SUMMARY AND STELLAR STABILITY

—Ch 7 :but only 7.1, 7.3, 7.4 (but not the derivations),7.5

Exercises: 7.1, 7.2,7.3

SUMMARY

H burning @ $T_c \sim 10^7$ K

$$4\,{}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2\,e^+ + 2\,\nu$$

p-p chain T_c $\quad \epsilon_{\rm pp} \propto X^2 \frac{\rho}{m_{\rm u}} T^4$

it starts with $\quad {}^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu$

$${}^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$$

$${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H}$$

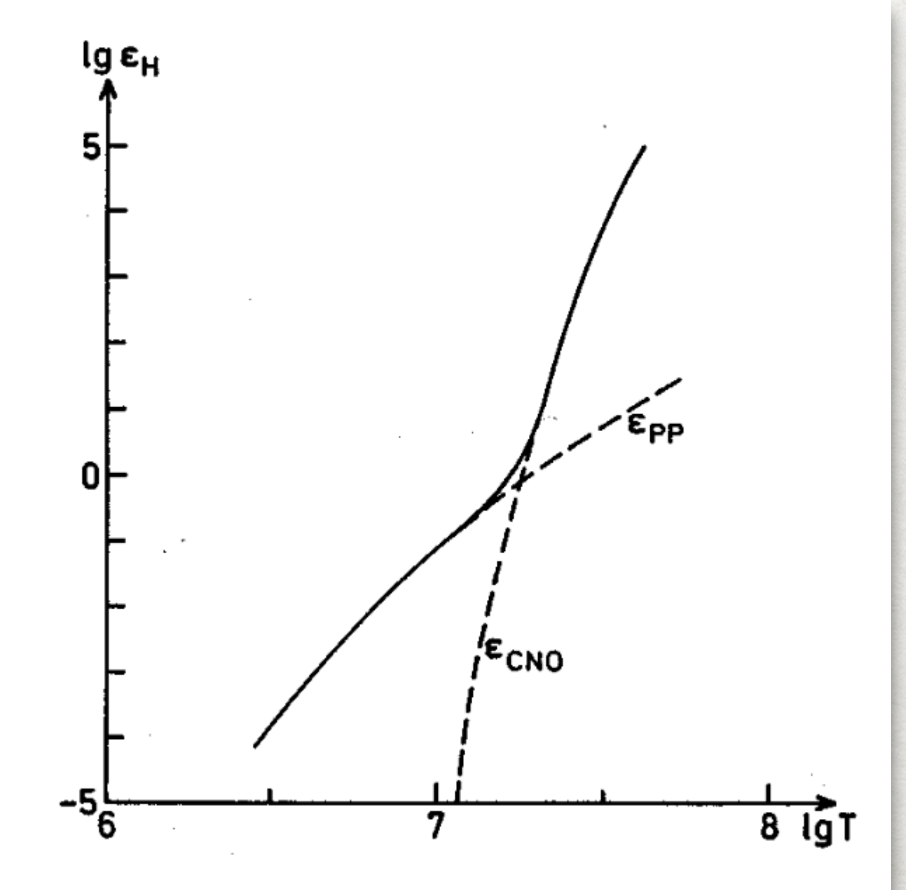
pp1

pp2 pp3

involving short lived Be, Li

$T_c < 1.5\ 10^7$ K

main energy source for Sun



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p-p chain

pp1 : main source of Sun

$$\epsilon_{\rm pp} \propto X^2 \frac{\rho}{m_{\rm u}} T^4$$

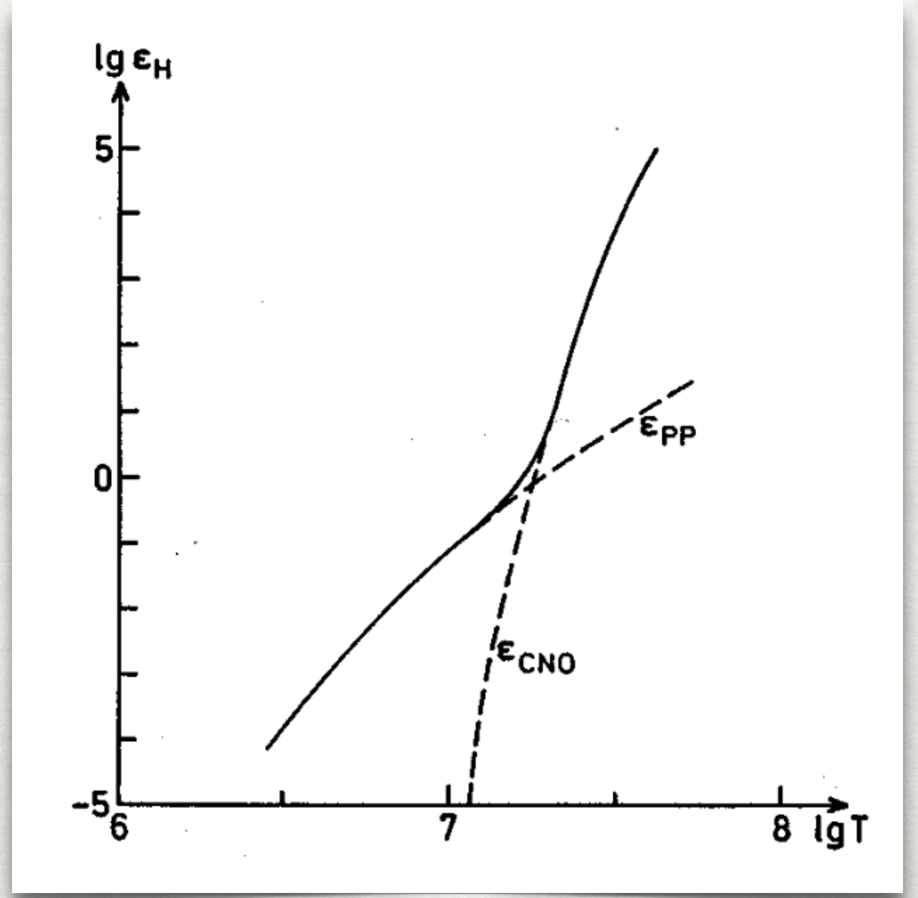
CNO chain $T_c > 1.5\ 10^7$ K

Starts with p captured by 12C

$$\epsilon_{\rm pp} \propto X X_{14} \frac{\rho}{m_{\rm u}} T^{18}$$

^{14}N+p—>^{15}O

slowest reaction: sets the rate



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He burning @ $T_c \sim 10^8$ K

it produces ${}^{12}C$ & ${}^{16}O$

$$\epsilon_{3\alpha} \propto Y^3 T^{40}$$

SUMMARY

H burning @ $T_c \sim 10^7$ K

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He burning @ $T_c \sim 10^8$ K $\quad$ it produces ${}^{12}C$ & ${}^{16}O$ $\quad \epsilon_{3\alpha} \propto Y^3 T^{40}$

Carbon (${}^{12}C$) burning: T> 5 10^8 K, leaves mostly ${}^{16}O$ ${}^{20}Ne$ ${}^{24}Mg$

Neon (${}^{20}Ne$) burning (photodisitegration for O): T>1.5 10^9 K leaves mostly ${}^{16}O$ ${}^{24}Mg$

Oxygen (${}^{16}O$) burning: T>2 10^9 K leaves mostly ${}^{28}Si$ ${}^{32}S$

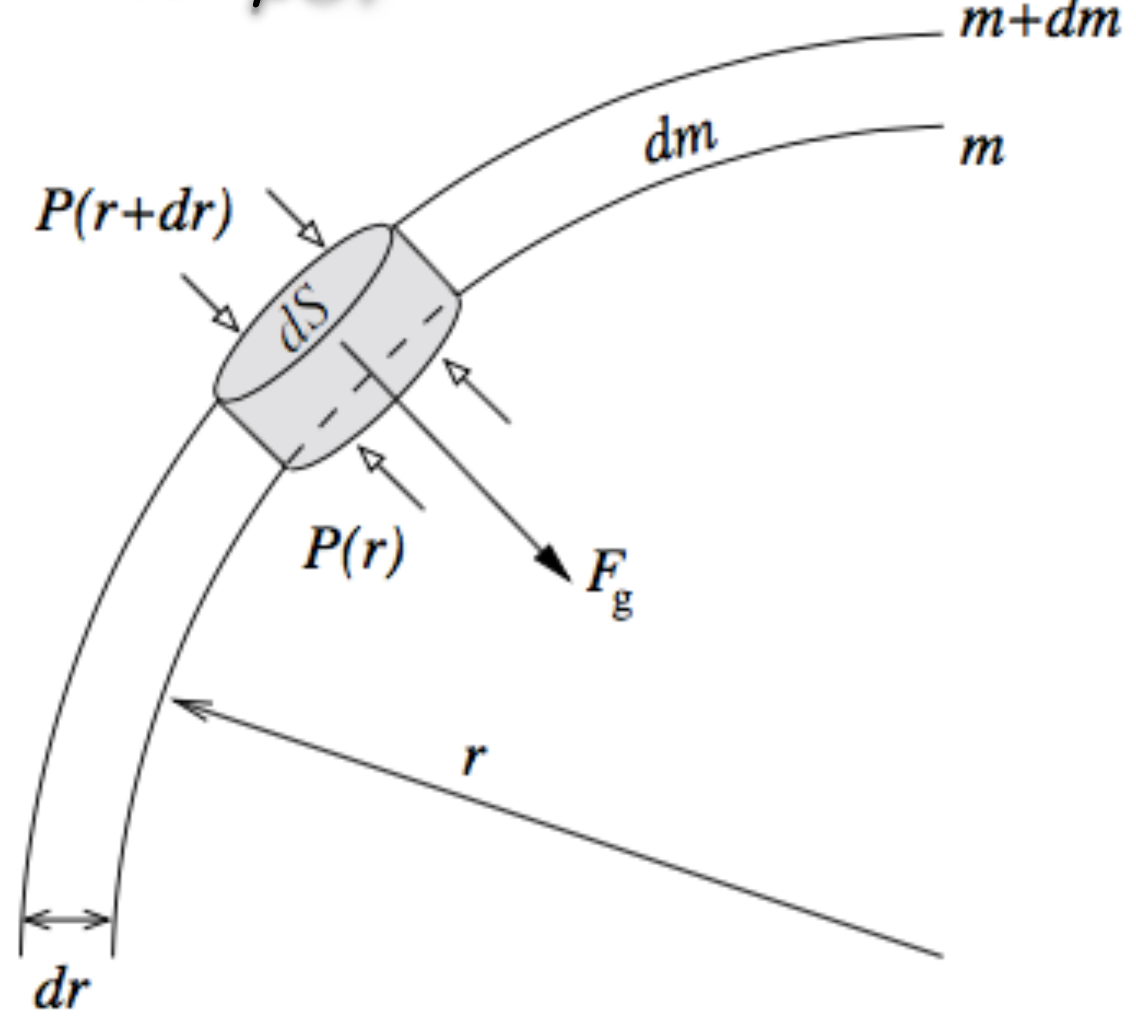
Silicon (${}^{28}Si$) burning: T>3 10^9 K leaves mostly ${}^{56}Fe$

