

Astronomical Observing Techniques 2019

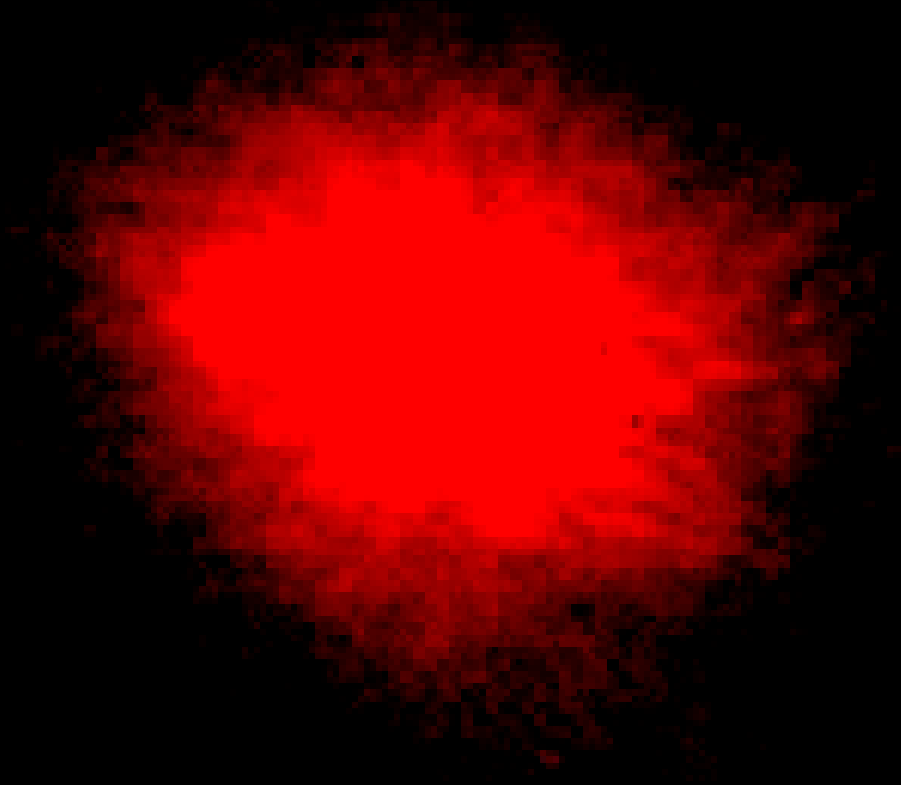
Lecture 13: Twinkle, twinkle little star ... No more!

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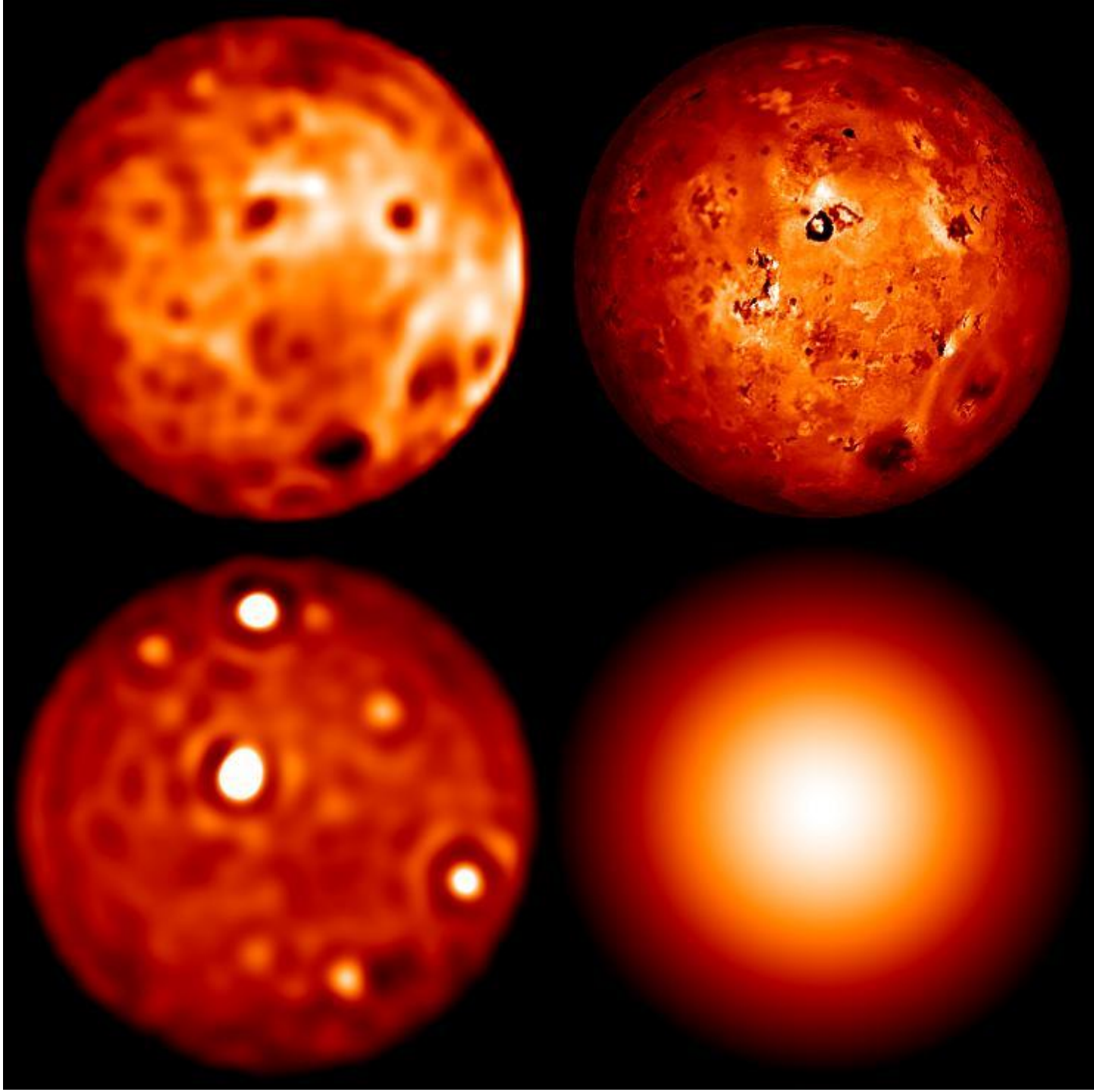
Overview

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The Power of Adaptive Optics

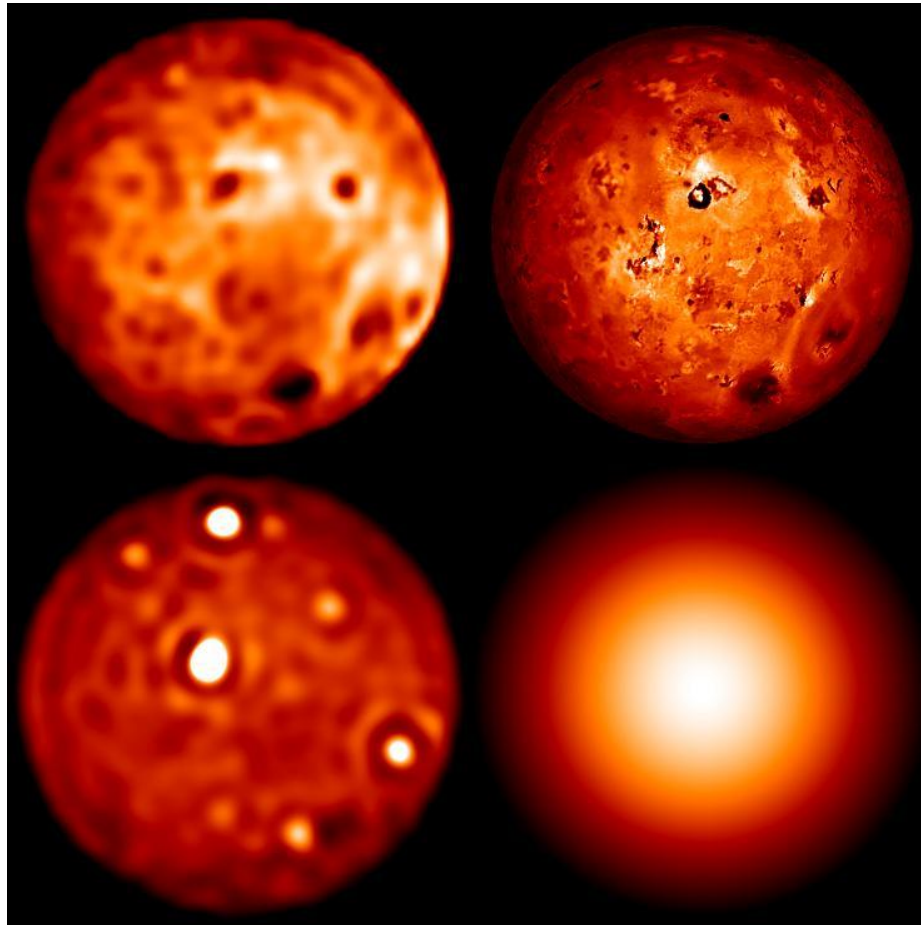


oldweb.lbto.org/AO/AOpressrelease.htm



Io with and without Keck AO

Io image taken
with Keck
adaptive
optics; K-band,
2.2micron



Io image based
on visible light
taken with
Galileo
spacecraft
orbiter.

Io image taken
with Keck
adaptive
optics; L-band,
3.5micron

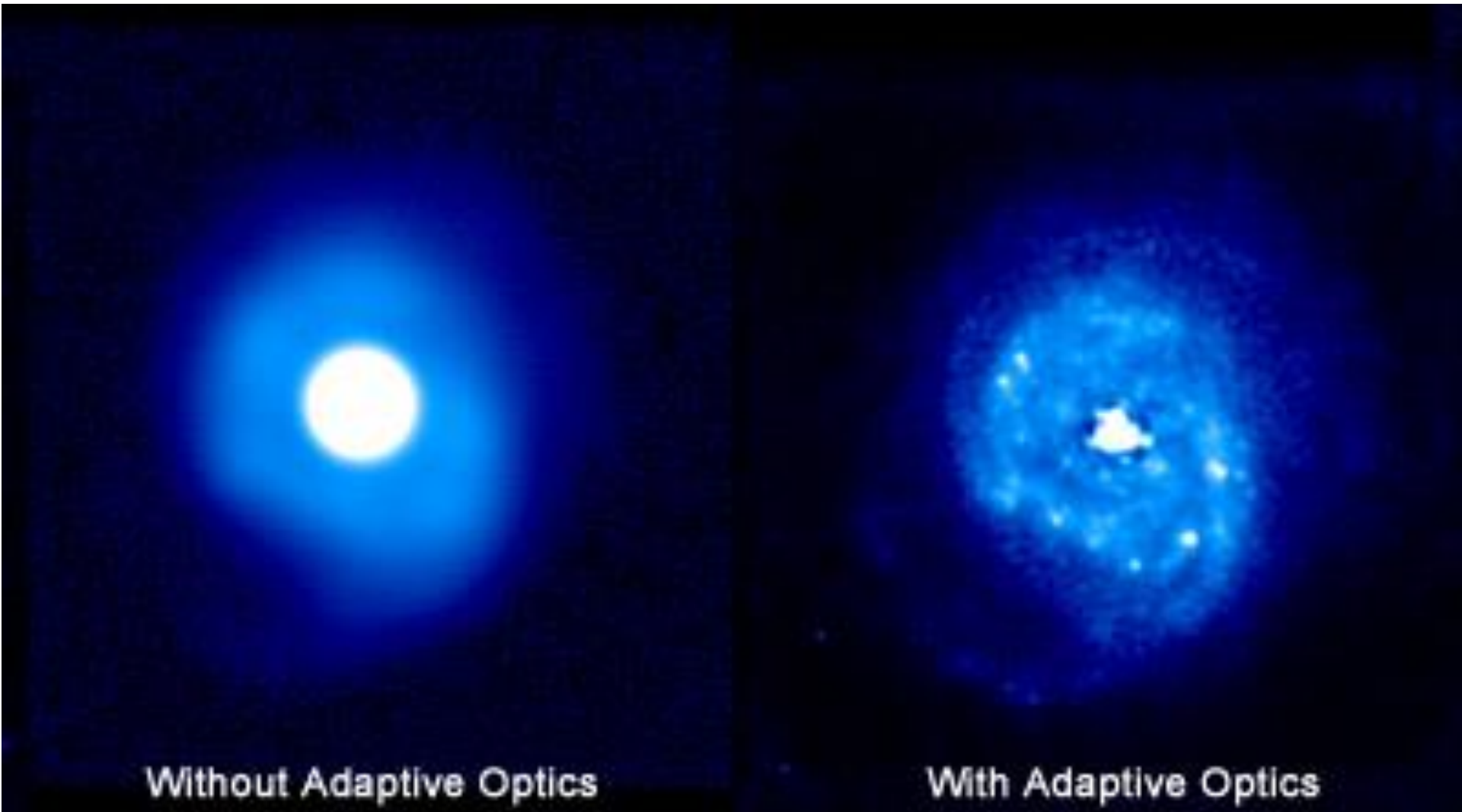
Io image taken
without Keck
adaptive
optics.

cfao.ucolick.org/pgallery/io.php

Io, volcanic moon of Jupiter

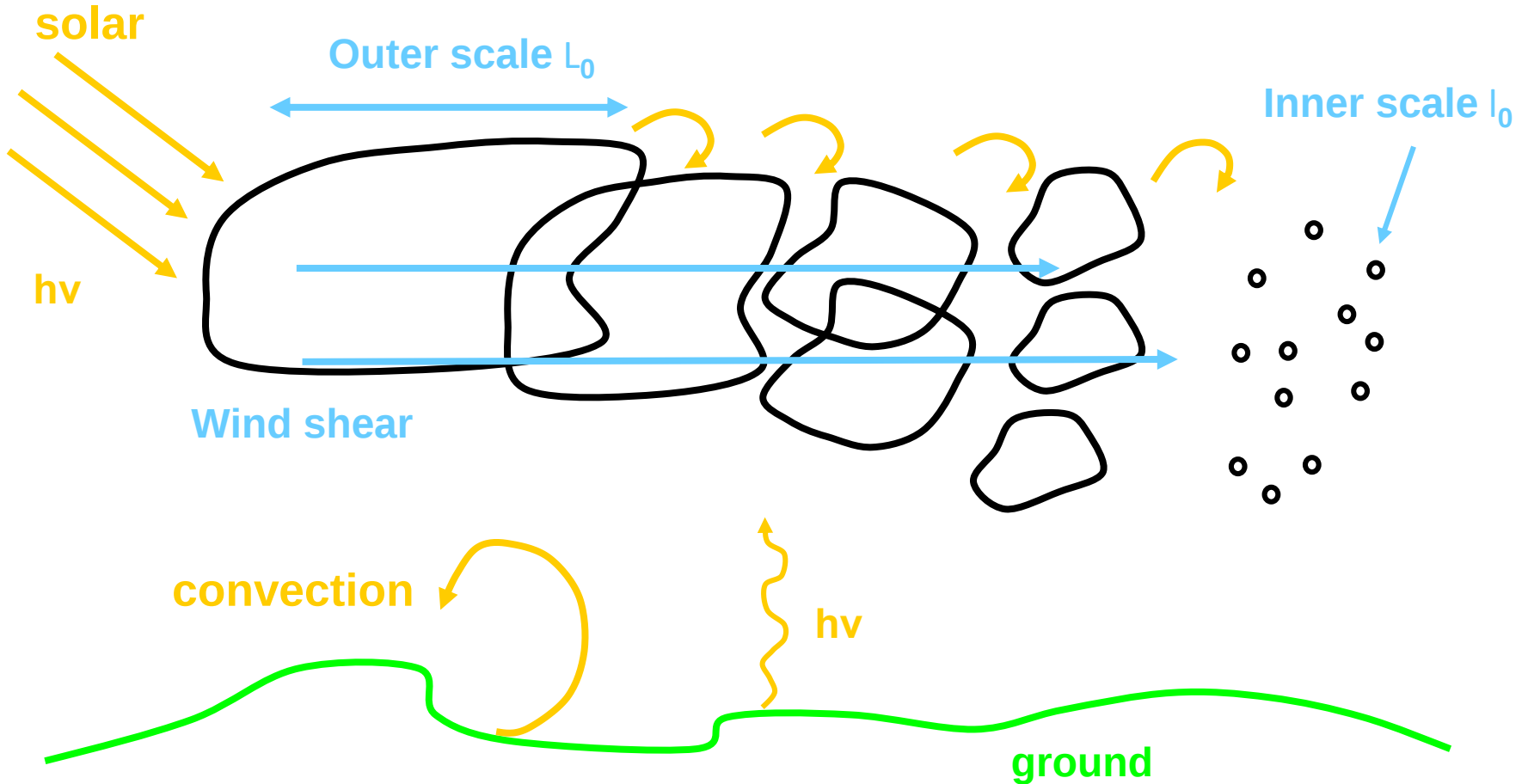
- Observations of [Io](#) by passing spacecrafts (the [Voyagers](#), [Galileo](#), [Cassini](#), and [New Horizons](#)) and Earth-based astronomers have revealed more than 150 active volcanoes.
- The heat source for Io's volcanism comes from [tidal heating](#) produced by its forced [orbital eccentricity](#).
- Lava flows on Io, tens or hundreds of kilometres long, have primarily [basaltic](#) composition, similar to lavas seen on Earth at [shield volcanoes](#) such as [Kīlauea](#) in [Hawaii](#).
- Ionian lava lakes are depressions partially filled with molten lava covered by a thin solidified crust. These lava lakes are directly connected to a magma reservoir lying below. The largest volcanic depression on Io is [Loki Patera](#) at 202 kilometres across.

