EVOLUTION OF STARS: A DETAILED PICTURE

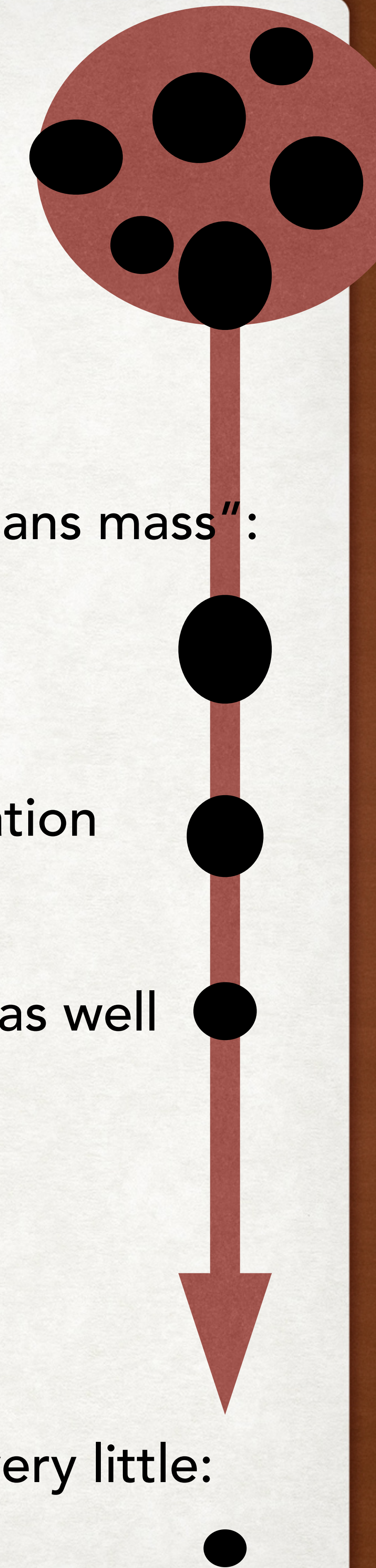
PRE-MAIN SEQUENCE PHASE

CH 9: 9.1

All questions 9.1, 9.2, 9.3, 9.4 at the end of this chapter are advised

PRE-PROTOSTELLAR PHASE

SELF-GRAVITATIONAL COLLAPSE



Evolution on $\tau_{\rm dy} \approx 1/\sqrt{G\rho} \approx 10^6 - 10^7 {\rm yr}$

In a medium in pressure equilibrium,
an overdensity in a molecular (H_2) cloud collapses if its mass exceeds the "Jeans mass":

$$M > M_{\rm j} = 2 \times 10^3\ {\rm M_{sun}} \left(\frac{T}{100\ {\rm K}}\right)^{2/3} \left(\frac{n}{300 {\rm cm}^{-3}}\right)^{-1/2}$$

- Transparent stage to photons and *cools* isothermally and more fragmentation occurs on smaller masses: details should give the star initial mss function

- Opaque phase to infrared photons and it *heats up* and pressure builds up as well towards HE, passing through unstable phases:

- Unstable dynamical phases due to non conservation of particles:
  - At T~2000 K H_2 is dissociated
  - At T~10^4 K H He ionisation start:
- in these phases potential energy is used and the temperature increases very little:

HE is not possible (these are phases of dynamical instabilities)

PROTOSTAR

EVOLUTION ON THE K-H TIMESCALE

$$\tau_{K-H} \approx 3 \times 10^7 \text{ yr} \left(\frac{M}{M_{sun}}\right)^2 \left(\frac{R}{R_{sun}}\right) \left(\frac{L}{L_{sun}}\right)^{-1}$$

- When H and He ionisation is almost complete, pressure balances gravity: the collapse proceed quasi-statically and $L \sim (GM/\dot{R})$

Let's estimate the protostellar radius:

assumption whole released potential energy used to ionising H, He, and H_2

$$\frac{\alpha}{2}\frac{GM^2}{R_p} \approx \frac{M}{m_u}\left(\frac{X}{2}\chi_{H_2} + X\chi_H + \frac{Y}{4}\chi_{He}\right) \equiv \frac{M}{m_u}\chi.$$

$$R_p \approx \frac{\alpha}{2}\frac{GMm_u}{\chi} \approx 50\, R_\odot \left(\frac{M}{M_\odot}\right).$$

X=0.7, Y=0.28, Z=0.02

$$\chi_{H_2} = 4.48 \text{ eV}$$

$$\chi_H = 13.6 \text{ eV}$$

$$\chi_{He} = 79 \text{ eV } 2e^-$$

$$\alpha \sim 1 \text{ e.g. for } n = \frac{3}{2}$$

PROTOSTAR

EVOLUTION ON THE K-H TIMESCALE

$$\tau_{\mathrm{K-H}} \approx 3 \times 10^7\ \mathrm{yr} \left(\frac{M}{M_{\mathrm{sun}}}\right)^2 \left(\frac{R}{R_{\mathrm{sun}}}\right) \left(\frac{L}{L_{\mathrm{sun}}}\right)^{-1}$$

- pressure balances gravity: the collapse proceeds (quasi-)hydrostatically statically
- Luminosity from gravity: $L \sim (GM/\dot{R})$

Protostellar radius:

$$R_{\mathrm{p}} \approx \frac{\alpha}{2} \frac{GMm_{\mathrm{u}}}{\chi} \approx 50\, R_{\odot} \left(\frac{M}{M_{\odot}}\right).$$

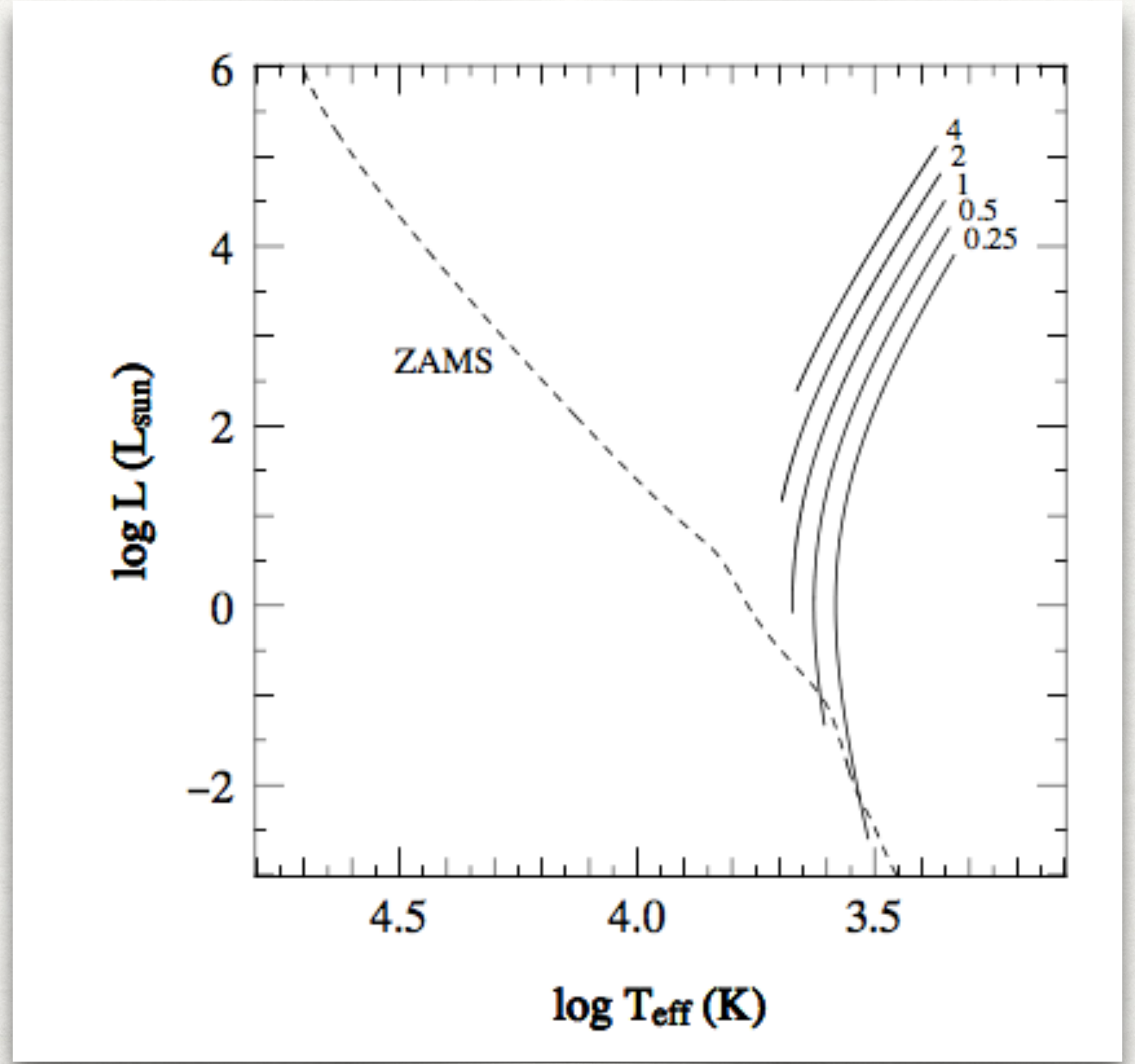
Virial theorem:

$$\bar{T} \approx \frac{\alpha}{3} \frac{\mu}{\mathcal{R}} \frac{GM}{R_{\mathrm{p}}} = \frac{2}{3} \frac{\mu}{k} \chi \approx 8 \times 10^4\, \mathrm{K},$$ independent of mass

→ low temperature i) no nuclear reaction yet, ii) **fully convective**

THE HAYASHI LINE/TRACK (EARLY 1960)

TRACK OF A FULLY CONVECTIVE PROTOSTAR



Note: a star is fully convective if T_{eff}< 9000 k => $\log T_{eff}$ < 3.95 (7.2.3)

Fully convective stars of a given mass trace an almost vertical line on the H-R diagram

- **to the right,** there is a forbidden zone for star in H-E
- **to the left**, stars cannot be fully convective