The differential equation of stellar structure

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

mass conservation

$$\partial m = 4\pi r^2 \rho \partial r$$



The differential equations of stellar structure

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

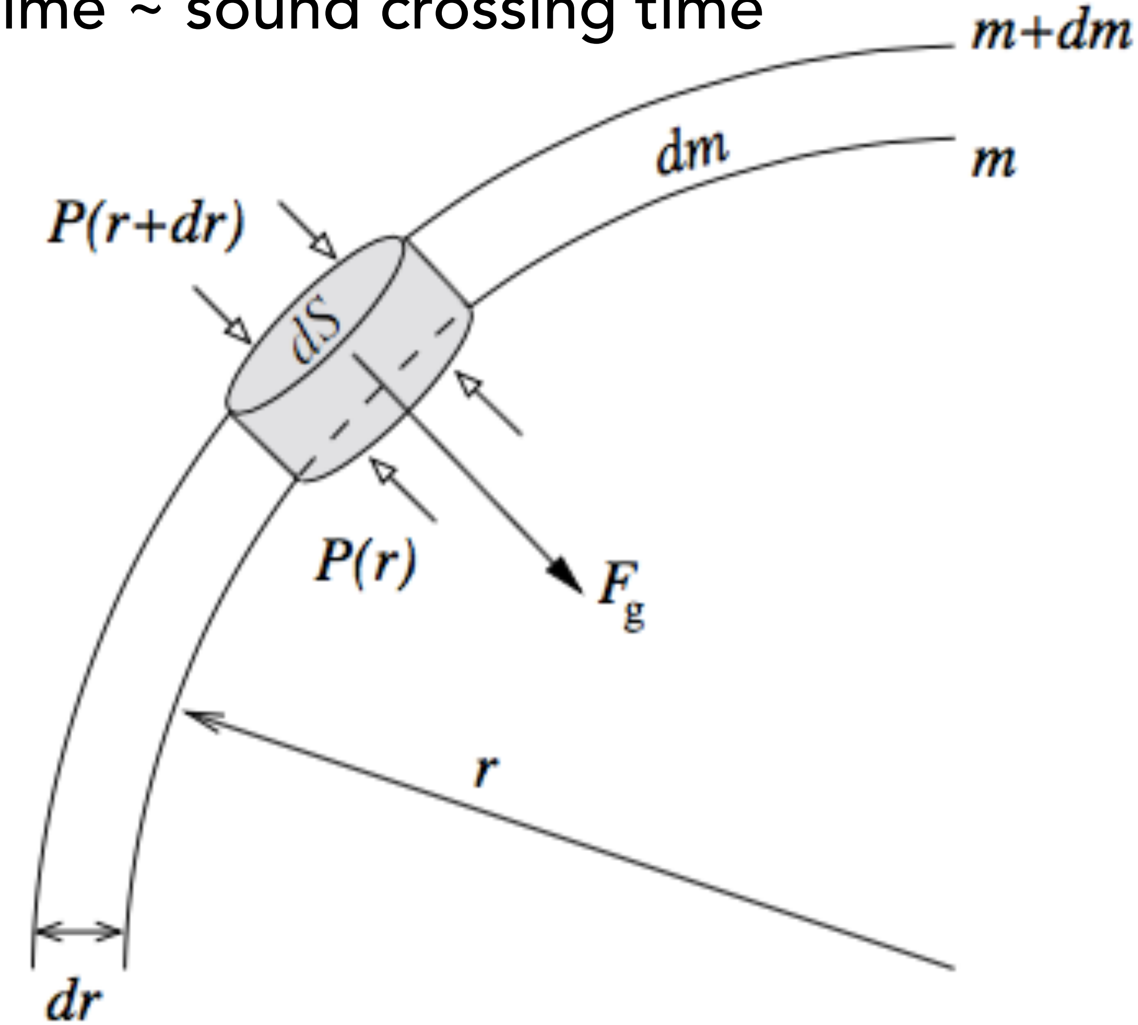
mass conservation

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \cancel{\frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}}$$

$$\tau_{\rm dyn} \ll \tau_{\rm nuc}$$

momentum conservation

free fall time ~ sound crossing time



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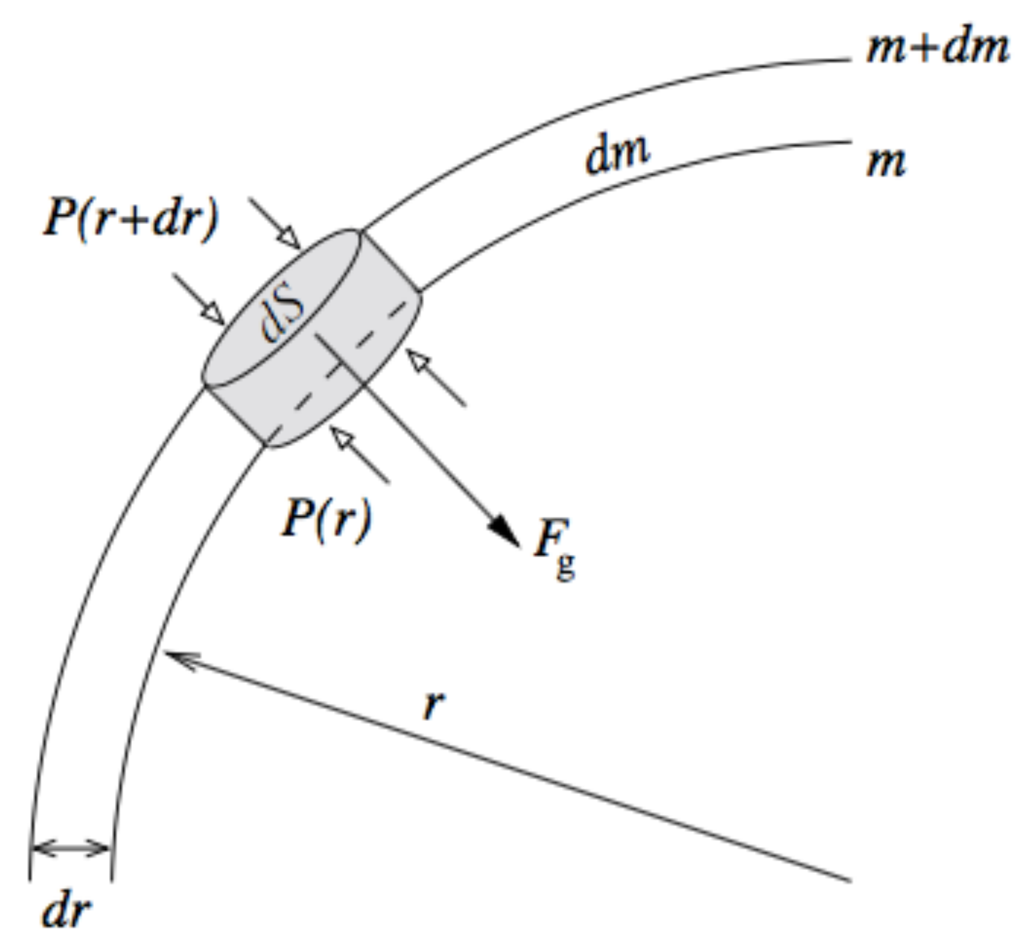
$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu - \cancel{T\frac{\partial s}{\partial t}}$$ energy conservation

$$\tau_{\rm KH} \ll \tau_{\rm nuc}$$

Changing in thermal state of the star from contraction (>0) or expansion (<0) on a thermal timescale



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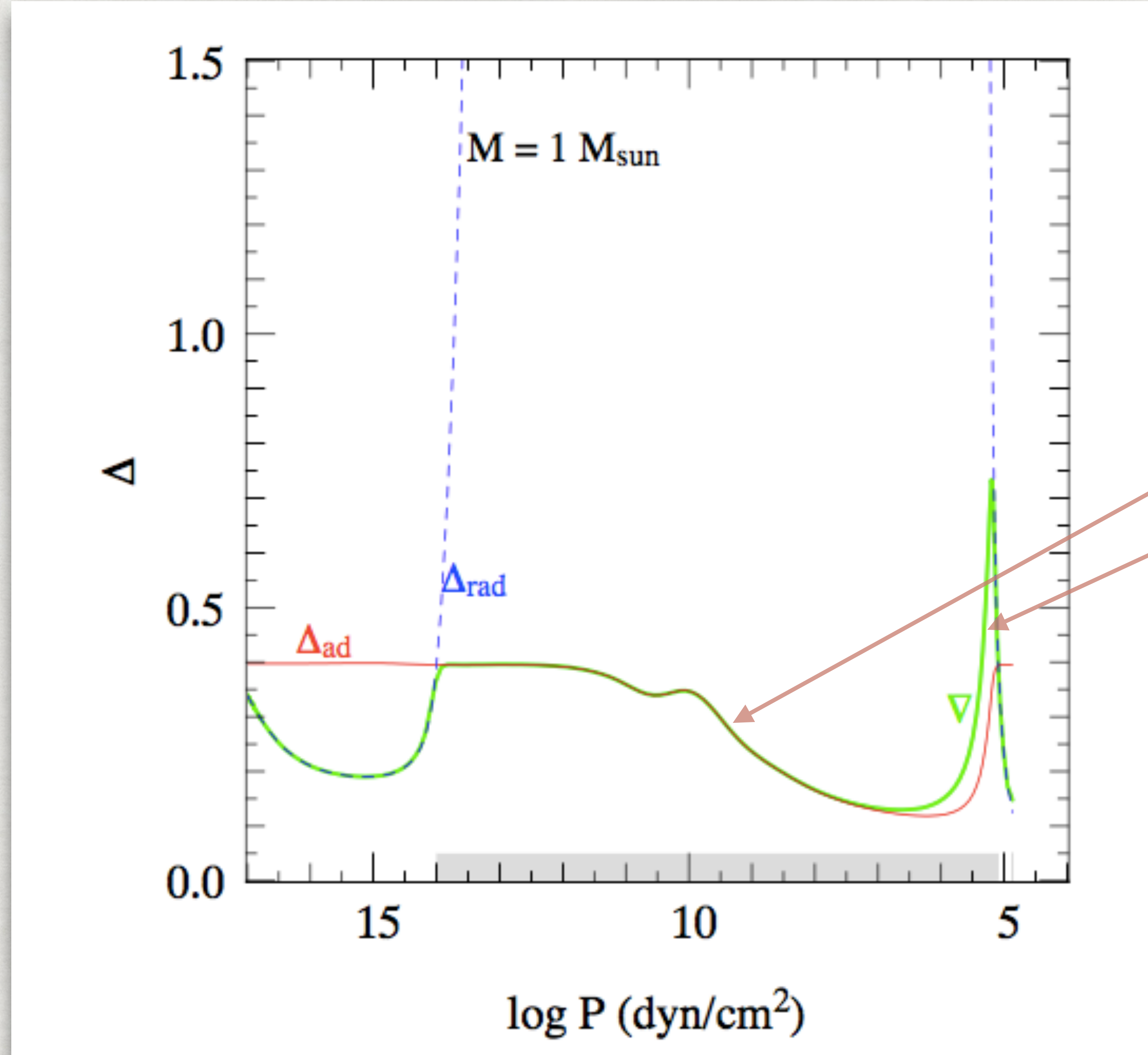
energy conservation

