Astronomical Observing Techniques 2019: Exercises on Fourier transforms (Due on 18 February 2019 at 11:00) Updated on 12 February 2019

February 12, 2019

This exercise set consists of analytical derivations to be done without Python, and computer exercises to be done in Python. For the latter, you'll need some supplementary files, that are available on the course website: `https://strw.leidenuniv.nl/~por/AOT2019`. Append your code to your submitted solutions.

1 Mathematical derivations

The Fourier pairs are $f(x)$ and $F(s)$. The expression for Fourier transform of $f(x)$ is:

$$\int_{-\infty}^{+\infty} f(x) e^{-2\pi i x s} dx = F(s). \tag{1}$$

The Fourier transform of $f(x)$ can also be denoted by: $\mathcal{F}(f(x)) = F(s)$. The inverse Fourier transform of $F(s)$ is denoted by $\hat{\mathcal{F}}(F(s)) = f(x)$, is $f(x)$ and it is defined as:

$$\int_{-\infty}^{+\infty} F(s) e^{2\pi i x s} ds = f(x). \tag{2}$$

a) Show that $\mathcal{F}(a(f(x)) + b(g(x))) = a\mathcal{F}(f(x)) + b\mathcal{F}(g(x))$

b) Show that $\mathcal{F}(f(x-a)) = e^{-2\pi i a s}\mathcal{F}(f(x))$.

c) Calculate $\mathcal{F}(\delta(x))$.

d) Calculate $\mathcal{F}(e^{-\pi x^2})$. Hint: use that $\int_{-\infty}^{\infty} e^{-ax^2} \mathrm{d}x = \sqrt{\frac{\pi}{a}}$.

e) Calculate $\mathcal{F}(\Pi(x))$ where:

$$\Pi(x) = \begin{cases} 1 & \text{if } |x| < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

2 Noise filtering using FFTs

a) Write a function that takes the Fourier transform of an array using Numpy. The Fourier transform in Numpy defines the zero frequency to be at index 0. We expect the zero frequency to be in the middle of the array. You can convert between these two conventions by using the `numpy.fft.fftshift` function. Do an fftshift before and after each Numpy Fourier transform.

b) Load in the time series from `time_series.fits` and plot the time series. The first column of the fits file contains time ranging from [-0.5,0.5] seconds. The second column contains the values of the time series.

c) Calculate the Fourier transform of the time series and plot as a function of frequency (determined using Numpy function `numpy.fft.fftfreq`).

d) What are the dominate frequencies of the signal?

3 Exoplanet detection with cross-correlation

Many astrophysical observations requires the detection of small signals buried in noise. In this exercise we will perform a similar analysis as those used for detection of water, methane and carbon dioxide on exoplanets.

a) Write a function that performs cross-correlation between two Numpy arrays using the Fourier transforms.

b) Load in the model and observed spectrum from `model_spectrum.fits` and `observed_spectrum.fits`, and plot the spectra as function of wavelength. The wavelength is given in Ångström. Can you see the spectral lines in the observed spectrum?

c) Cross-correlate the observed spectrum with the model spectrum. Also compute the autocorrelation of the model spectrum (ie. the cross-correlation of the model spectrum with itself). Plot both correlations including labels and units on the axes. Describe this plot qualitatively.

d) What is the shift in wavelength in the observed spectrum compared to the model spectrum? What redshift velocity does this correspond to?