Astronomical Observing Techniques 2019

Lecture 3: Everything You Always Wanted to Know About Optics

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1. Waves
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Waves from Maxwell Equations

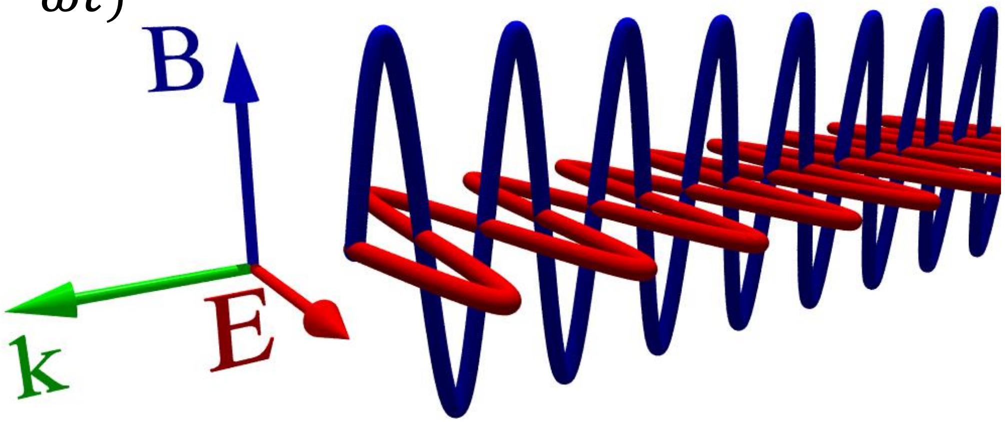
- Maxwell equations & linear material equations: differential equation for damped waves

$$\nabla^2 \vec{E} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{E}}{\partial t} = 0$$

- $\vec{E}$ electric field vector
- Material properties:
  - ε dielectric constant
  - μ magnetic permeability ($\mu = 1$ for most materials)
  - σ electrical conductivity (controls damping)
- c speed of light
- Same equation for magnetic field H

Plane Waves

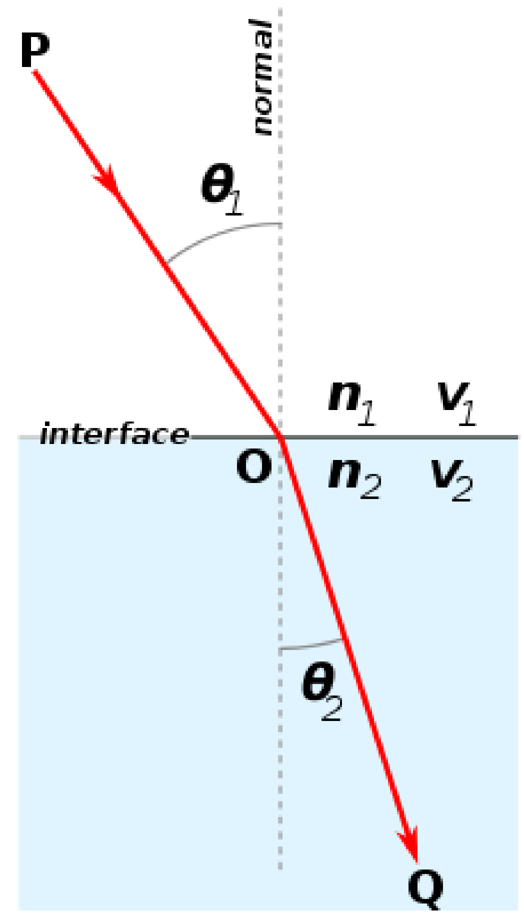
- Linear equation: sum of solutions is also solution
- Plane wave $\vec{E} = \overrightarrow{E_0} e^{i(\vec{k}\cdot\vec{x}-\omega t)}$
  - $\overrightarrow{E_0}$ complex constant vector (polarization)
  - $\vec{k}$ wave vector
  - $\vec{x}$ spatial location
  - ω angular frequency



- real electric field given by real part of E
- dispersion relation: $\vec{k} \cdot \vec{k} = \frac{\omega^2}{c^2}\tilde{n}^2, \tilde{n}^2 = \mu\left(\epsilon + i\,\frac{4\pi\sigma}{\omega}\right)$
- complex index of refraction $\tilde{n} = n + ik$
- $\vec{E}, \vec{H}, \vec{k}$ form right-handed triplet of orthogonal vectors

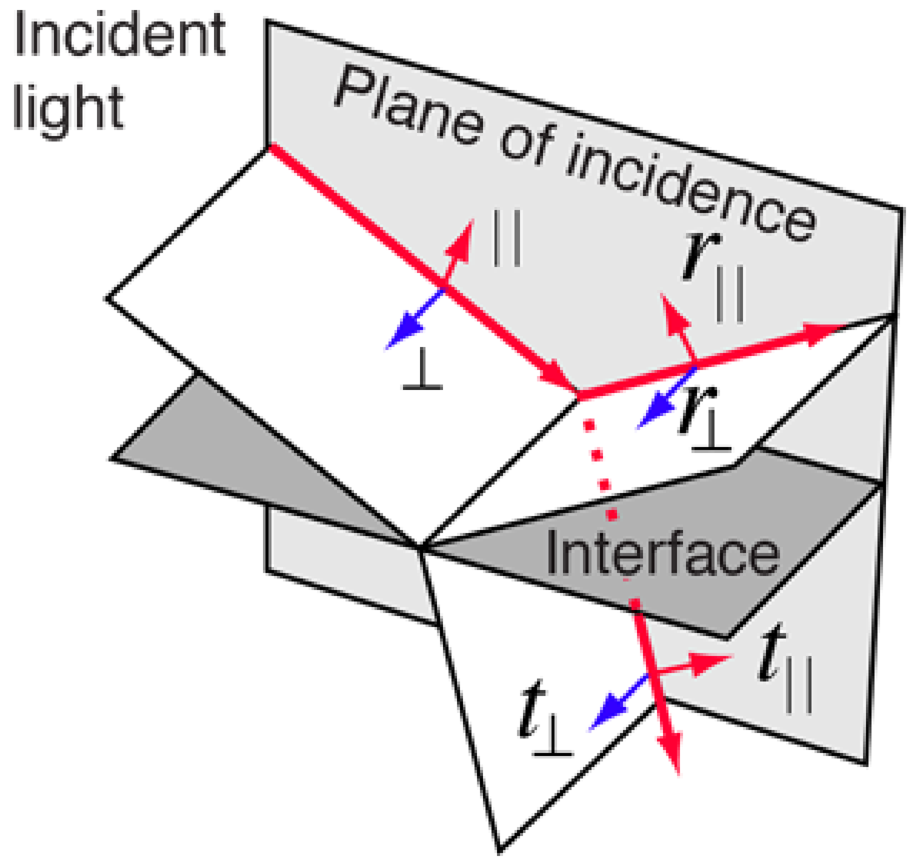
Snell's law

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$



Fresnel's equation

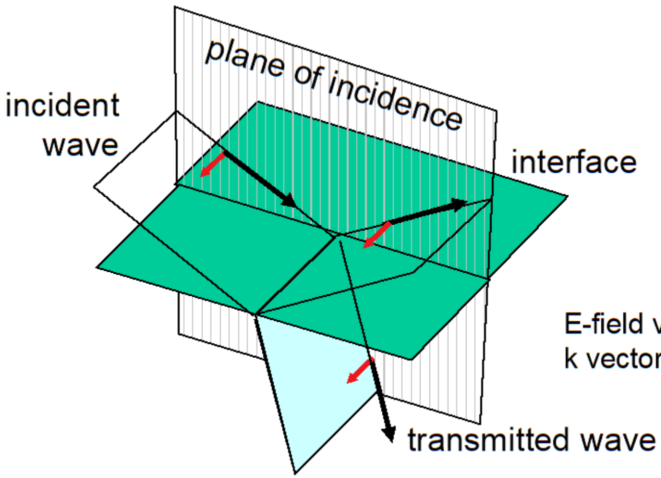
- 'snellius' for polarized plane waves

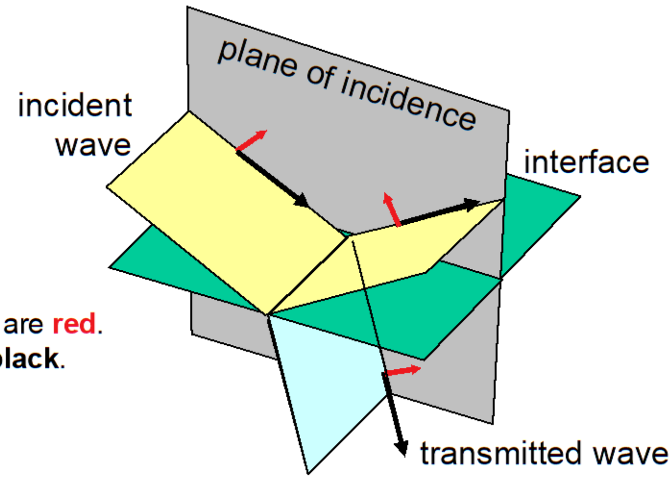


- See: https://www.brown.edu/research/labs/mittleman/sites/brown.edu.research.labs.mittleman/files/uploads/lecture13_0.pdf
- http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/freseq.html

- Given here for completeness, will (likely) not be used further

Fractions of reflected and transmitted light





E-field vectors are **red**.
k vectors are **black**.

s-polarized light:

$$r_{\perp} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_{\perp} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

p-polarized light:

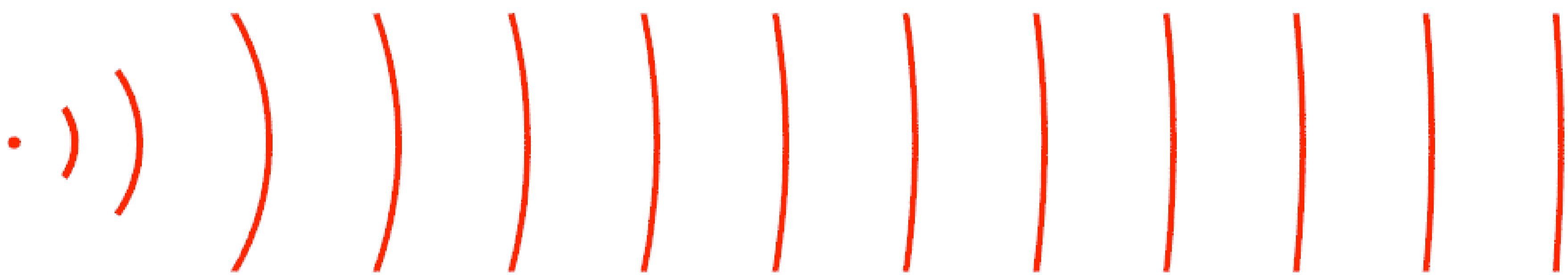
$$r_{\parallel} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

$$t_{\parallel} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

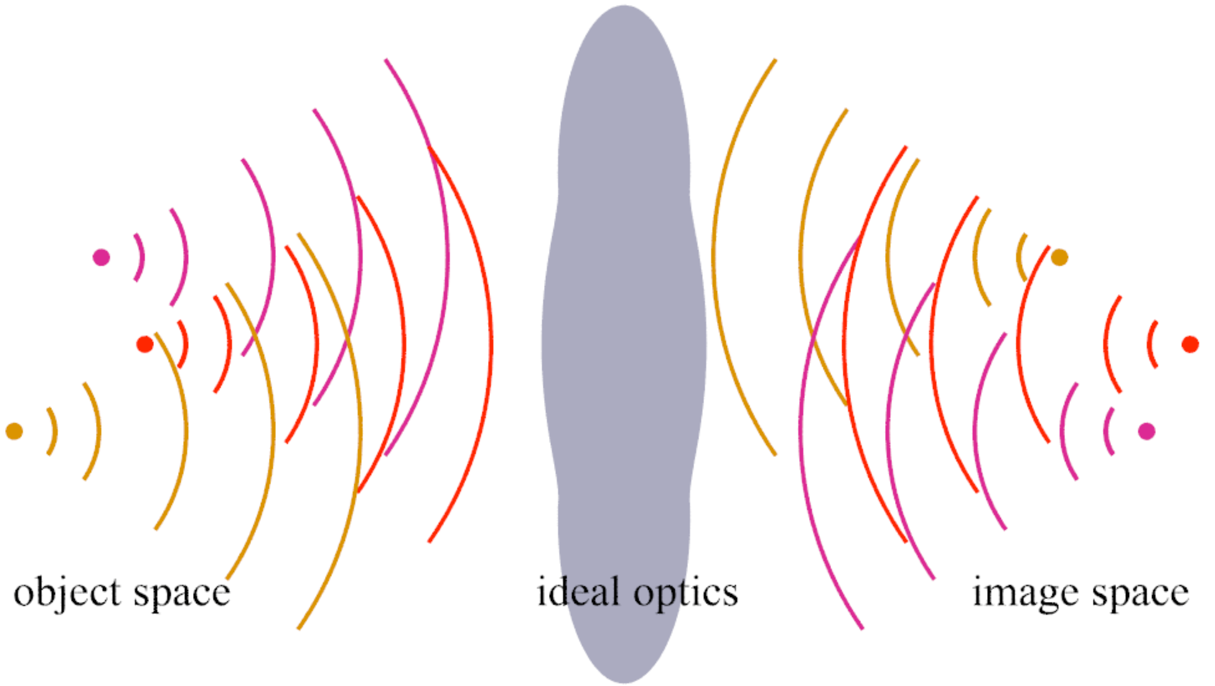
And, for both polarizations: $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

Spherical and Plane Waves

- light source: collection of sources of spherical waves
- astronomical sources: almost exclusively incoherent
- lasers, masers: coherent sources
- spherical wave originating at very large distance can be approximated by plane wave