THE HAYASHI LINE derivation

let's neglect the mass and thickness of the photosphere with respect to M and R

If we assume $$\frac{d\log T}{d\log P} \equiv \nabla = \nabla_{\mathrm{ad}} = 0.4 \rightarrow T \propto P^{0.4}$$

ideal gas, ignoring variation in partial ionisation zones

ignoring superadiabaticity in the sub-photospheric layers

$$T \propto P^{0.4}$$
$$T \propto P/\rho$$

$$P = K\rho^{5/3}$$

A fully convective star can be described by a n=3/2 polytrope

$$K = 0.42GM^{1/3}R$$

We have one free parameter R

Let's determine it by joining the fully convective interior to the radiative photospheree at the boundary r=R

THE HAYASHI LINE

a derivation

In the radiative layer:

Hydro equilibrium: $$\frac{dP}{dr} \approx -\rho \frac{GM}{R^2} \longrightarrow P_{\mathrm{R}} = \frac{GM}{R^2} \int_R^\infty \rho dr$$

optical depth: $$\int_R^\infty \kappa \rho dr = \langle \kappa \rangle \int_R^\infty \rho dr = k_0 \rho^a T_{\mathrm{eff}}^b \int_R^\infty \rho dr = 1$$

$$\Rightarrow \quad P_{\mathrm{R}} = \frac{GM}{R^2 \kappa 0} \rho_{\mathrm{R}}^{-a} T_{\mathrm{off}}^{-b}$$ radiative layer

$$P_{\mathrm{R}} = \left(0.42 G M^{1/3} R\right) \rho_{\mathrm{R}}^{5/3}$$ convective envelope

Perfect gas $$P_{\mathrm{R}} = (\mathcal{R}/\mu)\, \rho_{\mathrm{R}} T_{\mathrm{eff}}$$ this makes T_{eff} appears

Luminosity at photosphere $$L = 4\pi^2 \sigma T_{\mathrm{eff}}^4$$ this makes L appears

For a given M, four equations with 5 unknown : $P_{\mathrm{R}}, \rho_{\mathrm{R}}, T_{\mathrm{eff}}, L, R$

THE HAYASHI LINE a derivation

Combine the 4 equations to eliminate $P_{\mathrm{R}}, \rho_{\mathrm{R}}, R$

we obtain the Hayashi track:

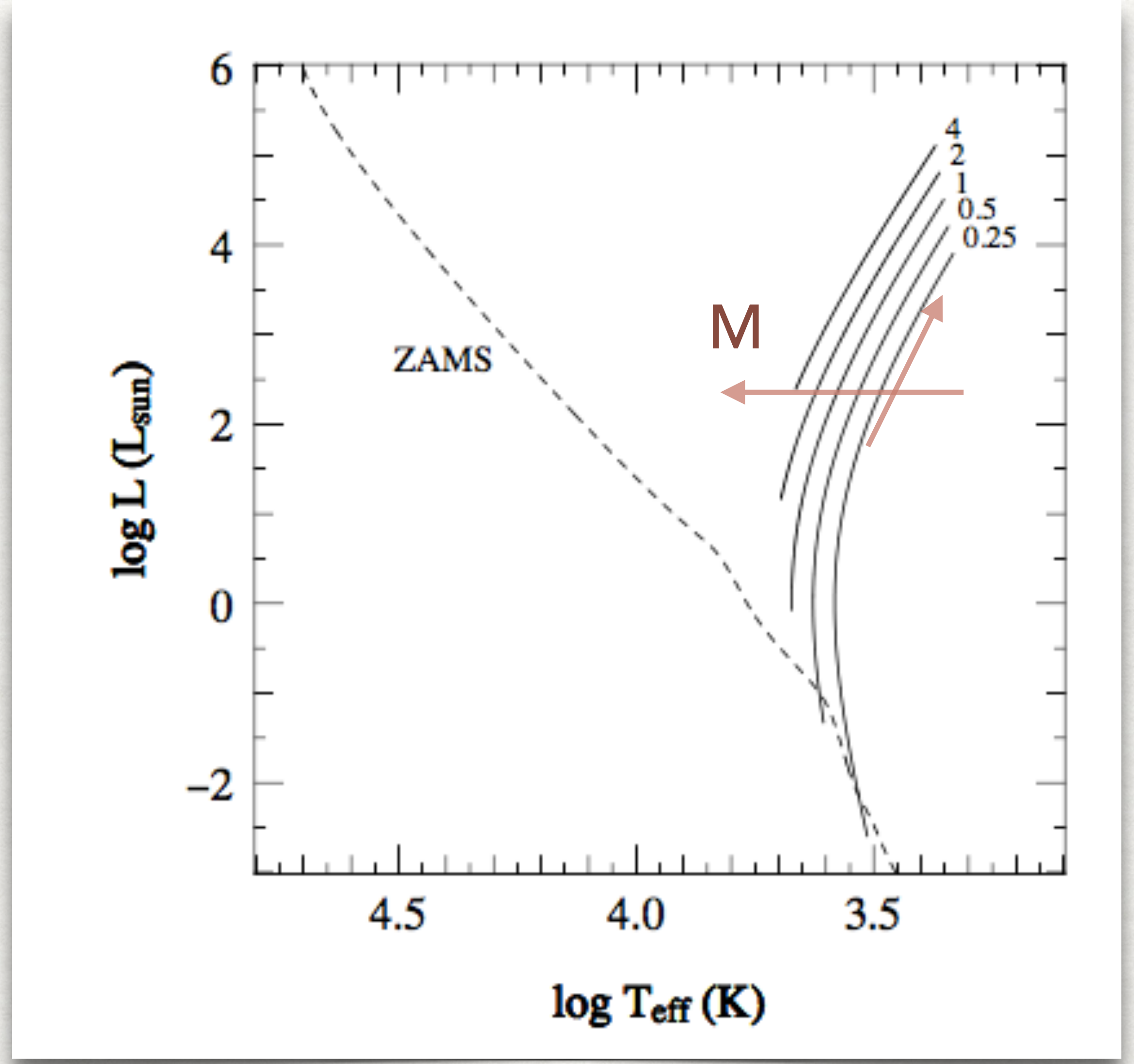
$$\log L = A \log T_{\mathrm{eff}} + B \log M + \mathrm{constant}$$

