Assignment 2

Feb 17, 2016

Exercise 1: Dynamical timescale

a) Derive the free-fall timescale for a star
b) What is the dynamical timescale for a neutron star ?
c) Can a pulsar be a pulsating neutron star ?

Exercise 2: Free fall timescale

Consider a spherically symmetric, homogeneous cloud with mass M and radius R, collapsing in free-fall (eg., no pressure forces). Write the equation of motion for a mass shell as,

$$\frac{d^2r}{dt^2} = -\frac{Gm_r}{r^2}$$

a. Introduce the velocity of the shell as, $u_r(t) = \frac{dr}{dt}$, and show that the velocity is given as,

$$u^2 = 2GM\left(\frac{1}{r} - \frac{1}{R}\right)$$

where we have assumed that initially the cloud is at rest.

b. This can be transformed into an integral for the time as a function of radius,

$$dt = -\left(\frac{3}{8\pi G\rho_0}\right)^{1/2}\left(\frac{\xi}{1-\xi}\right)^{1/2} d\xi$$

with $\xi = r/R$ and ρ_0 the initial density of the cloud.

c. Substitute $\xi = sin^2\emptyset$ and show that the free fall timescale for the cloud to contract from r=R to r=0 is given by,

$$t_{ff} = \left(\frac{3\pi}{32G\rho_0}\right)$$

Exercise 3: Virial theorem

Assume that a star with mass, M, has no nuclear energy sources.

a) Find the rate of contraction of its radius if it maintains a constant luminosity.
b) What is this rate for the Sun ?

c) As the Sun contracted towards the main sequence under the influence of gravity, its internal temperature changed from 30,000 K to 6,000,000 K. What is the total energy radiated away during this contraction ?

Exercise 4: Pressure and temperature in the Sun's interior

Using the virial theorem, make an estimate for the internal temperature and pressure of the Sun (assume constant density).

Exercise 5: Alternative derivation of the radiative energy transport equation

Consider two layers in a star separated by a mean free path.

a) Compare the black body flux from these two layers and write the net flux as a differential.
b) Write the mean free path in terms of the opacity.
c) Introduce the luminosity for the flux and arrive at the equation:

$$L(r) \sim -\frac{16 r^2 \sigma T^3}{\kappa \rho} \frac{dT}{dr}$$

The difference between this equation and the actual equation (a factor 4/3) stems from a proper integration over all solid angles.

Exercise 6: Energy diffusion

Consider radiative energy diffusion as a scattering process where the energy of a photon does not change (only the direction). Assume constant density and mean free path (ℓ =1 cm). How long does it take for a photon to travel from the core to the surface of the Sun ? How does that compare to the Kelvin-Helmholtz timescale?

Exercise 7: Mass-Luminosity relation

a) Assume that the opacity is due to electron scattering with κ=0.2(1+X) cm^2/g, where X is the hydrogen mass fraction (0.7). Evaluate the M-L relation and compare to the relation given in lecture 1.

b) Use the Mass-Luminosity relation to demonstrate that stars with M>1Msun will get brighter during their evolution on the main sequence.

Exercise 8: Electron conduction

Consider energy transport through electron conduction. You can use the same energy diffusion equation but replace the relevant expressions with those for electrons. Look over the lecture notes and – by considering the mean free path and velocity – make sure that you understand why electron conduction is not very important compared to radiation for solar type stars but becomes important for degenerate stellar cores