Ideal Optics

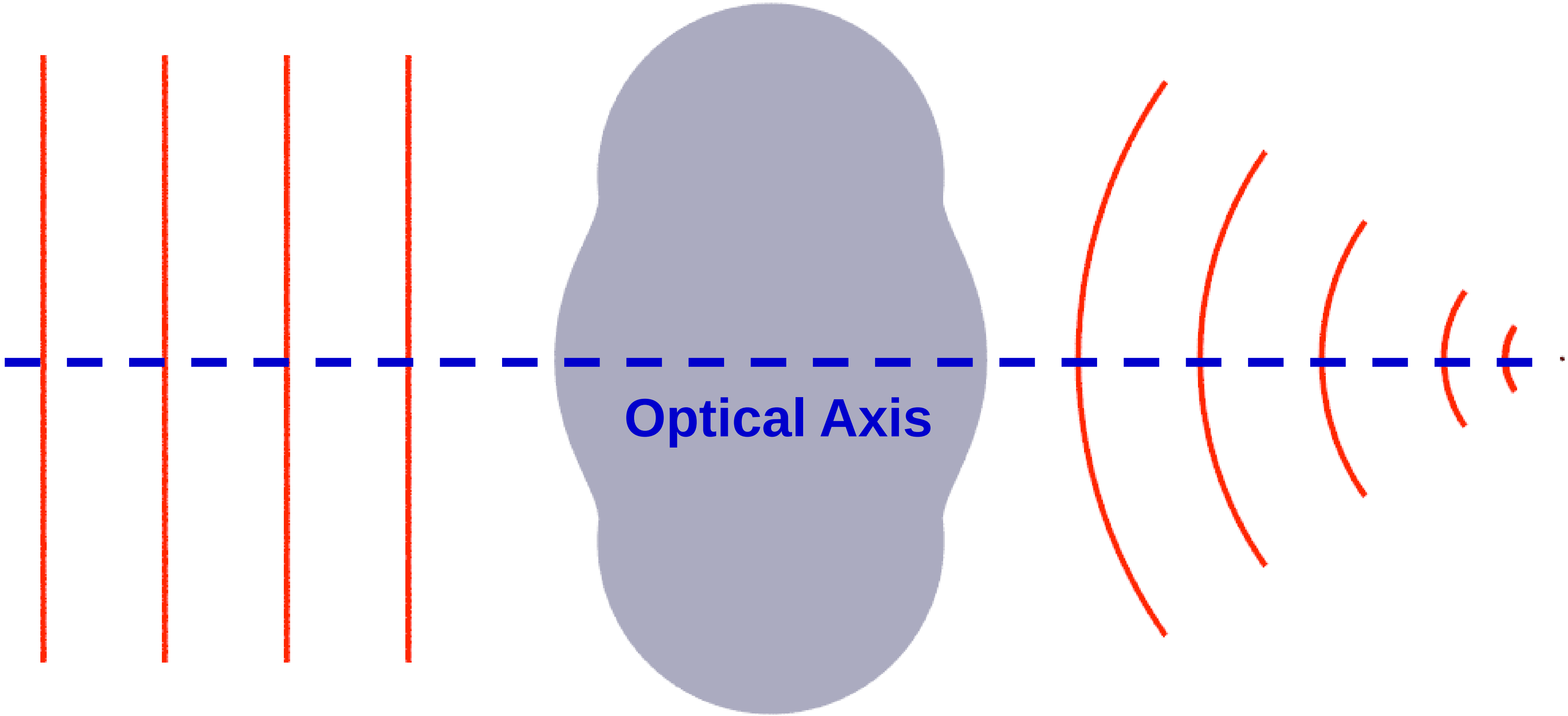


- ideal optics: spherical waves from any point in object space are
- imaged into points in image space
- corresponding points are called conjugate points
- focal point: center of converging or diverging spherical wavefront
- object space and image space are reversible

Ideal Optical System

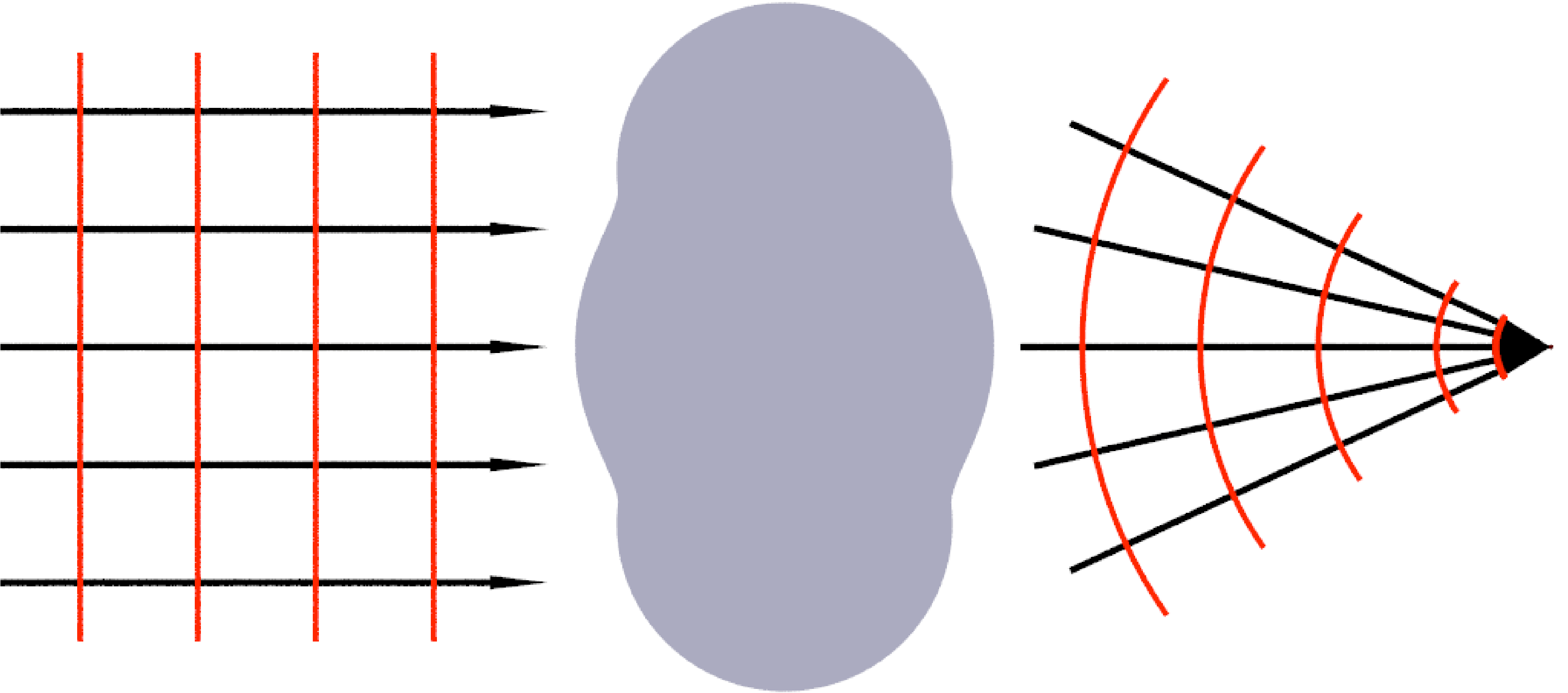
ideal optical system transforms plane wavefront into spherical, converging wavefront

Azimuthal Symmetry



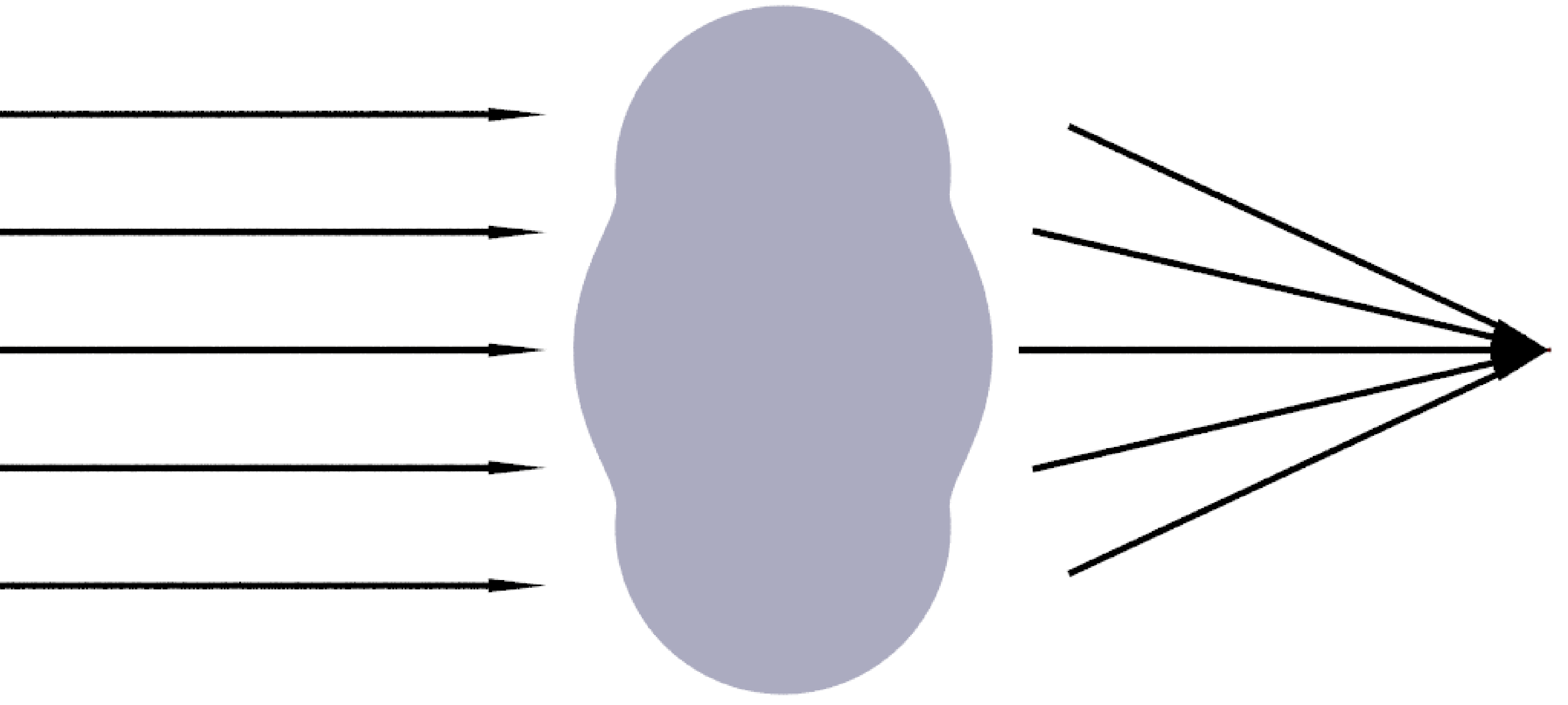
- most optical systems are azimuthally symmetric
- axis of symmetry is optical axis

Locally Flat Wavefronts



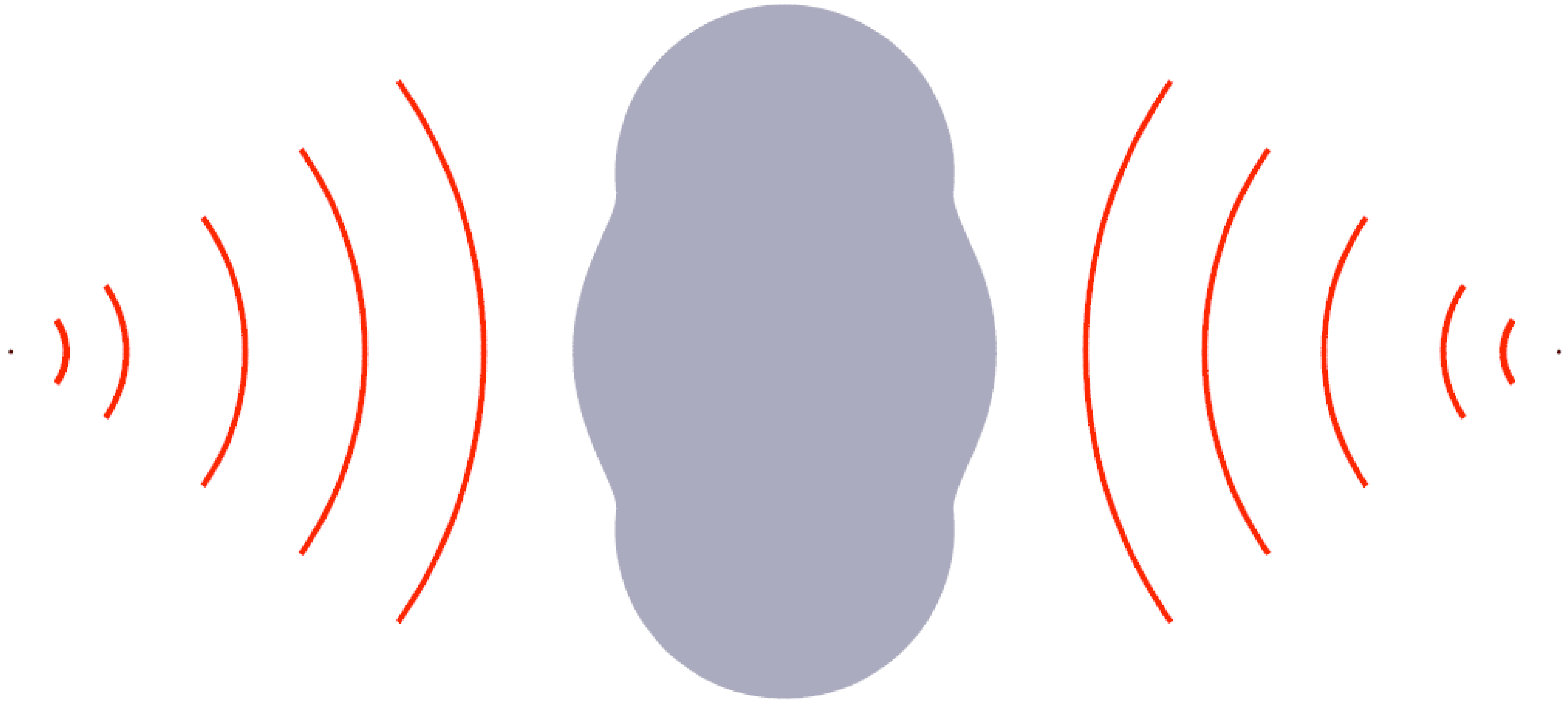
- rays normal to local wave (locations of constant phase)
- local wave around rays is assumed to be infinite, plane wave

Rays



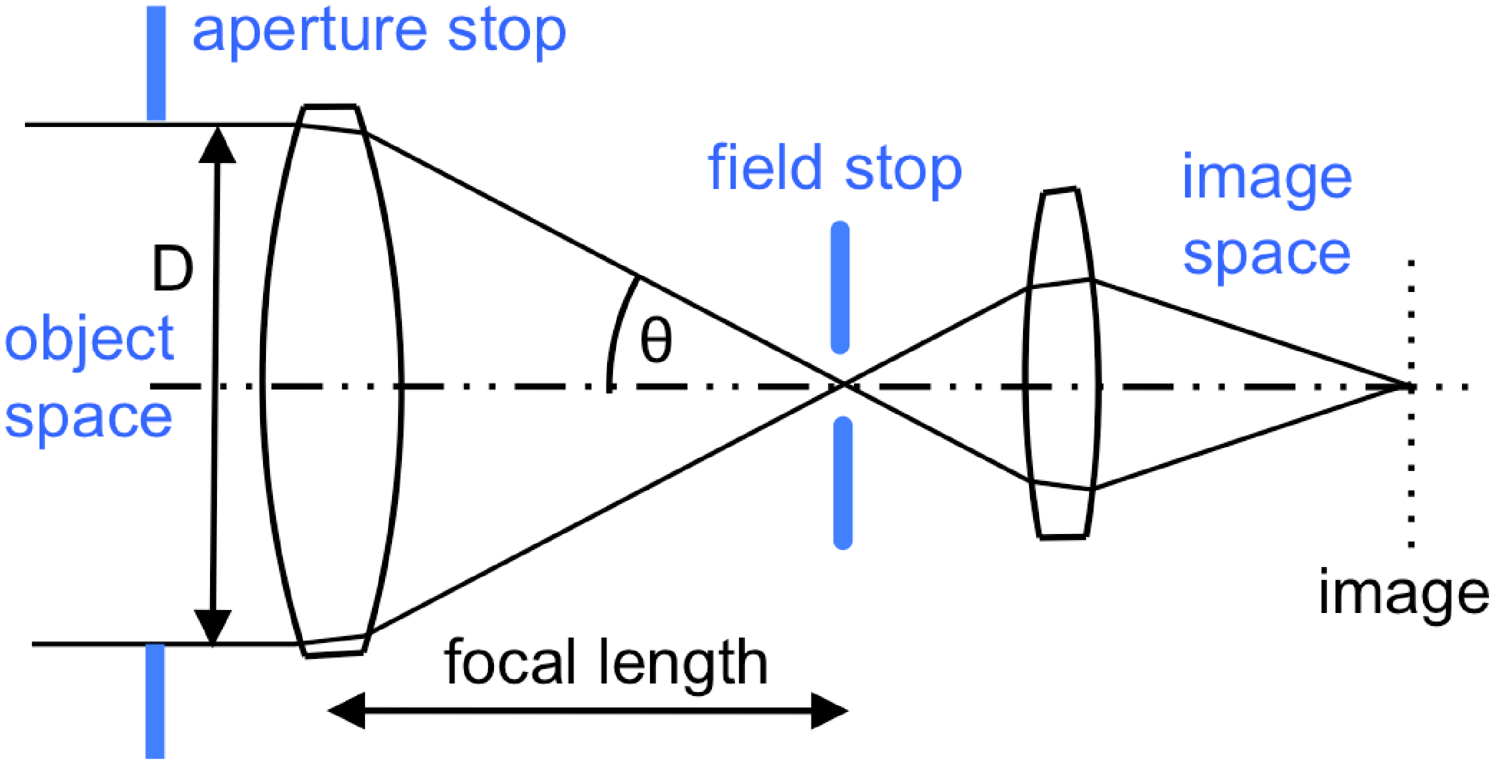
- geometrical optics works with rays only
- rays reflect and refract according to Fresnel equations
- phase is neglected (incoherent sum)

