Lecture 4-2: Nuclear Burning Stages

Literature: Prialnik chapter 4.3–4.10

a) History

Aston (1920): $m_{^4He} < 4m_p$ $\Rightarrow$ Eddington: nuclear reactions must be source of stellar energy

> ‘What is possible in the Cavendish Laboratory may not be too difficult in the Sun’
>
> ‘We do not argue with the critic who urges that the stars are not hot enough for this process. We simply tell him to go and find a hotter place’

- Sun: $T_c \sim 15\ 10^6$ K, $\rho_c \sim 100$ g/cm^3
- Main sequence stars: similar T_c, ρ_c
- Main sequence stars mostly hydrogen

Þ Universally accepted assumption: *Basic energy source on the main sequence is fusion of hydrogen into helium*

1. Proton-proton chain ($M<1.2M_\odot$)

Direct (but not simultaneous) combination of 4 protons to form α-particle (He nucleus) (von Weiszäcker 1937; Bethe 1938)

2. CNO-cycle ($M>1.2M_\odot$)

- Indirect fusion resulting from 4 successive proton captures by heavier nucleus, with eventual emission of α-particle
- Bethe (1939): lifetime against H-capture (at solar ρ_c and T_c)

^{2}H, ^{3}H, ^{6}Li, ^{9}Be, ^{10}B, ^{11}B: < 1 sec – 10^4 yr
C, N, O: $10^6 – 10^7$ yr
Heavier nuclei: $10^{10} – 10^{11}$ yr

Thus, indirect fusion possible with C, N, O as catalysts

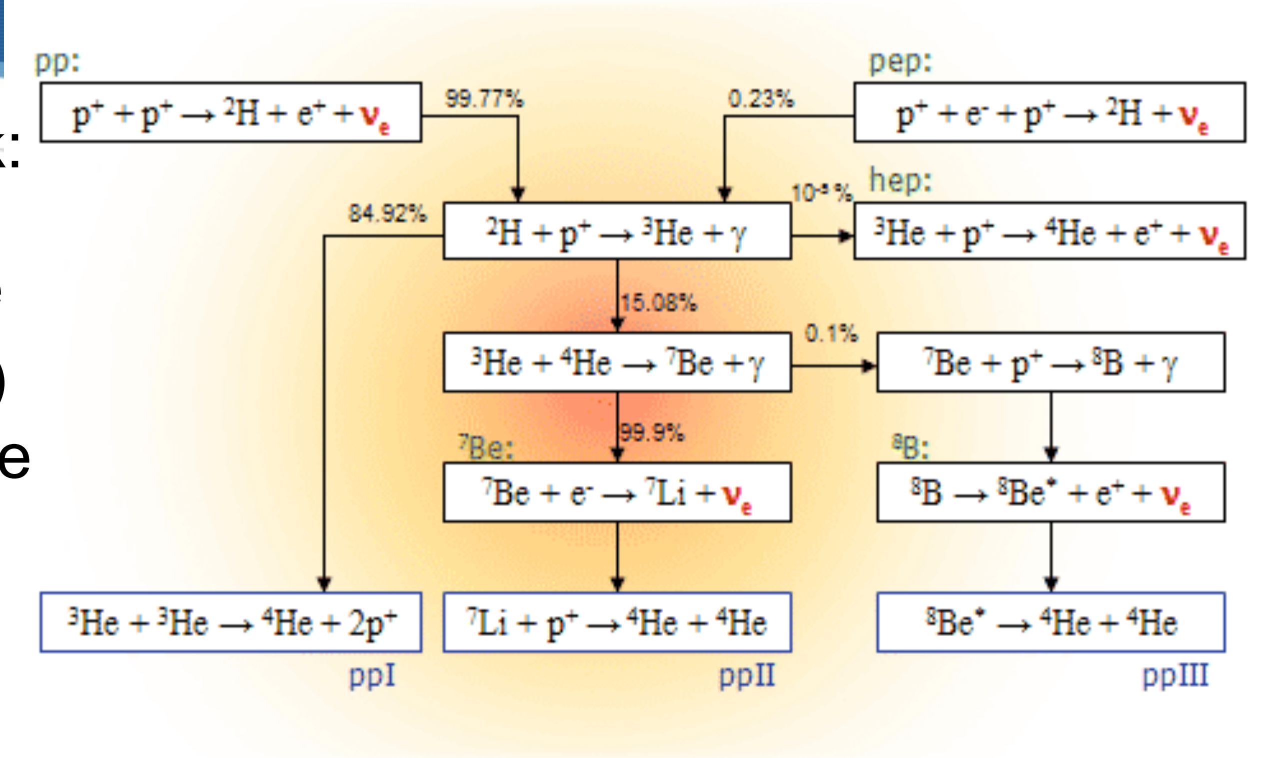
3. Other `burnings‘

Tunneling depends on $E_0 \propto Z_a^{2/3} Z_X^{2/3}$ (Ch4-1, 17) and hence different processes occur at significantly different central temperatures

Thus, well-separated phases during stellar evolution

b) Proton-proton chain

Reaction network:



- If He abundance
- very small: (pp2) & (pp3) negligible

- (pp1) in Sun $(T_c \approx 13 \times 10^6 K, \quad \rho X_H \approx 100\, g/cm^3, \quad \rho X_{^3He} \approx 0.01\, g/cm^3$

Reaction	Lifetime	Q(MeV)
$^1H + {}^1H \rightarrow {}^2H + e^+ + \boxed{?}$	14 10^9 yr	1.18 (times 2)
$^2H + {}^1H \rightarrow {}^3He + \boxed{?}$	6 sec	5.49 (times 2)
$^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$	10^6 yr	12.86

Q_{tot}=26.20 MeV

- First step very slow because: β-decay (weak interaction) is required at closest approach of 2 protons (see assignment)

- Reduced mass of ^{3}He+^{4}He > reduced mass of ^{3}He+^{3}He

  If ^{4}He abundance appreciable: (pp2) and (pp3) rates increase with T faster than (pp1), thus (pp1) dominates at T< 10^7 K, and (pp2)+(pp3) dominate at T > 10^7K

- Q_{tot} (pp2)=25.67 MeV Q_{tot}(pp3)=19.2 MeV

$$\varepsilon_{pp} = \psi \varepsilon_{pp1}$$

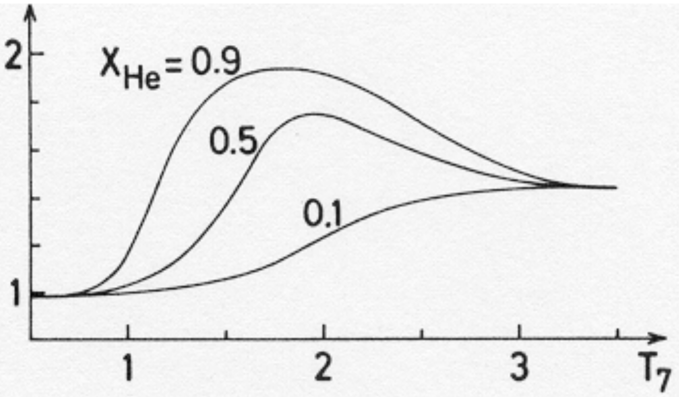


Fig. 18.7. The correction ψ for ε_{pp} as a function of T_7, for three different helium abundances. (After PARKER, BAHCALL, FOWLER, 1964)

