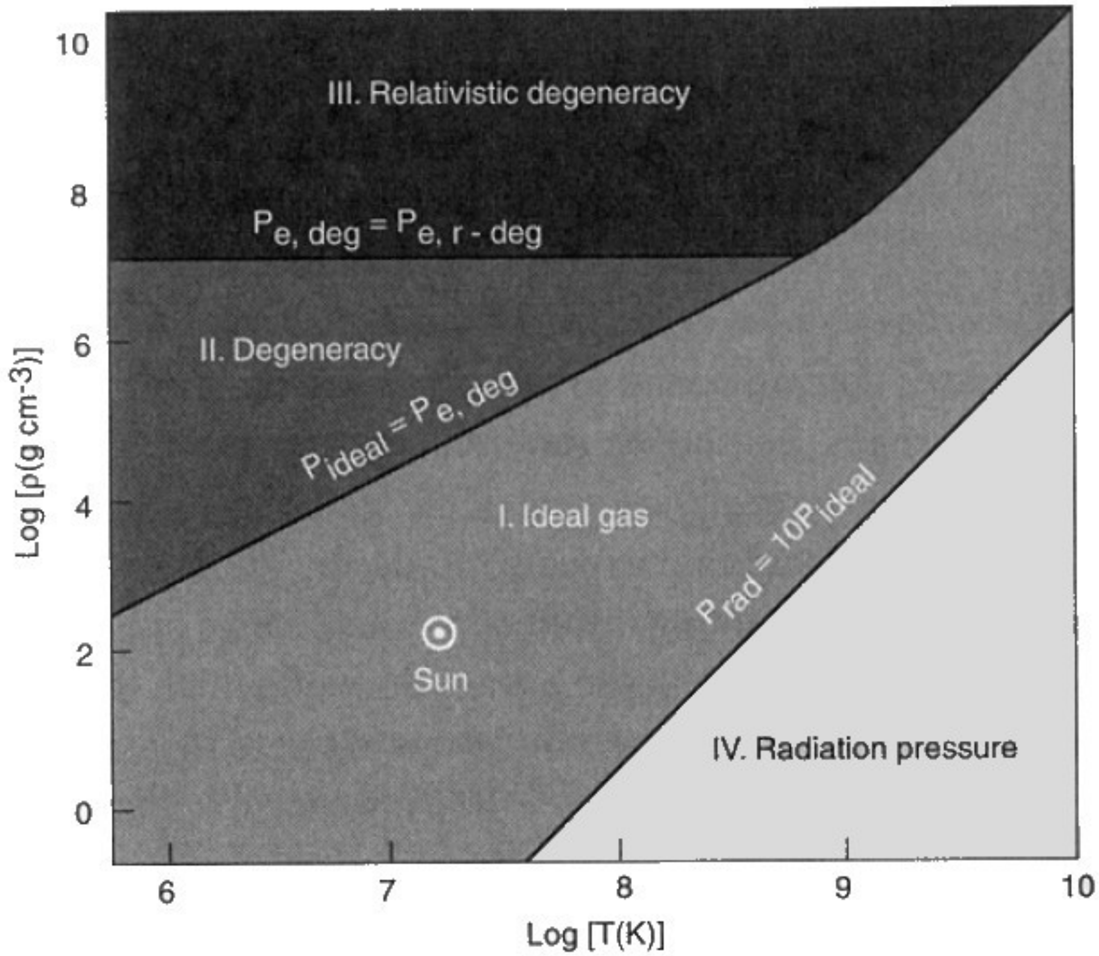
Lecture 6-1:
Schematic Evolution of Stars as seen from the core

Literature: KWW chapter 22
Pols chapter 7.1 & 7.2
[Prialnik Chapter 7]

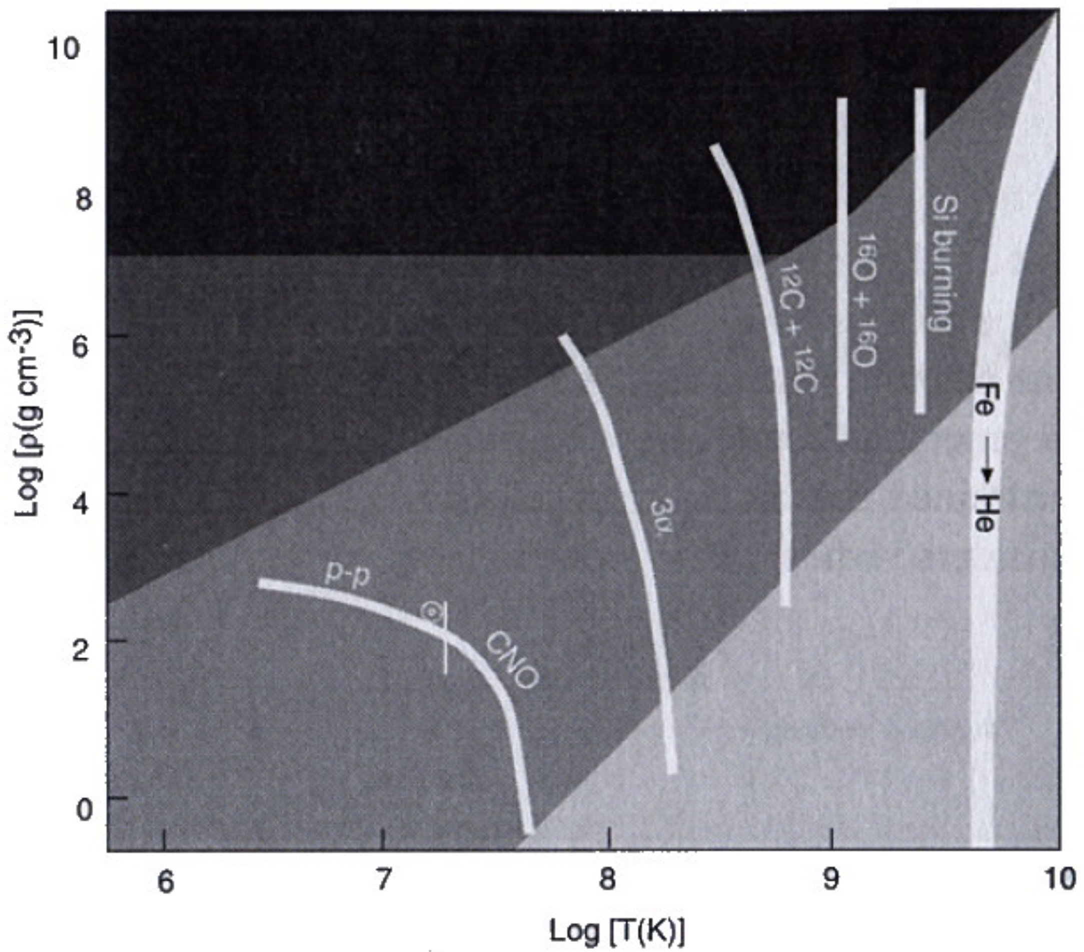
Core & Equation State

Slide 25 in Lecture notes 3-2 divided the density-temperature plane in different zones: ideal gas, degenerate (non-relativistic and extreme relativistic), radiation pressure. The boundaries are only approximate (exercise 2 assignment 4).



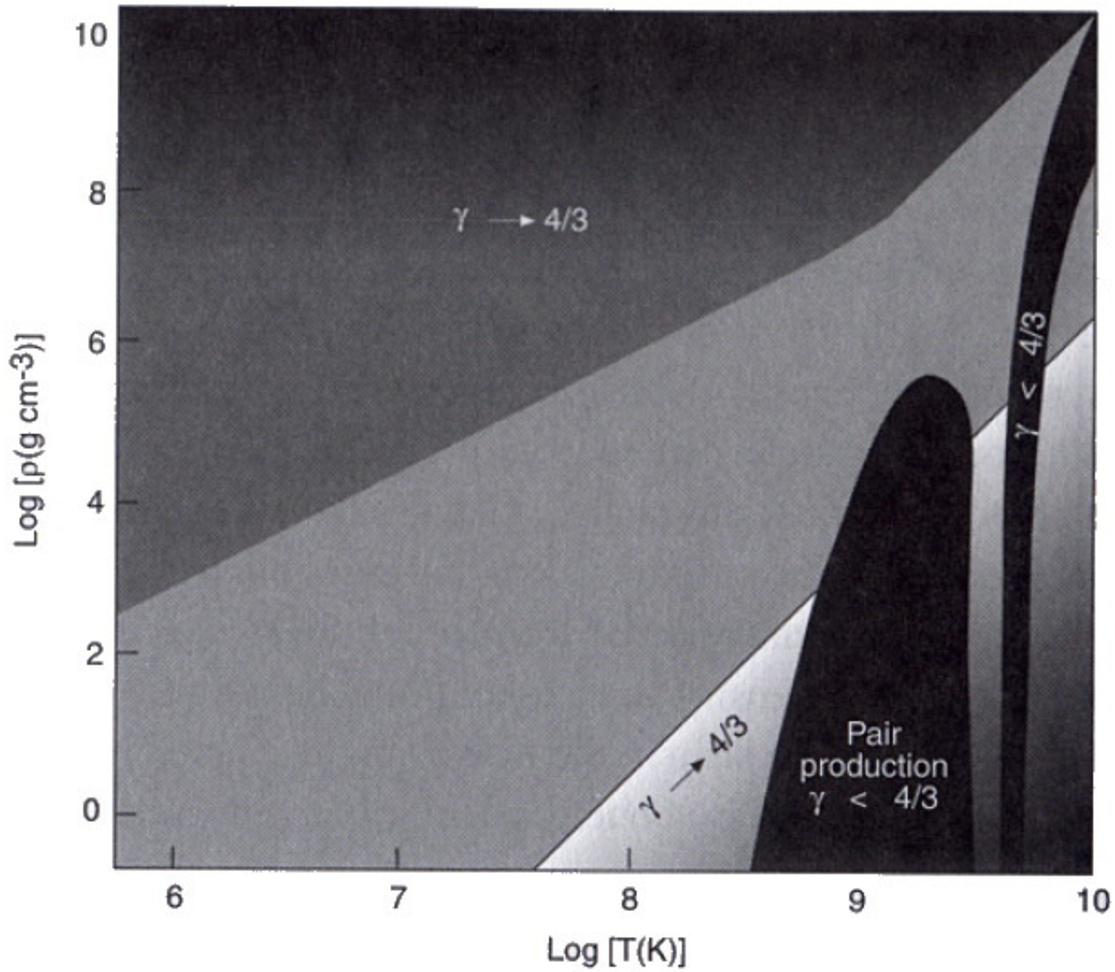
Core & Nuclear Burning Stages

Slide 21 in Lecture Notes 4-2 illustrated the nuclear burning stages on the density-temperature plane.



Core & Instability

We recognized instability for g<4/3 when we studied the virial theorem on slide 11 of lecture 2. We came across tis point again when we looked at the structure of white dwarfs and the Chandrasekhar mass (slides 20-22, lecture 5-3).



Evolutionary Path

For a polytrope, we have:

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

(Assignment 8 exercise 1)

with $B_n = 0.16 - 0.21$

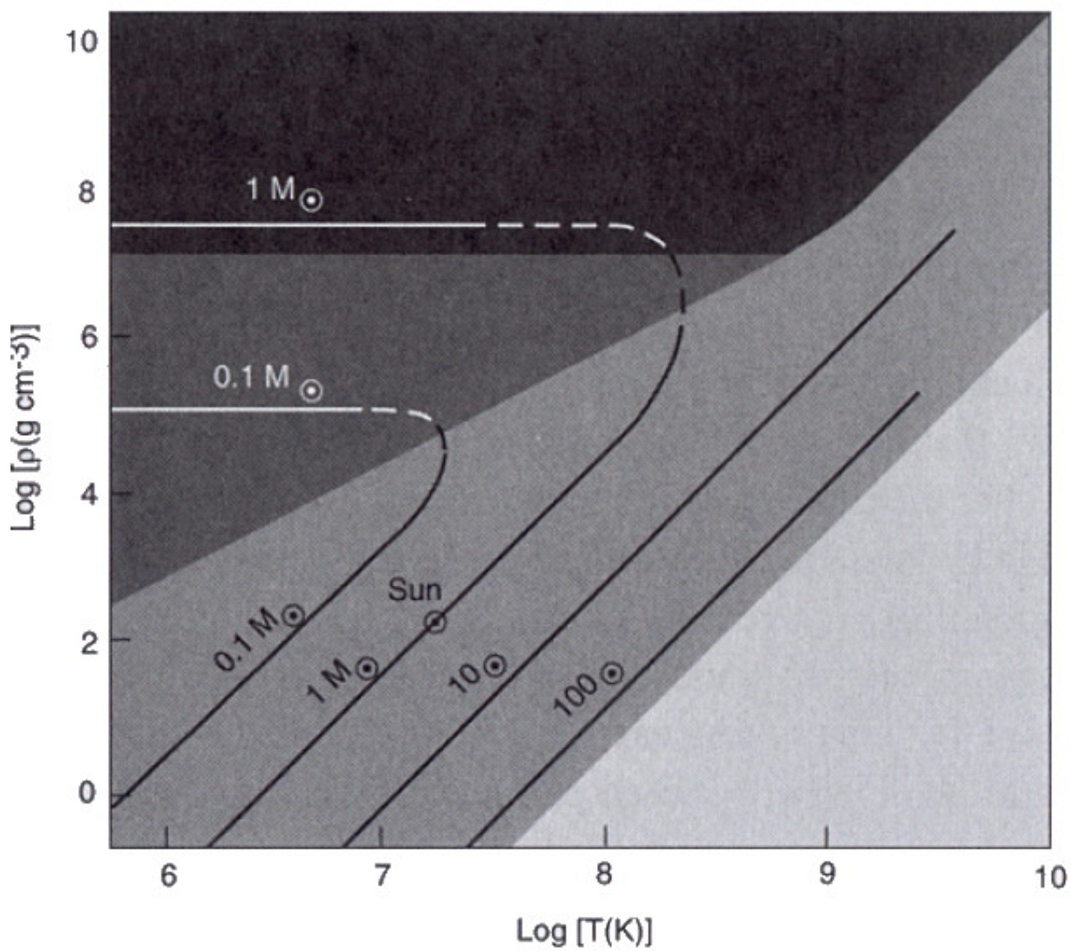
$$\rho_c = \frac{1}{4\pi}\left(\frac{k}{\mu m_u G B_n}\right)^3 \frac{T_c^3}{M^2} \qquad (1)$$

for an ideal gas.

$$\rho_c = 4\pi\left(\frac{B_{3/2} G}{K_{NR}}\right)^3 M^2 \qquad (2)$$

for a non-relativistic

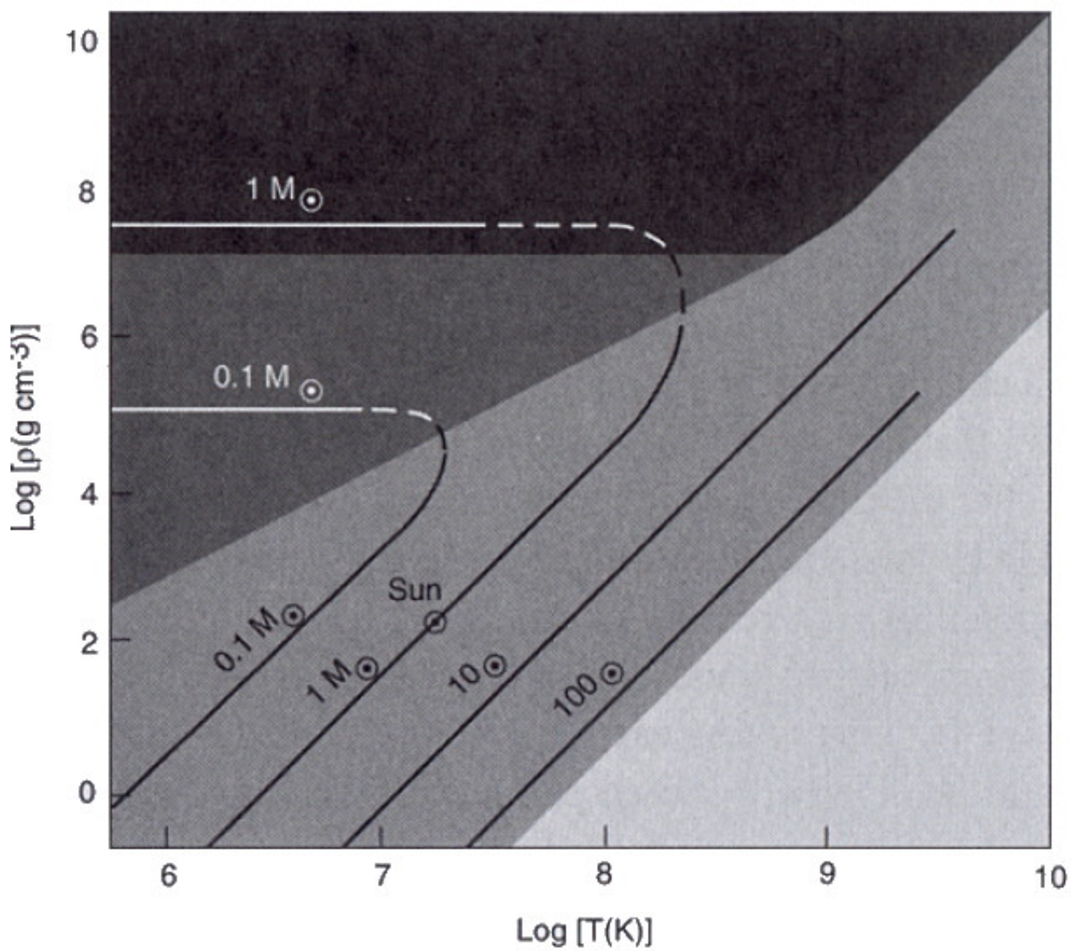
degenerate electron gas.



So, for different masses, parallel lines with slope 3 for (1) & horizontal lines for (2).

Evolutionary Path

For low mass stars, the two segments are connected by dashed lines. Tracks of higher mass stars will bend at higher temperatures and central density will approach infinity for the Chandrasekhar mass.

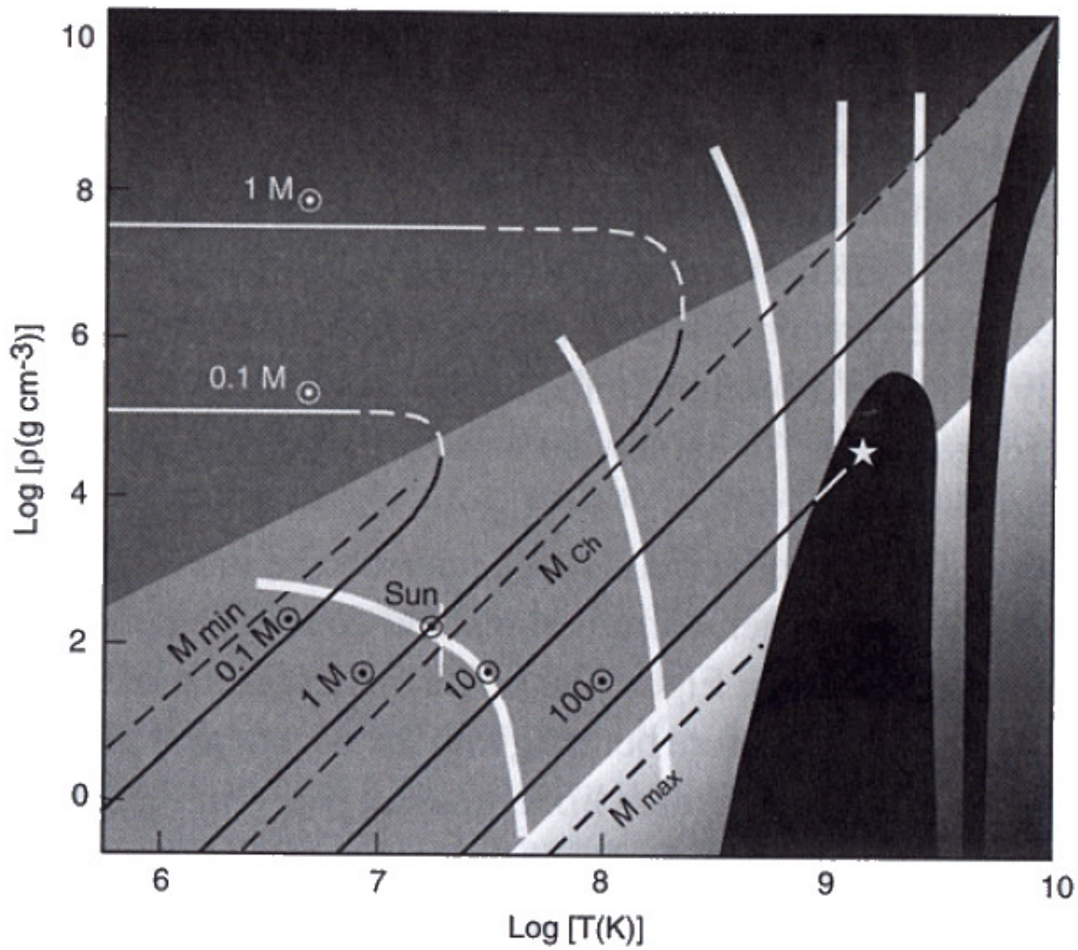


Summary

Hydrostatic equilibrium and the ideal gas law connects r_c and T_c. With increasing mass, the central pressure has to increase to support the star against gravity. As thermal pressure scales with r while hydrostatic pressure scales with $r^{4/3}$, a more massive star requires a higher central temperature (and/or lower density). For a non-relativistic degenerate gas, temperature doesn't enter. Now, thermal pressure scales with $r^{5/3}$ while hydrostatic pressure scales with $r^{4/3}$, and a more massive star requires a higher central density. And we have already seen that for an extreme relativistic gas, the central density has to be infinite and that hydrostatic equilibrium is only possible for a mass equal to the Chandrasekhar mass (well not really, as the star will collapse to make the density infinite).

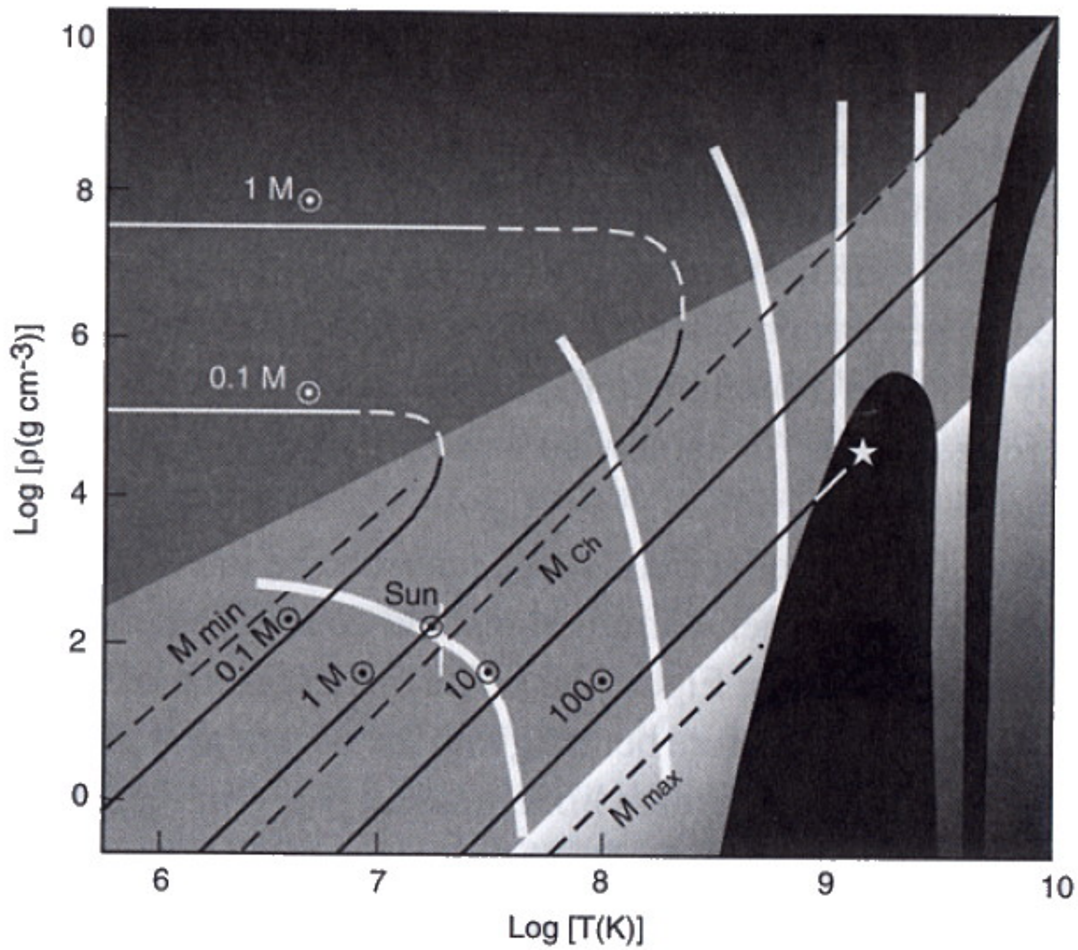
Evolution & ZAMS

Stars start their life on the Hayashi track at low central density and temperature (Lecture 5-3 slides 14–19). As the star contracts, the core heats up (virial theorem, lecture 2) and eventually reaches H-burning conditions. At that point contraction will halt, as nuclear energy generation can supplant the energy loss, and the star is in hydrostatic and thermal equilibrium.



