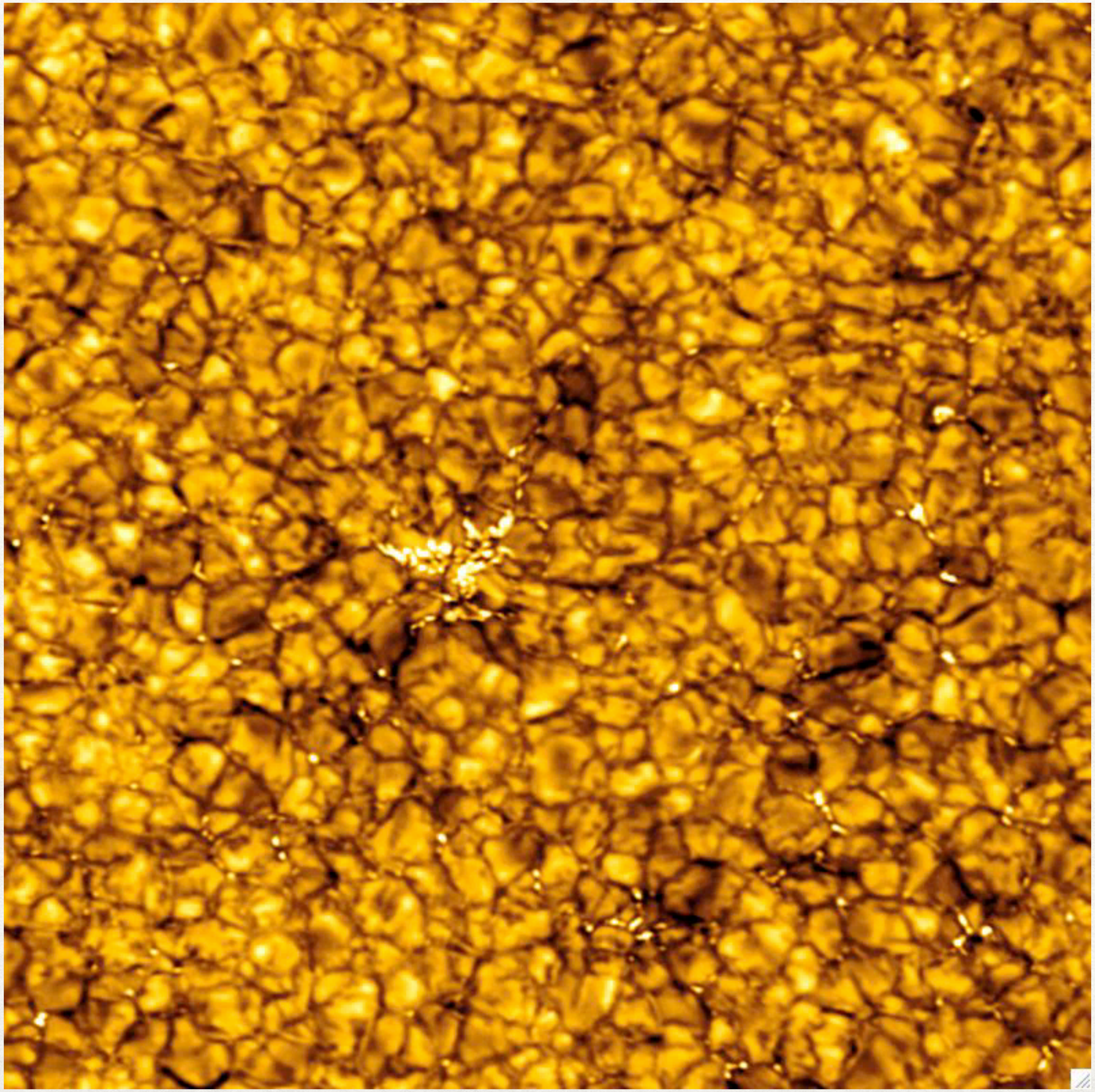