- Other completions of pp chain possible at high ρ_c:

$$^3He(e^-,\nu)^3H \rightarrow$$

$$^3H(p,\gamma)^4He \qquad or \qquad ^3H(^3He,np)^4He \qquad or \qquad ^3H(^3H,2n)^4He$$

First step endothermic by 18 keV

But ε_F > 18 keV for ρ_c > 2 10^4 g/cm^3

- $D + p \rightarrow {}^3He + \gamma$

This is a fast reaction and all (primordial) D is quickly destroyed at moderate T (strong nuclear interaction)

c) Reactions of protons with light nuclei

$^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma \qquad T_d \approx 0.5\times10^{6}\,K$

$^{6}Li + {}^{1}H \rightarrow {}^{3}He + {}^{4}He \qquad \approx 2.0\times10^{6}\,K$

$^{7}Li + {}^{1}H \rightarrow 2\,{}^{4}He \qquad \approx 2.4\times10^{6}\,K$

$^{9}Be + 2\,{}^{1}H \rightarrow 2\,{}^{4}He + {}^{3}He \qquad \approx 3.2\times10^{6}\,K$

$^{10}Be + 2\,{}^{1}H \rightarrow 3\,{}^{4}He \qquad \approx 4.9\times10^{6}\,K$

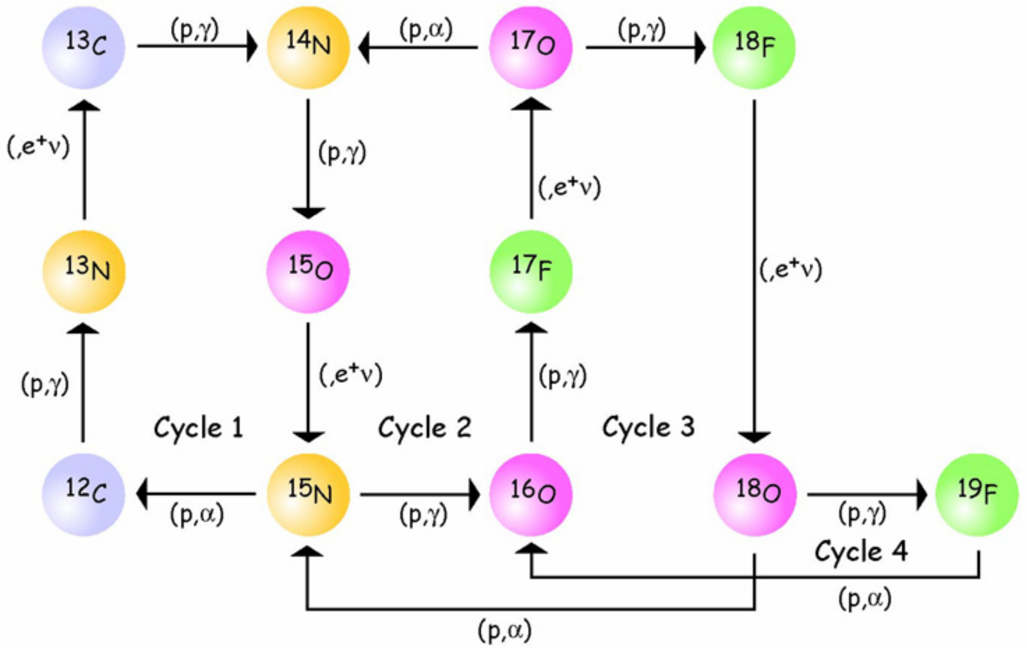
$^{11}B + {}^{1}H \rightarrow 3\,{}^{4}He \qquad \approx 4.7\times10^{6}\,K$

T_d = temperature at which abundance drops to 1/e of initial value in 10^9 yr

- These light elements *cannot* be present in the central regions of the Sun, as $T > T_d$
- Observations: $(Li/Fe)_{\odot} < 0.01\,(Li/Fe)_{earth}$

Reason: Sun has convective envelope, and $T > T_d$ at its bottom, so that Li is destroyed there

d) CNO-cycle



Reaction	Lifetime	Q(MeV)
^{12}C(p,γ)^{13}N	1.3 10^7 yr	1.94
^{13}N(,e+γ)^{13}C	7 min	1.51
^{13}C(p,γ)^{14}N	2.7 10^6 yr	7.54
^{14}N(p,γ)^{15}O	3.2 10^8 yr	7.29
^{15}O(,e+γ)^{15}N	82 sec	1.76
^{15}N(p,α)^{12}C	1.1 10^5 yr	4.96

Q_{tot}=25.02 MeV

(for equilibrium at T ~ 13 10^6 K)

Catalysis: Some ^{12}C is required to start the CNO cycle.

Cycle 1 is the main cycle but many other cycles may contribute

^{14}N(p,g)^{15}O is the bottle neck & ^{14}N abundance will pile up

Thus, initial CNO abundances change until in equilibrium:

$$\frac{X_{^{12}C}}{X_{^{13}C}} = 4.3 \qquad \frac{X_{^{14}N}}{X_{^{15}N}} = 2800 \qquad \frac{X_{^{14}N+^{15}N}}{X_{^{12}C+^{13}C}} = 21$$