$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla \qquad \text{with} \qquad \nabla = \begin{cases} \nabla_{\rm rad} = \dfrac{3\kappa}{16\pi acG}\dfrac{lP}{mT^4} & \text{if } \nabla_{\rm rad} \leq \nabla_{\rm ad} \\ \nabla_{\rm ad} + \Delta\nabla & \text{if } \nabla_{\rm rad} > \nabla_{\rm ad} \end{cases}$$



$\Delta\nabla$: superadiabaticity

- ~0 in internal layers
- when not zero need to be calculated with LMT or simulations
- close to photosphere convection is inefficient and $\nabla = \nabla_{\rm rad}$

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$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\sum_j (1+\delta_{ij})\, r_{ij} + \sum_{k,l} r_{kl,i}\right) \quad \text{[+ mixing terms]} \qquad i = 1 \ldots N$$

N equations, one for each nucleus

mixing due to convection

$\partial t \approx \tau_{\rm nuc}$

Change is on the nuclear timescale

Previous conservation eqs.

can be solve assuming $X^i=X^i(m,t=0)$ since $\tau_{\rm nuc} \gg \tau_{\rm KH} \gg \tau_{\rm dyn}$

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The differential equations of stellar structure for stars in mechanical and thermal equilibrium given an initial composition, ignoring neutrino emission, important only in late stages

$$\frac{\mathrm{d}r}{\mathrm{d}m} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{\mathrm{d}P}{\mathrm{d}m} = -\frac{Gm}{4\pi r^4}$$

mechanical structure

$$\frac{\mathrm{d}l}{\mathrm{d}m} = \epsilon_{\mathrm{nuc}}$$

thermal structure

energetic structure

$$\frac{\mathrm{d}T}{\mathrm{d}m} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla \quad \text{with} \quad \nabla = \begin{cases} \nabla_{\mathrm{rad}} = \dfrac{3\kappa}{16\pi acG}\dfrac{lP}{mT^4} & \text{if } \nabla_{\mathrm{rad}} \le \nabla_{\mathrm{ad}} \\ \nabla_{\mathrm{ad}} & \text{if } \nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} \end{cases}$$

- with: X_i(m,t=0) constant in time other detail of star formation not important!
  and two boundary conditions at m=0 and m=M
- 1 independent variables: m (in general m,t)
- 4 unknown: r, ρ, T, l for 4 equations (in general +N Xi)
- Need specifying $P = P(\rho, T, X_i)$ and ϵ_{nuc}

Note: when P=P(\rho), mechanical and thermal structure are decoupled

GLOBAL DYNAMICAL STABILITY OF STARS (7.5.1)

WHAT HAPPENS WHEN HYDROSTATIC EQUILIBRIUM (HE) IS PERTURBED ?

Let's assume a loss of pressure make star compress:

- **adiabatically** : compression happens on a timescale $\ll \tau_{\mathrm{KH}}$

$$\frac{P'}{P} = \left(\frac{\rho'}{\rho}\right)^{\gamma_{\mathrm{ad}}}$$

- **homologously**, i.e. keeping the same *relative* mass distribution:

$$r'(m) = r(m')\left(\frac{R'}{R}\right)$$

$$m' = m$$

$$\Rightarrow \quad \frac{\rho'}{\rho} = \left(\frac{R'}{R}\right)^{-3}$$

$$\left.\begin{array}{l} \dfrac{P'}{P} = \left(\dfrac{\rho'}{\rho}\right)^{\gamma_{\mathrm{ad}}} \\[2ex] \dfrac{\rho'}{\rho} = \left(\dfrac{R'}{R}\right)^{-3} \end{array}\right\} \frac{P'}{P} = \left(\frac{R'}{R}\right)^{-3\gamma_{\mathrm{ad}}}$$

GLOBAL DYNAMICAL STABILITY OF STARS (7.5.1)

WHAT HAPPENS WHEN HYDROSTATIC EQUILIBRIUM (HE) IS PERTURBED ?

- **adiabatic homologous** compression: $\frac{P'}{P} = \left(\frac{R'}{R}\right)^{-3\gamma_{\rm ad}}$

- It can be easily shown (7.4) that to preserve HE in an homologous contraction:

$$\frac{P'}{P} = \left(\frac{R'}{R}\right)^{-4}$$

Therefore if:

$\gamma_{\rm ad} > \frac{4}{3} \rightarrow P' > P'_{\rm HE}$ the star will re-expand to regain HE, going back to initial status

$\gamma_{\rm ad} < \frac{4}{3} \rightarrow P' < P'_{\rm HE}$ the star will keep contracting: unstable!

more precisely:

$$\int \left(\gamma_{\rm ad} - \frac{4}{3}\right)\frac{P}{\rho}\,{\rm d}m$$ >0 ——> stable equilibrium

CASES OF DYNAMICAL INSTABILITY $\gamma_{\rm ad} < \frac{4}{3}$

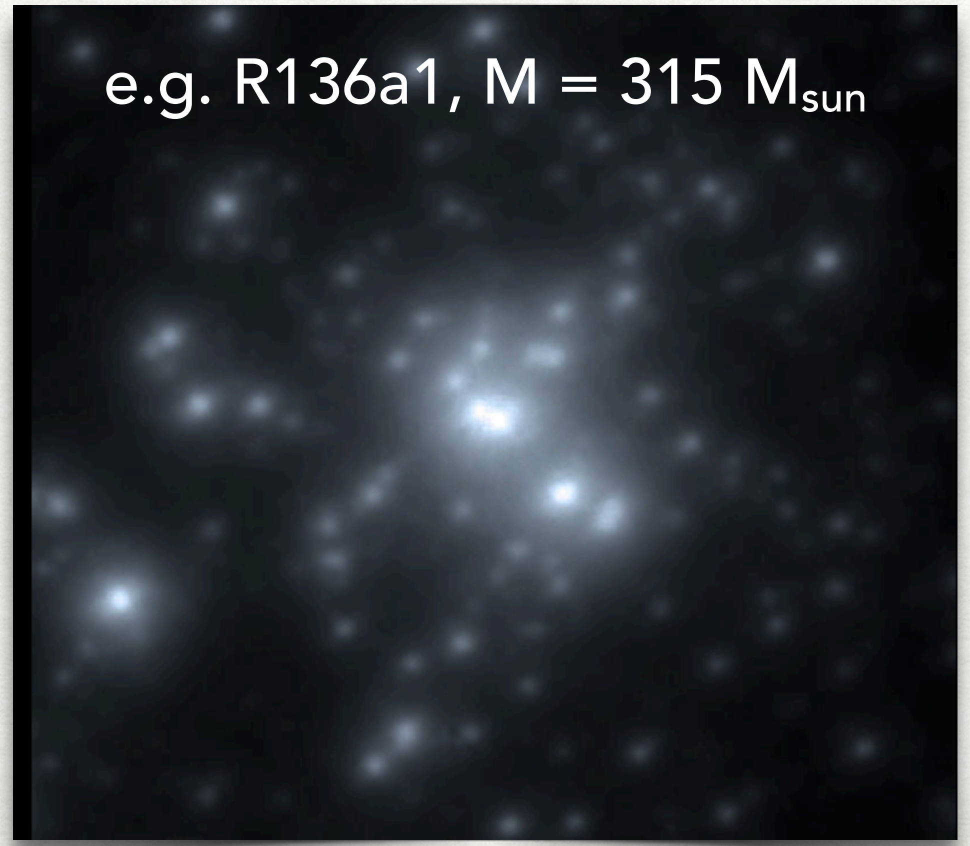
- A **relativistic -degenerate electron gas** $\gamma_{\rm ad} \to \frac{4}{3}$ and instability is expected when the total mass is higher than **the Chandrasekhar mass**: contraction will end up in collapse
- When **radiation pressure dominates** ($M > 100\ M_{sun}$) $\gamma_{\rm ad} \to \frac{4}{3}$ and the Virial Th.:

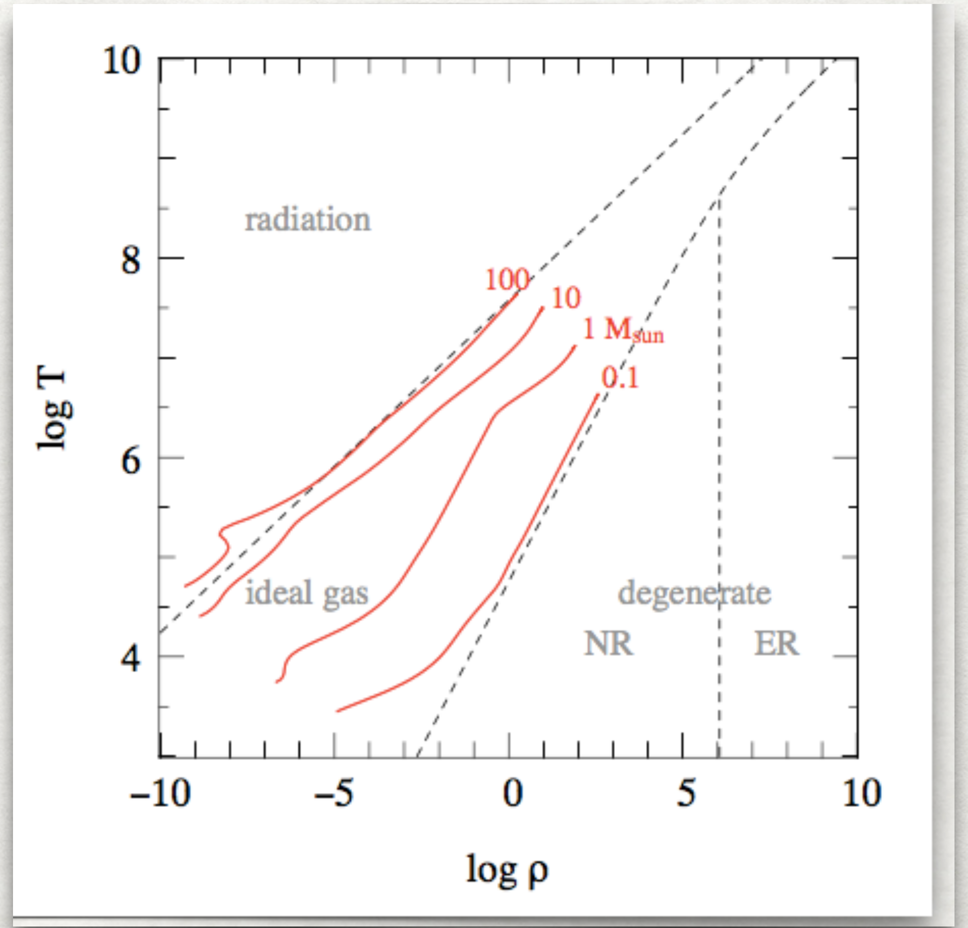
$$-E_{\rm int} \to E_{\rm gr}, \;\; E_{\rm tot} \to 0$$

The star becomes unbound. Ignoring rotation, this gives a maximum mass for a star of ~$100\ M_{sun}$.