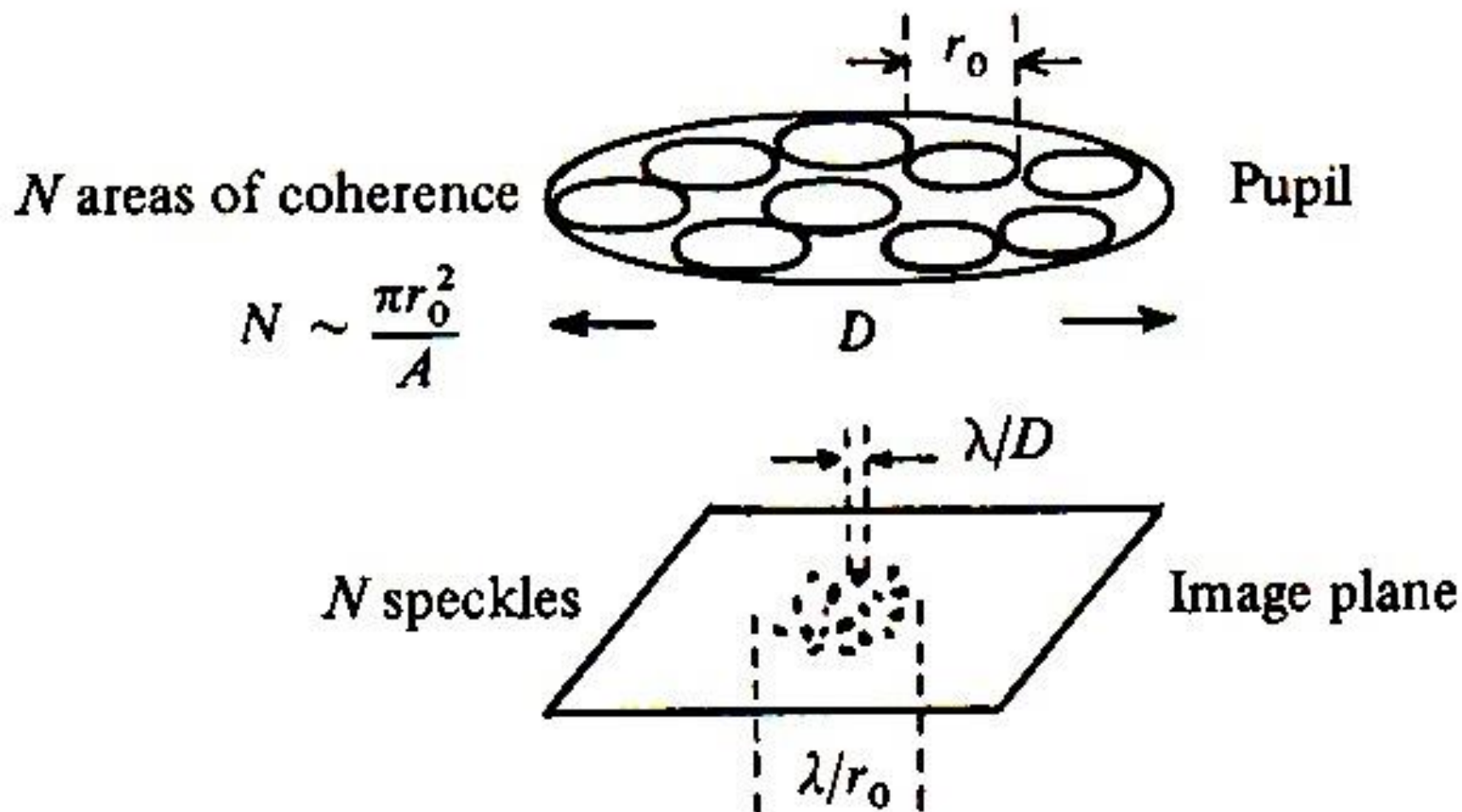
NGC 7469



cfao.ucolick.org/ao/why.php

Atmospheric Turbulence: Origin





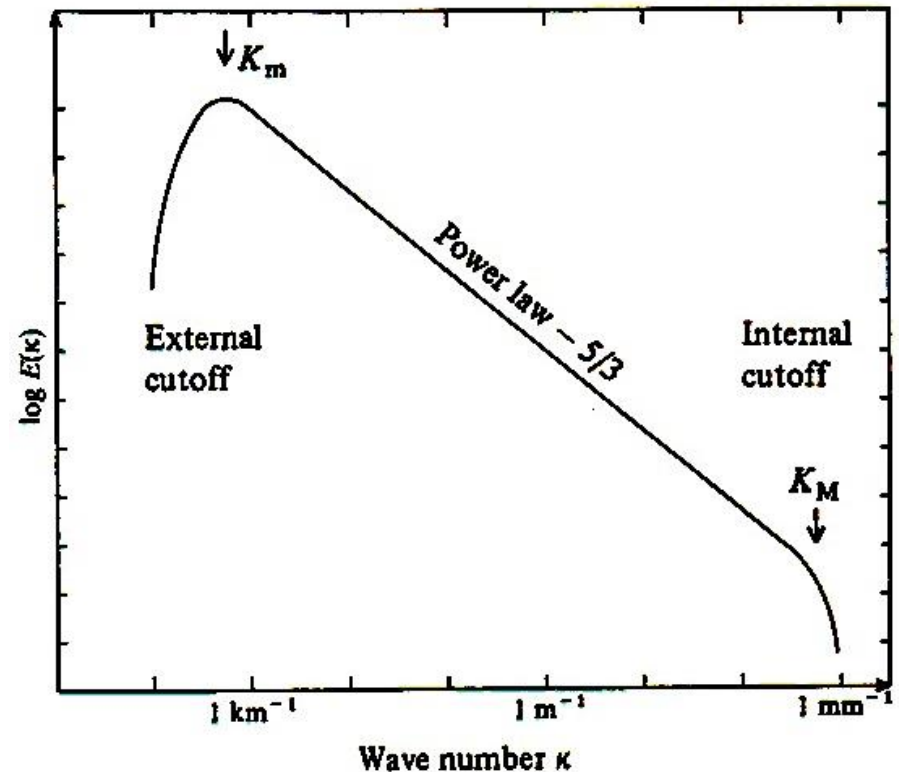
Kolmogorov Turbulence

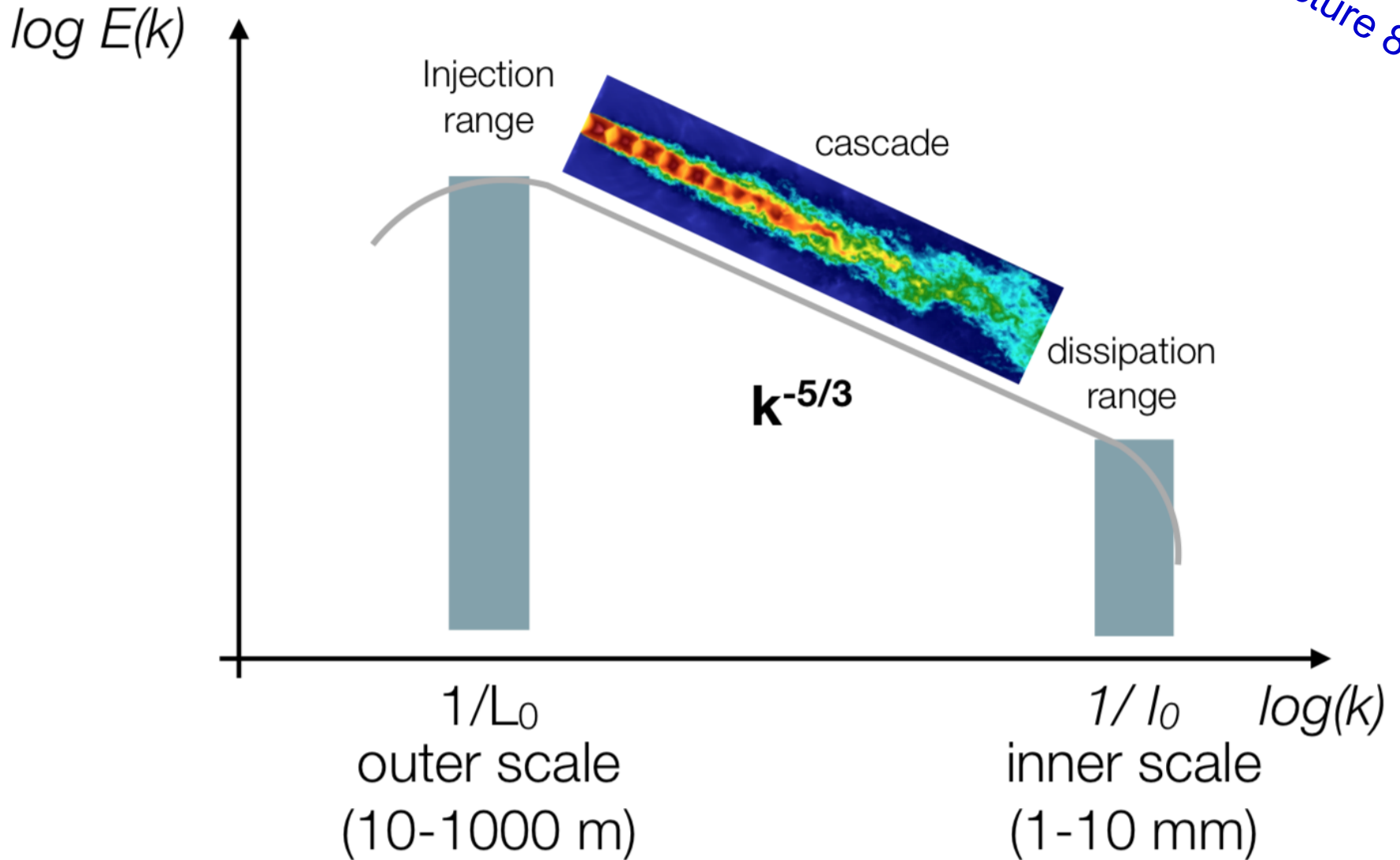
- kinetic energy of large scale ($\sim L$) movements transferred to smaller and smaller scales, down to minimum scale length l_0 , where energy is dissipated by viscous friction
- local velocity field decomposed into spatial harmonics of wave vector κ (Fourier domain)
- $1/\kappa$ is considered length scale
- mean spectrum of kinetic energy (Kolmogorov spectrum)

$$E(\kappa) \propto \kappa^{-5/3}$$

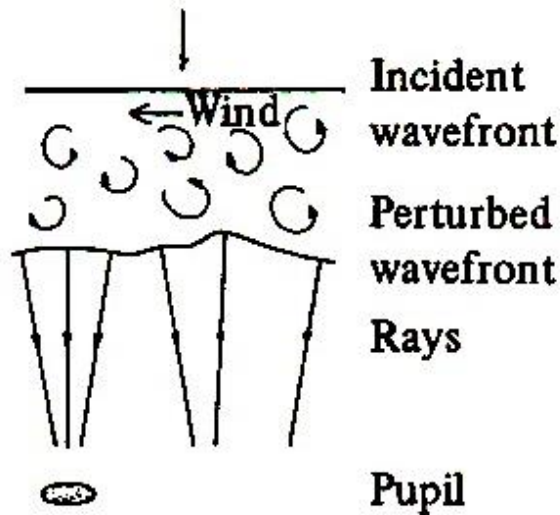
- l_0 = inner scale, L_0 = outer scale

$$L_0^{-1} < \kappa < l_0^{-1}$$





Aspects of Image Degradation



Scintillation

energy received by pupil varies in time (stars flicker)

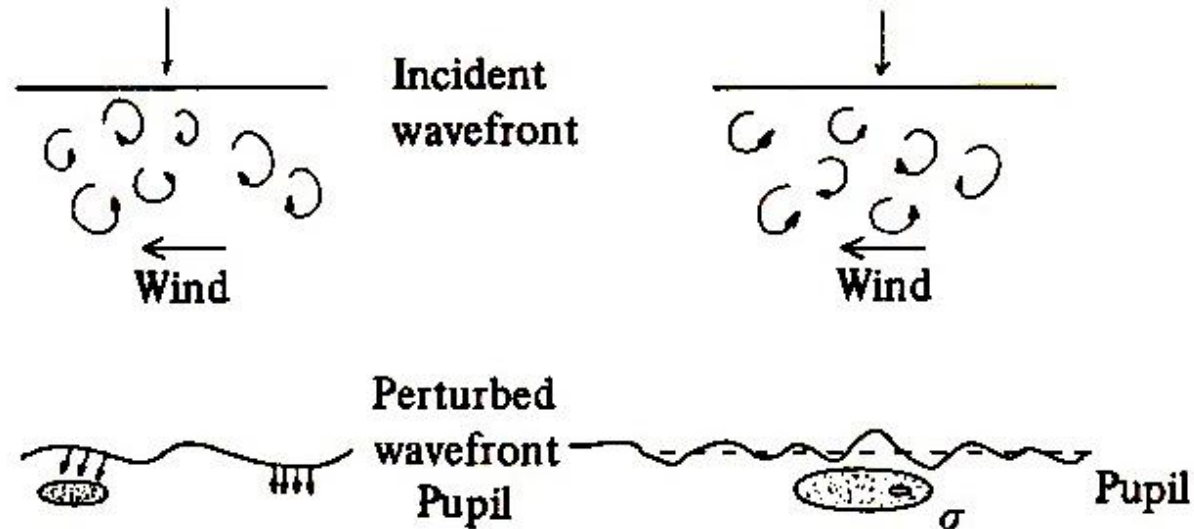


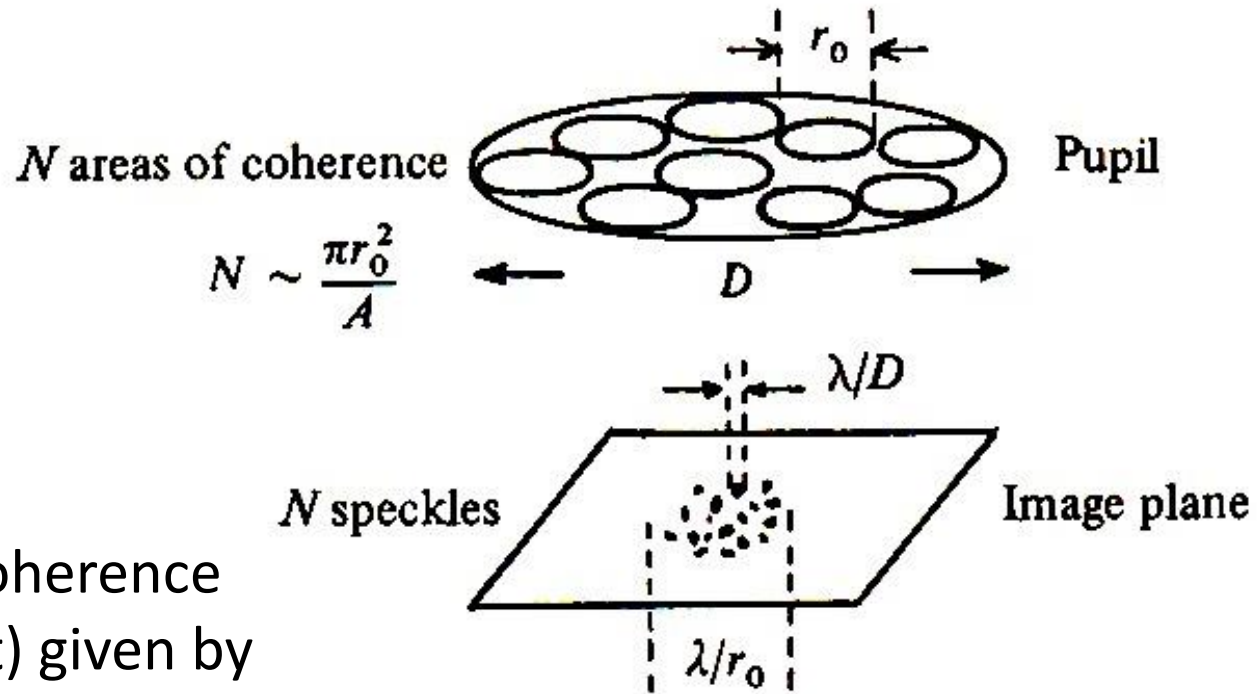
Image Motion

average slope of wavefront at pupil varies ("tip-tilt", stars move around)

Image Blurring

wavefront is not flat ("seeing")

Fried Parameter r_0



- radius of spatial coherence area (of wavefront) given by Fried parameter r_0 :

$$r_0(\lambda) = 0.185 \lambda^{6/5} \left[\int_0^\infty C_n^2(z) dz \right]^{-3/5}$$