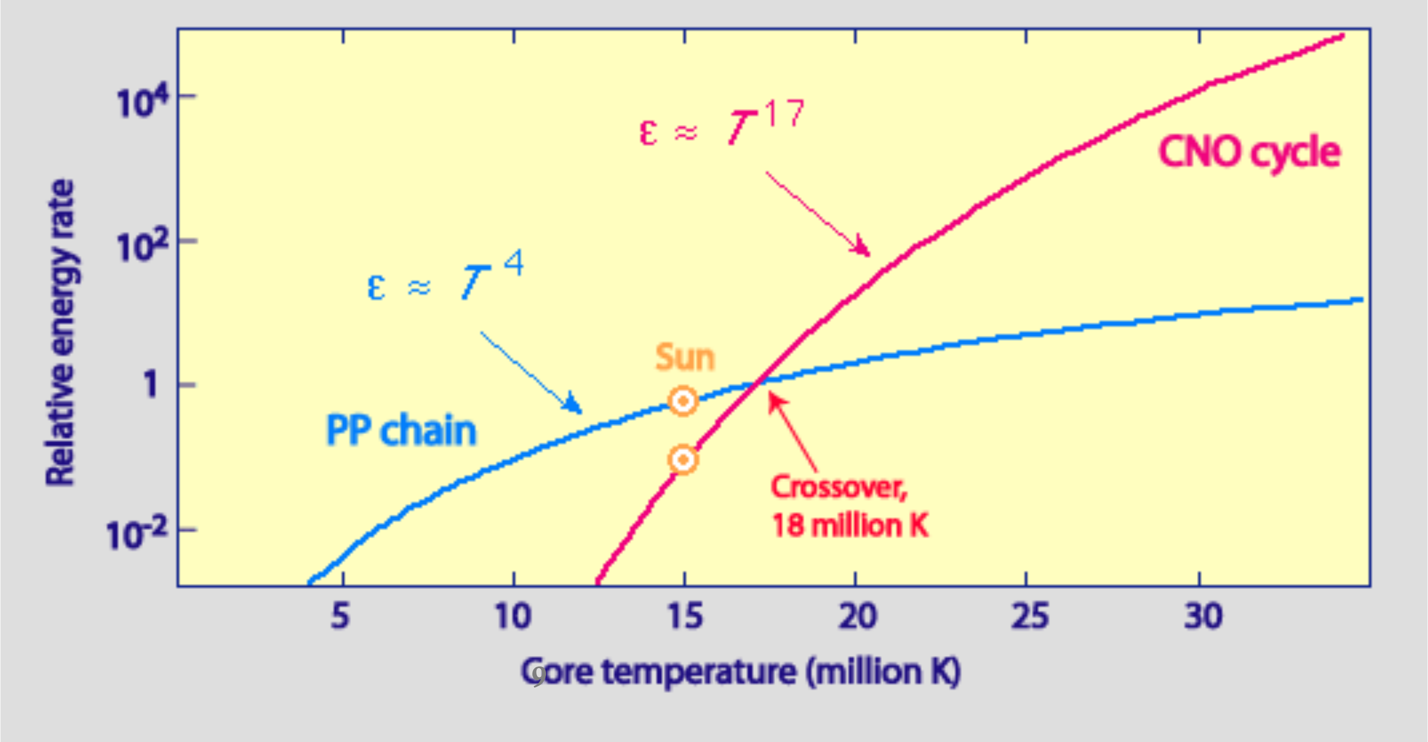
Earth: $X_{^{12}C}/X_{^{13}C} >> 4.3$

Sun: C:N:O=5.5:1:9.6; not made in CNO-cycle

Carbon stars: $X_{^{12}C}/X_{^{13}C}$ down to 4 effect of CNO-cycle

<u>Energy generation</u>

Steep T-dependence since reactions involve one proton and one heavier nucleus



e) Triple alpha process

No stable elements with A=5 and A=8
Neutron reactions are slow as neutron abundance very low
Further energy has to come from reaction with ^{4}He nuclei

$$^4He + {}^4He \rightarrow {}^8Be + \gamma \;\; Q = -0.092 \text{ MeV}$$

$$^8Be \rightarrow {}^4He + {}^4He \text{ lifetime } 2.6\times10^{-16} \text{ s}$$

$$X\left({}^8Be\right)/X\left({}^4He\right) \approx 10^{-8} \text{ for T} \approx 10^8 \text{ K and } \rho \approx 10^6 \text{ g/cm}^3$$

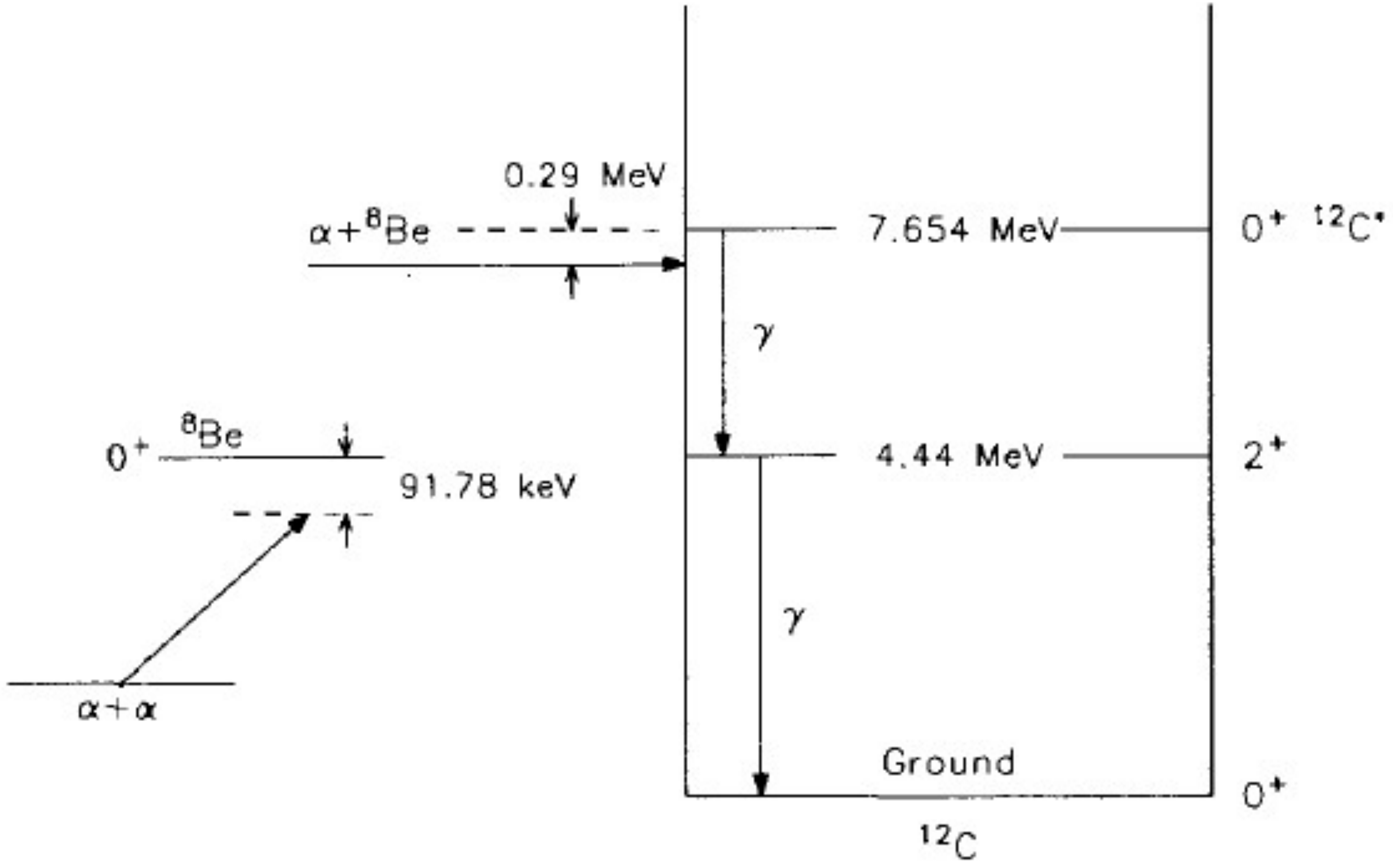
With some *^{8}Be* present, *^{12}C* formation can proceed through an excited state (see slide 11):

$$^8Be + {}^4He \leftrightarrow {}^{12}C^* \rightarrow {}^{12}C + 2\gamma \;\; Q = 7.367 \text{ MeV}$$

$$\varepsilon_{3\alpha} \approx 5.1\times10^8 \rho^2 Y^3 \left(T/10^9\right)^3 \exp\left[-4.40\times10^9 / T\right] \text{ erg/g/s}$$

$$\text{at T=}10^8 \text{ K, } \varepsilon_{3\alpha} \propto T^\nu \text{ with } \nu = \left(4.4\times10^9/T\right) - 3 \approx 41$$

