Astronomical Observing Techniques 2019:
Exercises on Optics
(Due on 25 February 2019 at 11:00)

February 18, 2019

1 Spherical aberration from singlet lenses

For a single, biconvex lens that images an object at infinity on the optical axis, show that the transverse spherical aberration is roughly proportional to F^{-3}, where F is the f-number.

Use the Touch Optical Design (TOD) software by Christoph Keller to perform ray-tracing simulations in your browser. This software is available at `http://strw.leidenuniv.nl/~keller/TOD/tod.min.html`. Determine the rms spot size in the best focus for a number of different beam diameters illuminating a single lens with a fixed focal length. Note that the location of best focus depends on the beam diameter, even if the lens has a constant focal length.

2 Point-spread functions

As you have seen, the Point-Spread Function (PSF) of an optical system can be calculated using Fourier transforms. In this exercise we will calculate PSFs for a perfect and an aberrated optical system.

a) Write a function that creates a telescope pupil. This pupil is a two-dimensional array where 1 indicates that light is transmitted, and 0 everywhere else. Use the following function signature:
`def make_circular_aperture(num_pixels, diameter)`.

b) Write a function that computes the PSF for a given aperture. The PSF is given by the absolute value squared of the Fourier transform of the aperture. Use two-dimensional FFTs for this and remember to add the appropriate FFTShifts and IFFTShifts. Use the following function signature:
`def compute_psf(aperture)`

c) Write a function that creates an obstructed circular pupil. Use the following function signature:
`def make_obstructed_circular_aperture(num_pixels, diameter, obscuration_ratio)`
Hint: use the `make_circular_aperture` function that you wrote before.

d) Calculate the PSF for various obstruction ratios ranging from 10% to 50%, show the results and describe them qualitatively. Make sure that the outer diameter of the pupil does not exceed a quarter of the array size. Use at least 1024 pixels in the aperture array.

3 Aberrations

Until now we have assumed a perfect optical system. Actual optical systems are not perfect, as you have seen in the first exercise. Here we will calculate some example aberrated PSFs. Common aberrations are focus, astigmatism and coma:

$$W_{\text{focus}}(x, y) = \sqrt{3}(2(x^2 + y^2) - 1), \tag{1}$$
$$W_{\text{astigmatism}}(x, y) = \sqrt{6}(x^2 - y^2), \tag{2}$$
$$W_{\text{coma}}(x, y) = \sqrt{8}(3x^3 + 3xy^2 - 2x), \tag{3}$$

where x and y are normalized by the radius of the pupil, so that $x^2 + y^2 \leq 1$ inside the pupil.

a) Write functions that compute the wavefront error for each of the aberrations mentioned above. Use the functions signatures:
```
def compute_focus(num_pixels, pupil_diameter)
def compute_astigmatism(num_pixels, pupil_diameter)
def compute_coma(num_pixels, pupil_diameter)
```
Show that aberrations correspond to those in the lecture slides.

b) Compute defocused PSFs for varying strengths of the defocus. You can add an aberration to a certain telescope pupil by multiplying it by the complex exponential of the aberration:

$$A = A_{\text{pup}} \exp{(i\alpha W_{\text{focus}})}. \tag{4}$$

where α is the amplitude (ie. strength) of the aberration. Calculate the Strehl ratio for each of these images.

c) Do the same for astigmatism and coma.

4 Binary stars

So far we have only created images of single point sources (ie. single stars). We can simulate the image from a double star by convolving a perfect image of the double star with the PSF of the telescope.

a) Write a function to convolve two images by using Fourier transforms. Again, make sure to put in appropriate FFTShifts and IFFTShifts. Use the following function signature:
```
def convolve(f, g)
```

b) Compute the perfect, non-telescopic image of a binary star. This image is zero everywhere except at two separate pixels, which indicate the positions of the two stars.

c) Compute the telescope image of the binary star. Alter the positions of the two stars so that in the telescope image they are close, but still visible as two separate stars (ie. still a "resolved" binary star).