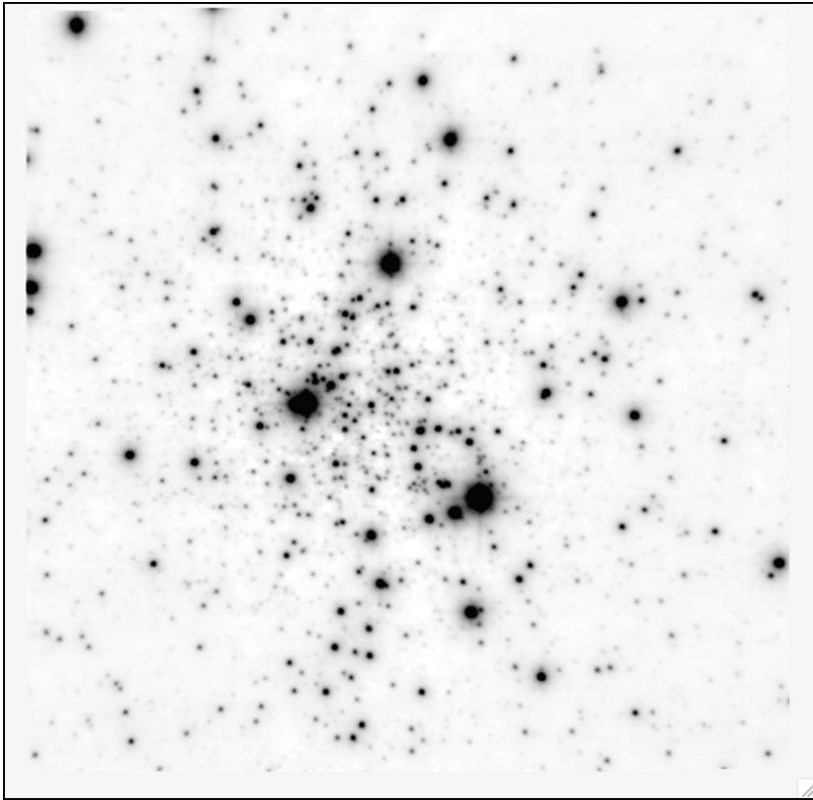
- With C_n^2 the atmospheric turbulent strength as function of altitude z

Seeing: τ_0 , θ_0

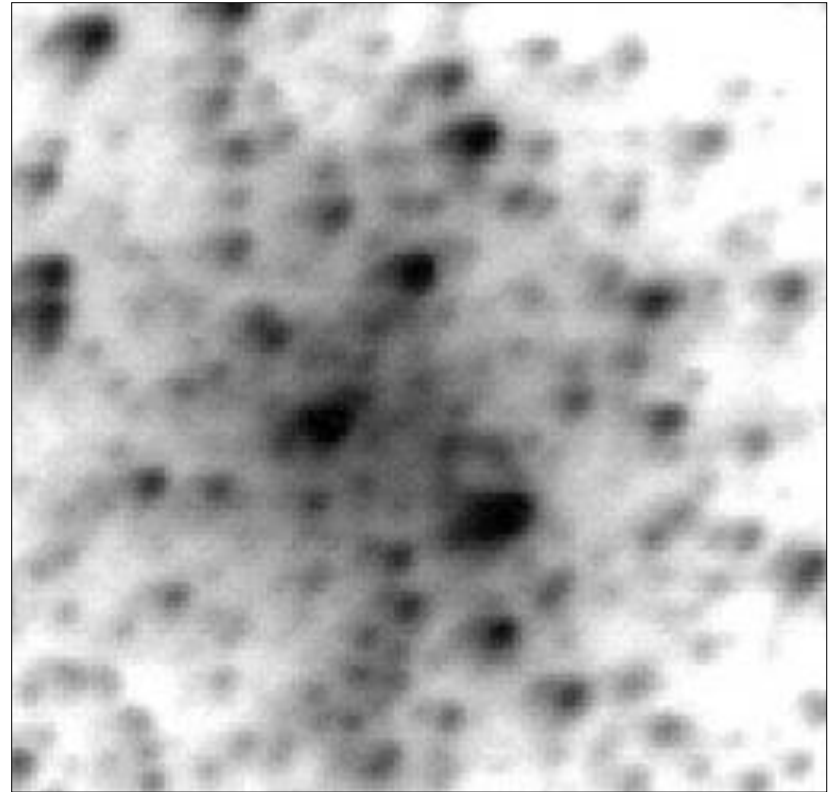
- **Seeing $\Delta\theta$** at good sites at $0.5\mu\text{m}$: 10 - 30 cm $\Delta\theta = \frac{\lambda}{r_0} \sim \lambda^{-1/5}$
- **atmospheric coherence time:**
 maximum time delay for RMS
 wavefront error to be < 1 rad ($\bar{v}$ is mean
 propagation velocity/wind) $t_0 = 0.314 \frac{r_0}{\bar{v}}$
- **Isoplanatic angle θ_0 :** angle over which
 RMS wavefront error is smaller than 1 rad,
 with $\bar{h}$ the average height of the turbulence $\theta_0 = 0.314 \cos \zeta \frac{r_0}{\bar{h}}$

Resolution & Sensitivity Improvement

1. Angular resolution: $\theta = \frac{\lambda}{r_0} \rightarrow \theta = \frac{\lambda}{D} \Rightarrow \text{gain} = \frac{D}{r_0}$
2. Point source sensitivity: $S / N \sim D^2 \Rightarrow \text{gain in } t_{\text{int}} \sim \frac{1}{D^4}$



PHARO Ks image using laser guide stars
500s integ., 40" FOV, 150 mas FWHM



WIRO H image
Kobulnicky et al. 2005, AJ 129, 239-250

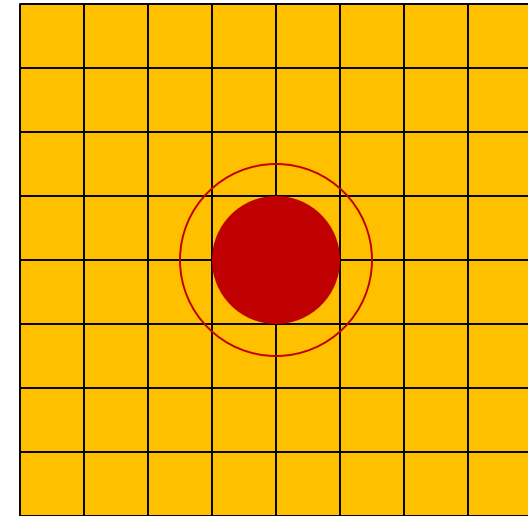
Case 3: Diffraction-limited “Point Source”

Signal* = S; Background* = B; Noise = N; Telescope diameter = D

$$“S/N = (S/N)_{\text{light bucket}} \cdot (S/N)_{\text{pixel scale}}”$$

(i) Effect of telescope aperture: $\rightarrow S/N \sim D$

- Signal $S \sim D^2$
- Background $B \sim D^2 \rightarrow N \sim D$



(ii) Effect of pixel FOV (if Nyquist sampled to θ_{diff}): $\rightarrow S/N \sim D$

- $S \sim \text{const}$ (pixel samples PSF = all source flux)
- $B \sim D^{-2} \rightarrow N \sim D^{-1}$

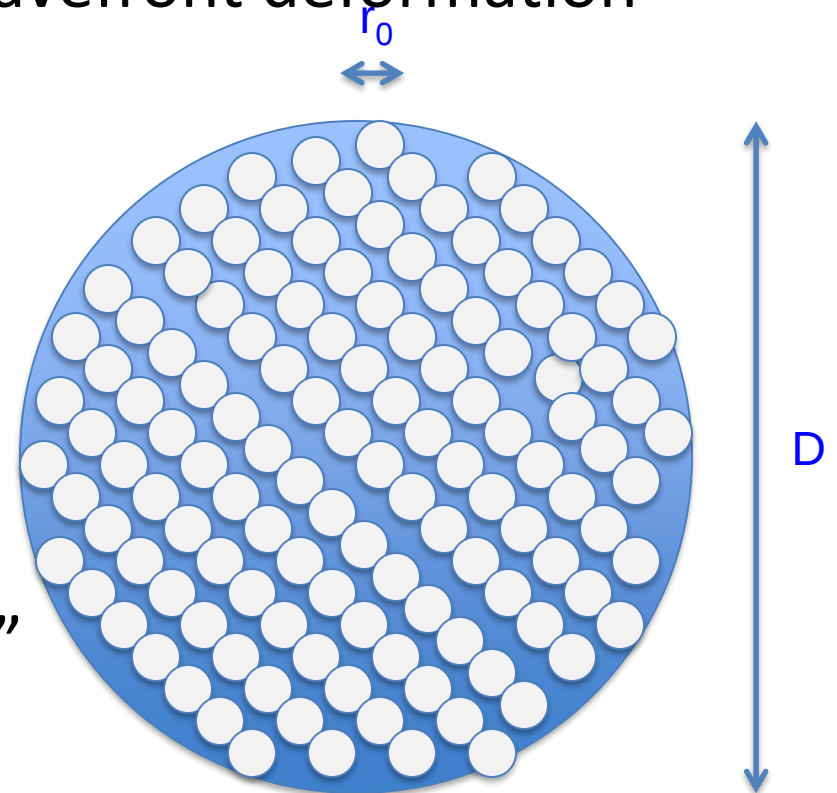
(i) and (ii) combined $S/N \sim D^2 \rightarrow t_{\text{int}} \sim D^{-4}$

$\rightarrow$ huge gain: 1hr ELT = 3 months VLT

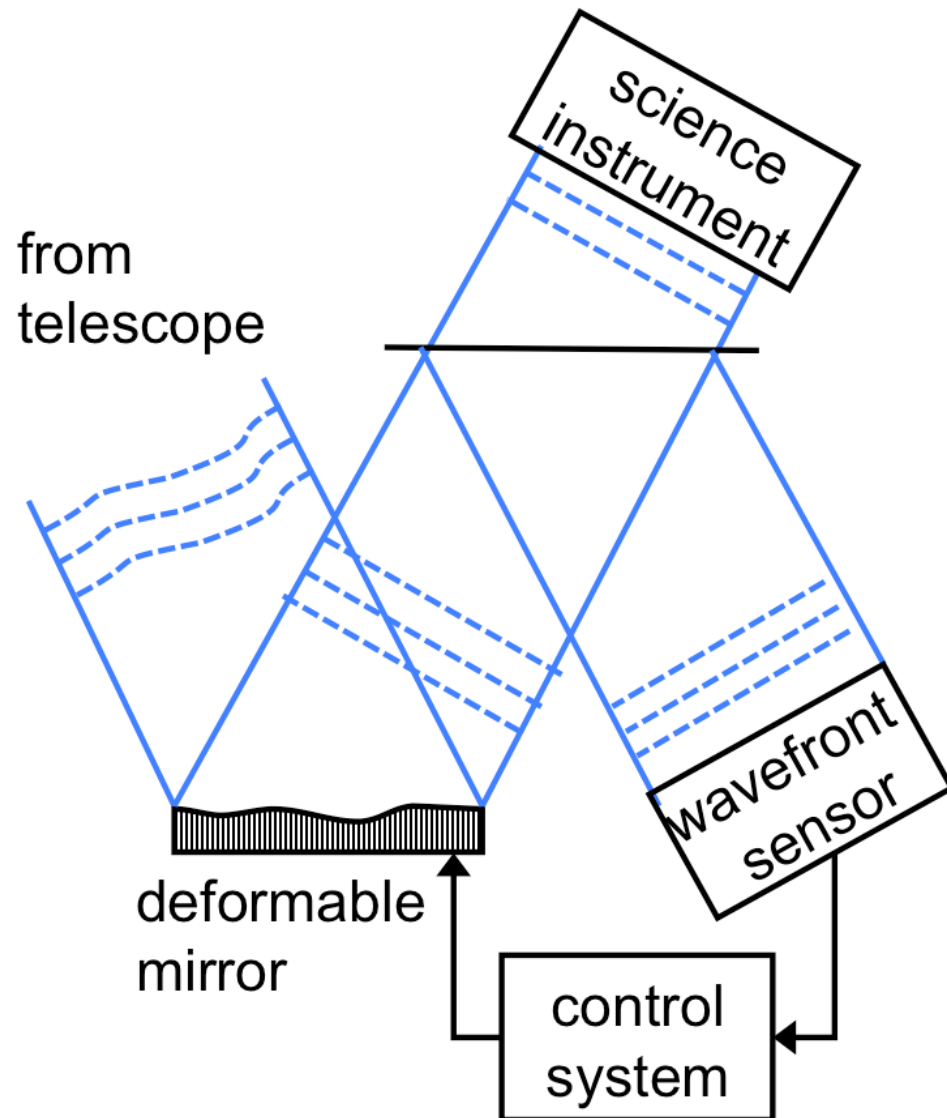
*per pixel

Adaptive Optics Principle

- Maximum scale of tolerated wavefront deformation is $r_0 \rightarrow$ subdivide telescope into apertures with diameter r_0
- Measure wavefront deformation
- Correct wavefront deformation by “bending back” patches of size r_0
- Number of subapertures is $(D/r_0)^2$ at observing wavelength $\rightarrow$ requires hundreds to thousands of actuators for large telescopes



Adaptive Optics Scheme



Wavefront Description: Zernike Polynomials

They are a sequence of polynomials $Z_n^m(\rho, \phi)$ that are orthogonal on the unit disk: $\sum a_{m,n} Z_n^m(\rho, \phi)$

There are **even and odd** Zernike polynomials. The even ones are defined as

$$Z_n^m(\rho, \varphi) = R_n^m(\rho) \cos(m \varphi)$$

and the odd ones as

$$Z_n^{-m}(\rho, \varphi) = R_n^m(\rho) \sin(m \varphi),$$

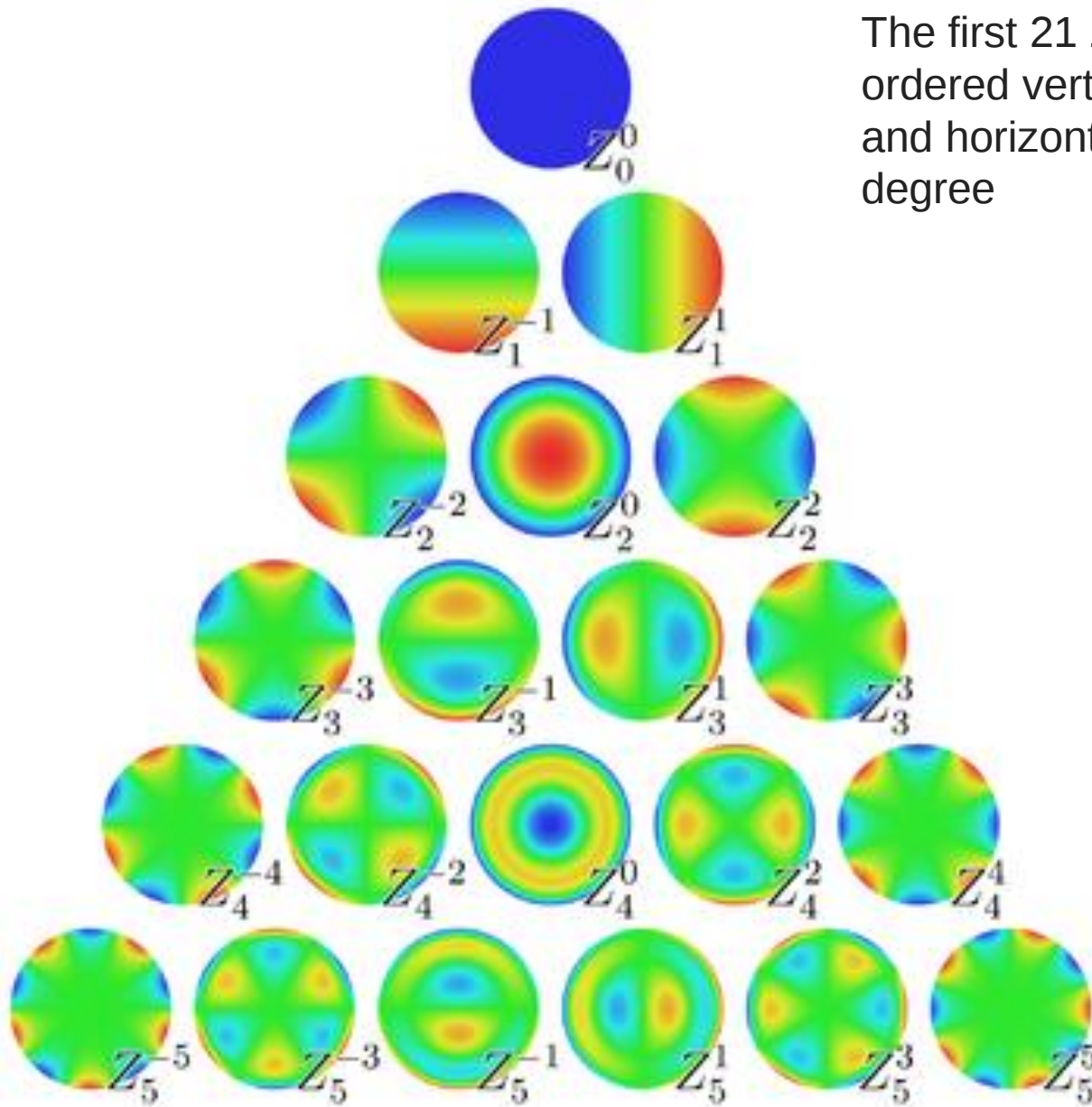
where m and n are nonnegative **integers** with $n \geq m$, ϕ is the **azimuthal angle**, ρ is the radial distance $0 \leq \rho \leq 1$, and R_n^m are the radial polynomials defined below. Zernike polynomials have the property of being limited to a range of -1 to $+1$, i.e. $|Z_n^m(\rho, \varphi)| \leq 1$. The radial polynomials R_n^m are defined as