However, a small degree of rotation stabilise the star to $M_{max} \sim 10^8\ M_{sun}$ (Fowler 1966)

unlike what the notes say





IONISATION TYPE PROCESSES

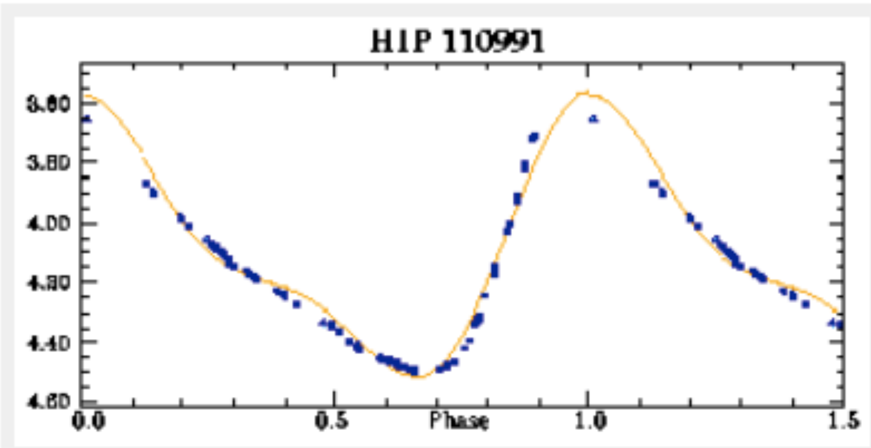
Whenever the number of particles is not conserved: e.g. ionisation/recombination

Compression in a star can enhance recombination, therefore the density increases less than in a fully ionised gas ==> P increases less steeply resulting in

$$\gamma_{\mathrm{ad}} < \frac{4}{3}$$

1) In star outer layers H,He partial ionisation may lead to $\gamma_{\mathrm{ad}} < \frac{4}{3}$ however there P/ρ are generally small enough that $\int \left(\gamma_{\mathrm{ad}} - \frac{4}{3}\right)\frac{P}{\rho}\mathrm{d}m > 0$

=> partial ionisation is related to photospheric oscillations in variable stars



δ Cephei light curve from HIPPARCOS.

Credit: ESTEC, ESA.

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$$\gamma_{\mathrm{ad}} < \frac{4}{3}$$

2) In core of massive stars at end of life : pair creation and photo-disintegration of heavy (Fe) nuclei lead to $\gamma_{\mathrm{ad}} < \frac{4}{3}$. Since P/ρ is very high

$$\int \left(\gamma_{\mathrm{ad}} - \frac{4}{3}\right)\frac{P}{\rho}\mathrm{d}m < 0$$

=> core collapse and consequently a supernova/Gamma ray bursts

We will come back to it…

THERMAL/ SECULAR STABILITY

Recall: in ideal gas + radiation stars the negative heat capacity ensures **stability**:

$\partial T > 0$ —-> $L_{nuc} > L$ —-> $\partial T < 0$ —-> $\partial L_{\mathbf{nun}} < 0$

(expansion decreases internal energy: Virial Theorem)

Recall: $\epsilon_{\rm nuc} \propto \rho^{\mu} T^{\nu}$

THERMAL/ SECULAR STABILITY

Recall: in degenerate stars or cores of evolved stars P independent of T. This can cause thermal INstability: **Thermonuclear Runaway!**

$\partial T > 0$ —-> L_{nuc} > L but $\partial \rho \sim 0$ —-> $\partial T > 0$ —-> $\partial L_{\rm nuc} > 0$ and so on…

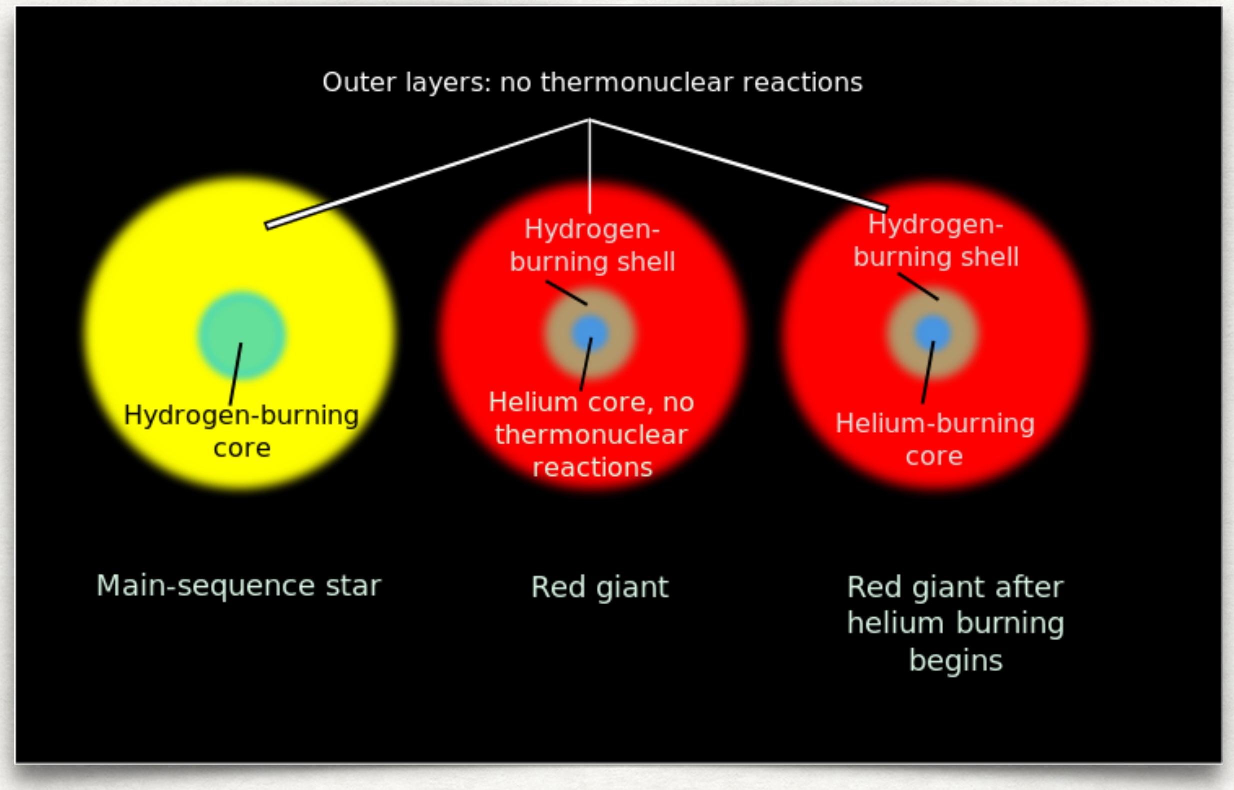
no expansion!

Recall: $\epsilon_{\rm nuc} \propto \rho^{\mu} T^{\nu}$

THERMAL/ SECULAR INSTABILITY

Recall: in degenerate stars or cores of evolved stars P independent of T. This can cause <u>thermal INstability</u>: **<u>Thermonuclear Runaway!</u>**

1): ignition of He fusion in degenerate core of stars < 2 M_{sun} : sudden increase in luminosity called``Helium flash" during ``red giant" phase



As the He core grows in mass by hydrogen burning in the shell outside it, it continues to contract and heats up but become degenerate before He burning sets in at 10^8 K

Recall: $\epsilon_{\rm nuc} \propto \rho^{\mu} T^{\nu}$

THERMAL/ SECULAR INSTABILITY

2) if hydrogen is accreted onto white dwarf surface and He fusion starts gives a flare
``nova burst"

Recall: $\epsilon_{\rm nuc} \propto \rho^{\mu} T^{\nu}$

THIN SHELL INSTABILITY

please, read yourself (end of Ch 7)

in evolved stars nuclear during can take place in thin shells around the core: thermal perturbation does not lead to sufficient expansion and pressure drop

runaway situation important during asymptotic giant branch for M< 8 Msun

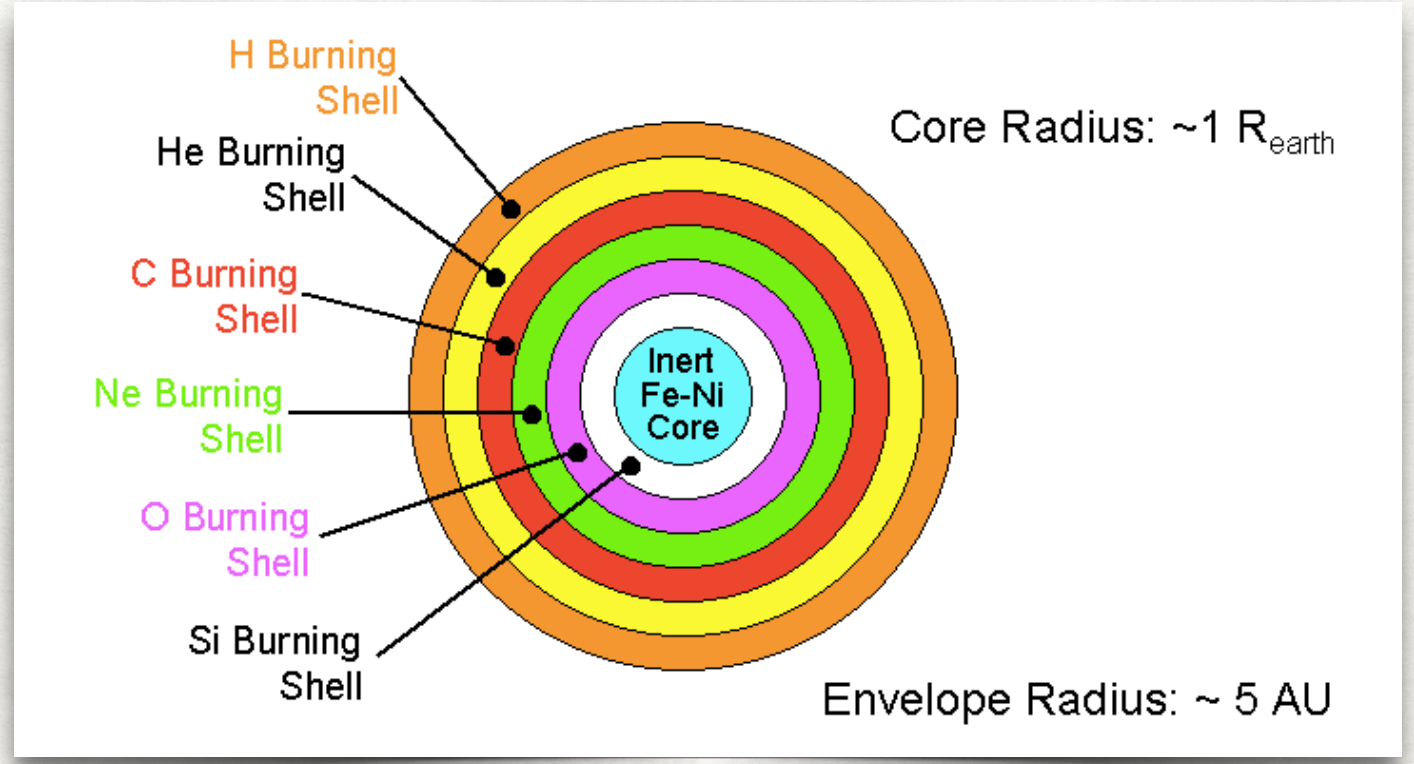
“STELLAR EVOLUTION: SCHEMATIC VIEW FROM THE <u>*CORE*</u>”

—*Ch 8 :*

Exercises: all

WHY THE CORE?