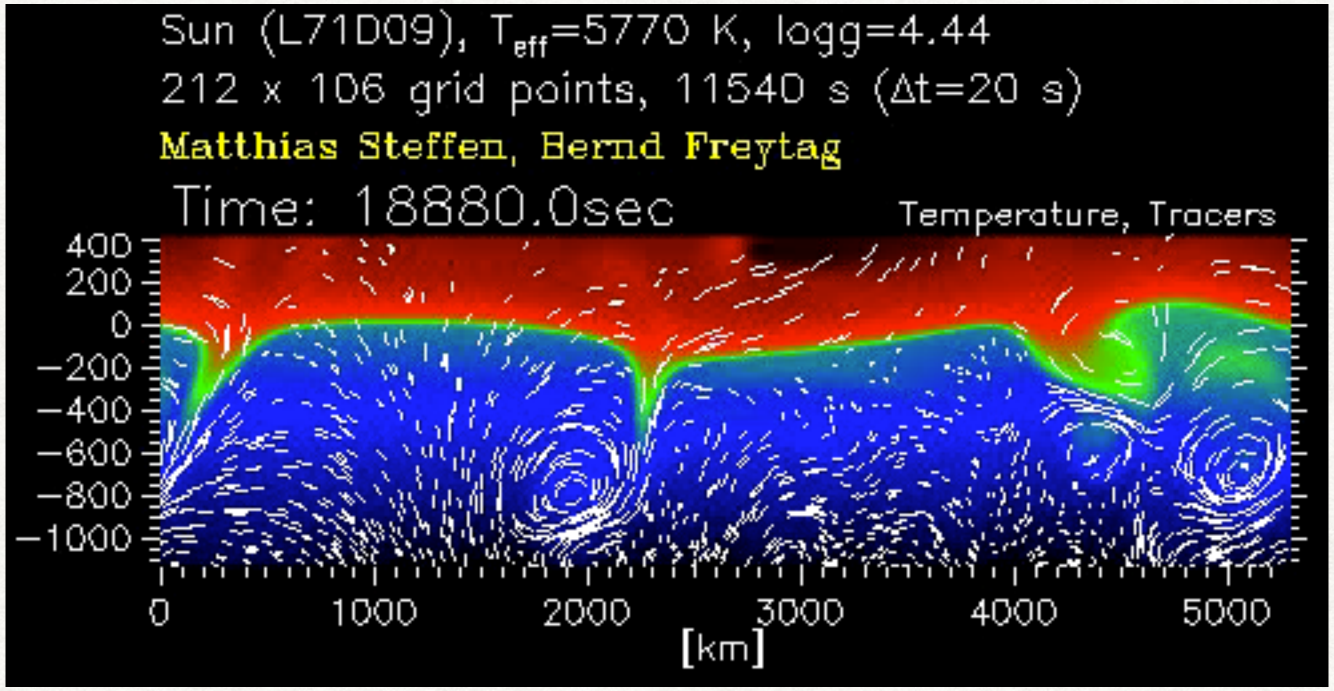
SUMMARY PREVIOUS CLASS

