Assignments 9

May 1, 2014

Exercise 1:

i. Peruse the behavior of the stars in the HR diagram on slide 31/32 as they move off the ZAMS. What is the origin of the different stages in the behavior of the 3 M_o star ? What causes the difference in global behavior of the tracks for stars above and below 2 M_o ?

ii. Compare and contrast the behavior of the core tracks in slide 48 with the schematic behavior outlined in the earlier slides of this lecture (slides 5–15). What differences and what similarities do you see ?

iii. Peruse the changes in the structure of stars as they evolve (slides 51/52) and describe their origin. How are these related to the tracks of the star in the HR diagram ?

iv. Compare the stellar structure of the 1 and 5 M_o stars in slides 61, 63 & 65. Describe the differences and similarities and their origin.

v. Examine slide 72, you should be able to describe and understand the different stages in the evolution of a 5 M_o star.

Exercise 2:

In the lecture, we derived an analytical core-mass (M_c) luminosity (L) relation on the RGB and compared it to numerical results. Here we will adopt $L = 125(M_c/0.3M_o)^7\ L_o$.

i. Write an equation for the rate at which the core mass grows in terms of the luminosity of the star and the energy generation per unit mass, Q.

ii. Solve for the time dependence of the core mass.

iii. Calculate the total time spend on the RGB for a 1 and a 2 M_o star, assuming that they arrive on the RGB with a core mass which is 15% of their mass and that they leave the RGB at 0.45 M_o.

iv. Compare these timescales to the main sequence time scales.

v. Show that the time spent by a low mass star evolving up the RGB (e.g., the number of stars) in a given absolute magnitude interval scales with $L^{-0.86}$. [In practice, globular clusters show a behavior more like $L^{-0.5}$].

Exercise 3:

Assume that the core of RGB star is isothermal with $T_c=10^7$ K and a core mass, $M_c=1$ M_o. Assume that the pressure in the core can be described by an ideal gas, that the opacity is given by $\kappa = \kappa_o \rho T^{-3.5}$ (Kramers opacity), and that the stellar luminosity is produced in a thin shell.

i. Write the pressure, P, temperature, T, density, ρ, and mass, M, dependence on r in the envelope as power laws with exponents, a, b, c, and d. Use the ideal gas law, mass continuity, hydrostatic equilibrium equation, and radiative energy transport equation to derive algebraic relationships between the power law exponents.

ii. Derive the values of these exponents.

iii. Use the equation of hydrostatic equilibrium at the core boundary to derive the core radius.

iv. Assume an effective temperature of 3700 K, appropriate for a RGB star on the Hayashi track, and derive the stellar radius.

v. Compare this value to the size of the solar system