The core is where the density, pressure and temperature are higher and therefore where nuclear burning proceeds faster: it sets the pace of the star evolution while the outer layers (envelope) lag behind



e.g. the core got to Fe while the outer layers are still fusing lighter nuclei

EVOLUTION IN $(P_{\rm c}, \rho_{\rm c})$ DIAGRAM

During an homologous contraction or expansion that keeps HE

the pressure and density in each shell scale as the central values

$$P(x) \propto P_{\rm c} \propto \frac{GM^2}{R^4}$$

$$\rho(x) \propto \rho_{\rm c} \propto MR^{-3}$$

the central Pressure evolves as $\boxed{P_c = C \cdot GM^{2/3}\rho_c{}^{4/3}}$ for any Equation of state

EVOLUTION IN (P_c, ρ_c) DIAGRAM

During an homologous contraction or expansion that keeps HE

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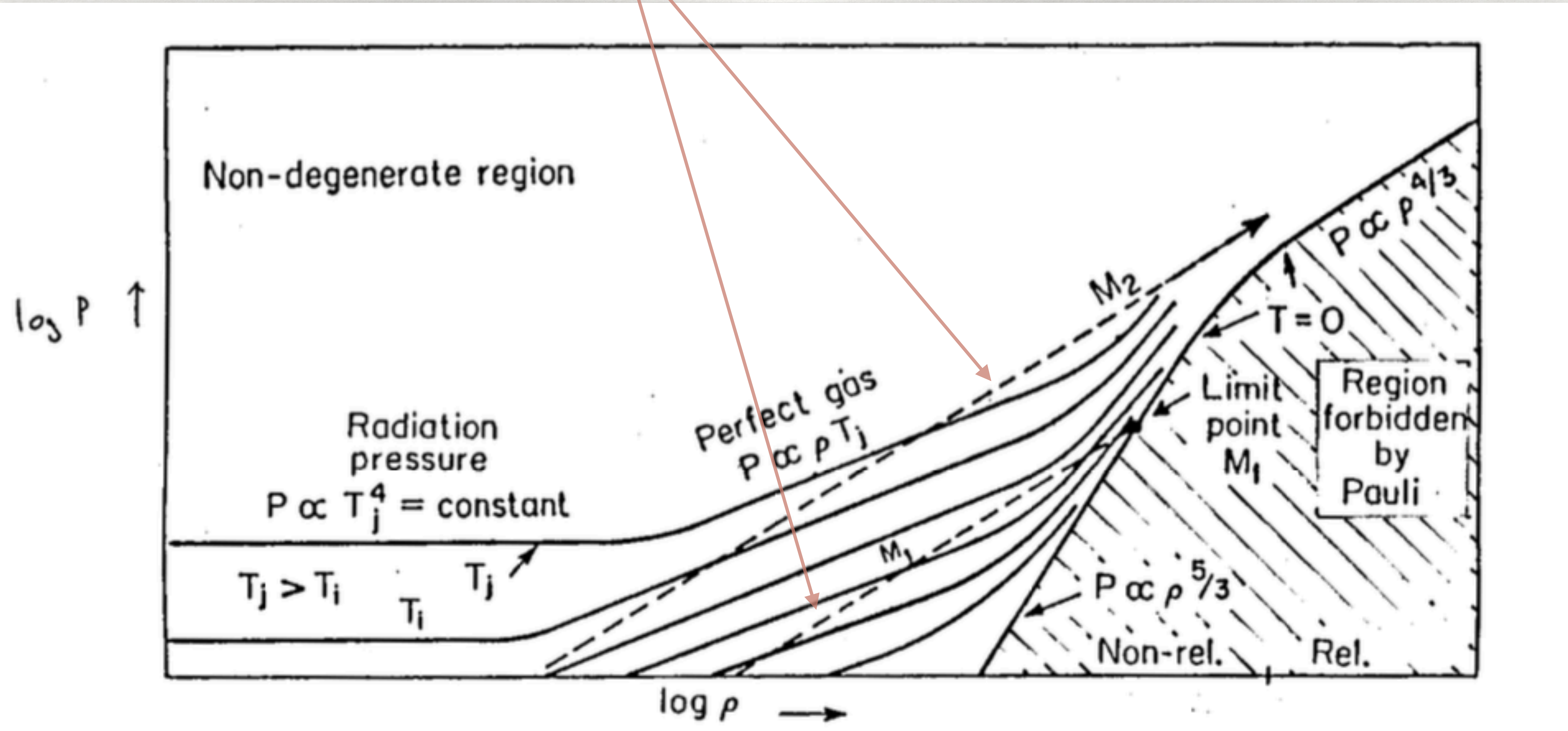

- higher mass, higher P for same rho
- higher mass, lower rho for same P

EVOLUTION IN (P_c, ρ_c) DIAGRAM

During an homologous contraction or expansion that keeps HE

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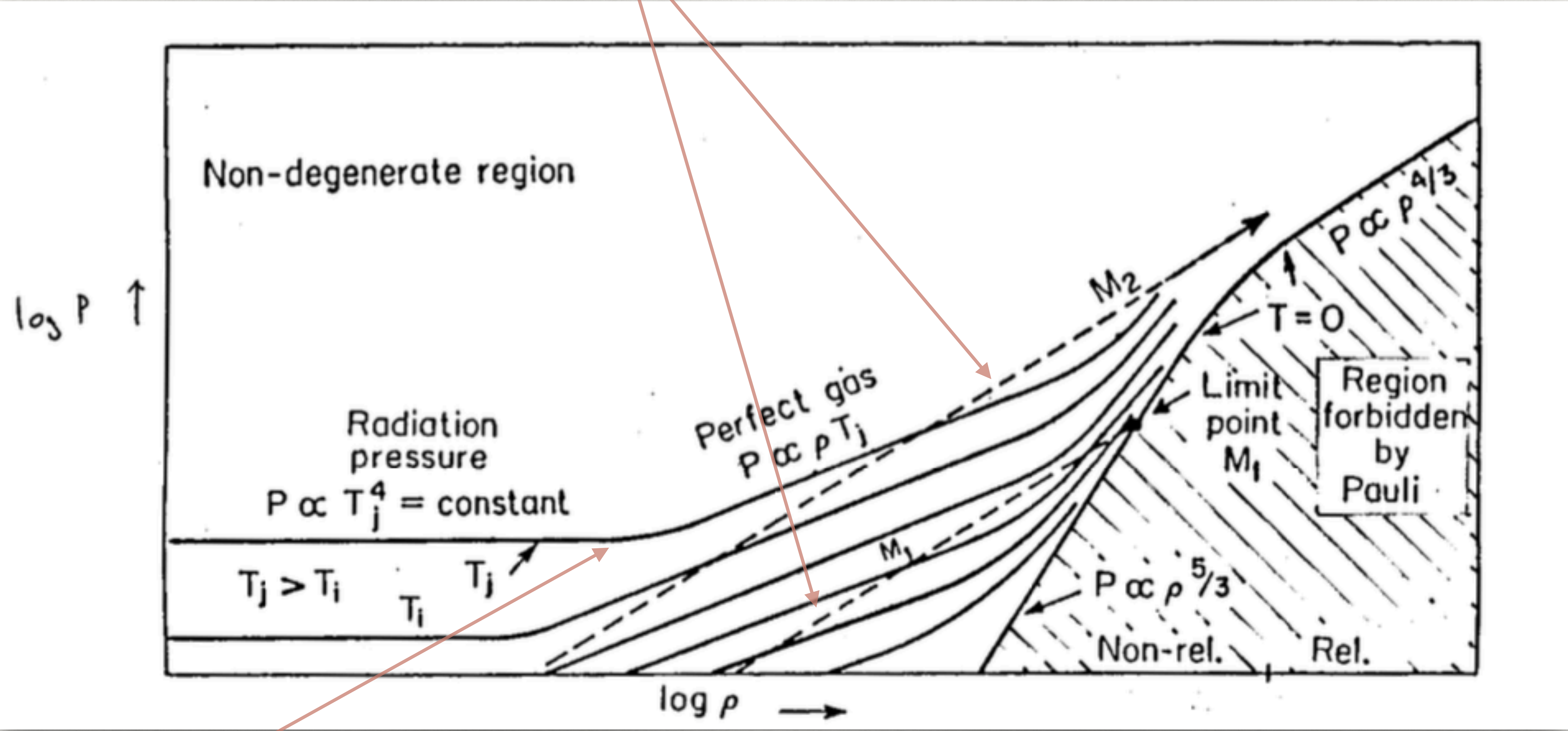
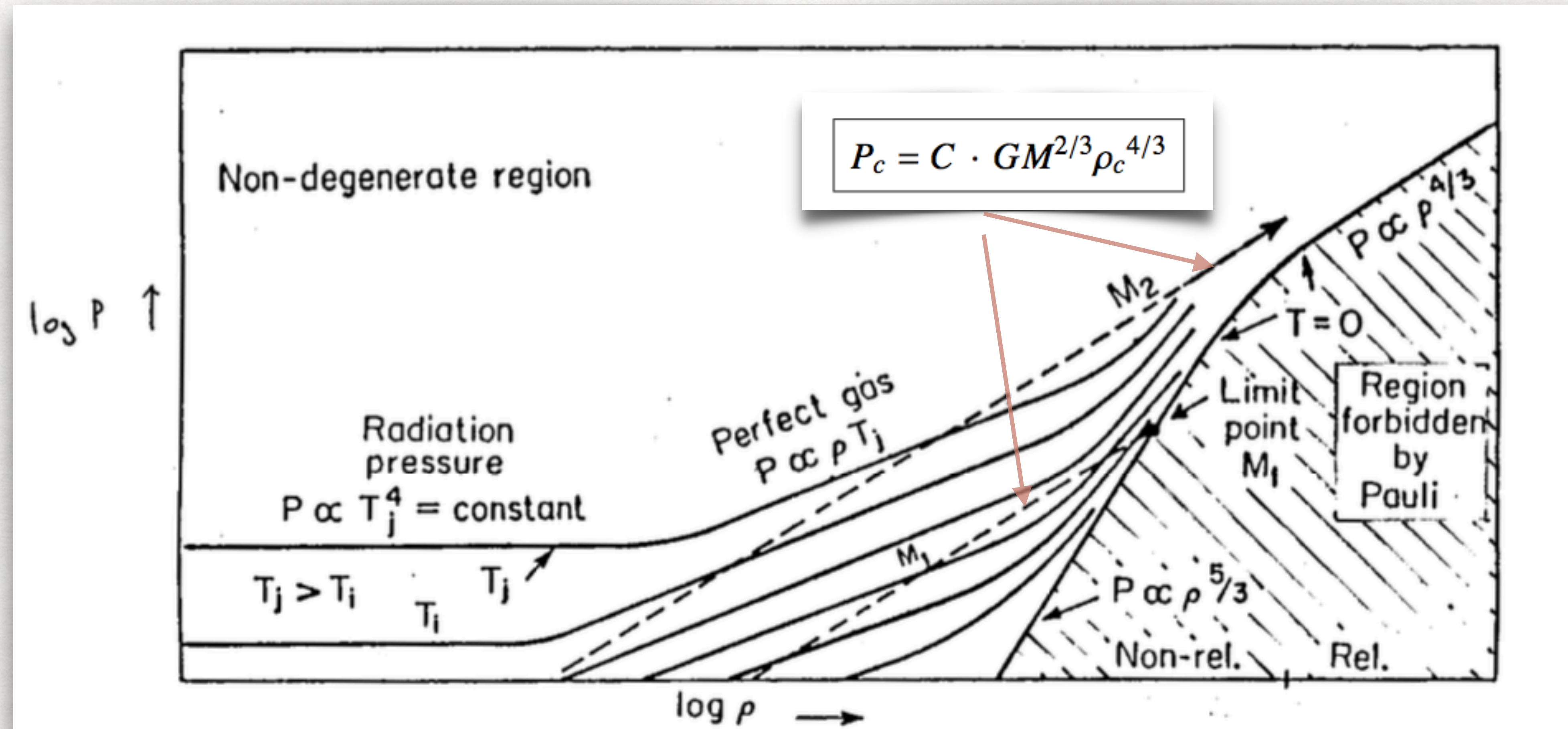


