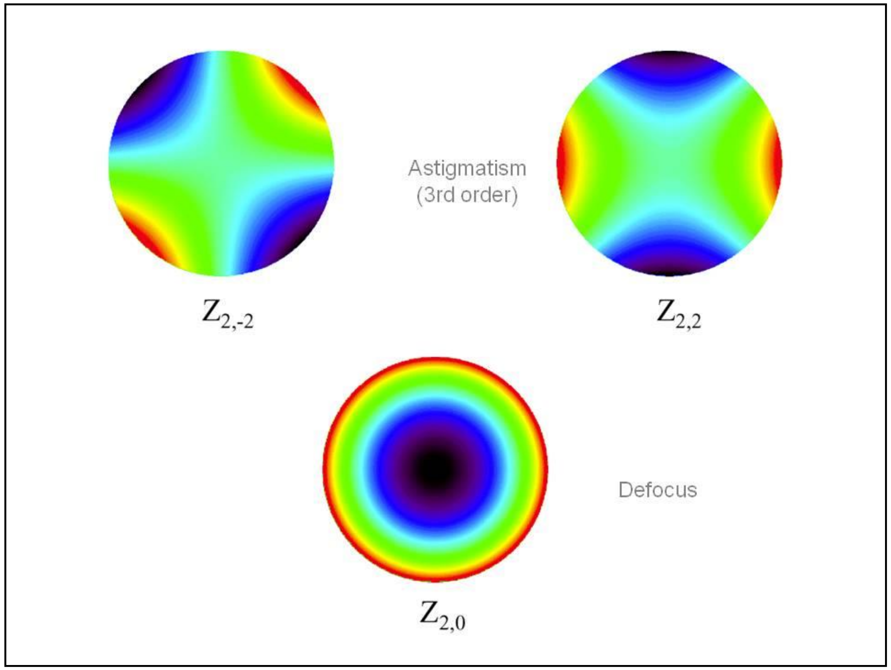
$$\left.\begin{aligned} Z_{1,-1} &= 2\,r\sin\theta \\ Z_{1,1} &= 2\,r\cos\theta \end{aligned}\right\} \qquad \text{tip/tilt}$$

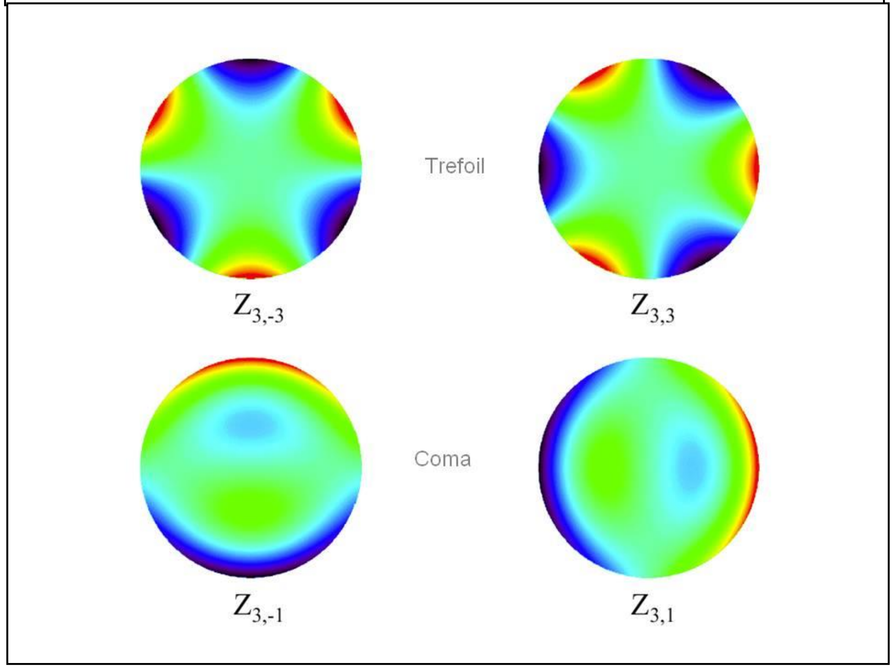
$$Z_{2,-2} = \sqrt{6}\,r^2\sin 2\theta \qquad \text{astigmatism}$$

