Assignments 6

March 23, 2014

Exercise 1:

Consider the nucleus as a droplet consisting of a degenerate "sea" of protons and neutrons. The radius of the droplet is given by $R \approx 1.4 \times 10^{-13} A^{1/3}$ cm. Adopt $Z \approx N \approx A/2$.

i. What is the energy of the highest occupied energy level in MeV ? Hint: use Fermi statistics of a non-relativistic completely degenerate proton/neutron gas.

ii. For a helium nucleus, what is the Coulomb barrier for a proton entering (or leaving) the well in MeV ?

iii. Calculate the typical energy of a proton in a core of a star with $T_c = 10^7$ K. in MeV

iv. Adopt the typical binding energy per nucleon, and sketch the potential well for a neutron and a proton.

Exercise 2:

Consider a nucleus with charge Z_A colliding with another nucleus with charge Z_B. The reduced mass is μ. Evaluate the probability of Coulomb barrier penetration from the Schrodinger equation.

$$\frac{\hbar^2}{2\mu}\frac{d^2\psi}{dr^2} = H = (V - E)\psi$$

with

$$V(r) = \frac{Z_A Z_B e^2}{r}$$

Consider that for E>V, the wave function will be oscillatory, while for E<V, ψ is exponential

$$\psi \propto exp[-(r_1 - r)/\ell]$$

with r_1 the classical turn around point ($V(r_1)=E$).

i. Derive an expression for the penetration probability.

ii. Determine the characteristic length, ℓ, by evaluating H at $r_1/2$.

iii. Evaluate this expression for $T=10^7$ K

Exercise 3:

Consider the reaction: $^4He + \ ^4He \leftrightarrow \ ^8Be \quad Q = 0.092MeV$ and
$^4He + \ ^{48}Be \leftrightarrow \ ^{12}C^* \longrightarrow \ ^{12}C + 2\gamma$ The C excited state is at an excited energy of 7.654 MeV with respect to the ground state. The forward reaction of each of the first two reactions can be considered in rapid equilibrium with the reverse reaction and the Saha equation can be used in the analysis.

i. Using the Gamow energy, E_0, calculate the temperature required for the initial reaction.
ii. Write down the Saha equation for ^4He, ^8Be system.
iii. Calculate the equilibrium abundance of ^8Be in a helium core (wrt ^{4}He) at a temperature, T=10^8 K, and a density of 10^6 g/cm^3. The ground states of ^4He and ^8Be have both zero spin.
iv. What is the number density of ^8Be nuclei ?
v. Write down the Saha equation for ^8Be, $^{12}C^*$ system in terms of the α abundance and the rest-mass energy difference between the 3α particles and the excited 12C nucleus (–0.3795 MeV).
vi. Calculate the fractional abundance of $^{12}C^*$ (wrt ^{4}He) at a temperature, T=10^8 K, and a density of 10^6 g/cm^3.
vii. The lifetime of $^{12}C^*$ is ~$2\text{x}10^{-16}$ s and only 1 in every 2500 $^{12}C^*$ decays to ^{12}C. The total energy released in the 3α reactions is 7.275 MeV. Calculate the energy production rate in erg/g/s.

Exercise 4:

Calculate how many 100 Watt bulbs are needed per m^3 in the Sun's core to provide all of the solar energy.

(Nuclear energy production can be very inefficient and still be effective because of sheer volume, resulting in long lifetimes.)

Exercise 5:

Explain qualitatively why the abundance of Li and Be is so low.