solid lines: isotherms (line of T=const.) in regions with different EOS

EVOLUTION IN (P_c, ρ_c) DIAGRAM



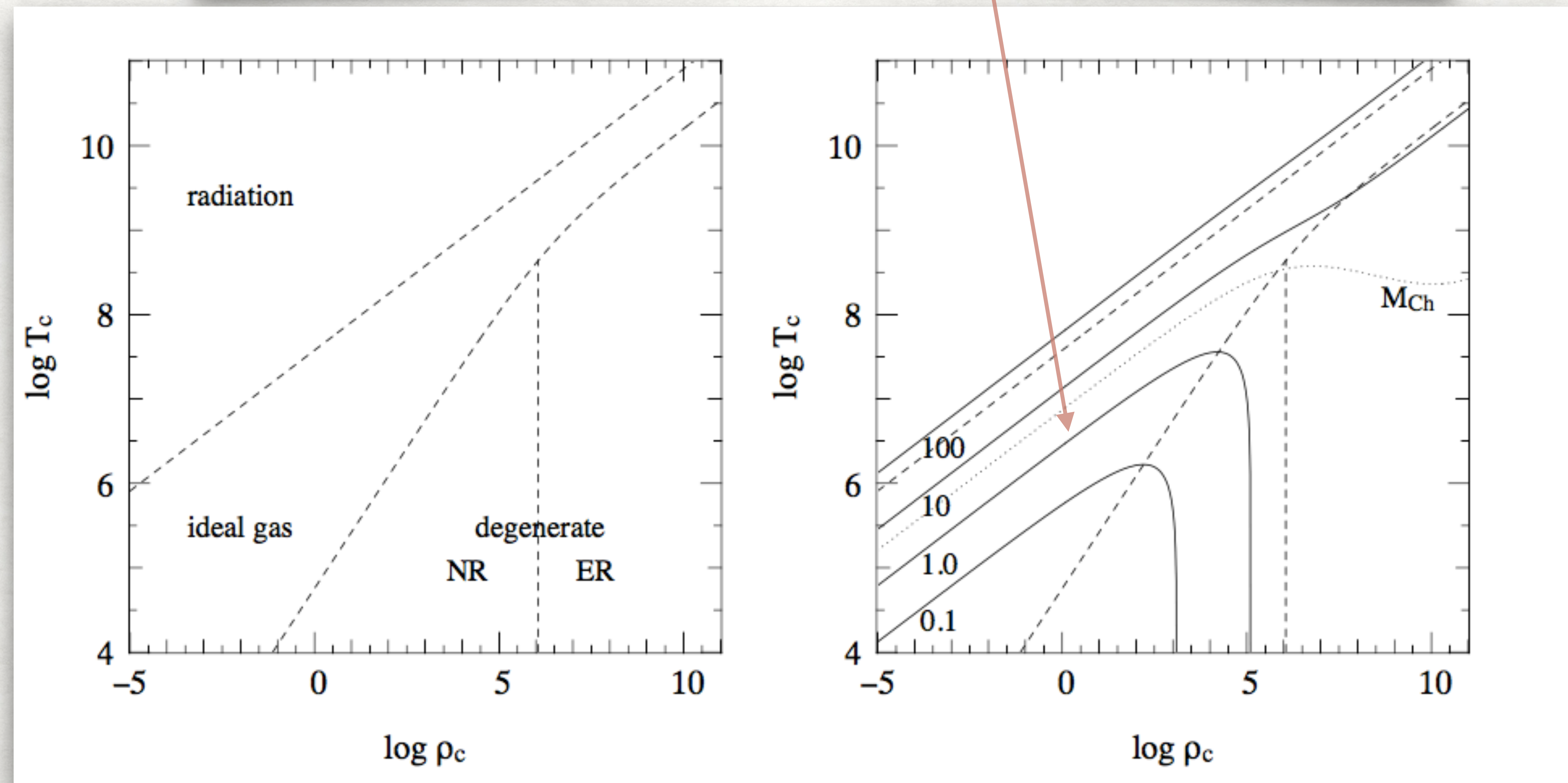
- in perfect gas regime stars that contract cross isotherm of higher temperature: heats up
- there is a critical mass above which stars do not hit the electron degenerate region because for high densities track parallel to degenerate EOS
- below critical mass stars reach a maximum density, T and P before degeneracy dependent on mass
  - critical mass: $M_{\mathrm{Ch}} = \frac{5.836}{{\mu_e}^2} M_\odot .$

EVOLUTION IN (T_c, ρ_c) DIAGRAM

$$\boxed{P_c = C \cdot GM^{2/3}\rho_c^{\ 4/3}}$$

ideal gas

$$\frac{\mathcal{R}}{\mu}T_c\,\rho_c = C\,GM^{2/3}\rho_c^{\ 4/3} \quad \rightarrow \quad \boxed{T_c = \frac{C\,G}{\mathcal{R}}\mu\,M^{2/3}\rho_c^{\ 1/3}}$$

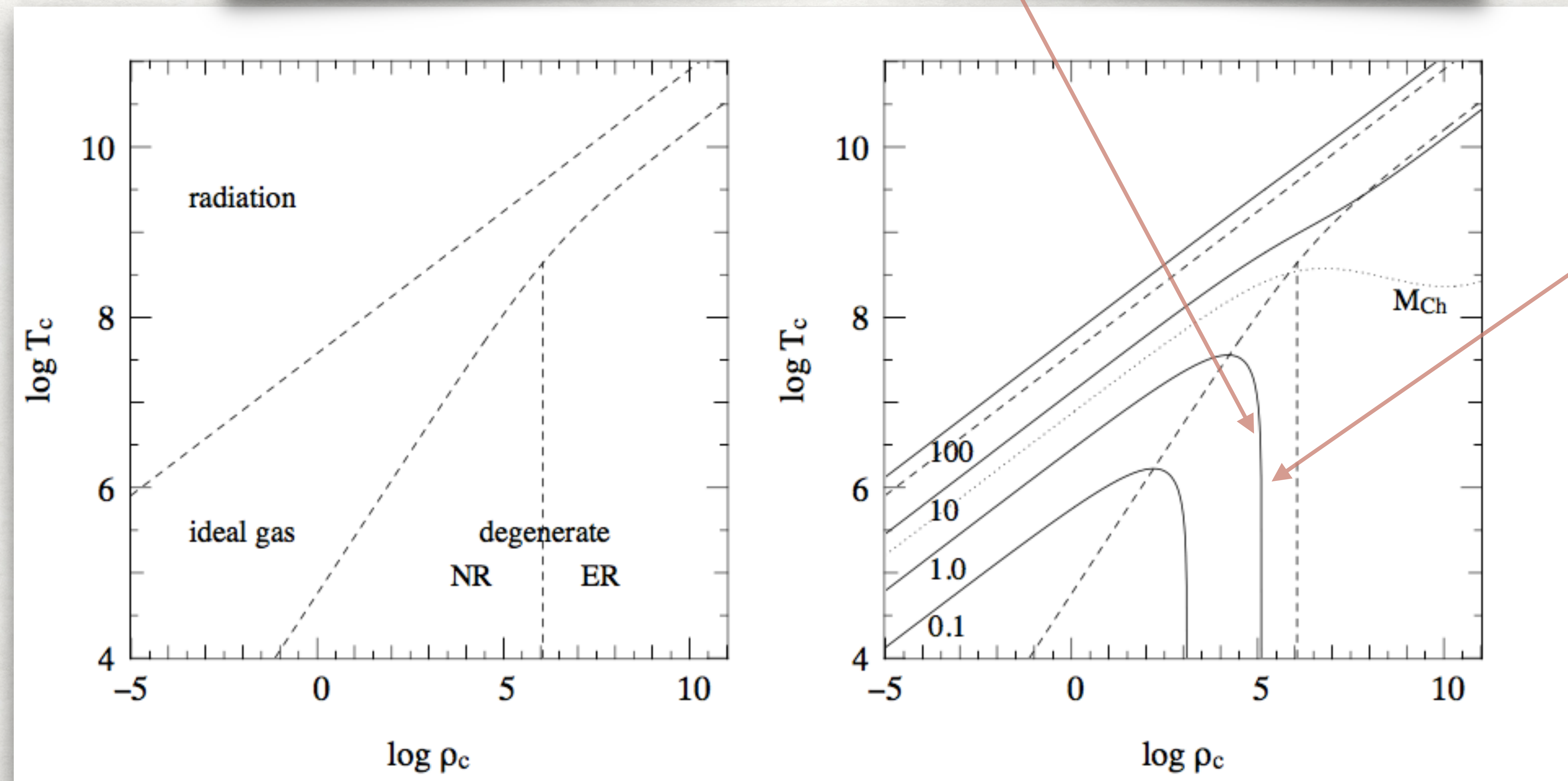


EVOLUTION IN (T_c, ρ_c) DIAGRAM

$$\boxed{P_c = C \cdot GM^{2/3} \rho_c^{4/3}}$$

degenerate NR electron gas

$$K_{NR} \frac{\rho_c^{1/3}}{\mu_e^{5/3}} = C\,G M^{2/3} \quad \rightarrow \quad \boxed{\rho_c = \left(\frac{C\,G}{K_{NR}}\right)^3 \mu_e^{5} M^2}$$



