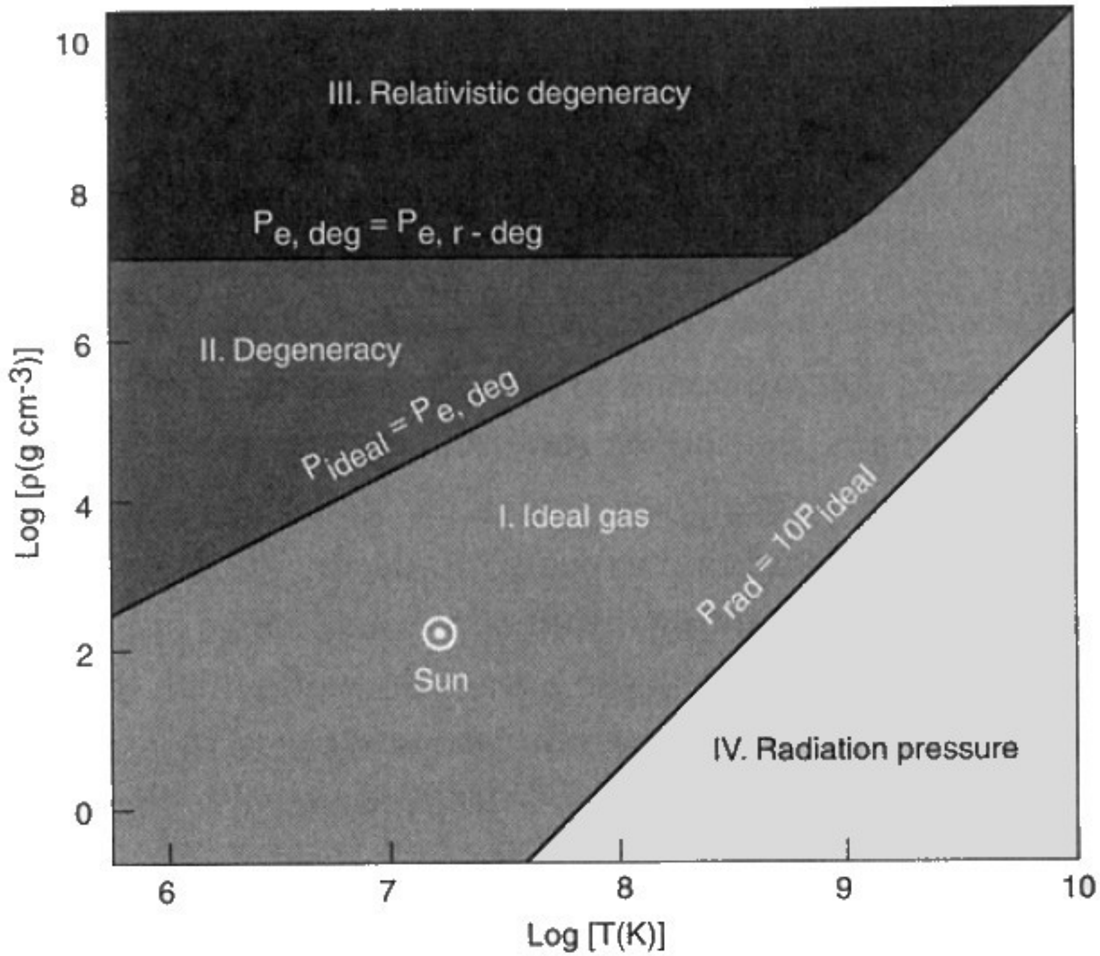
Lecture 6-1: Schematic Evolution of Stars as seen from the core

Literature: KWW chapter 22
Pols chapter 7.1 & 7.2
[Prialnik Chapter 7]

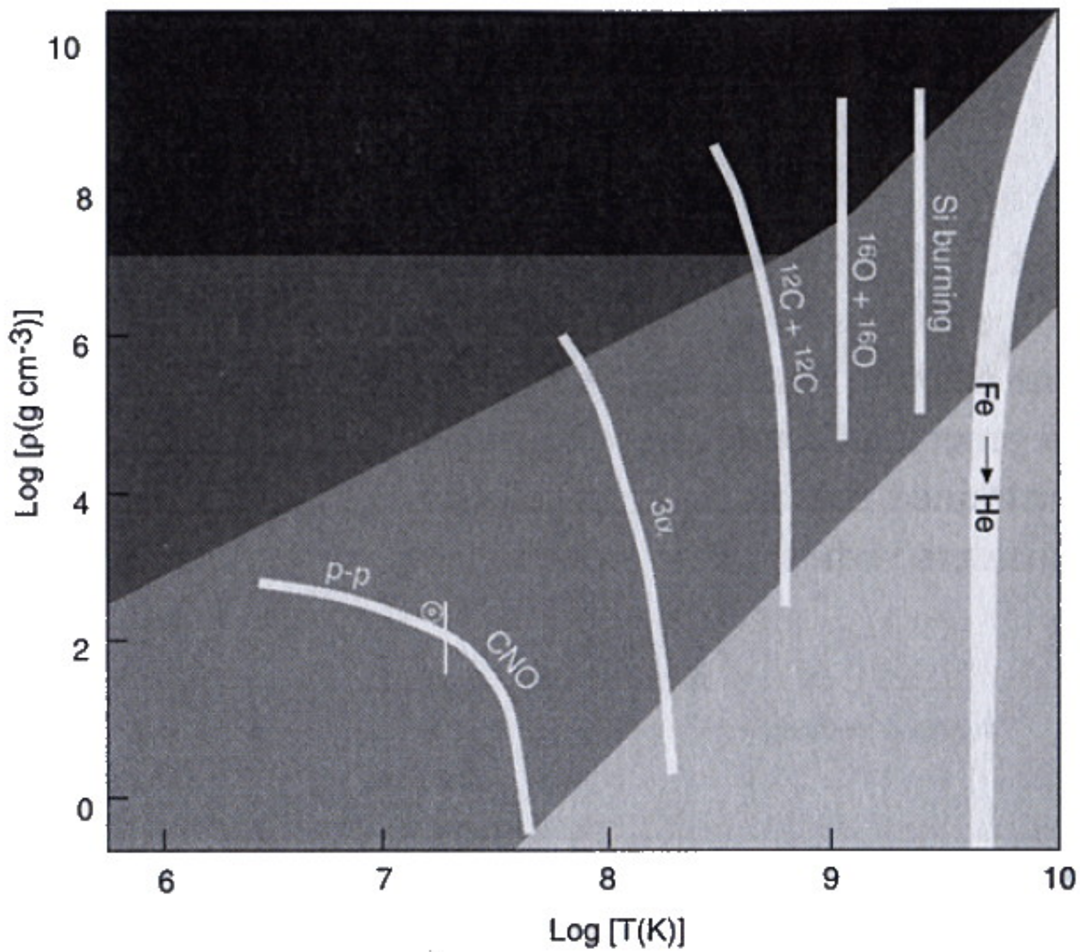
Core & Equation State

Slide 25 in Lecture notes 3-2 divided the density-temperature plane in different zones: ideal gas, degenerate (non-relativistic and extreme relativistic), radiation pressure. The boundaries are only approximate (exercise 2 assignment 4).



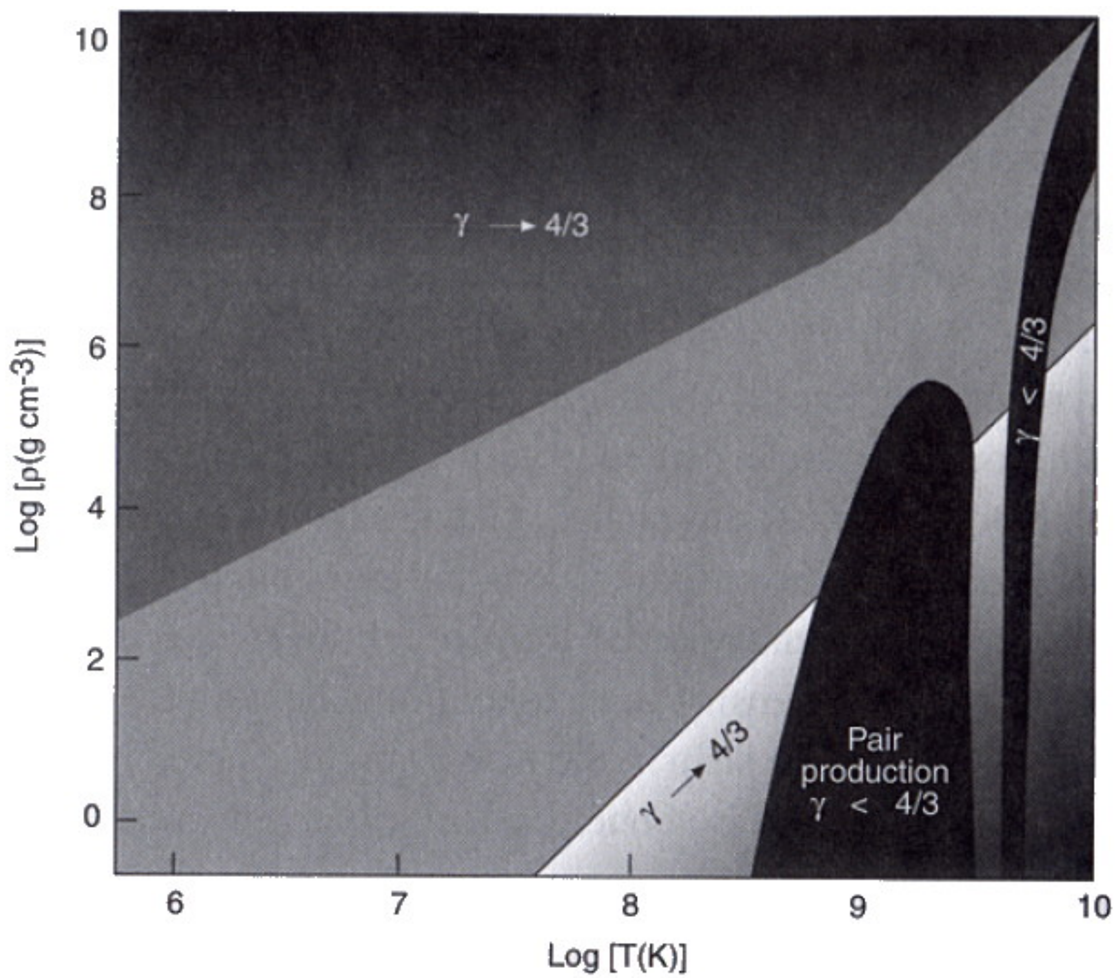
Core & Nuclear Burning Stages

Slide 21 in Lecture Notes 4-2 illustrated the nuclear burning stages on the density-temperature plane.



Core & Instability

We recognized instability for g<4/3 when we studied the virial theorem on slide 11 of lecture 2. We came across tis point again when we looked at the structure of white dwarfs and the Chandrasekhar mass (slides 20-22, lecture 5-3).