$$Z_{2,0} = \sqrt{3}\,(2r^2 - 1) \qquad \text{focus}$$

$$Z_{2,2} = \sqrt{6}\,r^2\cos 2\theta \qquad \text{astigmatism}$$

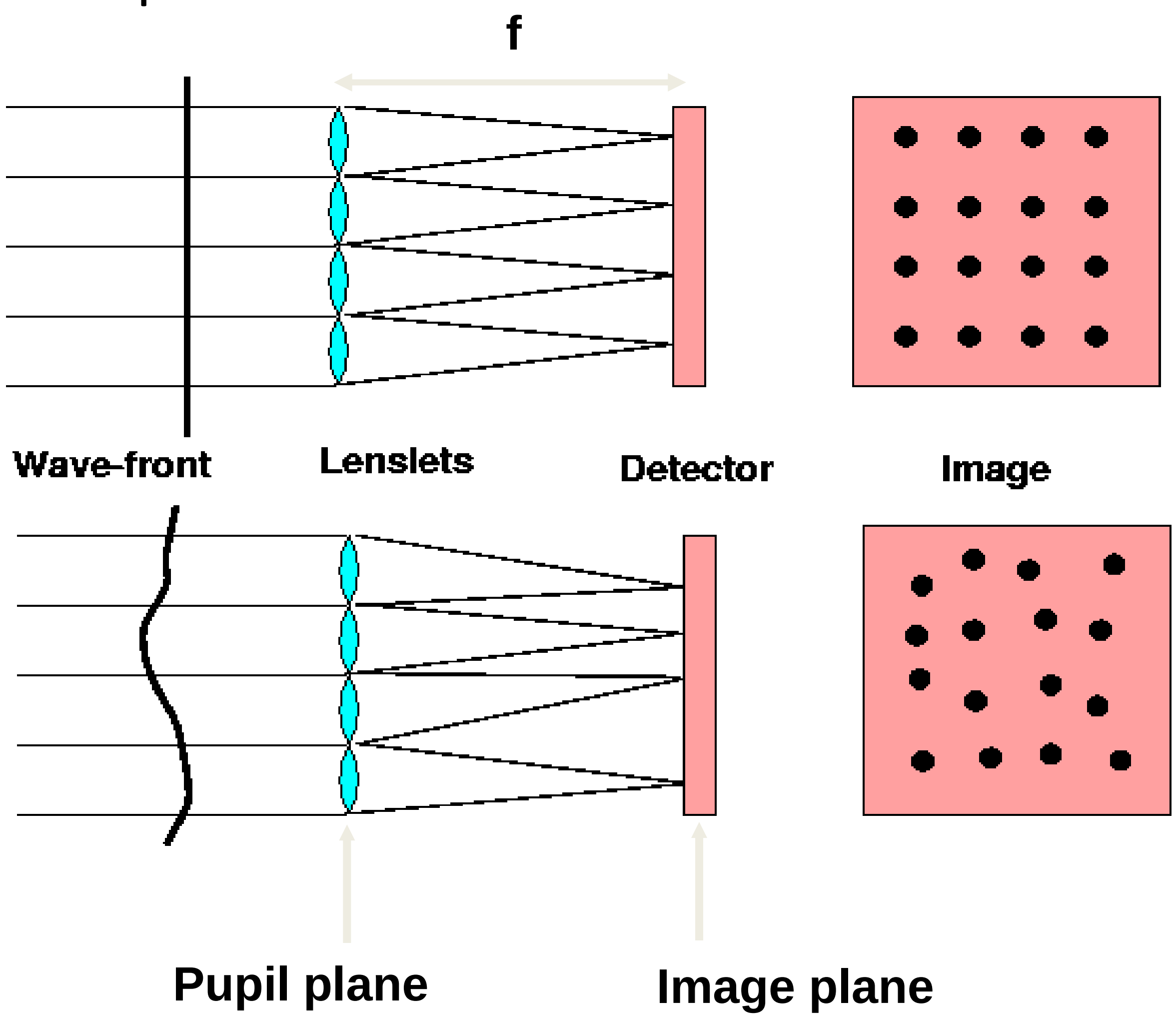




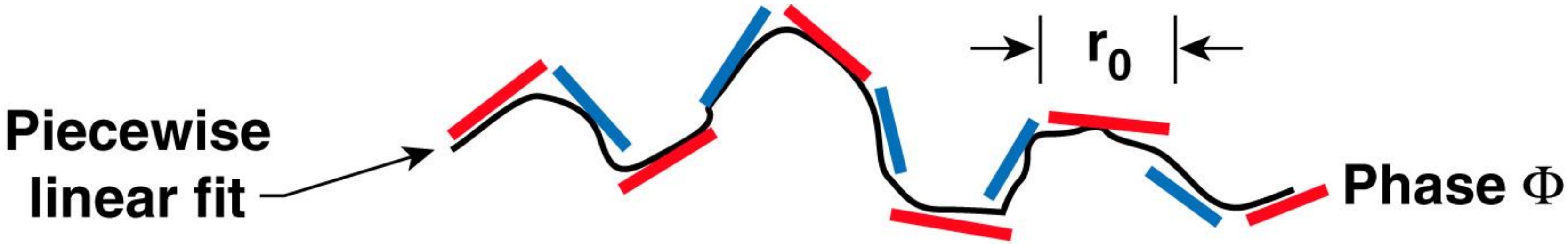