$$A = \frac{9a + 2b + 3}{(3/2)a - (1/2)} \qquad B = -\frac{a+3}{9a+2b+3}$$

The slope of the H-T depends on the dependence of opacity on T and density

The main opacity for very cool atmosphere is H^- : a =0.5 b =9 =>
A =100 B = -14

THE HAYASHI LINE

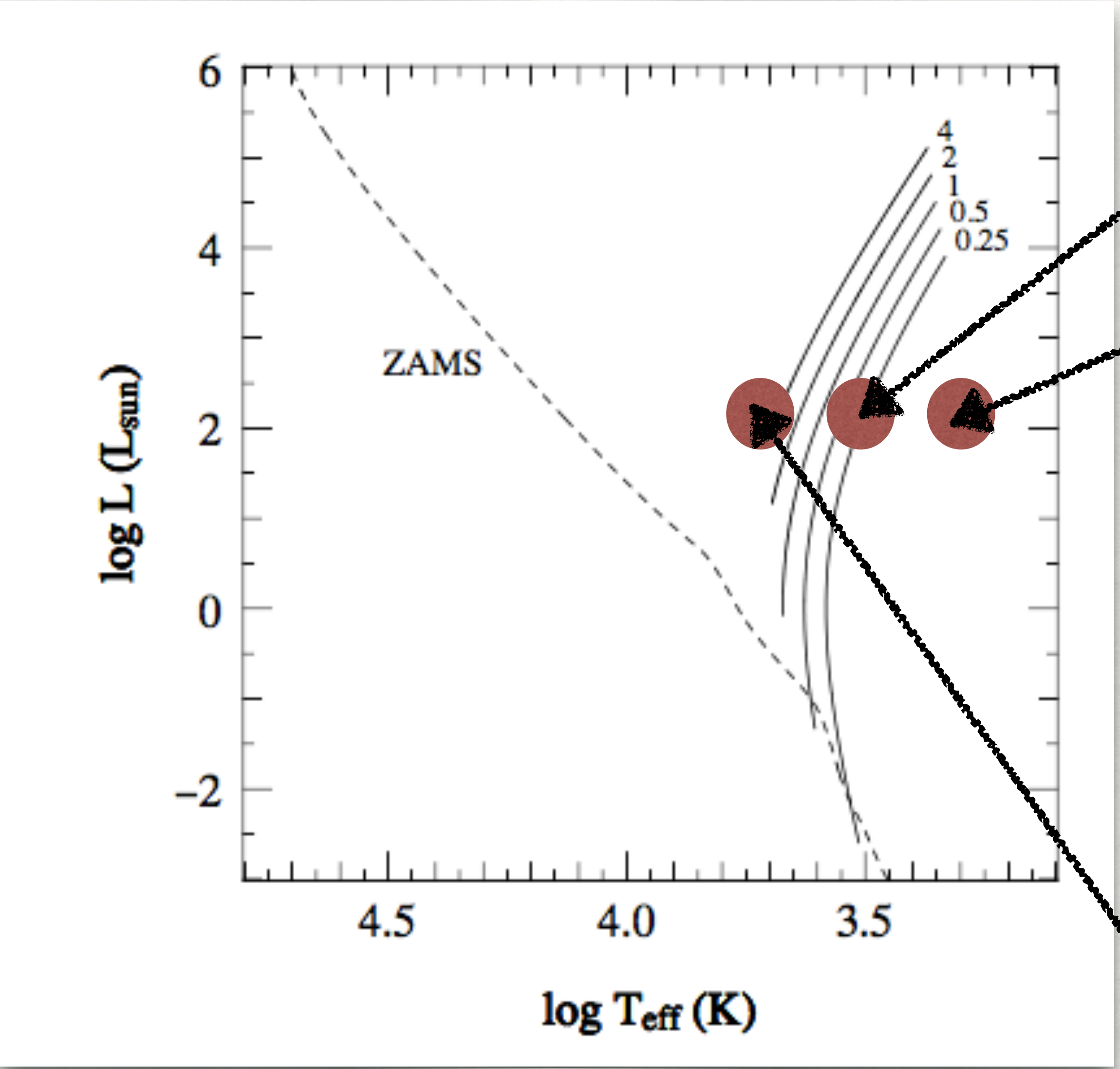


$$\log L = A \log T_{\rm eff} + B \log M + \text{constant}$$

Note: A =100 ==> the Hayashi line is a very steep line in L-T_{eff}

Note: B= negative ==> the Hayashi line moves to higher T_{eff} for higher M

FORBIDDEN HAYASHI ZONE



On the **Hayashi track** $\nabla = \nabla_{\rm ad}$

On **the right** $\langle\nabla\rangle > \nabla_{\rm ad}$

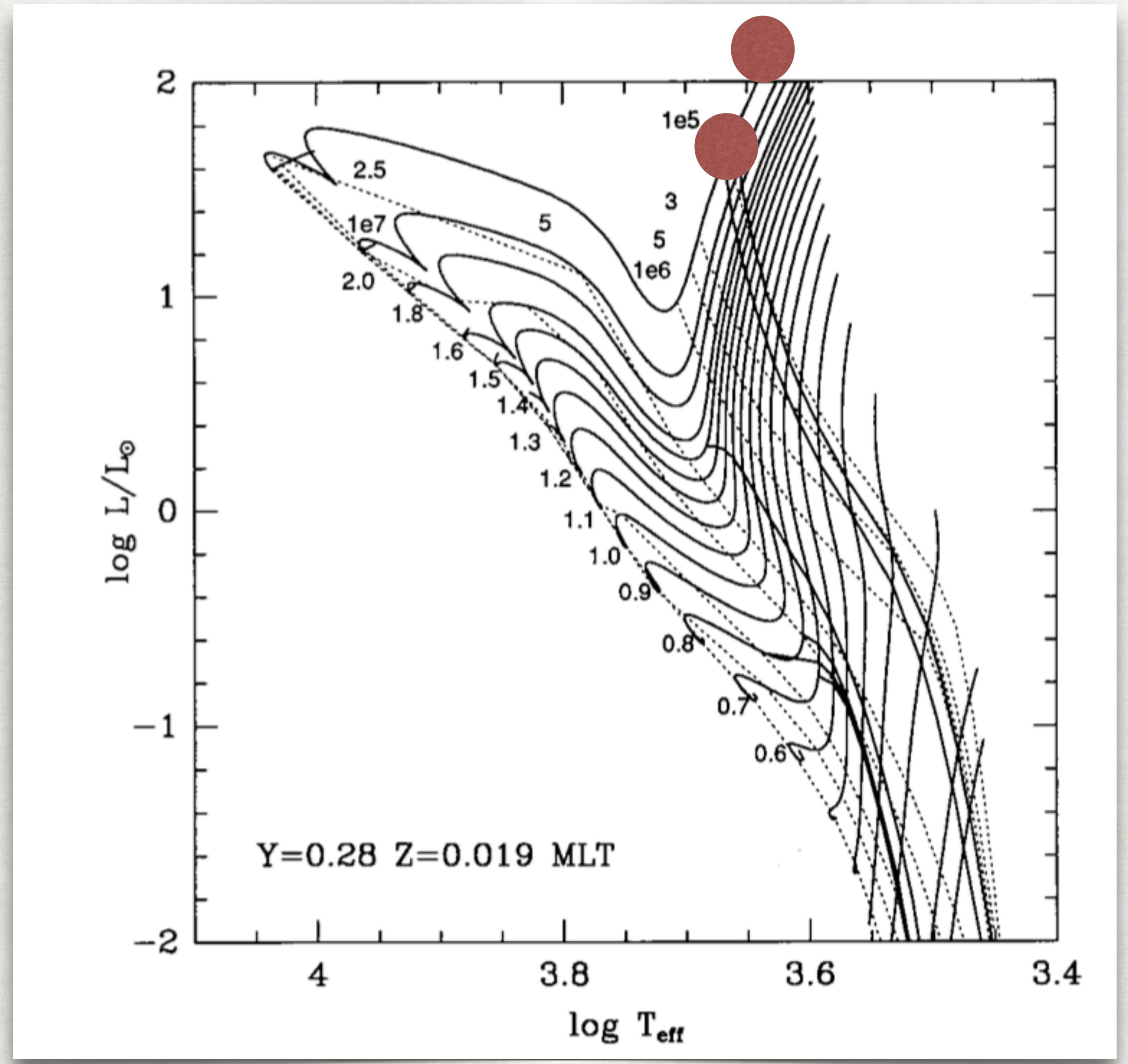
Large fraction of star has a superadiabatic temperature gradient
==> large convective flux
===> on a dynamical timescale star gradient gets smaller and star back to H. Track:
i.e. forbidden H Zone

On **the left** $\langle\nabla\rangle < \nabla_{\rm ad}$

First the core becomes **radiative**, at the far left the whole star is radiative

PRE-MAIN SEQUENCE (PMS) PHASE

A DESCRIPTION

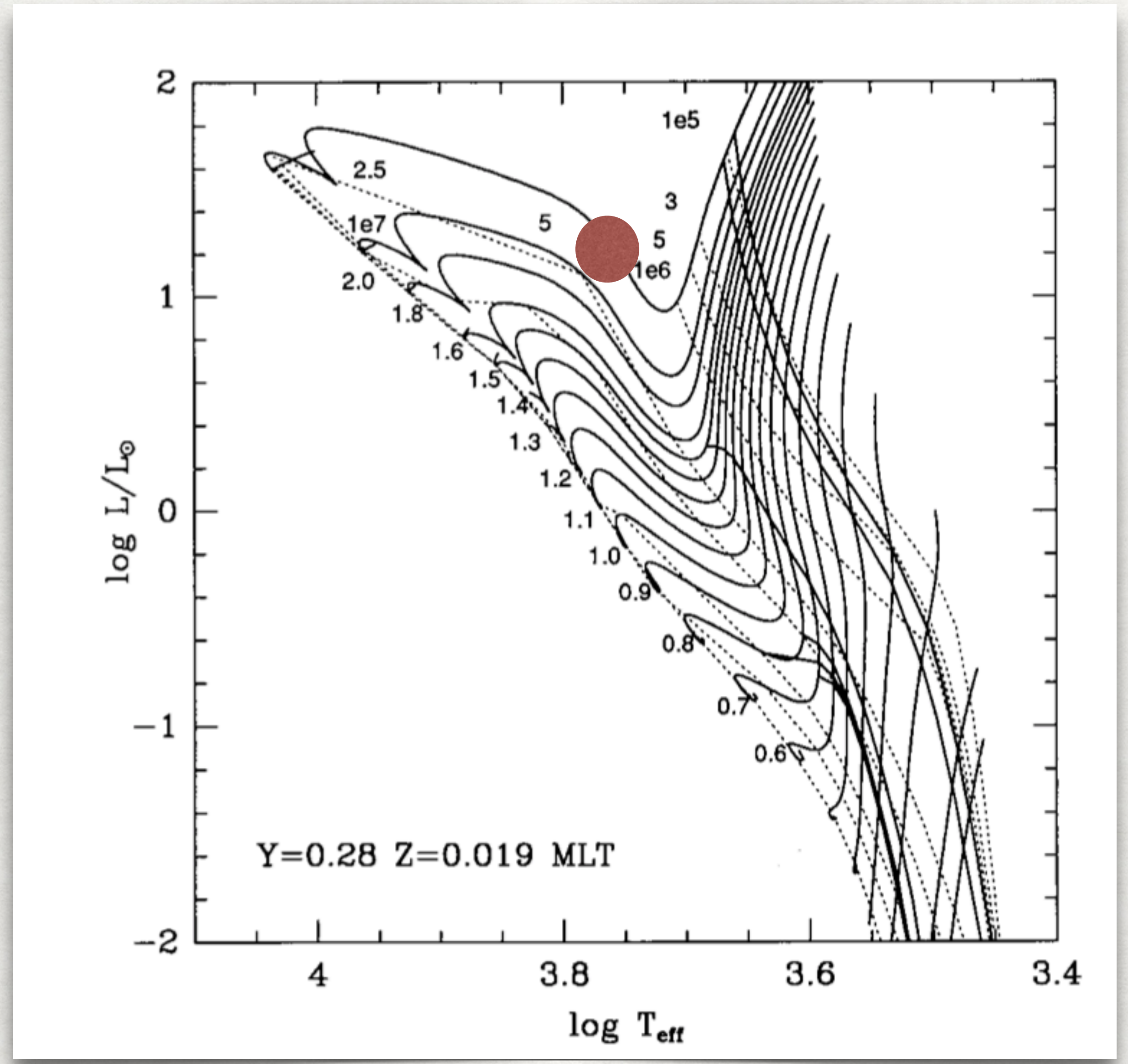


- Formation: dynamical collapse
- **Hayashi track:** pre-main sequence phase starts

  Along Hayashi track:
  - star contracts on t_{KH}
  - no (appreciable) nuclear reaction
  - luminosity given by gravity
  - **L ~ R^2 T_{eff}^4 ~R^2 decreases**
  - $T_c \propto \rho_c^{1/3} \propto 1/R$ increases
  - **opacity decreases inside**