Evolutionary Path

For a polytrope, we have:

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

(Assignment 8 exercise 1)

with $B_n = 0.16 - 0.21$

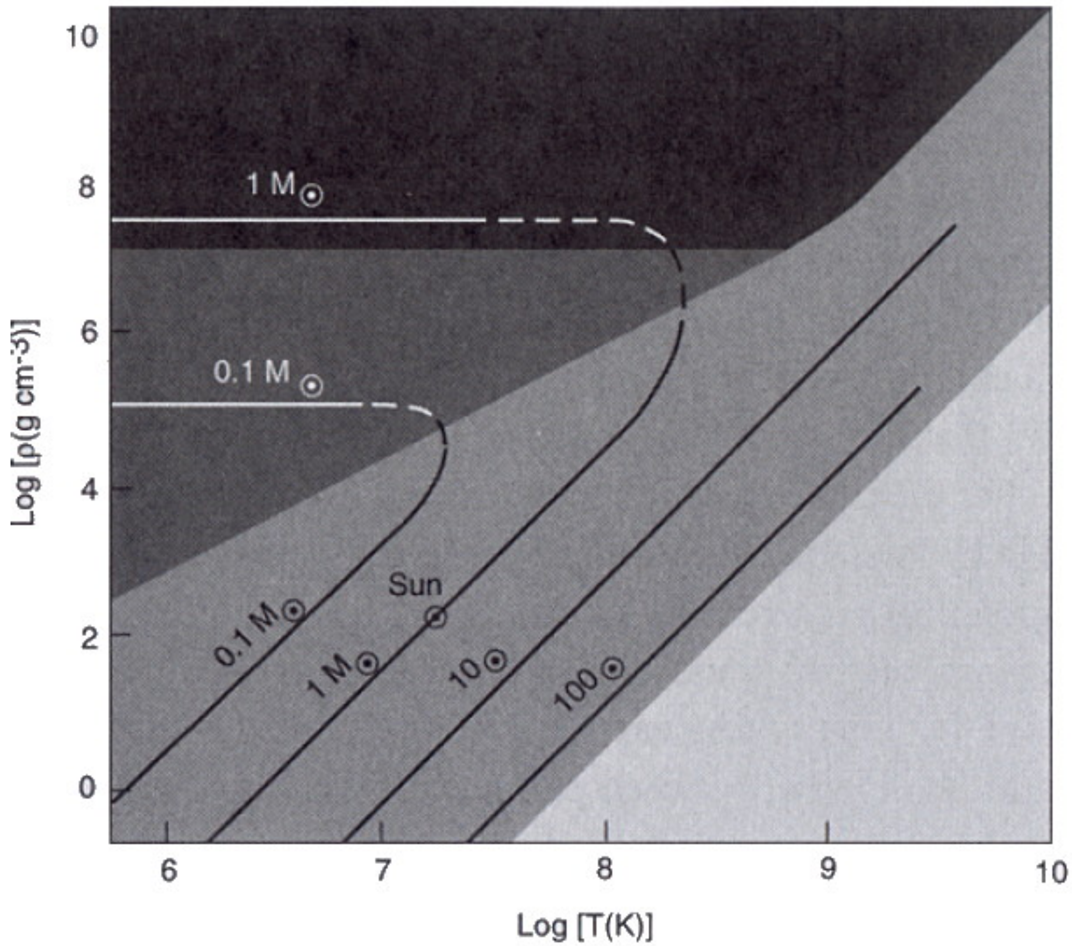
$$\rho_c = \frac{1}{4\pi}\left(\frac{k}{\mu m_u G B_n}\right)^3 \frac{T_c^3}{M^2} \quad (1)$$

for an ideal gas.

$$\rho_c = 4\pi\left(\frac{B_{3/2} G}{K_{NR}}\right)^3 M^2 \quad (2)$$

for a non-relativistic

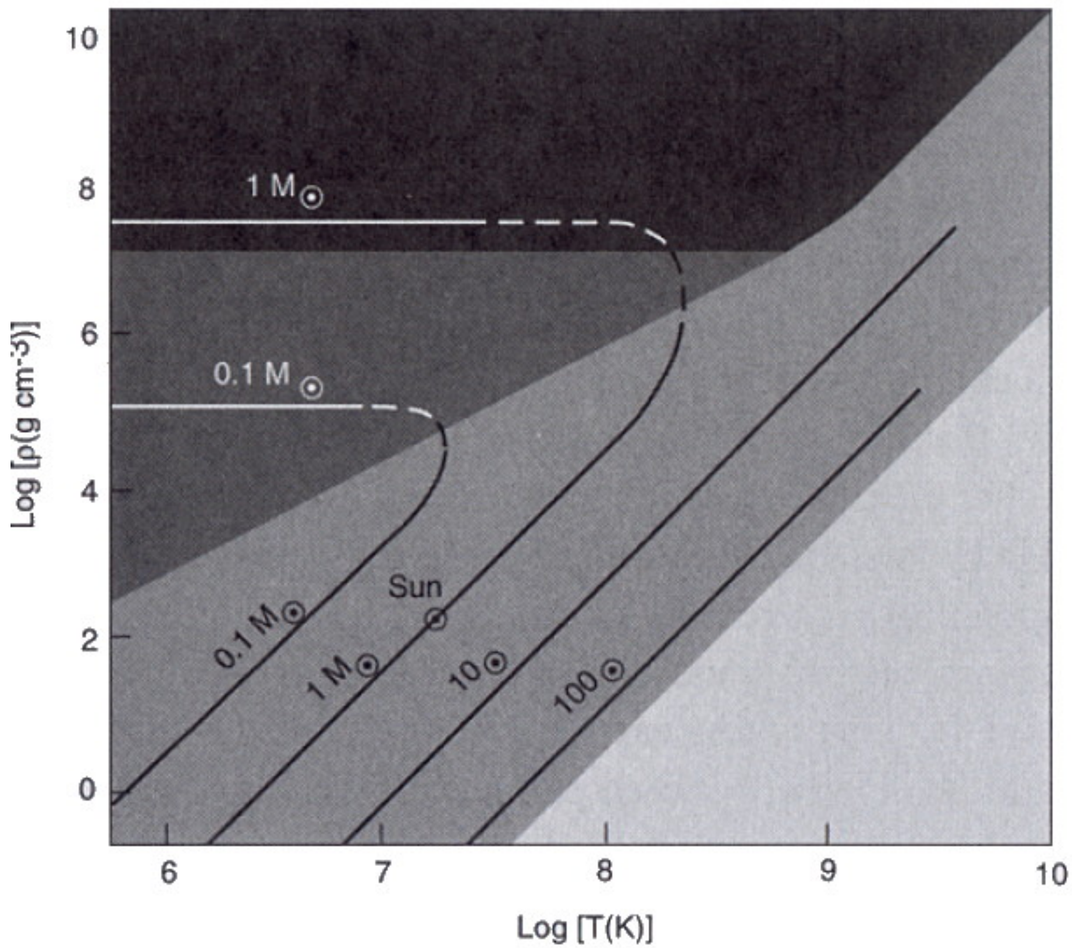
degenerate electron gas.



So, for different masses, parallel lines with slope 3 for (1) & horizontal lines for (2).

Evolutionary Path

For low mass stars, the two segments are connected by dashed lines. Tracks of higher mass stars will bend at higher temperatures and central density will approach infinity for the Chandrasekhar mass.

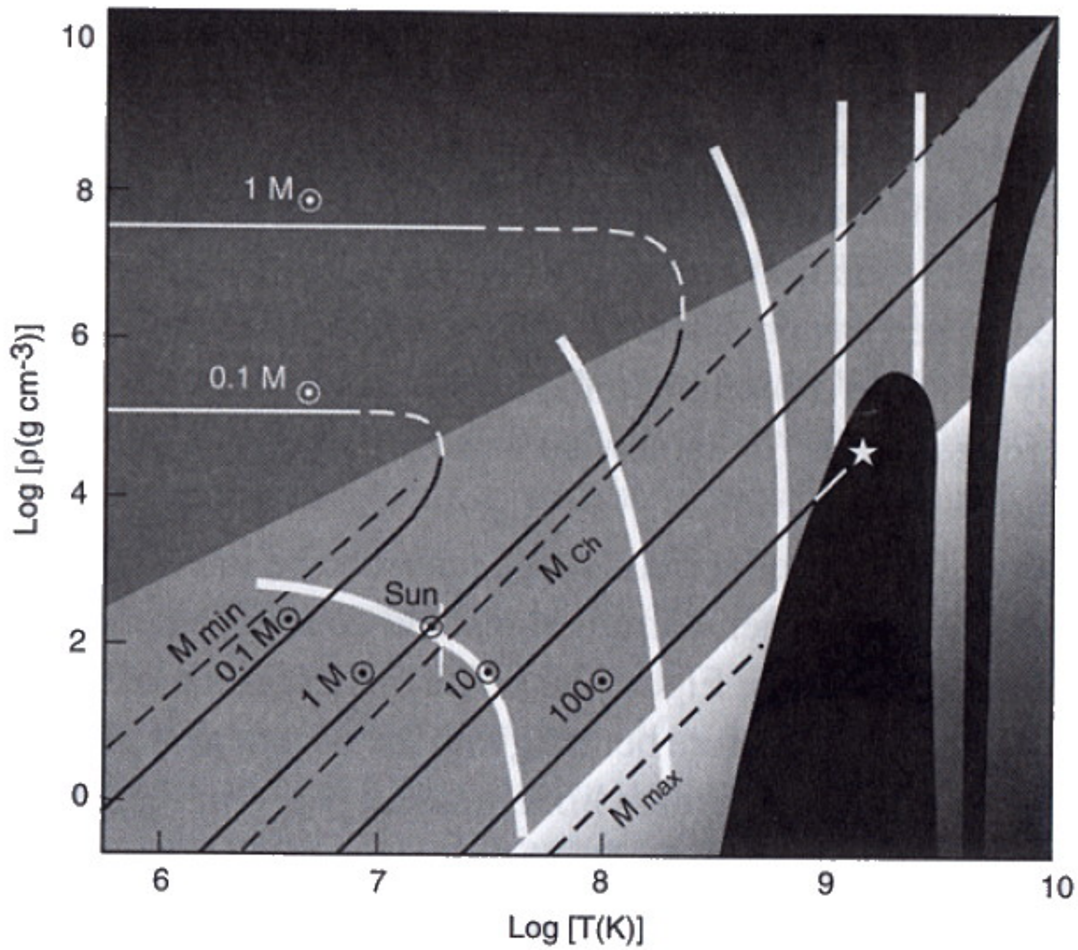


Summary

Hydrostatic equilibrium and the ideal gas law connects r_c and T_c. With increasing mass, the central pressure has to increase to support the star against gravity. As thermal pressure scales with r while hydrostatic pressure scales with $r^{4/3}$, a more massive star requires a higher central temperature (and/or lower density). For a non-relativistic degenerate gas, temperature doesn’t enter. Now, thermal pressure scales with $r^{5/3}$ while hydrostatic pressure scales with $r^{4/3}$, and a more massive star requires a higher central density. And we have already seen that for an extreme relativistic gas, the central density has to be infinite and that hydrostatic equilibrium is only possible for a mass equal to the Chandrasekhar mass (well not really, as the star will collapse to make the density infinite).