Finite Object Distance



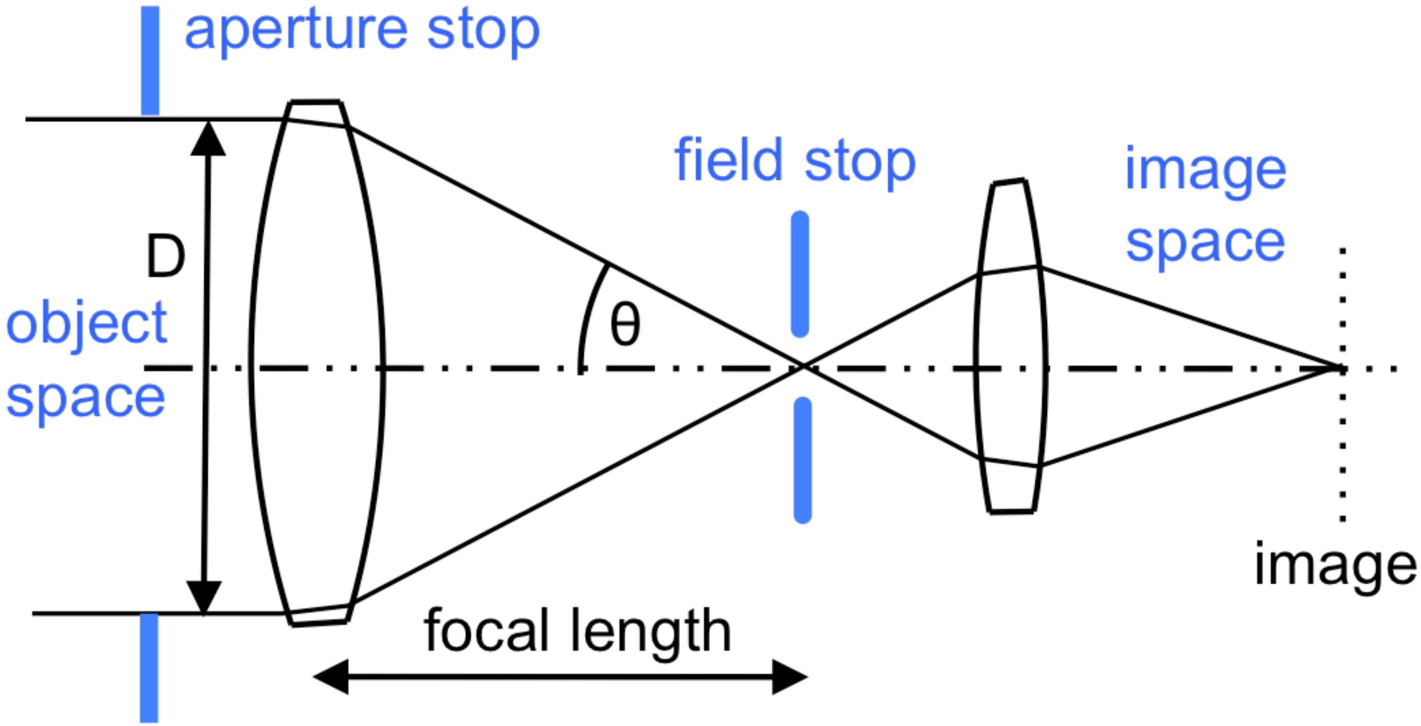
- object may also be at finite distance
- also in astronomy: reimaging within instruments and telescopes

Aperture and Field Stops



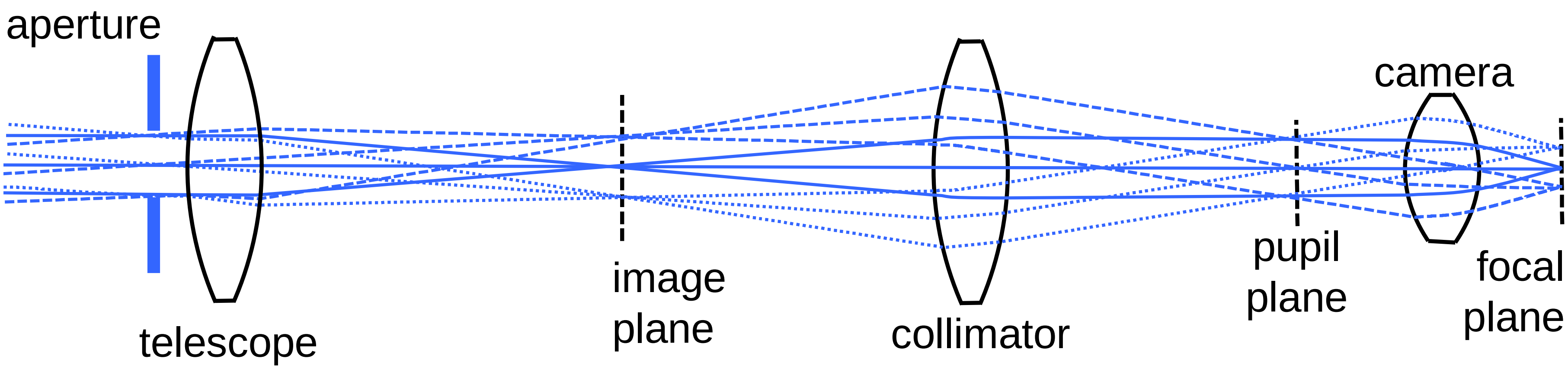
- Aperture stop: determines diameter of light cone from axial point on object.
- Field stop: determines the field of view of the system.

Images



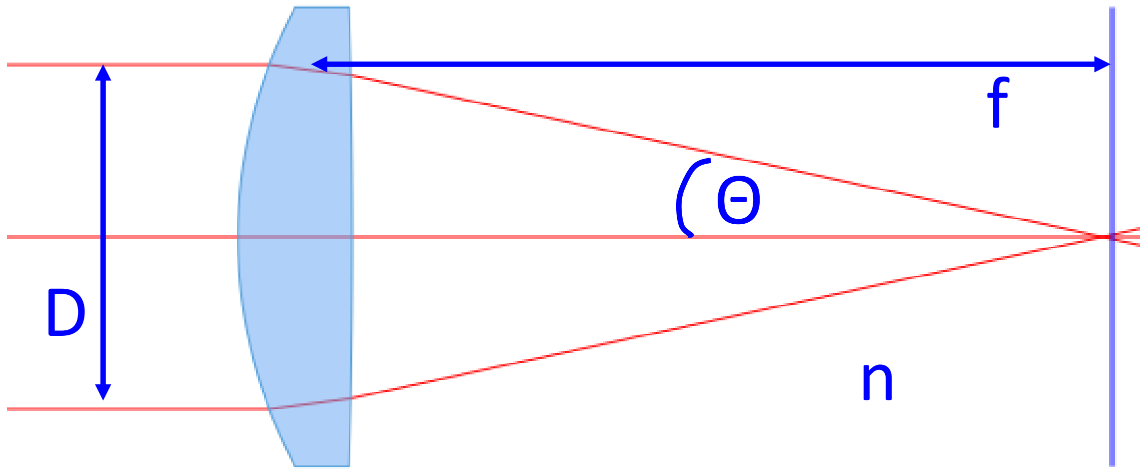
- every object point comes to focus in image plane
- light in image point comes from all pupil positions
- object information encoded in position, not angle

Pupils



- all object rays are smeared out over complete aperture
- light in one pupil point comes from different object positions
- object information is encoded in angle, not in position

Speed/F-Number/Numerical Aperture



Speed of optical system described by numerical aperture (NA) or *F*-number:

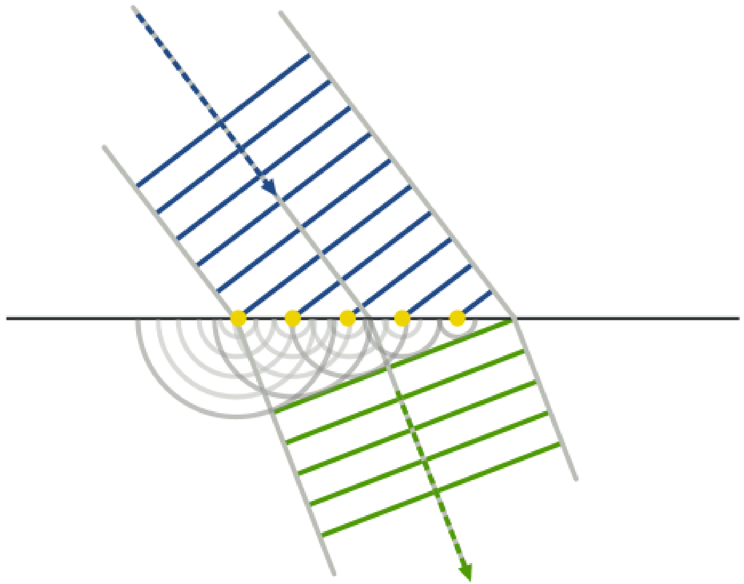
$$\mathrm{NA}{=}n \cdot \sin\theta \approx n\frac{1}{2F}, \qquad F = \frac{f}{D}$$

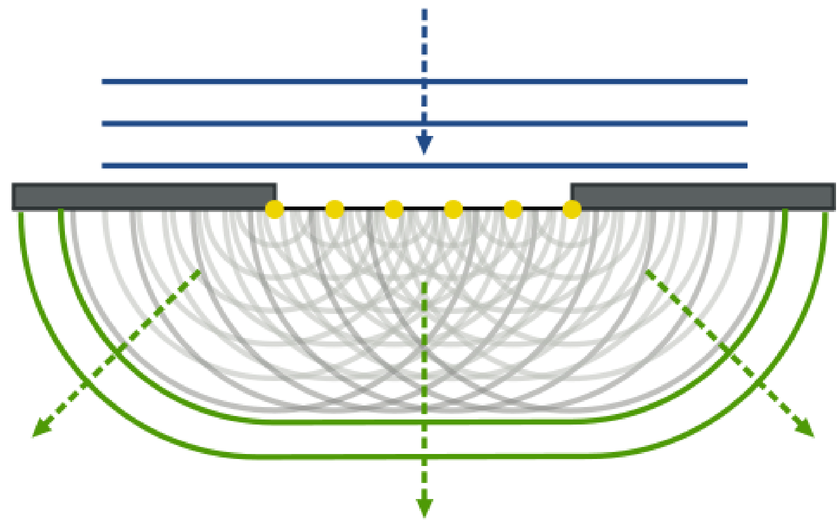
- fast optics (large NA, small F-number)
- slow optics (small NA, large F-number)
- f-ratio = 1/F, sometimes written as f/2.8

The Huygens-Fresnel Principle

Fermat's view: "A wavefront is a surface on which every point has the same OPD."

Huygens' view: "At a given time, each point on primary wavefront acts as a source of secondary spherical wavelets. These propagate with the same speed and frequency as the primary wave."





Reminder: Coherent Radiation

A light source may exhibit temporal and spatial coherence. The coherence function Γ_{12} between two points (1,2) is the cross-correlation between their complex amplitudes:

$$\Gamma_{12}(\tau) = \langle E_1(t+\tau) E_2^*(t) \rangle$$

The normalized representation is called the degree of coherence:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1 I_2}}$$

which leads to an interference pattern* with an intensity distribution of:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}[\gamma_{12}(\tau)]$$

where

$$|\gamma_{12}| = 1 \qquad \text{coherent}$$

$$|\gamma_{12}| = 0 \qquad \text{incoherent}$$

$$0 < |\gamma_{12}| < 1 \qquad \text{partial coherence}$$

and the visibility for $I_1 = I_2$.

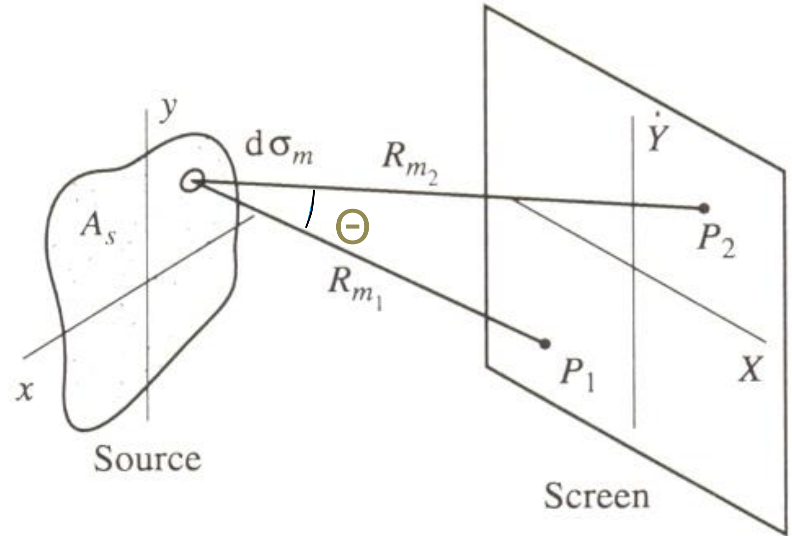
$$V = |\gamma_{12}(\tau)|$$

*e.g., from Young's double slit experiment

The Zernike-van Cittert Theorem (1)

Consider a monochromatic, extended, incoherent source A_s with intensity I(x,y).

Consider further a surface element dσ (dσ <<λ), which illuminates two points P_1 and P_2 at distances R_1 and R_2 on a screen.



The quantity measuring the correlation of the electric fields between P_1 and P_2 (for any surface element dσ at distance r) is:

$$\langle V_1(t) V_2^*(t) \rangle = \int_{A_s} I(r) \frac{\exp[ik(R_1 - R_2)]}{R_1 R_2} dr$$

The Zernike-van Cittert Theorem (2)

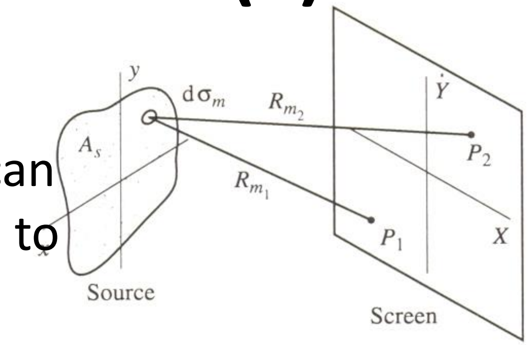
Generally, the degree of coherence is then given by the Zernike-van Cittert theorem:

$$\gamma_{12}(0) = \frac{1}{\sqrt{\langle |V_1|^2 \rangle \langle |V_2|^2 \rangle}} \int_{source} I(r) \frac{\exp[ik(R_1 - R_2)]}{R_1 R_2} dr$$

In words, the general Zernike-van Cittert theorem describes the relation between the degree of coherence between two points on the screen and the intensity distribution across the illuminating source A_s.