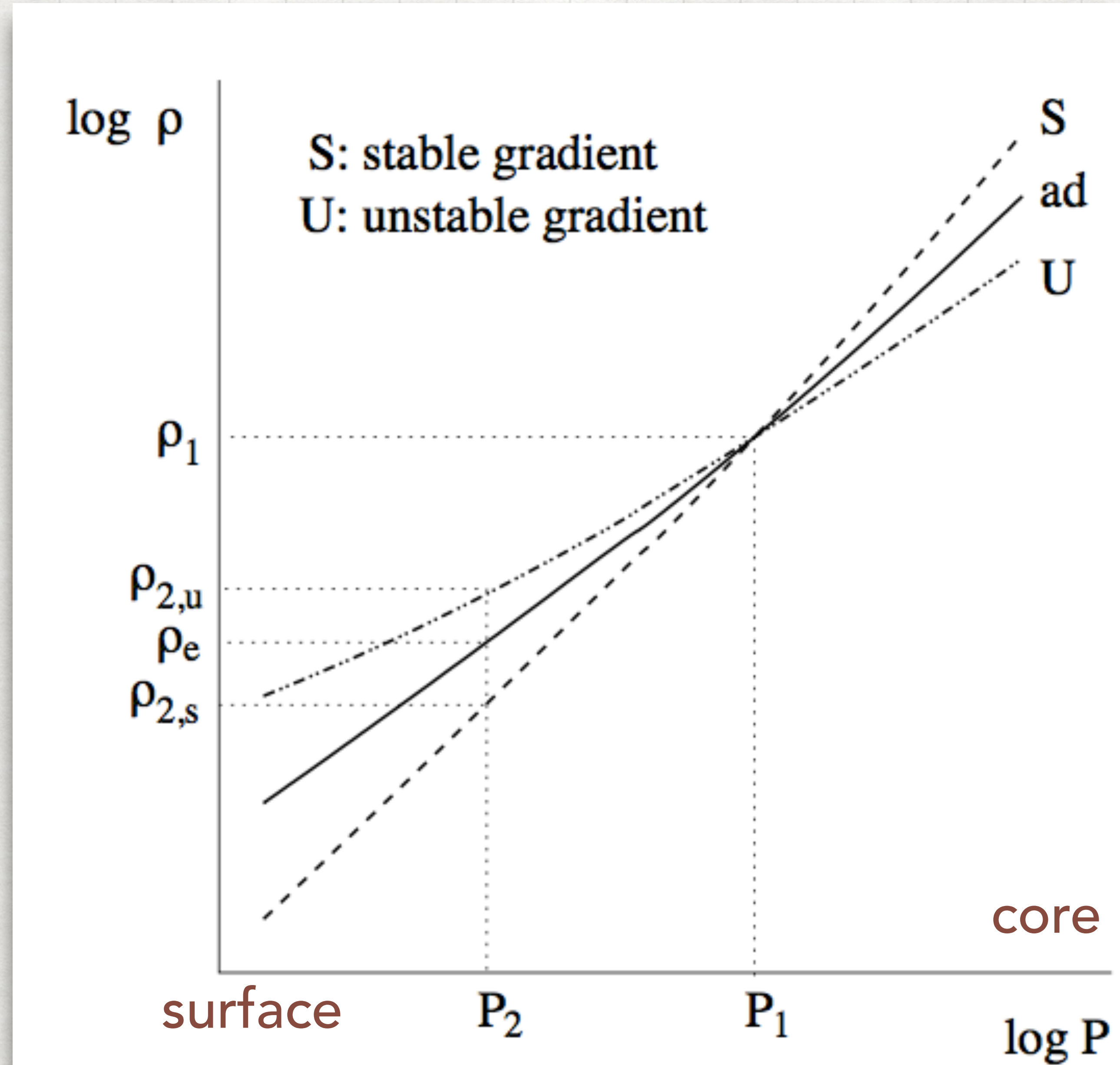
OBSERVATION OF SUN SURFACE WITH THE SWEDISH 1-M SOLAR TELESCOPE (SST)