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k! \left(\frac{n+m}{2} - k\right)! \left(\frac{n-m}{2} - k\right)!} \rho^{n-2k}$$

for $n - m$ even, and are identically 0 for $n - m$ odd.

https://en.wikipedia.org/wiki/Zernike_polynomials

The first 21 Zernike polynomials,
ordered vertically by radial degree
and horizontally by azimuthal
degree



$$Z_{0,0} = 1$$

piston

$$Z_{1,-1} = 2 r \sin\theta$$

$$Z_{1,1} = 2 r \cos\theta$$

tip/tilt

$$Z_{2,-2} = \sqrt{6} r^2 \sin 2\theta$$

astigmatism

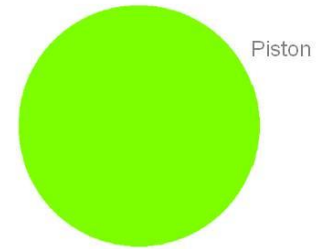
$$Z_{2,0} = \sqrt{3} (2r^2 - 1)$$

focus

$$Z_{2,2} = \sqrt{6} r^2 \cos 2\theta$$

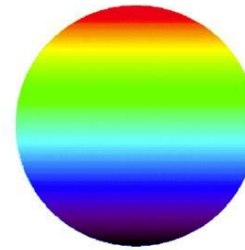
astigmatism

Gary Chanan, UCI

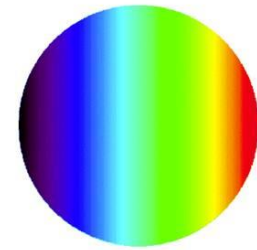


Piston

$Z_{0,0}$

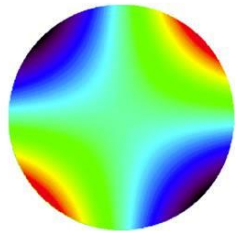


$Z_{1,-1}$

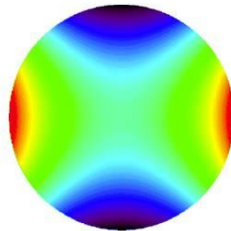


$Z_{1,1}$

Tip-tilt

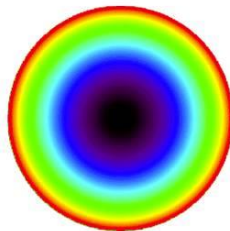


$Z_{2,-2}$



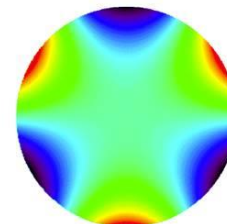
$Z_{2,2}$

Astigmatism
(3rd order)

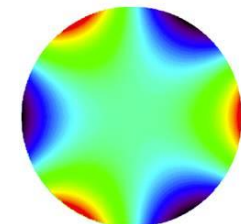


$Z_{2,0}$

Defocus

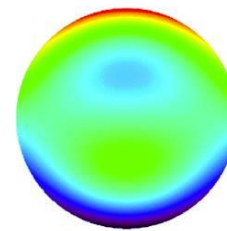


$Z_{3,-3}$

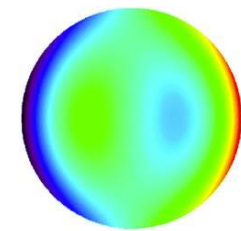


$Z_{3,3}$

Trefoil



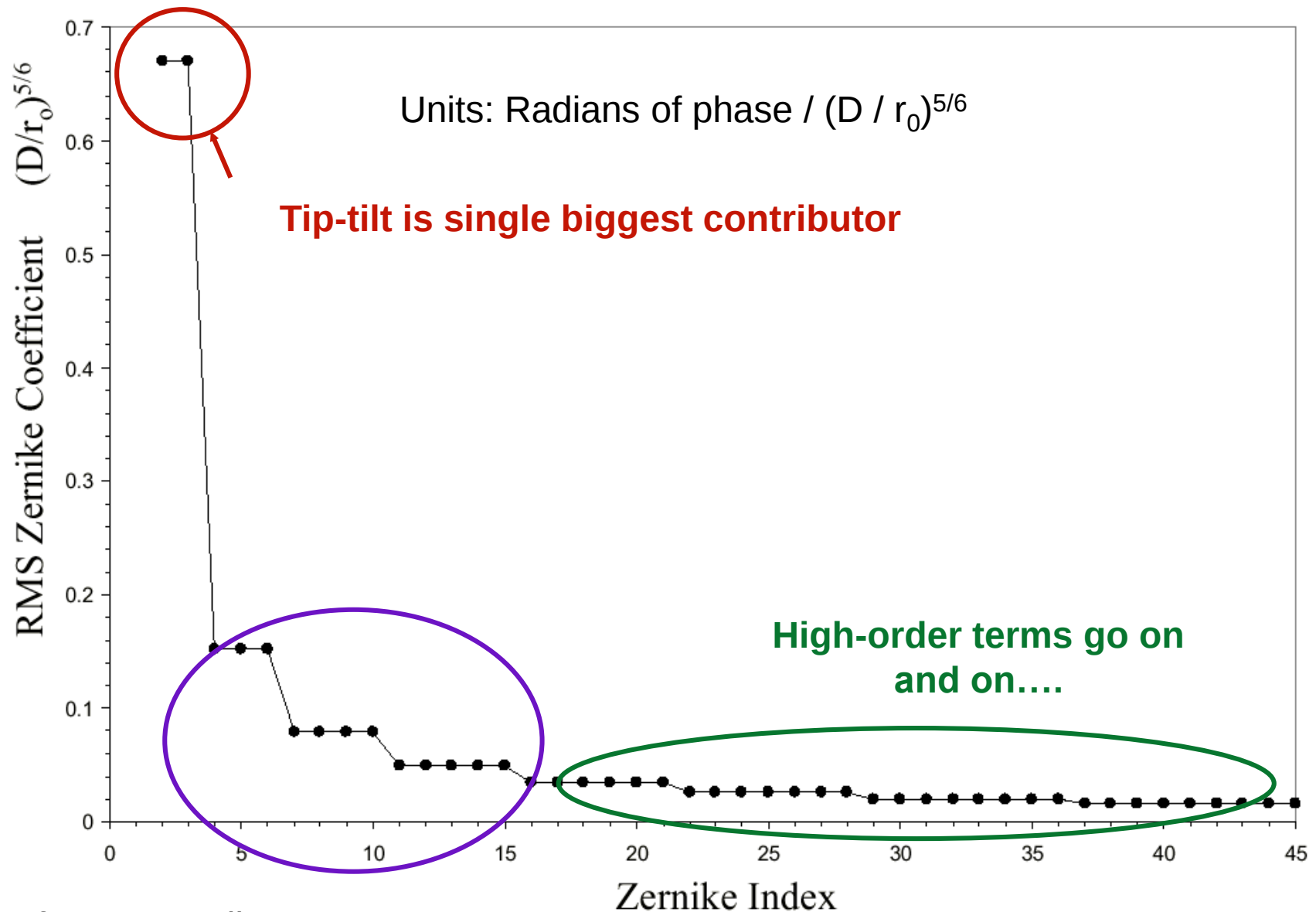
$Z_{3,-1}$



$Z_{3,1}$

Coma

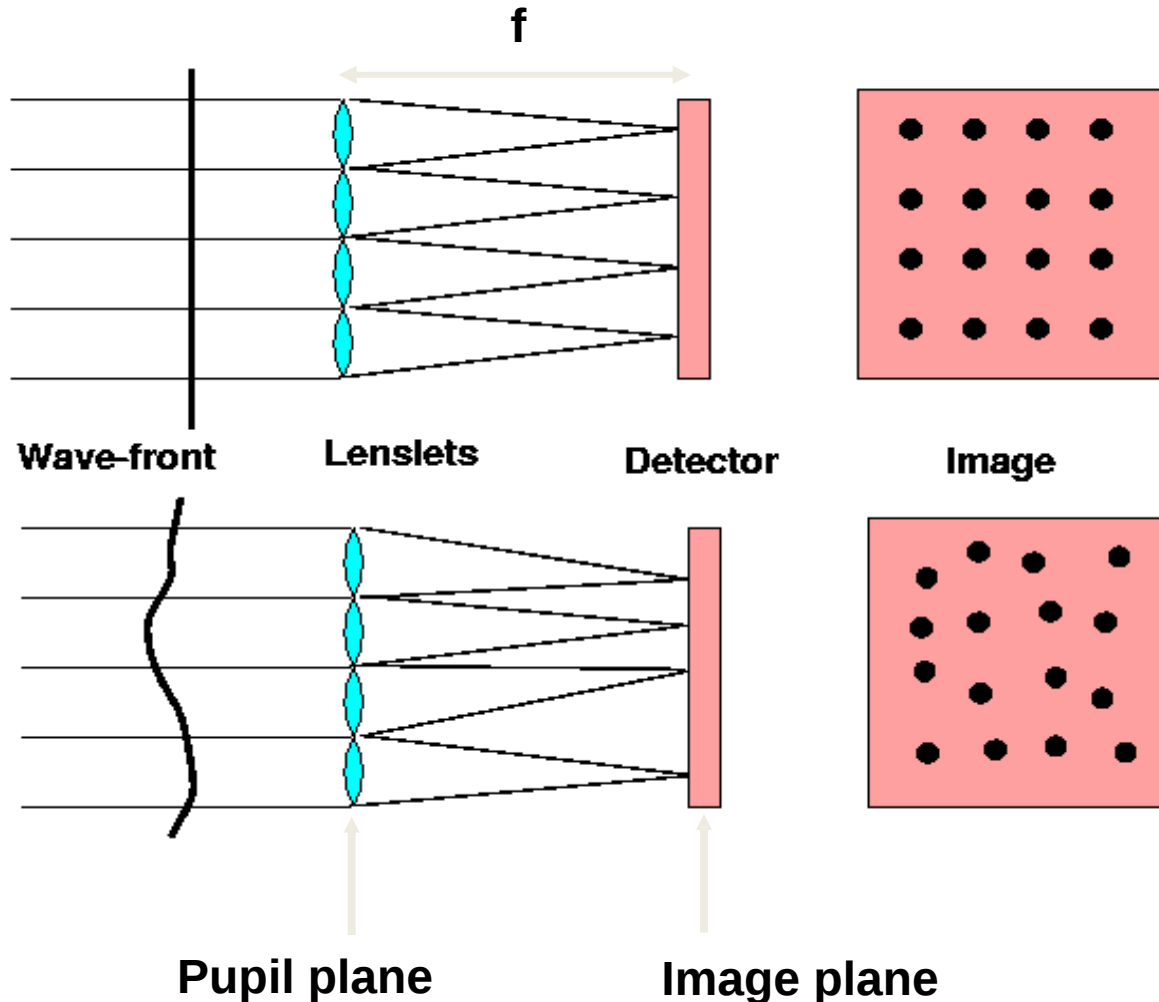
Zernike Amplitudes for Kolmogorov Turbulence

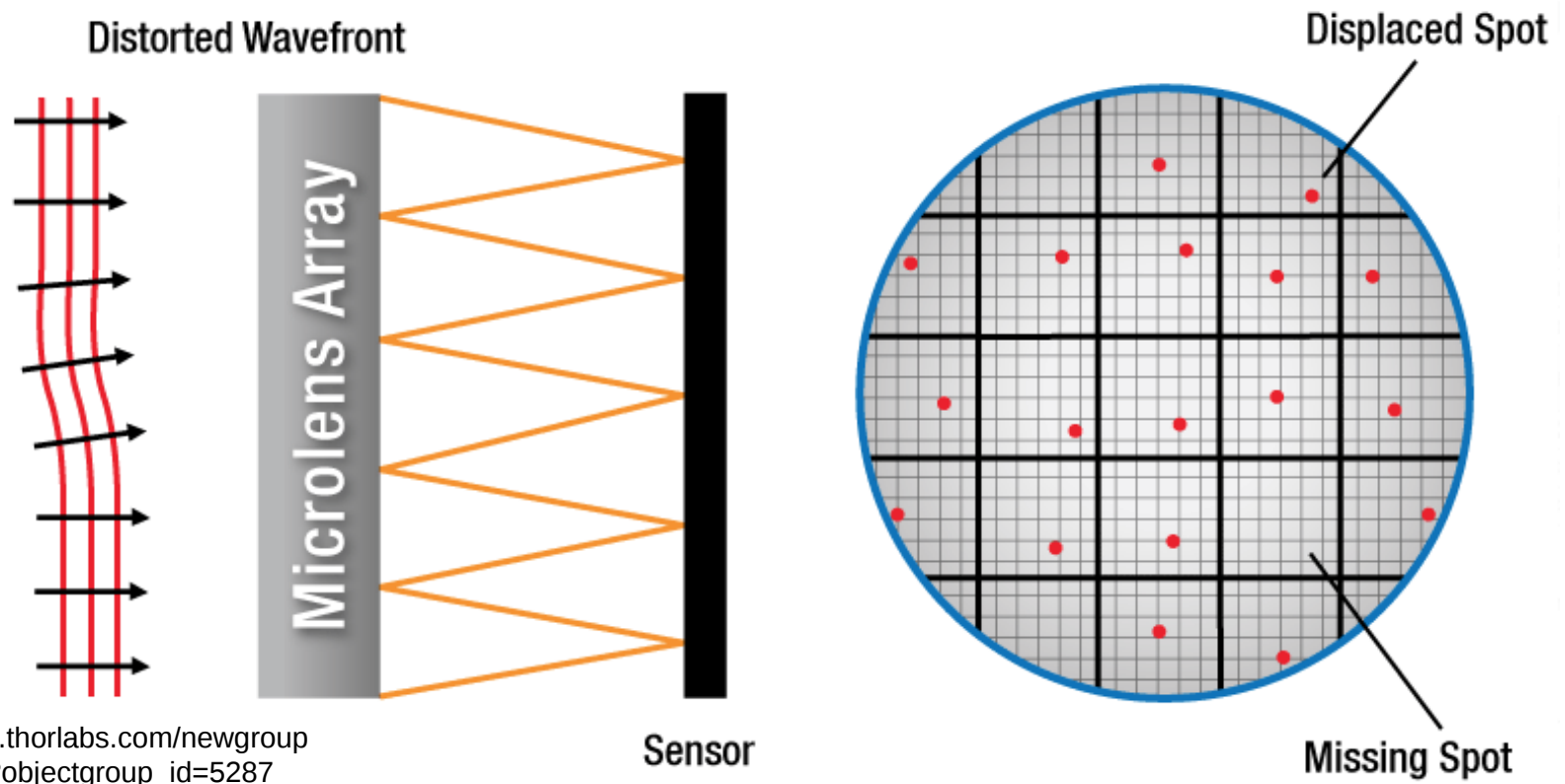


Reference: Noll

Wavefront Sensors – Shack Hartmann

Most common principle is the [Shack Hartmann](#) wavefront sensor measuring sub-aperture tilts:



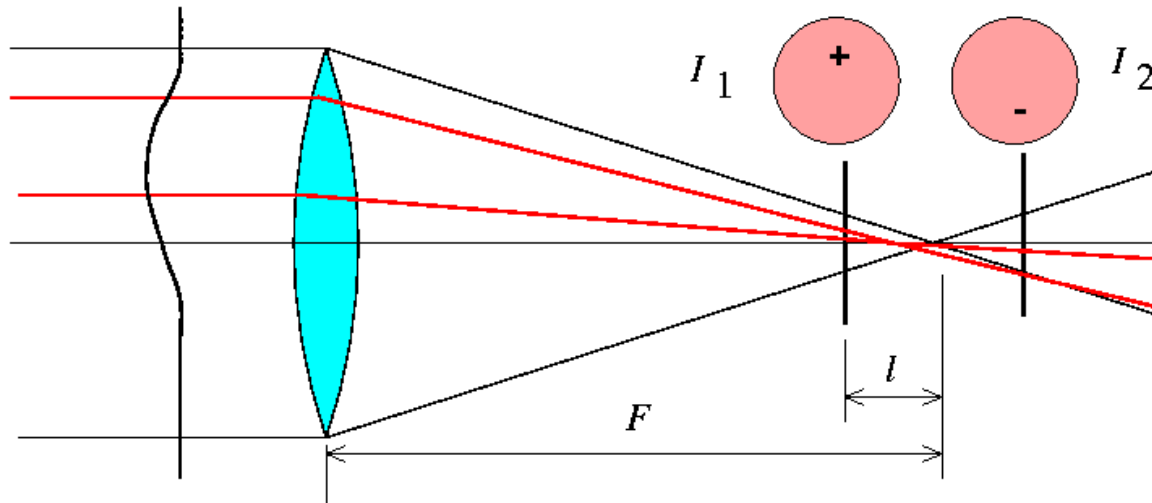


https://www.thorlabs.com/newgroup/page9.cfm?objectgroup_id=5287

A **Shack–Hartmann wavefront sensor (SHWFS)** is an optical instrument commonly used in [adaptive optics](#) systems. It consists of an array of lenses (called lenslets) of the same focal length. Each is focused onto a photon sensor (typically a [CCD](#)). The local tilt of the [wavefront](#) across each lens can then be calculated from the position of the focal spot on the sensor. Any phase [aberration](#) can be approximated by a set of discrete tilts. By sampling an array of lenslets, all of these tilts can be measured and the whole wavefront approximated.