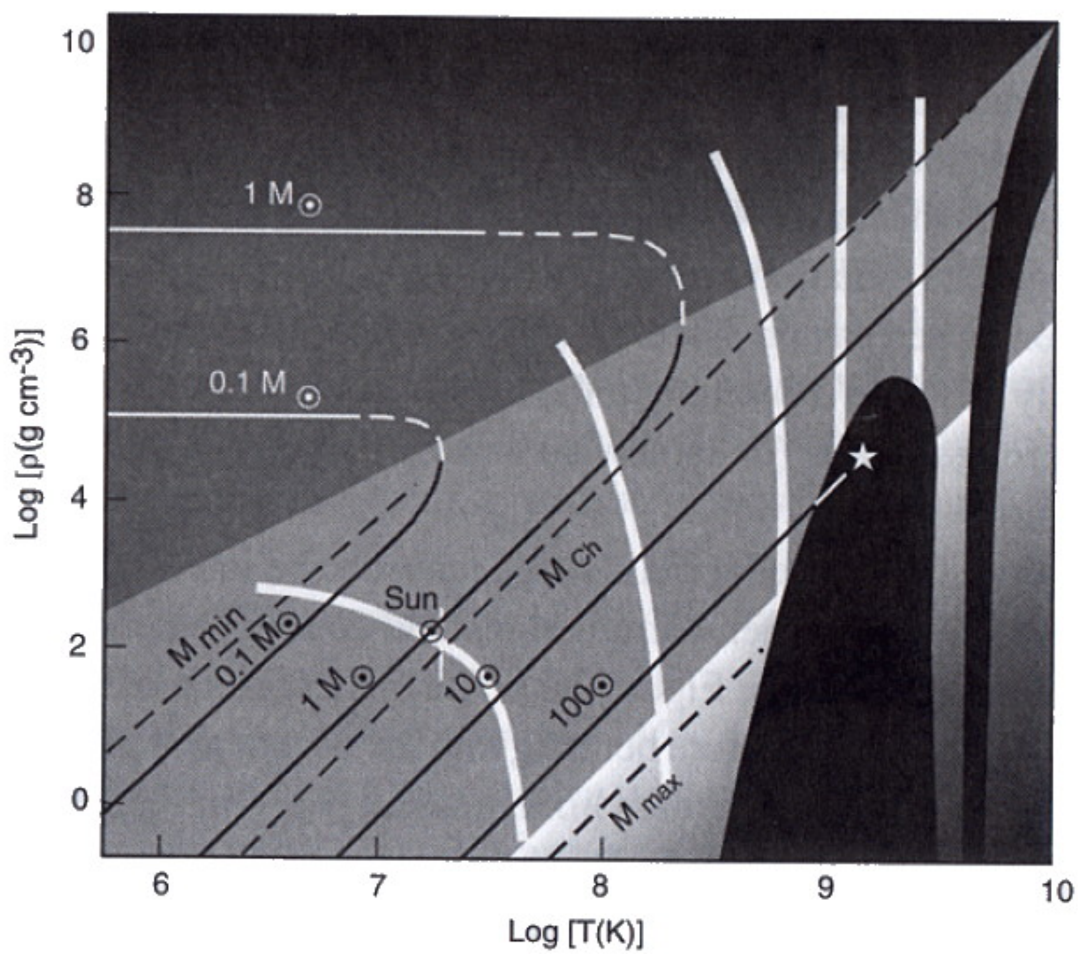
Evolution & ZAMS

Stars start their life on the Hayashi track at low central density and temperature (Lecture 5-3 slides 14–19). As the star contracts, the core heats up (virial theorem, lecture 2) and eventually reaches H-burning conditions. At that point contraction will halt, as nuclear energy generation can supplant the energy loss, and the star is in hydrostatic and thermal equilibrium.



Evolution & ZAMS

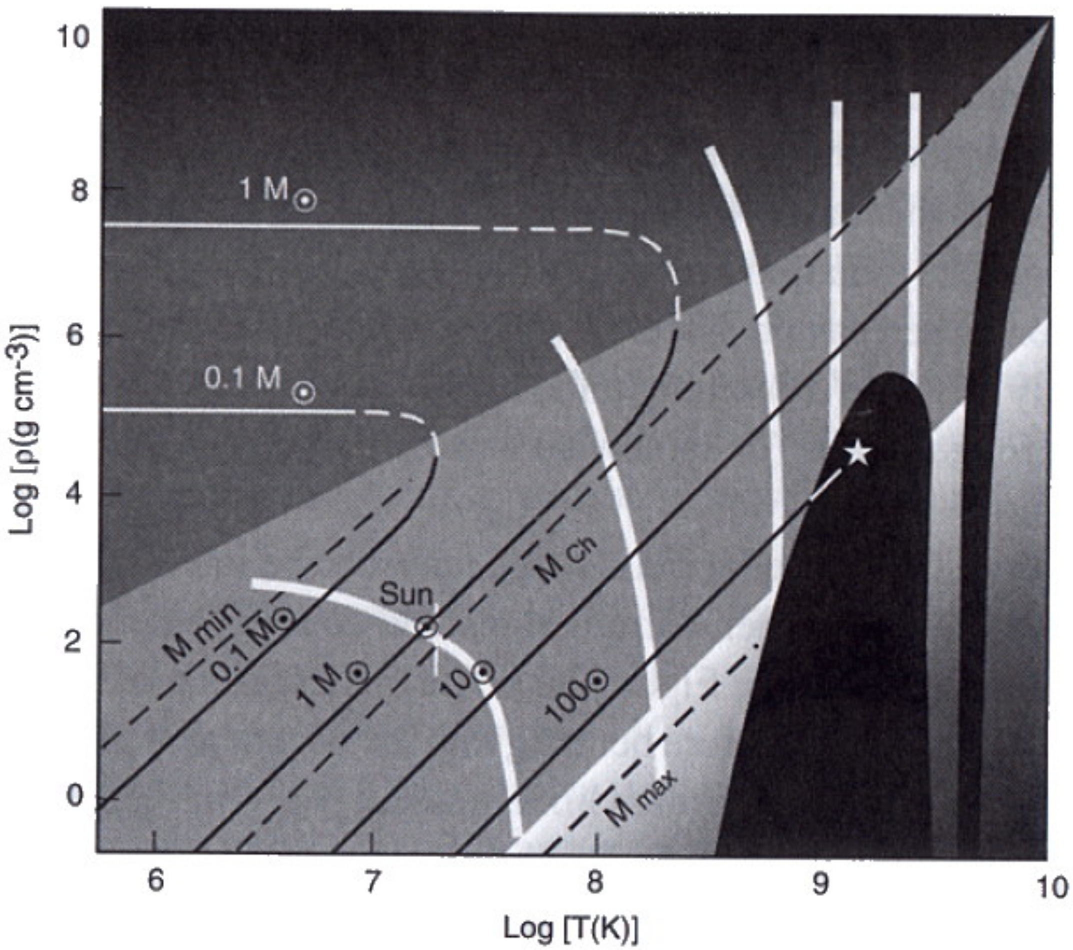
The star has now entered its life on the ZAMS. Note that low mass stars burn with the p-p chain, while high mass stars burn through the CNO cycle with very different T-dependencies.



Evolution & ZAMS

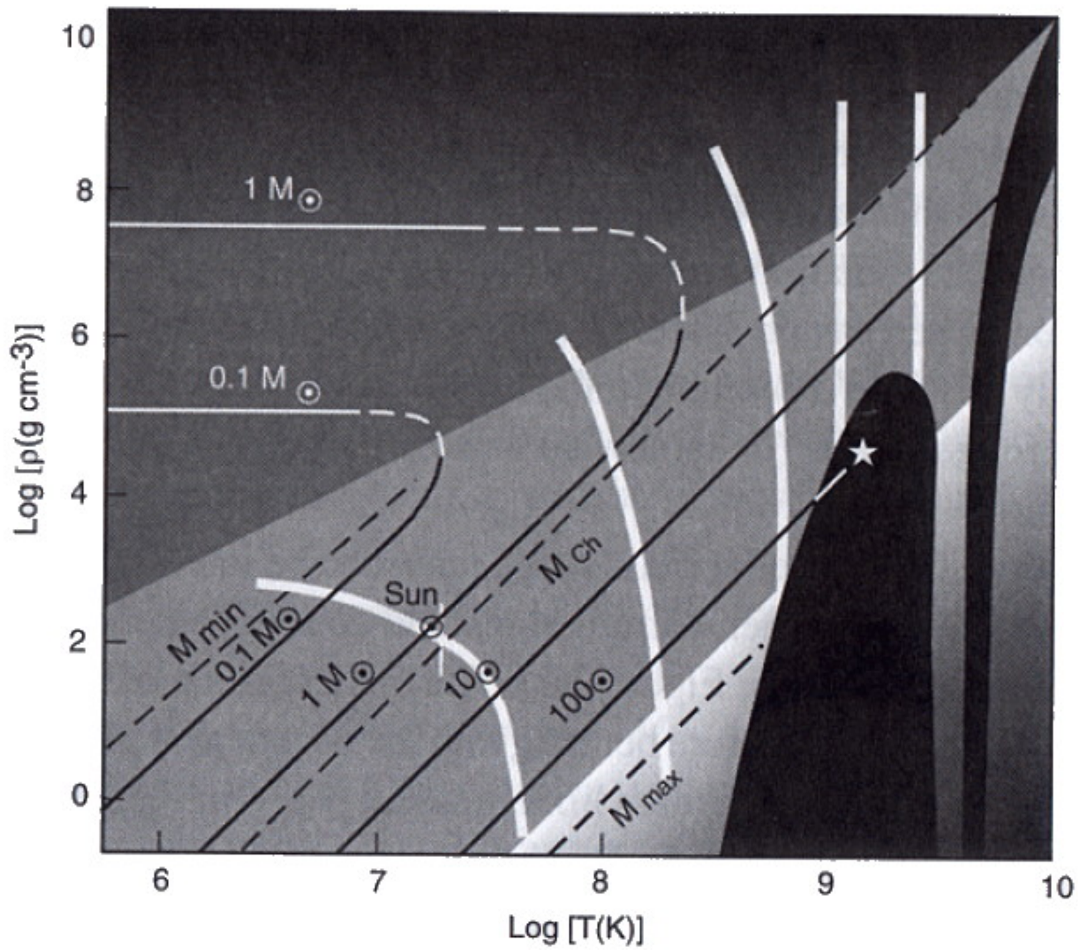
Maximum mass is set by radiation pressure; e.g., g=4/3 and E_t=0 (Lecture 2 slide 11). Any disturbance in the "force" will trigger an eruption (LBV). The precise upper mass is not well known but is somewhere between 100 and 300 M_o (Lecture 3-4, slide 22ff).

Minimum mass limit is set by minimum temperature required to ignite H ($T \sim 3x10^6$ K), which yields $M \approx 0.08$ M_o.(see slide 25).