Frits Zernike (1888-1966) : Dutch physicist and winner of the Nobel prize for physics in 1953 for his invention of the phase contrast microscope,

The Z-vC Theorem for Large Distances (3)



For large distances from source to screen (relative to the distance between P_1 and P_2 and the size of the source) we can use angular variables [x/R=α, y/R=β, Θ=(α,β), and $\Delta X = X_2 - X_1$] to describe the source as seen from the screen.

Then the general Z-vC theorem simplifies (Lena p. 211) to:

$$|\gamma_{12}(0)| = \left| \frac{\iint_{source} I(\theta) \exp\left[-\frac{i2\pi}{\lambda} (\alpha \Delta X + \beta \Delta Y) \right] d\theta}{\iint_{source} I(\theta) d\theta} \right|$$

For large distances, the modulus* of the degree of coherence $|\gamma_{12}|$ between two points is the modulus of the normalized Fourier transform of the source intensity distribution.

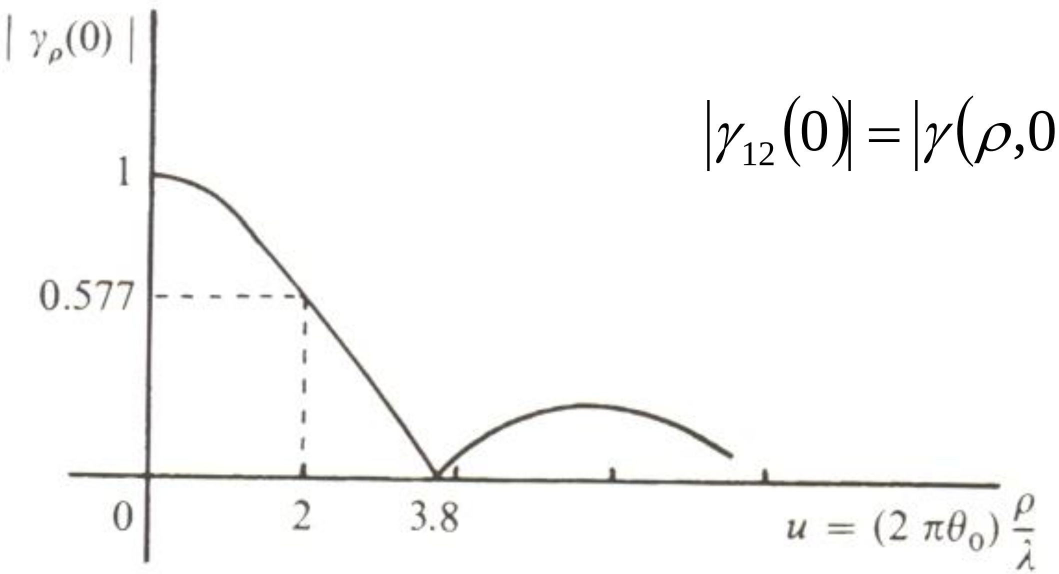
*absolute value of a complex number

The Z-vC Theorem for a Circular Source (4)

Now: calculate the complex degree of coherence for a circular source of radius r_0.
Let P_1 be at the center of the screen and P_2 at distance ρ where $\Theta = r_0/R$.

$$I(\theta) = \Pi\left(\frac{r}{2r_0}\right) = \Pi\left(\frac{\theta}{2\theta_0}\right)$$

Then the modulus of the degree of coherence for a circular source is:



$$|\gamma_{12}(0)| = |\gamma(\rho,0)| = \left|\frac{J_1\left(2\pi\theta_0\frac{\rho}{\lambda}\right)}{\pi\theta_0\frac{\rho}{\lambda}}\right| = \frac{2|J_1(u)|}{u}$$

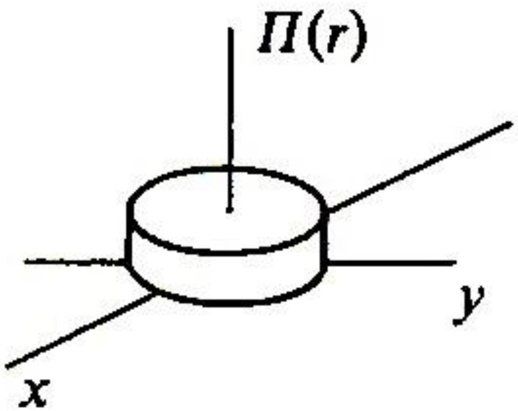
Θ_0 = angular size of circular source
ρ = distance of P_2 from center
λ = wavelength
J_1 = 1st order Bessel function

Fourier Pair in 2-D: Box Function

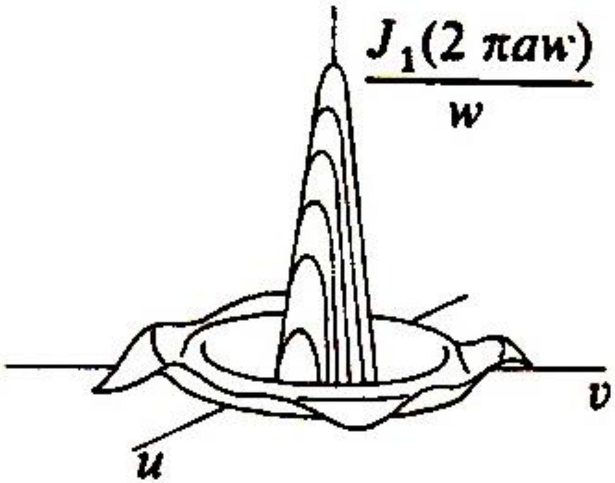
2-D box function with $r^2 = x^2 + y^2$: $$\Pi\left(\frac{r}{2}\right) = \begin{cases} 1 & \text{for } r < 1 \\ 0 & \text{for } r \geq 1 \end{cases}$$

Fourier Transform: $$\Pi\left(\frac{r}{2}\right) \Leftrightarrow \frac{J_1(2\pi\omega)}{\omega}$$ (1st order Bessel function J_1)

Electric Field in Telescope Aperture:



Electric Field in Focal plane:

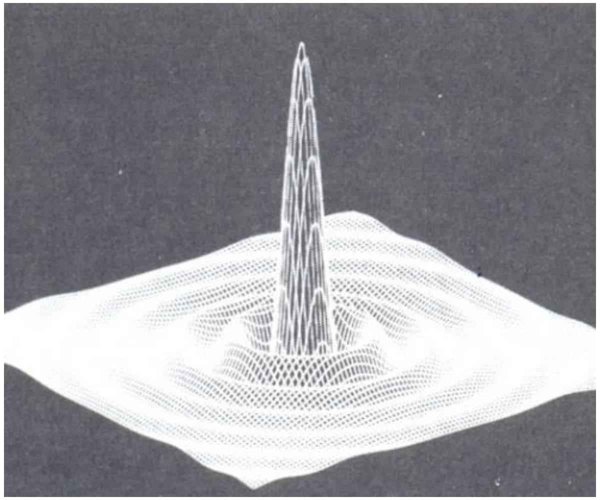


Larger telescopes produce smaller Point Spread Functions (PSFs)!

Point Spread Function

First dark ring (minimum) at:

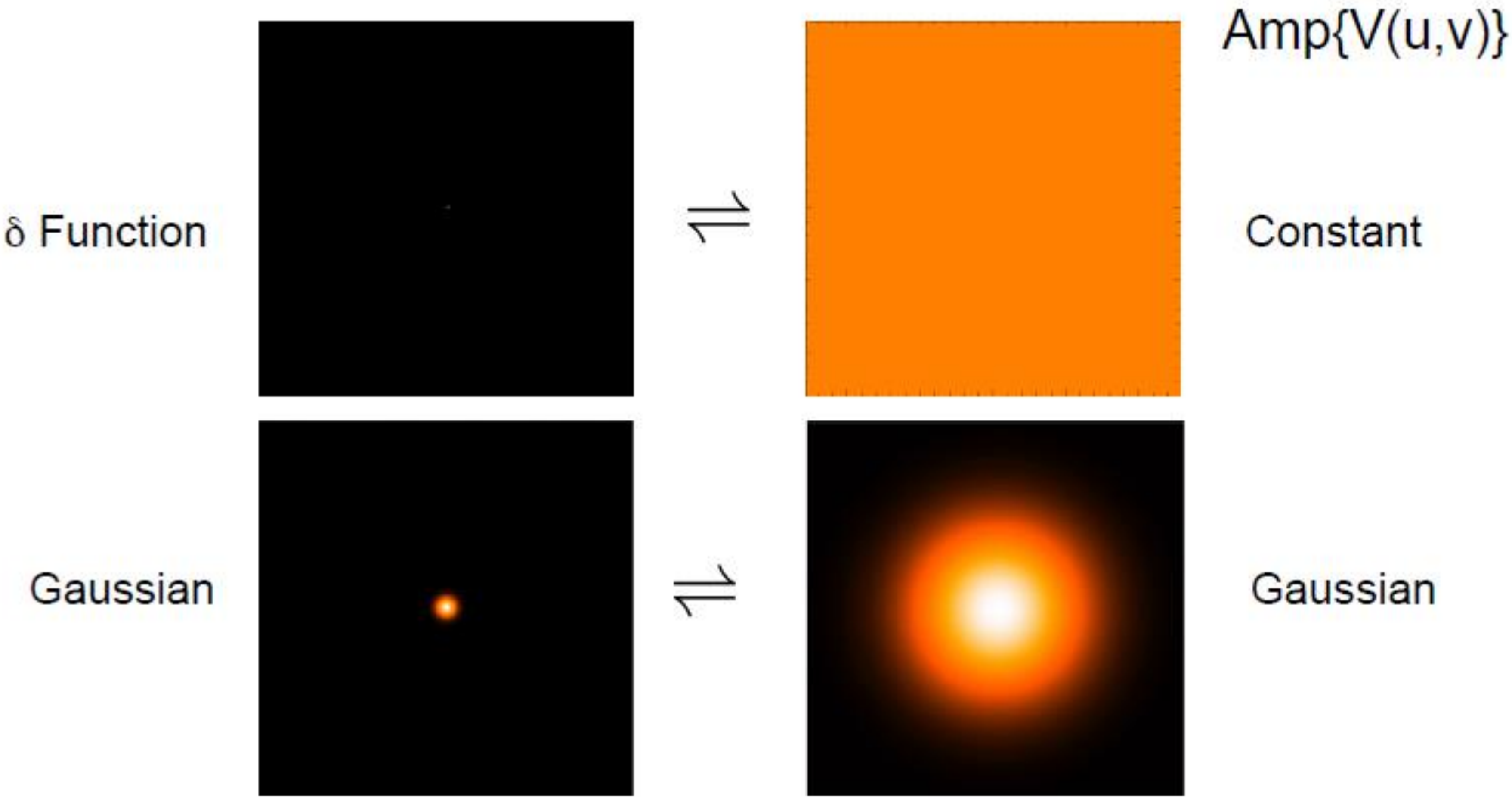
$$r_1 = 1.22\lambda F \qquad \text{or} \qquad \alpha_1 = \frac{r_1}{f} = 1.22\frac{\lambda}{D}$$



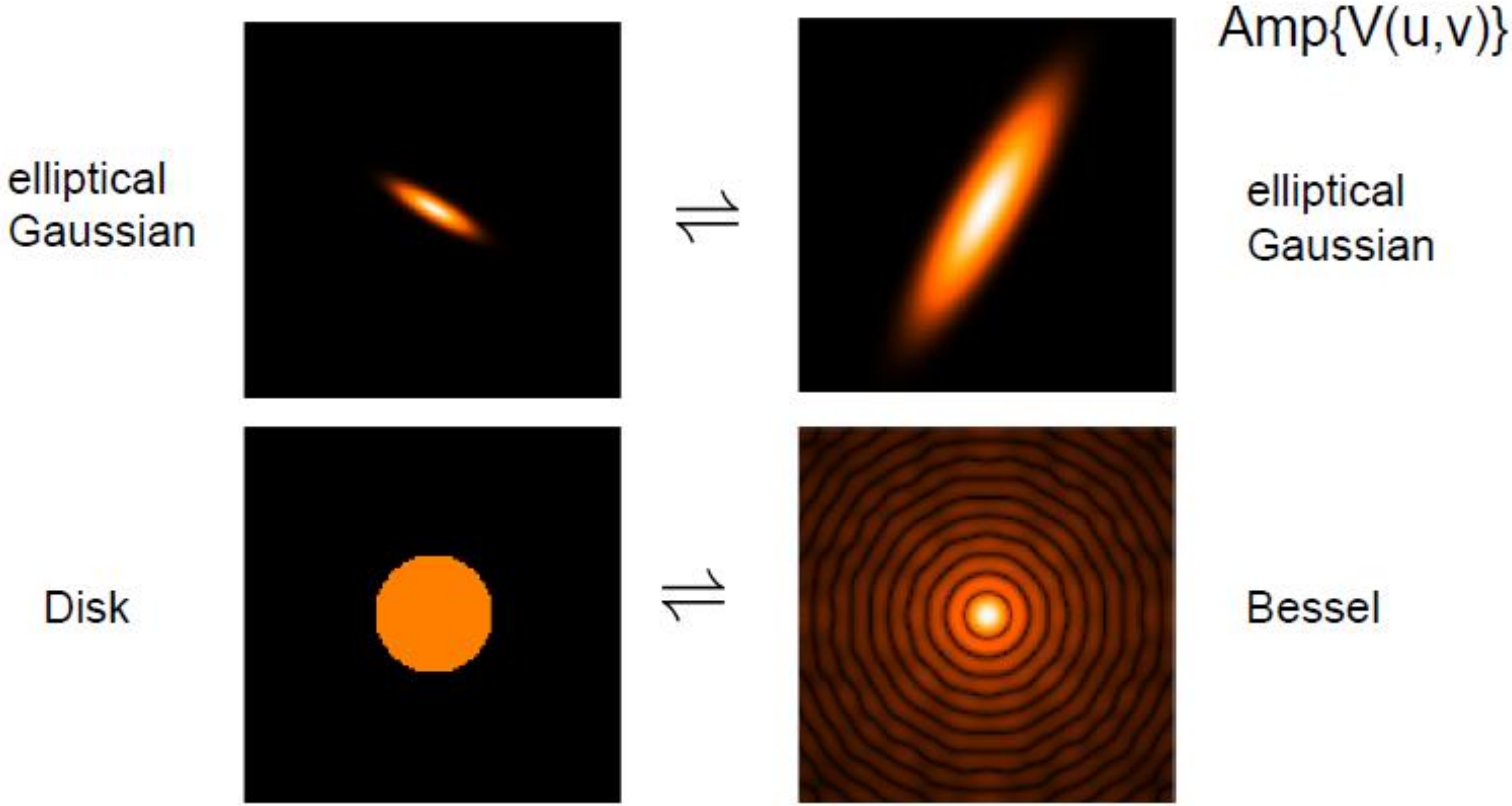
PSF often characterized by Full Width at Half Maximum (FWHM) in angular units. Airy function: FWHM=1.028 λ/D

Nyquist sampling theorem requires sampling at least every 0.5 FWHM. Airy function: at least every 0.5λ/D