PRE-MAIN SEQUENCE (PMS) PHASE

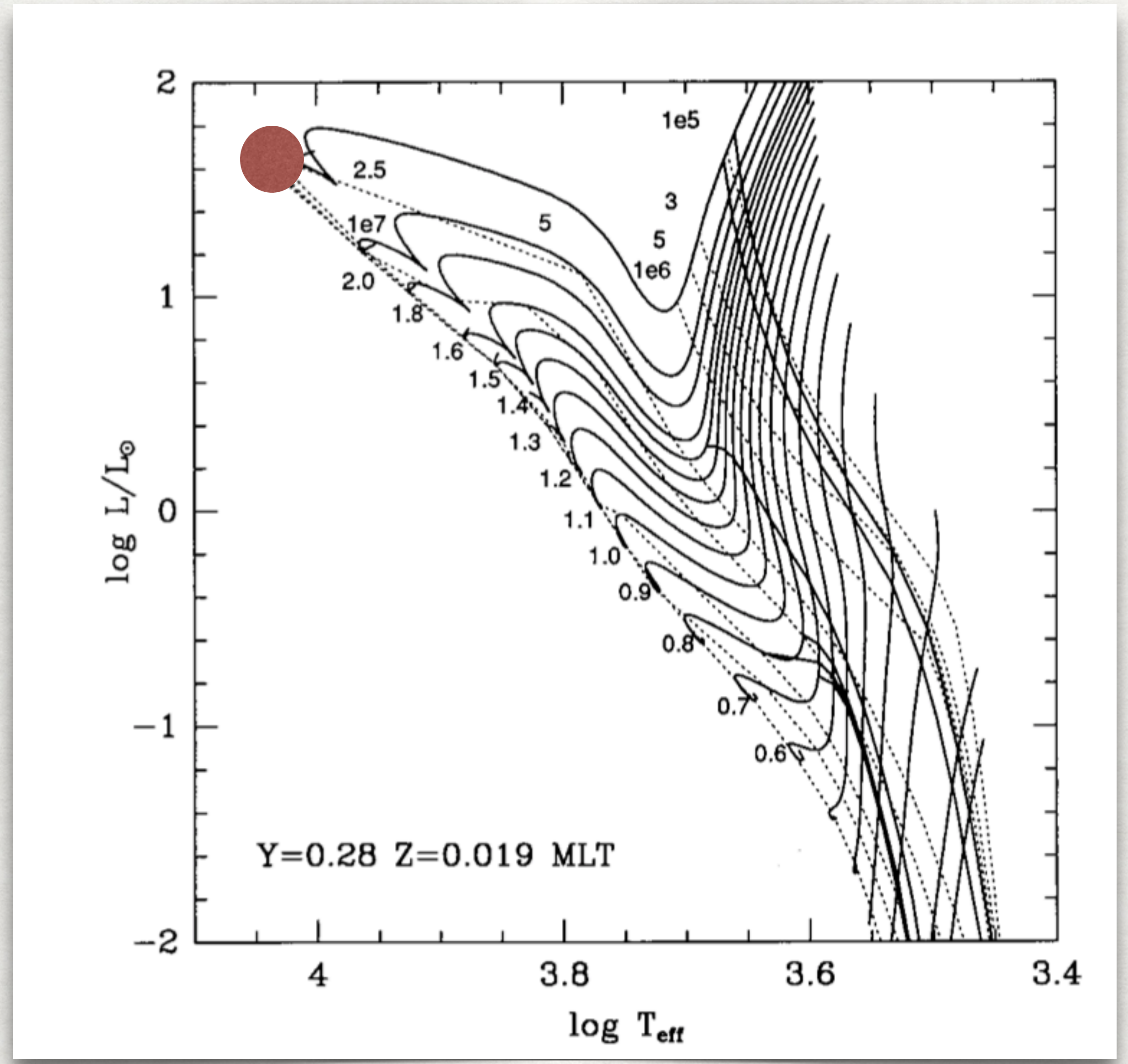
A DESCRIPTION



- Formation: dynamical collapse
- Hayashi track: pre-main sequence phase starts
- **Radiative core develops** : star moves to left
  - Temperature and L increase

PRE-MAIN SEQUENCE (PMS) PHASE

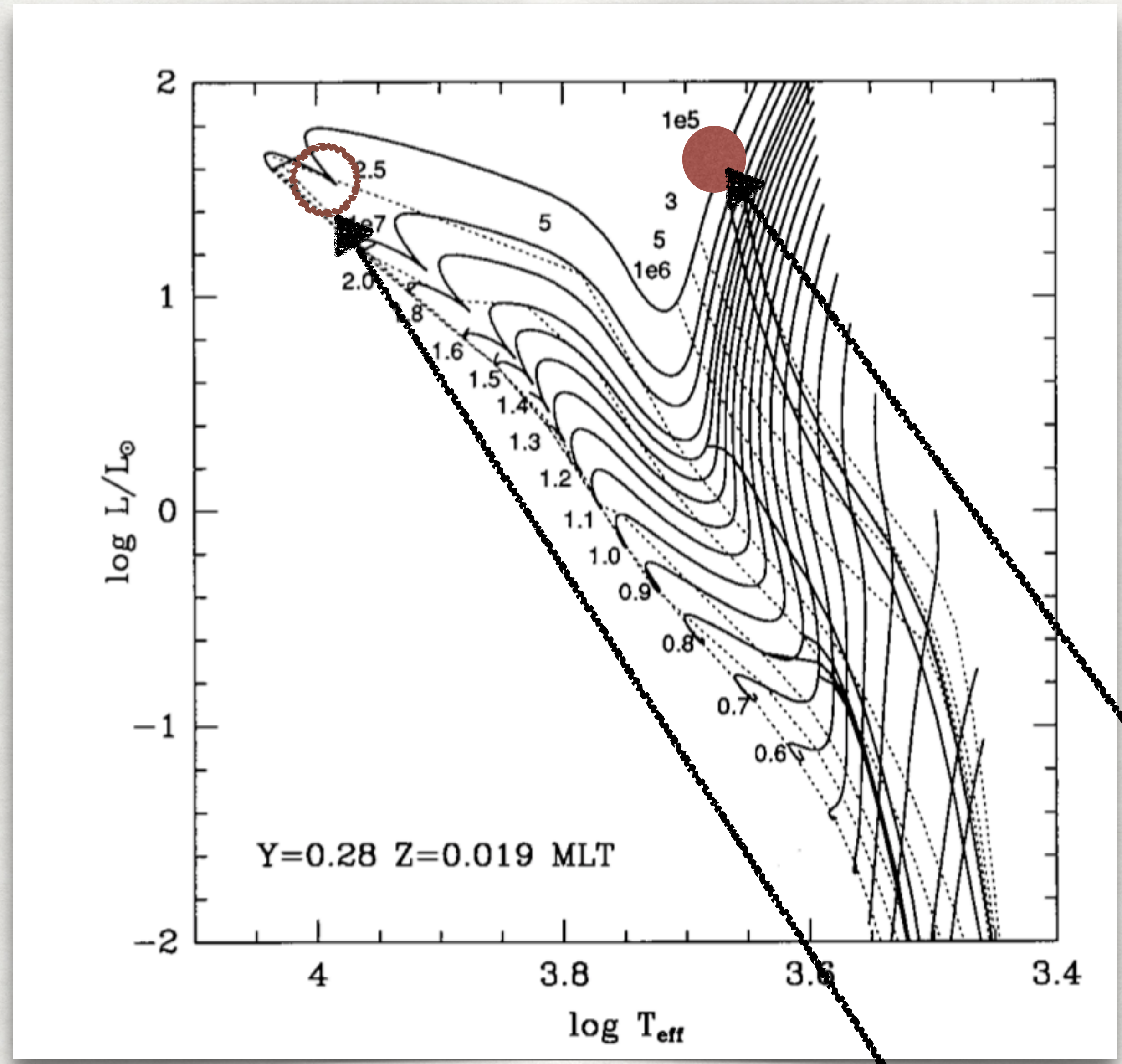
A DESCRIPTION



- Formation: dynamical collapse
- Hayashi track: pre-main sequence phase starts
- Radiative core develops : star moves to left
- **H burning and thermal equilibrium**: star on Zero Age Main Sequence (ZAMS)

PRE-MAIN SEQUENCE (PMS) PHASE

A DESCRIPTION



- Formation: dynamical collapse
- Hayashi track: pre-main sequence phase starts
- Radiative core develops : star moves to left
- H burning and thermal equilibrium: star on ZAMS

Other nuclear reactions along the way

- Deuterium ^2H, $X_D \sim 10^{-5}$ at $T \sim 10^6$ K on H Trak

$$^2H +^1 H \rightarrow^3 H_e + \gamma \text{ (pp)}$$

5.5 MeV per reaction: halts contraction for ~10^5 yr

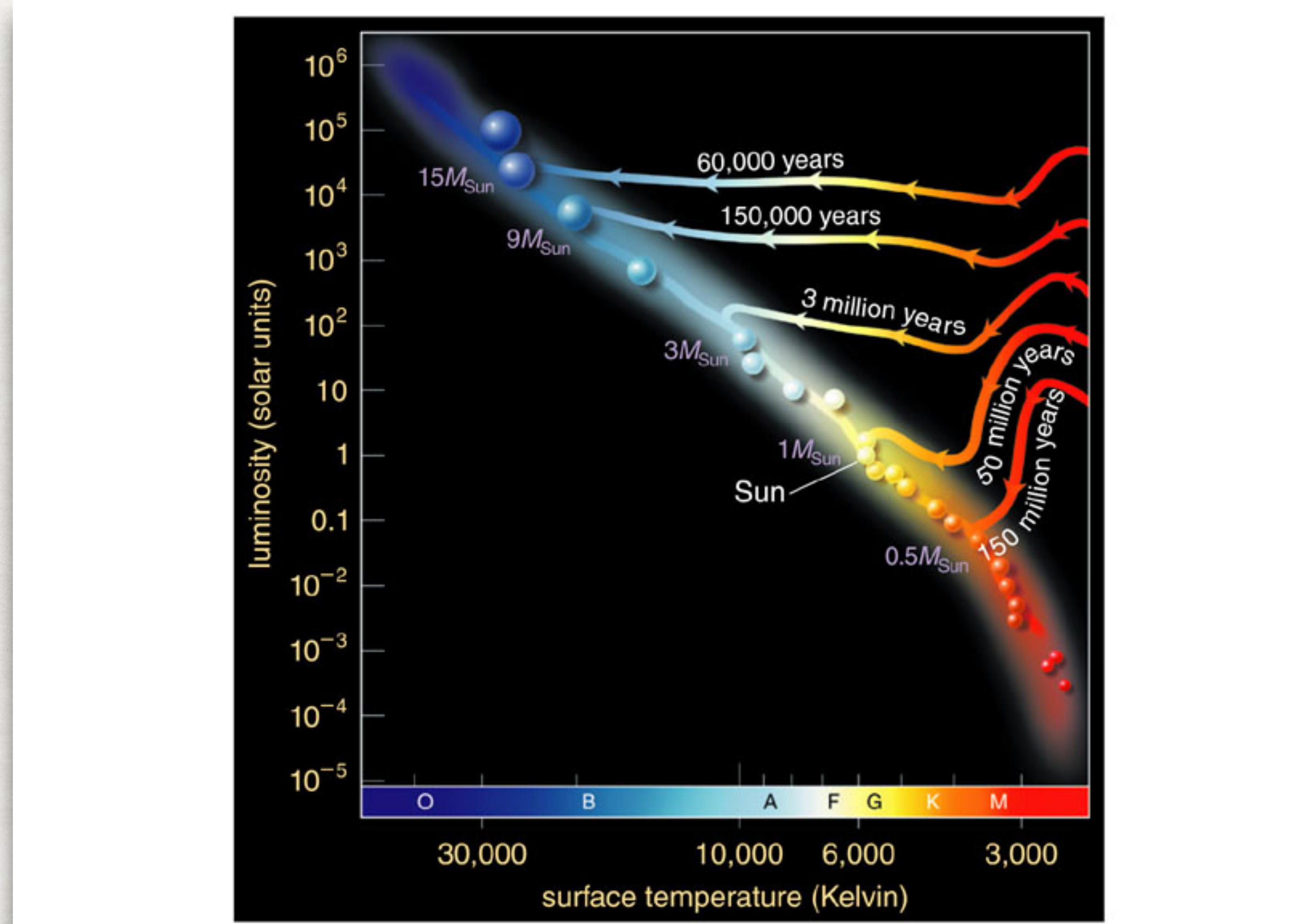
- Almost all ^{12}C is converted into ^{14}N before ZAMS

$$^{12}C +^1 H \rightarrow^{13} N + \gamma \text{ (CNO)}$$

it causes "V" in tracks also for stars ``pp-cycle dominated"