Evolution & ZAMS

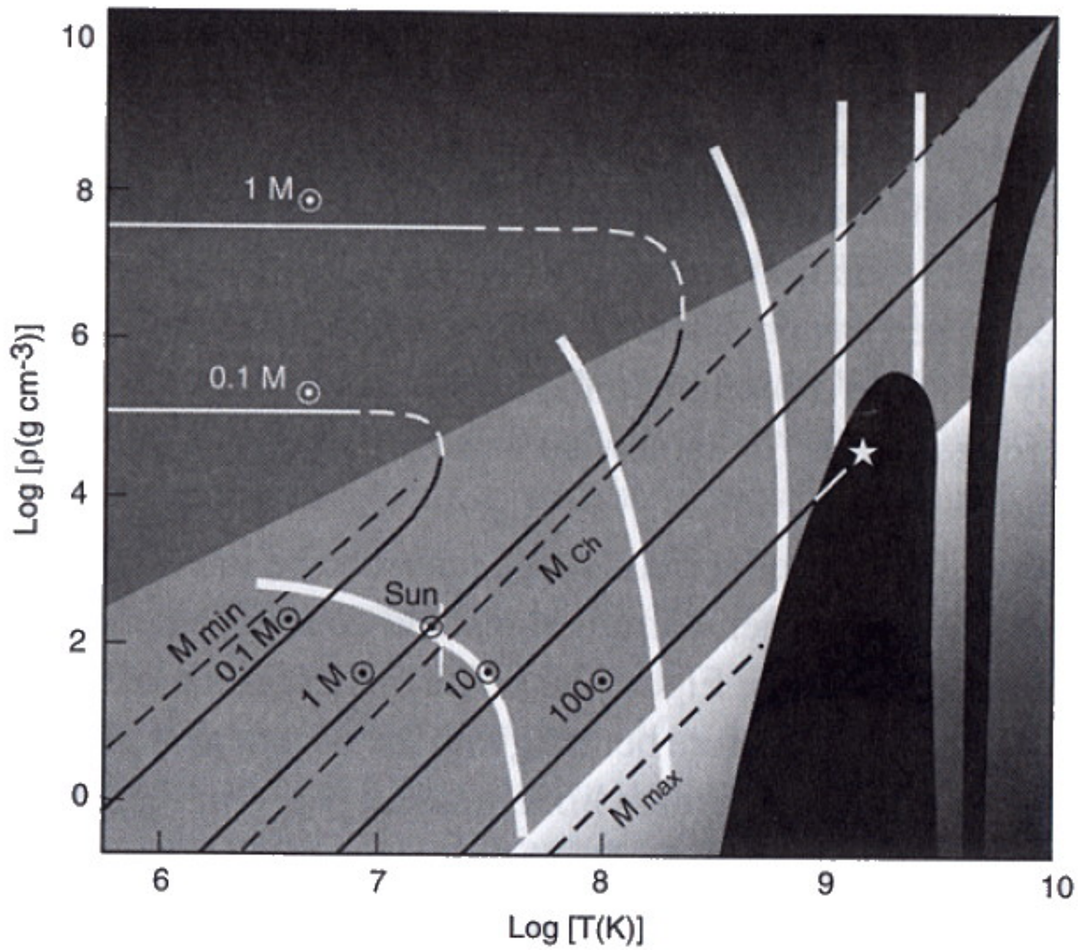
The star has now entered its life on the ZAMS. Note that low mass stars burn with the p-p chain, while high mass stars burn through the CNO cycle with very different T-dependencies.



Evolution & ZAMS

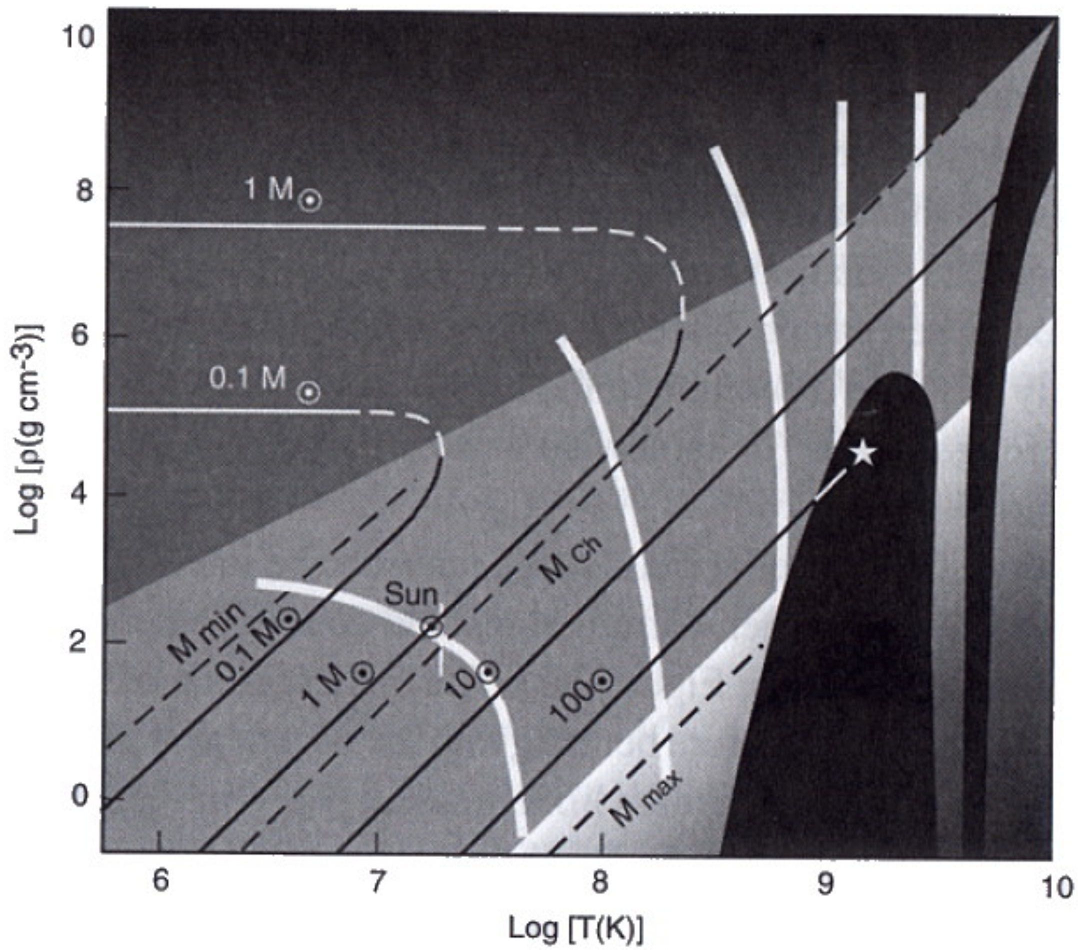
Maximum mass is set by radiation pressure; e.g., g=4/3 and E_t=0 (Lecture 2 slide 11). Any disturbance in the "force" will trigger an eruption (LBV). The precise upper mass is not well known but is somewhere between 100 and 300 M_o (Lecture 3-4, slide 22ff).

Minimum mass limit is set by minimum temperature required to ignite H (*T~$3x10^6$* K), which yields *M≈0.08* M_o.(see slide 25).



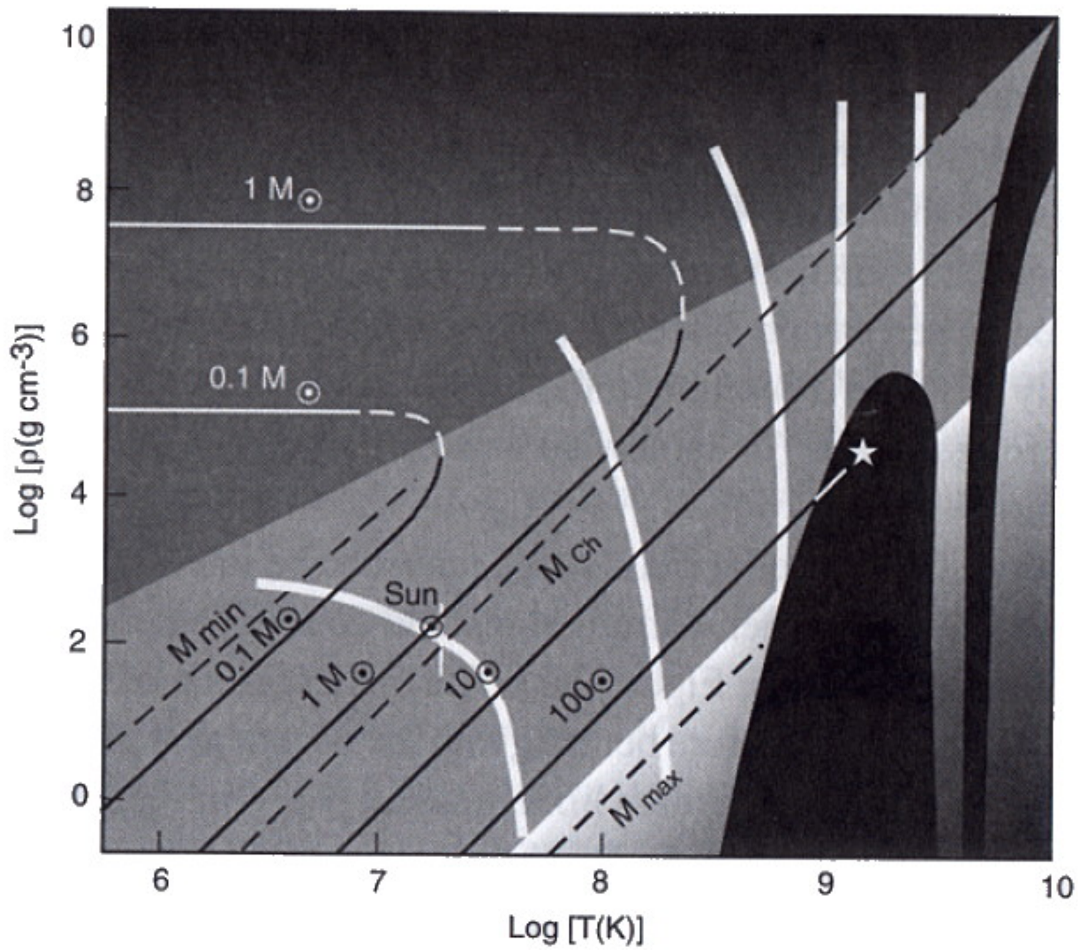
Evolution & beyond the ZAMS

When H is exhausted, the star will have to shrink to supply the energy lost and the core contracts and heats up. Very low mass stars will bend to the left into the degenerate regime. The star is now stabilized by degenerate electron pressure and will slowly cool as a He white dwarf with a density and radius determined by its mass.



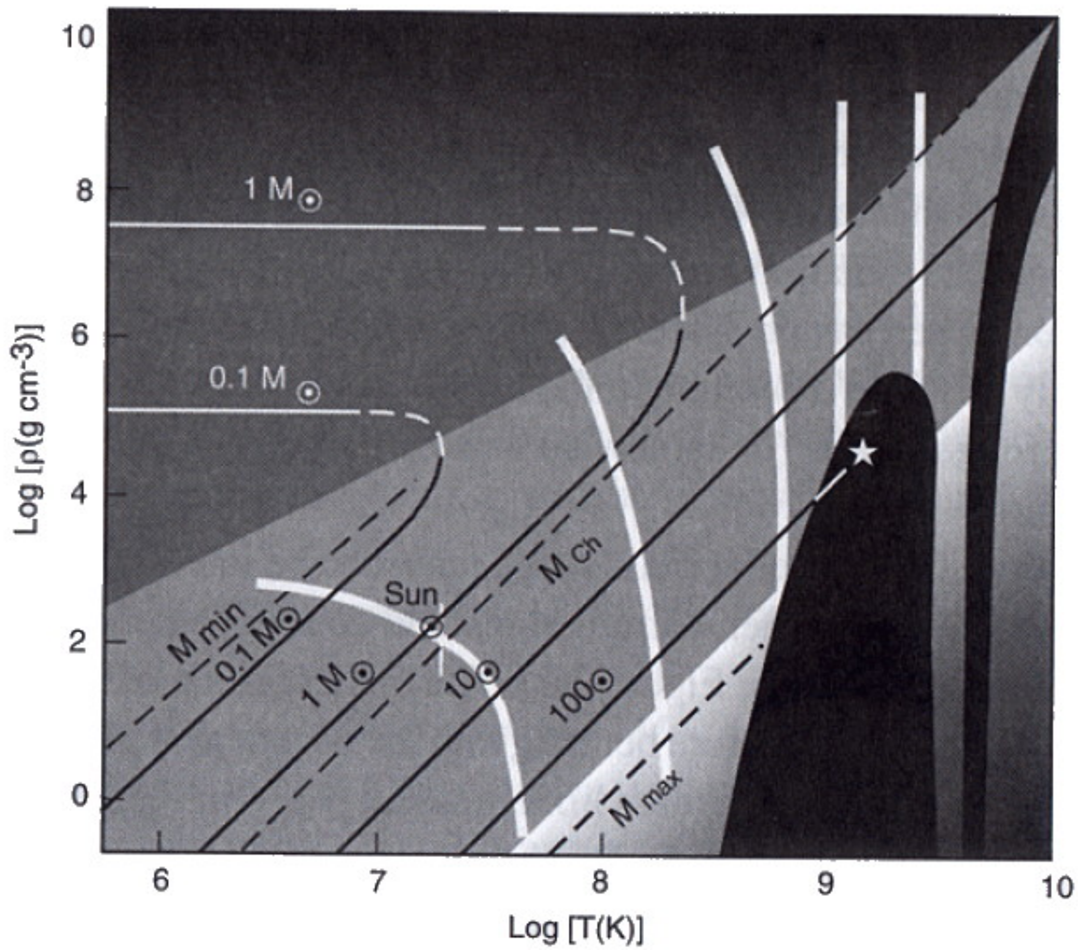
Evolution & beyond the ZAMS

Ignoring for the moment shell burning ,for larger mass, when the core is hot enough, He will ignite with another phase of stable burning. If this happens near the degenerate border, a thermal instability happens (g-4/3; *P* independent of *T*). This story continues with the lower mass stars developing a degenerate C,O core with a density and size set by its mass, while for the more massive stars, the core contracts, heats up, and a new energy source ignites.



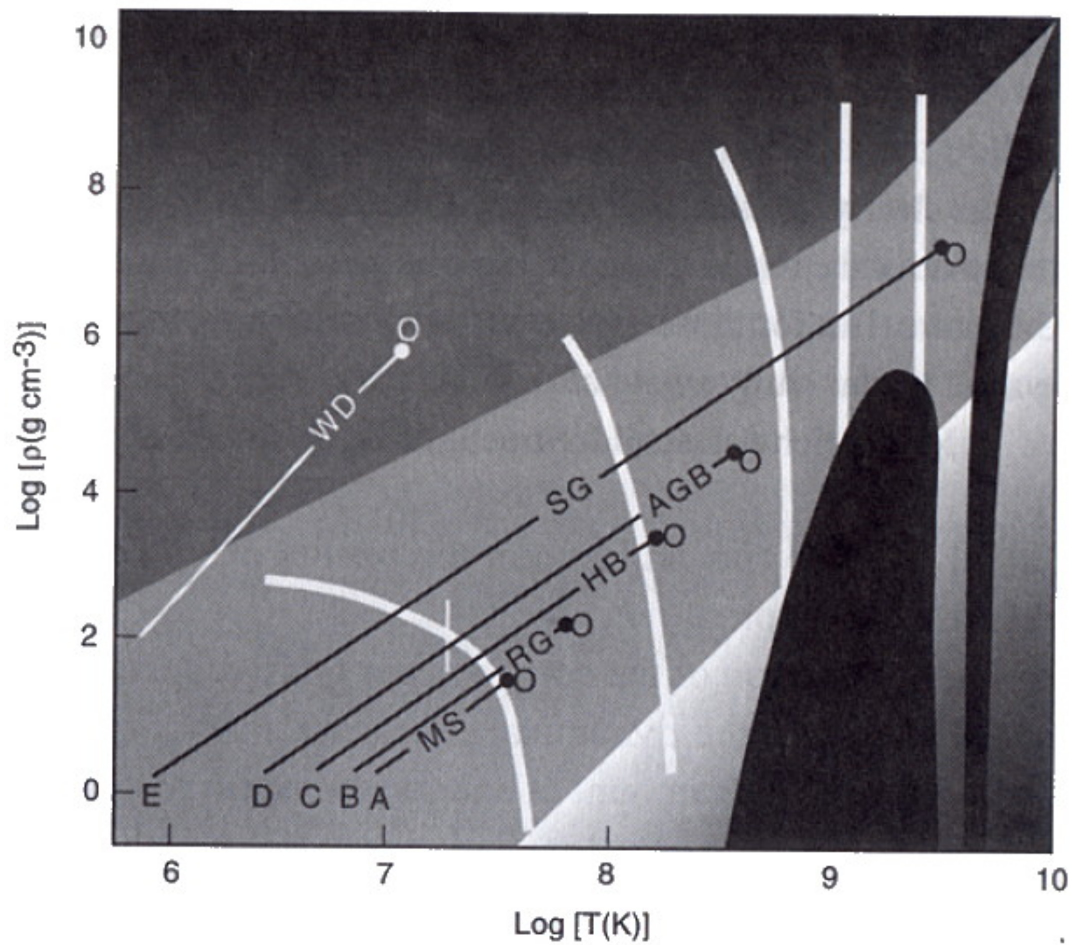
Evolution & beyond the ZAMS

Stars with cores more massive than the Chandrasekhar mass, will go through all nuclear burning stages up to Fe. Eventually, the core will enter the photodisintegration zone and the result will be catastrophic (g=4/3). Very massive stars enter the pair-production instability zone that could end the star much earlier in their life.



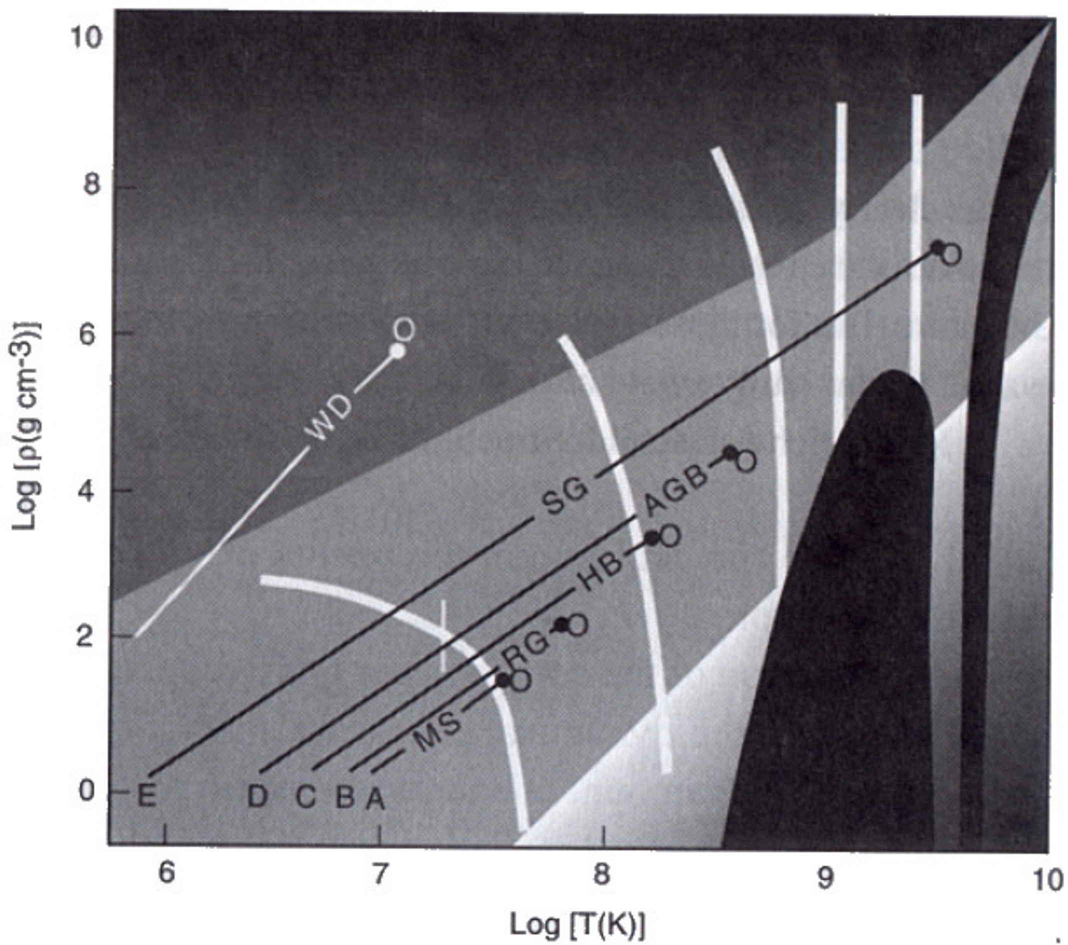
Shell Burning & Giants

Any point in the star will describe a track in the ESO diagram (to the left of the core-track). After the MS (track A), while the core keeps contracting, the shell can cross a nuclear burning stage, leading to a phase of stable shell burning (line B). If during this phase, thermal equilibrium is maintained, core contraction will be compensated by envelope expansion and cooling (virial theorem; gravitational & thermal energy is constant) and the star moves to lower effective temperature.



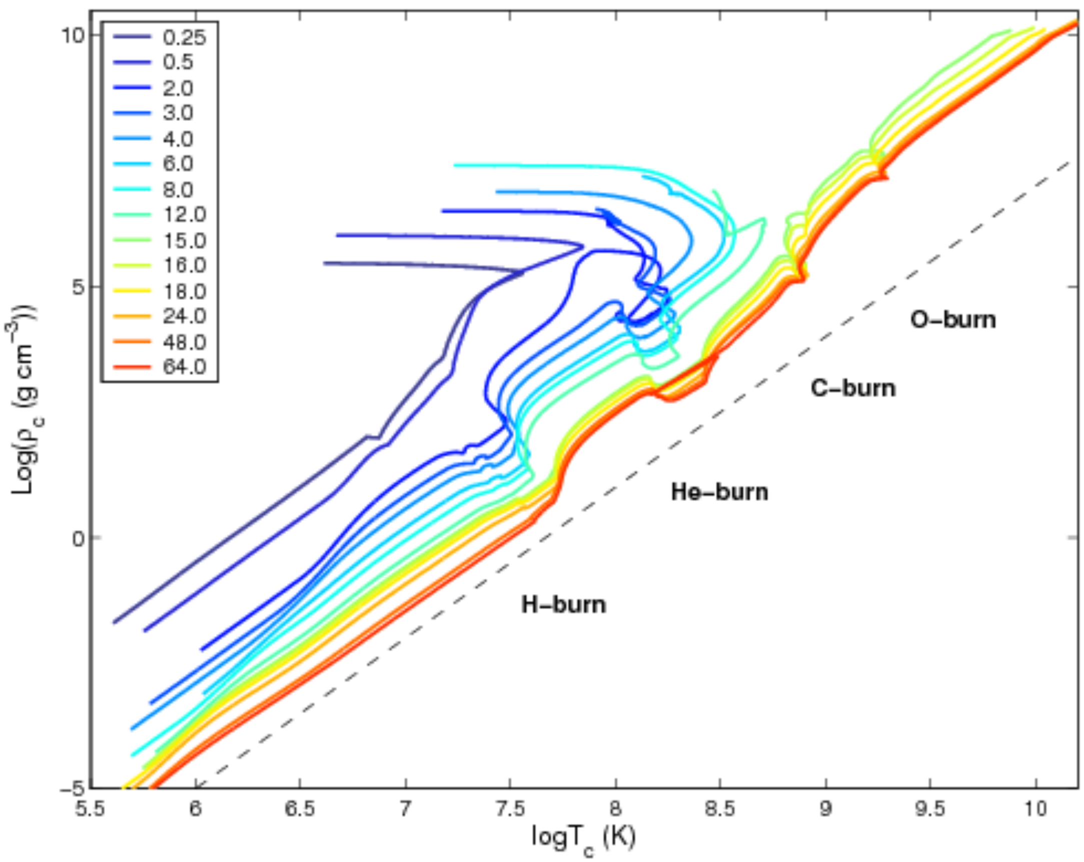
The Helium Main Sequence

When the core reaches He-ignition, the star will have two zones of nuclear burning: a core burning He(main) and a shell burning H (minor). When He is exhausted, the core will contract again and the envelope expands again, the AGB phase (track D) with a C,O core surrounded by a He burning shell, a He layer, and a H burning shell.