Telescope Aperture ⇔ Focal Plane 1

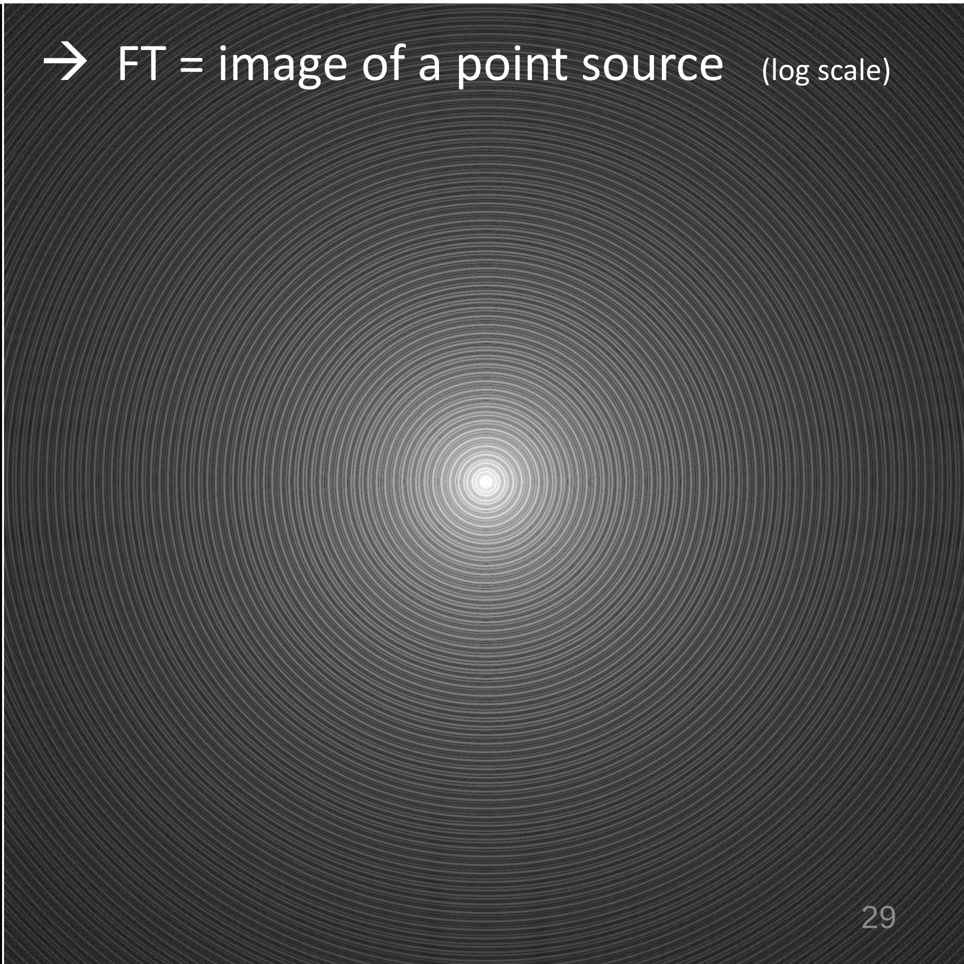


Telescope Aperture ⇔ Focal Plane 2

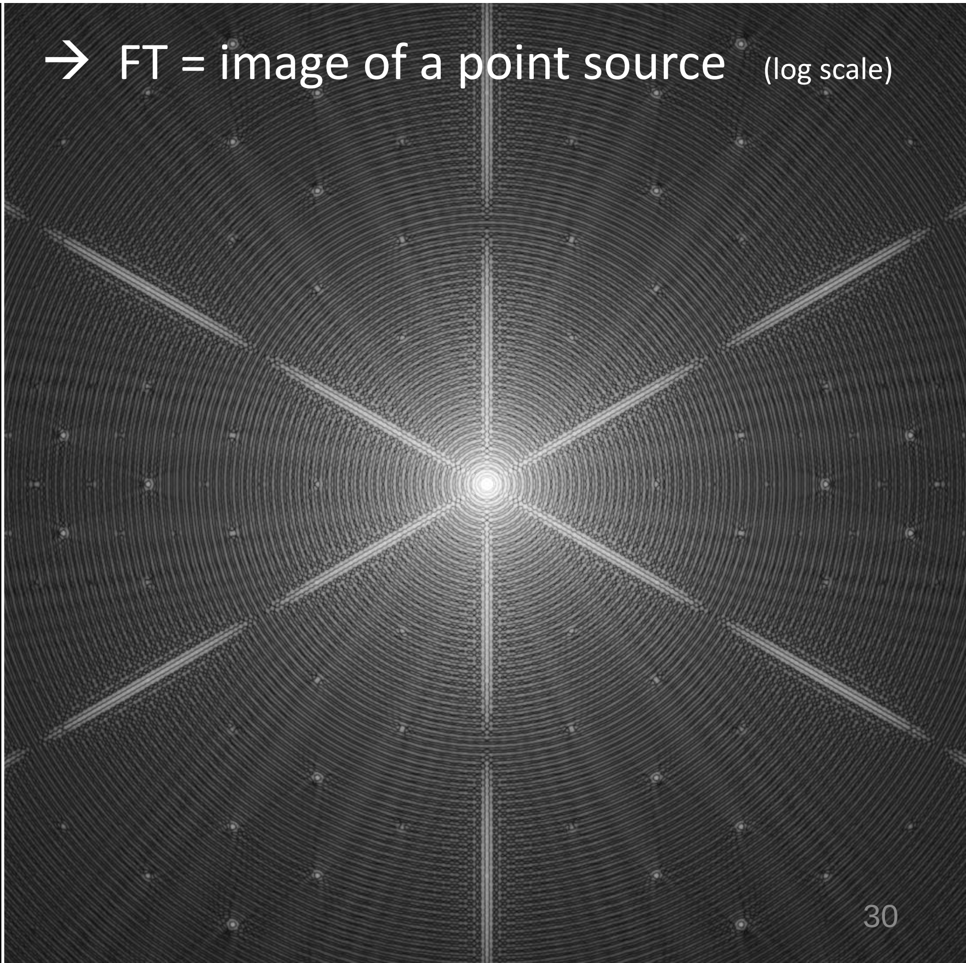


PSF Example

central obscuration,
monolithic mirror (pupil)
no support-spiders



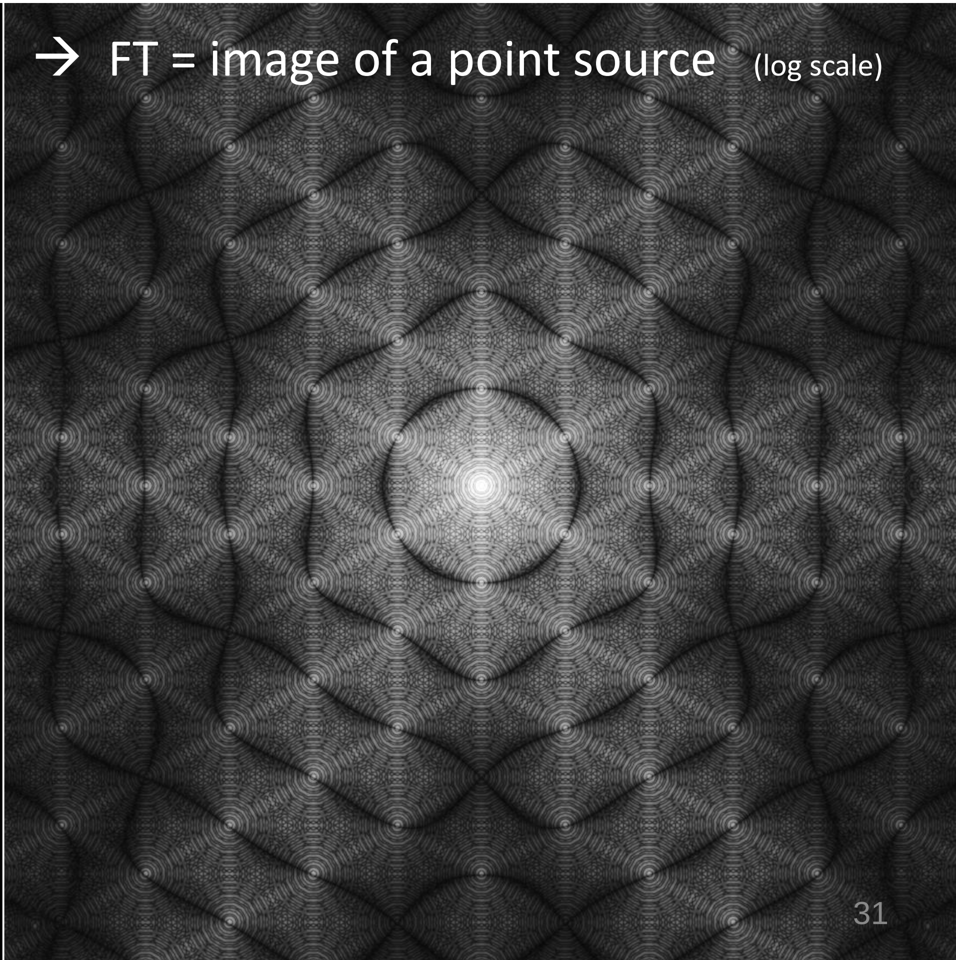
central obscuration,
monolithic mirror (pupil)
with 6 support-spiders



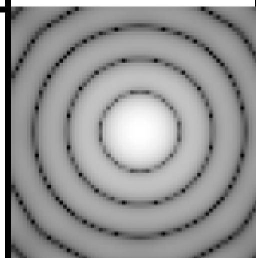
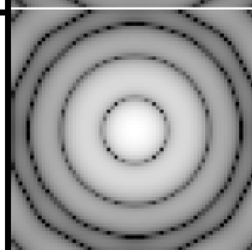
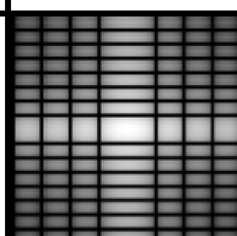
central obscuration,

segmented mirror (pupil)

no support-spiders



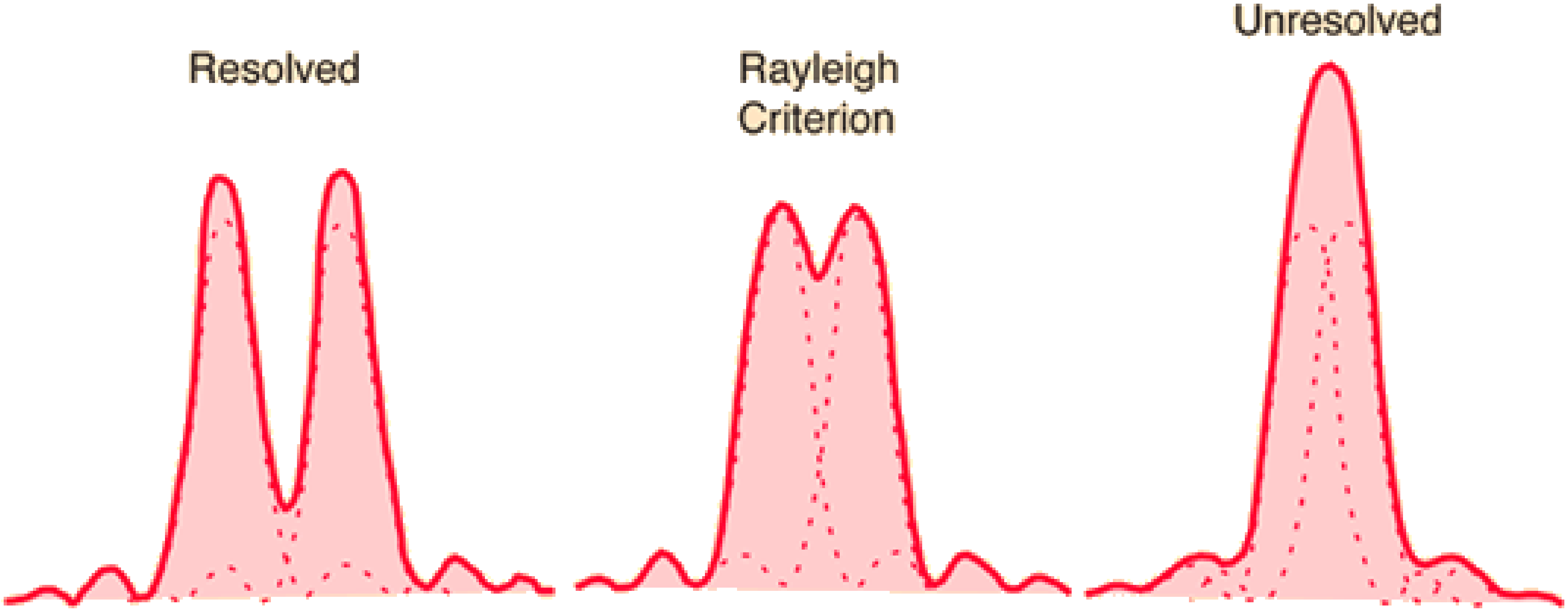
Point Spread Functions

Aperture	PSF	PSF equation
round, diameter d_x		$\left(\frac{2J_1(x)}{x}\right)^2$
obscured round, diameter d_x, obscuration ratio ε		$\frac{1}{(1-\varepsilon^2)^2}\left(\frac{2J_1(x)}{x}-\frac{2J_1(\varepsilon x)}{x}\right)^2$
rectangle, sides $d_{x,y}$		$\left(\frac{\sin x}{x}\right)^2\left(\frac{\sin y}{y}\right)^2$

Angular Resolution: Rayleigh Criterion

Two sources can be resolved if the peak of the second source is no closer than the 1st dark Airy ring of the first source.

$$\sin\Theta = 1.22\frac{\lambda}{D}$$



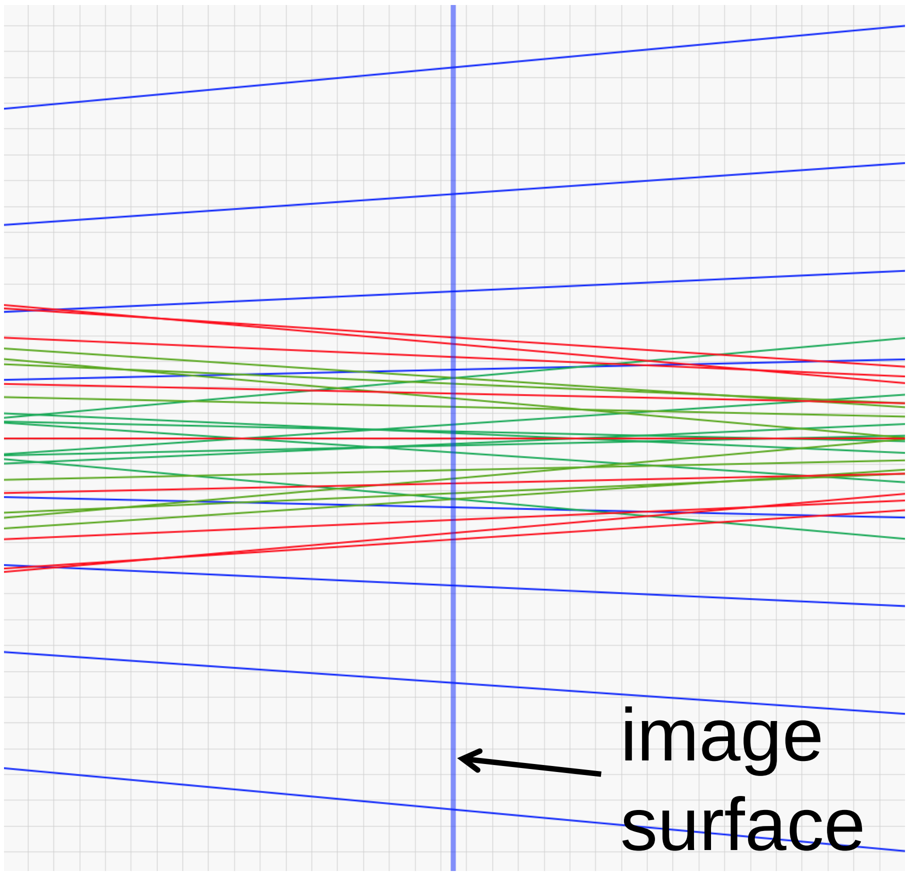
http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/Raylei.html