Wavefront Sensors – Shack Hartmann

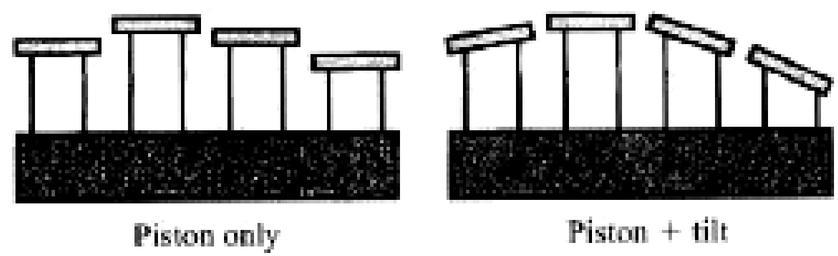
Most common principle is the Shack Hartmann wavefront sensor measuring sub-aperture tilts:

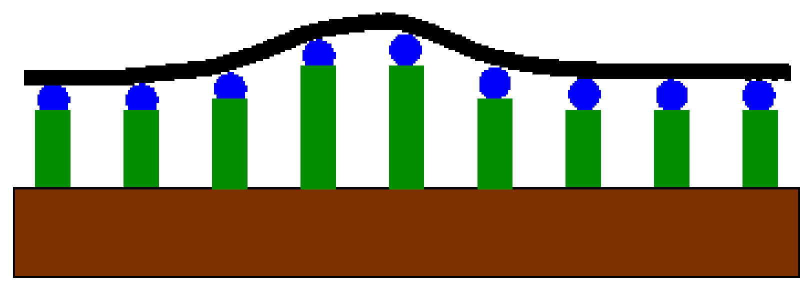


Deformable Mirrors (DM)