https://en.wikipedia.org/wiki/Shack–Hartmann_wavefront_sensor

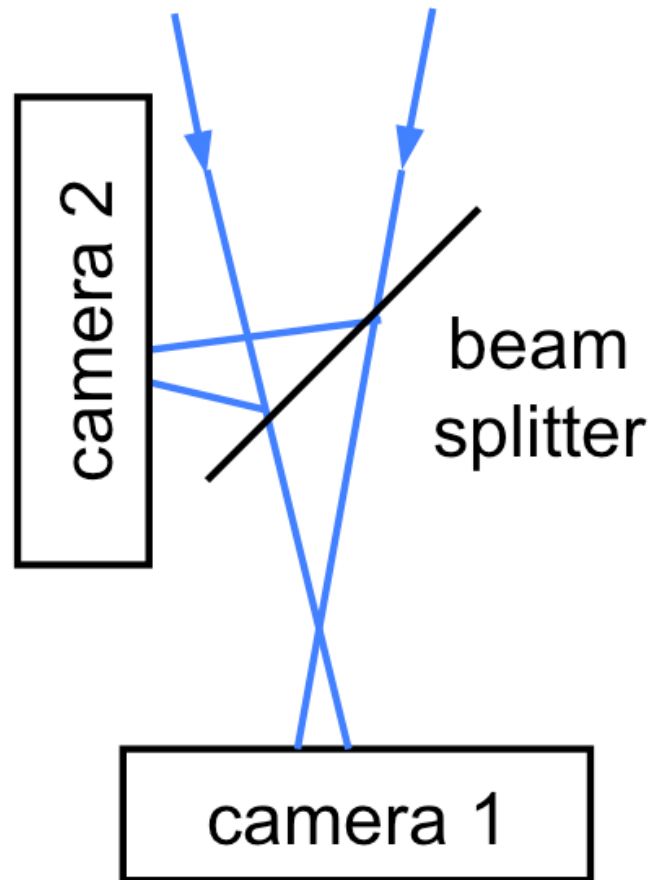
Curvature Wavefront Sensor



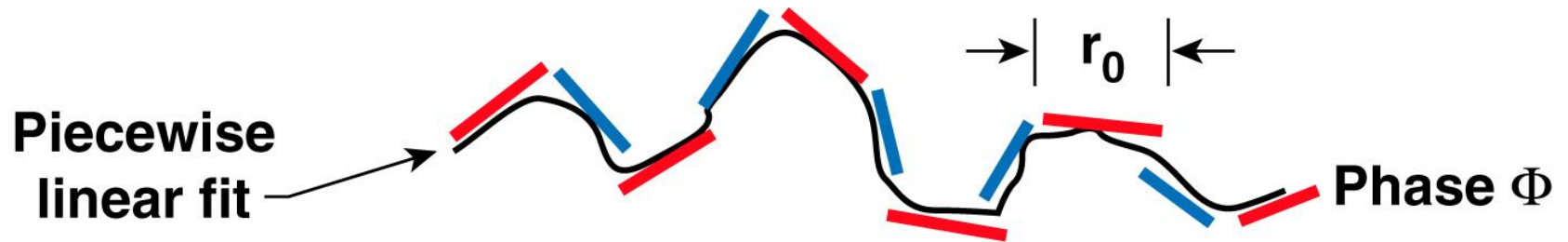
<http://www.ctio.noao.edu/~atokovin/tutorial/part3/wfs.html>

- two images, one on each side of the focus
- difference of images is proportional to wavefront curvature

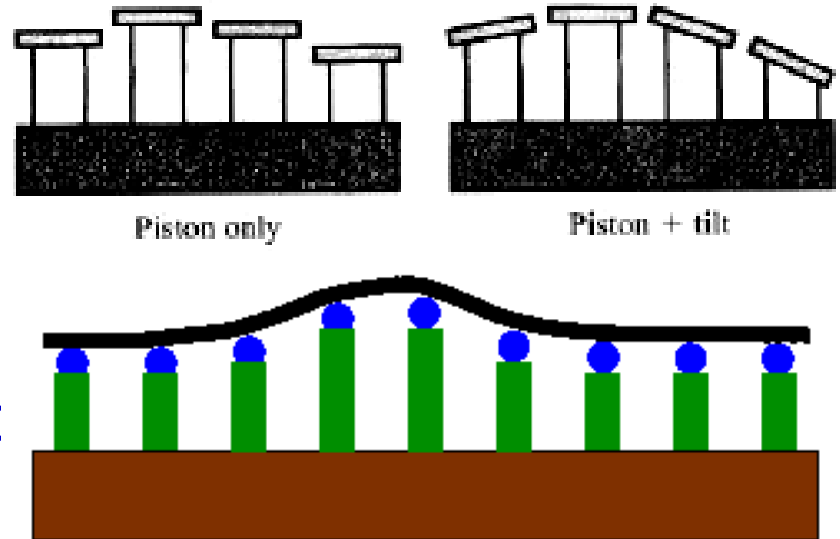
An implementation Curvature Wavefront Sensor



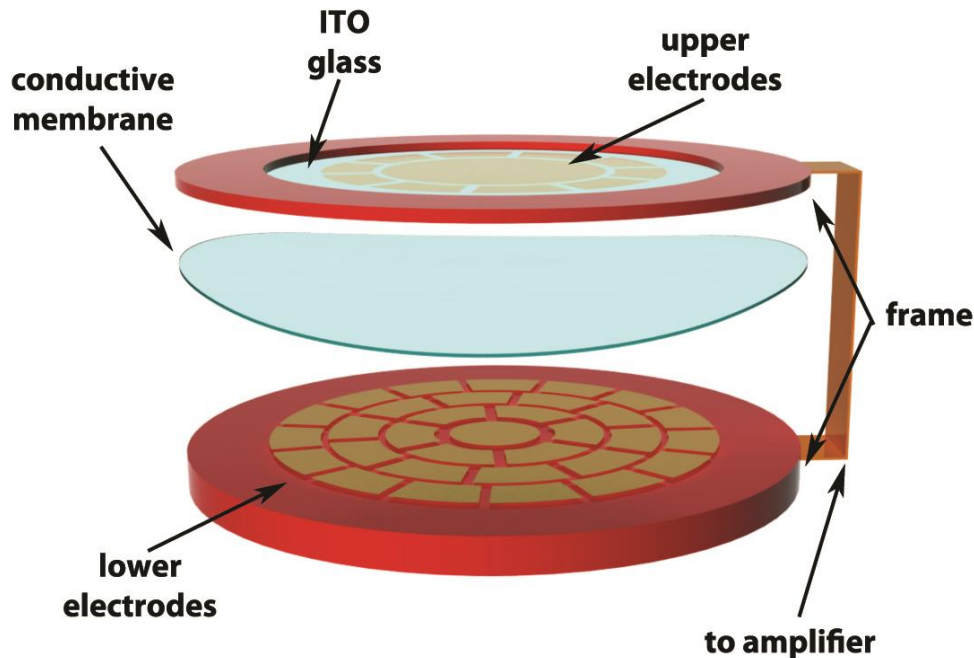
Deformable Mirrors (DM)



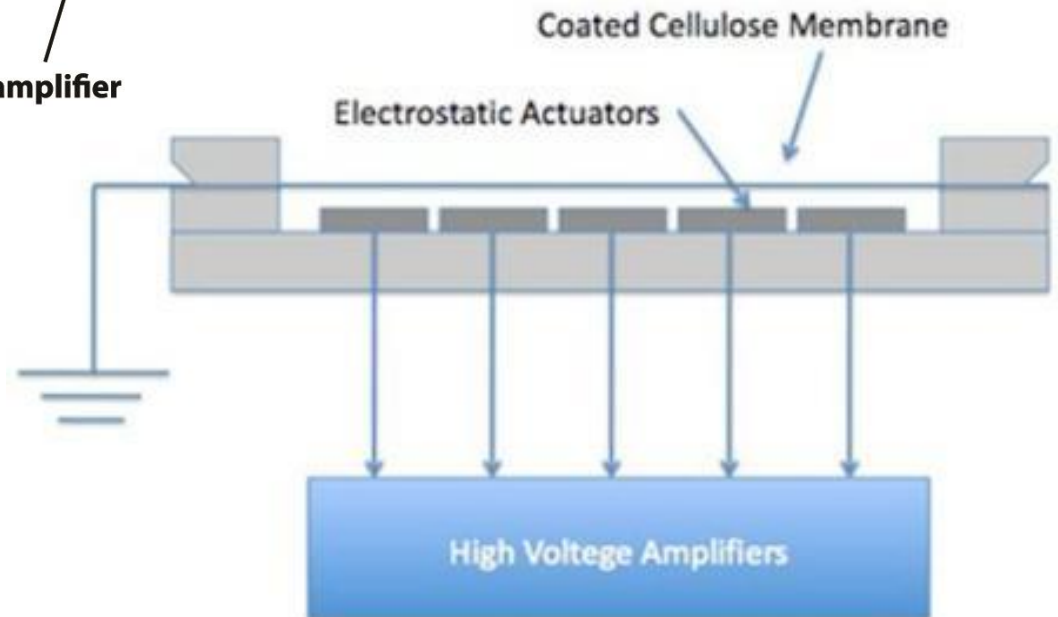
- Fit mirror surface to wavefront
- r_0 sets number of **degrees of freedom**
- segmented mirrors rarely used anymore
- mostly **continuous face-sheet mirrors**



Membrane Deformable Mirror



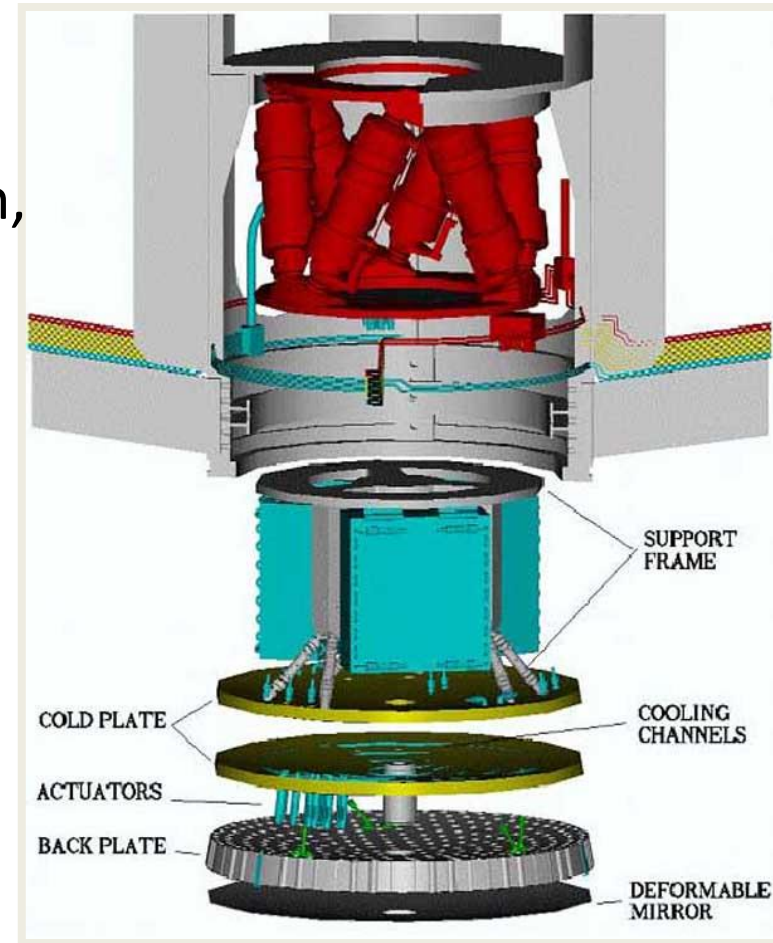
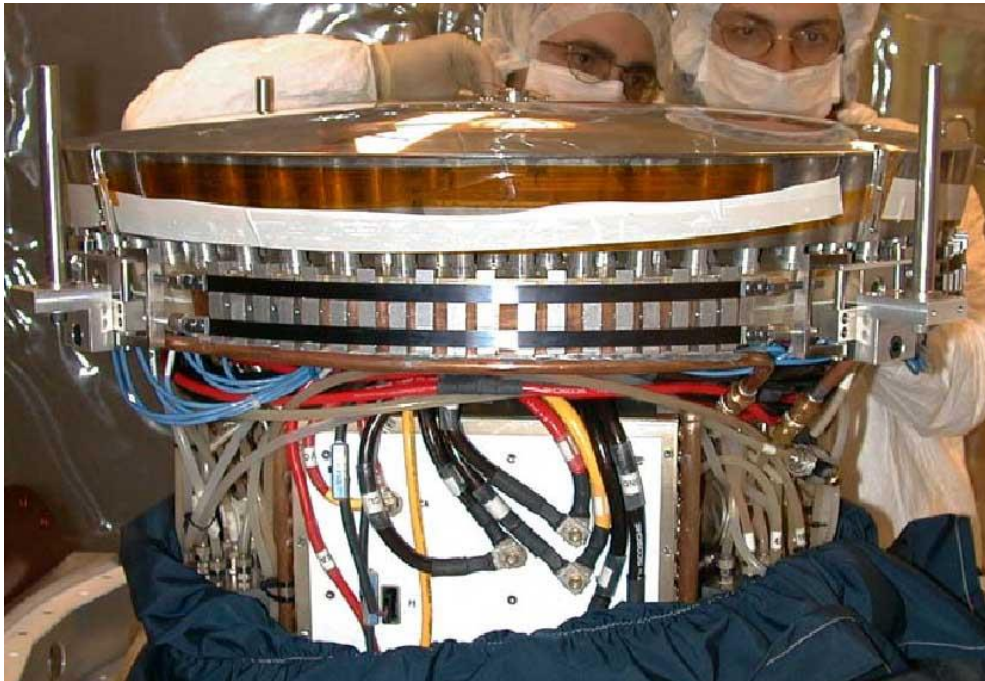
<https://optica.weblog.tudelft.nl/2012/02/14/new-publication-on-method-to-control-deformable-mirrors/>



<https://www.azooptics.com/Article.aspx?ArticleID=353>

Adaptive Secondary Mirrors

- DM part of telescope → **adaptive secondary mirrors**
- no additional optics → lower emission, higher throughput
- more difficult to build, control, and handle



DM for MMT Upgrade

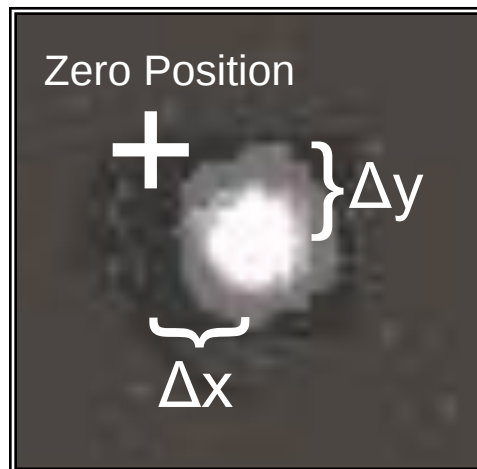
Shack-Hartmann in practice

Wavefront Analysis

- centroid (center of gravity) calculation on each subaperture:

$$D_x = \frac{\sum_{i=i_{\min}}^{i_{\max}} \sum_{j=j_{\min}}^{j_{\max}} I(i, j) \cdot i}{\sum_{i=i_{\min}}^{i_{\max}} \sum_{j=j_{\min}}^{j_{\max}} I(i, j)}$$

$$D_y = \frac{\sum_{i=i_{\min}}^{i_{\max}} \sum_{j=j_{\min}}^{j_{\max}} I(i, j) \cdot j}{\sum_{i=i_{\min}}^{i_{\max}} \sum_{j=j_{\min}}^{j_{\max}} I(i, j)}$$



Influence Matrix

- slope of local wave front *and* Shack-Hartmann star positions are proportional to actuator position
- linear relationship between position actuators a_k and star positions c_n as determined by the x- and y-offsets

$$c_n = \sum_{k=1}^N a_k b_{nk}$$

- combine equations for each spot position n into matrix equation:

$$C = BA$$

C = star positions

A = control vector determining actuator positions

B = **influence matrix** describing influence of specific actuator position on star positions

Determining the Control Vector

- Find control vector A to correct for error in wavefront

$$A = B^{-1}C$$

- Overdetermined system
 - More centroid measurements than actuators
 - No exact solution A exists for given set of centroids
 - No exact B^{-1} exists (B is rectangular)
- Approximate B^{-1} in best possible way to minimize wavefront errors