MASSIVE STARS EVOLVE FASTER

$$\tau_{\mathrm{K-H}} \approx 3 \times 10^7 \text{ yr} \left(\frac{M}{M_{\mathbf{sun}}}\right)^2 \left(\frac{R}{R_{\mathrm{sun}}}\right) \left(\frac{L}{L_{\mathrm{sun}}}\right)^{-1}$$



Contraction toward the main sequence takes only ~1% of star life

Stars spend ~80% of star life on the Main Sequence

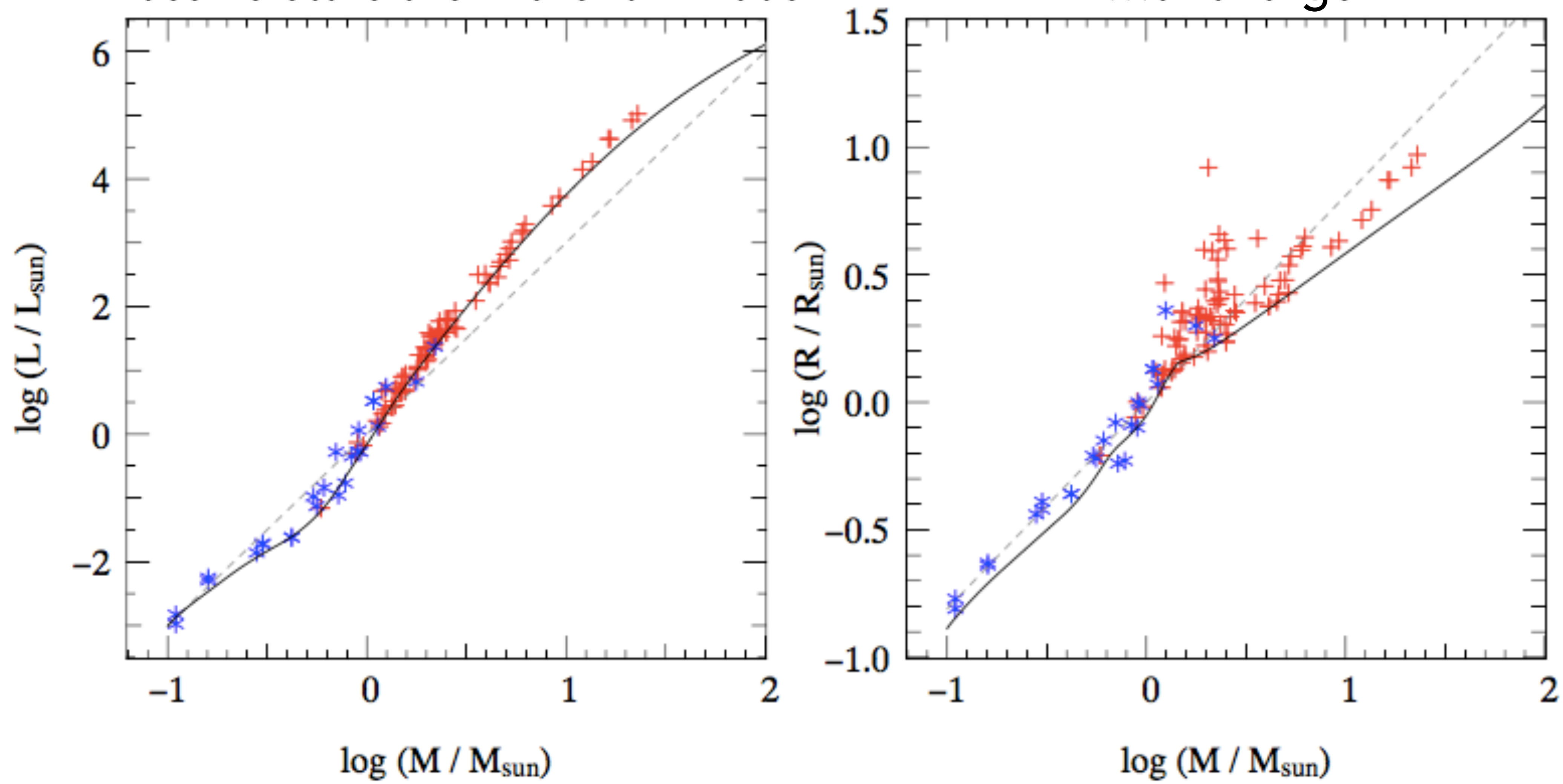
EVOLUTION OF STARS: FROM ZERO AGE TO END OF MAIN SEQUENCE

CH 9: 9.2, 9.3

just read 9.3.3 and 9.3.4

THE ZERO AGE MAIN SEQUENCE

- The main sequence phase is characterised by core-hydrogen burning to helium
- The star has amble time to reach hydrostatic and thermal equilibrium and ``forgets" about the initial contraction phase, expect for initial metallicity
  - massive stars are more luminous
  - ...and larger



- Detailed models using equations we derived can explain observations (solid lines)

HOMOLOGOUS RELATIONS (7.4)

Assuming: along the main sequence stars can be simply homologously rescaled WITH a different mass: i.e. the relative mass distribution is identical

$$\frac{r_1(x)}{r_2(x)} = \frac{R_1}{R_2}$$

$$x = \frac{m_1}{M_1} = \frac{m_2}{M_2}$$

$$\frac{\rho_2(x)}{\rho_1(x)} = \frac{M_2}{M_1}\left(\frac{R_2}{R_1}\right)^{-3}.$$

If we now assume that they are all (see 7.4.1, 7.4.2):

- radiative stars
- homogenous composition
- hydrostatic equilibrium
- thermal equilibrium
- constant opacity
- ideal gas EOS
- hydrogen burning:

$$\epsilon_{\rm nuc} = \epsilon_0 \rho T^{\nu}$$

$$\boxed{L \propto \frac{1}{\kappa}\mu^4 M^3}$$

Mass-Luminosity relation

$$\boxed{R \propto \mu^{(\nu-4)/(\nu+3)}\, M^{(\nu-1)/(\nu+3)}}$$

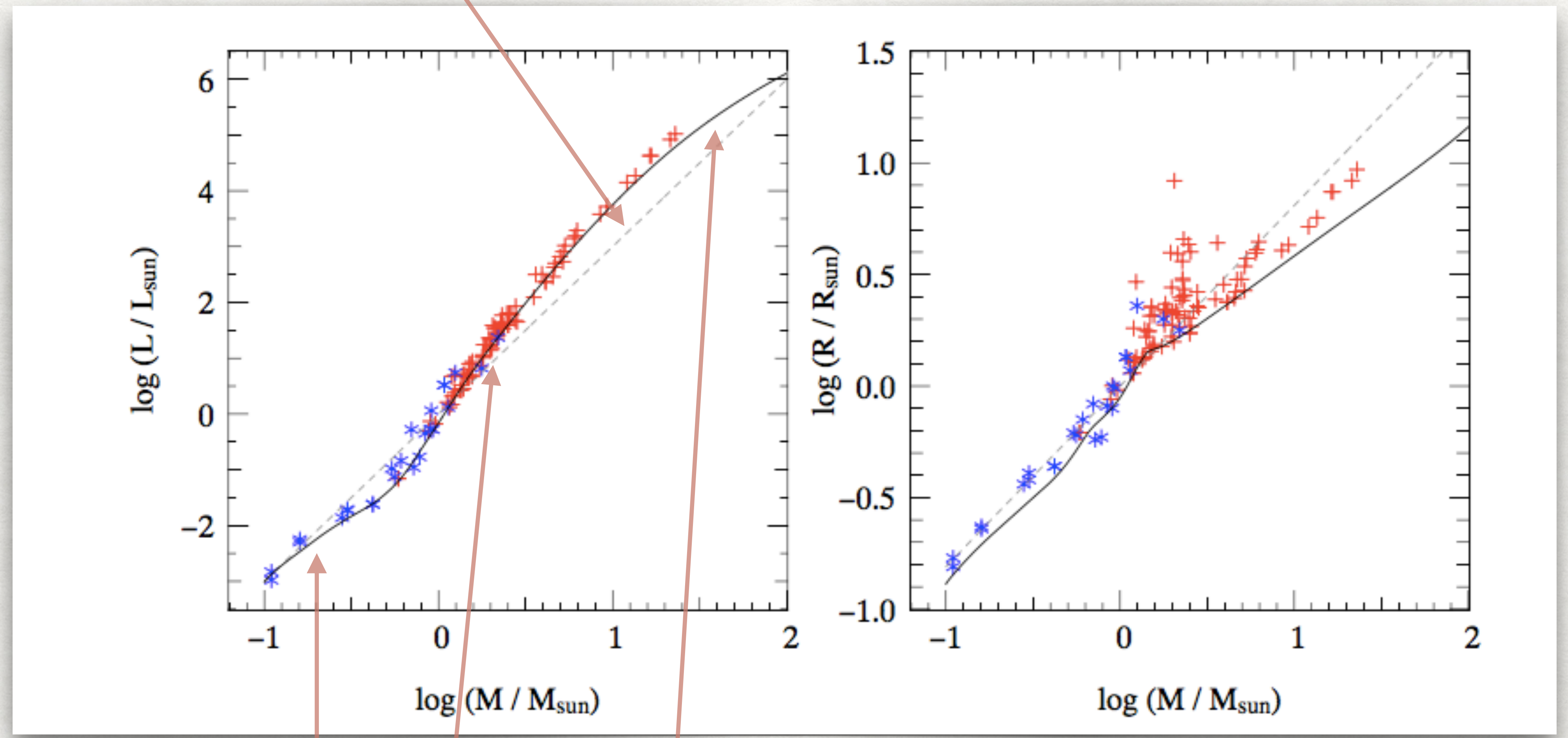
$$\nu = 18 \rightarrow \quad R \propto M^{0.81}$$

$$\nu = 4 \rightarrow \quad R \propto M^{0.43}$$

Mass-Radius relation

SPOT THE DIFFERENCES!