The triple a reaction energy level diagram



Once ${}^{12}C$ has been formed further reactions with a particles become possible:

$$\alpha + {}^{12}C \rightarrow {}^{16}O + \gamma$$

$$\alpha + {}^{16}O \rightarrow {}^{20}Ne + \gamma$$

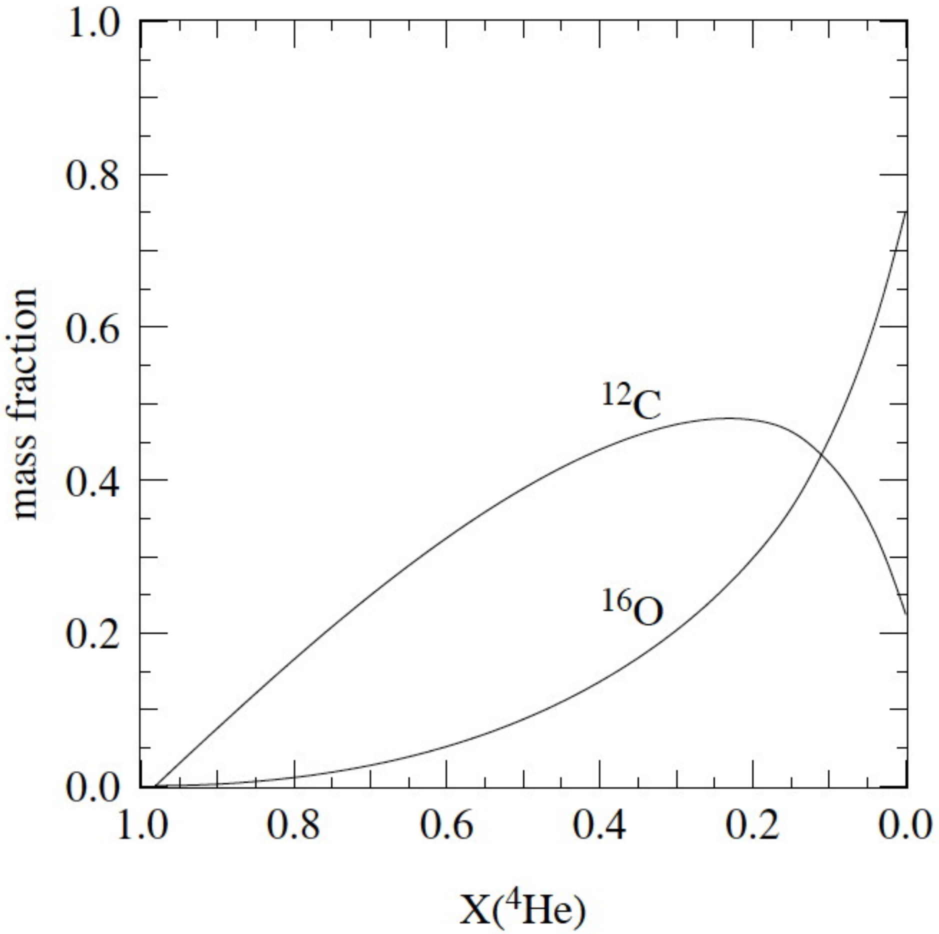
$$\alpha + {}^{20}Ne \rightarrow {}^{24}Mg + \gamma$$

The first reaction is of course particularly important to us. It proceeds through a resonance and its rate is uncertain.

^{12}C and ^{16}O formation in star of intermediate mass

$$\varepsilon_{3\alpha} \propto Y^3 \rho^2 T^{40}$$

$$\varepsilon_{\alpha C} \propto Y^2 X_{12} \rho T^{20}$$



f) Carbon burning

$$\left.\begin{array}{llll} & & Q[\text{MeV}] & Y \\ {}^{12}C+{}^{12}C\rightarrow & {}^{23}Na+p & 2.238 & 0.56 \\ \rightarrow & {}^{20}Ne+\alpha & 4.616 & 0.44 \end{array}\right\}\ \text{Most probable}$$

$$\begin{array}{lll} \rightarrow & {}^{24}Mg+\gamma & 13.931 \\ \rightarrow & {}^{23}Mg+n & -2.605 \\ \rightarrow & {}^{16}O+2\alpha & -0.114 \end{array}$$

This reaction is followed by ^{23}Na(p,a)^{20}Ne and ^{23}Na(p,g)^{24}Mg using the protons from the first reaction. The alpha particles are used to burn ^{16}O to ^{20}Ne or ^{20}Ne to ^{24}Mg. Net result, formation of ^{20}Ne with lesser amounts of ^{23}Na, ^{24}Mg.

Conditions: T=0.5-1x10^9 K r≈3x10^6 g/cm^3

$$\varepsilon_{CC} \approx 1.43\times 10^{42} Q(MeV) Y_{CC} \rho X_{12}^2 / T_9^{3/2} \exp[-84.165/T_9^{1/3}] \ \text{erg/g/s}$$

g) Neon burning

^{16}O is very stable and neon burning occurs first. As T is very high ($T_9 \approx 1.5$), photons are in the MeV range. This leads to photodisintegration processes:

$$^{20}Ne + \gamma \to {}^{16}O + \alpha \quad Q = \text{-4.73 MeV}$$

The α particle is used in:

$$^{20}Ne + \alpha \to {}^{24}Mg + \gamma \quad Q = +9.3 \text{ MeV}$$

or effectively,

$$2\,^{20}Ne + \gamma \to {}^{16}O + {}^{24}Mg + \gamma \quad Q = +4.583 \text{ MeV}$$

h) Oxygen burning

	Q[MeV]	Y
$^{16}O + {}^{16}O \rightarrow {}^{28}Si + \alpha$	9.539	0.21
$\rightarrow {}^{31}P + p$	7.678	0.61
$\rightarrow {}^{31}S + n$	1.500	0.18

$$\varepsilon_{OO} \approx 1.3 \times 10^{52} Q(MeV) Y_{OO} \rho X_{16}^{2} / T_9^{3/2} \exp[-135.93 / T_9^{1/3}] f(T_9) \text{ erg/g/s}$$

with

$$f\left(T_9\right) = \exp\left[-0.629 T_9^{2/3} - 0.445 T_9^{4/3} + 0.0103 T_9^{2}\right]$$

i) Silicon burning

Silicon burning proceeds through photodisintegration (α,γ) and α-capture (α,γ) processes when $T_9 \geq 3$. Thus, part of the ^{28}Si "melts" into lighter nuclei while the other part captures the released a-particles to make heavier nuclei. Most of these reactions are in equilibrium and can be described by the equivalent of the Saha equation. When $T_9 \geq 4$, this results in nuclear statistical equilibrium.