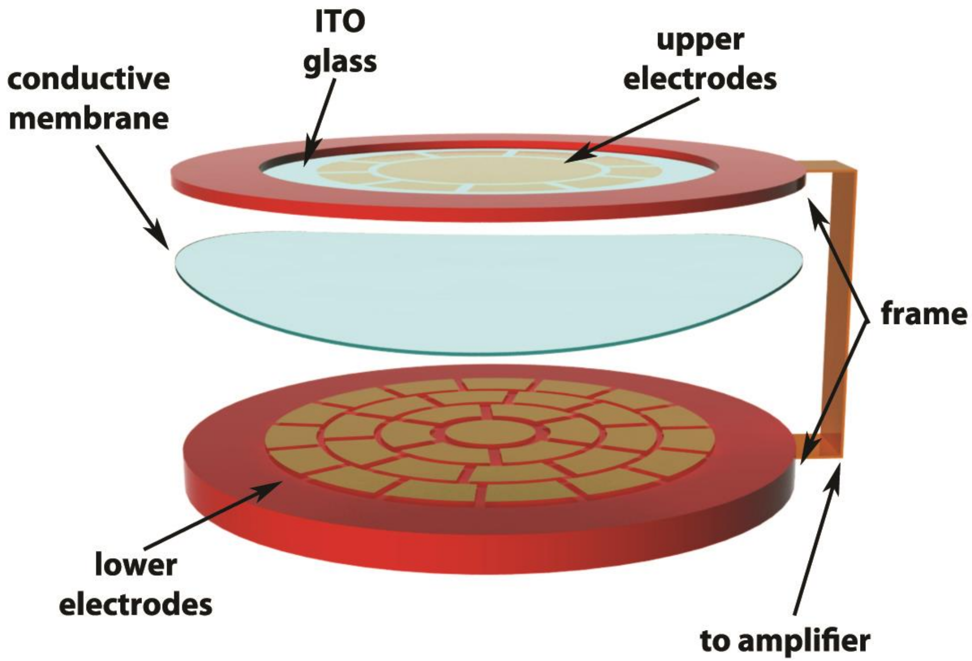


- Fit mirror surface to wavefront
- r_0 sets number of degrees of freedom
- segmented mirrors rarely used anymore
- mostly continuous face-sheet mirrors

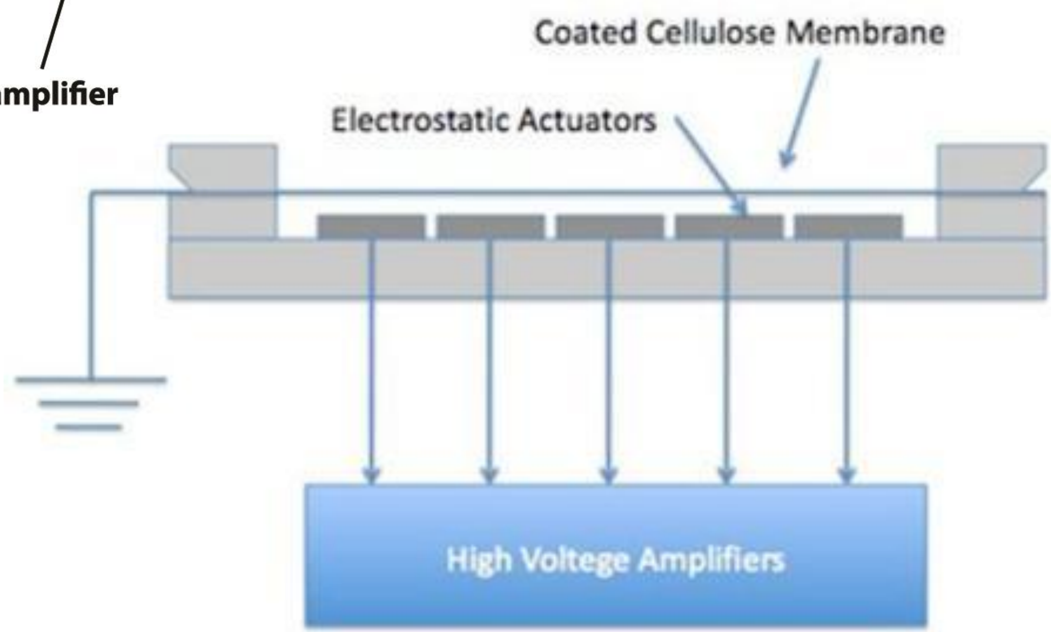




Membrane Deformable Mirror



ıttps://optica.weblog.tudelft.nl/2012/
.2/14/new-publication-on-method-to-
:ontrol-deformable-mirrors/



