

Astronomical Observing Techniques 2019:
Exercises on Detectors
(Due on 29 April 2019 at 11:00)

April 7, 2019

1 Fermi Distribution

The Fermi distribution for the probability of finding an electron with an energy E in a given physical system is given by:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \quad (1)$$

1. What is the probability of finding an electron in the conduction band at zero temperature? Explain this.
2. What is the probability of finding an electron with the Fermi energy at zero temperature? And at 100 K? What does this imply about the Fermi energy? Make a drawing of $f(E)$ for $T=0$ and $T>0$.

The Fermi distribution gives a probability, but in a physical system, each energy has several states associated with it. To account for the actual number of electrons within an energy range dE , we need to know the density of states $N(E)$. The concentration of electrons n_0 in the energy range $[E_1, E_2]$ is

$$n_0 = \int_{E_1}^{E_2} f(E)N(E)dE \quad (2)$$

For the conduction band, $N(E)$ can be derived as

$$N(E) = \frac{1}{2\pi^2} \left(\frac{2m_{eff}}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2} \quad (3)$$

with $\hbar = h/2\pi$.

3. Which famous equation do you use to derive a density of states?
4. What are (E_1, E_2) for the electron concentration in the conduction band?
5. Show that in the limit where $E - E_F \gg kT$, the concentration of electrons is given by:

$$n_0 = 2 \left(\frac{2\pi m_{eff} kT}{h^2} \right)^{3/2} e^{(E_F - E_c)/kT} \quad (4)$$

Tip:

$$\int_0^\infty x^{1/2} e^{-ax} dx = \frac{\sqrt{\pi}}{2a\sqrt{a}} \quad (5)$$

6. The bandgap energy for pure silicon is 1.11 eV. What is $E_c - E_F$, using the definition of the Fermi energy?
7. Calculate the electron concentration in the conduction band at $T = 300K$ and $T = 30K$ for a “pixel” of volume 1 mm^3 . Note that m_{eff} for silicon is $\sim 1.1 m_e$.
8. Explain why both the $f(E)$ and the $N(E)$ are essential for building a photo-conductor.

2 Gain Determination

The analog-to-digital converter in CCD cameras converts the electrical signal into digital numbers, so-called Arbitrary Digital Units (ADU). When calibrating a CCD camera, one needs to know the gain G , which relates the ADU A and the number of electrons e via $A = G \cdot e$. Show that $G = \frac{\sigma_A^2}{A}$ where σ_A^2 is the measured variance in ADU due to the Poisson statistics of photon-generated electrons.

3 Backside Illuminated CCDs

There are two different ways to operate a CCD: in front and in back illumination mode (see figure below). In front illumination, the metal electrode is replaced by heavily doped silicon (since pure metal would block the incoming photons), but it is still not transparent at blue and UV wavelengths. Therefore, for detectors used at short wavelengths, backside illumination is used.

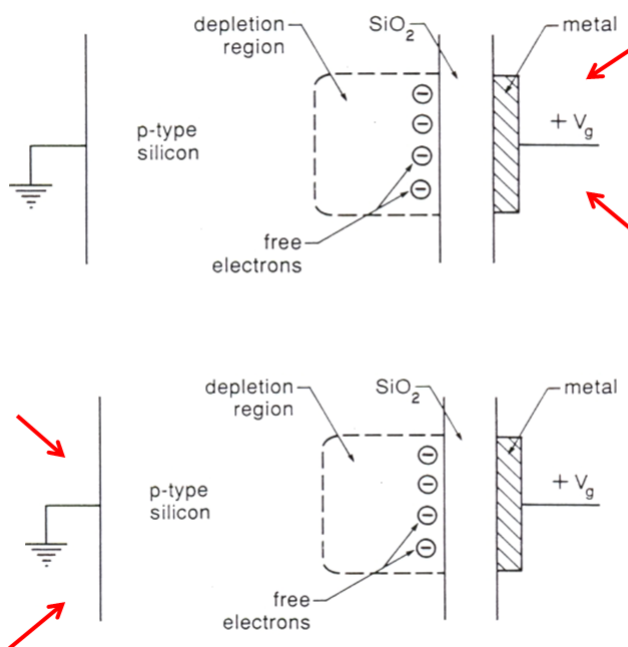


Figure 1: Front (top) and back illuminated (bottom) CCDs, arrows indicate the incoming photons.

1. Although the absorption of UV/blue photons in the p-type silicon is efficient, the quantum efficiency of a back-illuminated CCD, as illustrated in the above figure, is generally low. Explain why this is the case.
2. Most backside-illuminated CCD are “thinned” (reduced thickness of the p-type silicon). Explain the objective of the “thinning” and list considerations on minimum/maximum thickness.