Detailed Tracks

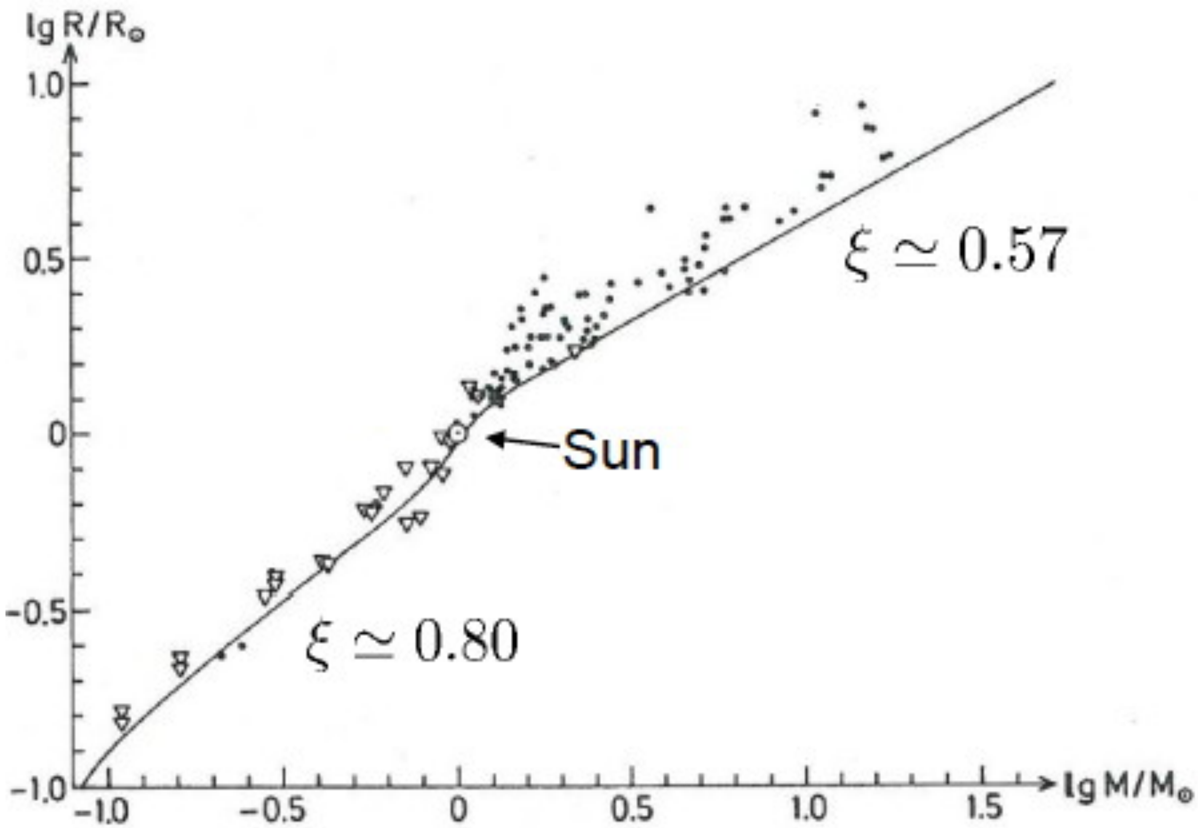
General trends are very similar: recognize premainsequence contraction, H-burning in the core, H-shell-burning on the red giant branch, He-core burning, H- & He-shell burning on the AGB, White Dwarf formation for low mass stars, Further burning stages for massive stars, and the importance of mass loss



Lecture 6-2:
Main Sequence

Literature: KWW Chapter 22, 29, 30

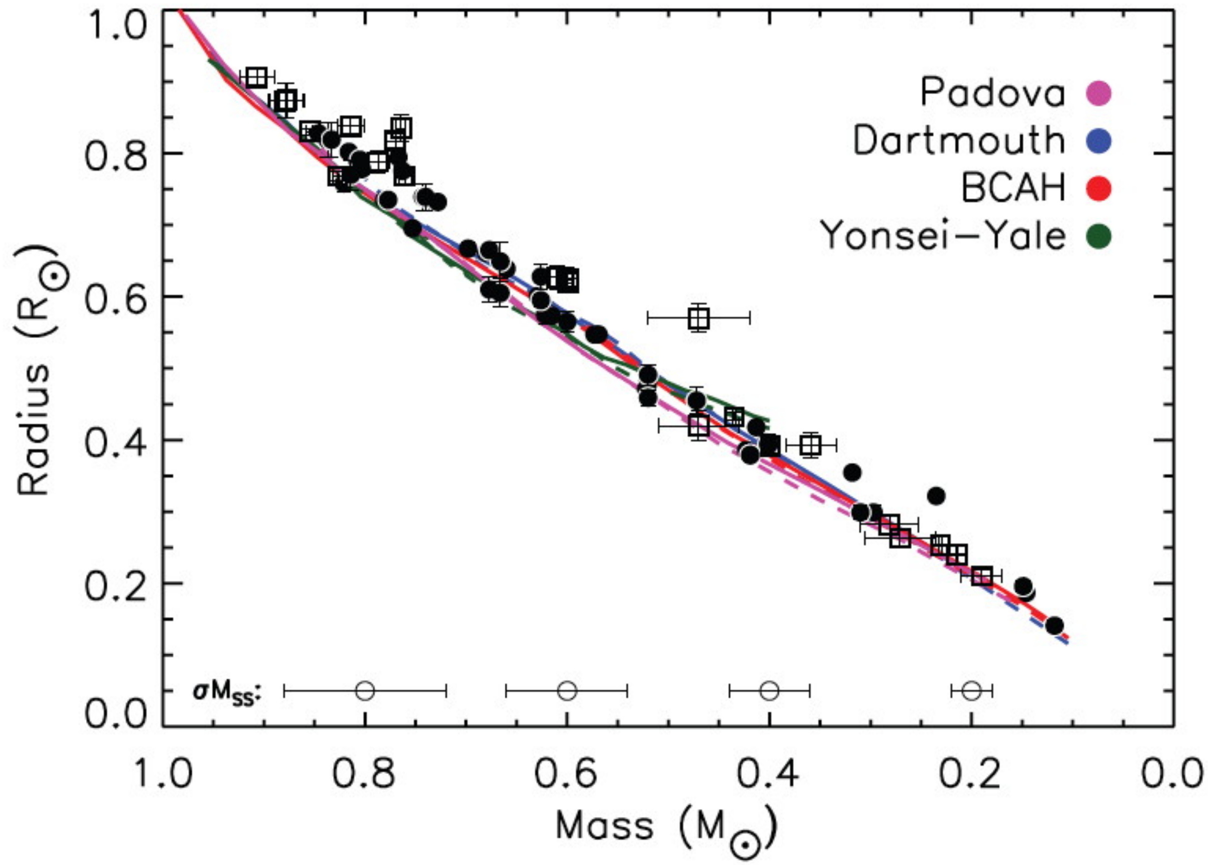
a) Mass-Radius Relation



Triangles: visual binaries; dots detached binaries; line models

Note break around $1M_o$ (convective envelope)

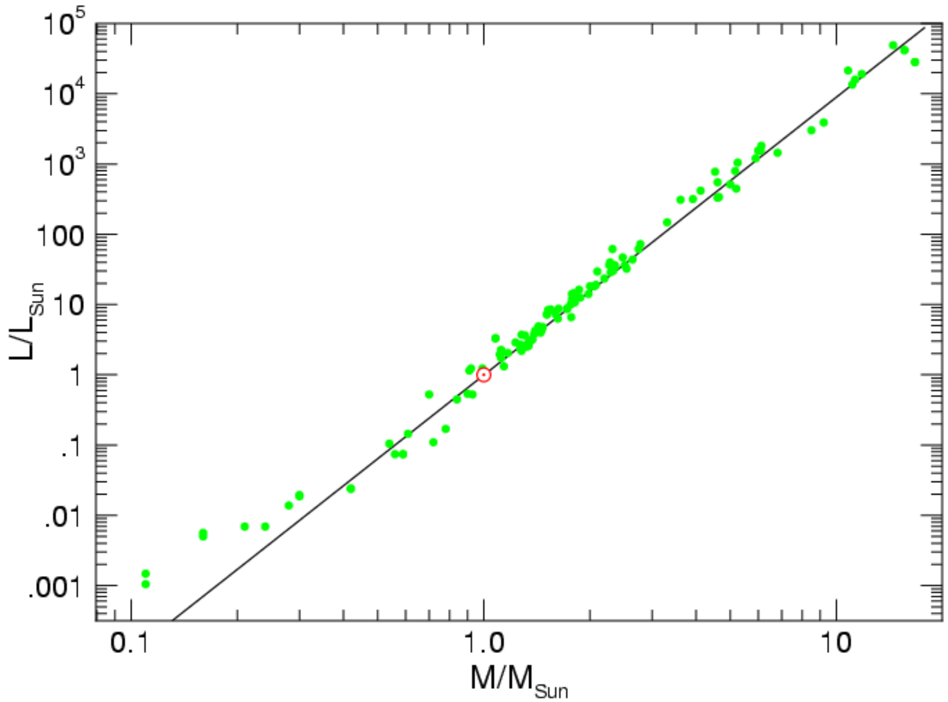
Mass-Radius Relation



dots: interferometry; squares: binaries; lines models
Note the x-axis runs in the opposite direction

Mass-luminosity relation: Slide 11, lecture 1

Measure individual masses for double-lined eclipsing variables



Popper 1980 ARA&A
Martin & Mignard 1998 AA 330, 585

Recent measurements:

Break in slopes due to Convection/radiation pressure (assignment)

$$\frac{L}{L_O} = \begin{cases} 0.66\left(M/M_O\right)^{2.5} & 0.08M_O \le M \le 0.5M_O \\ 0.92\left(M/M_O\right)^{3.55} & 0.5M_O \le M \le 40M_O \\ 300\left(M/M_O\right)^{2} & 40M_O \le M \le 130M_O \end{cases}$$

b) Mass-luminosity relation: theory

We encountered this in exercise 1, assignment 3 where we used hydrostatic equilibrium and virial theorem to derive this relation for a star in radiative equilibrium. Examining this again

$$P \propto \frac{GM}{R}\rho \ \&\ T \propto \frac{P}{\rho} \text{ (gas pressure dominated)} \Rightarrow T \propto \frac{\mu M}{R}$$

$$L \propto \frac{R^2}{\kappa\rho}\frac{T^4}{R} \ \&\ \kappa \propto \rho^n T^{-\alpha} \Rightarrow L \propto \mu^{\alpha+4} R^{3n-\alpha} M^{\alpha-n+3}$$

$$\text{Kramers opacity } (n=1, \alpha=3.5)\text{: } L \propto \mu^{7.5}\frac{M^{5.5}}{R^{0.5}}$$

$$\text{Thompson opacity } (n=0, a=0)\text{: } L \propto \mu^4 M^3$$

$$\text{Radiation pressure dominated } (P \sim P_{rad} \propto T^4 \propto \frac{M^2}{R^4}) \text{ \& Thompson}$$

$$L \propto R^2 \frac{R^3}{M}\frac{M^2}{R^5} \propto M$$

Mass-Radius relation: theory

Set the luminosity equal to the thermonuclear power generation in the core:

$\varepsilon \propto \rho_c T_c^s$

Very massive stars (>100 M_o; radiation pressure & CNO cycle($s = 14$)):

$$L \propto \varepsilon M \propto \frac{M}{R^3}\left(\frac{M^2}{R^4}\right)^{14/4} M \propto \frac{M^9}{R^{17}} \propto M \Rightarrow R \propto M^{8/17}$$

Massive stars (2-100M_o; gas pressure & CNO cycle):

$$L \propto \varepsilon M \propto \frac{M}{R^3}\left(\frac{M}{R}\right)^{14} M \propto \frac{M^{16}}{R^{17}} \propto M^3 \Rightarrow R \propto M^{13/17}$$

For low mass stars (<2 M_o), convection becomes more and more important. Efficient energy transport and star shrinks (central pressure, temperature, and density go up).For a fully convective star, $M \propto R^{-3}$ (slide 9 ch 5-3; slides 8-10 ch 5-2).

Mass-Radius relation: theory

Looking at it from the Mass-luminosity relation: $L \propto M^{\eta}$

and virial equilibrium,

$$T_c \propto \frac{M}{R} \Rightarrow T_c^3 \propto \left(\frac{M}{R}\right)^3 \propto \rho_c M^2$$

We can write:

$$L \propto \varepsilon M \propto \rho_c T_c^s M \propto \rho_c \left(\rho_c^{1/3} M^{2/3}\right)^s M \propto M^{\eta} \quad \text{Or:}$$

$$\log \rho_c \propto \left(\frac{3\eta - 2s - 3}{s+3}\right) \log M \qquad \log T_c \propto \left(\frac{\eta + 1}{s+3}\right) \log M$$

For high mass stars, $s = 18$ & $\eta = 3$, the exponents are $-30/21$ & $4/21$

For low mass stars, $s = 5$ & $\eta = 2$, the exponents are $-7/8$ & $3/8$

Around $1M_o$, $s = 4$ & $\eta = 4$, the exponents are $1/7$ & $5/7$

Putting it together in the HR diagram:

$$R = R(M) \propto M^{\xi}$$

with

$$\xi \approx 0.57 \quad M \geq M_O$$

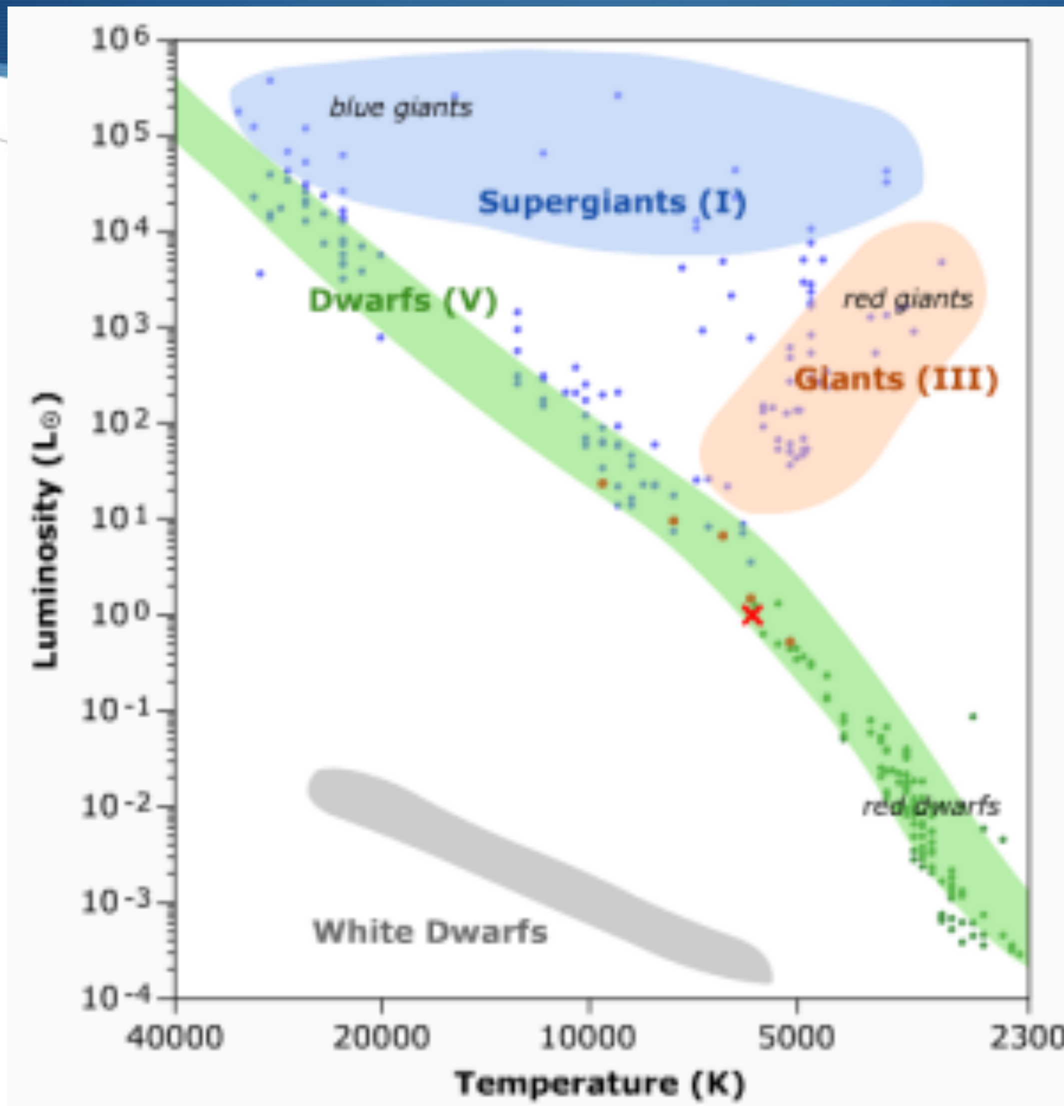
$$\xi \approx 0.80 \quad M \leq M_O$$

$$\left.\begin{aligned} L &= L(M) \propto M^{\eta} \quad (\eta \approx 3.2) \\ L &= 4\pi R^2 \sigma T_{eff}^4 \end{aligned}\right\} \Rightarrow$$

$$L \propto T_{eff}^{\varsigma} \quad with$$

$$\varsigma = \frac{4}{1 - 2\xi / \eta} = \begin{cases} 6.2\ (M \geq M_O) \\ 8.0\ (M \leq M_O) \end{cases}$$

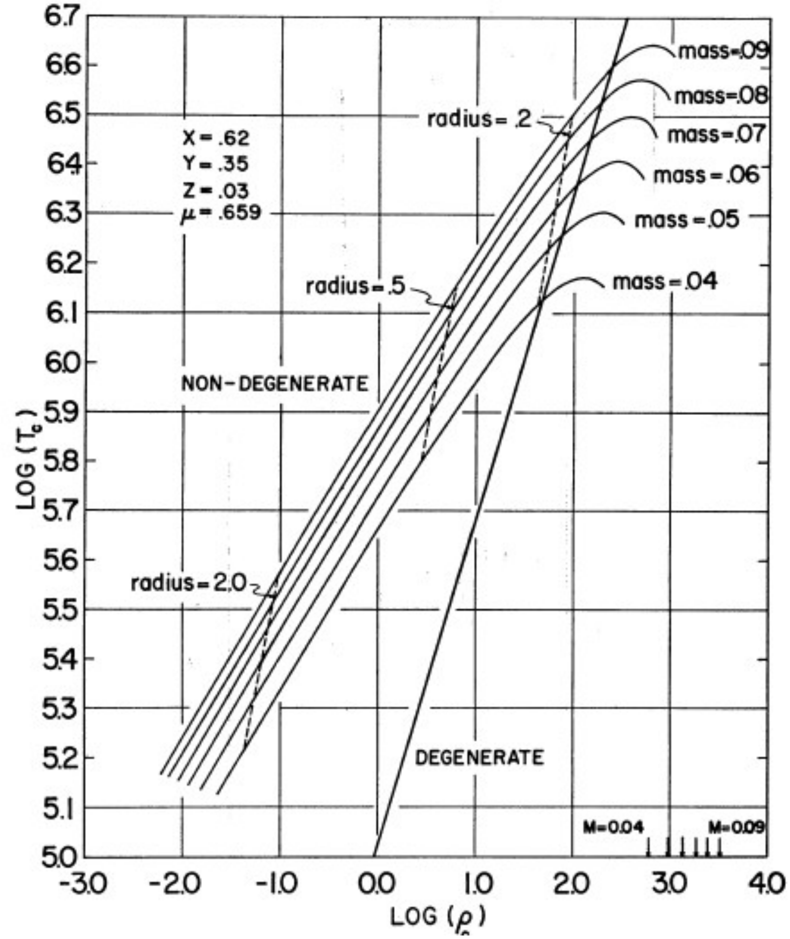
Steep dependence on L and T_{eff} in the HR diagram as observed



blue giants
Supergiants (I)
Dwarfs (V)
red giants
Giants (III)
red dwarfs
White Dwarfs
Luminosity ($L_\odot$)
Temperature (K)
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
40000
20000
10000
5000
2300

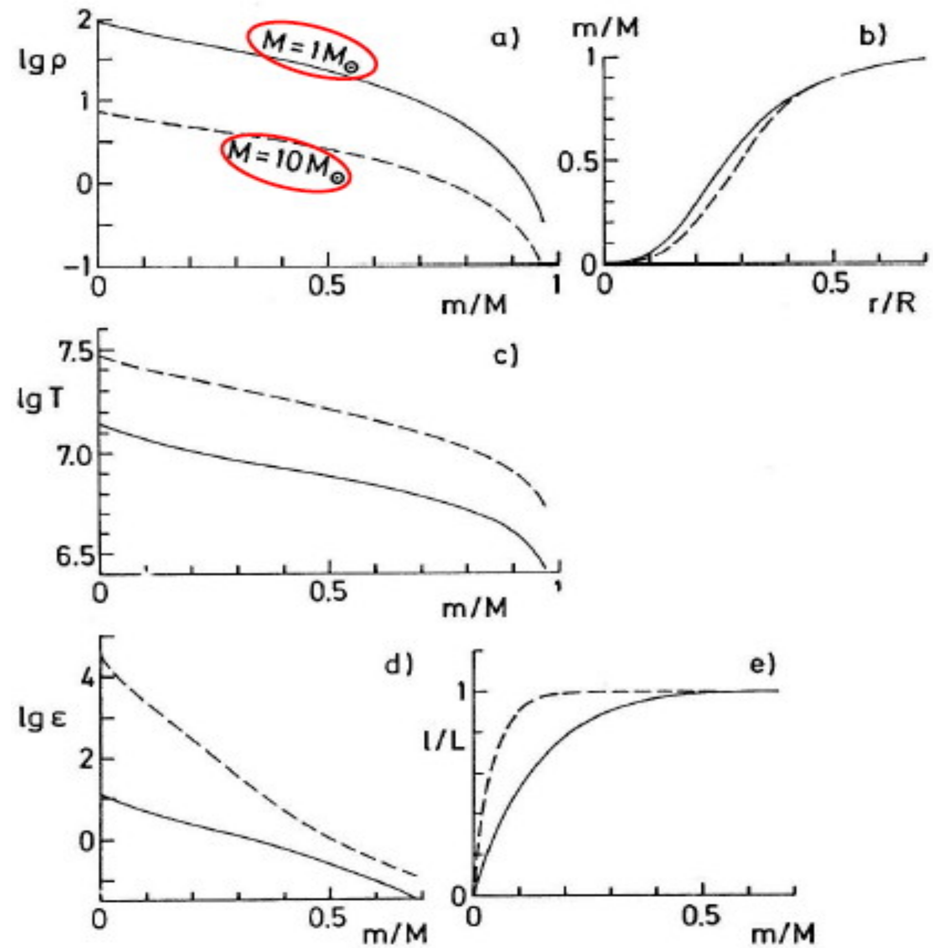
c) Minimum stellar mass

Minimum stellar mass is set by H burning ignition which requires a central temperature of 3×10^6 K. This corresponds to a minimum mass of ~0.08 M_o (see assignment).