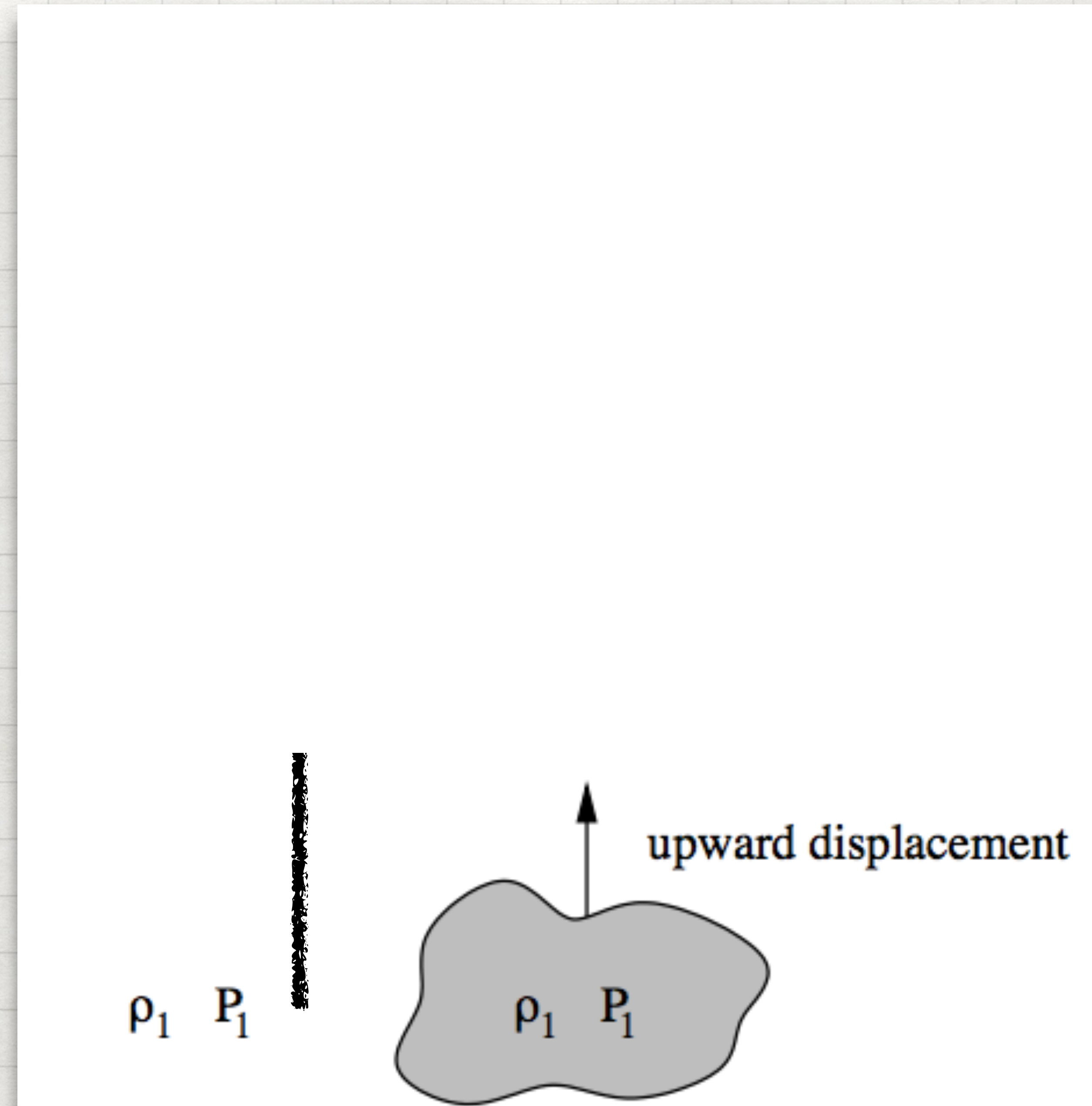




- Cyclic macroscopic motions of the gas causing <u>a net heat flux</u> against the direction of gravity without net mass displacement

We can treat it as an instability in the star: This macroscopic motion starts when