https://www.azooptics.com/A
rticle.aspx?ArticleID=353

Typical AO Error Terms

- Fitting errors from insufficient approximation of wavefront by deformable mirror due to finite number of actuators (*d*: actuator spacing)

$$s_{fit}^{2} \approx 0.3\left(\frac{d}{r_0}\right)^{5/3}$$

- Temporal errors from time delay between measurement and correction, mostly due to exposure and readout time

$$\sigma_{temp}^{2} \approx \left(\frac{t}{\tau_0}\right)^{5/3}$$

- Measurement errors from wavefront sensor

$$\sigma_{measure}^{2} \sim S/N$$

- Calibration errors from non-common aberrations between wavefront sensing optics and science optics

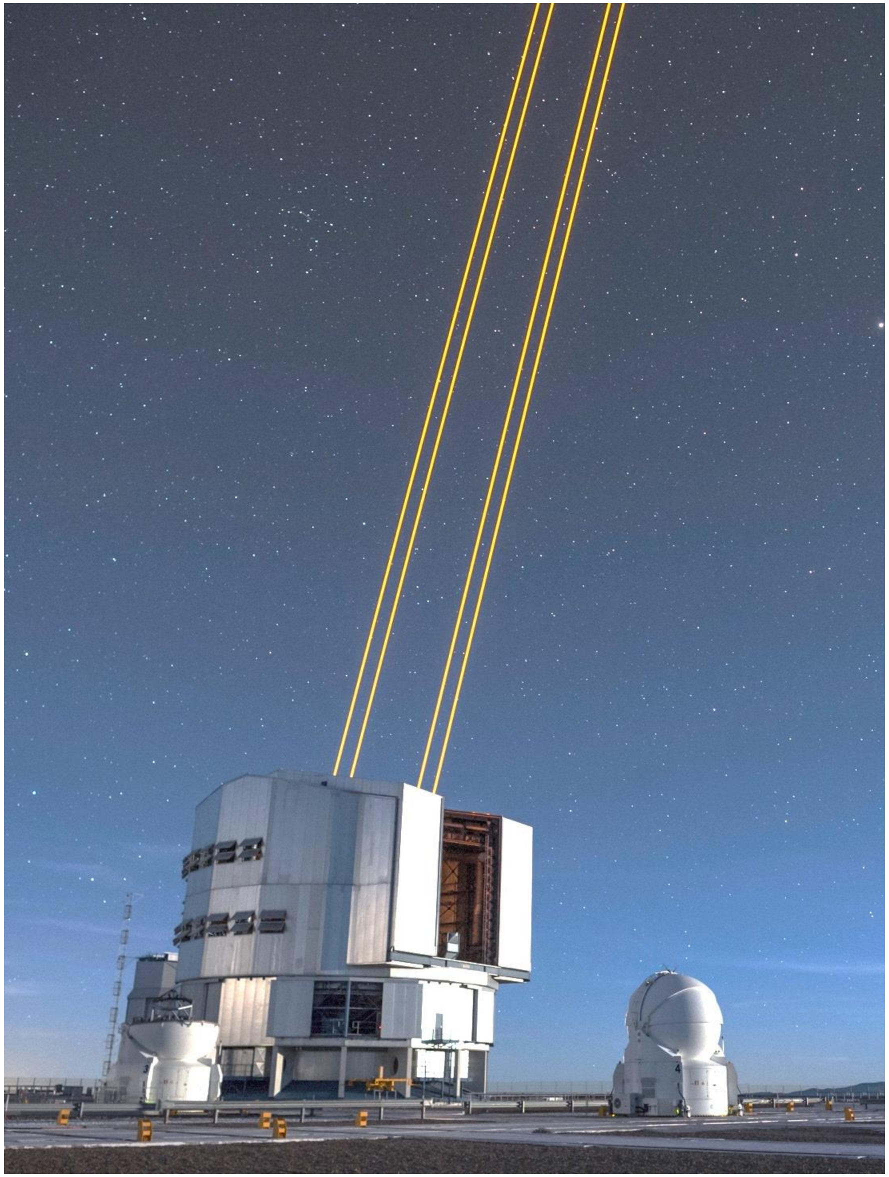
$$s_{calibration}^{2} \sim ???$$

- Angular anisoplanatism from sampling different lines of sight through the atmosphere, mostly limits field of view