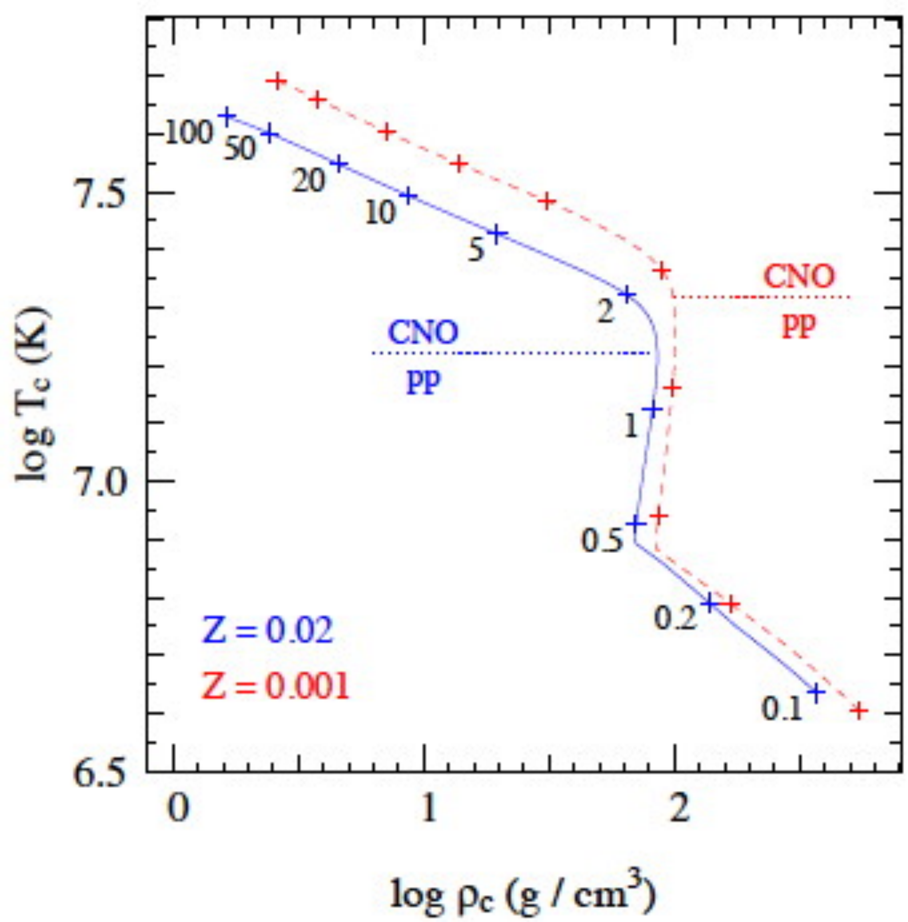
d) Stellar structure main sequence stars

- Density increases by a factor one billion with m/M (or r)
- More massive stars are less dense (slide 8, Ch 6-1)
- Mass is highly concentrated: 60% of the mass in the core (30% of the radius, 3% of the volume)
- Temperature increases by a factor ~3000 with m/M (or r)
- More massive stars are hotter
- Energy generation: 90% of L generated in inner 30% of M for $1M_o$
- 90% of L generated in inner 10% of M for $10M_o$



Stellar structure main sequence stars

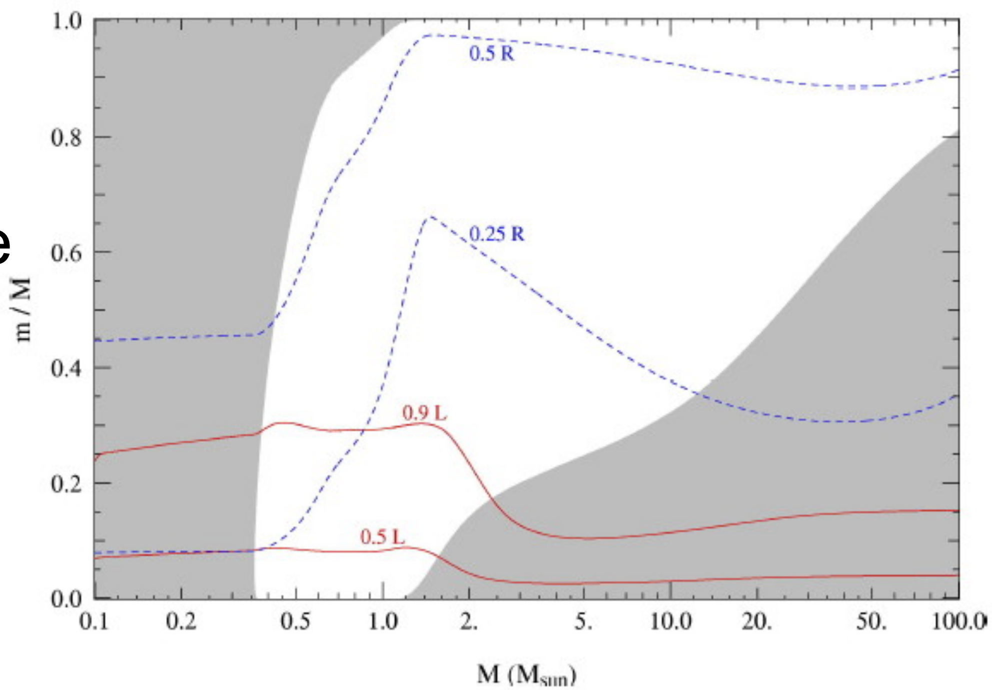
- Look back at slide 23
- Slow increase in T_c with increasing M
- Rapid decrease in r_c with increasing M
- Rapid change between 0.3 and 2 M_o because core is radiative
- Low mass stars become degenerate (y=0 for 0.3M_o, slide 19 in lecture 3-2)
- Radiation pressure is important for M>50 M_o (b=0.5)

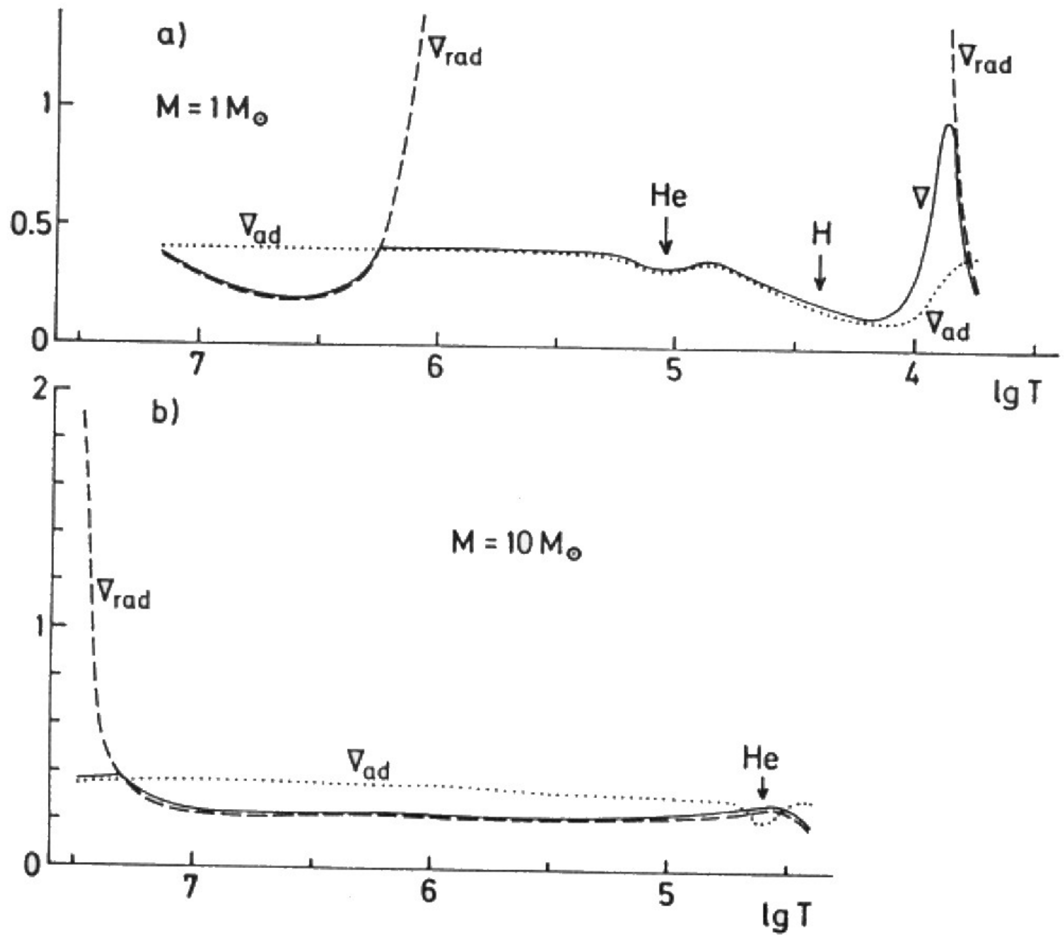


e) Convection

Two types of models

- Convective core, radiative envelope (upper MS)
- Radiative core, convective envelope (lower MS)
- Transition at ~1 M_o
- pp→CNO energy generation centrally condensed (ϵ~T^{16})→L(r)/M(r) large→large ∇_{rad}
- Massive stars, increased P_{rad} decreases ∇_{ad}
- Low mass stars have high opacity due to ionization zones and low $\nabla_{ad} < 0.4$

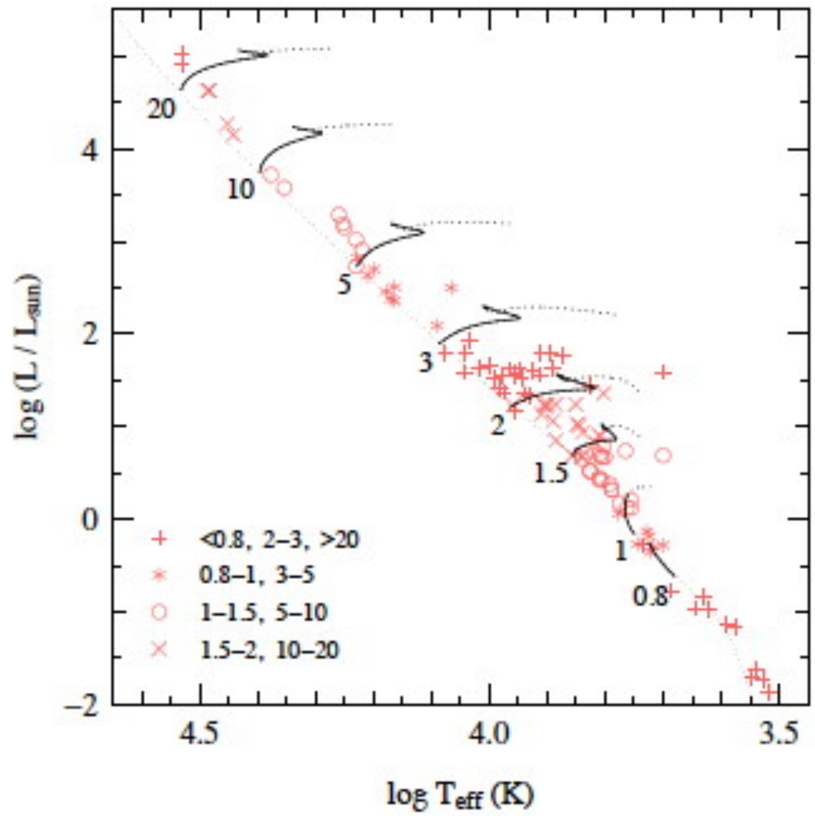




- 1 M_o: Effects of partial ionization on radiative and adiabatic temperature gradients near the surface are obvious
- 10M_o: energy generated within 10% of mass
- Check slides 39/16 in Lecture 3-2/3-3

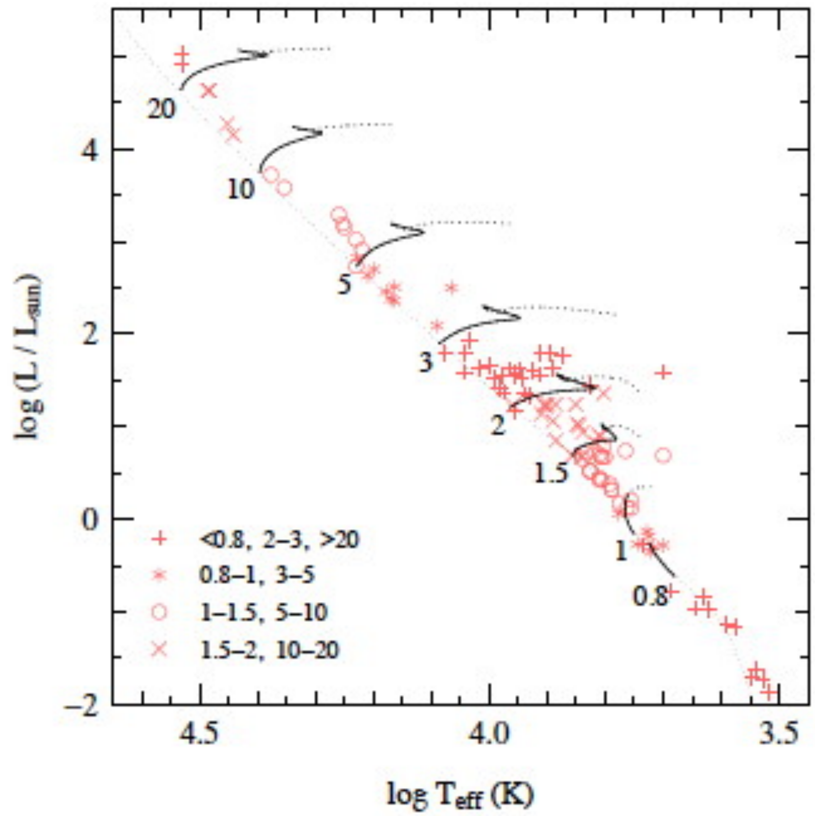
f) Luminosity evolution on the main sequence

- H converted into He and hence *m* increases and therefore *L* increases & P_c decreases (see slides 33-35)
- CNO cycle very temperature (and density) sensitive and in HE and TE, star has to reduce the "weight" of the envelope layers → expansion of the envelope and lower Teff
- pp chain less sensitive to temperature and density and hence T_c increases more and envelope expands less than for CNO cycle



Luminosity evolution on the main sequence

- The hook for high mass stars occurs when H is exhausted in the convective core. The star is no longer in TE and will contract (virial) – red extreme point in track – until shell burning can start – blue end point of track.
- Low mass stars have radiative cores and energy generation in relatively large core: H is depleted gradually with a smooth transition to shell burning.



Luminosity evolution

Due to H-burning, the number of particles decreases in the core and rT has to increase (HE). The larger temperature gradient and the reduced (Kramer) opacity leads to a higher luminosity. We can estimate the luminosity change and its timescale:

$$\text{Virial: } T \propto \mu \frac{M}{R} \text{ or } T \propto \mu \rho^{1/3} M^{2/3}$$

$$\text{Radiative energy transport: } L \propto -\frac{R^2 T^3}{\kappa\rho}\frac{dT}{dr} \propto \frac{RT^4}{\kappa\rho}$$

$$\text{Kramers opacity: } L \propto \frac{RT^{15/2}}{\rho^2} \propto M^{16/3}\rho^{1/6}\mu^{15/2}$$

$$L(t) = L_0 \left(\frac{\mu(t)}{\mu_0}\right)^{\psi} \text{ with } \psi = 15/2$$

Luminosity evolution

$$\mu(t) \approx \left[\mu_e^{-1} + \mu_i^{-1}\right]^{-1} = \frac{4}{3+5X}$$

$$\frac{dX}{dt} \propto -\frac{L(t)}{MQ}$$ with Q energy released per gram ($6x10^{18}$ erg)

When we neglect the weak dependence on ρ, we have

$$\frac{dL}{dt} = \left(\frac{dL}{d\mu}\right)\left(\frac{d\mu}{dX}\right)\left(\frac{dX}{dt}\right) = \left[\frac{\psi L_0}{\mu_0}\mu^{\psi-1}\right]\left[\frac{5\mu^2}{4}\right]\left[\frac{L}{MQ}\right]$$

$$\frac{dL}{dt} = \frac{5\psi\mu_0}{4L_0^{1/\psi}MQ}L^{2+1/\psi}$$

$$L = L_0\left[1 - \frac{5}{4}(\psi+1)\frac{\mu_0 L_0}{MQ}t\right]^{-\psi/(\psi+1)}$$

Luminosity evolution

$$\mu_0 = 0.6 \text{ and } \psi = 15/2$$

$$\frac{L}{L_{sun}} = \frac{L_0}{L_{sun}}\left[1 - 0.3\frac{L_0}{L_{sun}}\frac{t}{\tau_{sun}}\right]^{-15/17}$$

With t_{sun} the age of the sun. So, at $t=t_{sun}$, $L= L_{sun}$ and hence $L_0=0.79L_{sun}$; ie., the sun is now brighter by ~25% than on the ZAMS.

g) Intermezzo: Solar neutrinos

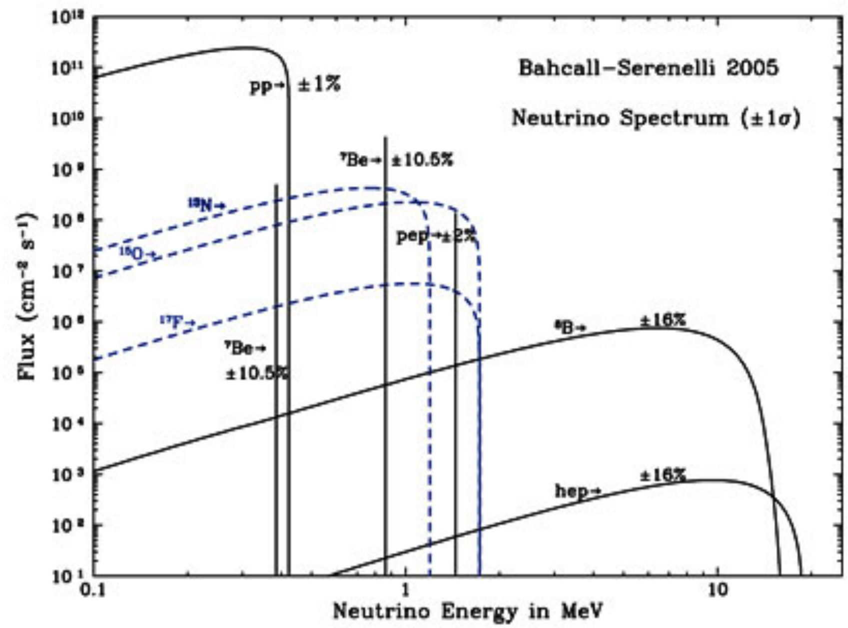
Gallium

Chlorine

SuperK, SNO

pp-chain produces neutrinos

Reaction	Spectrum
$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu_e$	continuous
$^7Be + e^- \rightarrow {}^7Li + \nu_e$	2 lines
$^8B \rightarrow {}^8Be + e^+ + \nu_e$	continuous
$^1H + {}^1H + e^- \rightarrow {}^2H + \nu_e$	line
$^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$	continuous
$^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$	continuous
$^3He + {}^1H \rightarrow {}^4He + e^+ + \nu_e$	continuous



Chlorine experiment

Use $^{37}Cl + \nu \rightarrow {}^{37}Ar + e^-$ as ‘detector’ (Davis 1965)

^{37}Ar is radioactive and decays to $^{37}Cl + \nu_e + e^+$ (half life 35 days)

Count number of radioactive Ar nuclei and you measure flux of high energy (^{8}B) neutrinos from the Sun

Results (robust for over 30 years)

Measured: 2.6 ± 0.2 SNU (1.3 x 10^{-36} captures per second/nucleus)

Model: 8.5 ± 1.8 SNU (Bahcall, 1965 onwards)

Discrepancy of about a factor 3: *Solar neutrino problem*

Many solutions proposed over the years

Reduce ^{4}He content ⇒ decrease T_c ⇒ fewer ^{8}B decays??

Sun not in TE ⇒ energy (and ν) generation too small??

….

Particle physics: neutrino oscillations occur if ν's have mass (ν_e's produced in Sun mixed over the 3 flavors ν_e, ν_μ, ν_τ in Sun, and on the way to Earth ⇒ flux reduced by factor 3)

Nineties: new experiments (Gallium, D_2O, …), sensitive to different energy ranges ⇒ different discrepancy

$^{71}Ga + \nu_e \rightarrow {}^{71}Ge + e^+$ Predict 131±11, measure 69±4

In 2004, the 35-year old problem was solved:

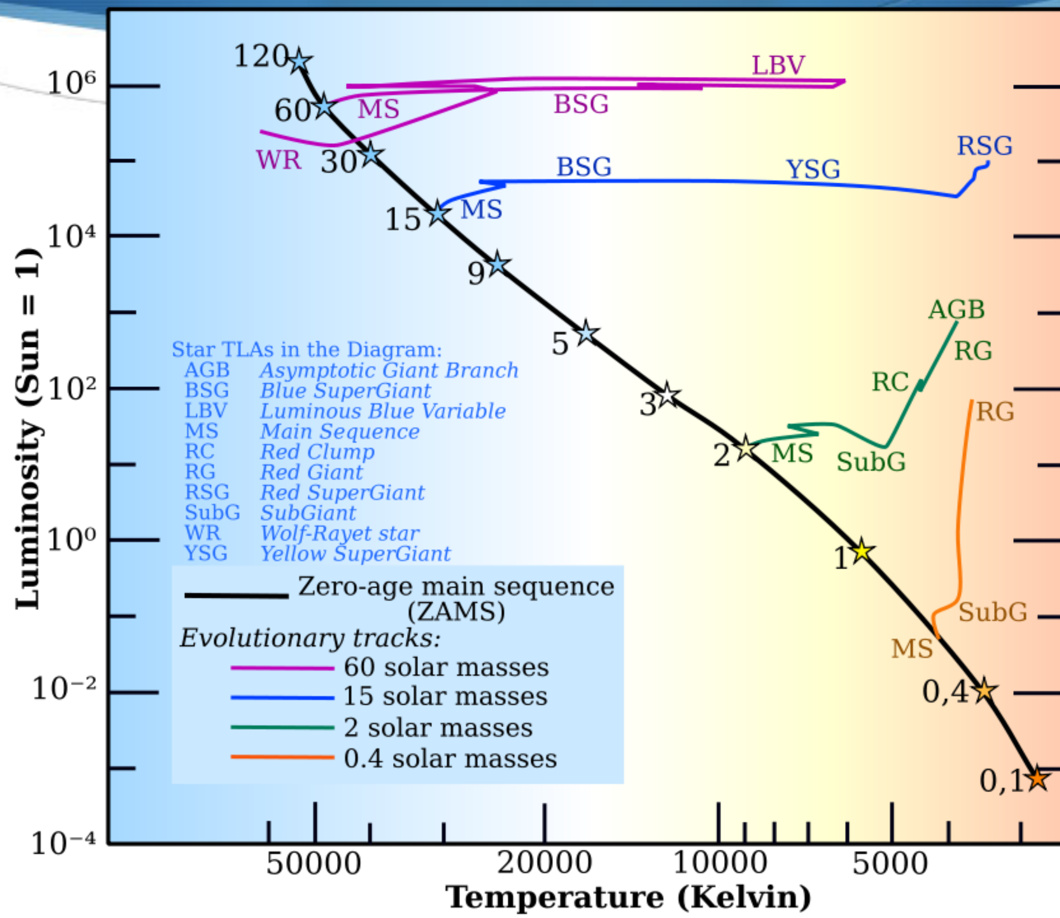
- Both Davis and Bahcall were right all along
  - Bahcall computed the right number of ν_e produced in Sun
  - Davis measured the right number of ν_e measured at Earth
- Set of experiments, including those at Sudbury Neutrino Observatory (heavy water) give: flux of ν_e is 0.340 ± 0.038 times flux of $\nu_e + \nu_\mu + \nu_\tau$ (which were all ν_e in Sun)
- This implies:
  - Neutrinos must have mass (and contribute to dark matter in Universe)
  - 'Oscillate' on the way from center of Sun to Earth
- Solar model is accurate, and fusion of light nuclei powers Sun
- Standard model of particle physics requires modification

Bahcall JN, Peña-Garay C, New Journ. of Phys. 6(2004) 63

Lecture 6-3:
Post-Main Sequence

Literature: KWW 32, 33, 34

Post-Main Sequence Evolutionary Tracks



a) Red giant phase

H-burning takes place in a shell feeding the (exhausted) He core. So, the core grows slowly in mass. No energy generation, core is isothermal and is pressure-supported by the density gradient. Note that an isothermal sphere corresponds to polytrope with index, $n=\infty$. The truncated isothermal sphere is described by the Bonnor-Ebert sphere (often encountered in astrophysics). For a given stellar mass, there is a limiting core mass – the Schönberg-Chandrasekhar mass – that can support the weight of the envelope. When the core mass exceeds this limit, it will start to contract rapidly.

a-1) Schonberg-Chandrasekhar mass

Start with the virial theorem for a finite core with a finite external pressure, P_s :

$$q\frac{GM_c^2}{R_c} = 3\int_0^{V_c} P\,dV = \frac{k}{\mu_c m_u}T_c M_c - 3P_s V_c$$

$$P_s\left(R_c\right) = \frac{3}{4\pi}\frac{kT_c}{\mu_c m_u}\frac{M_c}{R_c^3} - \frac{qG}{4\pi}\frac{M_c^2}{R_c^4}$$

For $\mathrm{P}_s = 0,\ R_0 = \dfrac{qGm_u}{3k}\dfrac{\mu_c M_c}{T_c}$

The maximum pressure corresponds to a radius $(dP_s / dR_c = 0)$,

$R_1 = \dfrac{4qGm_u}{9k}\dfrac{\mu_c M_c}{T_c}$ and is $P_{s,\max}\left(M_c\right) = C_1\dfrac{T_c^4}{\mu_c^4 M_c^2}$

with $C_1 = \dfrac{2187}{16\pi}\left(\dfrac{k}{m_u}\right)^4\left(4qG\right)^{-3}$

So the maximum surface pressure of the core decreases strongly with the mass.