cooling at constant density

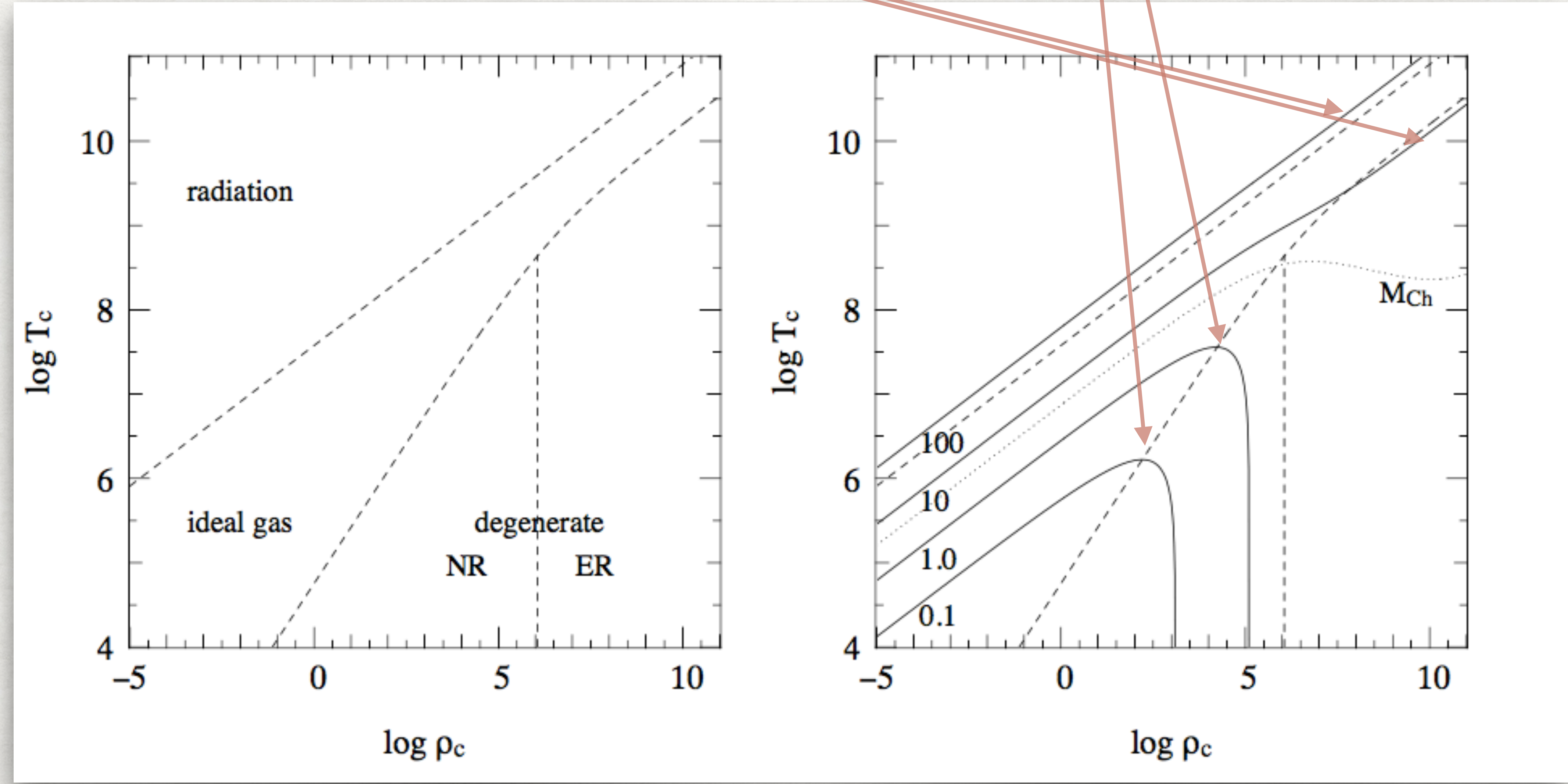
EVOLUTION IN (T_c, ρ_c) DIAGRAM

maximum temperature depends on the stellar mass

$M < M_{\rm Ch}$ T_{max} is when $P_{ideal}=P_{degenerate}$

$$T_{c,max} = \frac{C^2 G^2}{4\mathcal{R} K_{NR}} \mu \mu_e^{5/3} M^{4/3}.$$

$M > M_{\rm Ch}$ T keeps increasing $> 10^{10}$ K



cooling at constant density

BROWN DWARFS

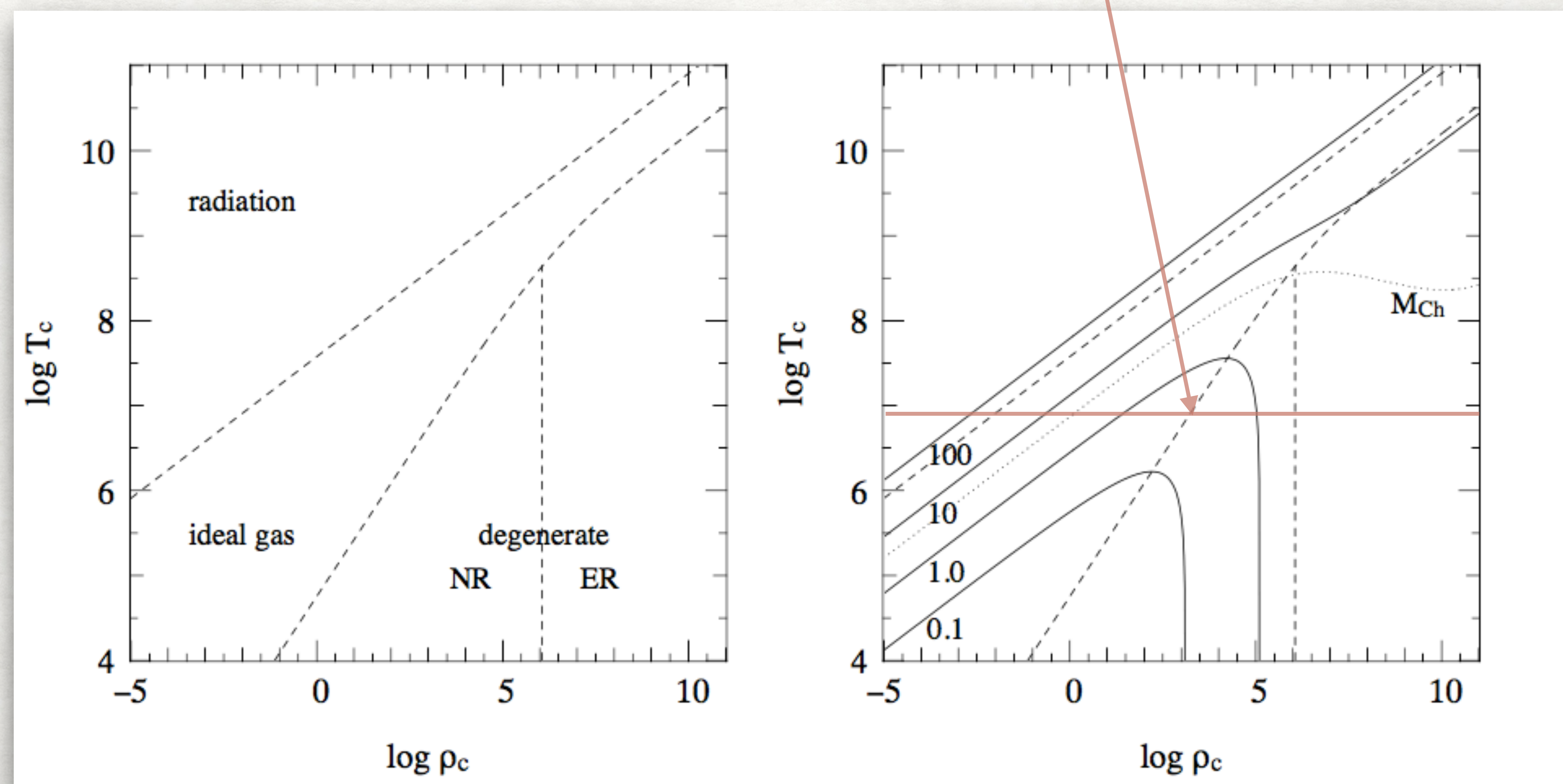
consider protostars contracting in HE :

$M < M_{\rm Ch}$ $T_{c,max}$ is when $P_{ideal}=P_{degenerate}$

$$T_{\rm c,max} = \frac{C^2 G^2}{4\mathcal{R}\, K_{\rm NR}}\, \mu\, \mu_{\rm e}^{5/3}\, M^{4/3}.$$