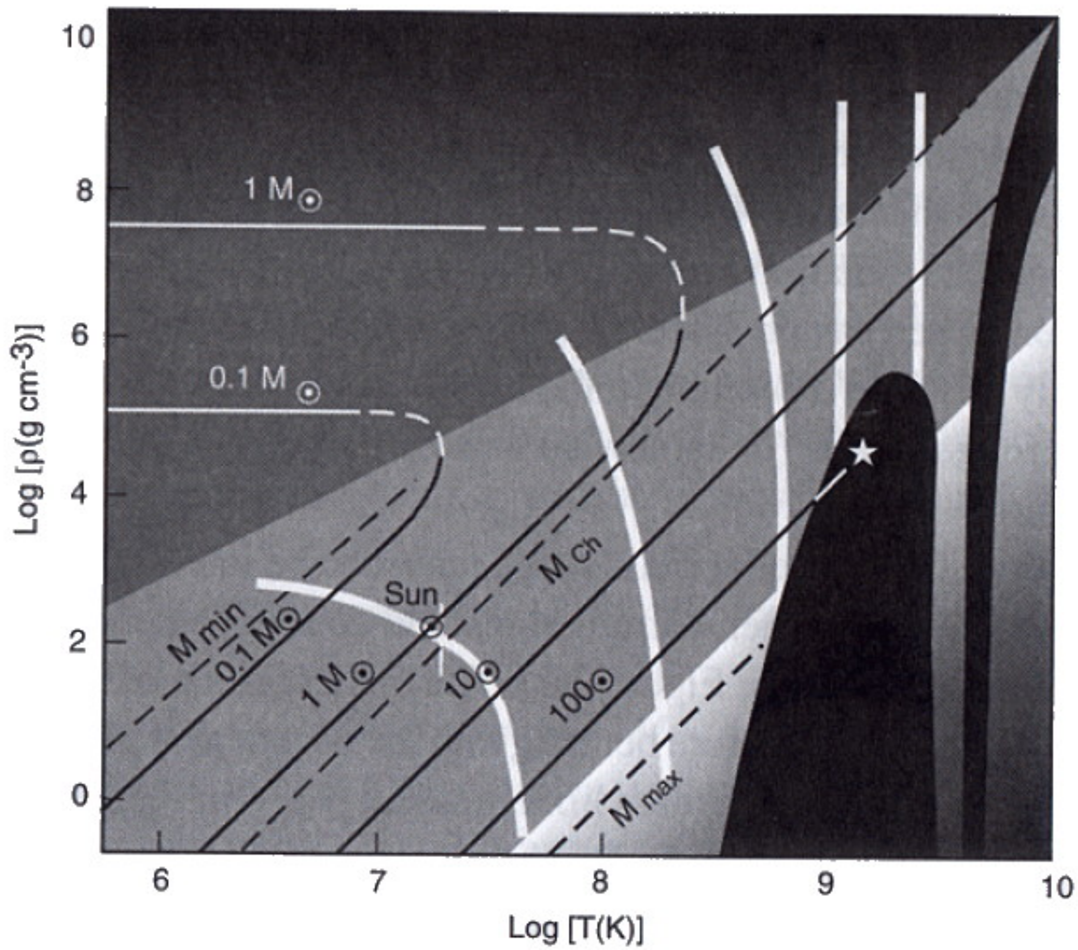
Evolution & beyond the ZAMS

When H is exhausted, the star will have to shrink to supply the energy lost and the core contracts and heats up. Very low mass stars will bend to the left into the degenerate regime. The star is now stabilized by degenerate electron pressure and will slowly cool as a He white dwarf with a density and radius determined by its mass.



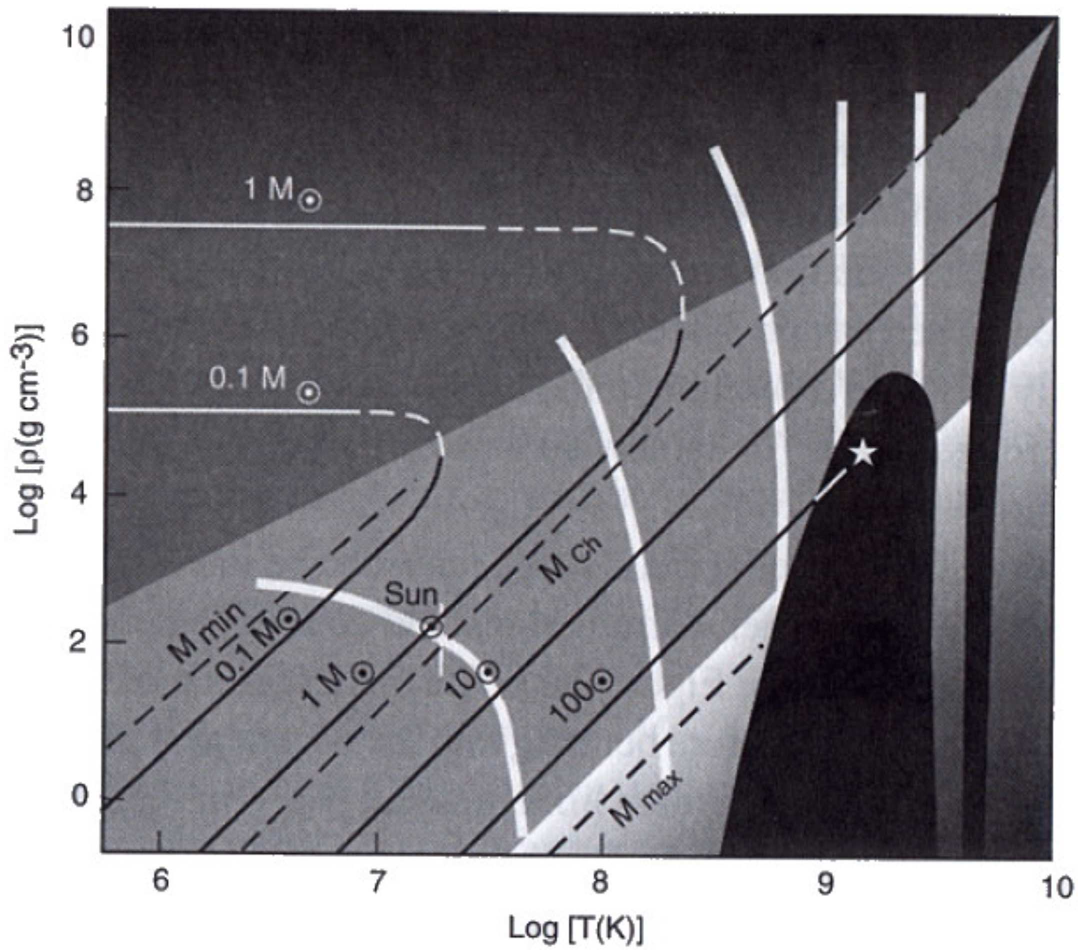
Evolution & beyond the ZAMS

Ignoring for the moment shell burning ,for larger mass, when the core is hot enough, He will ignite with another phase of stable burning. If this happens near the degenerate border, a thermal instability happens (g-4/3; *P* independent of *T*). This story continues with the lower mass stars developing a degenerate C,O core with a density and size set by its mass, while for the more massive stars, the core contracts, heats up, and a new energy source ignites.



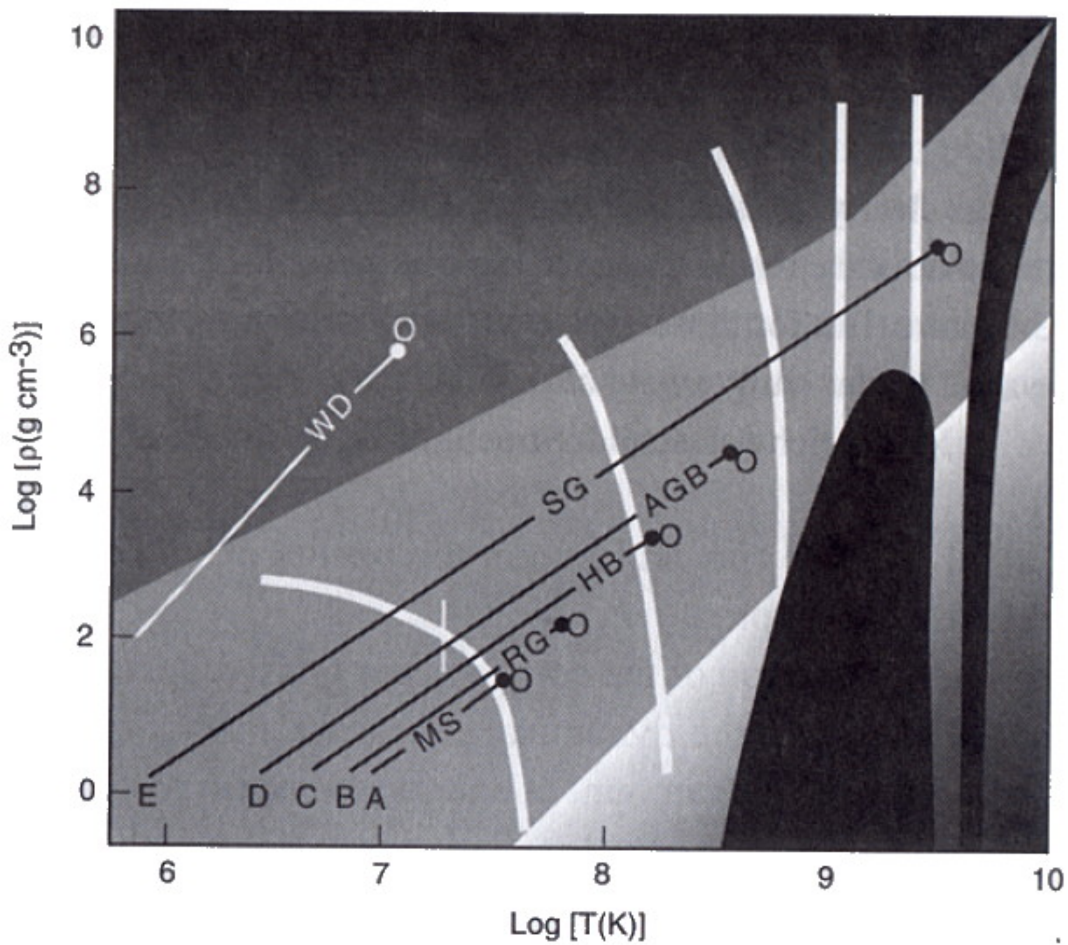
Evolution & beyond the ZAMS

Stars with cores more massive than the Chandrasekhar mass, will go through all nuclear burning stages up to Fe. Eventually, the core will enter the photodisintegration zone and the result will be catastrophic (g=4/3). Very massive stars enter the pair-production instability zone that could end the star much earlier in their life.