From the virial theorem, the pressure of the envelope, $P_{env} \approx GM^2 / 8\pi R^4$ (lecture 2 slide 16), and this pressure cannot exceed this maximum pressure.

$$P_{env} = \frac{GM^2}{8\pi R^4} = P_{s,\max}\left(M_c\right) = C_1\frac{T_c^4}{\mu_c^4 M_c^2}$$

Also, of course, we have $T_c = T_{env} = \dfrac{P_{s,\max}\mu_{env}m_u}{k\rho_{env}} = \dfrac{\mu_{env}m_u}{k\rho_{env}}C_1\dfrac{T_c^4}{\mu_c^4 M_c^2}$

a-1) continued

Or,

$$T_c^3 = \frac{k}{C_1 m_u} \frac{\mu_c^4 M_c^2 \rho_{env}}{\mu_{env}}$$

with $\rho_{env} \approx 3M / 4\pi R^3$

$$T_c^3 = \frac{3k}{4\pi C_1 m_u} \frac{\mu_c^4 M_c^2 M}{\mu_{env} R^3}$$

plugging this back into the stability equation, we have

$$\frac{GM^2}{8\pi R^4} = \frac{C_1}{\mu_c^4 M_c^2} \left(\frac{3k}{4\pi C_1 m_u} \frac{\mu_c^4 M_c^2 M}{\mu_{env} R^3} \right)^{4/3}$$

So, stability is only possible for a core mass fraction less than a critical value:

$$q_{sc} \equiv \frac{M_c}{M} \leq \sqrt{\frac{27}{2048 q^3}} \left(\frac{\mu_{env}}{\mu_c} \right)^2$$

A more careful analysis yields:

$$q_{sc} \leq 0.37 \left(\frac{\mu_{env}}{\mu_c} \right)^2 \text{ with } \mu_{env} \approx 0.6 \text{ and } \mu_c \approx 4/3\text{: } q_{sc} \approx 0.1$$

Stellar evolution depends on q_{core}:
$q_{core} < q_{sc}$: isothermal core can support weight of the envelope.
$q_{core} > q_{sc}$: isothermal core cannot support weight of the envelope. Core must contract, which releases gravitational energy and heats up the core. Core is no longer isothermal.

Stars more massive than 2 M_o will have convective cores on the main sequence more massive than the Schonberg-Chandrasekhar limit and their cores will start contracting after H-exhaustion. The core will start collapsing, develop a temperature gradient and (virial theorem) contraction and heating up will continue on a Kelvin Helmholtz timescale (the hook at the start of the RGB phase of the stellar tracks on lecture 6-2 slide 32).

Let’s first consider stars with $\underline{M > 6M}_{\odot}$

$q_{core} > q_{SC}$ at ignition of H-burning shell

$\boxed{?}$ core cannot become isothermal

$\boxed{?}$ continuing gravitational contraction on $\boxed{?}_{KH}$

$\boxed{?}$ T_c rises until He ignites at $\sim 10^8$ K

For stars in the mass range, $\underline{2.5\ M_{\odot}}\ \underline{< M < 6\ M_{\odot}}$,

$q_{core} < q_{SC}$ at ignition of H-burning shell

$\boxed{?}$ isothermal core develops with T_c ~ T(H-burning shell)

As H is burned, q_{core} steadily increases and R_c decreases slightly so that $\boxed{?}_c$ increases and partial degeneracy increases.

But $q_{core} > q_{SC}$ before core becomes fully degenerate

$\boxed{?}$ rapid core contraction until He ignites

For low mass stars ($M < 2.5M_\odot$)

Core degenerates before $q_{core} > q_{SC}$ $\boxed{?}$ q_{SC} does not apply

Degeneracy pressure allows q_{core} to become very large and remain in thermal equilibrium

As q_{core} increases, core contracts slightly and T_c rises slowly

Depending on stellar mass, we can distinguish the following scenarios:

1) $M < 0.33\ M_\odot$

T_c never exceeds 10^8 K

H shell continues to burn outwards

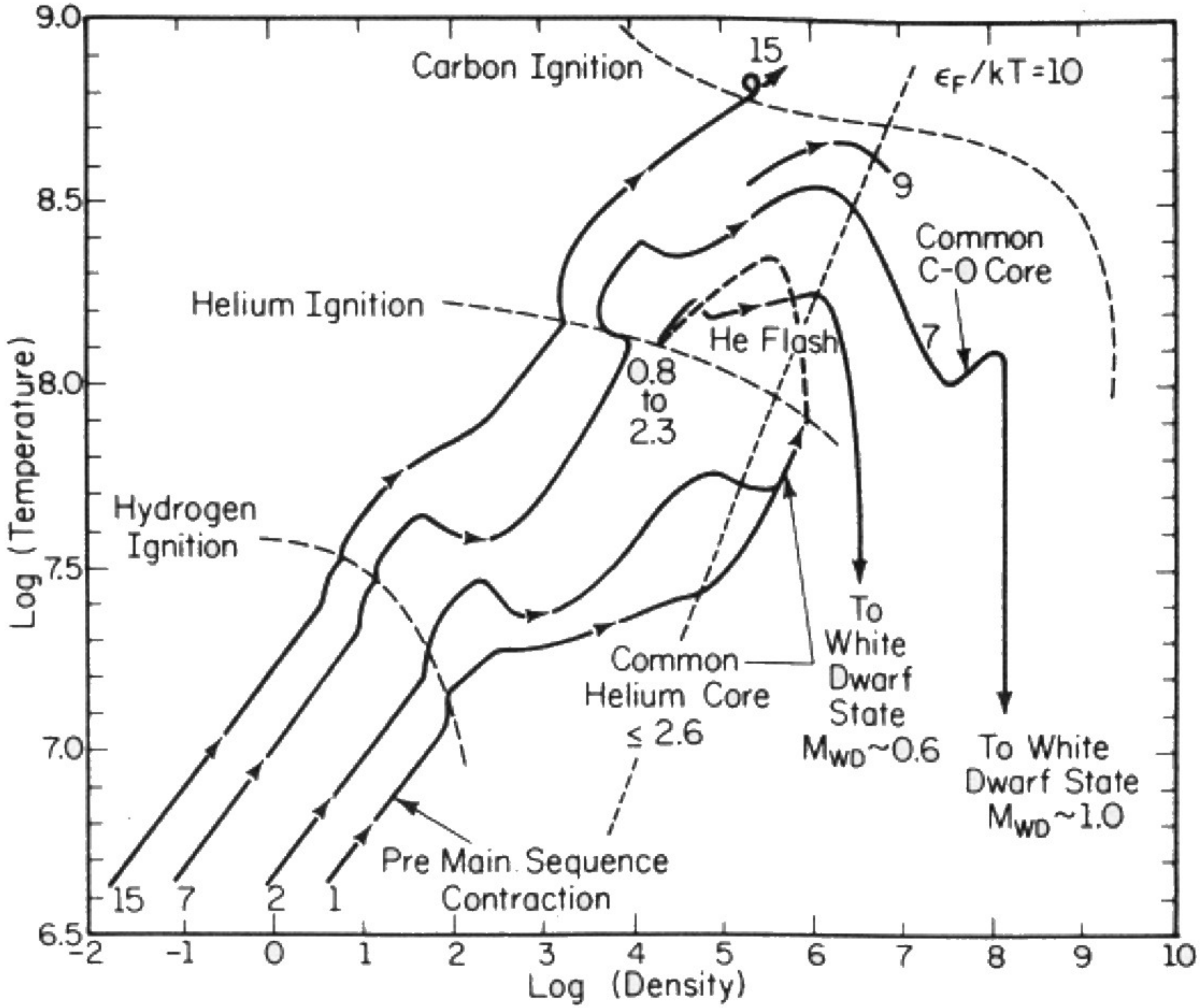
Result is a degenerate star composed of He: He white dwarf

2) $0.33\ M_\odot < M < 2.5M_\odot$

Cores all evolve to about the same degenerate state

When T_c exceeds 10^8K, He ignites under degenerate conditions, leading to the *helium flash*

a-3) Evolution of the core in the log T_c – log ?$_c$ plane



a-4) The envelope

Hydrostatic and thermal equilibrium lead to the mirror principle for stars which derive their energy from shell burning:

Nuclear burning acts as a thermostat keeping the temperature of the shell constant. Contraction would lead to heating and so the shell will tend to remain at constant radius. Hence when the core contracts, the density of the shell – and therefore the pressure – will drop. Thus, the pressure of the overlying envelope must decrease. The envelope will expand. The converse holds true when the core expands.

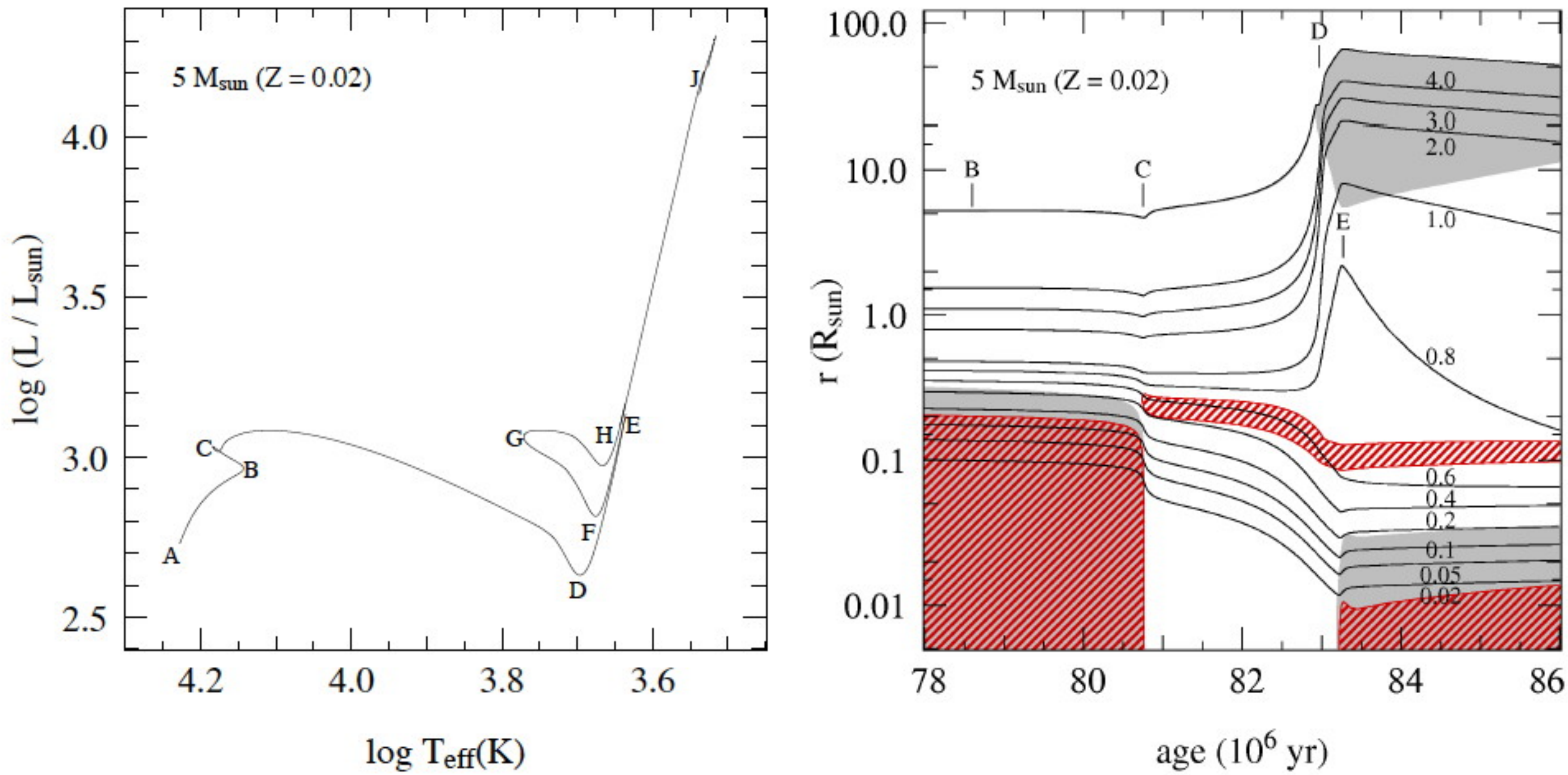
- Core contraction → envelope expansion
- Core expansion → envelope contraction

In this, keep also in mind that when the luminosity is largely generated by nuclear burning, the total stellar gravitational and thermal energy are conserved. Core contraction has to be compensated by envelope expansion !

As the envelope expands, the effective temperature decreases and the star moves to the right in the HR diagram. This changes when the star reaches the Hayashi limit. The star is fully convective and in Hydrostatic and Thermal Equilibrium. The star will then have to move up along the Hayashi track, increasing in luminosity at constant effective temperature: The Red Giant Branch. Note that this is set by the H^- opacity in the envelope (slide 10-13 Ch 5-3)

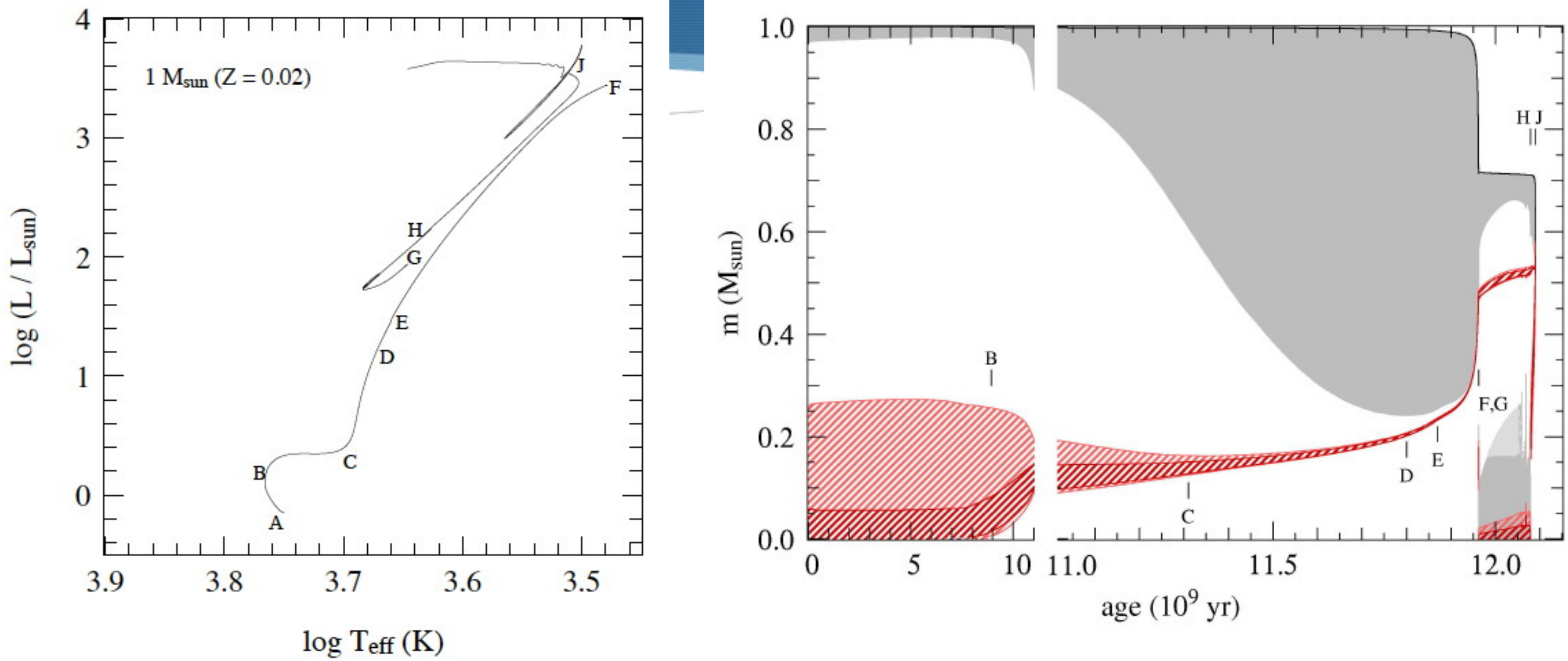
These points are readily recognized in the evolution of the structure of a $5M_o$ star on the next slide.

a-4-1) Structure & evolution of 5 M_o star



Left: Evolutionary track in the HR diagram. Right: Radial variations in the star's structure. Letters correspond to the HR phases in the left diagram. Curves are labeled by their mass in M_o. Red hashed indicate energy generation zones. Gray hashed indicates convection zones.

a-4-2) Structure & evolution of 1 M_o star



Left: Evolutionary track in the HR diagram. Right: Radial variations in the star's structure. Letters correspond to the HR phases in the left diagramRed hashed indicate energy generation zones. Gray hashed indicates convection zones

Structure & evolution of 1 M_o star

Low mass stars have degenerate cores and remain in Thermal Equilibrium with an isothermal core. In contrast to a $5M_o$ star, evolution will be on nuclear not Kelvin-Helmholtz timescale.

Evolution of low mass stars along the RGB is almost independent of stellar mass. The envelope exerts so little pressure that the degenerate He core sets the structure; e.g.,

$$\frac{dP}{dm} = -\frac{GM_c}{4\pi R_c^4} << 0$$

If the core were fully degenerate, the core mass and radius would scale as one over the core radius cubed, $M_c \propto R_c^{-3}$ (see the white dwarf discussion, Lecture 5-3, slide 21). However, actually, the core top layer is non-degenerate and more extended. Analysis yields, $M_c \propto R_c^{-6}$

Following the Eddington standard model approach (Lecture 5-3, slides 3-6), we can derive a mass-luminosity relation. Consider a He-degenerate core surrounded by a H-burning shell (sole energy source) and a tenuous, convective envelope. The shell and convective envelope are separated by a thin radiative zone with a simple structure in which the temperature and density drop by many orders of magnitude. We will assume that $M_r \sim M_c$ and $L_r = L$.

$$\frac{dP}{dr} = \frac{GM_c}{r^2}\rho \qquad \& \qquad \frac{d\left(aT^4/3\right)}{dr} = -\frac{\kappa L}{4\pi c r^2}\rho$$

$$P_{rad} = \left(1-\beta\right)P = \frac{\kappa L}{4\pi c G M_c}P + cst \approx \frac{L}{L_{edd,c}}P$$

where we realize that the integration constant is small except near the base of the convective zone.

$$L_{edd,c} = \frac{4\pi c G M_c}{\kappa} \approx \frac{65000}{1+X}\left(\frac{M_c}{\mathrm{M_o}}\right) \quad \mathrm{L_o}$$

and $L = (1-\beta) L_{edd,c}$

Because of shell burning, the core mass will increase, leading to an increase in luminosity. The core temperature and the luminosity can then be evaluated as follows:

The equation of state can be rewritten to relate the density and temperature through *b*.

$$P = \frac{k}{\mu m_u}\rho T + \frac{a}{3}T^4 \Rightarrow \rho = \frac{a\mu m_u}{3k}\frac{\beta}{1-\beta}T^3$$

$$\frac{dT}{dr} = -\frac{\mu m_u}{4k}\frac{\kappa L}{4\pi c r^2}\frac{\beta}{1-\beta} = -\frac{\mu m_u}{4k}\frac{GM_c}{r^2}$$

Integration leads to,

$$T = \frac{\mu m_u}{4k}\frac{GM_c}{r} + cst \approx \frac{\mu m_u}{4k}\frac{GM_c}{r}$$

and we wind up with:

$$T \approx 5.8\times 10^6 \beta\mu \frac{M_c}{M_0}\frac{R_o}{r} \quad \text{K} \quad \& \quad \rho \approx 6\times 10^{-3}\frac{\beta^4}{1-\beta}\mu^4\left(\frac{M_c}{M_0}\right)^3\left(\frac{R_o}{r}\right)^3 \quad \text{g/cm}^3$$

The luminosity can now be calculated using CNO burning:

$$L = \int_{r_{sh}}^{\infty} 4\pi r^2 \rho \varepsilon \, dr \approx 4\pi \varepsilon_0 \int_{r_{sh}}^{\infty} r^2 \rho^2 T^{\nu} \, dr$$

$$L \propto \frac{\beta^{\nu+8}}{\left(1-\beta\right)^2} M_c^{\nu+6} \int_{r_{sh}}^{\infty} r^{-\nu-4} \, dr \propto \frac{\beta^{\nu+8}}{\left(1-\beta\right)^2} M_c^{\nu+6} r_{sh}^{-\nu-3}$$

Where we realize that the shell is very thin:

$$\varepsilon\left(2r_{sh}\right) / \varepsilon\left(r_{sh}\right) \approx 10^{-6}$$

We now have:

$$\left(1-\beta\right)M_c \propto \frac{\beta^{\nu+8}}{\left(1-\beta\right)^2} M_c^{\nu+6} r_{sh}^{-\nu-3}$$

$$\frac{\left(1-\beta\right)^3}{\beta^{\nu+8}} \propto M_c^{(7\nu+39)/6}$$

with ν=13, we find, $L=\left(1-\beta\right)M_c \sim M_c^{8.2}$

Numerical studies reveal:

$$L \approx 125\left(\frac{M_c}{0.3\mathrm{M}_\mathrm{o}}\right)^7 \ \mathrm{L}_o \ \text{ for } \ 0.3<\frac{M_c}{\mathrm{M}_\mathrm{o}}<0.45$$

with slighly different fitting values for the range 0.15 to 0.3 M_o

a-5) First dredge up

On the RGB, the envelope will become convective, reaching down to zones which contain hydrogen burning products and this enriched material is mixed to the surface. This first dredge up changes the CNO abundances:

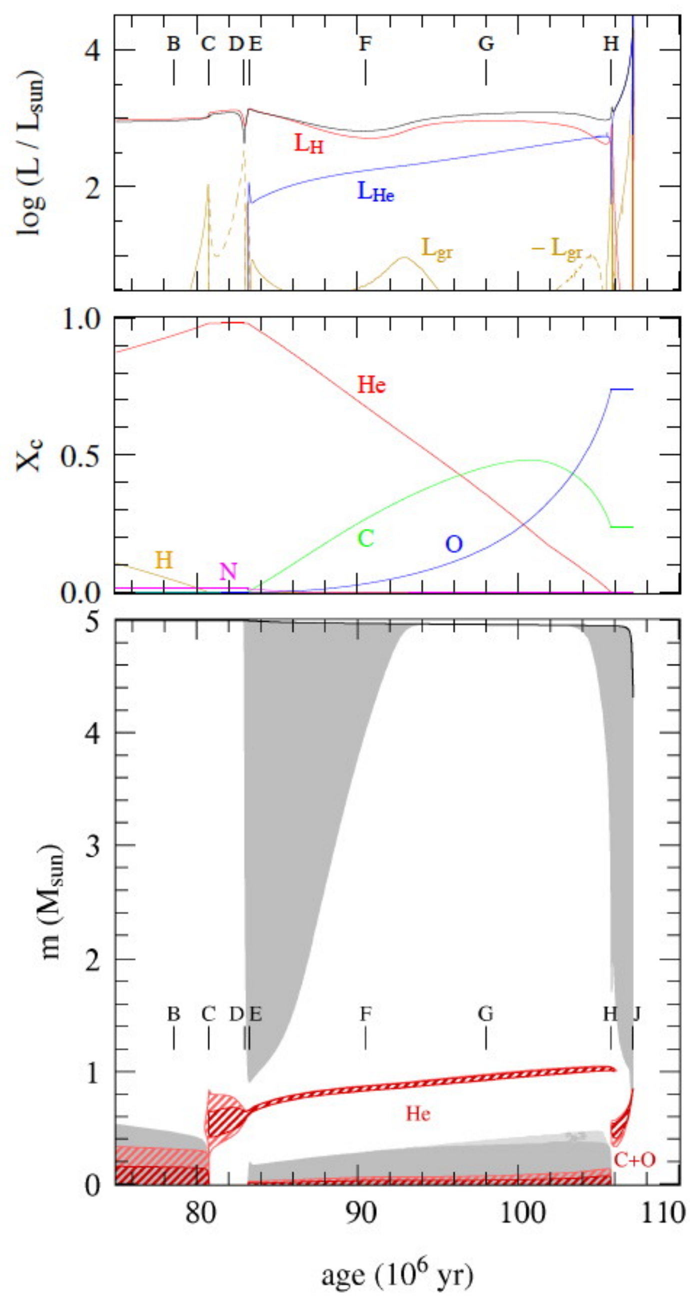
$$CNO = 1/2 : 1/6 : 1 \text{ initial to } CNO = 1/3 : 1/3 : 1 \text{ after dredge up}$$

b) Core Helium burning

As the temperature in the core approaches 10^8 K, He ignites. For a massive star ($>2.5M_o$), this occurs in point E (slide 51). This is a stable – thermostated – phase of burning where an increase in *T* leads to an increase in *P* and expansion and this decreases *T*. The 3a process is very temperature sensitive, therefore, concentrated towards the center of a highly convective core. Eventually when He is mostly burned to ^{12}C, $^{12}C(a,g)^{16}O$ takes over (Lecture 4-3, slide 13).