The minimum mass for stars is given by $T_{c,max}$ ~10^7 K ==> 0.15 M_{sun}

Detailed calculations gives 0.08 $M_{sun:}$ BROWN DWARFS



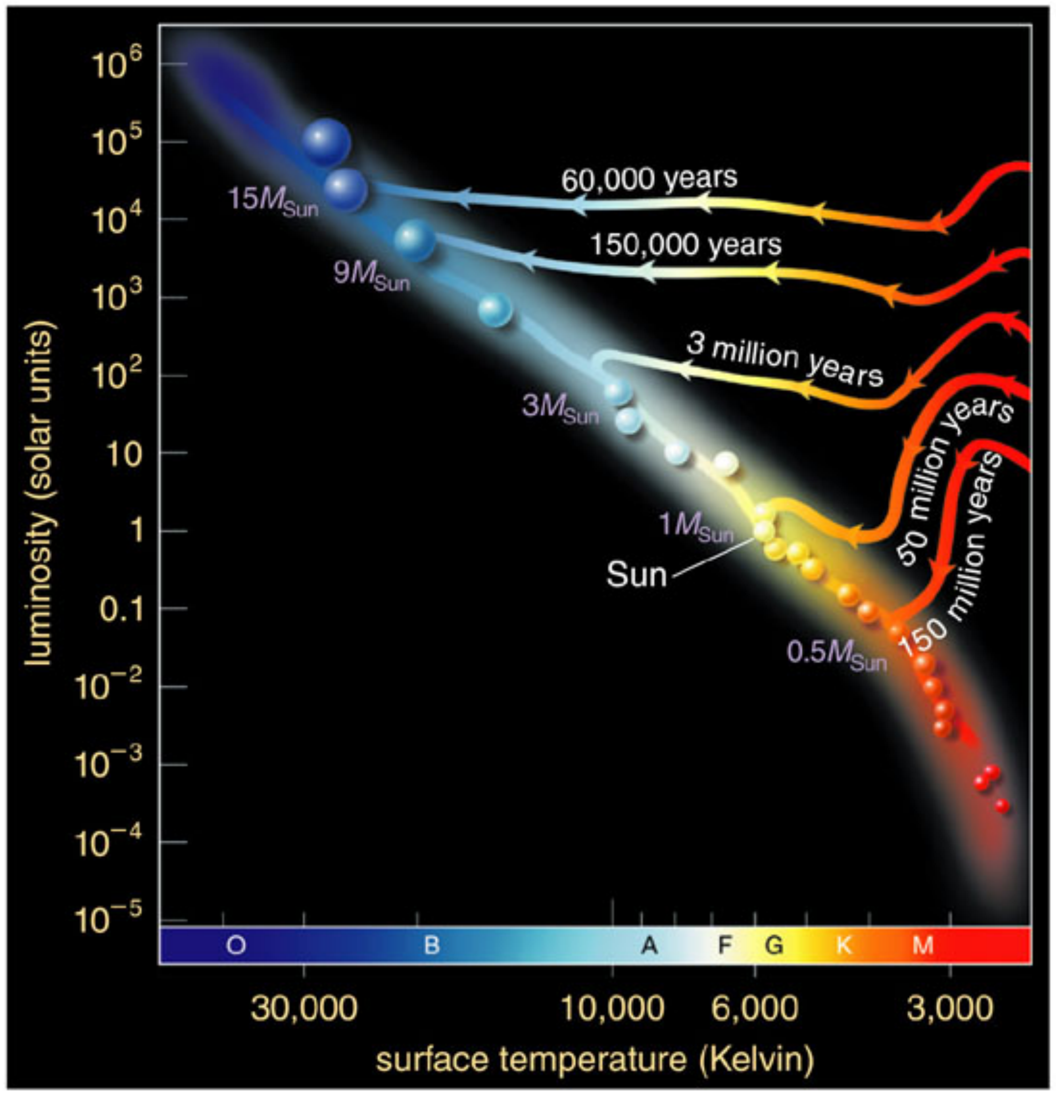
FROM PROTOSTAR TRACK TO MAIN SEQUENCE

A protostar = self gravitating sphere with no nuclear burning

It contracts and heats up as it radiates from surface

$$L = -\dot{E}_{\rm tot} = \dot{E}_{\rm in} = -\tfrac{1}{2}\dot{E}_{\rm gr} \approx \frac{E_{\rm gr}}{\tau_{\rm KH}}$$

protostar
contraction on KH timescale



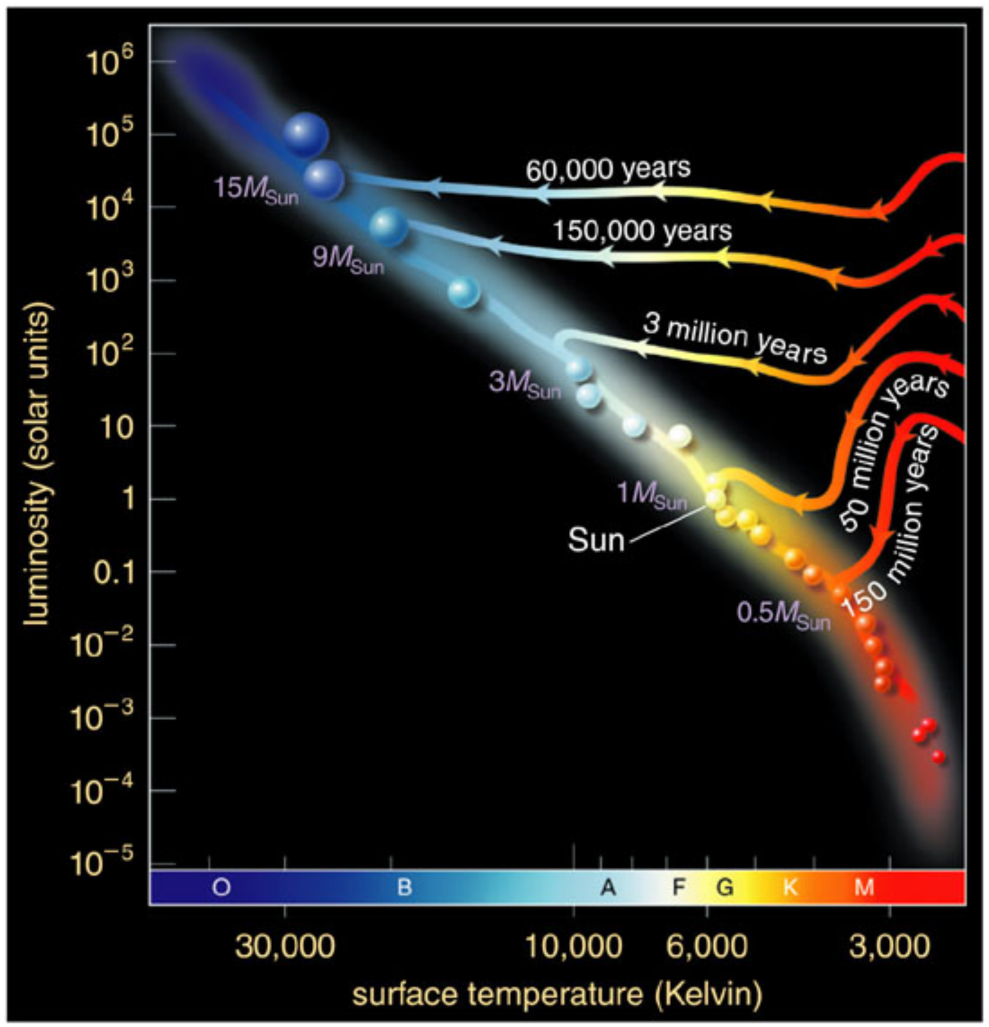
FROM PROTOSTAR TRACK TO MAIN SEQUENCE

protostar contracts until $T_c \sim 10^7$ K and H burning starts and

$$L = -\dot{E}_{\mathrm{nuc}} \approx \frac{E_{\mathrm{nuc}}}{\tau_{\mathrm{nuc}}}$$

star on main sequence for a nuclear timescale

T_c and density remains constant at value needed for H fusion



BEYOND MAIN SEQUENCE TO HE FUSION

when H is exhausted in core, He core of 10% in mass re-start contracting

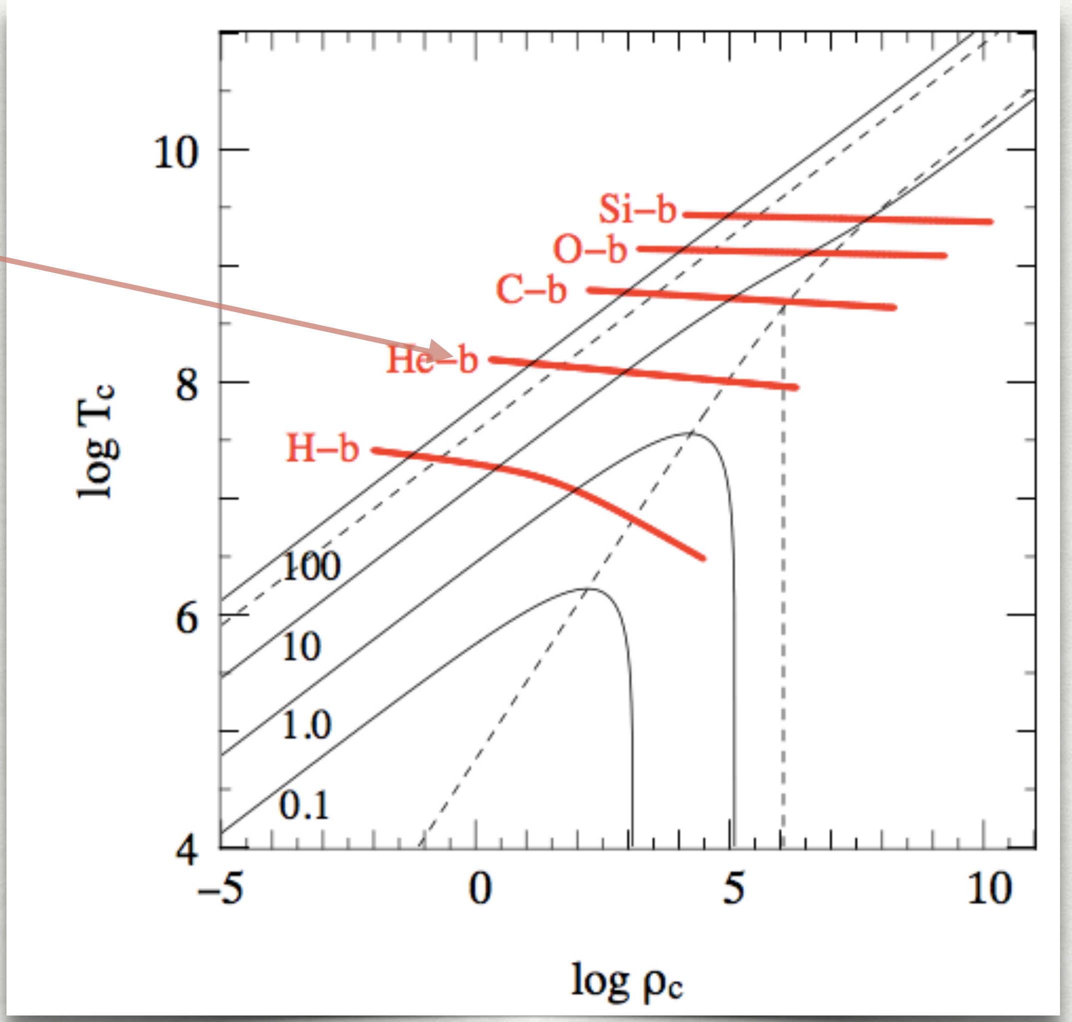
$$L_{\rm core} \approx \dot{E}_{\rm in,core} \approx -\frac{1}{2}\dot{E}_{\rm gr,core} \approx \frac{E_{\rm gr,core}}{\tau_{\rm KH,core}}$$

and releases potential energy that causes external layers to expand

when $T_c \sim 10^8$ K He burning sets in

- there is a minimum mass for He burning

detailed calculations with varying composition give core > 0.3 M_{sun}



THE SUN

- For core < 0.3 M_{sun} , core becomes degenerate,H burning around the core increases its mass up to 0.5 M_{sun} and He ignites in a flash