$$L \propto \mu^4 M^3 \qquad R \propto \mu^{2/3} M^{0.81} \quad \text{(CNO)}$$



Qualitative good agreement but:

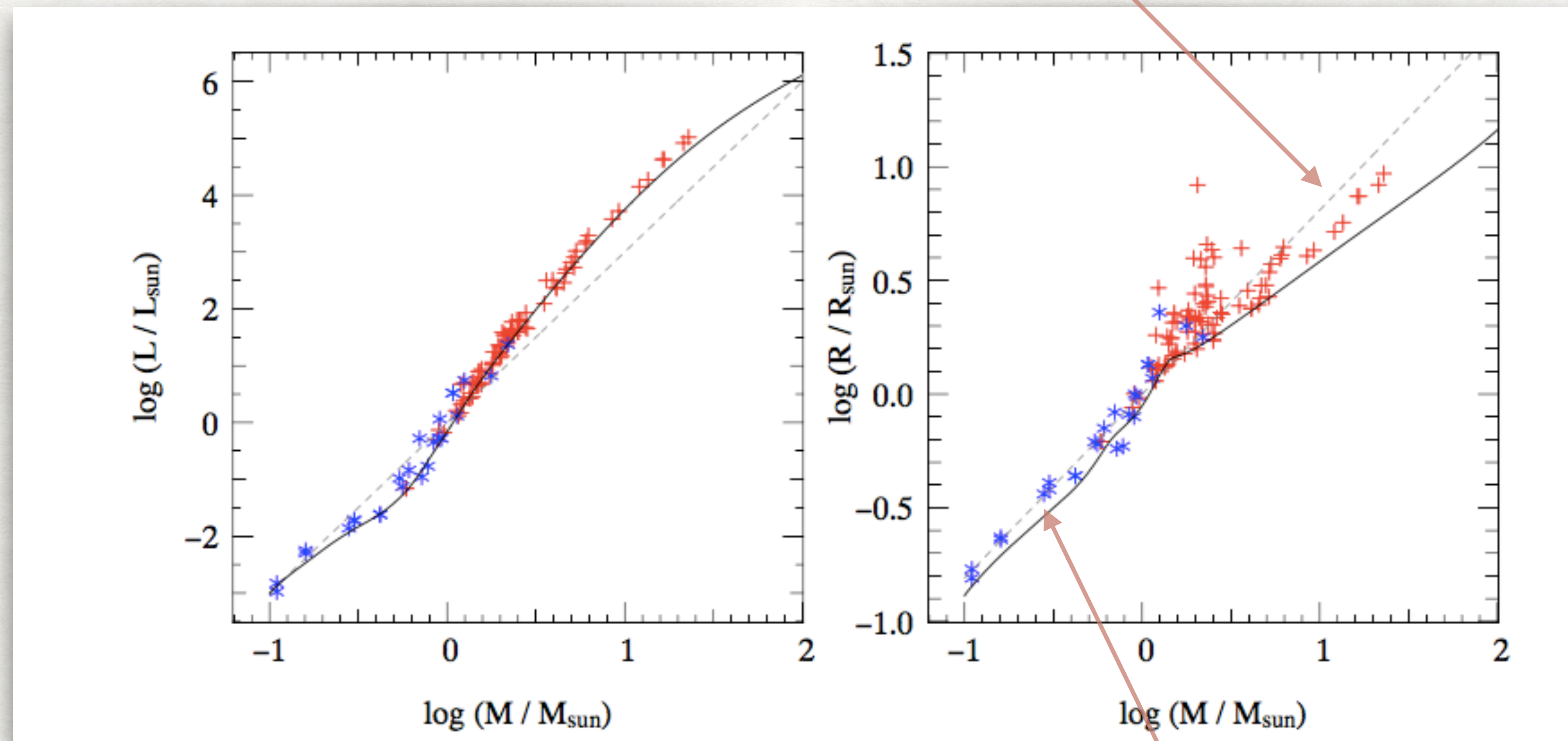
< 1 M convection becomes dominant

1-10 M radiative envelope but opacity (free-free and bound free) increases towards atmosphere

>50 M radiation pressure becomes not negligible

SPOT THE DIFFERENCES!

$$L \propto \mu^4 M^3 \qquad R \propto \mu^{2/3} M^{0.81} \quad \text{(CNO)}$$



Qualitative good agreement but:

Same reasons for disagreement + < 1 M pp dominates

ZERO MAIN SEQUENCE ON H-R

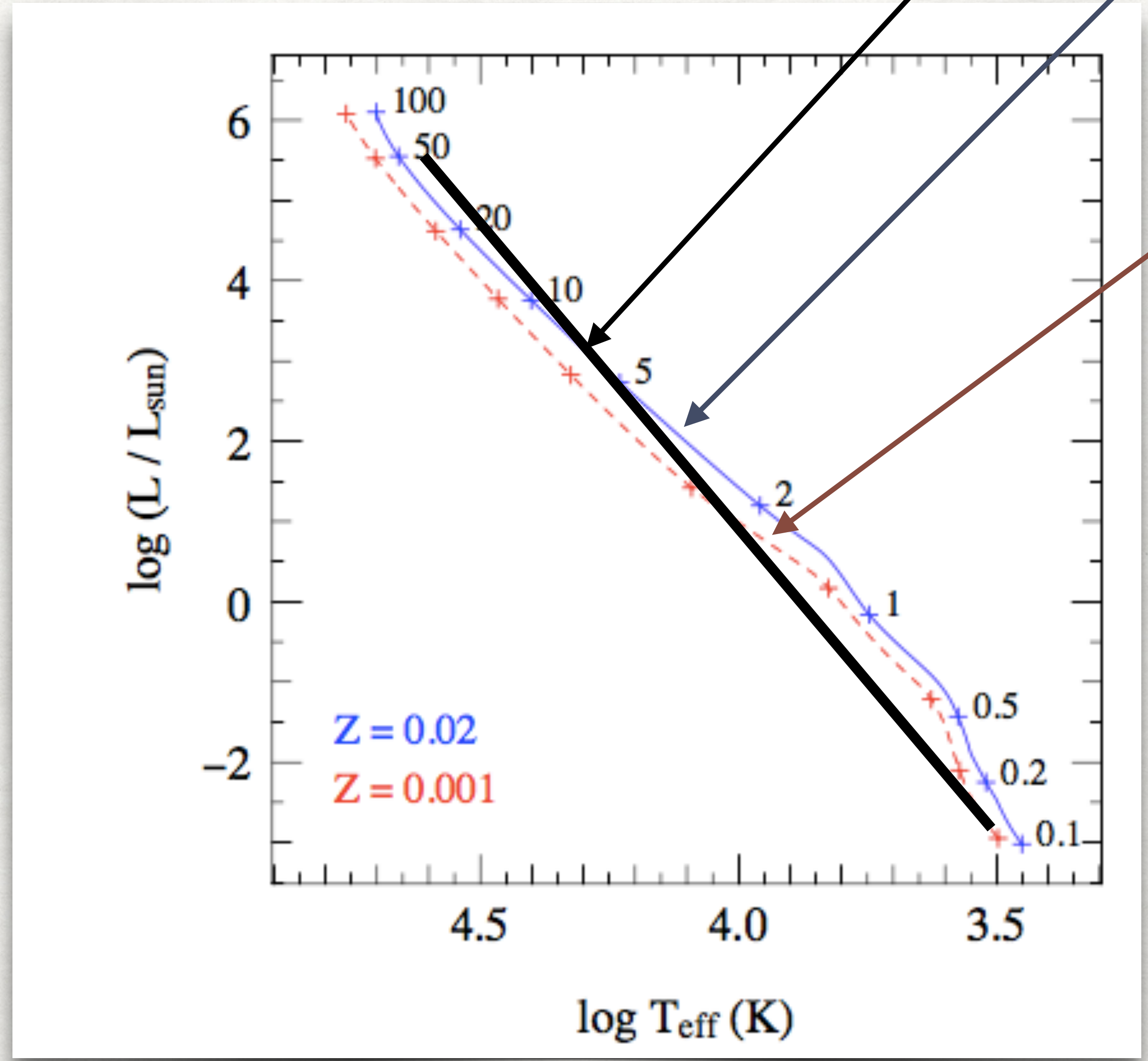
$$L \propto \mu^4 M^3$$

$$L \propto T_{\rm eff}^4 R^2 \propto T_{\rm eff}^4 M^{1.6}$$

$$L \propto T_{\rm eff}^{8.6} \mu^{-4.6}$$

$$\mu^{-1} \approx 0.55$$

$$\mu^{-1} \approx 0.53$$



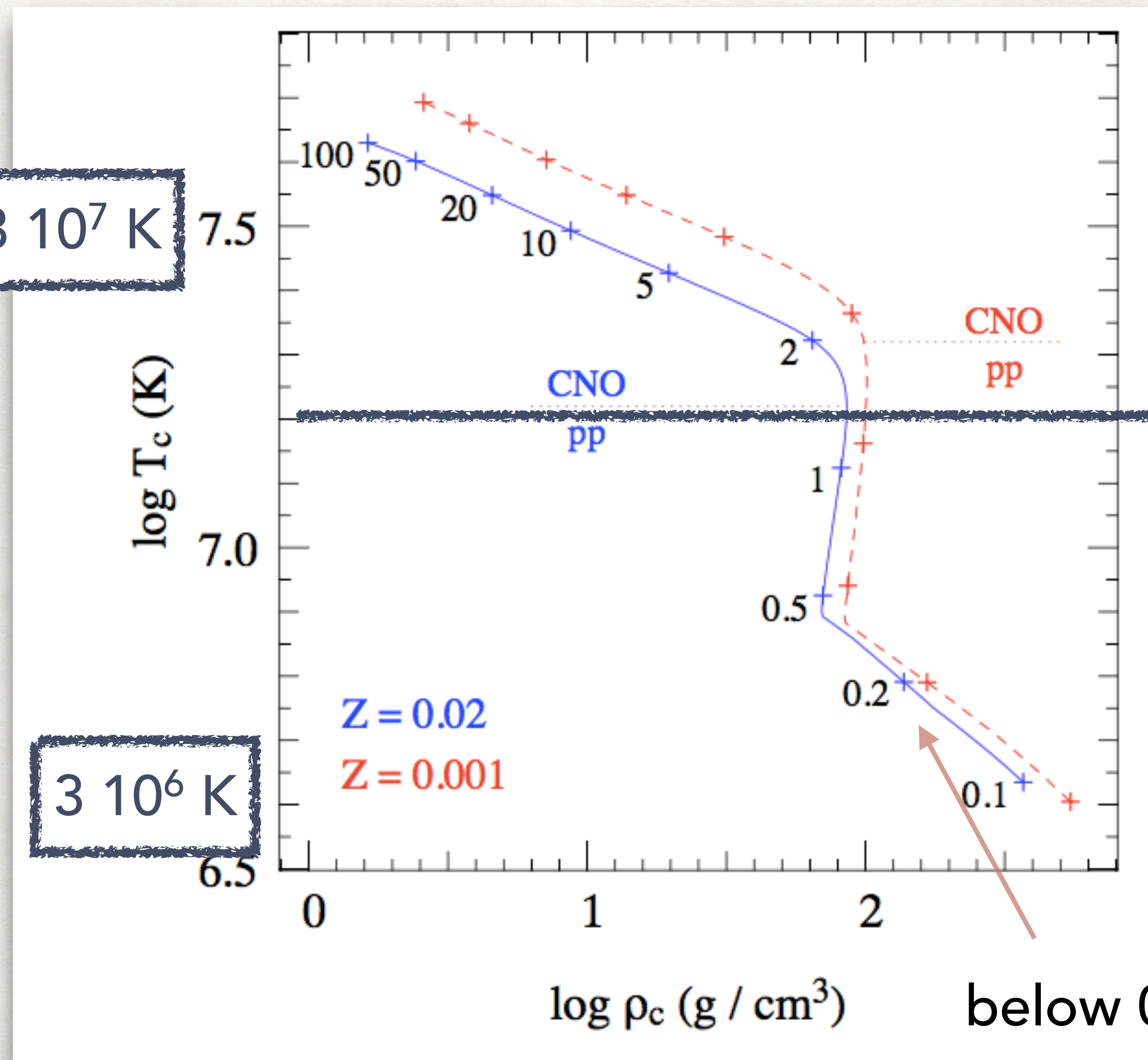
CENTRAL CONDITIONS ON THE M-S

Homologous modelling with same assumptions as before gives

pp $\quad T_c \propto M^{0.57} \qquad \rho_c \propto M^{-0.3}$

CNO $\quad T_c \propto M^{0.21} \qquad \rho_c \propto M^{-1.4}$

3 10^7 K



3 10^6 K

1.7 10^7 K

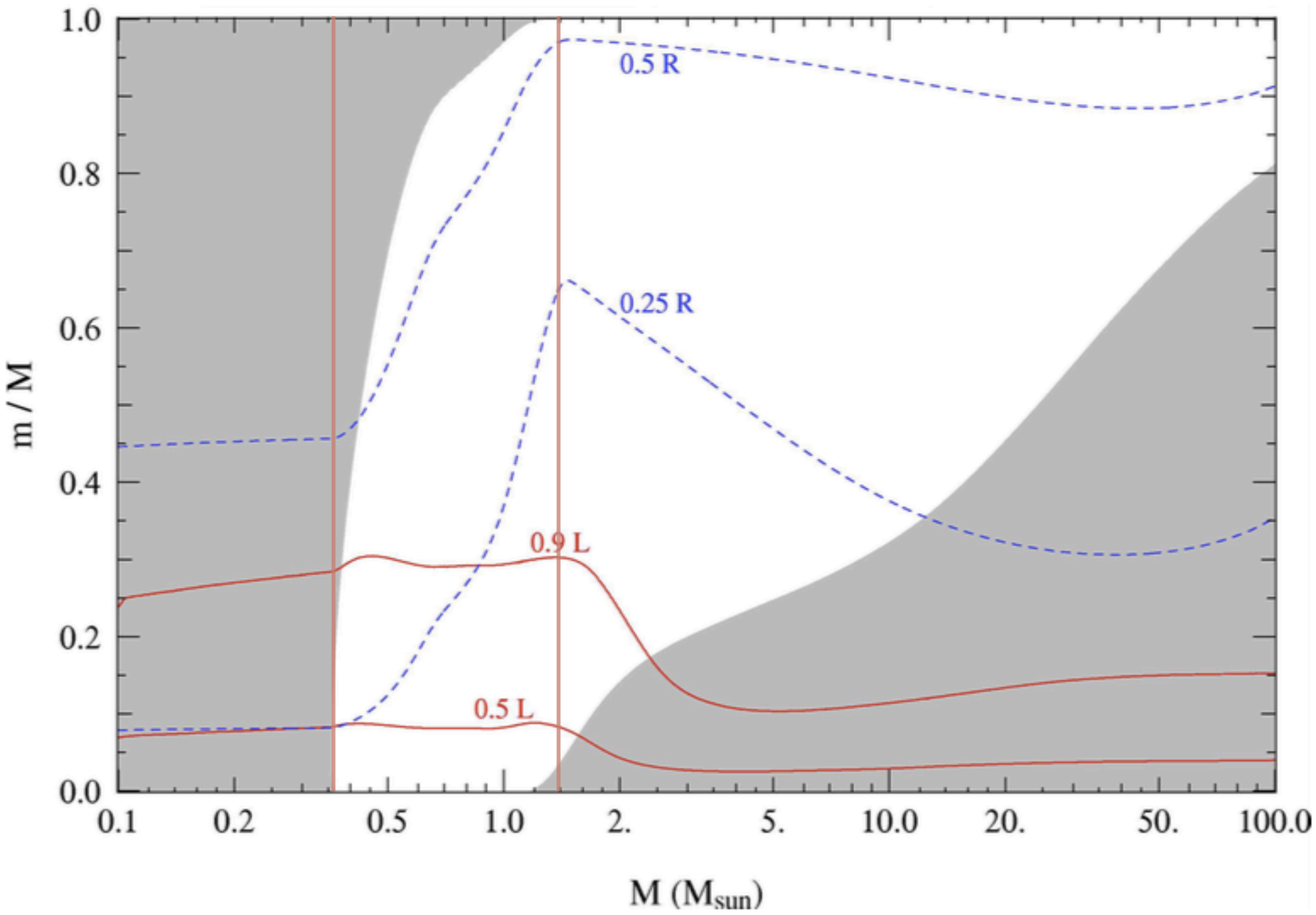
: transition for solar composition

M>1.3 M_{sun} —> CNO

M< 1.3 M_{sun} —> pp

below 0.4 Msun fully convective

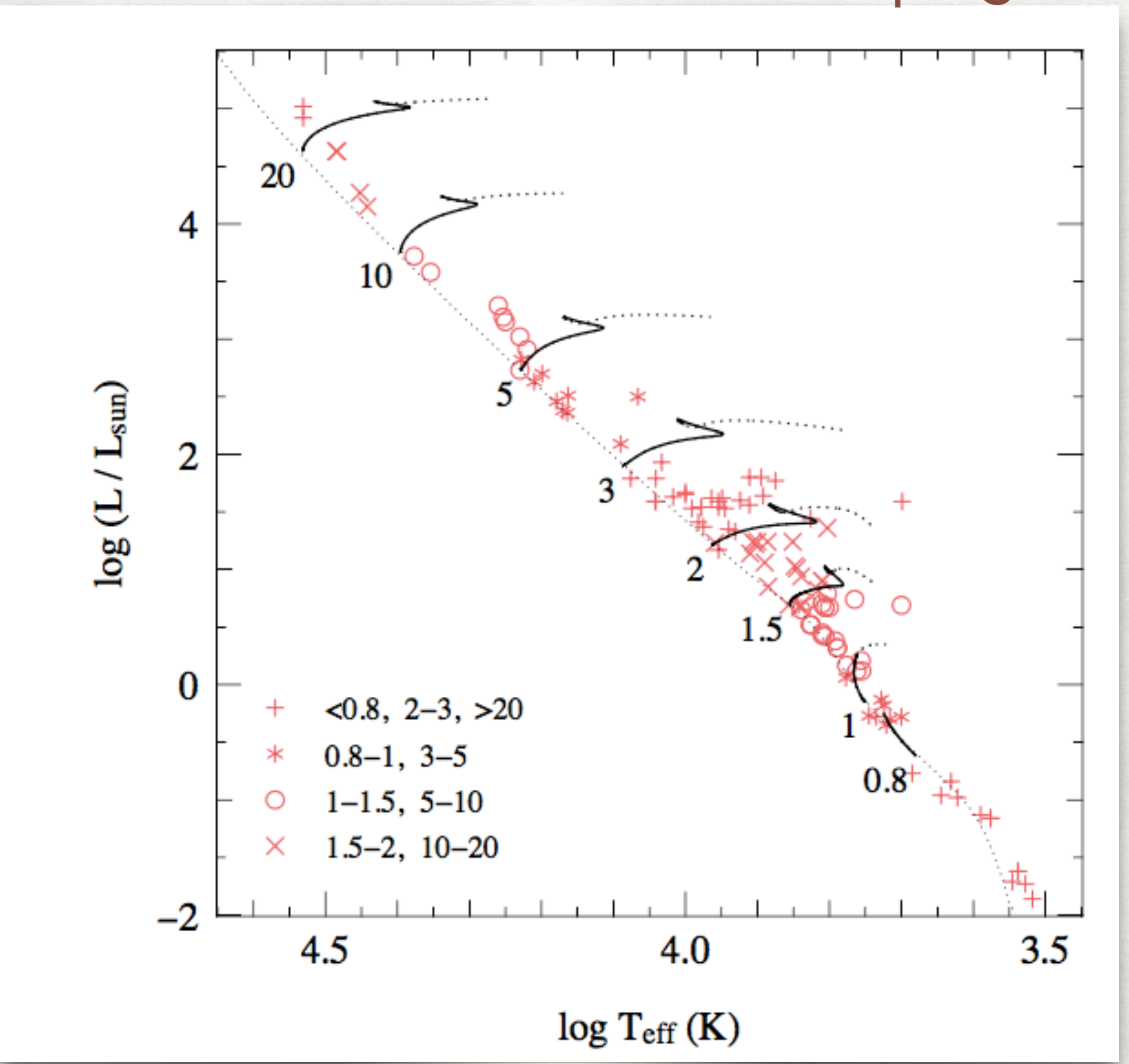
CONVECTIVE REGIONS



- completely convective, for $M < 0.35\ M_\odot$,
- radiative core + convective envelope, for $0.35\ M_\odot < M < 1.2\ M_\odot$,
- convective core + radiative envelope, for $M > 1.2\ M_\odot$.