Note that most of the luminosity actually comes from the H-burning shell. The stellar structure adjusts, the core expands somewhat and the envelope shrinks. When the envelope becomes radiative, the star will leave the Hayashi track, and move to the left in the HR diagram.

Evolution of a $5M_o$ star from the RGB to the AGB (see also slide 51).
Top: The different contributions to the luminosity. Letters refer to the stellar tracks of slide 51 (dashed lines are negative contributions)
Middle: Central abundances.
Bottom: Stellar structure. Red hash are nuclear burning. Gray hash are convection zones

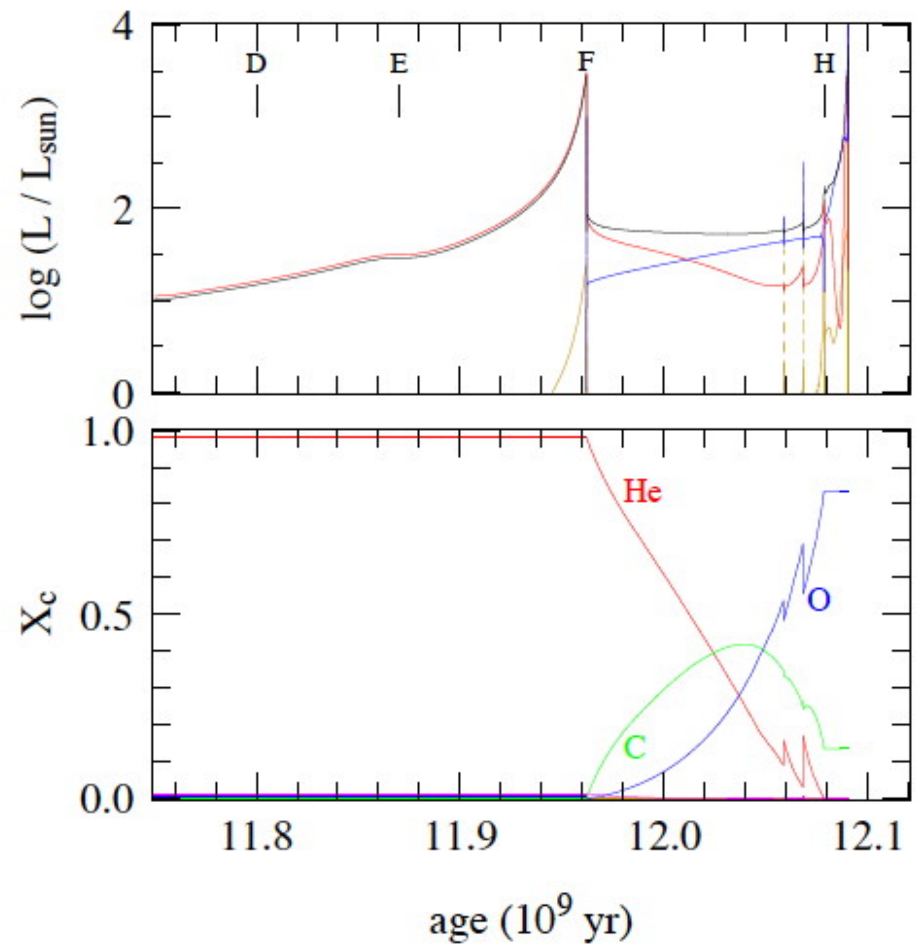


For a low mass ($<2.5M_o$), the core is supported by an degenerate electron gas at He-ignition. The increased energy generation leads to an increased temperature (of the ions) but has no influence on the pressure (electrons). This leads to a thermonuclear runaway: a He-bomb which burns some 40% of the helium core in a few seconds: The He flash. The resulting energy corresponds to $\sim10^{11}$ L_o – equal to the energy of a galaxy – but is absorbed by the envelope. The increased core temperature will lift degeneracy and the runaway ends. Models show a sequence of minor He flashes before degeneracy is completely lifted. As the core expands, the envelope contracts (see slide 49). The star will settle on the Horizontal Branch with a core burning He surrounded by a reignited H-burning shell.
PS: HB of globular clusters indicates masses of 0.7 M_o but these stars should still be on the MS → mass loss at the top of the RGB

Evolution of a $1M_o$ star from the RGB to the AGB (see also slide 52).

Top: The different contributions to the luminosity. Letters refer to the stellar tracks of slide 52 (dashed lines are negative contributions)

Bottom: Central abundances.



c) Helium shell burning

The AGB phase starts when when He is exhausted in the core (point H in slides 51/52). The CO core will contract heating the star until He burning ignites in a shell. At this point, the CO core is surrounded by a He burning shell, a quiescent He shell, a H-burning shell, and an envelope. As the core contracts, the He shell expands (mirror effect, slide 49), the envelope contracts (mirror effect, slide 49) but T of H-burning shell decreases until H-shell extinguishes. Now the He + H envelope expands.

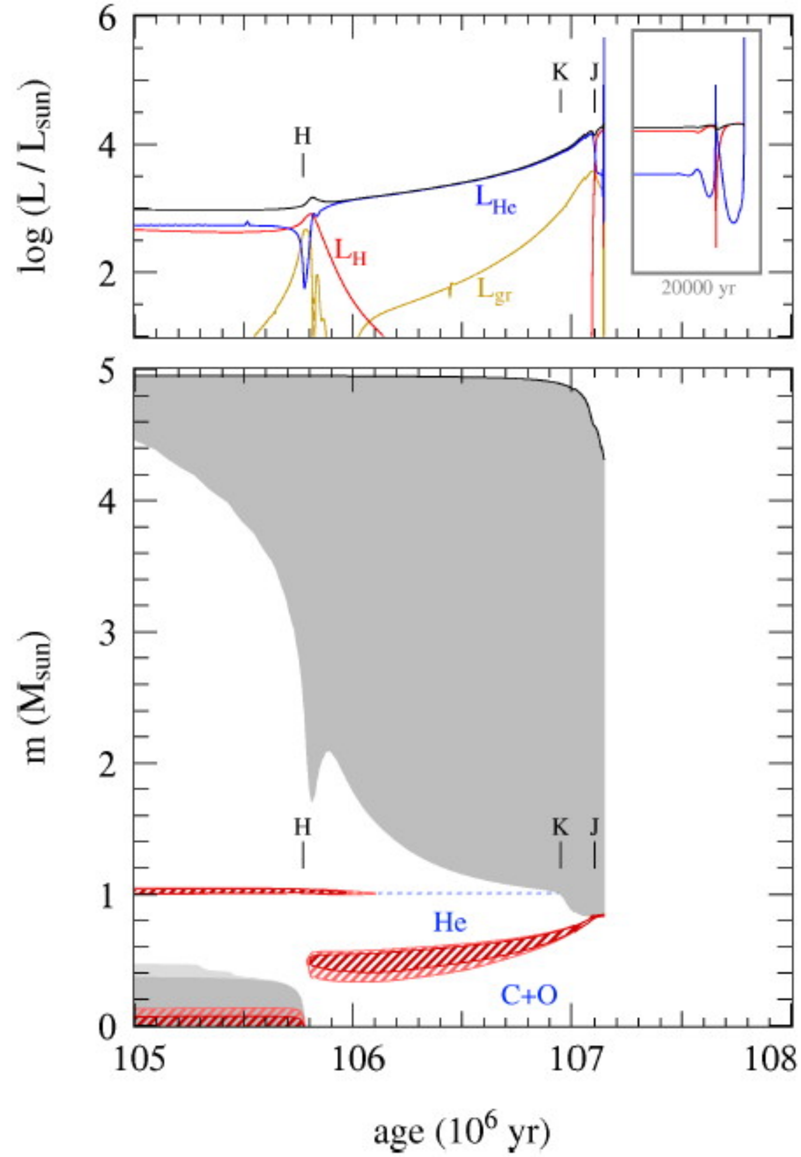
c-1) Second dredge up

In massive stars (>4M_o), as the envelope expands and cools, the convective envelope penetrates into the He-rich layers, once again changing the surface abundances:

$$CNO = 1/3 : 1/3 : 1 \text{ after } 1^{st} \text{ dredge up}$$

$$CNO = 0.34 : 0.58 : 1 \text{ after } 2^{nd} \text{ dredge up}$$

Evolution of a $5M_o$ star from the RGB through the AGB (see also slide 51).
<u>Top</u>: The different contributions to the luminosity. Letters refer to the stellar tracks of slide 51 (dashed lines are negative contributions)
<u>Bottom</u>: Stellar structure. Red hash regions are nuclear burning zones. Gray hash regions are convective zones.



c-1) Thermal pulses

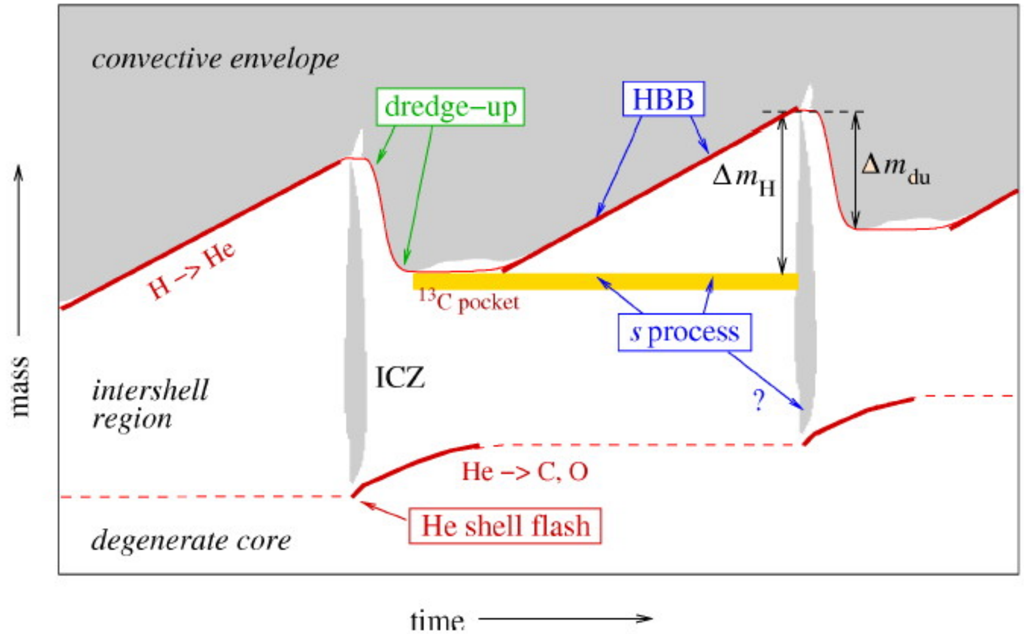
When the He shell runs out of fuel, the luminosity decreases, the envelope contracts, and reignites H-burning in a shell. As H-burning and He-burning do not run in step, the He-burning shell will undergo thermal pulses (see schematic figure on slide 67). Most of the time the He shell is inactive but as the He shell mass increases due to H-shell burning, the pressure and temperature of the He shell increase and He ignites.

For a thin shell this is unstable and leads to a thermal runaway – a helium shell flash – as the fractional drop in density is larger than the fractional drop in pressure and the temperature has to go up to maintain HE:

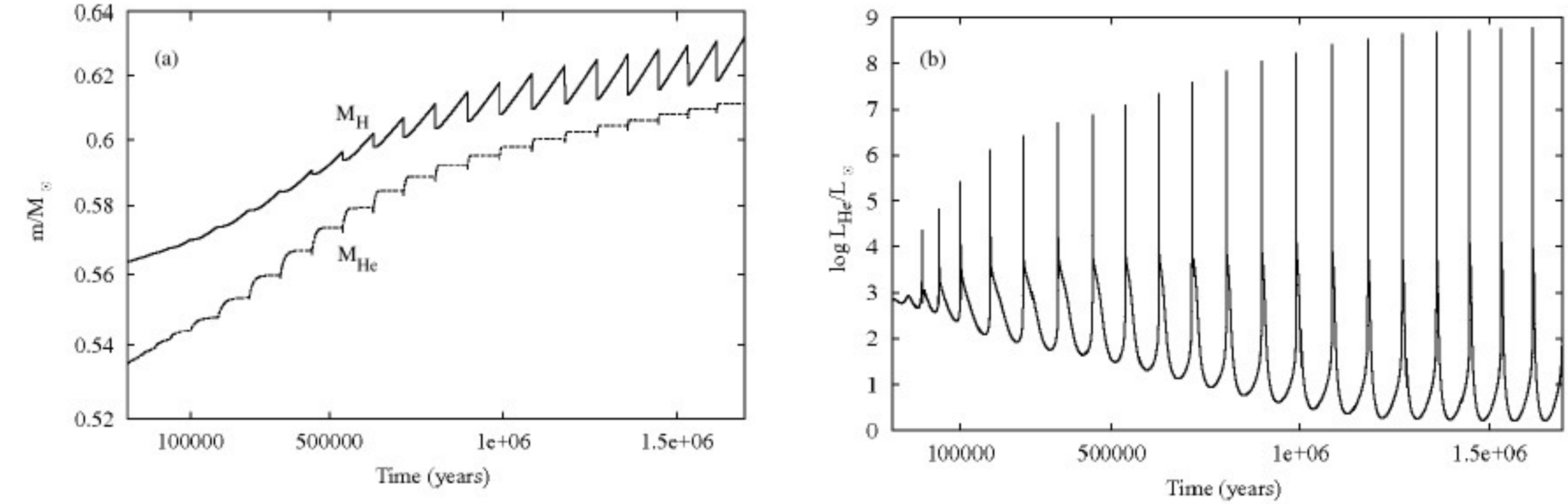
Hydrostatic equilibrium: $\frac{dP}{P} = -4\frac{dr}{r}$

Shell mass conservation: $\frac{d\rho}{\rho} = -\frac{dl}{l} = -\frac{dr}{l} = -\frac{dr}{r}\frac{r}{l}$

ideal gas: $\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$ and thus $\frac{dT}{T} = \left(\frac{4l}{r} - 1\right)\frac{d\rho}{\rho}$



The resulting high luminosity (10^8 L_o for 1 yr) leads to convection and mixing of the intershell. This energy is used to expand the He shell and eventually quenches the He flash and stable He burning ensues while the H-shell extinguishes. The outer convective envelope can penetrate through the (extinct) H-burning shell into the intershell, mixing nuclear products to the surface (3rd dredge up). Eventually H ignites again and the He shell extinguishes, starting a new cycle.



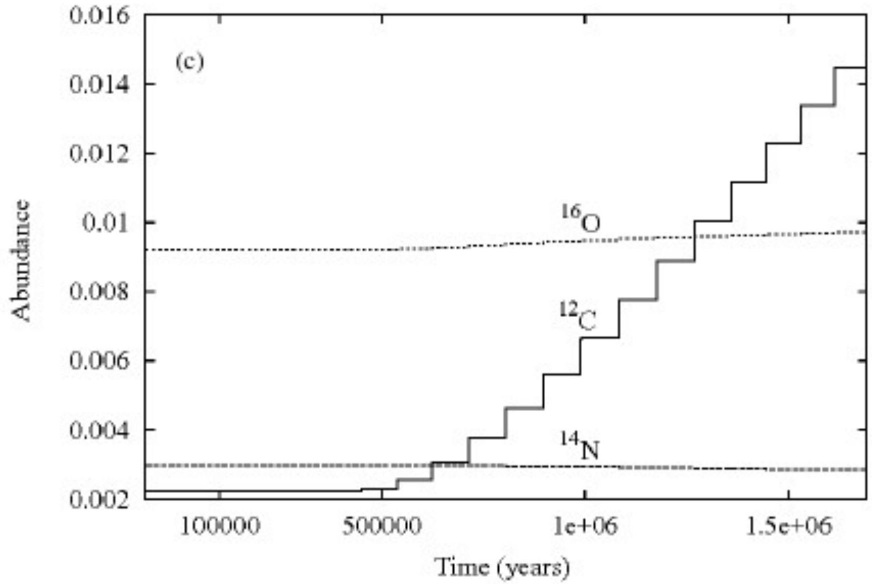
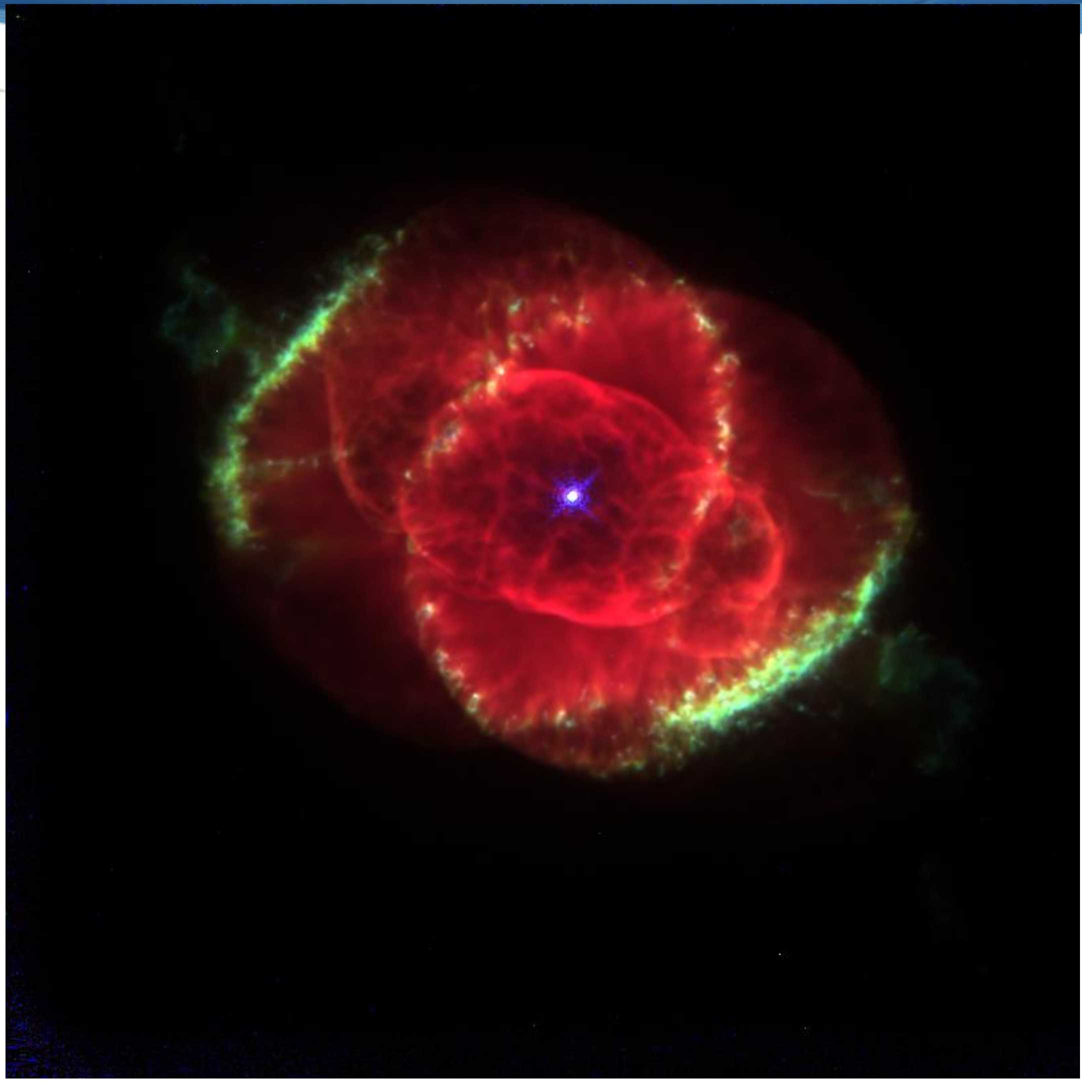


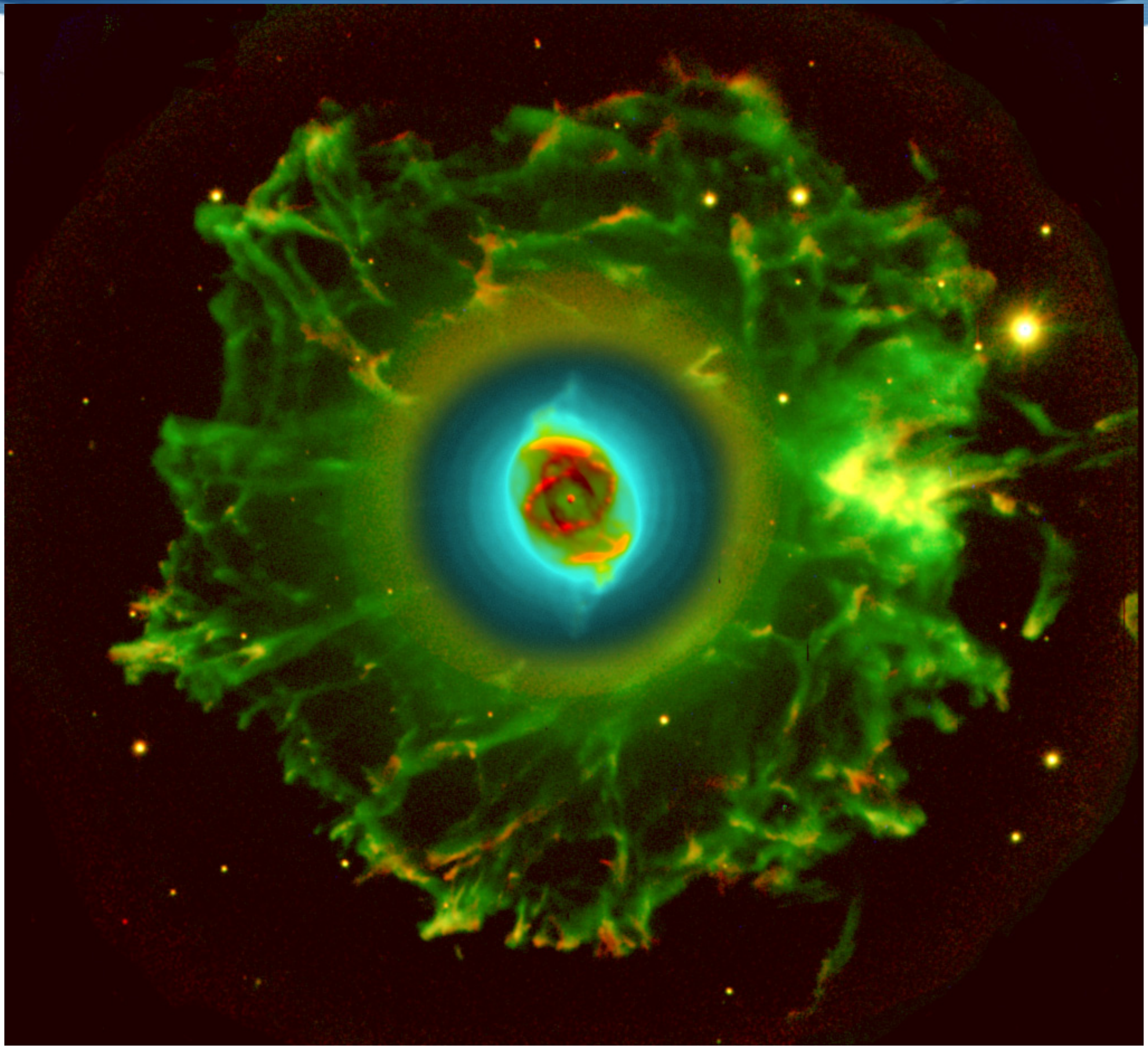
Figure 10.4. Evolution of a $3\,M_{\odot}$ star with $X = 0.7, Z = 0.02$ during the TP-AGB phase. Time is counted since the first thermal pulse. The three panels show (a) the growth of the hydrogen-exhausted core mass and helium-exhausted core mass, (b) the He-burning luminosity and (c) the changes in surface abundances by mass fraction of ^{12}C, ^{14}N and ^{16}O. Except for the first few pulses, each thermal pulse is followed by a dredge-up episode (sudden drop in core mass) and a sudden increase in ^{12}C abundance. Figure adapted from Stancliffe et al. (2004).

d) Evolution beyond the AGB

Evolution is now controlled by mass loss. For intermediate mass stars, the H-envelope is rapidly removed during the Mira phase on the AGB. The details are not well understood but involves stellar pulsations transporting material to great heights above the photosphere where small dust particles can nucleate and grow. Radiation pressure on the dust pushes them out and the gas is dragged along, forming a low velocity (10-20km/s) wind with mass loss rates of 10^{-6}–10^{-4} M_o/yr. Eventually, for stars with $M<8M_o$, a remnant white dwarf is left embedded in the AGB ejecta. If the transition to the WD stage is rapid, the ejecta are close enough to be ionized and set to glow. Very low mass stars do not make it to a CO white dwarf state (slide 47). We have discussed the physics and evolution of white dwarf stars in lecture 5-3.