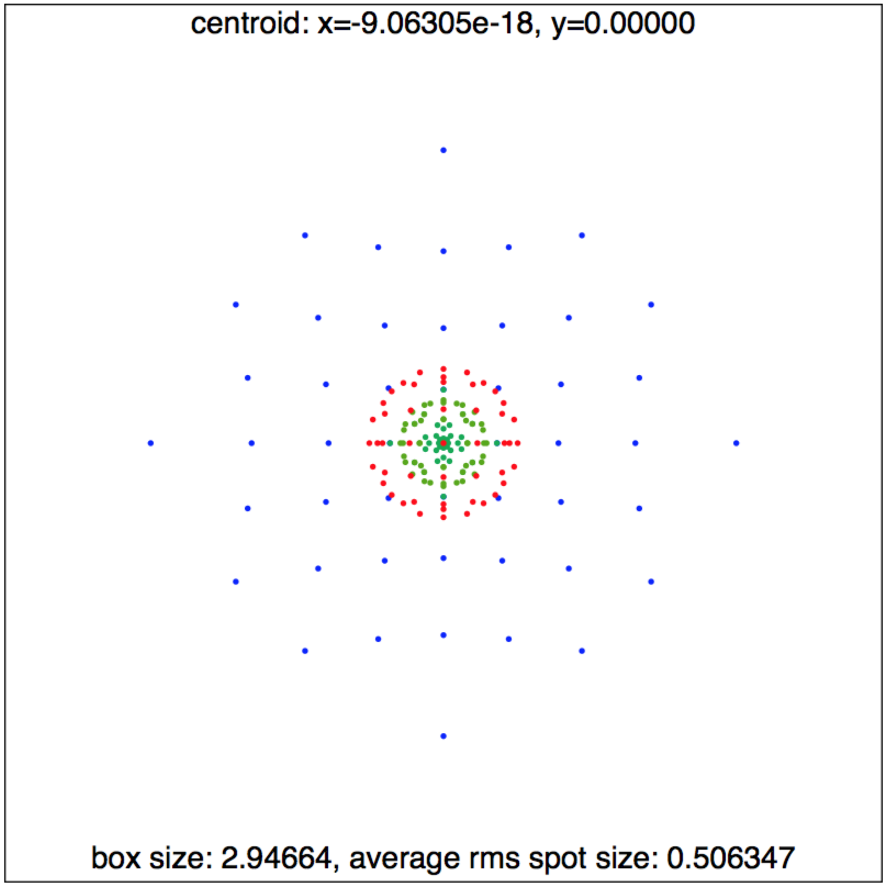
Aberrations

Aberrations are departures of the performance of an optical system from the ideal optical system.

1. On-axis aberrations: aberrations that can be seen everywhere in the image, also on the optical axis (center of the image)
2. Off-axis aberrations: aberrations that are absent on the optical axis (center of the image)
   a) Aberrations that degrade the image
   b) Aberrations that alter the image position

Spot Diagram

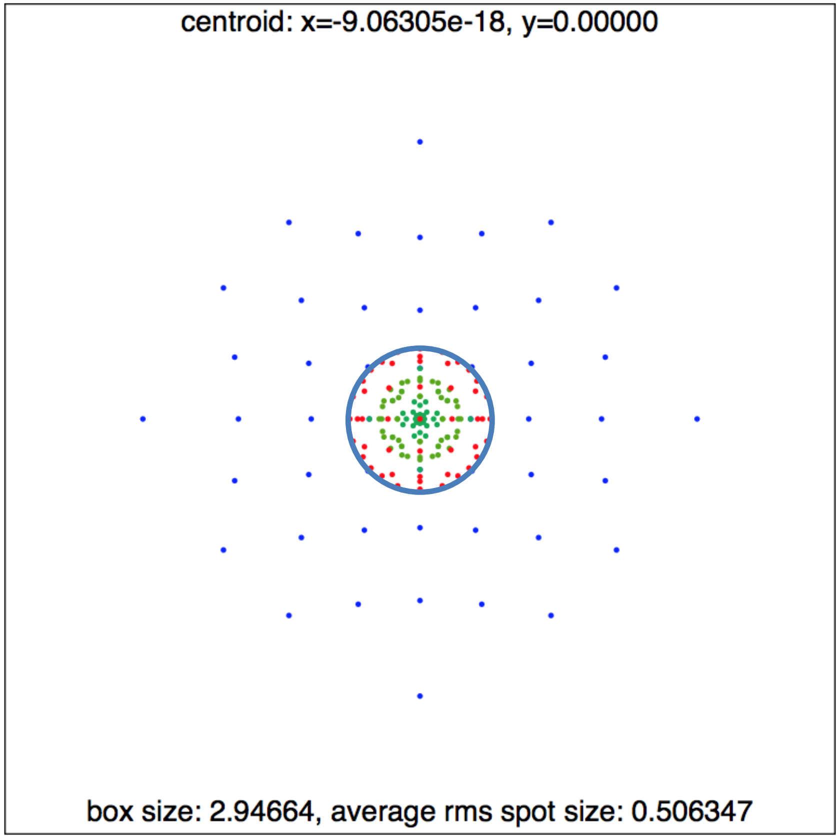




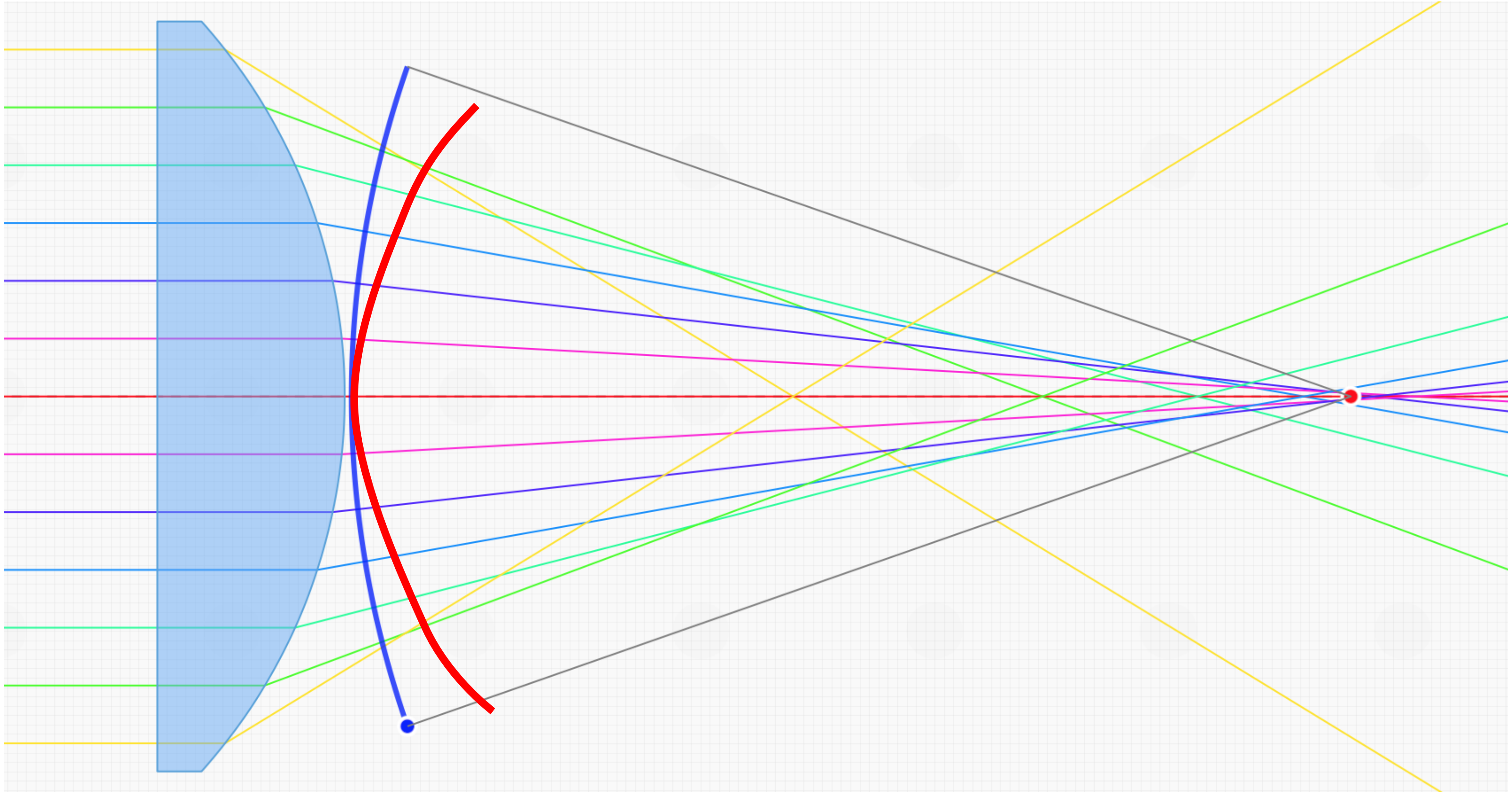
intersection of rays with image surface

RMS Spot Radius

- calculate rms radius of all spots from the perfect center
- provides a rough measure of image quality
- optics are virtually perfect if rms spot radius $\leqq \lambda/2D$

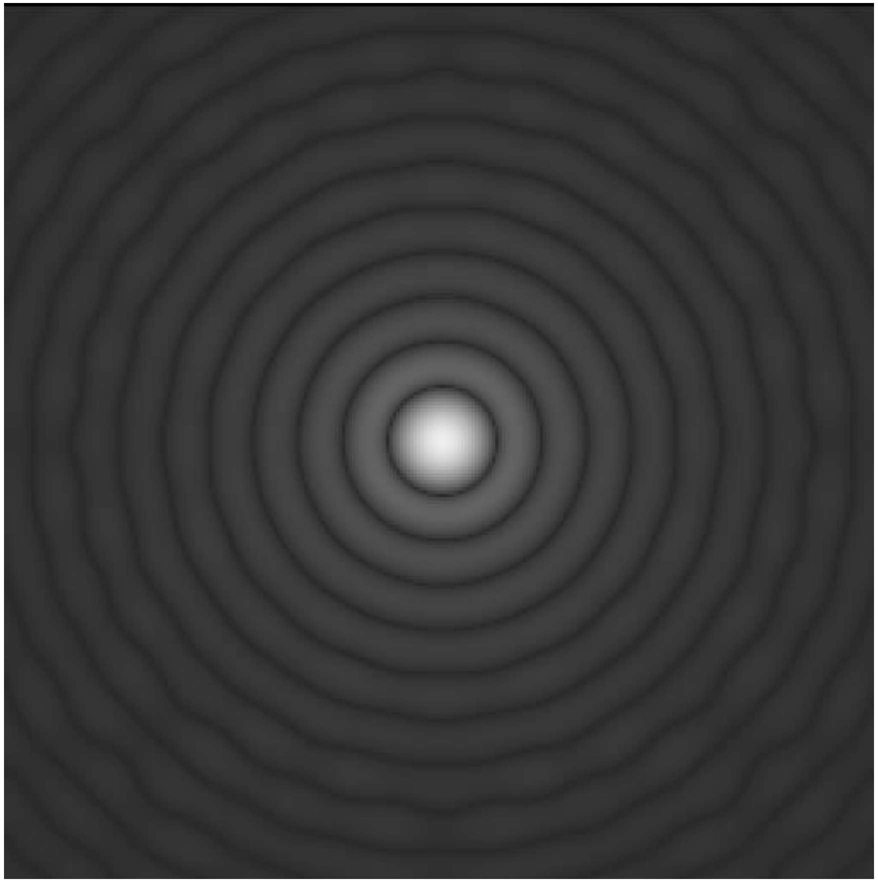


Wavefront Error

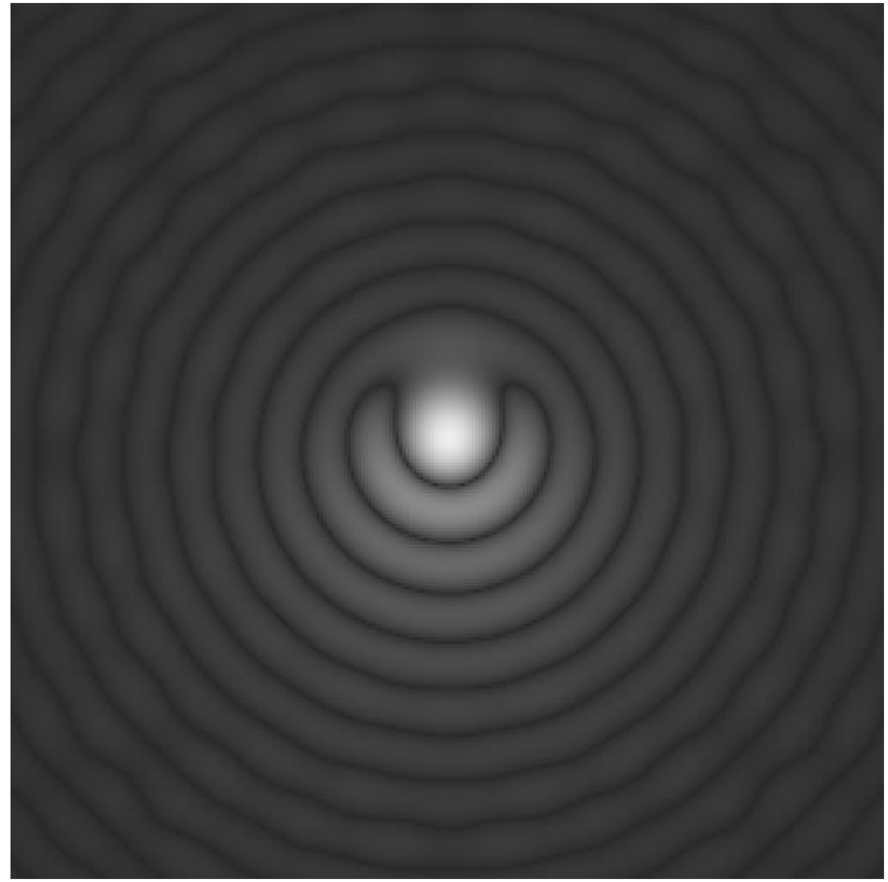


- deviation of surface that is normal to rays and spherical reference surface (distance between red & blue line)
- often shown as grayscale image or 3D surface