Typical AO Error Terms

- **Fitting errors** from insufficient approximation of wavefront by deformable mirror due to finite number of actuators (d : actuator spacing)
- **Temporal errors** from time delay between measurement and correction, mostly due to exposure and readout time
- **Measurement errors** from wavefront sensor
- **Calibration errors** from non-common aberrations between wavefront sensing optics and science optics
- **Angular anisoplanatism** from sampling different lines of sight through the atmosphere, mostly limits field of view

$$S_{fit}^2 \approx 0.3 \left(\frac{d}{r_0} \right)^{5/3}$$

$$\sigma_{temp}^2 \approx \left(\frac{t}{\tau_0} \right)^{5/3}$$

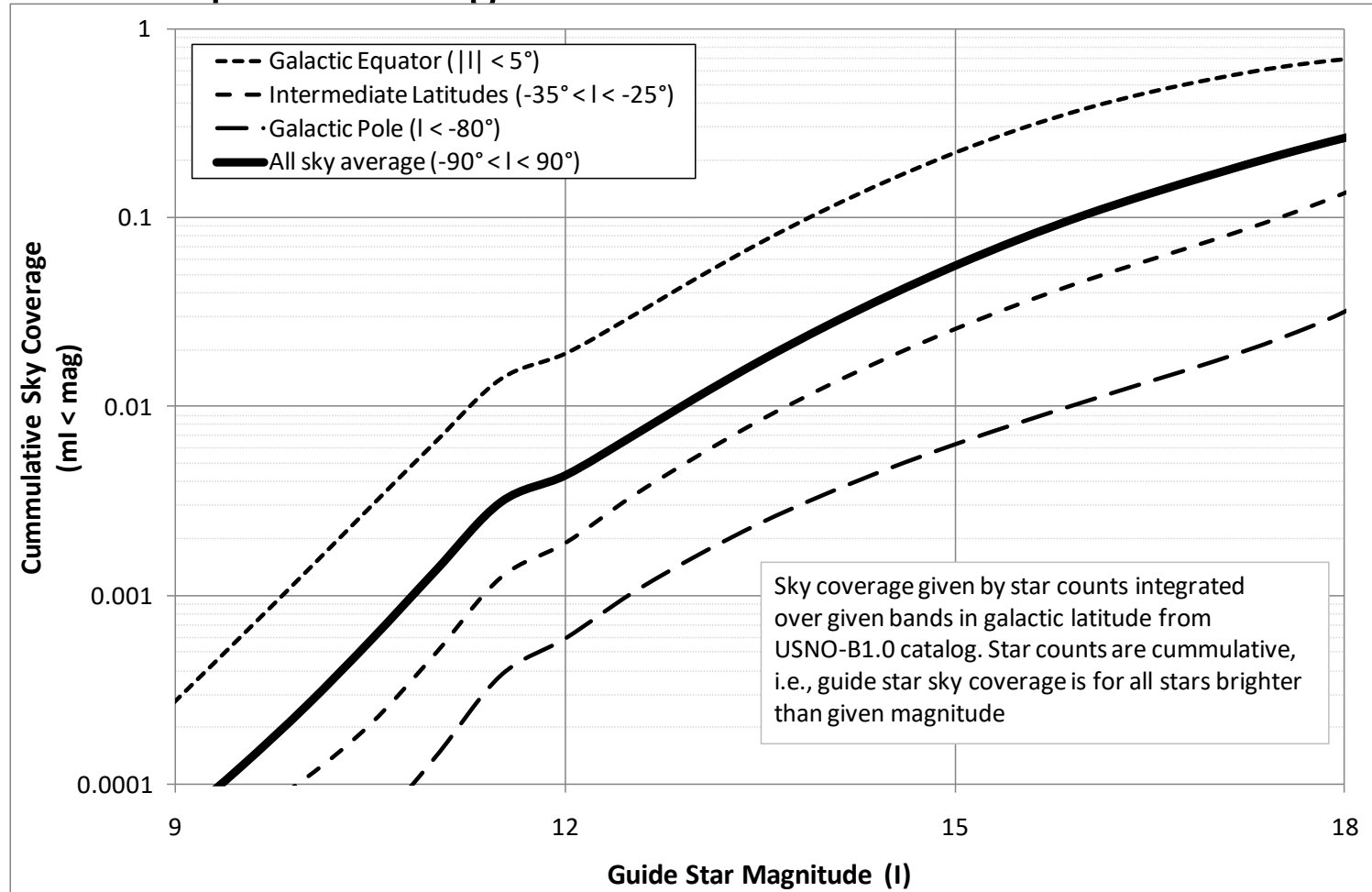
$$\sigma_{measure}^2 \sim S / N$$

$$S_{calibration}^2 \sim ???$$

$$\sigma_{aniso}^2 \approx \left(\frac{\theta}{\theta_0} \right)^{5/3}$$

Sky Coverage

To sense the wavefront one needs a bright reference/guide star within the isoplanatic angle.



Cumulative sky coverage, i.e., the chance of finding stars brighter than given magnitude, for a random target as a function of I-band magnitude using the USNO-B1.0 catalogue.

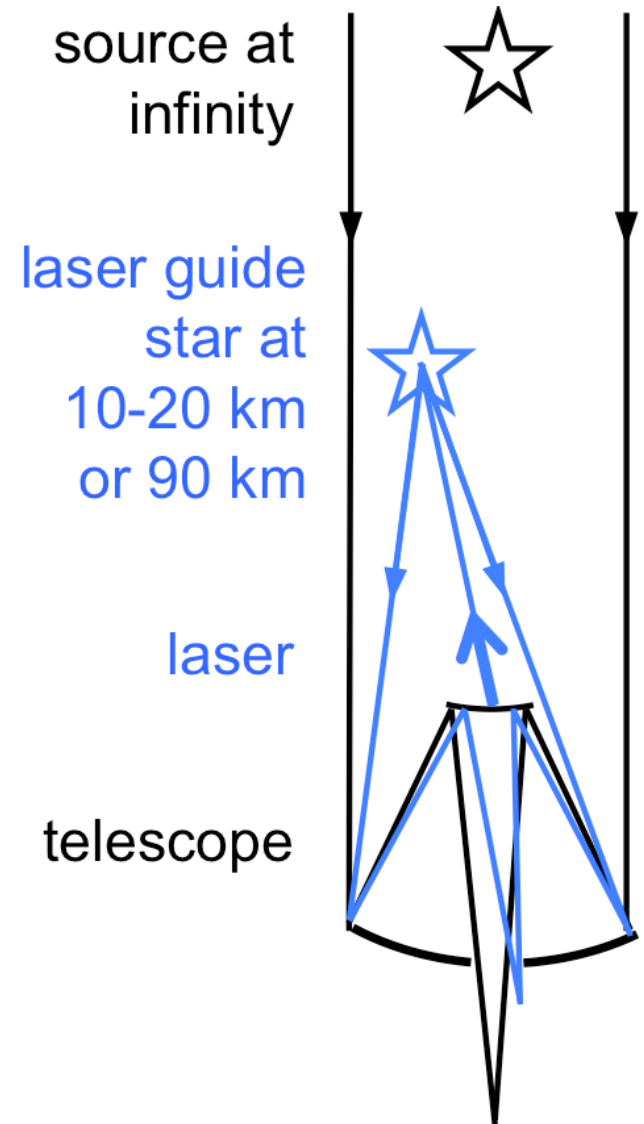
Laser Guide Stars (LGSs)

Solution to the sky coverage problem:

create your own guide star

- **Sodium LGS** – excite atoms in “sodium layer” at altitude of ~ 95 km.
- **Rayleigh beacon LGS** – scattering from air molecules sends light back into telescope, $h \sim 10$ km

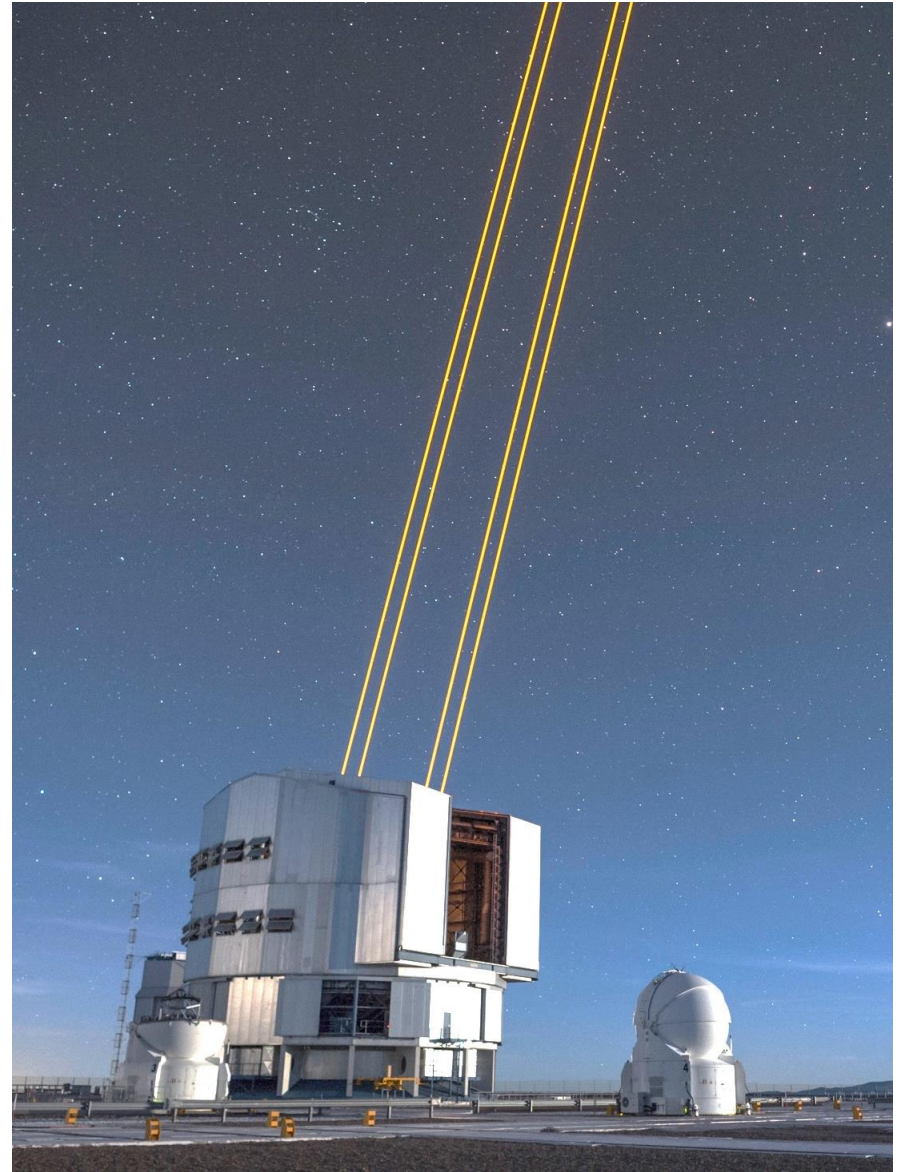
Since the beam travels twice (up and down) through the atmosphere, tip-tilt cannot be corrected $\rightarrow$ LGS-AO **still needs a natural guide star**, but this one can be **much fainter (~ 18 mag)** as it is only needed for tip-tilt sensing



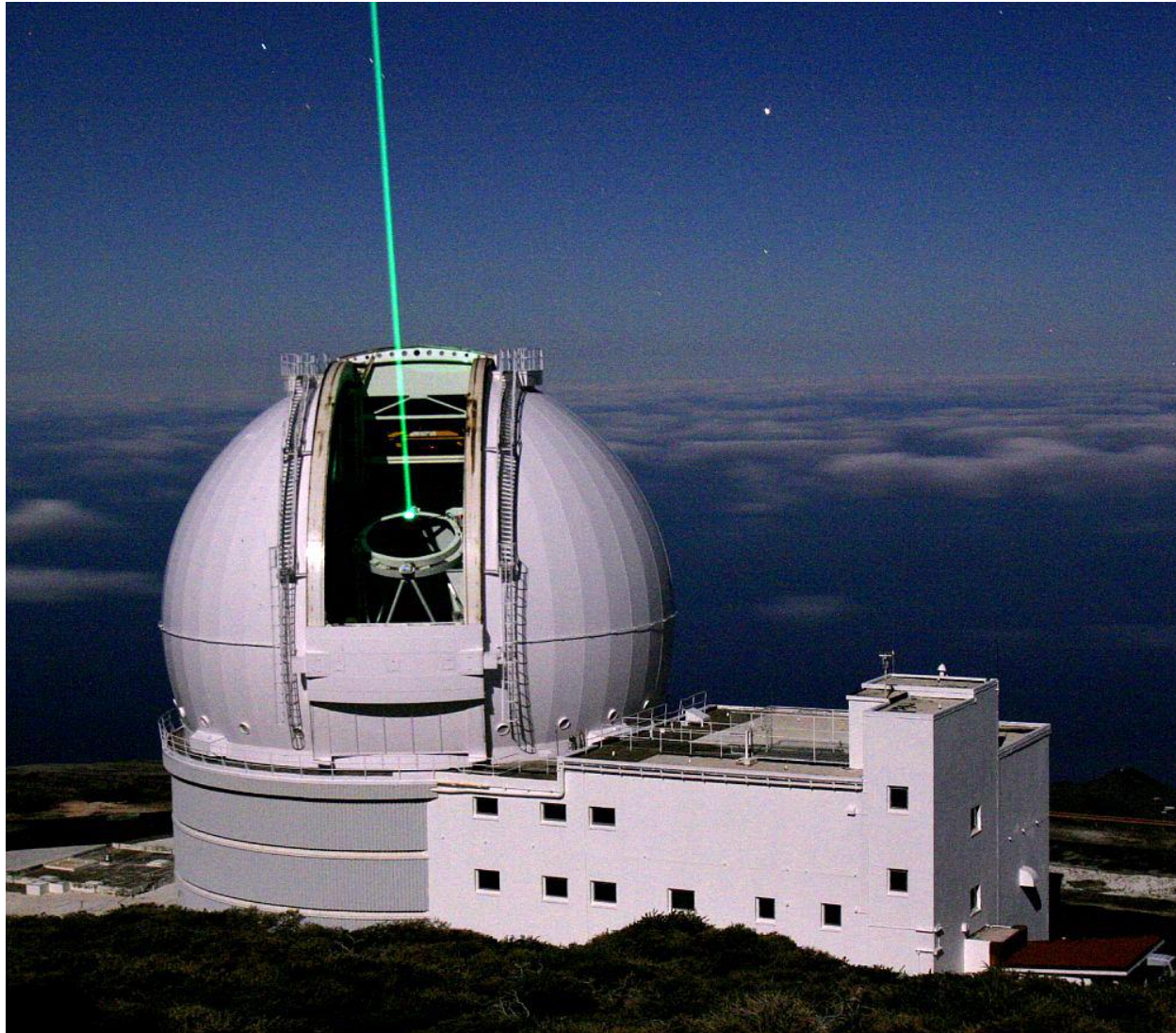
Sodium Beacons

- Layer of neutral sodium atoms in mesosphere (height ~ 95 km, thickness ~ 10 km) from smallest meteorites
- **Resonant scattering** occurs when incident laser is tuned to D2 line of Na **at 589 nm**.

www.eso.org/public/images/eso1613n/

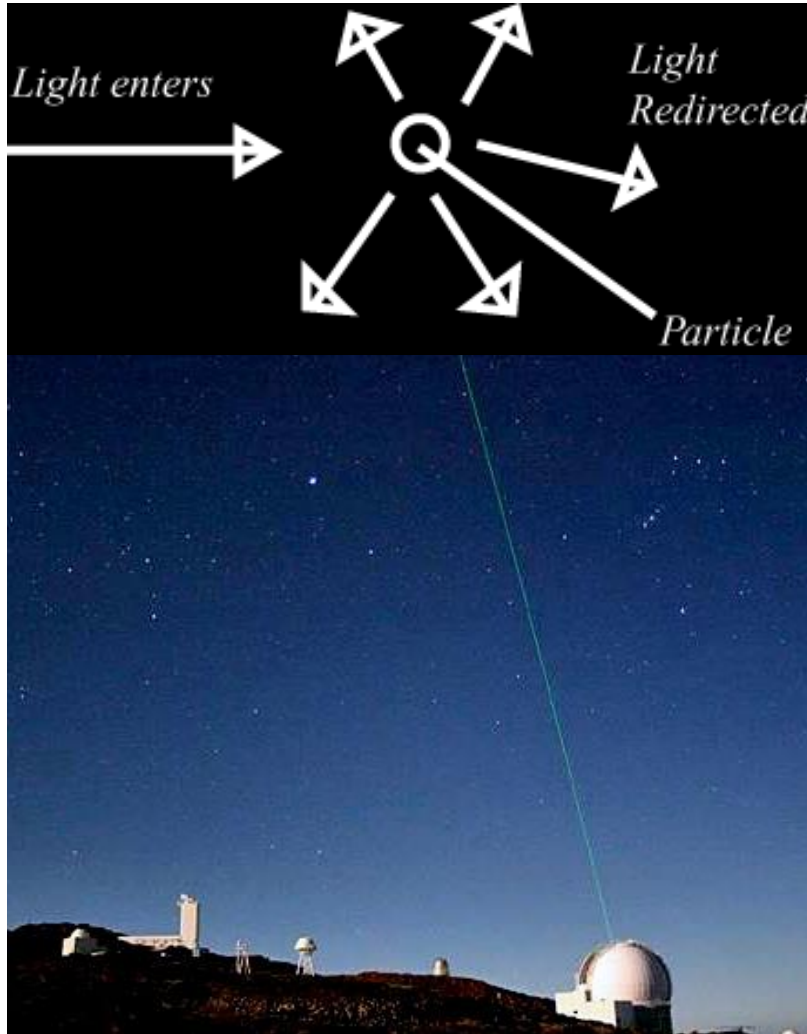


WHT Rayleigh Guide Star



Rayleigh Beacons

Due to **interactions** of the electromagnetic wave from the laser beam with **molecules in the atmosphere**.