Cat’s eye



Cat’s eye

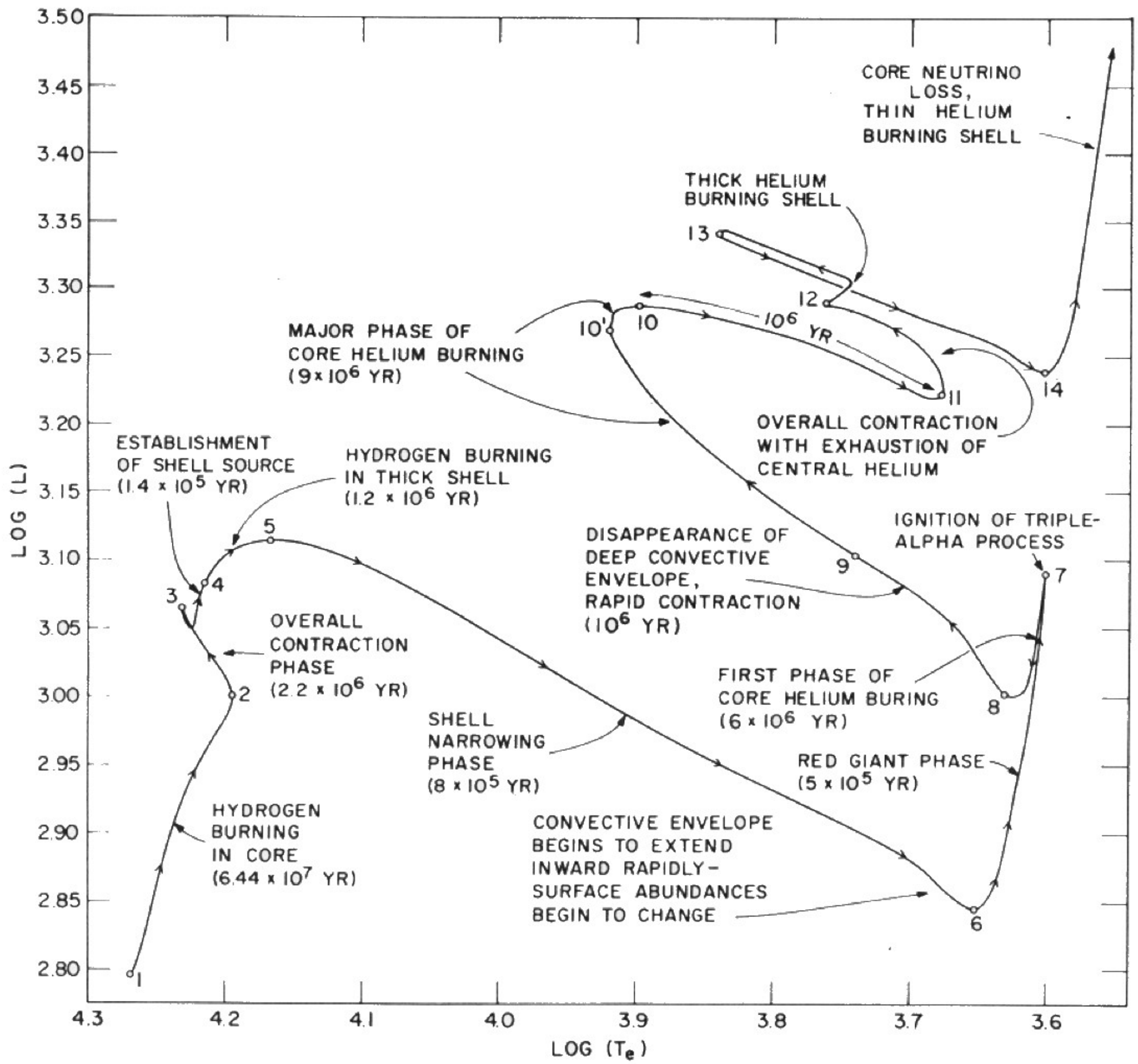
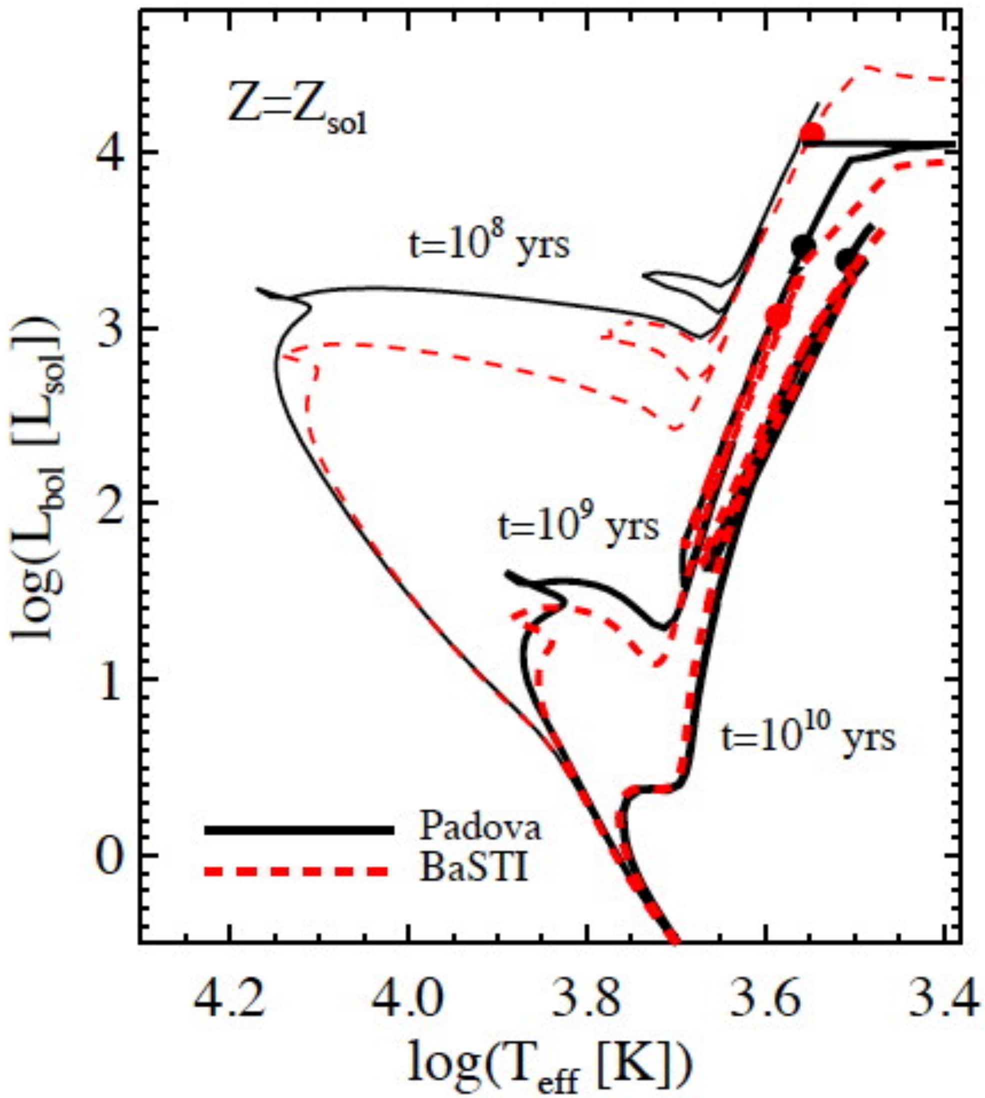


FIG. 2.—The track in the H-R diagram of a theoretical model star of mass 5 $M_\odot$ and of Population I composition. Text beside various portions of the track describe an important physical process occurring within the star at the indicated position. From Iben (1967c).

Comparison of tracks from two often used stellar evolution model codes illustrating the limits of our understanding.



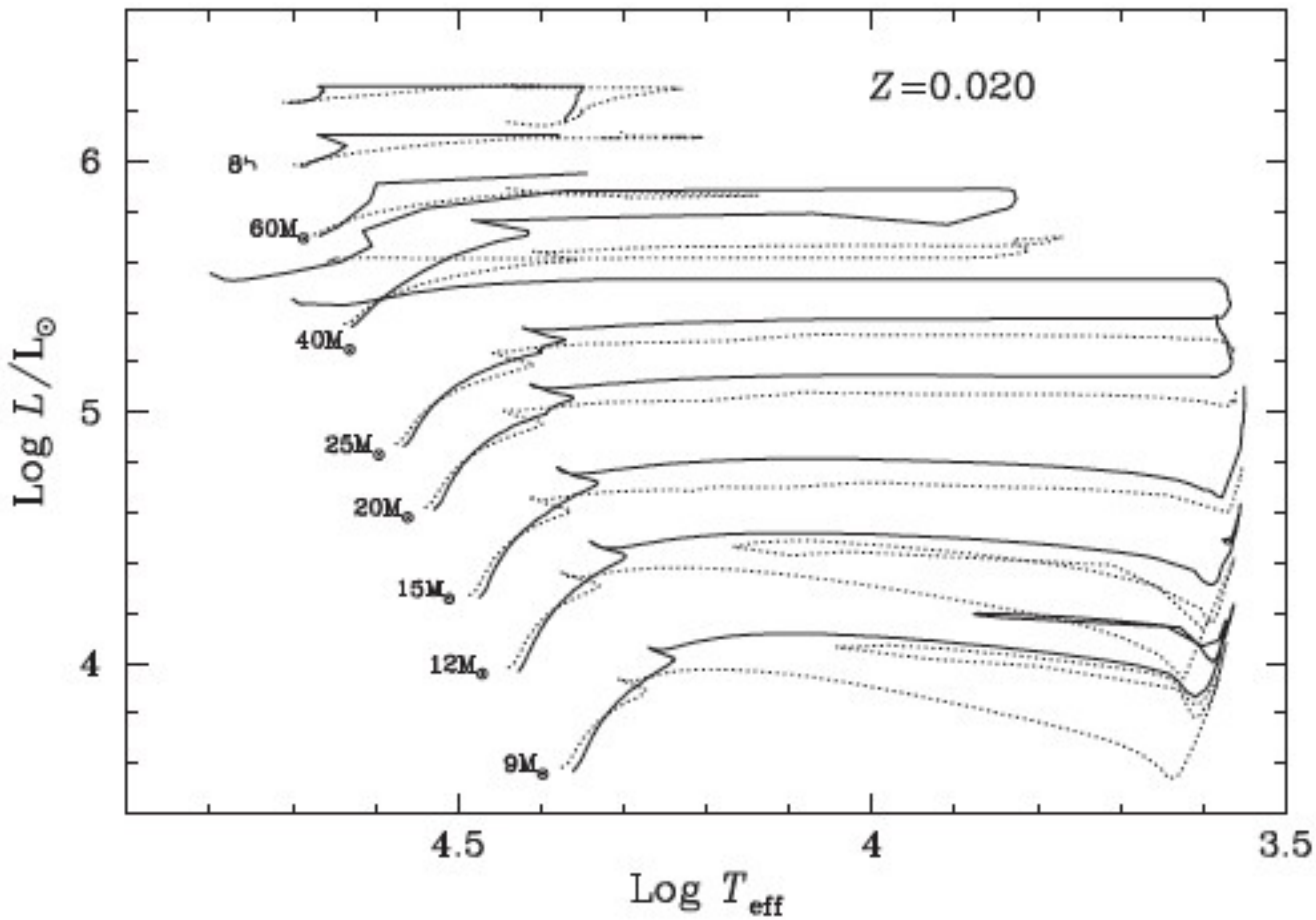
e) Evolution of massive stars

Massive stars (>8M_o) will not develop degenerate cores and can evolve further going through sequential stages of Ne, O, and Si burning before running out of fuel with an iron core. Very massive stars may run into the pair instability first. In either case, a SN results.

The details are very uncertain as mass loss dominates the evolution of these stars and the details of mass loss are not fully understood. Empirical mass loss rates have been derived but those are very uncertain (factor 3) due to unknown clumping.

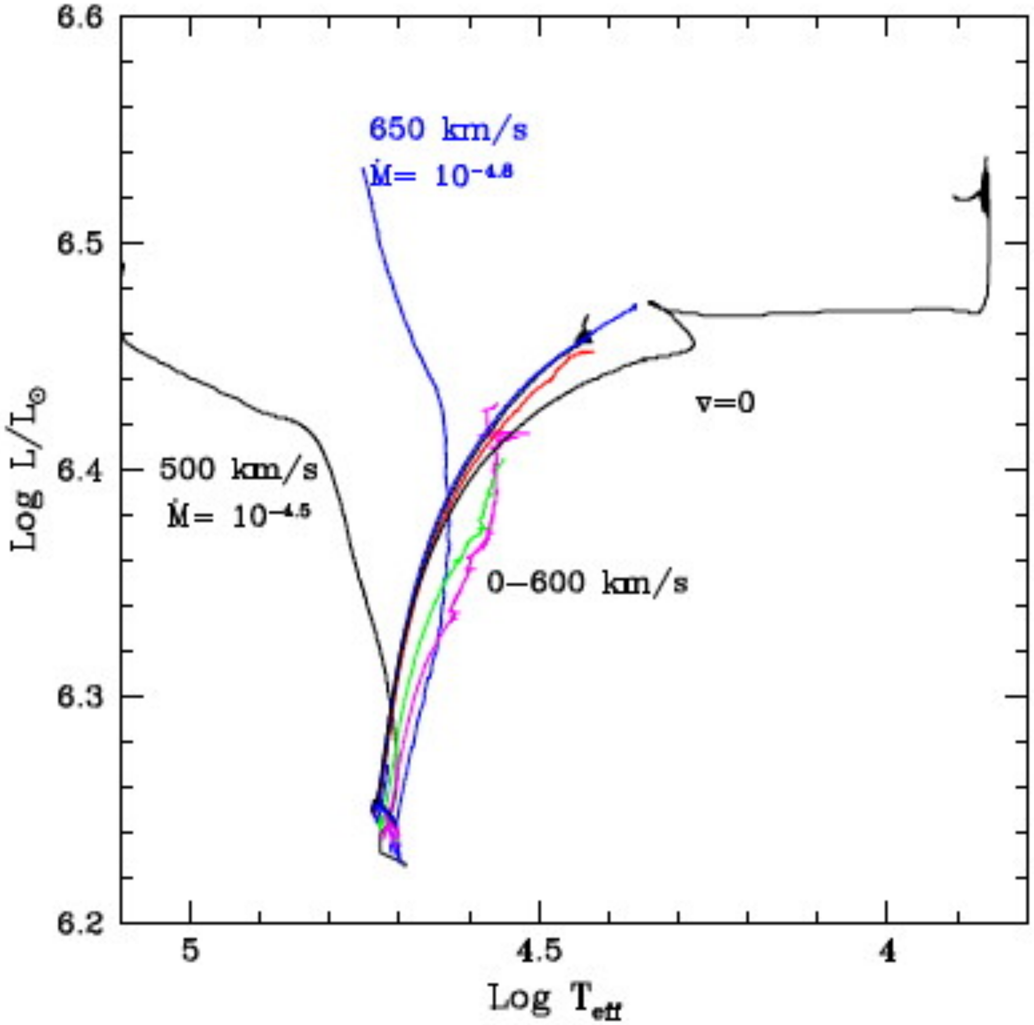
In addition, rotation leads to mixing and that can prolong burning stages.

e-1) The effects of rotation



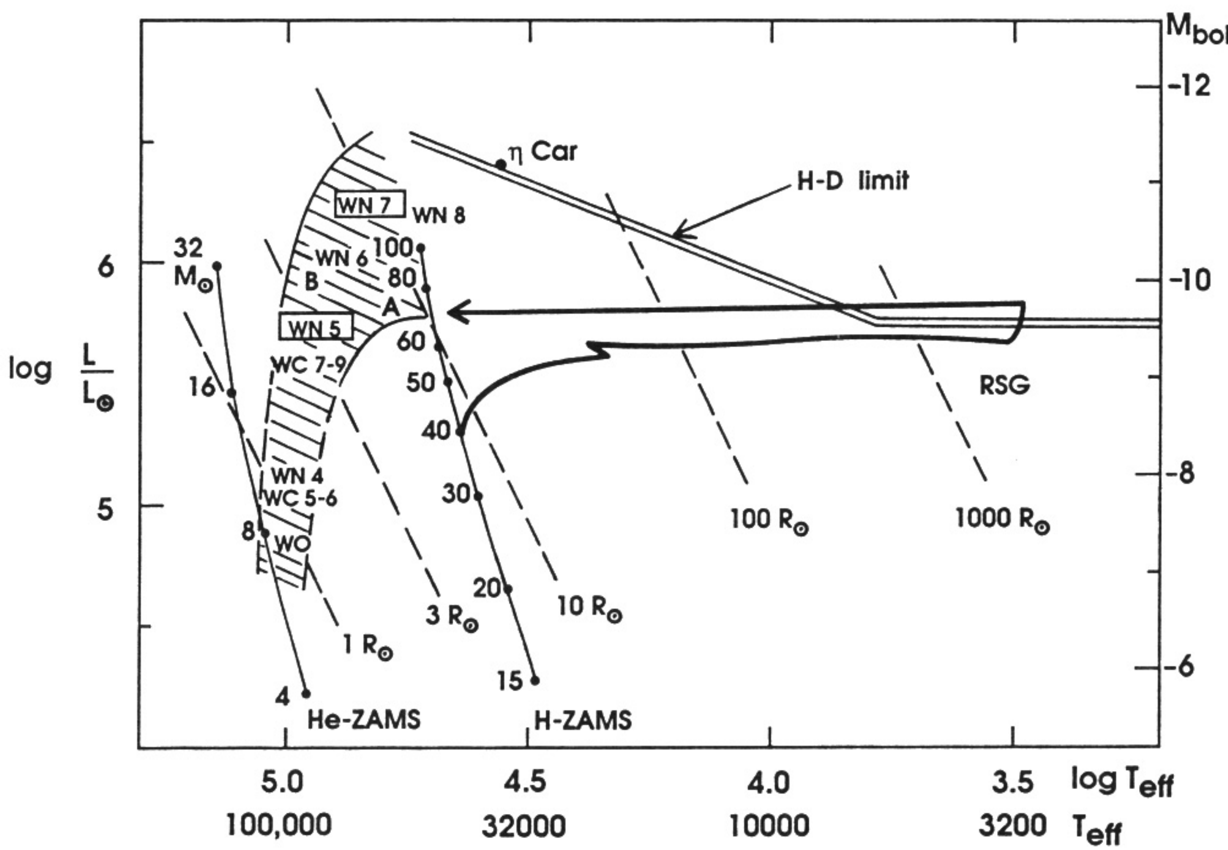
Solid lines: rotating stars with initial velocity 300 km/s
Dotted lines: no rotation

e-2) the effects of rotation and mass loss



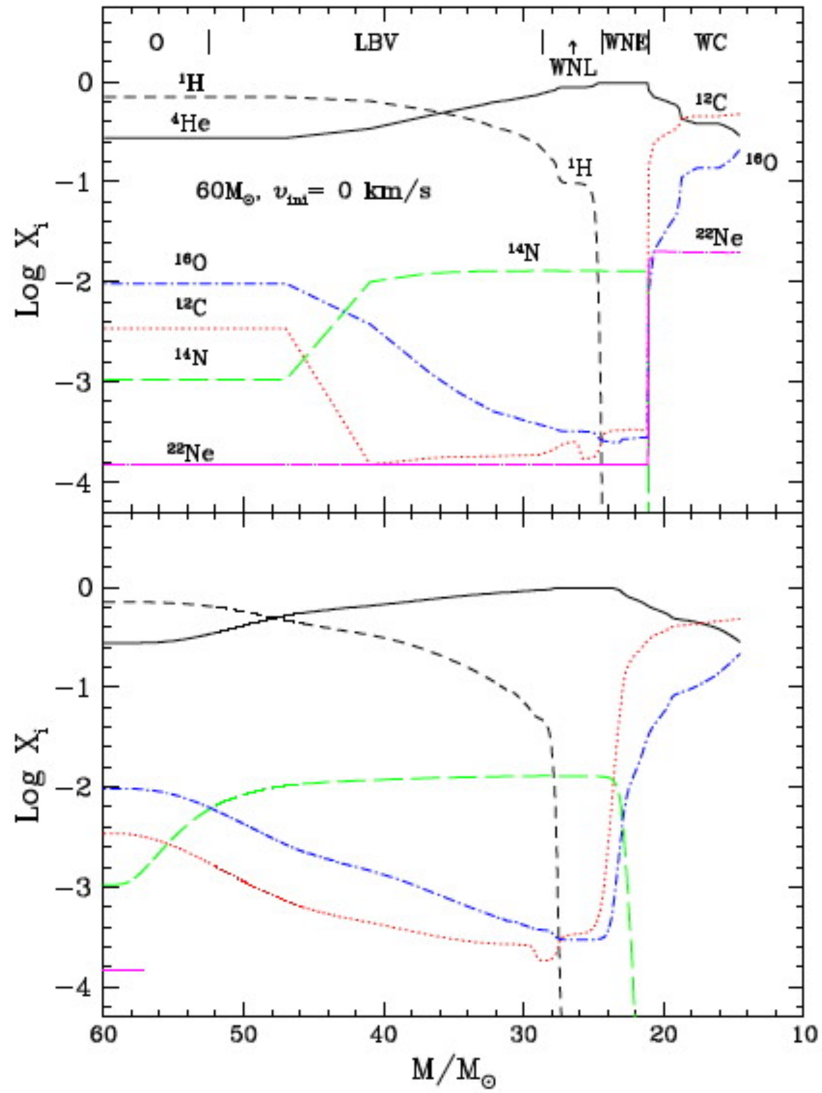
M=120 M_{sun}

e-3) Stellar evolution & mass loss



As the envelope is stripped away by mass loss during stellar evolution, nuclear ashes are exposed on the surface and the star evolves to the Wolf-Rayet phase

e-4) Stellar structure with and without rotation (& mass loss)



Initial mass is 60 M_{sun}
Top: with rotation
Bottom: without rotation

e-5) Stages in stellar evolution & mass loss

$M > 90\,M_\odot$: O – Of – WNL – (WNE) – WCL – WCE – SN (hypernova at low Z?)