Nuclear Statistical Equilibrium: Strong nuclear and EM interactions are in equilibrium
Nuclear abundances are function of T, ρ, and neutron-richness: most abundant nuclei are those with highest binding energy. This carries composition to iron peak.

j) The end of nuclear burning

$$^{28}Si + \alpha \rightarrow {}^{32}S + \gamma$$

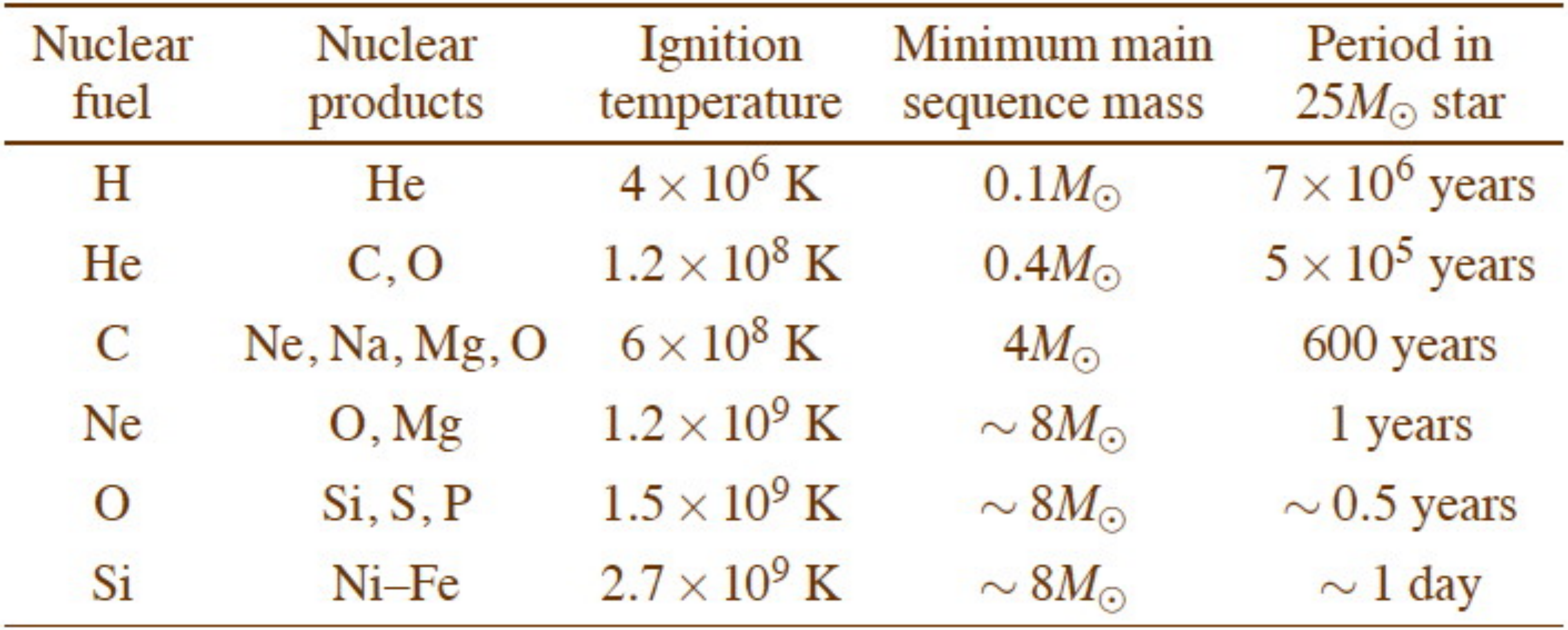
$$^{32}S + \alpha \rightarrow {}^{36}Ar + \gamma$$

$$\vdots$$

$$^{52}Fe + \alpha \rightarrow {}^{56}Ni + \gamma$$

Most of the energy is carried away by neutrino’s
Core will collapse to nuclear densities (& SN explosion)

k) Nuclear burning timescales



Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_\odot$ star
H	He	4×10^6 K	$0.1M_\odot$	7×10^6 years
He	C, O	1.2×10^8 K	$0.4M_\odot$	5×10^5 years
C	Ne, Na, Mg, O	6×10^8 K	$4M_\odot$	600 years
Ne	O, Mg	1.2×10^9 K	$\sim 8M_\odot$	1 years
O	Si, S, P	1.5×10^9 K	$\sim 8M_\odot$	~ 0.5 years
Si	Ni–Fe	2.7×10^9 K	$\sim 8M_\odot$	~ 1 day

Short timescales beyond the main sequence because:

Small amount of energy released per reaction

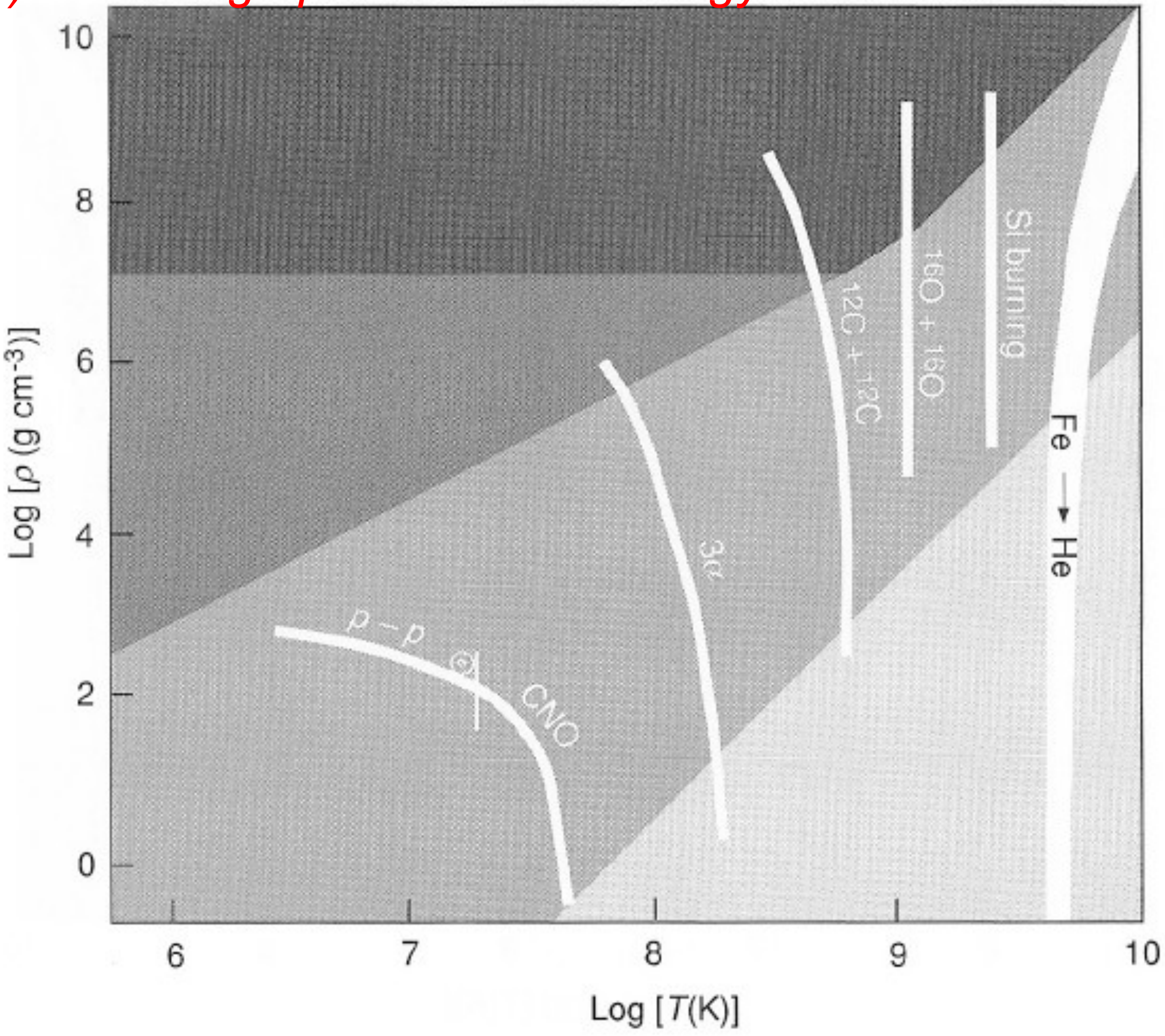
Few nuclei available

High burning temperature, which imply large energy losses

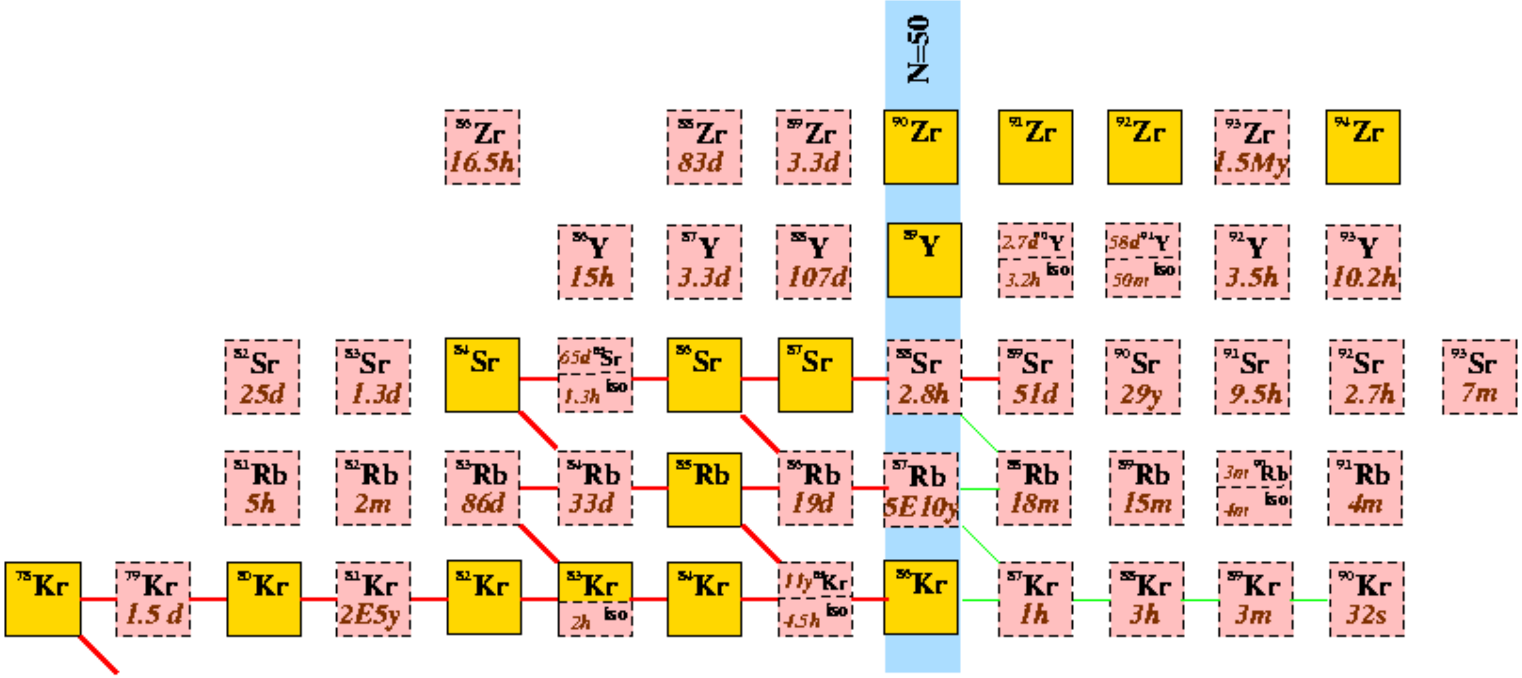
I) Beyond the iron core

The iron group elements are the most tightly bound elements and no more energy can be gained by further nuclear reactions. Energy is lost through neutrino production (see slide 23). The core will shrink, the increased density will lead to recombination of protons and electrons to neutrons (inverse beta process) and neutrinos. The heavy energy losses will lead to core collapse. This forms a ‘rigid’ neutron star on which the infalling outer layers will bounce.

m) Summing up the nuclear energy sources



n) Slow neutron process in AGB stars



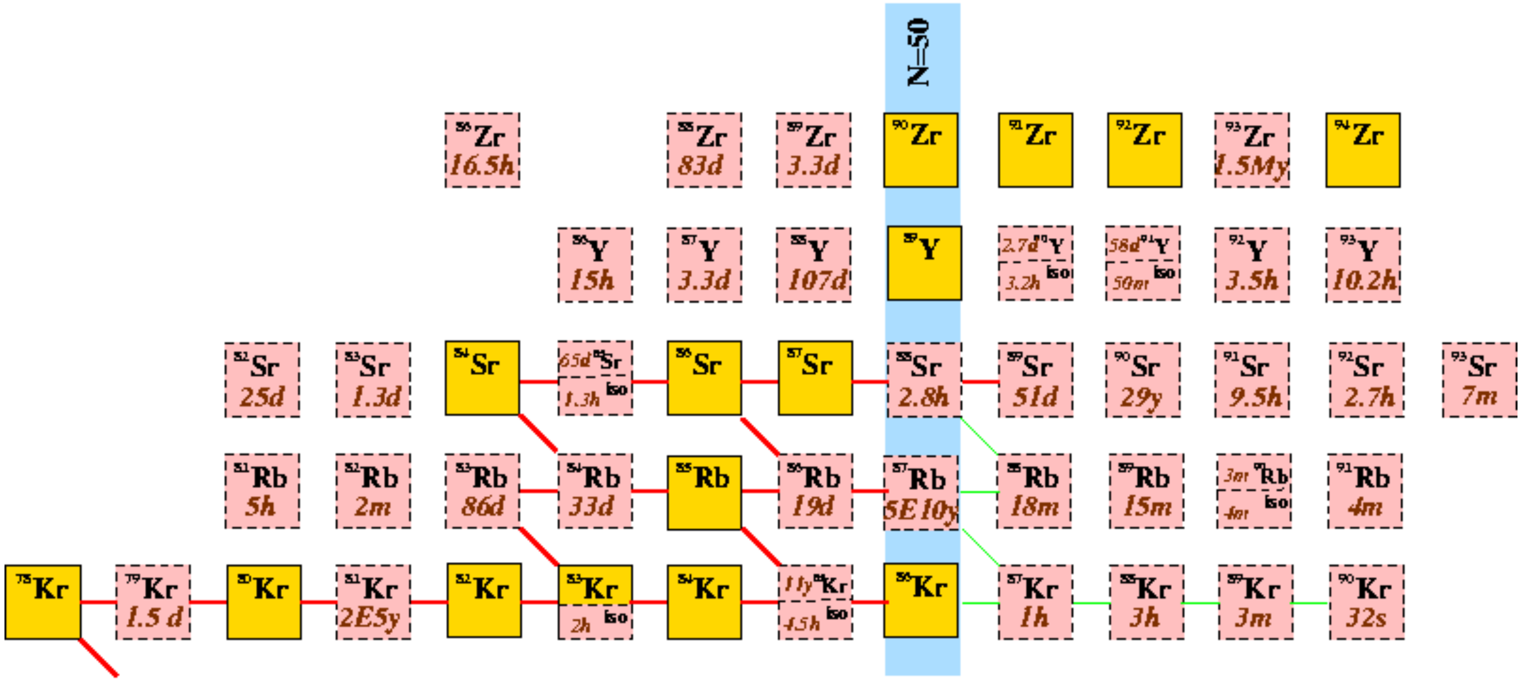
Slow neutron process: neutron capture until unstable element is formed that decays through beta decay