Advantages:

- cheaper and easier to build
- higher power
- independent of Na layer

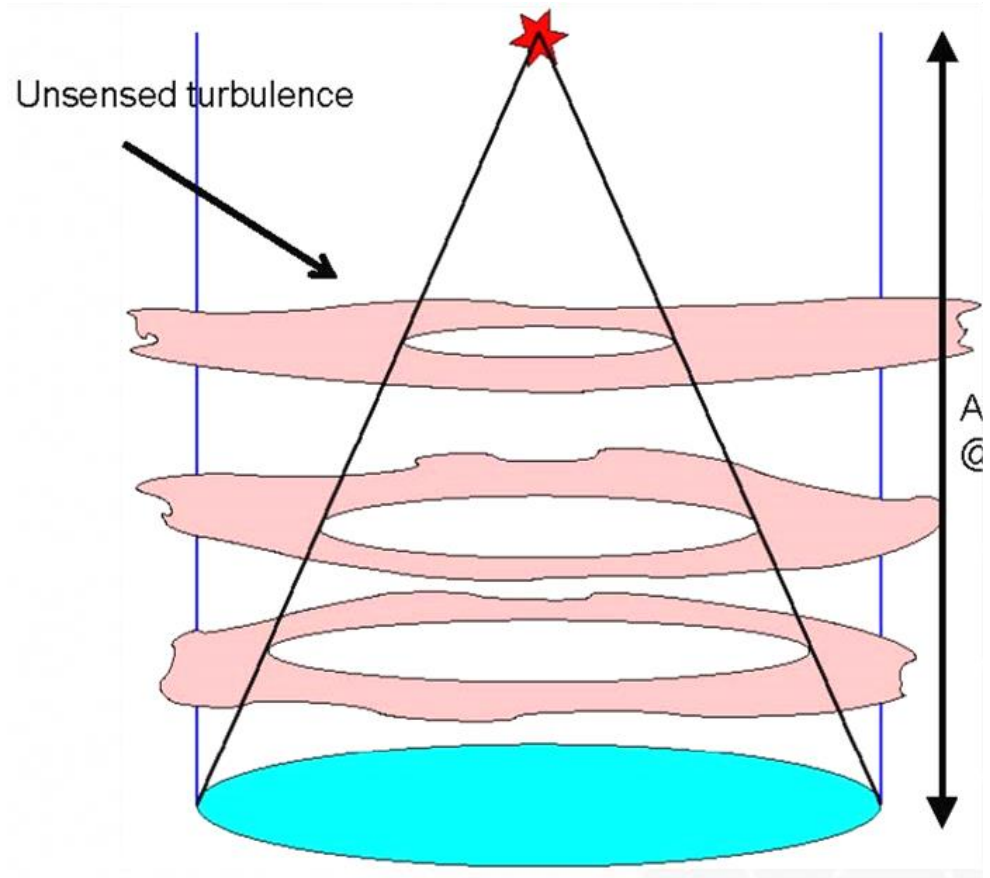
Disadvantages:

- larger **focus anisoplanatism**
- laser pulses → timing

Focus Anisoplanatism

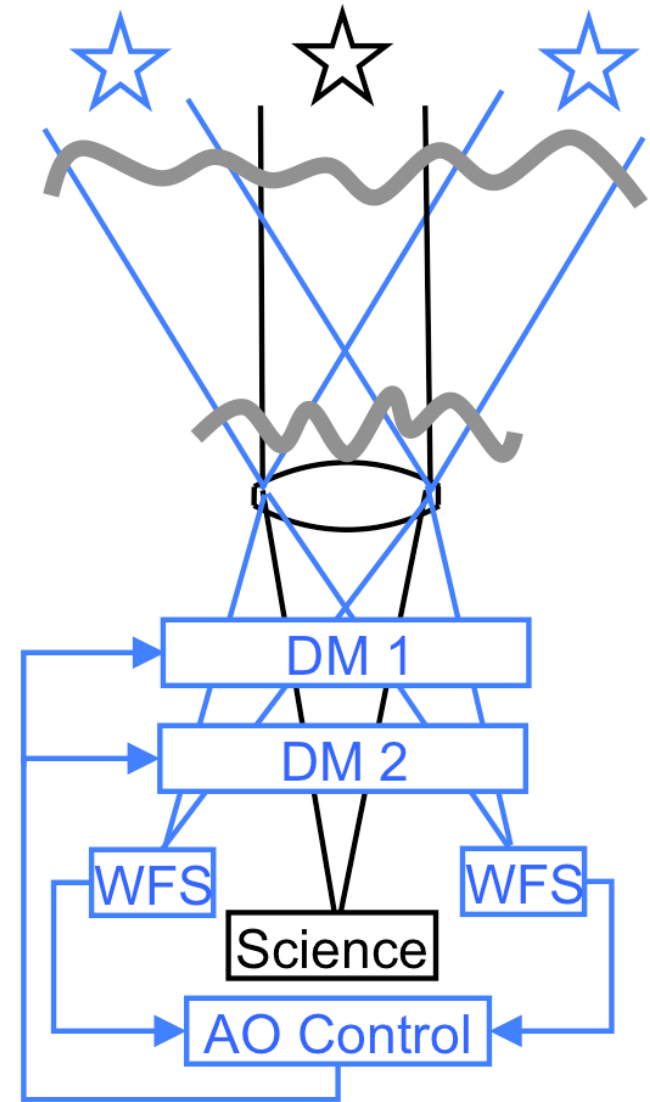
The LGS is at finite distance H above the telescope and does not sample all turbulence and not the same column of turbulent atmosphere (“cone effect”):

→ very large telescopes need multiple LGSs due to this cone effect.



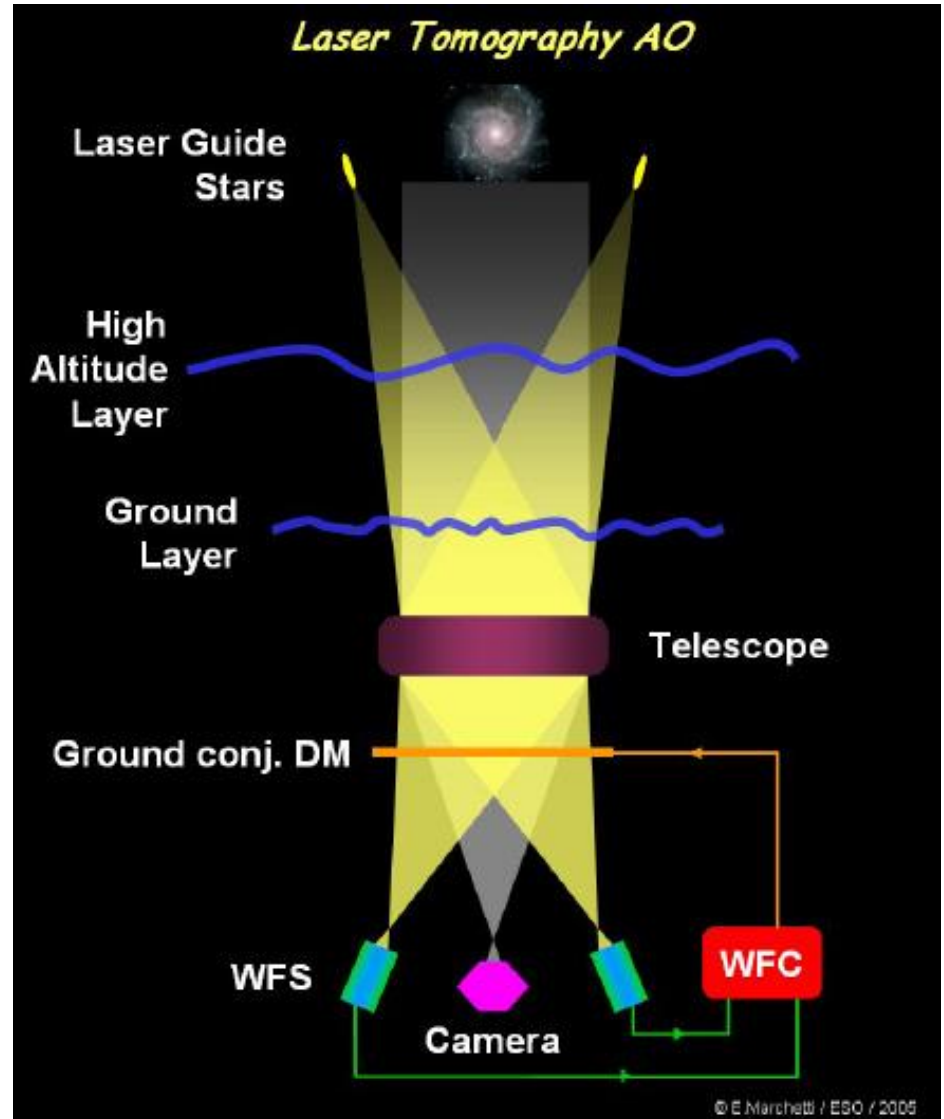
Multi-Conjugate AO – MCAO

- to overcome anisoplanatism, the basic limitation of single guide star AO
- MCAO uses multiple natural guide stars or laser guide stars
- MCAO controls several DMs
- each DM is conjugated to a different atmospheric layer at a different altitude
- at least one DM is conjugated to the ground layer
- best approach to larger corrected FOV



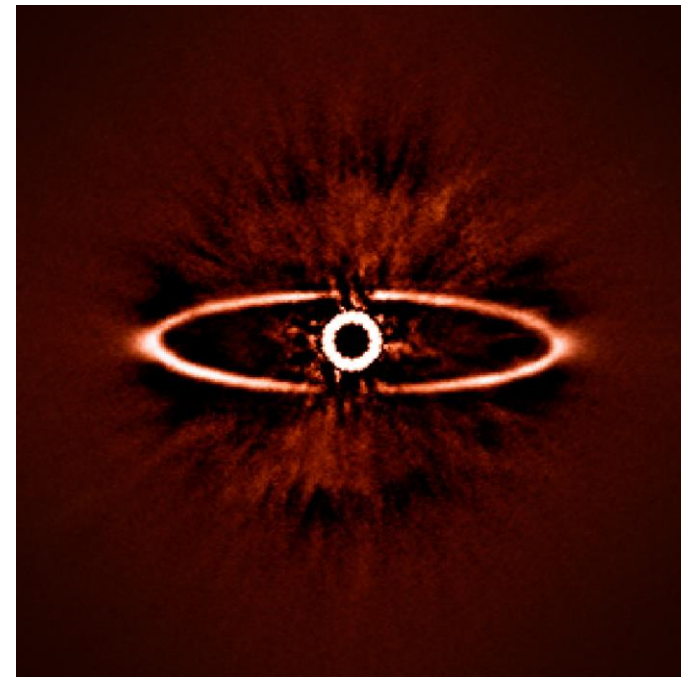
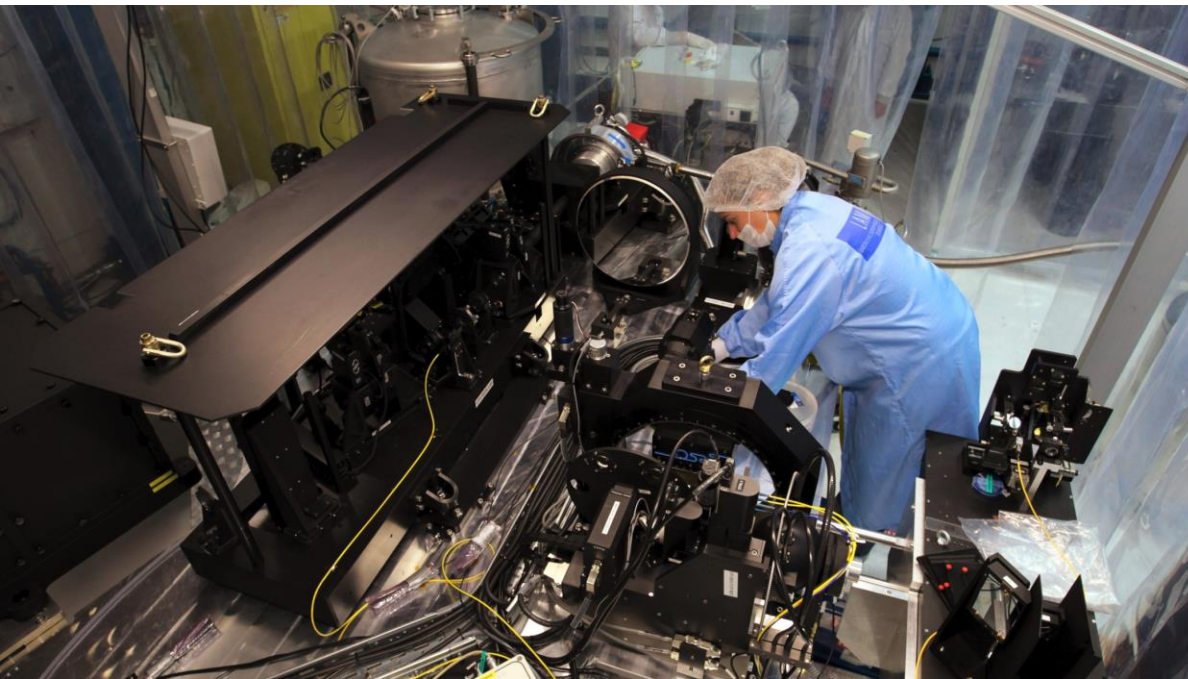
Tomography AO

- Uses multiple laser beacons or reference star
- combined information is used to optimize the correction by one DM on-axis.
- reduces the cone effect
- system performance similar to natural guide star AO but at much higher sky coverage.



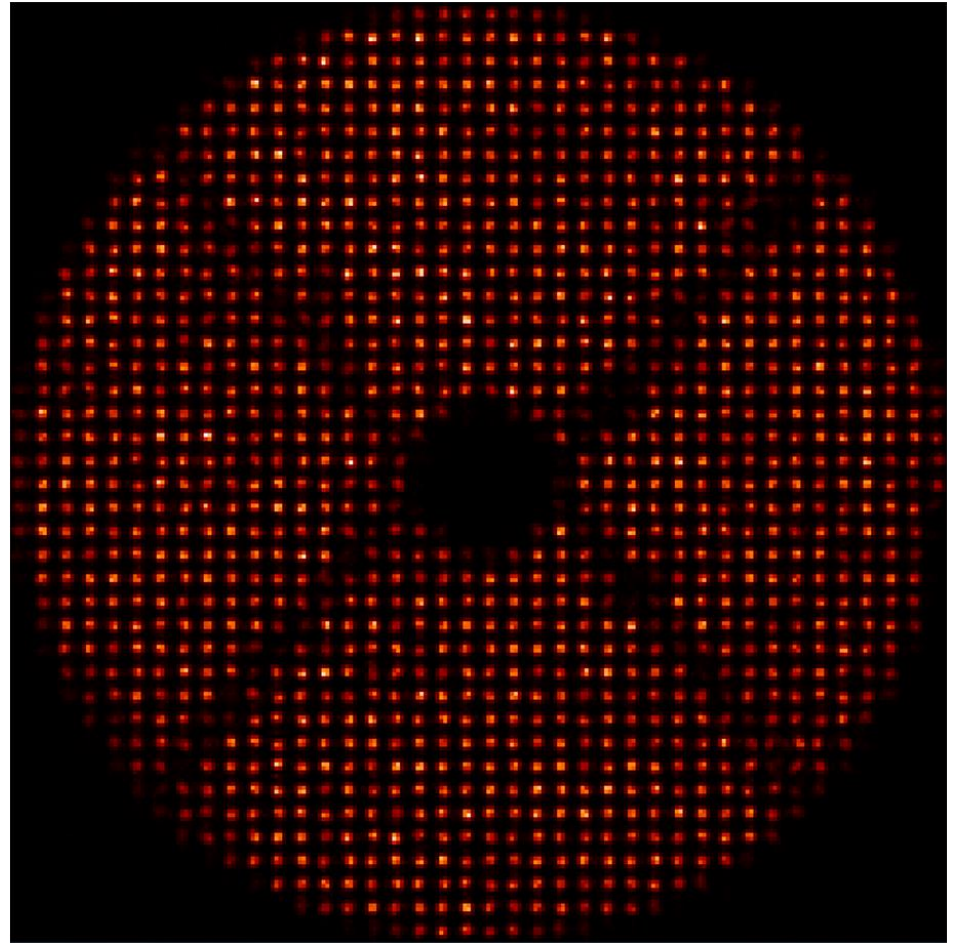
Extreme AO – XAO

- high Strehl ratio on-axis and small corrected FOV
- requires thousands of DM actuators
- requires minimal optical and alignment errors
- main application: search for exoplanets, SPHERE on VLT, GPI on Gemini