Aberrated Point-Spread Function



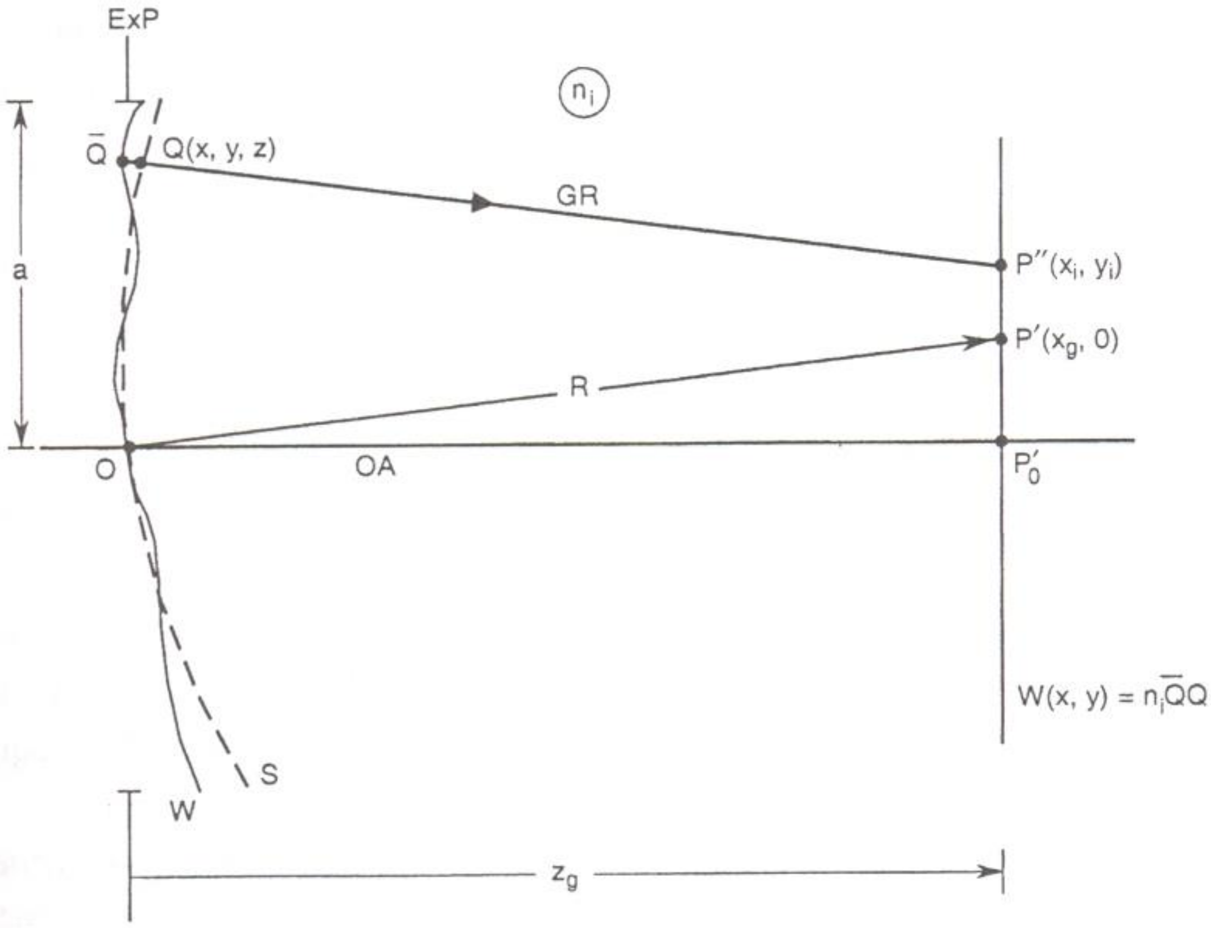
perfect



aberrated

Wave and Ray Aberrations

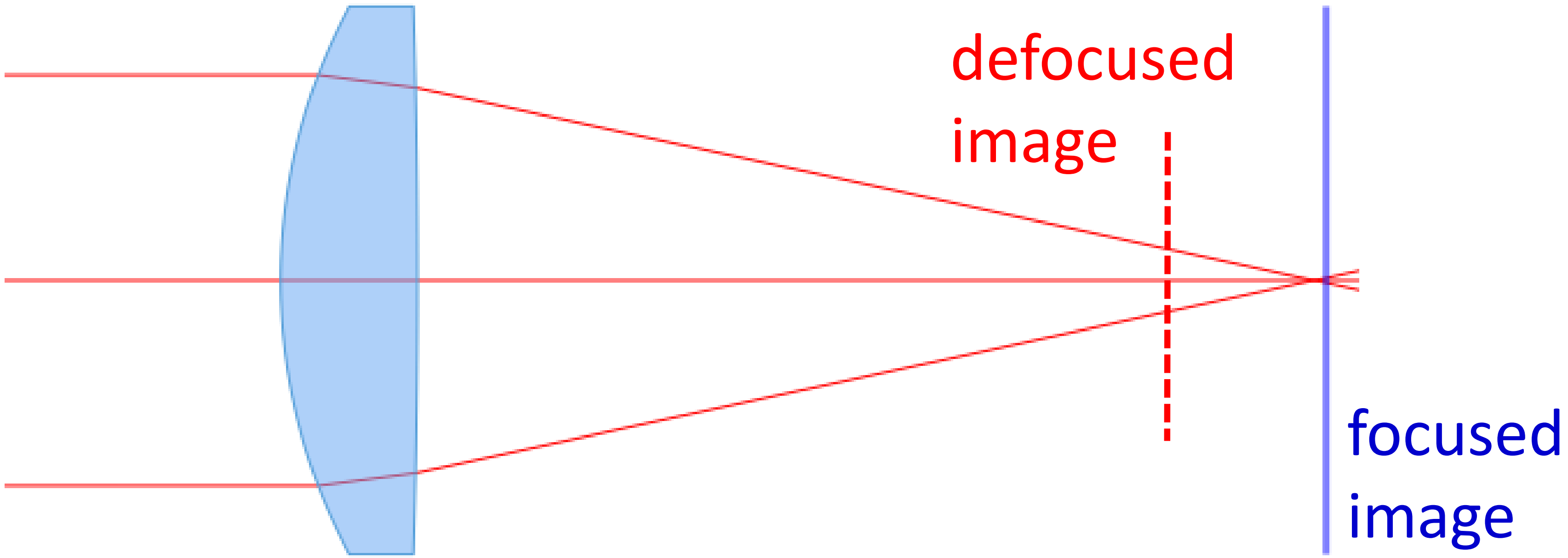
- Reference sphere S with radius R for off-axis point P' and aberrated wavefront W
- "Aberrated" ray from object intersects image plane at P"
- Ray aberration is P'P"
- Wave aberration is $n_i \times Q\bar{Q}$



Small FOV, radially symmetric wavefront W(r)

$$r_i = \frac{R}{n_i}\frac{\partial W(r)}{\partial r}$$

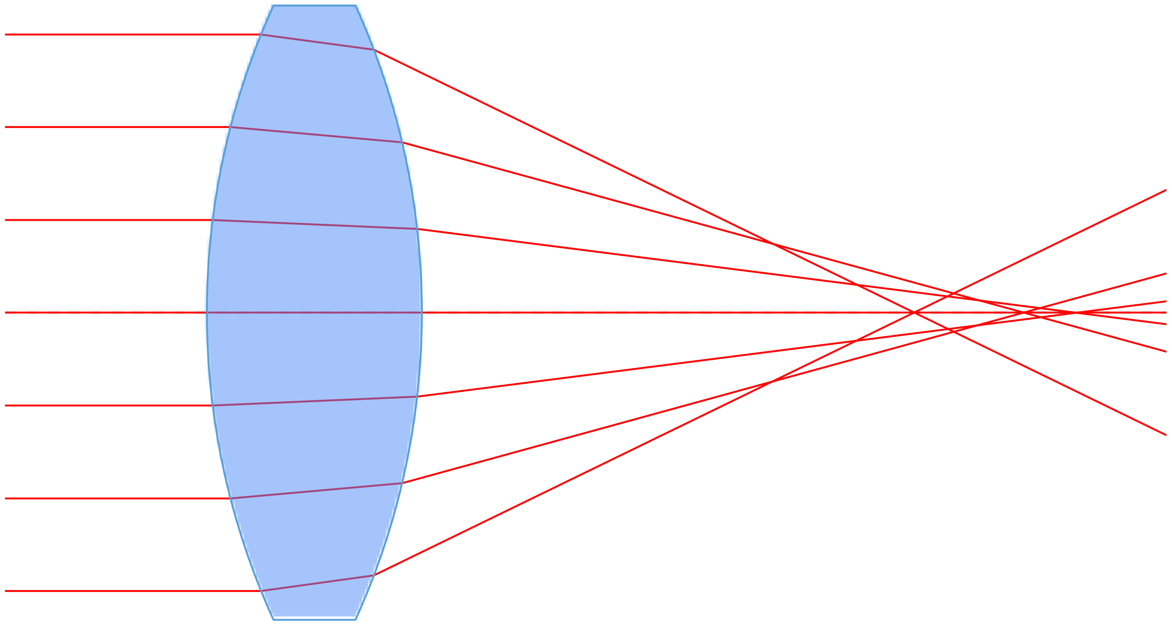
Defocus (Out of Focus)

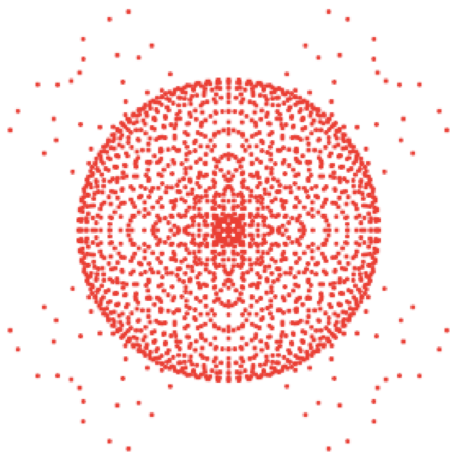


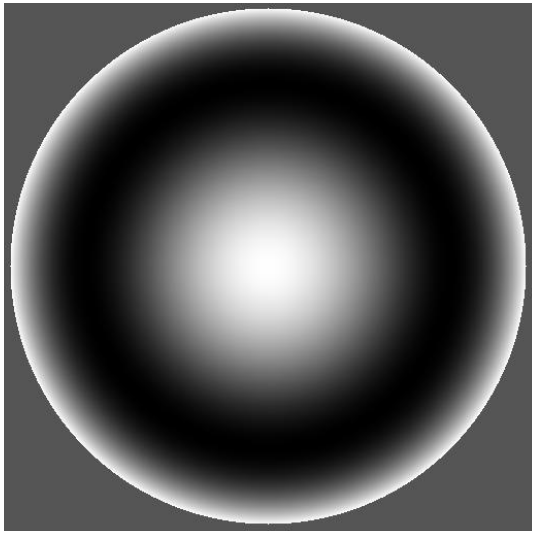
Depth of focus: $$\delta = 2\lambda F^2 = \frac{\lambda}{2}\left(\frac{1}{\mathrm{NA}}\right)^2$$

Usually refers to optical path difference of $\lambda/4$.

Spherical Aberration

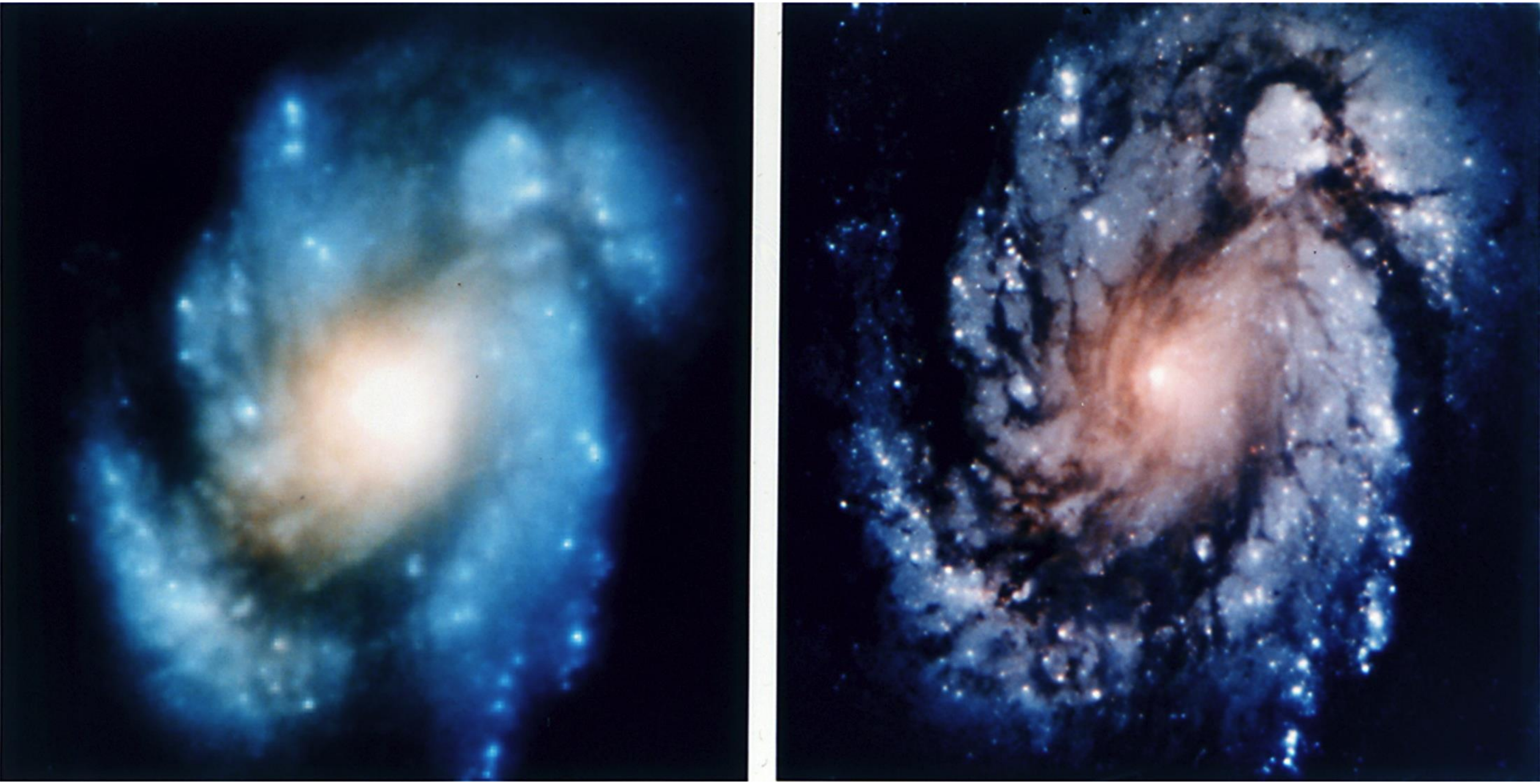






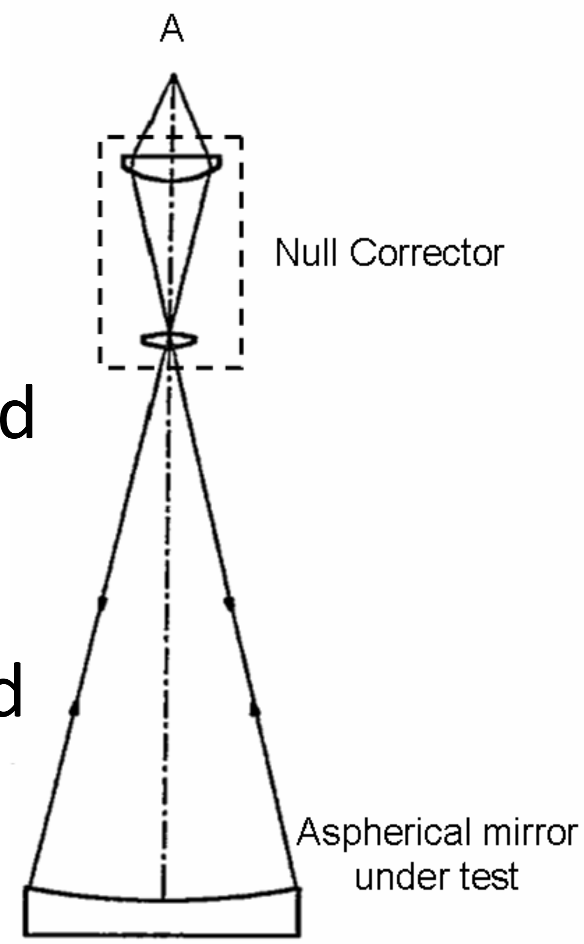
Rays further from the optical axis have a different focal point than rays closer to the optical axis.