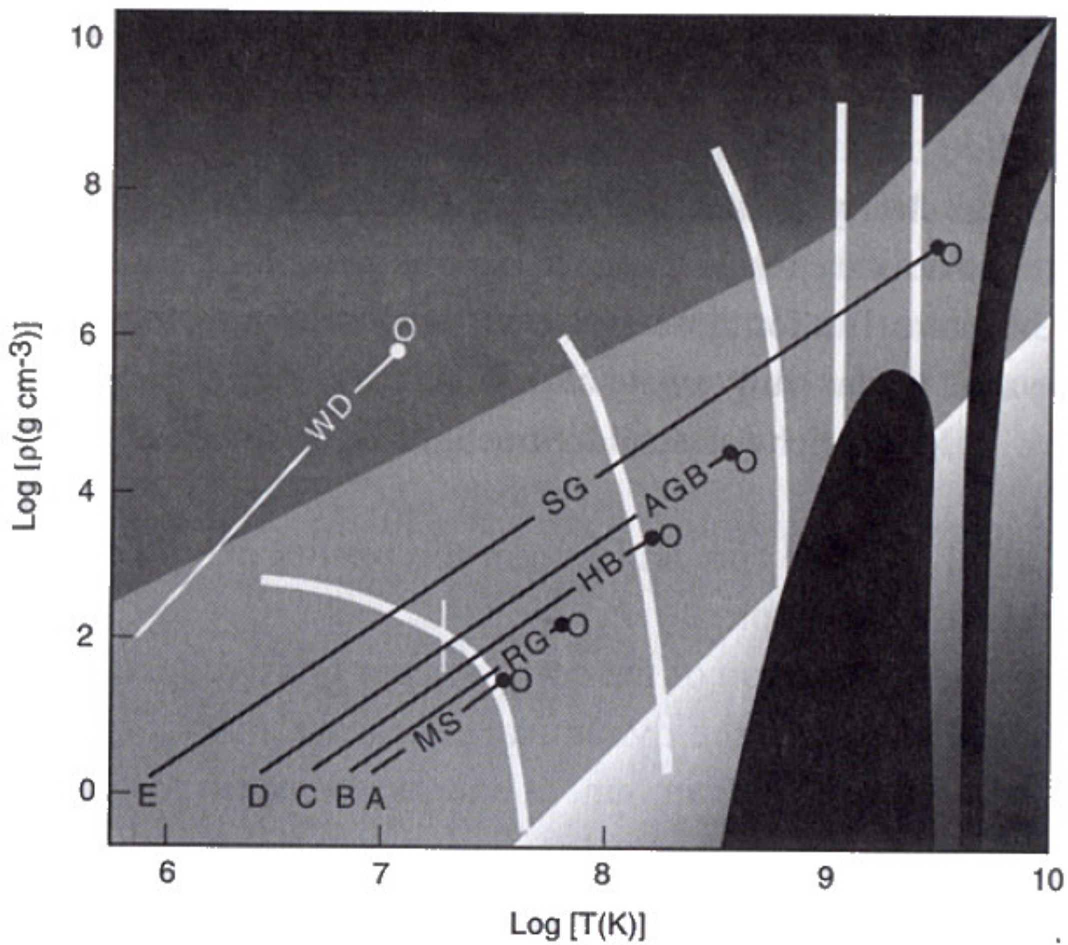
Shell Burning & Giants

Any point in the star will describe a track in the ESO diagram (to the left of the core-track). After the MS (track A), while the core keeps contracting, the shell can cross a nuclear burning stage, leading to a phase of stable shell burning (line B). If during this phase, thermal equilibrium is maintained, core contraction will be compensated by envelope expansion and cooling (virial theorem; gravitational & thermal energy is constant) and the star moves to lower effective temperature.



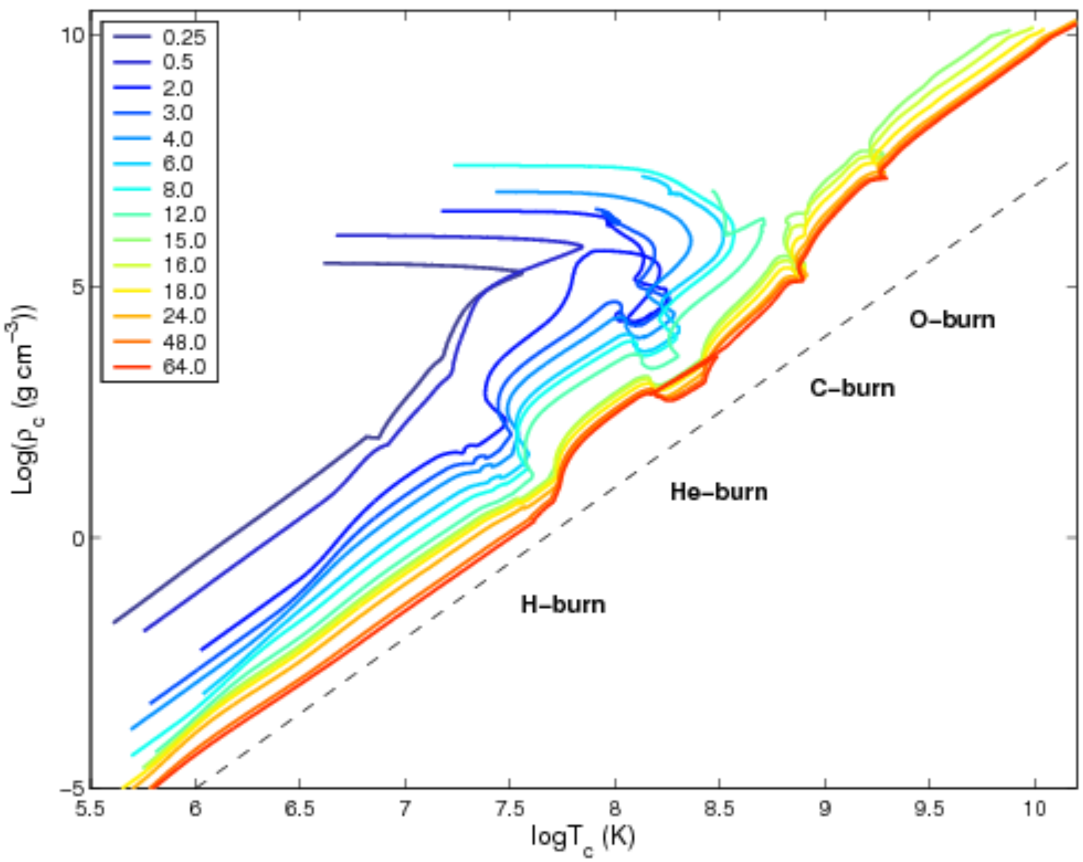
The Helium Main Sequence

When the core reaches He-ignition, the star will have two zones of nuclear burning: a core burning He(main) and a shell burning H (minor). When He is exhausted, the core will contract again and the envelope expands again, the AGB phase (track D) with a C,O core surrounded by a He burning shell, a He layer, and a H burning shell.



Detailed Tracks

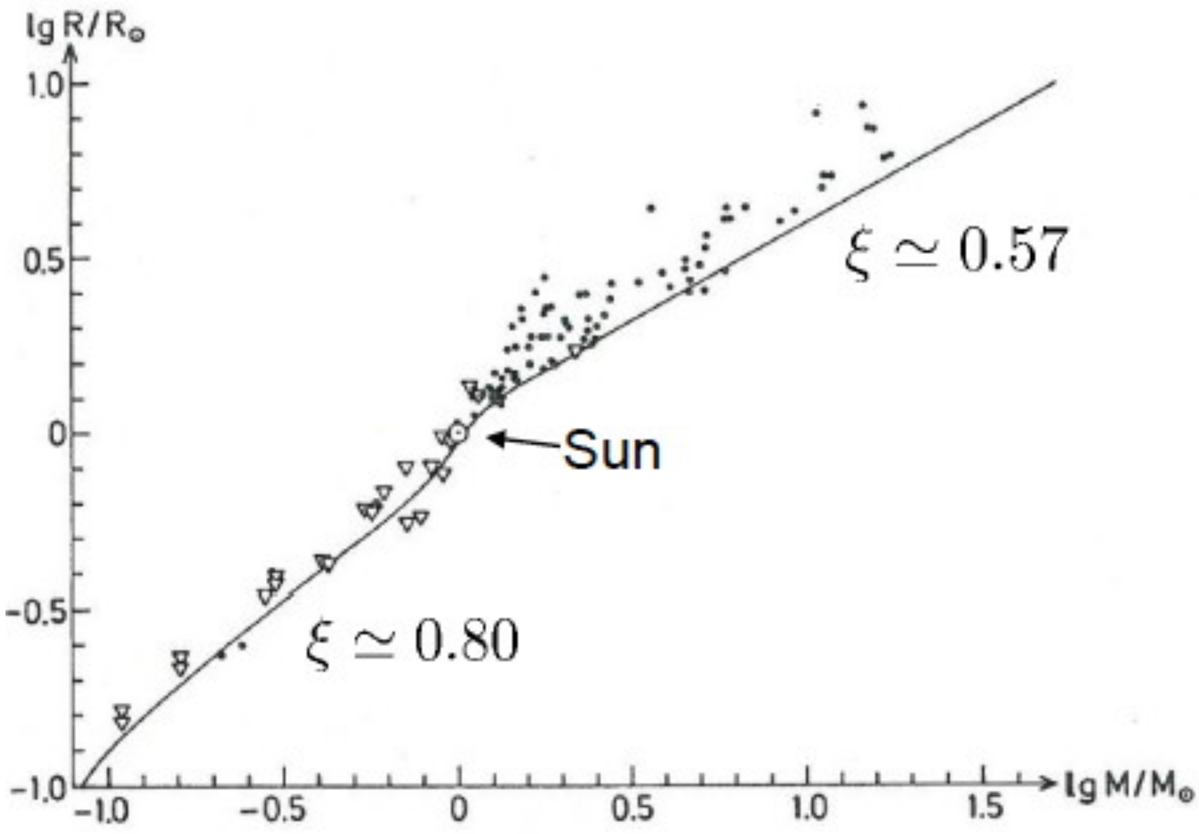
General trends are very similar: recognize premainsequence contraction, H-burning in the core, H-shell-burning on the red giant branch, He-core burning, H- & He-shell burning on the AGB, White Dwarf formation for low mass stars, Further burning stages for massive stars, and the importance of mass loss



Lecture 6-2: Main Sequence

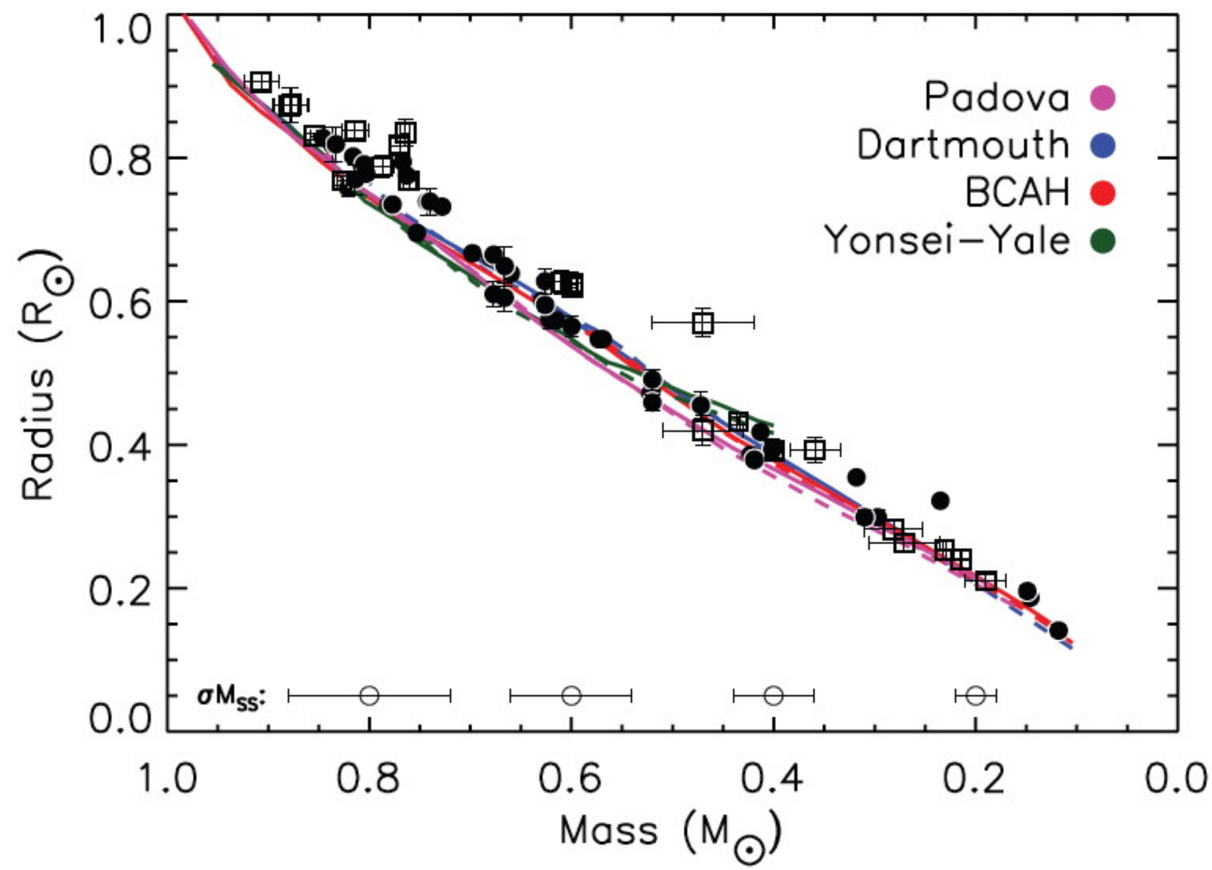
Literature: KWW Chapter 22, 29, 30

a) Mass-Radius Relation



Triangles: visual binaries; dots detached binaries; line models
Note break around $1M_o$ (convective envelope)