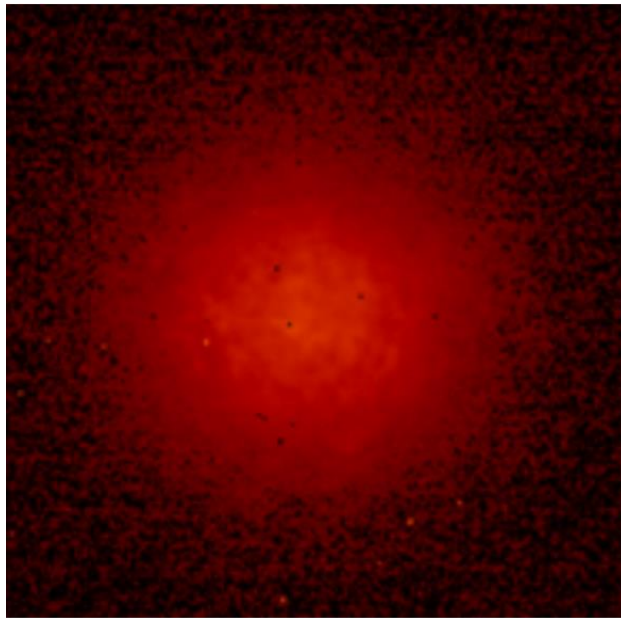


SPHERE Extreme AO

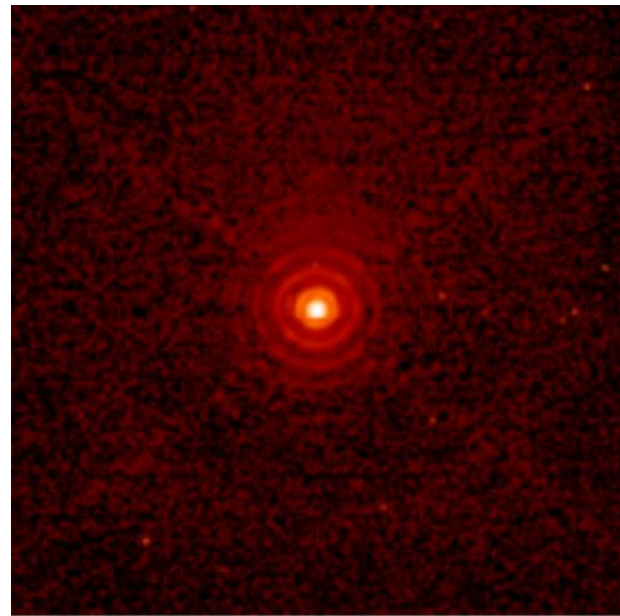
- Piezo-electric deformable mirror with 41 by 41 actuators
- Shack-Hartmann wavefront sensor with 40 by 40 subapertures
- EMCCD with 240 by 240 pixels, >1200Hz frame rate, <0.1e⁻ readout noise



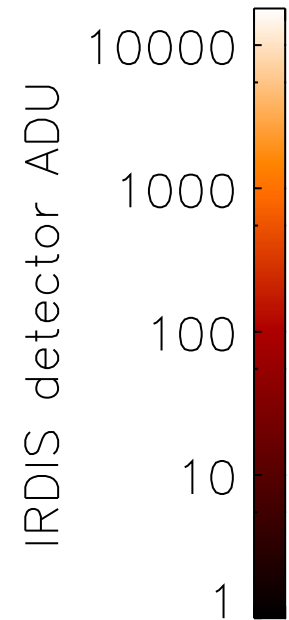
SPHERE



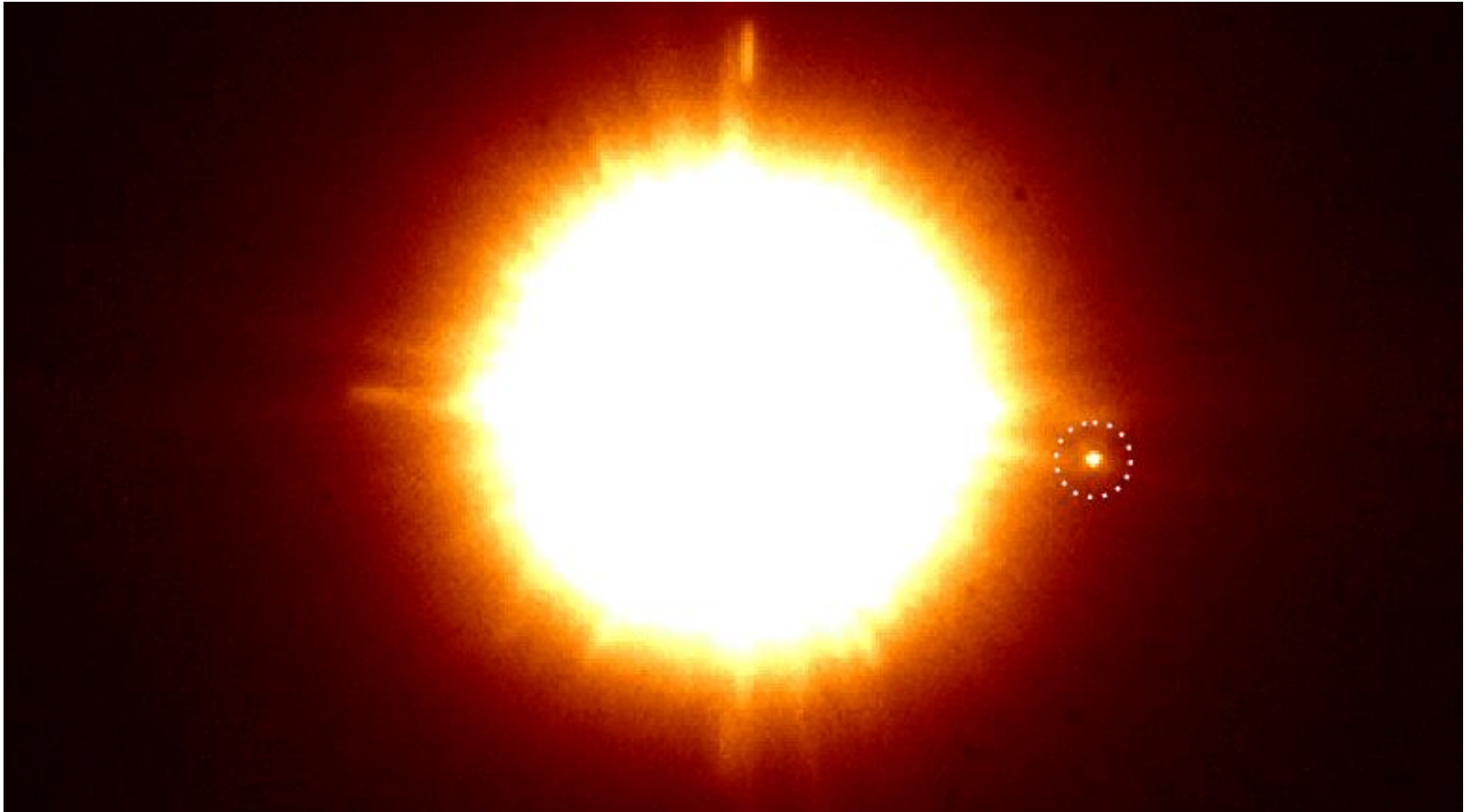
Seeing limited image
 $5.2 \pm 2\%$ SR



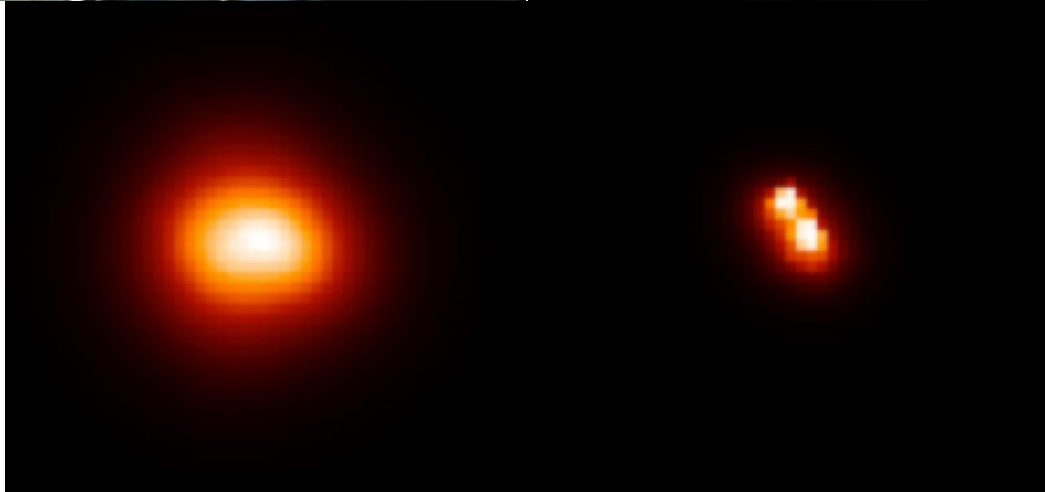
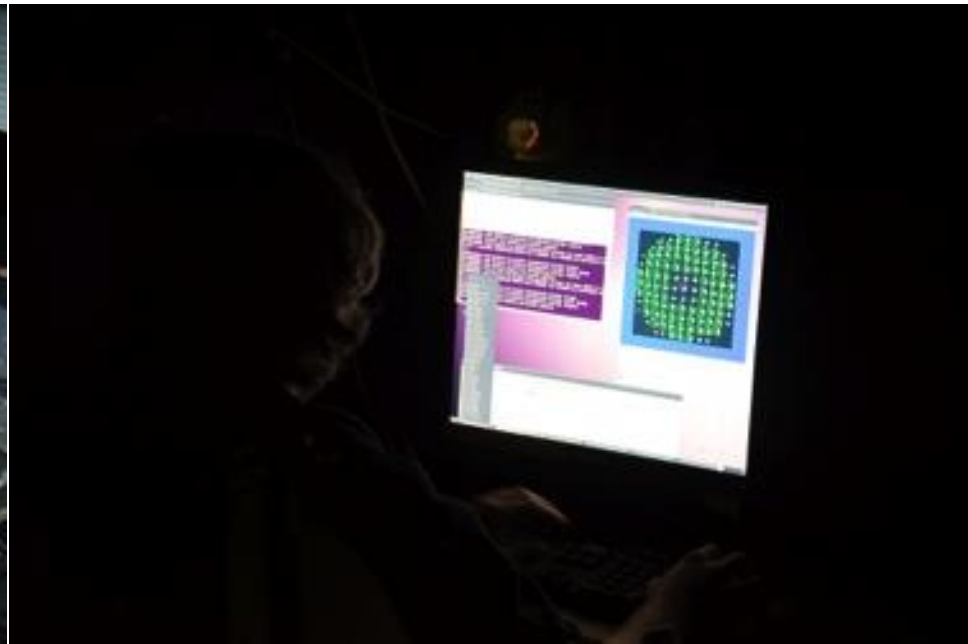
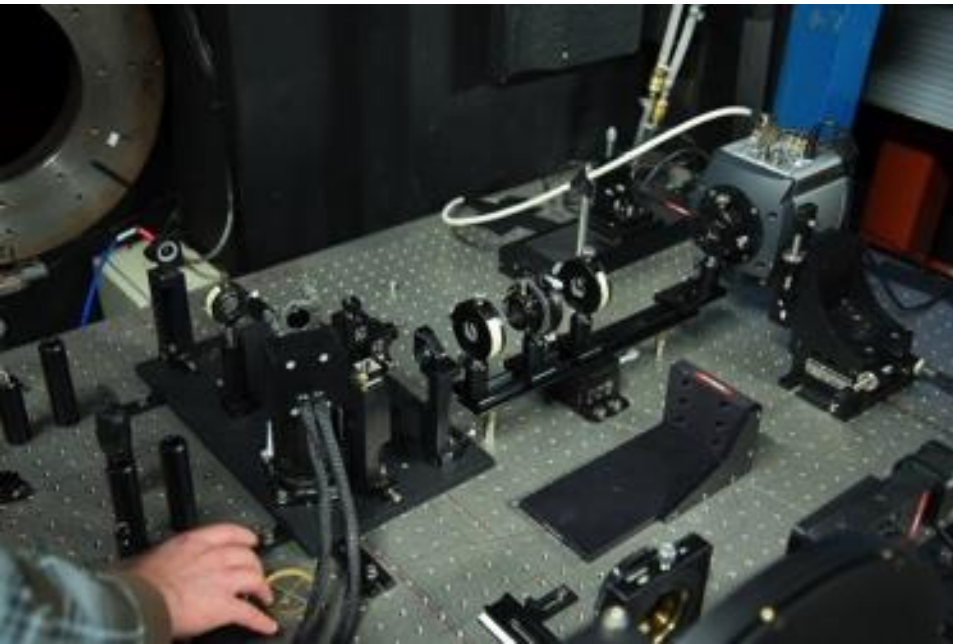
AO corrected image
 $90.3 \pm 2\%$ SR



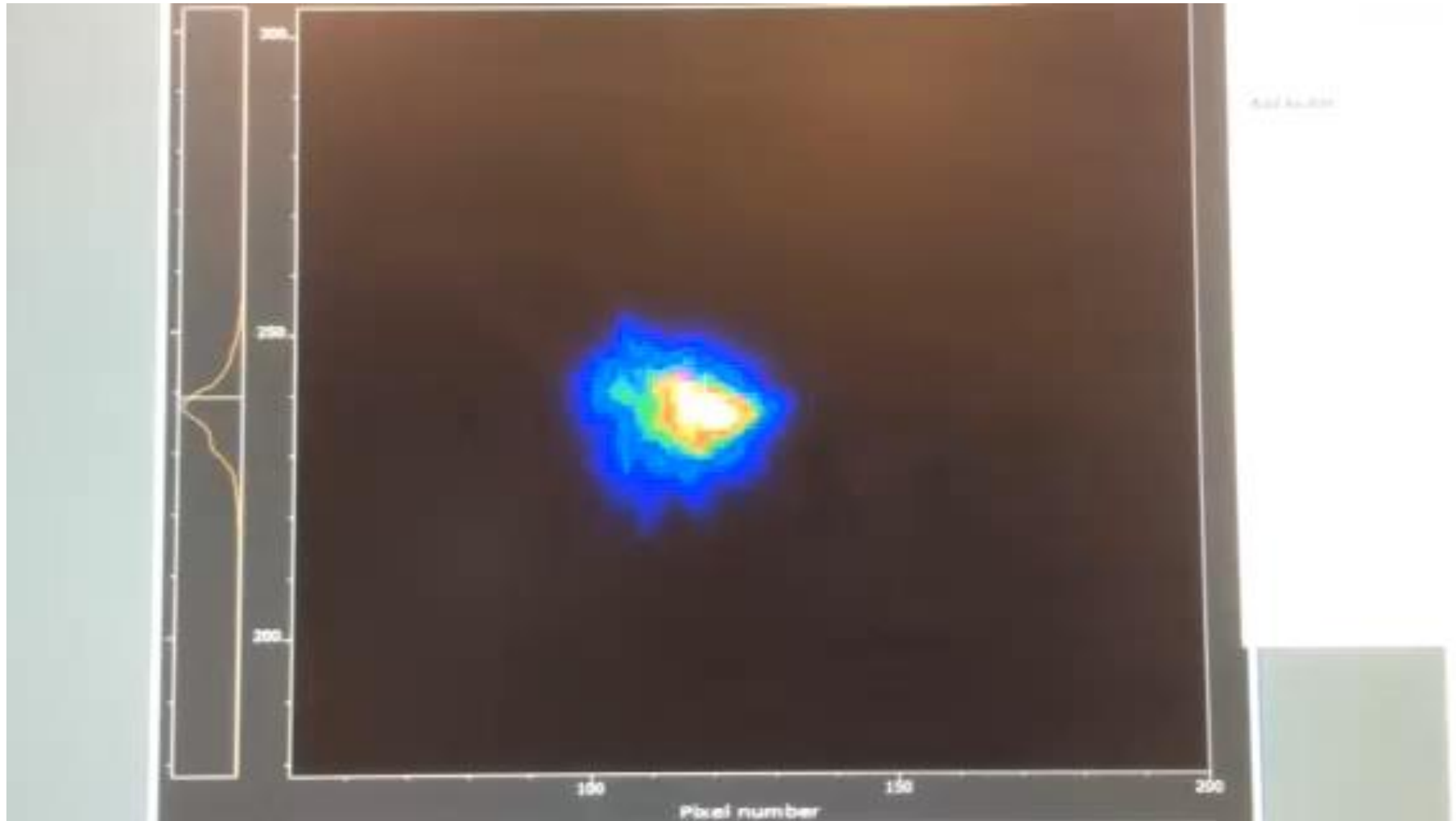
CS Cha (Ginski et al. 2018)



Student-Built ExPo Adaptive Optics



ExPo Adaptive Optics at WHT



MSc Astronomy & Instrumentation

- lectures:
 - Astronomical Telescopes and Instruments
 - Detection of Light
 - High Contrast Imaging
 - Project Management
- option to take courses at TU Delft (not required)
- option for major research thesis in industry

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