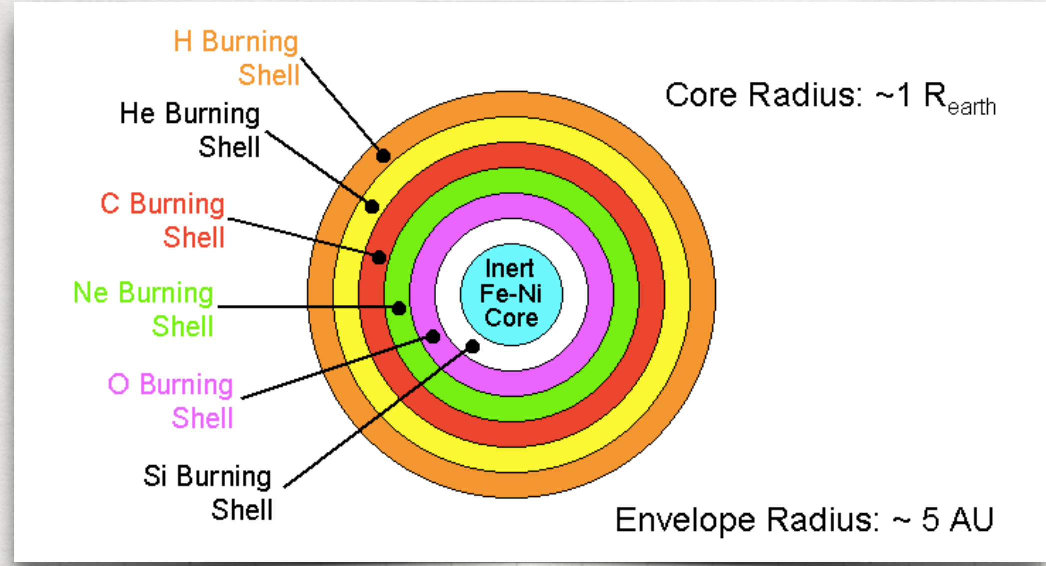




$$\boxed{\frac{\mathrm{d}\log\rho}{\mathrm{d}\log P} > \frac{1}{\gamma_{\mathrm{ad}}}.}$$

$$\gamma_{\mathrm{ad}} = \left(\frac{\partial\log P}{\partial\log\rho}\right)_{\mathrm{ad}}$$