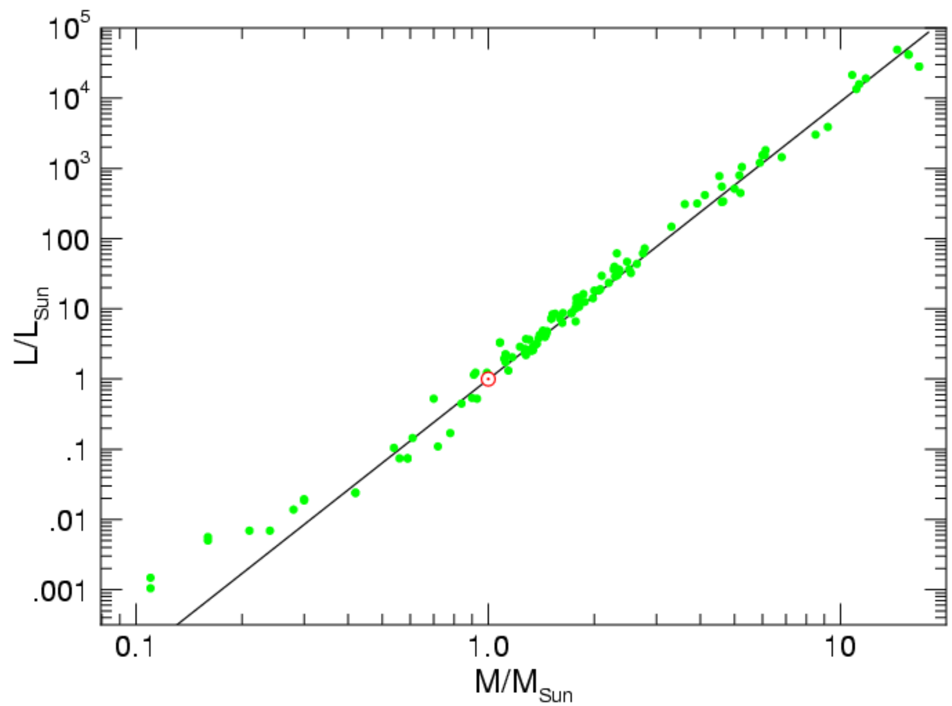
Mass-Radius Relation



dots: interferometry; squares: binaries; lines models
Note the x-axis runs in the opposite direction

Mass-luminosity relation: Slide 11, lecture 1

Measure individual masses for double-lined eclipsing variables



Popper 1980 ARA&A

Martin & Mignard 1998 AA 330, 585

Recent measurements:

Break in slopes due to Convection/radiation pressure (assignment)

$$\frac{L}{L_O} = \begin{cases} 0.66\left(M/M_O\right)^{2.5} & 0.08M_O \le M \le 0.5M_O \\ 0.92\left(M/M_O\right)^{3.55} & 0.5M_O \le M \le 40M_O \\ 300\left(M/M_O\right)^{2} & 40M_O \le M \le 130M_O \end{cases}$$

b) Mass-luminosity relation: theory

We encountered this in exercise 1, assignment 3 where we used hydrostatic equilibrium and virial theorem to derive this relation for a star in radiative equilibrium. Examining this again

$$P \propto \frac{GM}{R}\rho \;\&\; T \propto \frac{P}{\rho} \text{ (gas pressure dominated)} \Rightarrow T \propto \frac{\mu M}{R}$$

$$L \propto \frac{R^2}{\kappa\rho}\frac{T^4}{R} \;\&\; \kappa \propto \rho^n T^{-\alpha} \Rightarrow L \propto \mu^{\alpha+4} R^{3n-\alpha} M^{\alpha-n+3}$$

$$\text{Kramers opacity } (n=1, \alpha=3.5)\text{: } L \propto \mu^{7.5}\frac{M^{5.5}}{R^{0.5}}$$

$$\text{Thompson opacity } (n=0, a=0)\text{: } L \propto \mu^4 M^3$$

$$\text{Radiation pressure dominated } (P \sim P_{rad} \propto T^4 \propto \frac{M^2}{R^4}) \text{ \& Thompson}$$

$$L \propto R^2 \frac{R^3}{M}\frac{M^2}{R^5} \propto M$$

Mass-Radius relation: theory

Set the luminosity equal to the thermonuclear power generation in the core:

$\varepsilon \propto \rho_c T_c^s$

Very massive stars (>100 M_o; radiation pressure & CNO cycle($s = 14$)):

$$L \propto \varepsilon M \propto \frac{M}{R^3}\left(\frac{M^2}{R^4}\right)^{14/4} M \propto \frac{M^9}{R^{17}} \propto M \Rightarrow R \propto M^{8/17}$$

Massive stars (2-100M_o; gas pressure & CNO cycle):

$$L \propto \varepsilon M \propto \frac{M}{R^3}\left(\frac{M}{R}\right)^{14} M \propto \frac{M^{16}}{R^{17}} \propto M^3 \Rightarrow R \propto M^{13/17}$$

For low mass stars (<2 M_o), convection becomes more and more important. Efficient energy transport and star shrinks (central pressure, temperature, and density go up).For a fully convective star, $M \propto R^{-3}$ (slide 9 ch 5-3; slides 8-10 ch 5-2).

Mass-Radius relation: theory

Looking at it from the Mass-luminosity relation: $L \propto M^{\eta}$

and virial equilibrium,

$$T_c \propto \frac{M}{R} \Rightarrow T_c^3 \propto \left(\frac{M}{R}\right)^3 \propto \rho_c M^2$$

We can write:

$$L \propto \varepsilon M \propto \rho_c T_c^s M \propto \rho_c \left(\rho_c^{1/3} M^{2/3}\right)^s M \propto M^{\eta} \quad \text{Or:}$$

$$\log \rho_c \propto \left(\frac{3\eta - 2s - 3}{s+3}\right) \log M \qquad \log T_c \propto \left(\frac{\eta + 1}{s+3}\right) \log M$$

For high mass stars, $s = 18$ & $\eta = 3$, the exponents are $-30/21$ & $4/21$

For low mass stars, $s = 5$ & $\eta = 2$, the exponents are $-7/8$ & $3/8$

Around $1M_o$, $s = 4$ & $\eta = 4$, the exponents are $1/7$ & $5/7$

Putting it together in the HR diagram:

$$R = R(M) \propto M^{\xi}$$

with

$$\xi \approx 0.57 \quad M \geq M_O$$

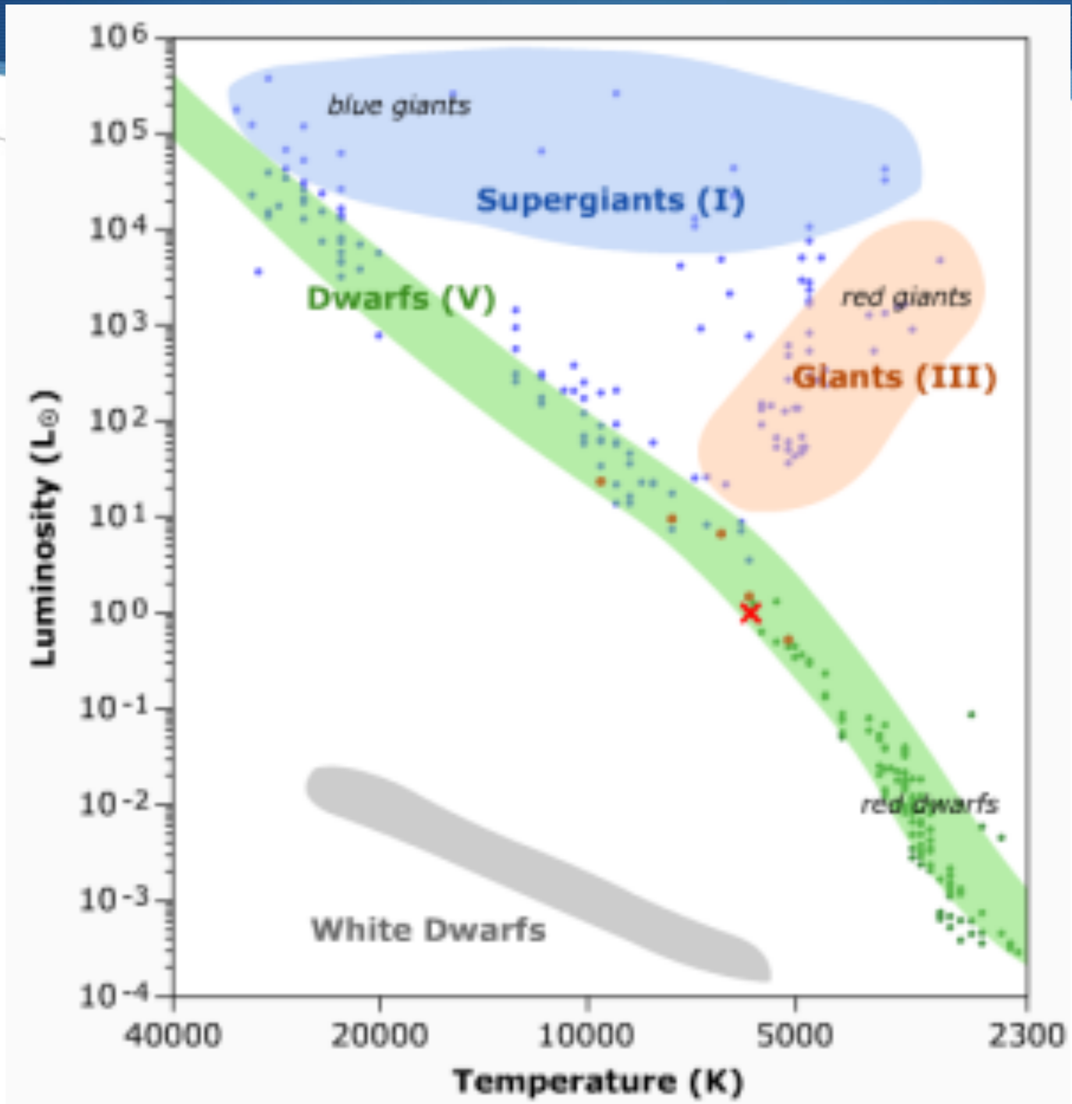
$$\xi \approx 0.80 \quad M \leq M_O$$

$$\left.\begin{array}{l} L = L(M) \propto M^{\eta} \quad (\eta \approx 3.2) \\ L = 4\pi R^2 \sigma T_{eff}^4 \end{array}\right\} \Rightarrow$$

$$L \propto T_{eff}^{\varsigma} \quad with$$

$$\varsigma = \frac{4}{1 - 2\xi / \eta} = \begin{cases} 6.2\,(M \geq M_O) \\ 8.0\,(M \leq M_O) \end{cases}$$