Spherical Aberration: Hubble Trouble



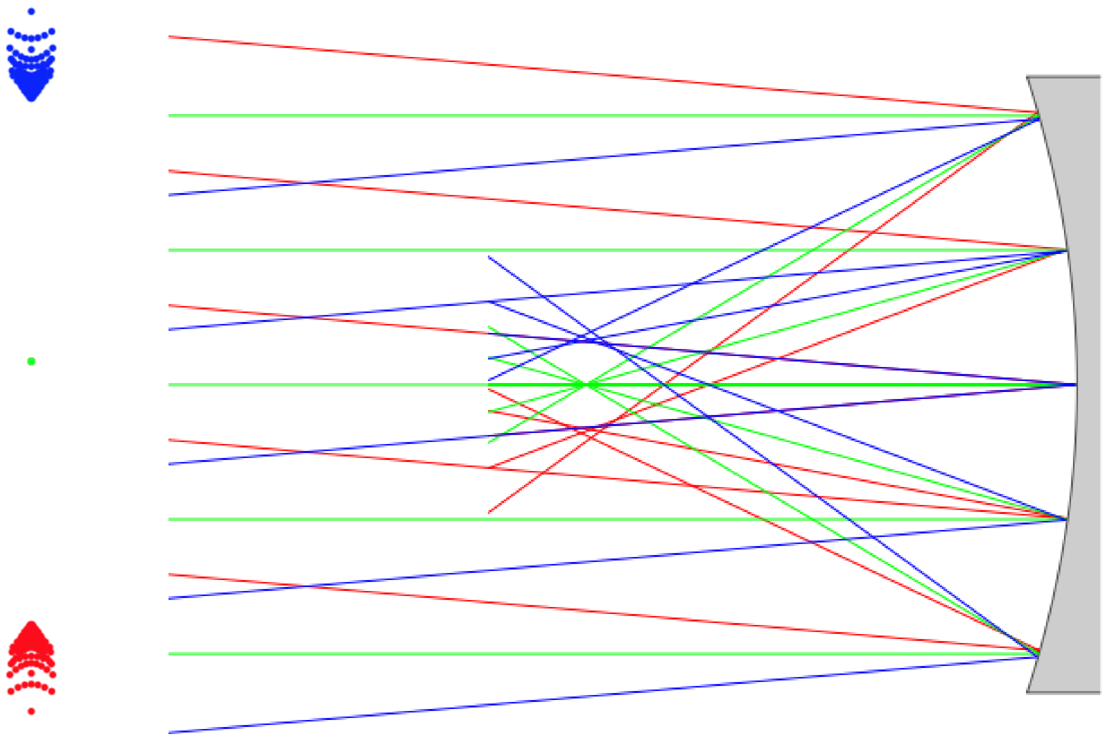
HST Primary Mirror Aberration

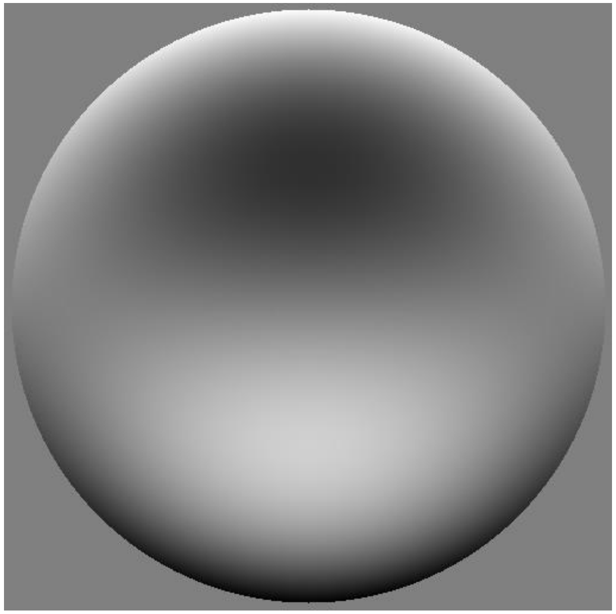
- Null corrector cancels non-spherical portion of aspheric mirror shape. Viewed from point A, combination looks precisely spherical
- Null corrector had one lens misplaced by 1.3 mm
- Manufacturer analyzed surface with other null correctors, which indicated the problem, but ignored results because they were believed to be less accurate



Coma

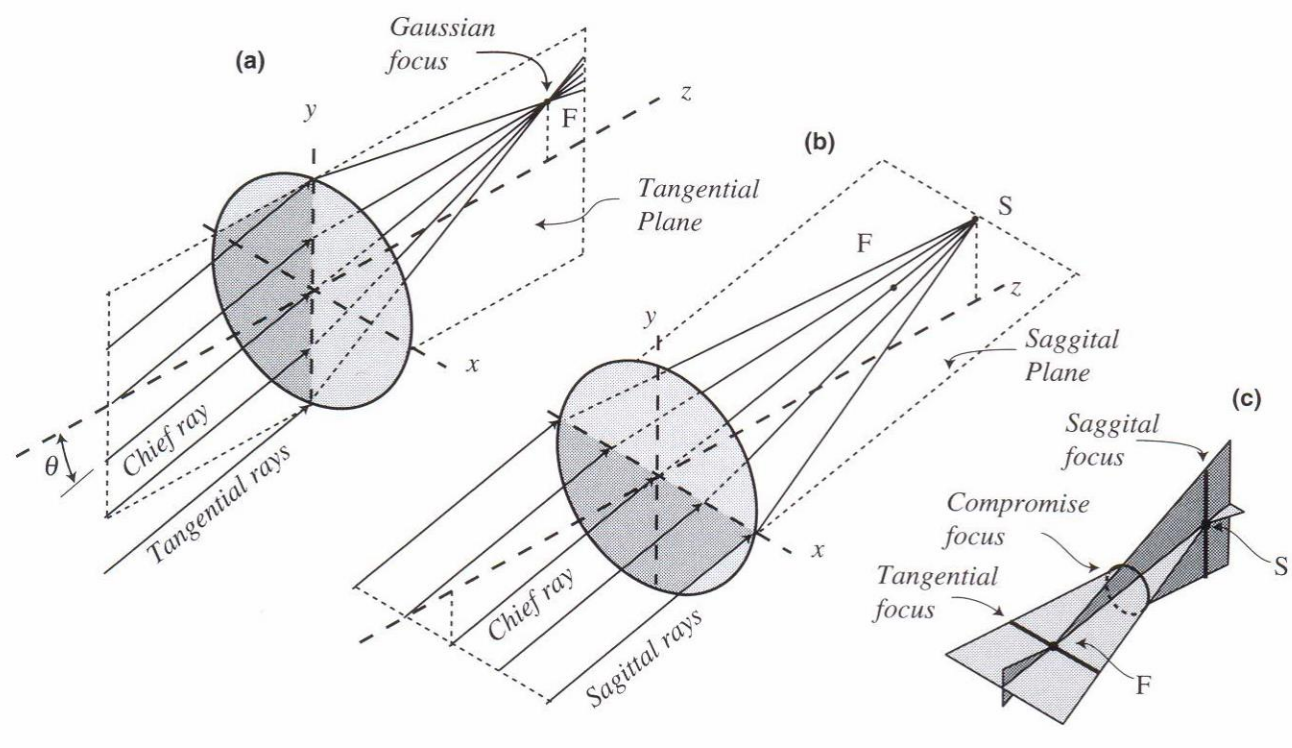
Variation of magnification across entrance pupil. Point sources will show a come-like tail. Coma is an inherent property of telescopes using parabolic mirrors

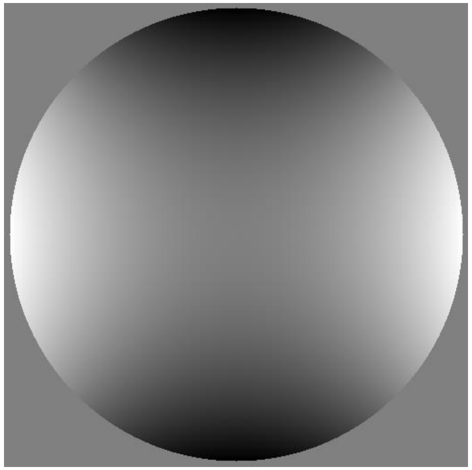




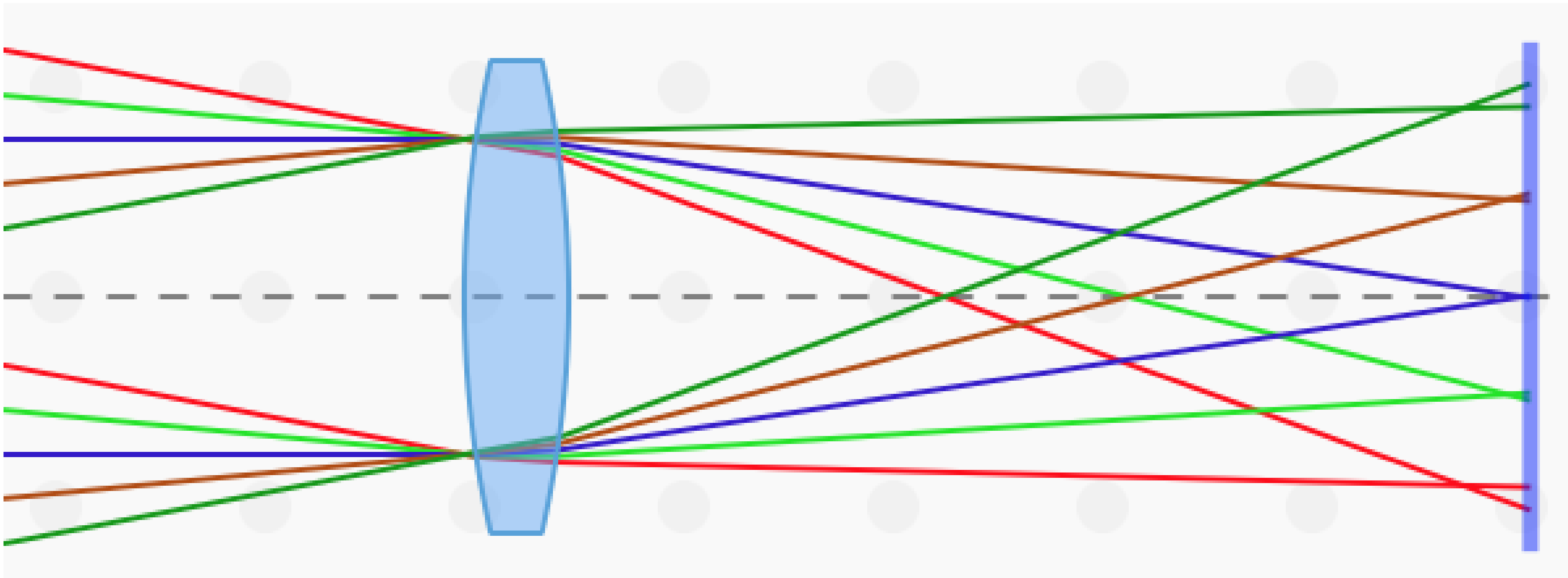
Astigmatism

From off-axis point A lens does not appear symmetrical but shortened in plane of incidence (tangential plane).
Emergent wave will have a smaller radius of curvature for tangential plane than for plane normal to it (sagittal plane) and form an image closer to the lens.





Field Curvature

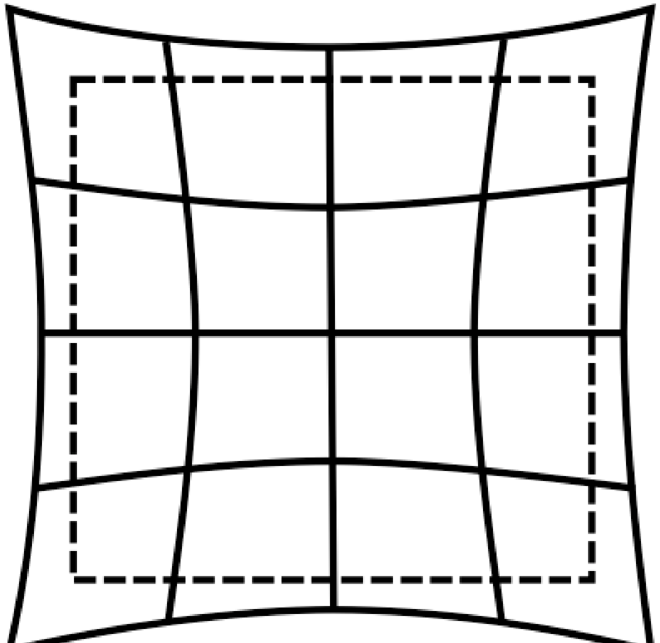


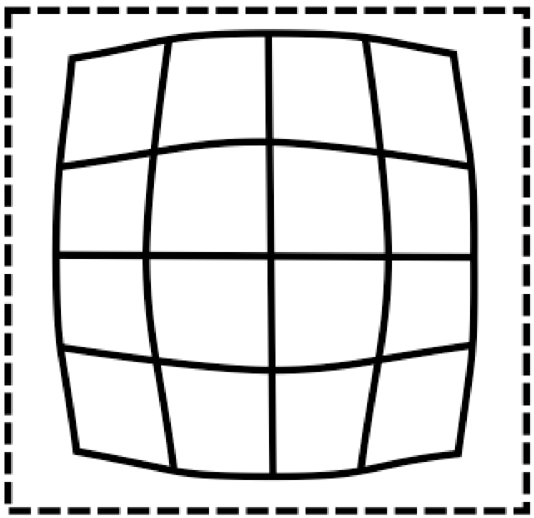
Only objects close to optical axis will be in focus on flat image plane. Off-axis objects will have different focal points.

Distortion

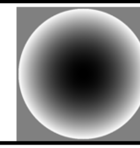
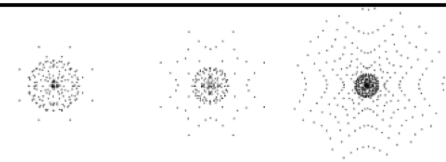
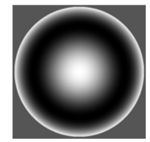
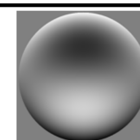
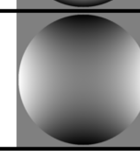
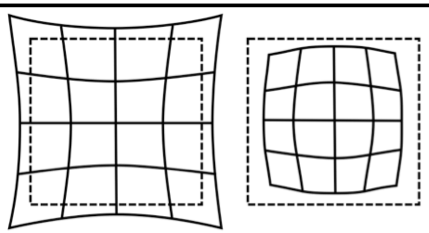
Straight line on sky becomes curved line in focal plane because magnification depends on distance to optical axis.

1. Outer parts have larger magnification→ pincushion
2. Outer parts have smaller magnification → barrel





Aberrations Summary

aberration	spot diagram / image	wavefront	scaling	
perfect			-	-
focus			$1/F^2$	-
spherical			$1/F^3$	-
coma			$1/F^2$	y
astigmatism			$1/F^2$	y^2
field curvature			$1/F^2$	y^2
distortion			-	y^3

Chromatic Aberration

Refractive index variation with wavelength n(λ) results in focal length of lens f(λ) to depend on wavelength; different wavelengths have different foci