Astronomical Observing Techniques 2019:
Exercises on Atmosphere
(Due on 8 April 2019 at 11:00)

March 31, 2019

1 Airmass

The flux of a star is reduced by absorption in the Earth's atmosphere: $I = I_0 e^{-\tau}$, with $\tau = A \int \rho(z)\kappa(z)dz$. A is the airmass, κ the absorption coefficient and z the altitude. The airmass is given by $1/\cos(\theta)$, with θ the zenith angle. The optical depth, τ, is difficult to calculate in practice as $\rho(z)$ and $\kappa(z)$ are not precisely known. Show that we can find I_0, i.e. the flux before the star light enters the Earth's atmosphere, if we carry out two measurements of the received flux (I_1 and I_2) at different airmasses (A_1 and A_2), assuming the properties of the atmosphere do not change between the two measurements.

2 Sky Background

1. Calculate the spectral radiance (at zenith) of the sky background in the L band (3.4μm). Assume that the optical depth is $\tau = 0.15$, which is much smaller than 1. Use wavelength units and assume that the average temperature of the atmosphere is $T = 250$ K.
2. Calculate the sky brightness in mag arcsec^{-2}. For mag$_L = 0$, the spectral irradiance is 8.1*10^{-11} W m$^{-2}\mu$m^{-1}.

3 Refraction

The direction of light passing through the atmosphere changes because of the changing index of refraction with hight. The amount of change is given by Snell's law: $n_1 \sin(z_1) = n_2 \sin(z_2)$. Let z_t be the true zenith angle, z_0 the observed zenith angle, z_i the observed zenith angle at layer i in the atmosphere, $n_0(\lambda)$ the index of refraction at the surface, and $n_i(\lambda)$ the index of refraction at layer $i(i = 1....N)$.

1. Show that the refraction only depends on the index of refraction near the Earth's surface.
2. We define astronomical refraction, R, to be the angular amount that the object is displaced by the refraction of the Earth's atmosphere. Derive the following approximation to the refraction $R(z_0)$ as a function of the observed zenith angle z_0:

$$R = (n - 1)\tan(z_0)$$

   (Hint: Use $\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$ and $R \ll 1$).
3. How large is this effect for an object observed at a zenith angle of 45°? Take a typical index of refraction of 1.00029.
4. We want to observe a source in the L band ($\lambda = 3.45\mu$m, bandwidth = 472 nm) with a diffraction-limited 15-m telescope. Do we need to worry about distortion due to the dispersion for a zenith angle of 45°? And for 85°? Hint: the diffraction limit of a telescope is given by $1.22\lambda/D$.