$$\sigma_{aniso}^{2} \approx \left(\frac{\theta}{\theta_0}\right)^{5/3}$$

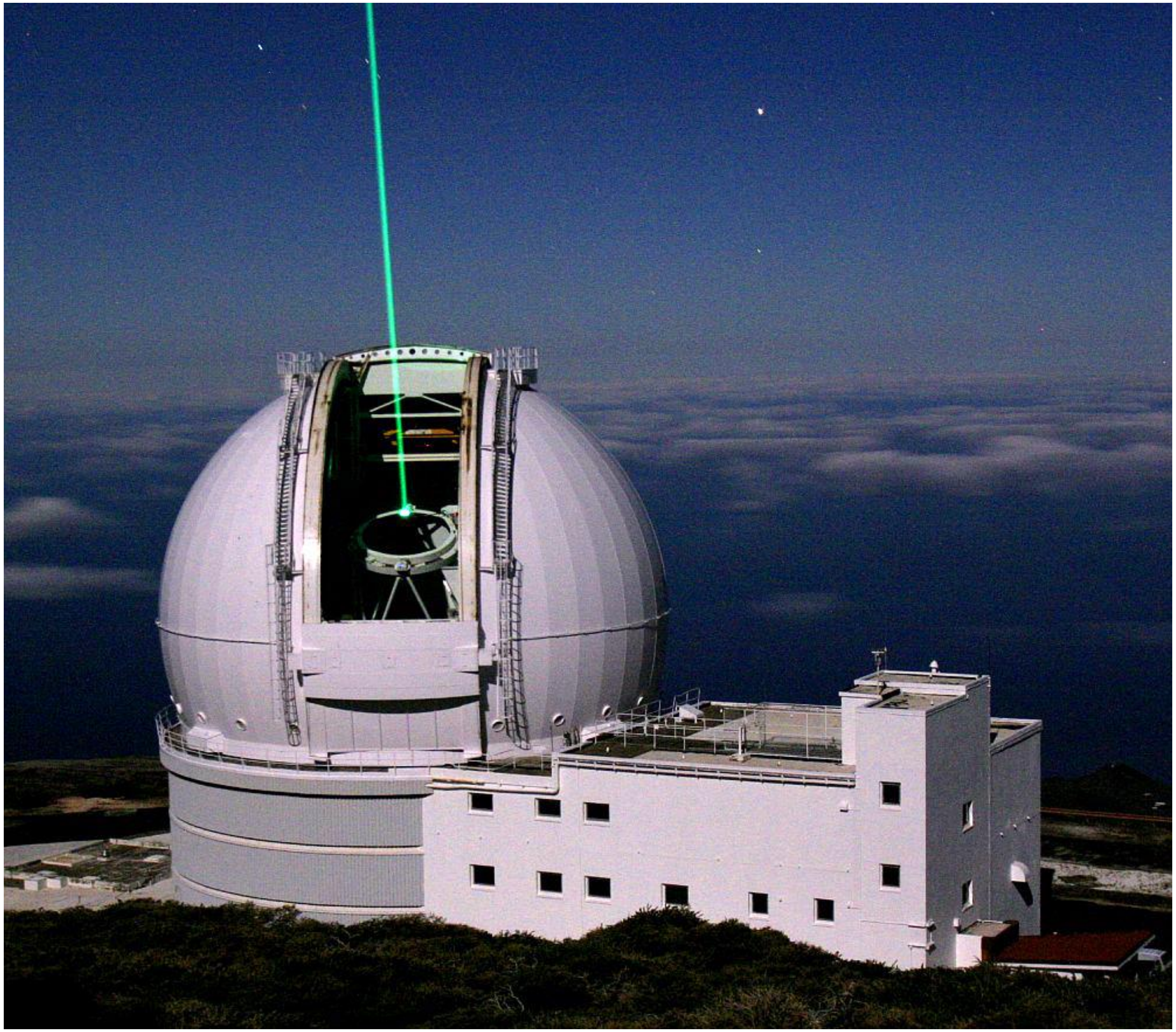
Sodium Beacons

- Layer of neutral sodium atoms in mesosphere (height ~ 95 km, thickness ~10km) from smallest meteorites
- Resonant scattering occurs when incident laser is tuned to D2 line of Na at 589 nm.