$60 - 90\,M_\odot$: O – Of/WNL ⇔ LBV – WNL (H poor)- WCL-E – SN (SNIIn?)

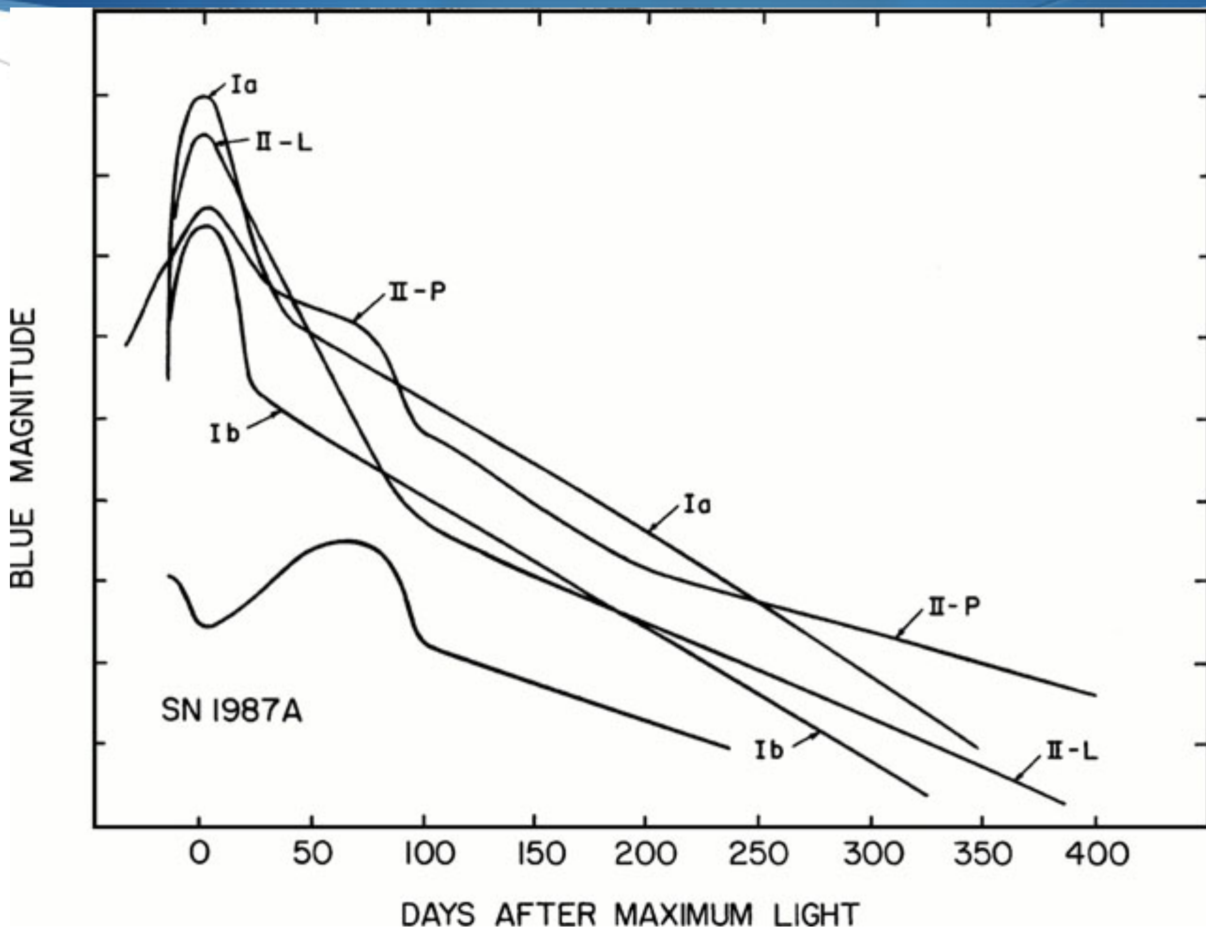
$40 - 60\,M_\odot$: O – BSG – LBV ⇔ WNL -(WNE) – WCL-E – SN (SNIb) or – WCL-E – WO SN (SNIc)

$30 - 40\,M_\odot$: O – BSG – RSG – WNE – WCE – SN (SNIb) or – RSG – OH/IR ⇔ LBV?

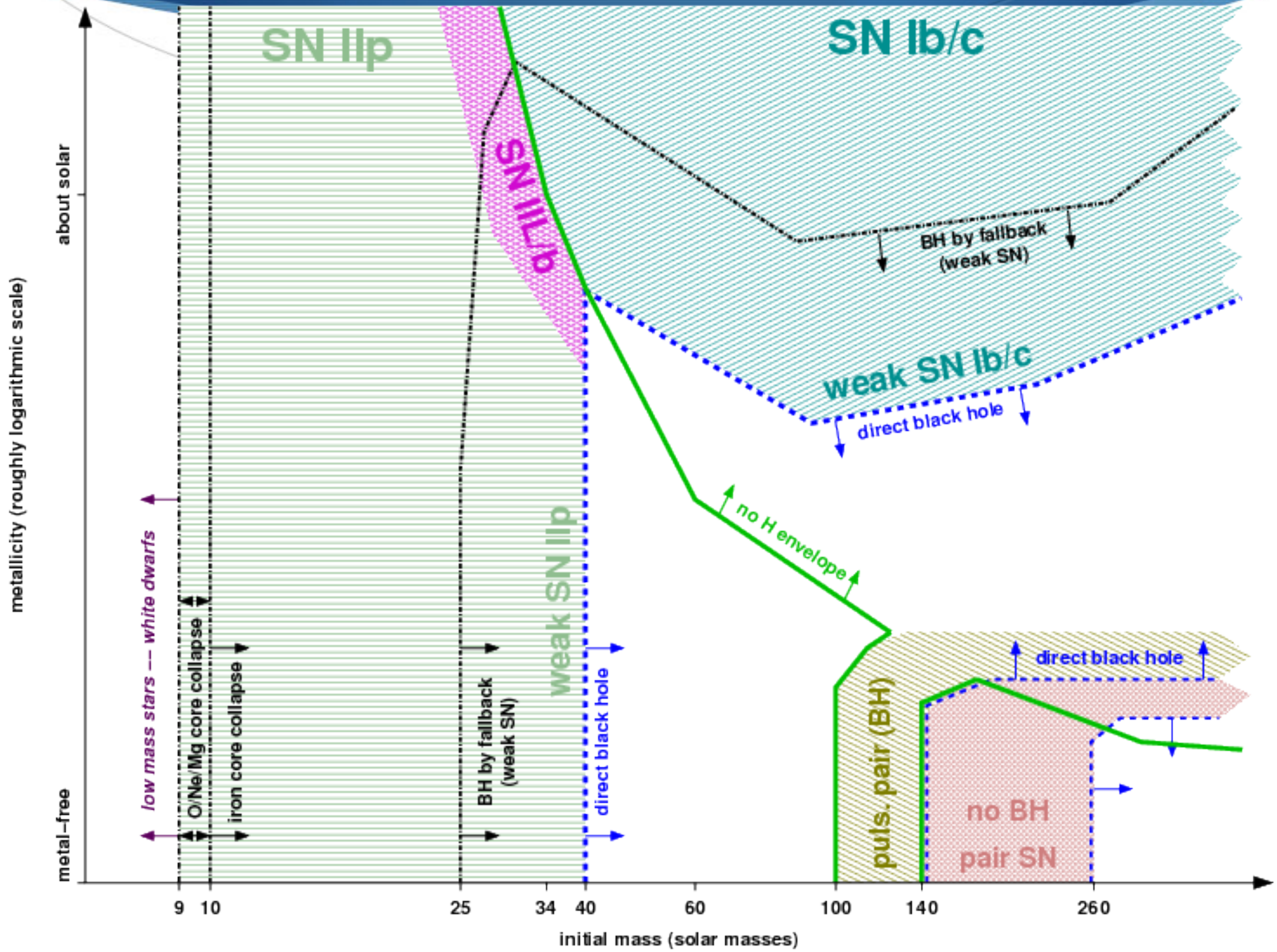
$25 - 30\,M_\odot$: O -(BSG)- RSG – BSG (blue loop) – RSG – SN (SNIIb, SNIIL)

$10 - 25\,M_\odot$: O – RSG – (Cepheid loop, $M < 15\,M_\odot$) RSG – SN (SNIIL, SNIIp)

Supernova Light Curves

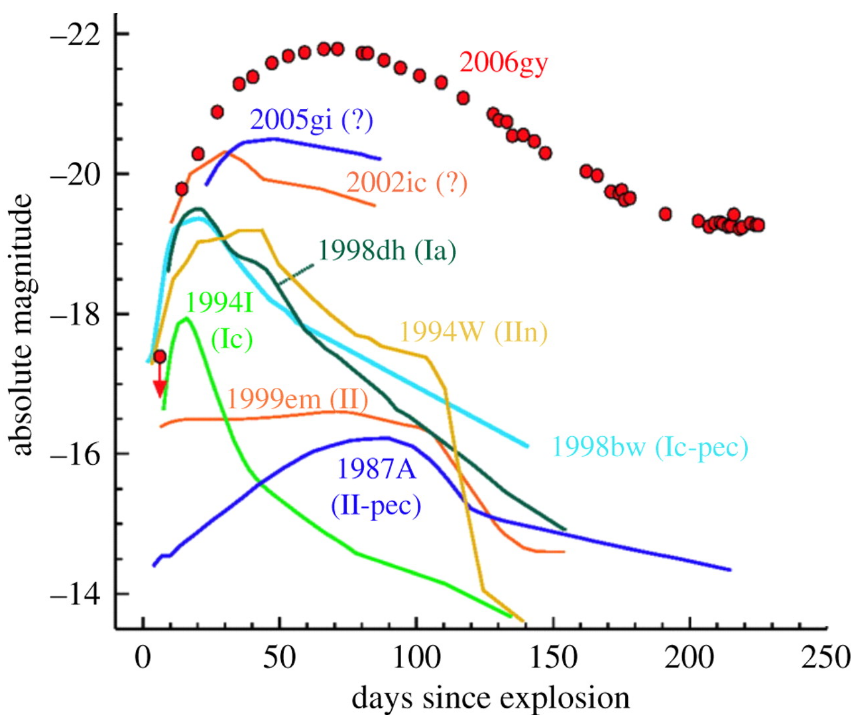


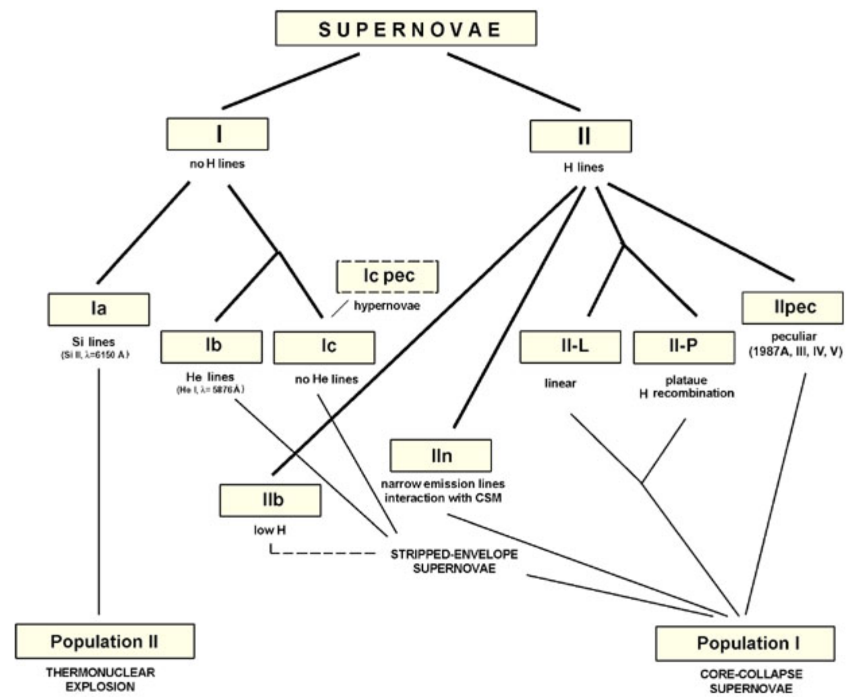
e-6) The bewildering zoo of SNe



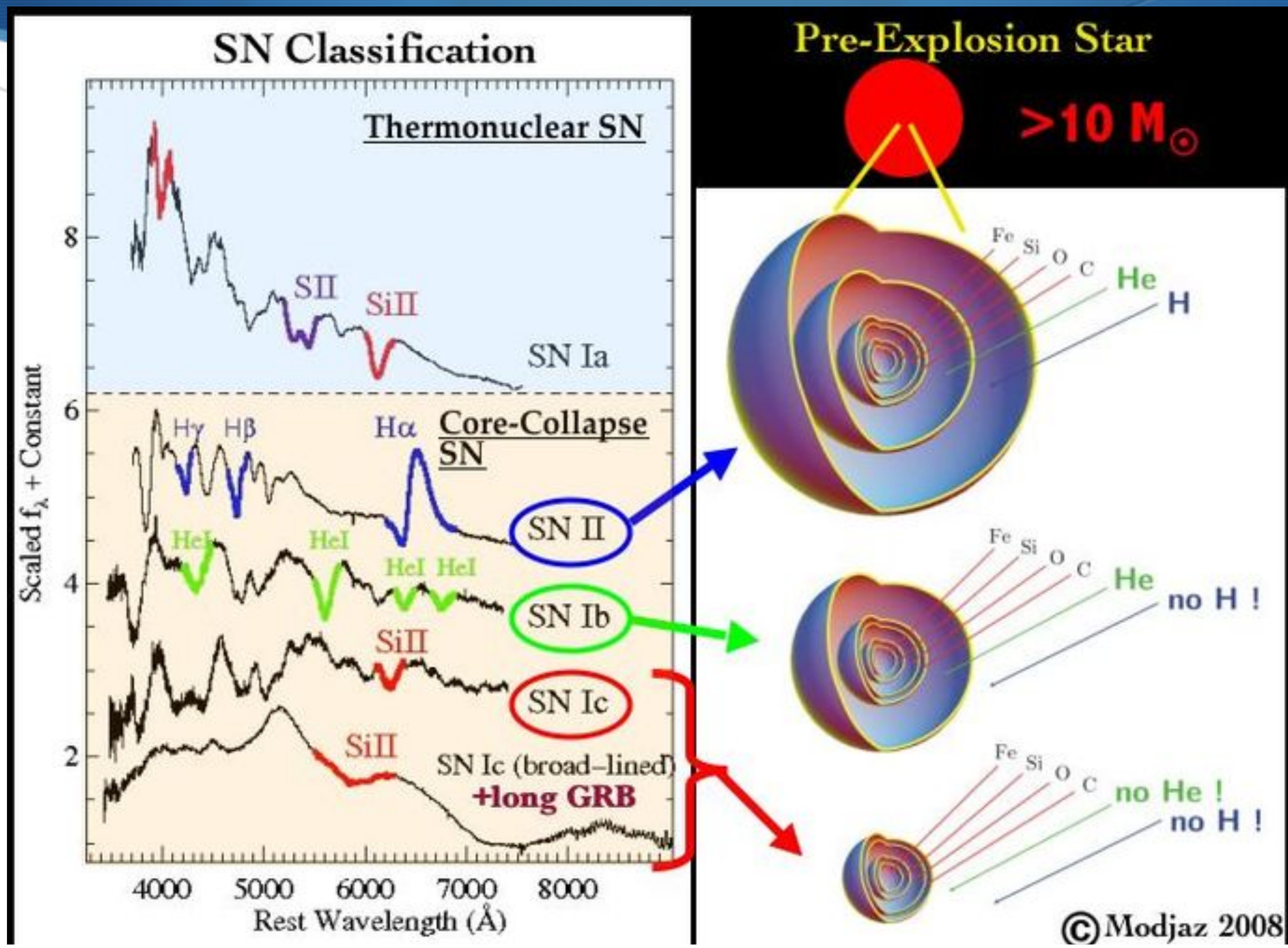
Supernova Properties

- Light curve: Rise time of a few weeks and decline in a few months.
- Spectral classification





Supernova Properties II



Explosion

- Supernova explosion: Release of nuclear energy on a timescale that is fast compared to the dynamical timescale (R/v)

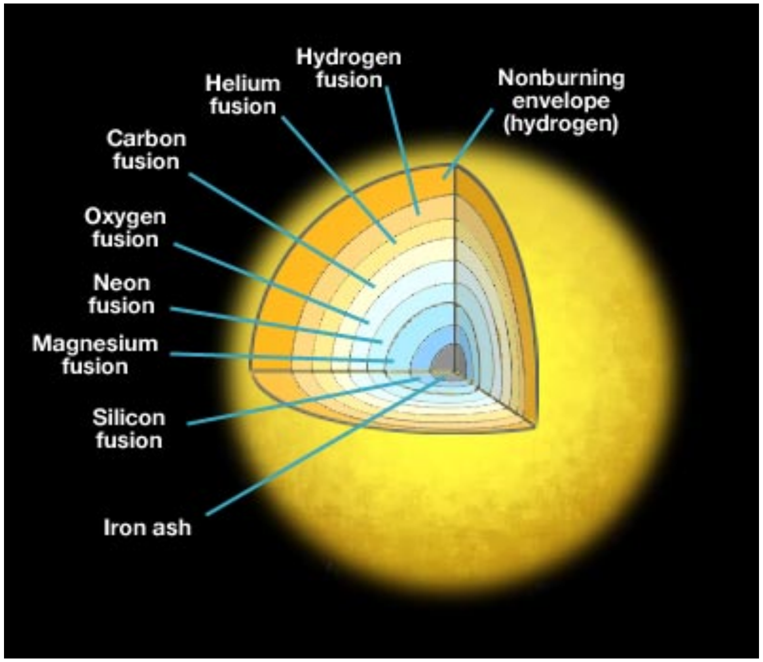
Evolution of high mass stars

***M* > 8 *M*$_\odot$**

TIMESCALES FOR NUCLEAR BURNING

Hydrogen – 10 Myr
Helium – 1 Myr
Carbon – 1000 yr
Neon ~ 10 yr
Oxygen ~ 1 yr
Silicon ~ 1 day

Structure of type II SN just before explosion



Iron core grows to 1.2-1.4 Msun (Chandrasekhar mass) and collapse sets in. Core heats up and nucleosynthesis is reversed through two energy consuming processes:

1) Photodisintegration of iron followed by helium

$$^{56}\mathrm{Fe}+\gamma \rightarrow 13\,^{4}\mathrm{He}+4\mathrm{n}-124\mathrm{MeV}$$

$$^{4}\mathrm{He}+\gamma \rightarrow 2\mathrm{p}+2\mathrm{n}-24\mathrm{MeV}$$

2) This is followed by neutronisation

$$\mathrm{p}+\mathrm{e} \rightleftarrows \mathrm{n}+\nu_e-1\mathrm{MeV}$$

This also robs the core of pressure support. Note that neutrinos become trapped and that limits proton conversion.

The neutron gas becomes degenerate at $\rho \approx 10^{15}\,\mathrm{g/cm^3}$ – the density of a nucleus – and the strong nuclear force comes into play. Inner part of the core overshoots, bounces, and infalling gas is accelerated outwards by the shock.

Energetics

Gravitational energy: $\Delta E_{grav} = \frac{GM_c^2}{R_{nc}} \approx 3\times 10^{53}$ erg

Nuclear energy: $\Delta E_{nucl} = \frac{M_c}{m_H} 6 \text{ MeV} \approx 2\times 10^{52}$ erg

Radiative energy: $\Delta E_{rad} = L\Delta t \approx 10^9 \times 10^7 \approx 4\times 10^{49}$ erg

Envelope gravitational binding energy: $\Delta E_{grav} = \frac{GM_c\left(M-M_c\right)}{R_{nc}} \approx 5\times 10^{51}$ erg

Kinetic energy ejecta: $\Delta E_{kin} = \frac{1}{2}\left(M-M_c\right)v_{ej}^2 \approx 1-8\times 10^{51}$ erg

Most of the energy carried by neutrino's

Shock & neutrino's

Shock sweeps up its own mass in ~30 msec and shock will stall. Interaction with neutrino's from the core will rejuvenate the shock.

Neutrino interaction cross section: $\sigma_\nu \approx 10^{-44} \left(E_\nu / \mathrm{MeV}\right)^2 \ \mathrm{cm}^2$

Mean free path: $\lambda = 1 \big/ n\sigma_\nu \approx 0.5 \left[\left(10^{12}\ \mathrm{g/cm^3}\right) \big/ \rho \right]^{5/3}$ km becomes comparable

to core radius when $\rho \approx 10^{11}\,\mathrm{g/cm^3}$

Diffusion timescale: $\tau_{dif,\nu} = \dfrac{N_{sca}\lambda}{c} \approx \dfrac{R^2}{\lambda c} \approx 10 \dfrac{\rho}{10^{15}\ \mathrm{g/cm^3}}$ s

Shock will heat the envelope to $5\mathrm{x}10^9$ K and matter will consist of equal parts of p and n. This will form ^{56}Ni which is unstable and will decay to ^{56}Co (half-life=6.1 days) and then to ^{56}Fe (half-life=77.1 days). Nucleosynthesis will also form the heaviest nuclei through the r-process.

Black hole or Neutron Star

As for white dwarfs, there is a maximum mass for a stable neutron star. The Oppenheimer-Volkoff mass whose value depends on nuclear physics aspects which are not well known: $M_{ov} \approx 2-3\mathrm{M}_{\odot}$

Cores more massive than this will collapse to a black hole; e.g., a singularity of "zero volume and infinite mass". The singularity will have an event horizon, the Schwarzschild radius:

$$R_{sch} = \frac{2GM}{c^2} \approx 3\left(\frac{M}{\mathrm{M}_{\odot}}\right) \text{ km}$$

Lightcurve

Consider an ejected mass, M, expanding at velocity, v, (e.g., $R = vt$)

and with opacity κ. The diffusion timescale is then:

$$\tau_{dif,\gamma} = \frac{N\lambda}{c} \approx \frac{R^2}{\lambda c} \approx \frac{\kappa M}{Rc}$$

The diffusion timescale becomes comparable to the age when

$$\tau_{dif,\gamma} \approx \sqrt{\frac{\kappa M}{vc}} \approx 10-20 \text{ days (Rise time)}$$

Until that time, expansion is adiabatic and since it is radiation dominated

with γ=4/3, $PV^{4/3} = \text{cst}$ and $U = 3P = aT^4$ ($T^4R^4 = \text{cst}$). Hence, $E_{th} = UV \propto R^{-1}$

$$L = 4\pi R^2 \frac{c}{\kappa\rho}\frac{daT^4}{dr} \approx \frac{R^3 E_{rad}}{\tau_{dif,\gamma}}$$

Lightcurve

During the adiabatic phase, $L \sim \frac{R_o^4 ac T_o^4}{\kappa M} \sim \frac{E_{SN} c R_o}{\kappa M}$

For type IIP with $L = 10^9 \ \mathrm{L}_\odot$ and R=1AU, and

adopting E=10^{51} erg, we have $M = 10\text{-}20 \ \mathrm{M}_\odot$

Light curve SN 1987A