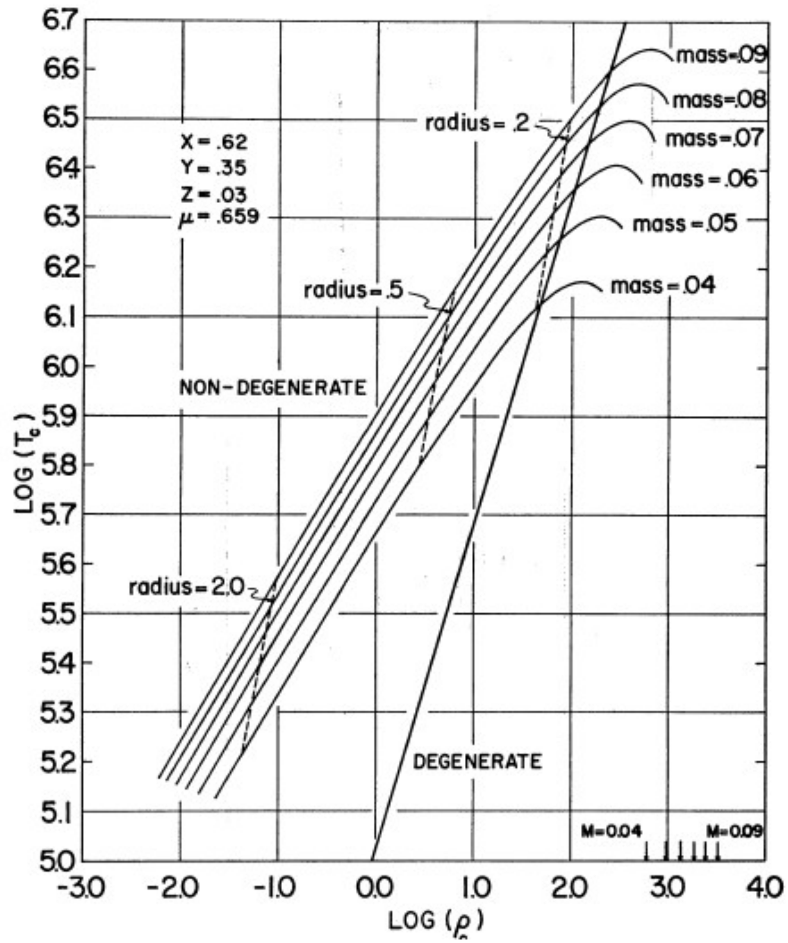
Steep dependence on L and T_{eff} in the HR diagram as observed



blue giants
Supergiants (I)
Dwarfs (V)
red giants
Giants (III)
red dwarfs
White Dwarfs
Luminosity (L☉)
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
40000
20000
10000
5000
2300
Temperature (K)

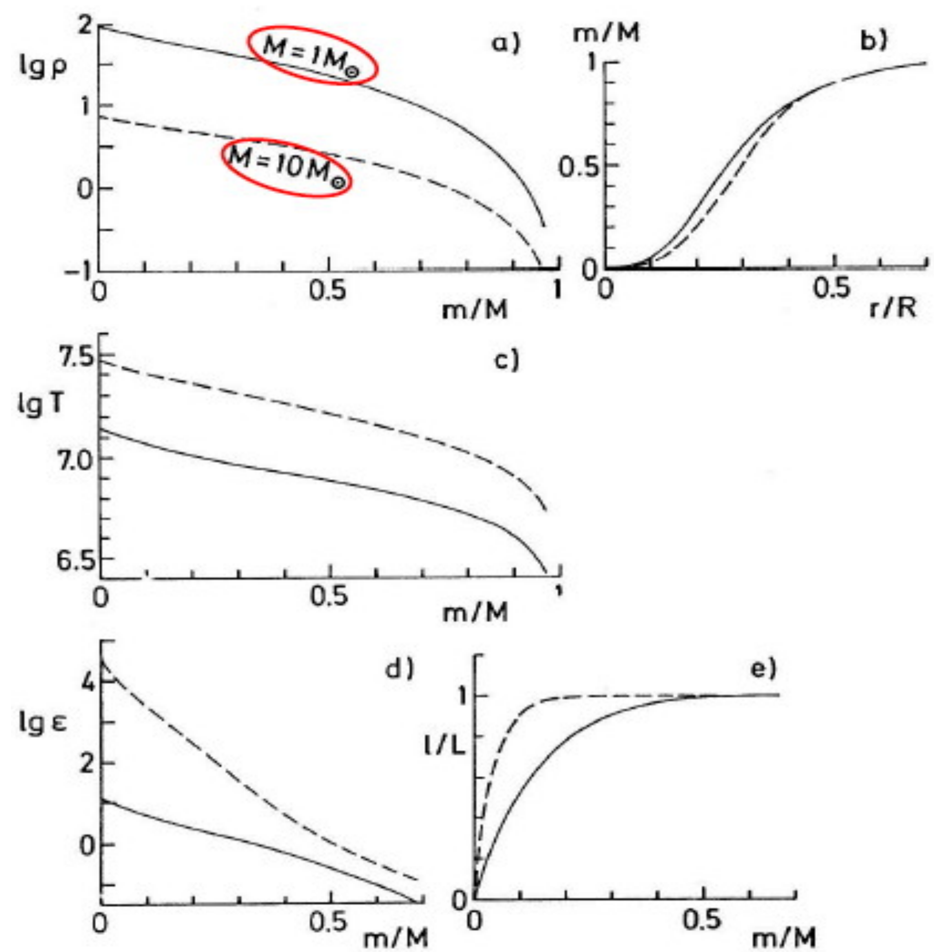
c) Minimum stellar mass

Minimum stellar mass is set by H burning ignition which requires a central temperature of $3x10^6$ K. This corresponds to a minimum mass of ~0.08 M_o (see assignment).



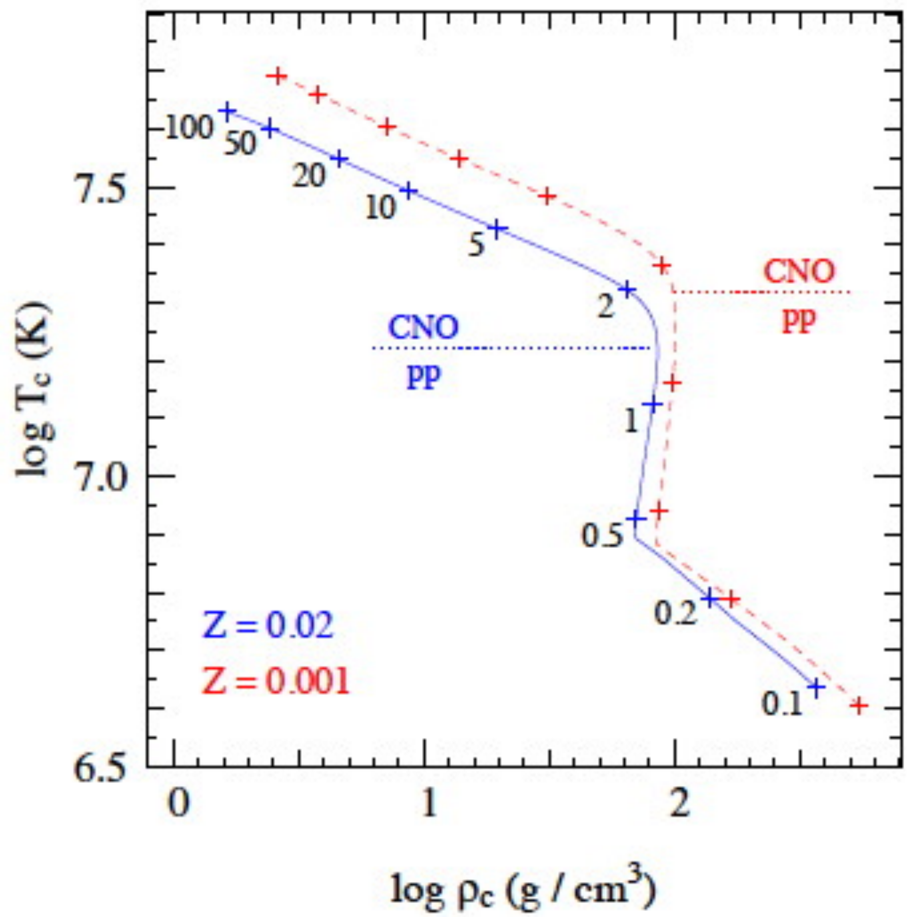
d) Stellar structure main sequence stars

- Density increases by a factor one billion with m/M (or r)
- More massive stars are less dense (slide 8, Ch 6-1)
- Mass is highly concentrated: 60% of the mass in the core (30% of the radius, 3% of the volume)
- Temperature increases by a factor ~3000 with m/M (or r)
- More massive stars are hotter
- Energy generation: 90% of L generated in inner 30% of M for $1M_o$
- 90% of L generated in inner 10% of M for $10M_o$



Stellar structure main sequence stars

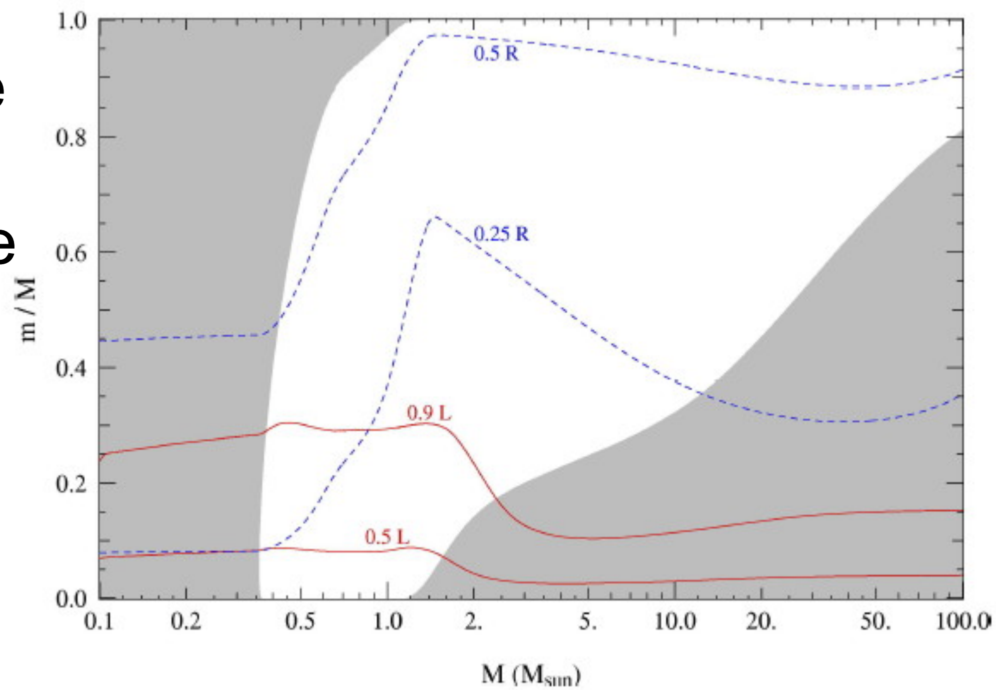
- Look back at slide 23
- Slow increase in T_c with increasing M
- Rapid decrease in r_c with increasing M
- Rapid change between 0.3 and 2 M_o because core is radiative
- Low mass stars become degenerate (y=0 for 0.3M_o, slide 19 in lecture 3-2)
- Radiation pressure is important for M>50 M_o (b=0.5)