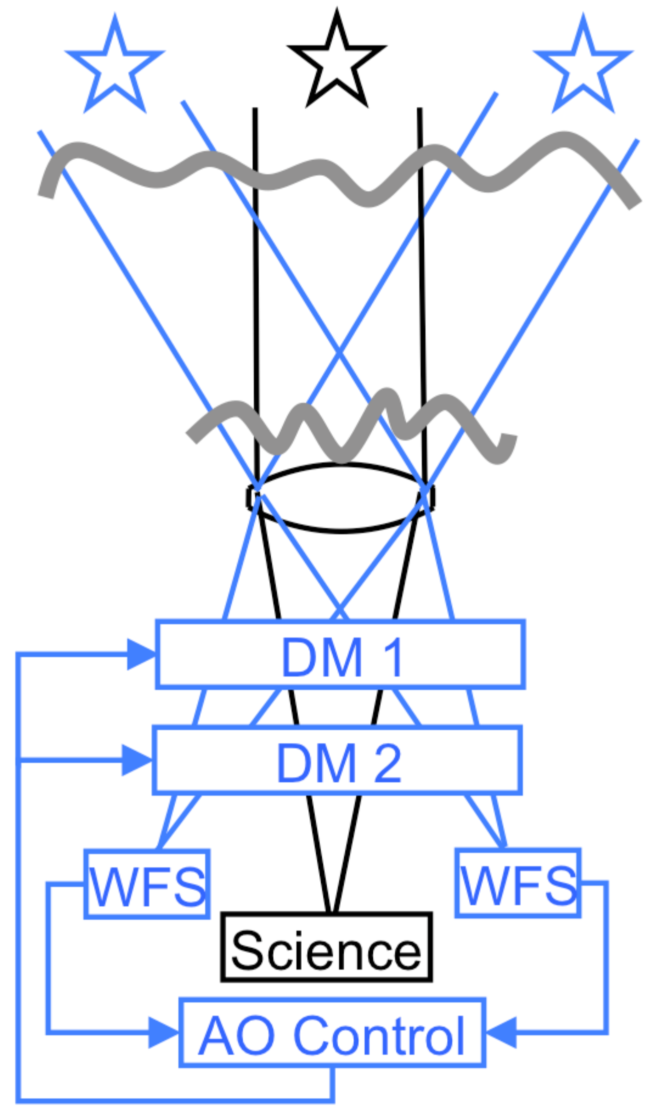
www.eso.org/public/images/eso1613n/

WHT Rayleigh Guide Star



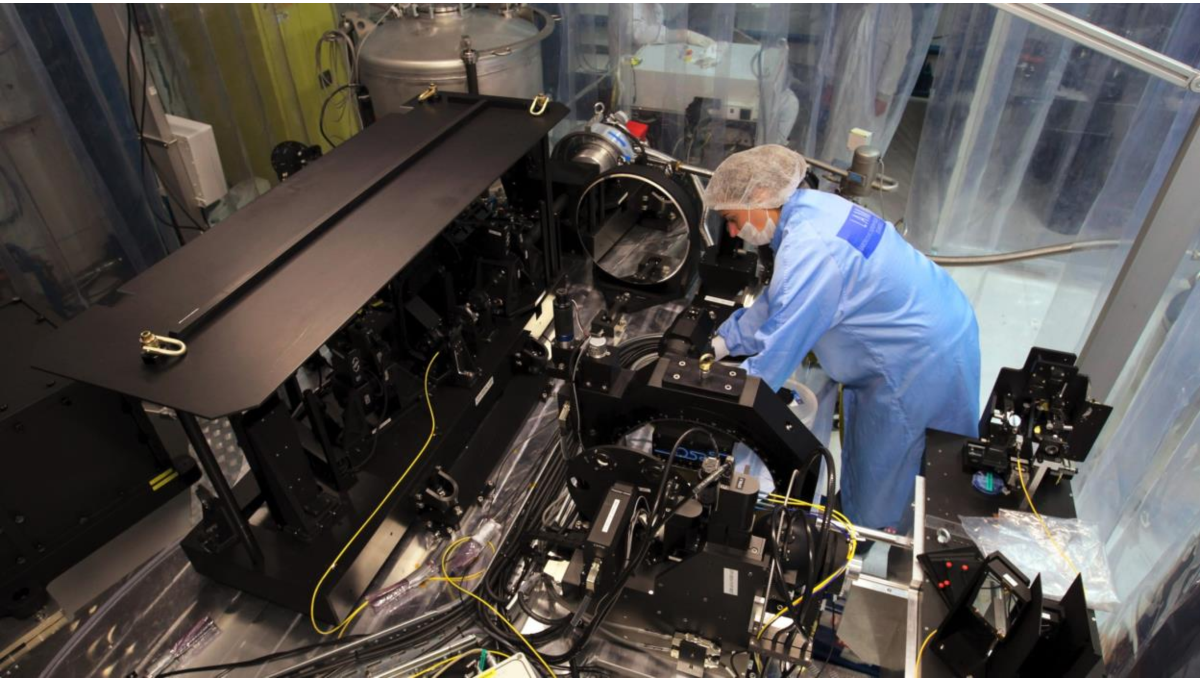
Multi-Conjugate AO – MCAO

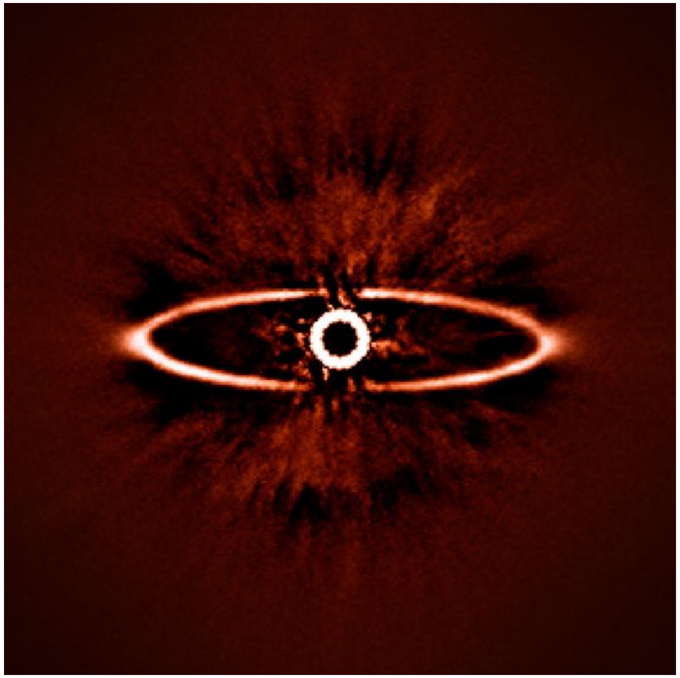
- to overcome anisoplanatism, the basic limitation of single guide star AO
- MCAO uses multiple natural guide stars or laser guide stars
- MCAO controls several DMs
- each DM is conjugated to a different atmospheric layer at a different altitude
- at least one DM is conjugated to the ground layer
- best approach to larger corrected FOV



Extreme AO – XAO

- high Strehl ratio on-axis and small corrected FOV
- requires thousands of DM actuators
- requires minimal optical and alignment errors
- main application: search for exoplanets, SPHERE on VLT, GPI on Gemini





SPHERE Extreme AO

- Piezo-electric deformable mirror with 41 by 41 actuators
- Shack-Hartmann wavefront sensor with 40 by 40 subapertures
- EMCCD with 240 by 240 pixels, >1200Hz frame rate, <0.1e$^-$ readout noise