Typical s-process elements are: Zr, Sr, Ba, Pb

AGB stars produce neutrons through $^{13}C(\alpha,n)^{16}O$ and $^{22}Ne(\alpha,n)^{25}Mg$

Mixed to the surface by `Helium flash’

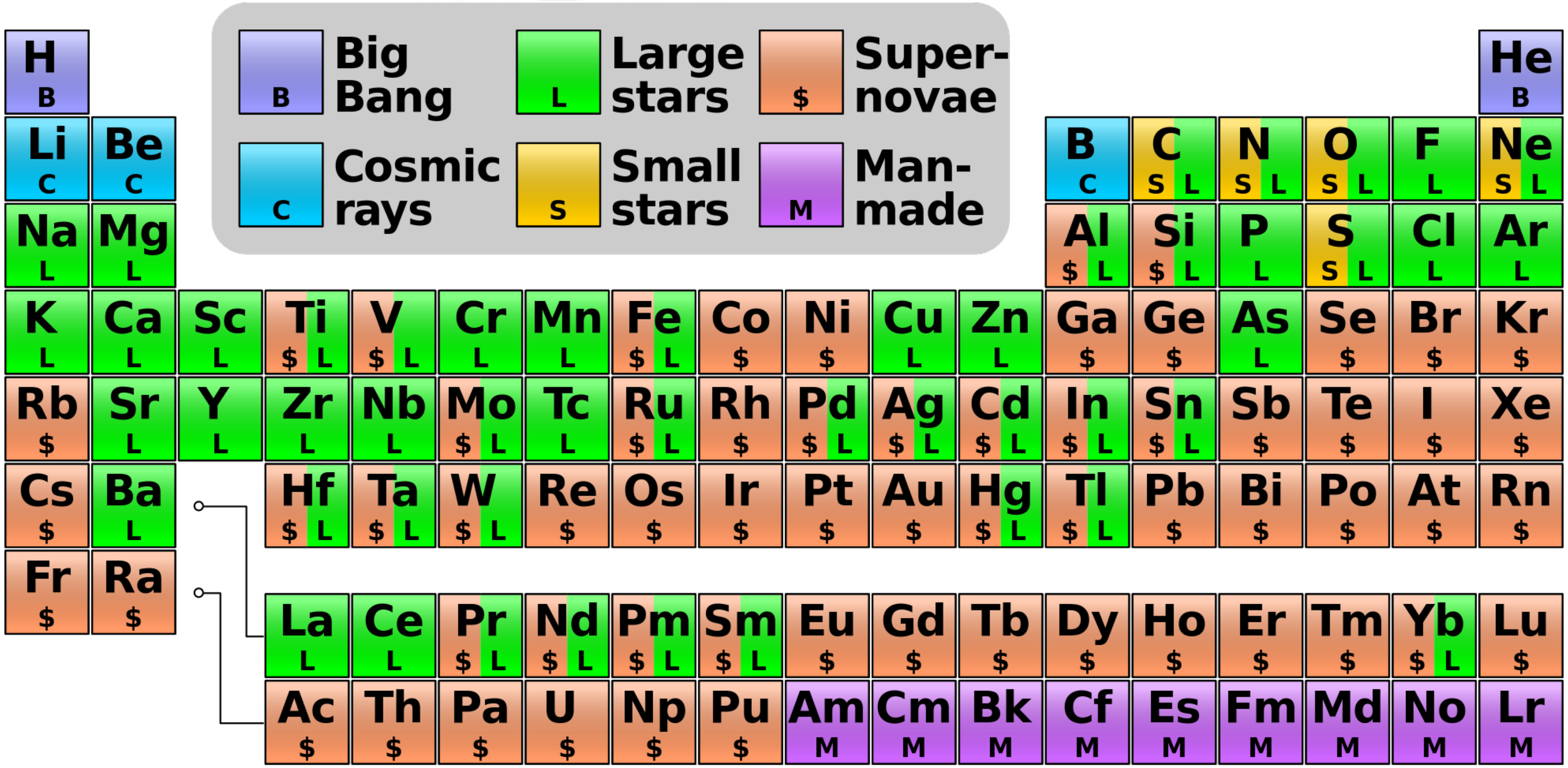
o) Rapid neutron process in supernovae



Rapid neutron process: neutron capture without time for beta decay. When neutron source is turned of, beta decay to stable nuclei