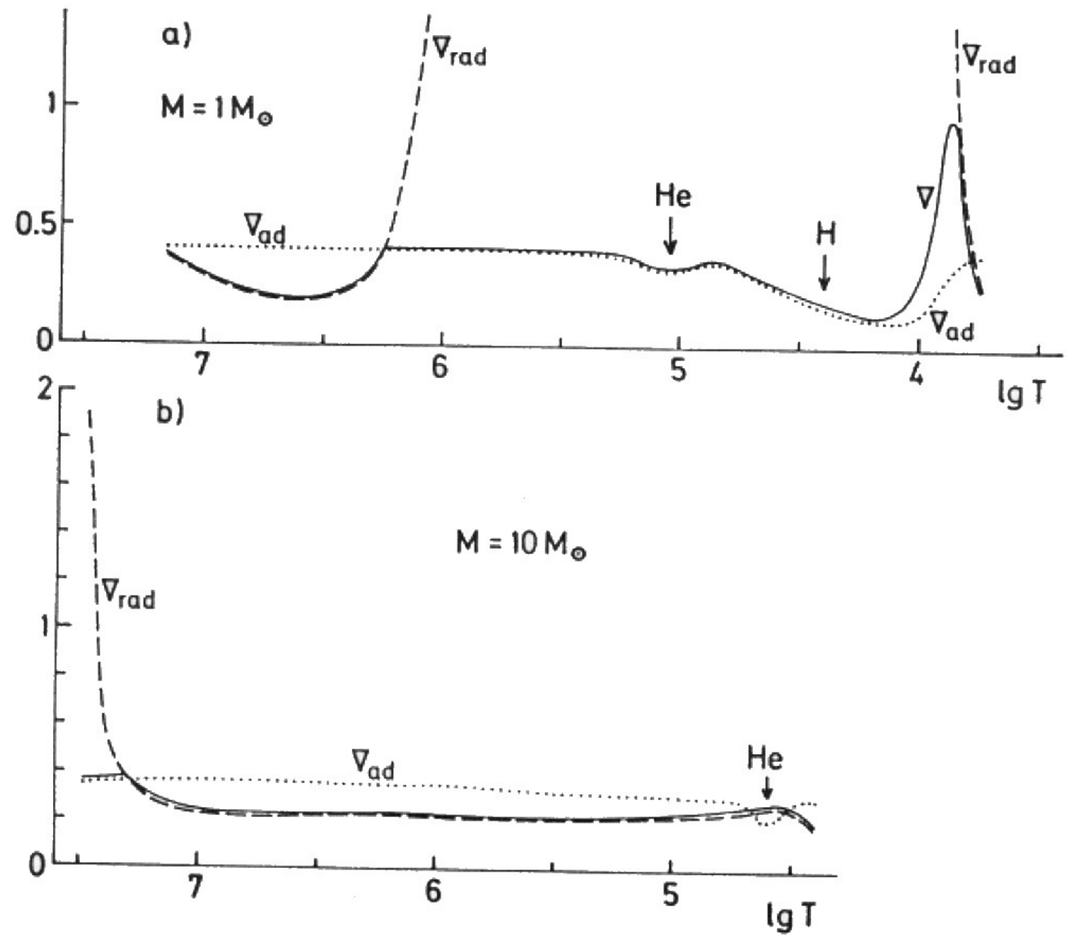


e) Convection

Two types of models

- Convective core, radiative envelope (upper MS)
- Radiative core, convective envelope (lower MS)
- Transition at ~1 M_o
- pp→CNO energy generation centrally condensed ($e \sim T^{16}$)→$L(r)/M(r)$ large→large ∇_{rad}
- Massive stars, increased P_{rad} decreases ∇_{ad}
- Low mass stars have high opacity due to ionization zones and low $\nabla_{ad} < 0.4$

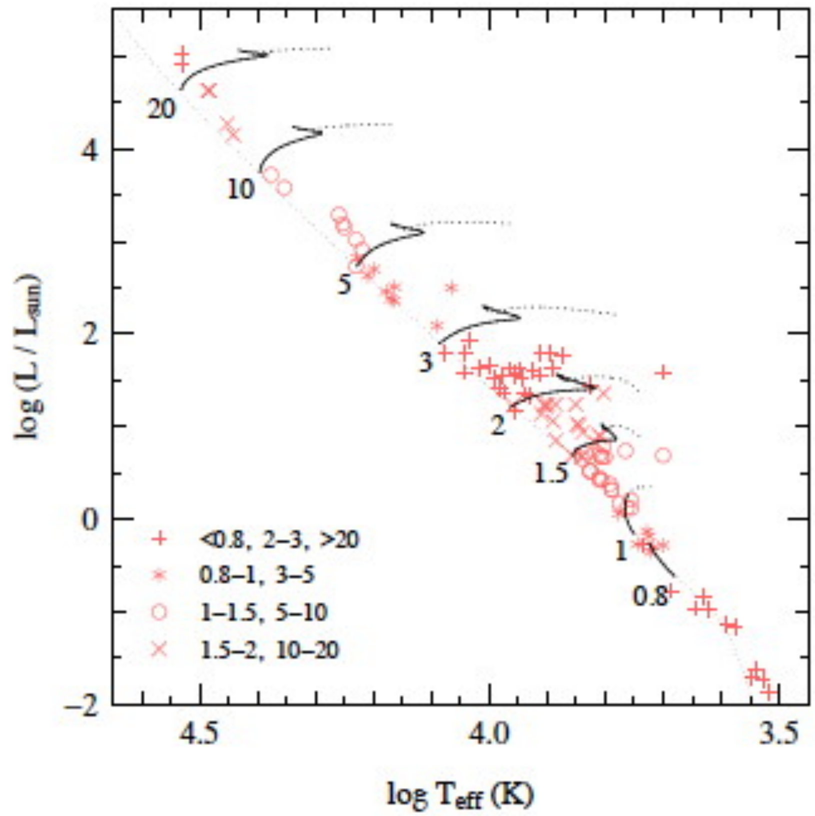




- 1 M_o: Effects of partial ionization on radiative and adiabatic temperature gradients near the surface are obvious
- 10M_o: energy generated within 10% of mass
- Check slides 39/16 in Lecture 3-2/3-3

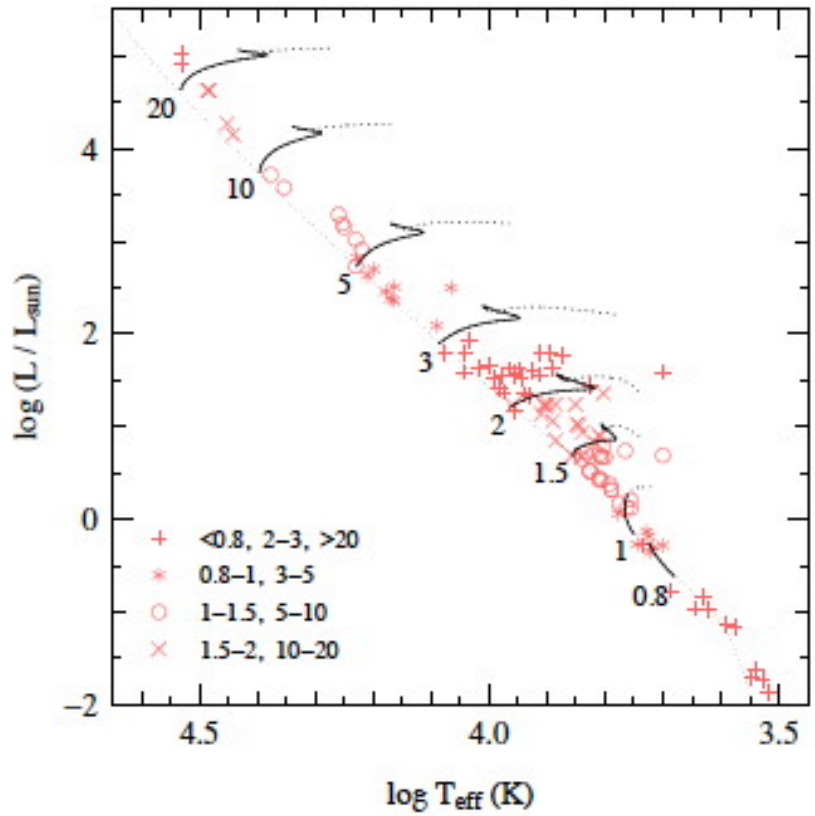
f) Luminosity evolution on the main sequence

- H converted into He and hence *m* increases and therefore *L* increases & P_c decreases (see slides 33-35)
- CNO cycle very temperature (and density) sensitive and in HE and TE, star has to reduce the "weight" of the envelope layers → expansion of the envelope and lower Teff
- pp chain less sensitive to temperature and density and hence T_c increases more and envelope expands less than for CNO cycle



Luminosity evolution on the main sequence

- The hook for high mass stars occurs when H is exhausted in the convective core. The star is no longer in TE and will contract (virial) – red extreme point in track – until shell burning can start – blue end point of track.
- Low mass stars have radiative cores and energy generation in relatively large core: H is depleted gradually with a smooth transition to shell burning.



Luminosity evolution

Due to H-burning, the number of particles decreases in the core and ρT has to increase (HE). The larger temperature gradient and the reduced (Kramer) opacity leads to a higher luminosity. We can estimate the luminosity change and its timescale:

$$\text{Virial: } T \propto \mu \frac{M}{R} \text{ or } T \propto \mu \rho^{1/3} M^{2/3}$$

$$\text{Radiative energy transport: } L \propto -\frac{R^2 T^3}{\kappa\rho}\frac{dT}{dr} \propto \frac{RT^4}{\kappa\rho}$$

$$\text{Kramers opacity: } L \propto \frac{RT^{15/2}}{\rho^2} \propto M^{16/3}\rho^{1/6}\mu^{15/2}$$

$$L(t) = L_0\left(\frac{\mu(t)}{\mu_0}\right)^{\psi} \text{ with } \psi = 15/2$$

Luminosity evolution

$$\mu(t) \approx \left[\mu_e^{-1} + \mu_i^{-1}\right]^{-1} = \frac{4}{3+5X}$$

$$\frac{dX}{dt} \propto -\frac{L(t)}{MQ}$$ with Q energy released per gram ($6x10^{18}$ erg)

When we neglect the weak dependence on ρ, we have

$$\frac{dL}{dt} = \left(\frac{dL}{d\mu}\right)\left(\frac{d\mu}{dX}\right)\left(\frac{dX}{dt}\right) = \left[\frac{\psi L_0}{\mu_0}\mu^{\psi-1}\right]\left[\frac{5\mu^2}{4}\right]\left[\frac{L}{MQ}\right]$$

$$\frac{dL}{dt} = \frac{5\psi\mu_0}{4L_0^{1/\psi}MQ}L^{2+1/\psi}$$

$$L = L_0\left[1 - \frac{5}{4}(\psi+1)\frac{\mu_0 L_0}{MQ}t\right]^{-\psi/(\psi+1)}$$

Luminosity evolution

$$\mu_0 = 0.6 \text{ and } \psi = 15/2$$

$$\frac{L}{L_{sun}} = \frac{L_0}{L_{sun}}\left[1 - 0.3\frac{L_0}{L_{sun}}\frac{t}{\tau_{sun}}\right]^{-15/17}$$

With t_{sun} the age of the sun. So, at $t = t_{sun}$, $L = L_{sun}$ and hence $L_0 = 0.79 L_{sun}$; ie., the sun is now brighter by ~25% than on the ZAMS.