EVOLUTION DURING H-BURNING

the evolution towards up-right is given by **the change in composition**



μ increases in the core as H —> He

- Stars get brighter:

$$L \propto \mu^4 M^3$$

- Radius increases

because:

Nuclear reaction act as a thermostat

$\epsilon_{\rm pp} \propto \rho T^4$ $\epsilon_{\rm CNO} \propto \rho T^{18}$

$$\frac{P_{\rm c}}{\rho_{\rm c}} \propto T_{\rm c}/\mu \quad \text{decreases}$$

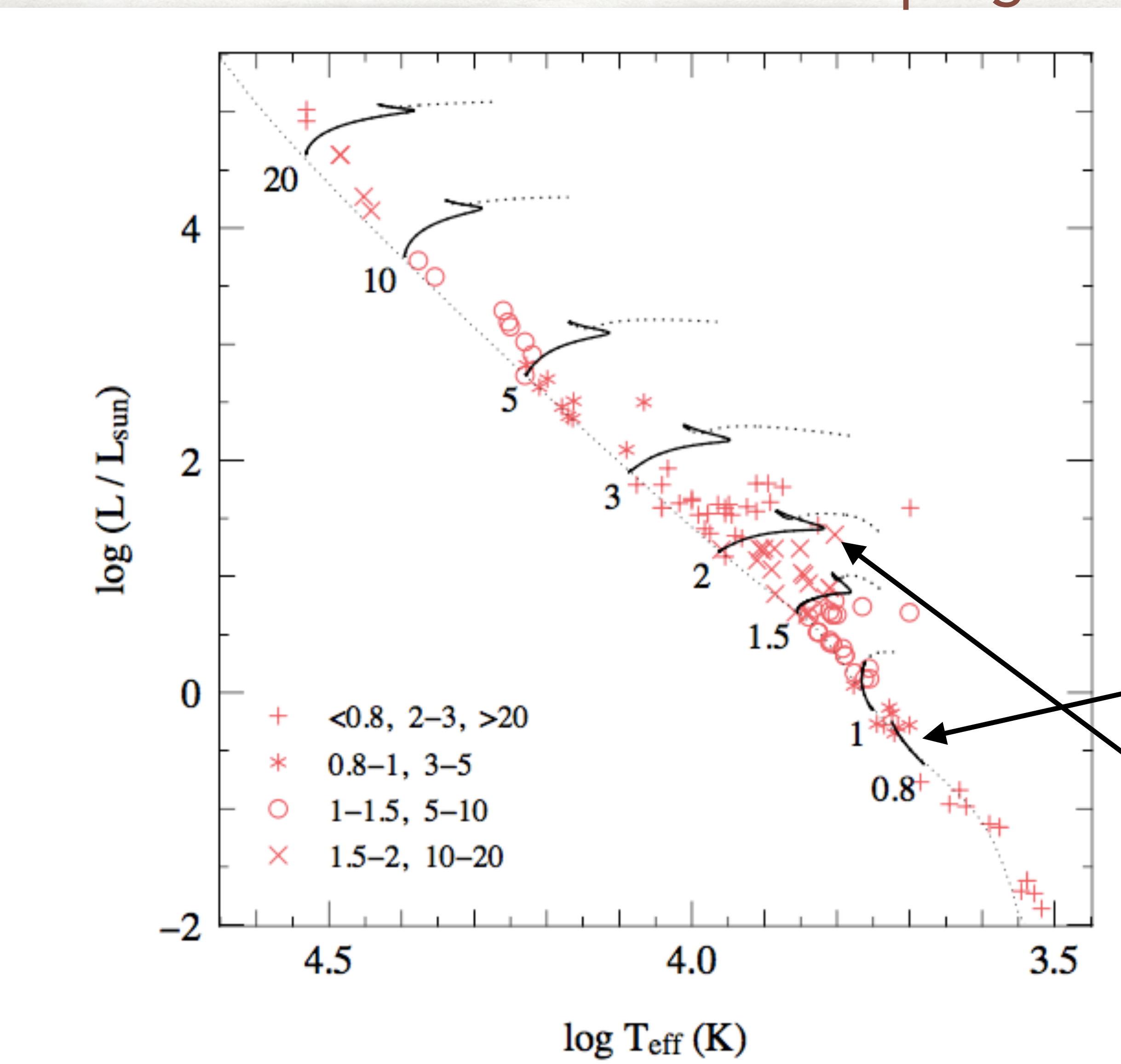
$$P_{\rm env} = \int_{\rm m_c}^{M} \frac{Gm}{4\pi r^4} dm \quad \text{expansion}$$

Chapter 9.3.1 - 9.3.2

- M >1.3 M_{sun} bigger by a factor of 2-3
- M <1.3 M_{sun} more modest increase

EVOLUTION DURING H-BURNING

the evolution towards up-right is given by **the change in composition**



μ increases in the core as H —> He

brighter

$$\sigma T_{\rm eff} = \frac{L}{4\pi R^2}$$

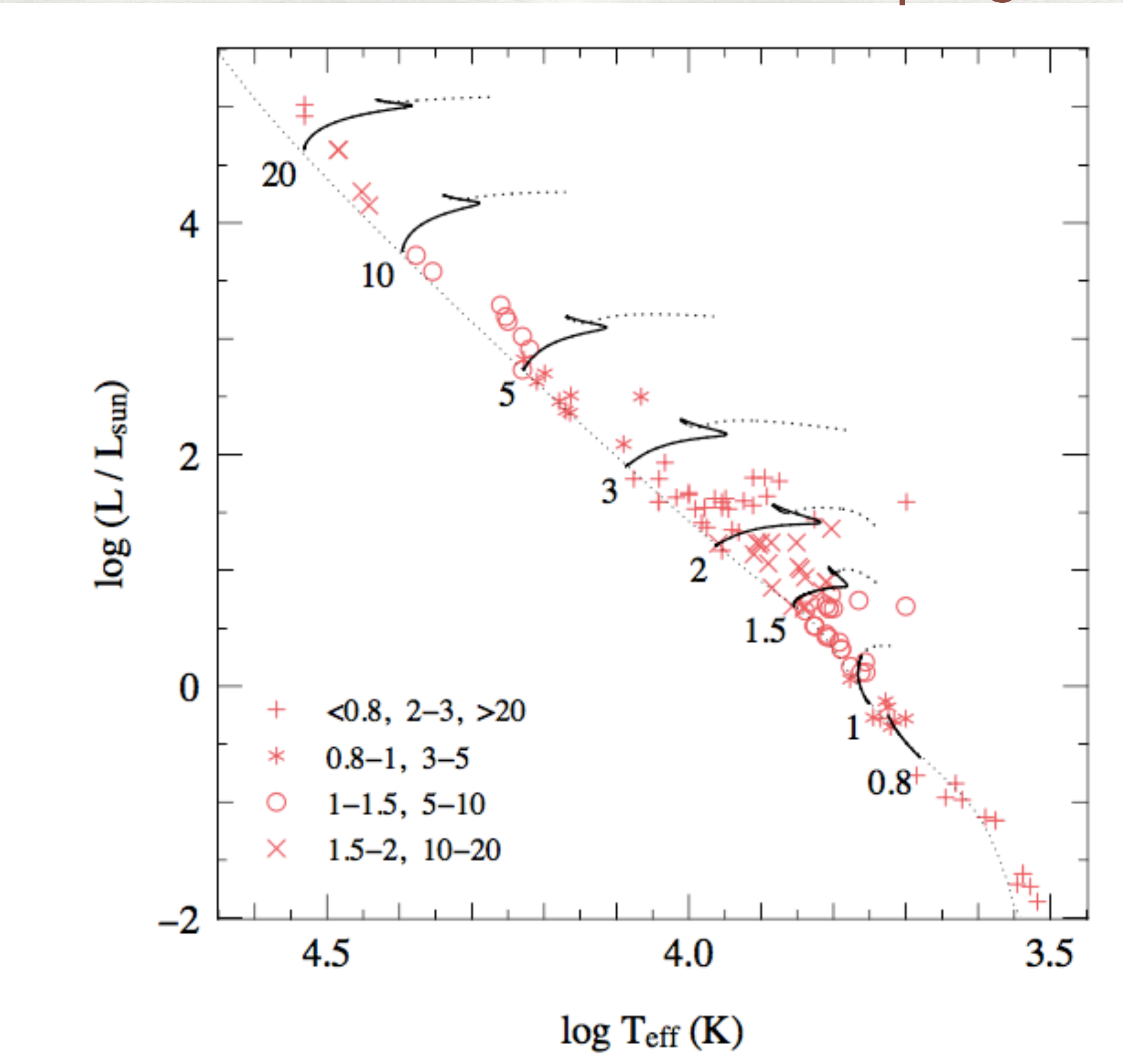
Consequence on surface temperature:

- M >1.3 M_{sun} : bigger ==> cooler
- M <1.3 M_{sun} : modest increase in R => hotter

Chapter 9.3.1 - 9.3.2

EVOLUTION DURING H-BURNING

the evolution towards up-right is given by **the change in composition**



- M >1.3 M_{sun} : large energy concentration of nuclear energy in the centre implies high l/m
- —> convective core
- —> mixed homogeneously
- —-> increase of lifetime of H-burning phase

- M <1.3 M_{sun} :
- —-> radiative cores
- —-> H abundance gradient : increases outward

Chapter 9.3.1 - 9.3.2