Typical r-process elements are: Eu, Au, Xe, Pt

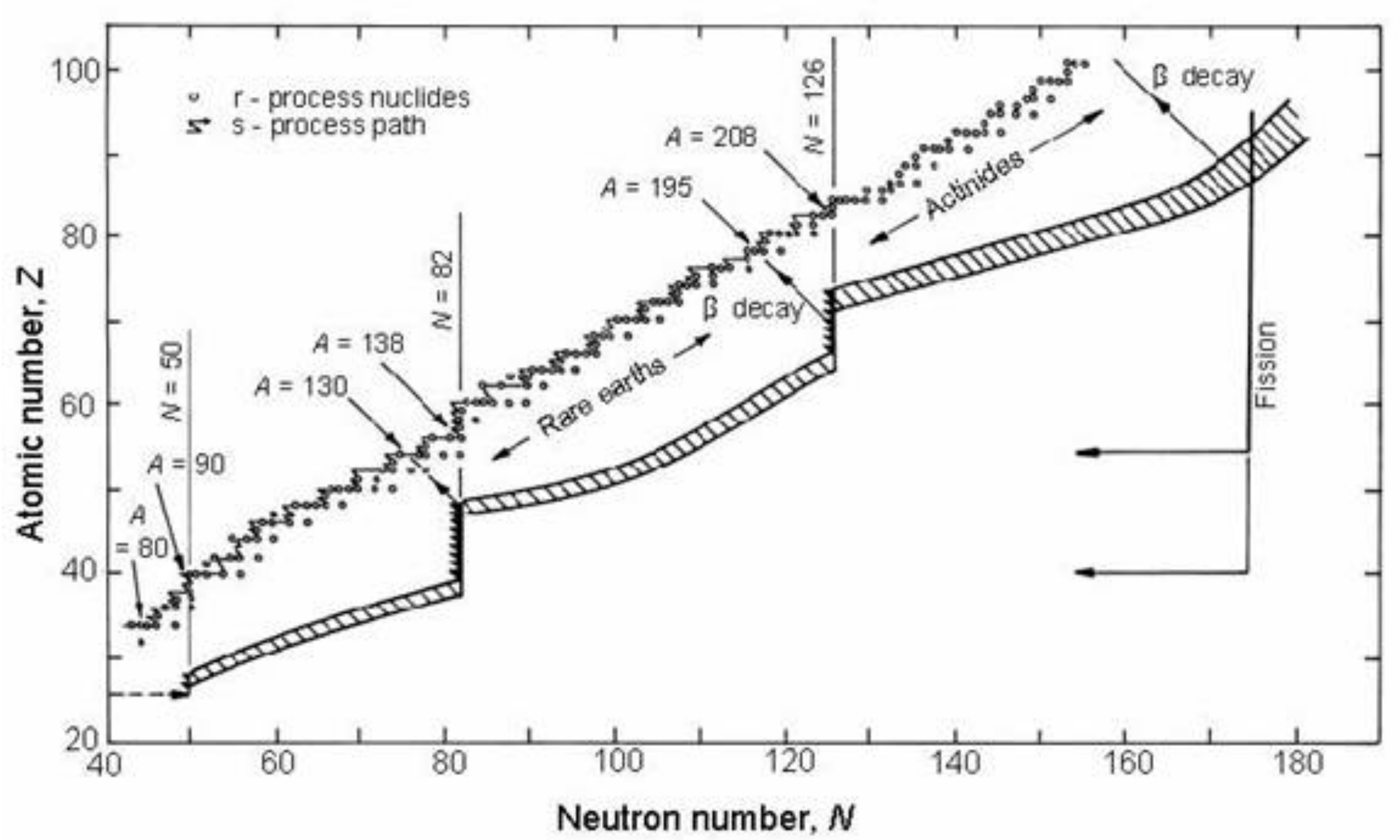
p) Summing up the light elements



B Big Bang
L Large stars
$ Supernovae
C Cosmic rays
S Small stars
M Man-made

H B																	He B
Li C	Be C											B C	C S L	N S L	O S L	F L	Ne S L
Na L	Mg L											Al $ L	Si $ L	P L	S S L	Cl L	Ar L
K L	Ca L	Sc L	Ti $ L	V $ L	Cr L	Mn L	Fe $ L	Co $	Ni $	Cu L	Zn L	Ga $	Ge $	As L	Se $	Br $	Kr $
Rb $	Sr L	Y L	Zr L	Nb L	Mo $ L	Tc L	Ru $ L	Rh $	Pd $ L	Ag $ L	Cd $ L	In $ L	Sn $ L	Sb $	Te $	I $	Xe $
Cs $	Ba L		Hf $ L	Ta $ L	W $ L	Re $	Os $	Ir $	Pt $	Au $	Hg $ L	Tl $ L	Pb $	Bi $	Po $	At $	Rn $
Fr $	Ra $																
			La L	Ce L	Pr $ L	Nd $ L	Pm $ L	Sm $ L	Eu $	Gd $	Tb $	Dy $	Ho $	Er $	Tm $	Yb $ L	Lu $
			Ac $	Th $	Pa $	U $	Np $	Pu $	Am M	Cm M	Bk M	Cf M	Es M	Fm M	Md M	No M	Lr M

q) Summing up the heavy elements

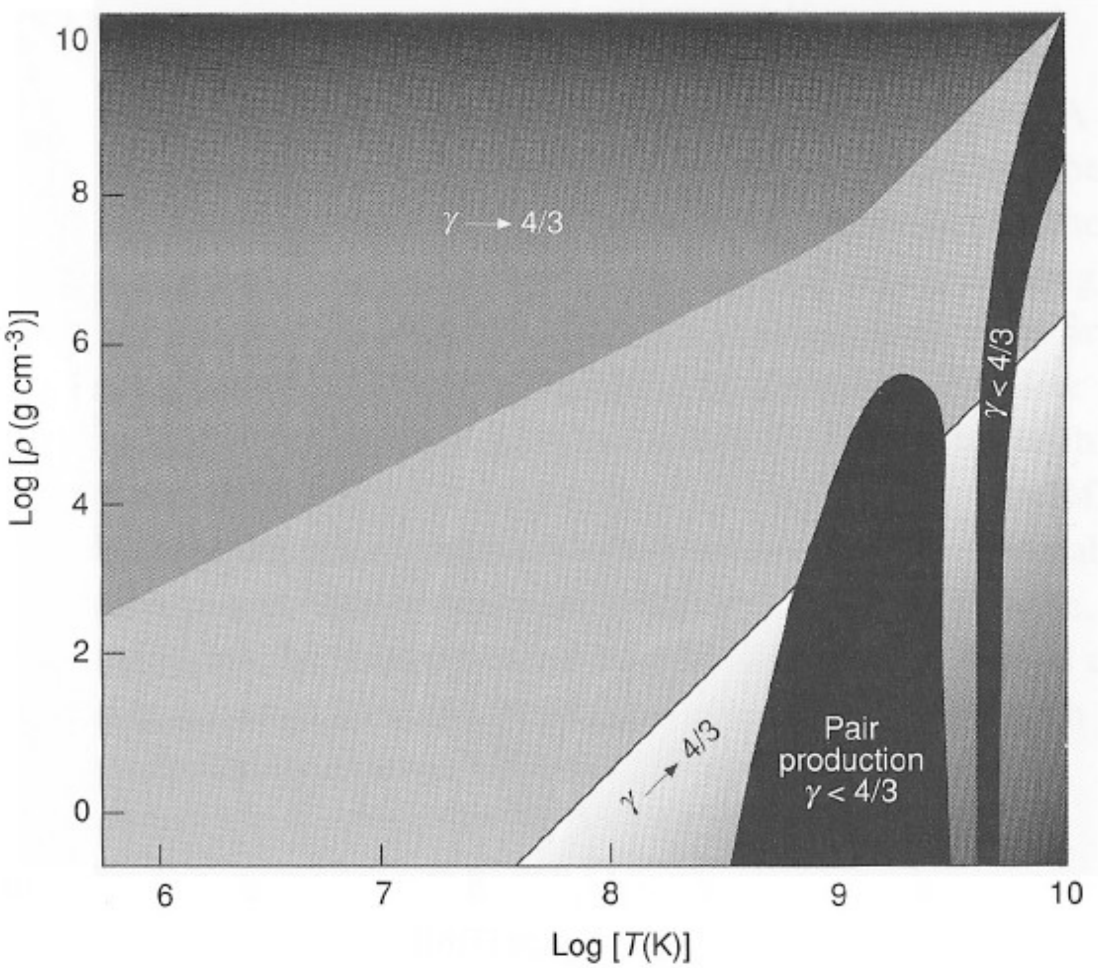


r) Neutrinos

Generally, neutrinos are an energy drain for stars. This has to be taken into account in evaluating the energy generation rate. Note that this may differ for the separate branches of the same general nuclear reaction (cf., pp-reactions; slides 4 & 5). In the later stages of nuclear burning, capture of energetic electrons (near the Fermi level) by protons in nuclei becomes important, resulting in large neutrino losses and a shift towards neutron-rich nuclei. Neutrinos can also be produced through pair production:

$$\gamma + \gamma \rightarrow e^- + e^+$$

$$e^- + e^+ \leftrightarrow \nu_e + \bar{\nu}_e$$


Log [ρ (g cm^{-3})]
10
8
6
4
2
0
Log [T(K)]
6
7
8
9
10
$\gamma \to 4/3$
$\gamma \to 4/3$
Pair production $\gamma < 4/3$
$\gamma < 4/3$