$$\boxed{\partial\rho_{\mathrm{e}} > \frac{d\rho}{dr}\Delta r}$$

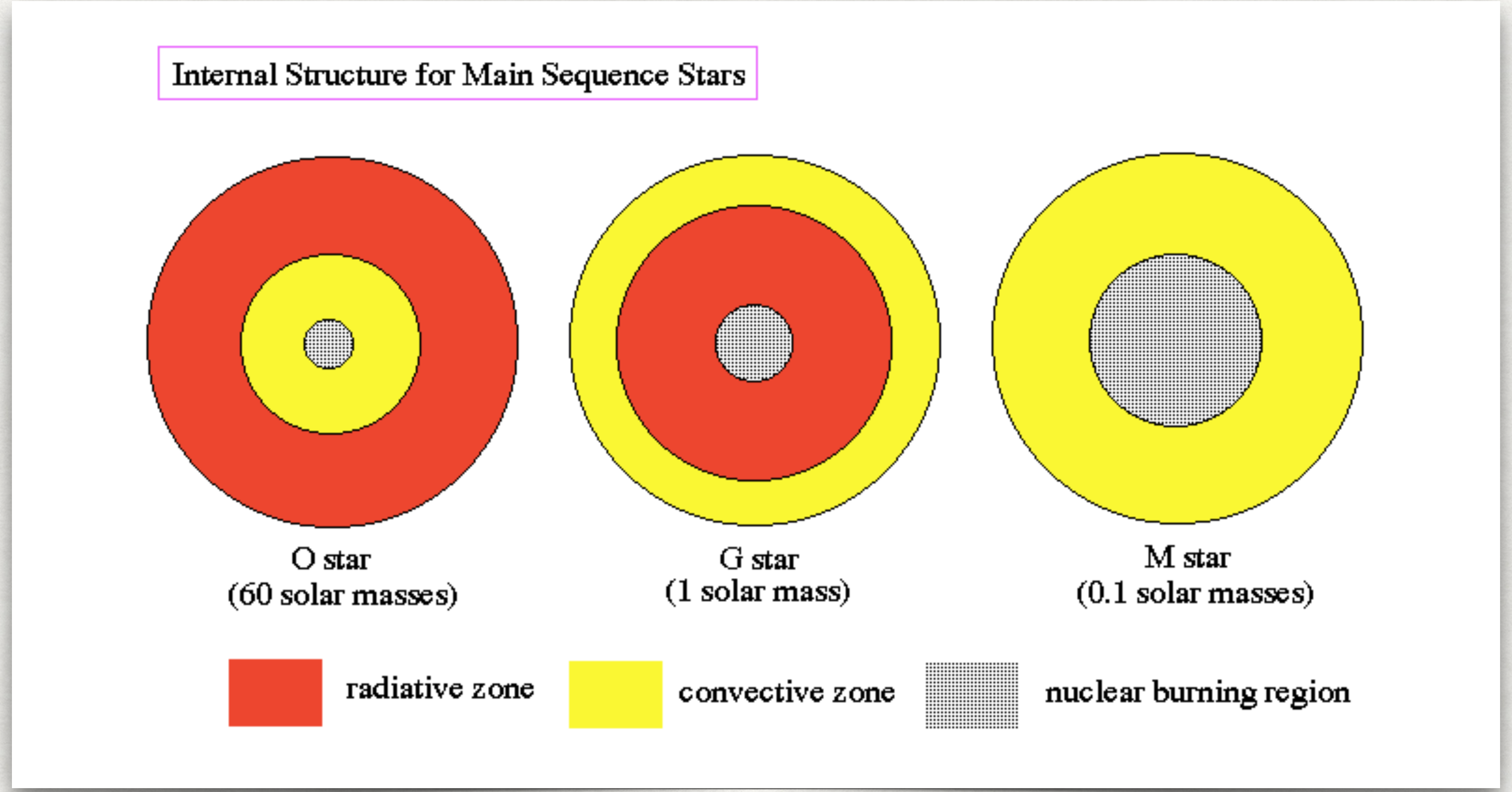


“NUCLEAR PROCESSING IN STARS”

Ch 6 notes (but 6.5)

e.x. 6.1

SUMMARY


Internal Structure for Main Sequence Stars
O star
(60 solar masses)
G star
(1 solar mass)
M star
(0.1 solar masses)
radiative zone
convective zone
nuclear burning region

ENERGY GENERATION RATES

Energy generation per unit mass is

Energy

$$\epsilon_{ij} = \frac{Q_{ij} r_{ij}}{\rho}.$$

rate

$$\epsilon_{\text{nuc}} = \sum_{i,j} \epsilon_{ij}.$$

<u>It is the total nuclear energy rate we used in the thermal balance.</u>

BASICS

- A reaction is denoted with $X + a \rightarrow Y + b$ or $X(a, b)Y$
- Charges and baryon number are conserved:

charge number: number of protons

$$Z_X + Z_a = Z_Y + Z_b \quad \text{and} \quad A_X + A_a = A_Y + A_b.$$

baryon or mass number: protons+neutrons

Table 6.1. Atomic masses of several important isotopes.

element	Z	A	M/m_u	element	Z	A	M/m_u	element	Z	A	M/m_u
n	0	1	1.008665	C	6	12	12.000000	Ne	10	20	19.992441
H	1	1	1.007825		6	13	13.003354	Mg	12	24	23.985043
	1	2	2.014101	N	7	13	13.005738	Si	14	28	27.976930
He	2	3	3.016029		7	14	14.003074	Fe	26	56	55.934940
	2	4	4.002603		7	15	15.000108	Ni	28	56	55.942139
Li	3	6	6.015124	O	8	15	15.003070				
	3	7	7.016003		8	16	15.994915				
Be	4	7	7.016928		8	17	16.999133				
	4	8	8.005308		8	18	17.999160				

A/Z ~1/2 for > H

- Lepton number (e.g. in weak interaction) is conserved.
- **mass is NOT conserved**

BINDING ENERGY

NOT EQUAL TO SUM OF MASSES X C^2; USUALLY IN MEV

For nucleus "i":

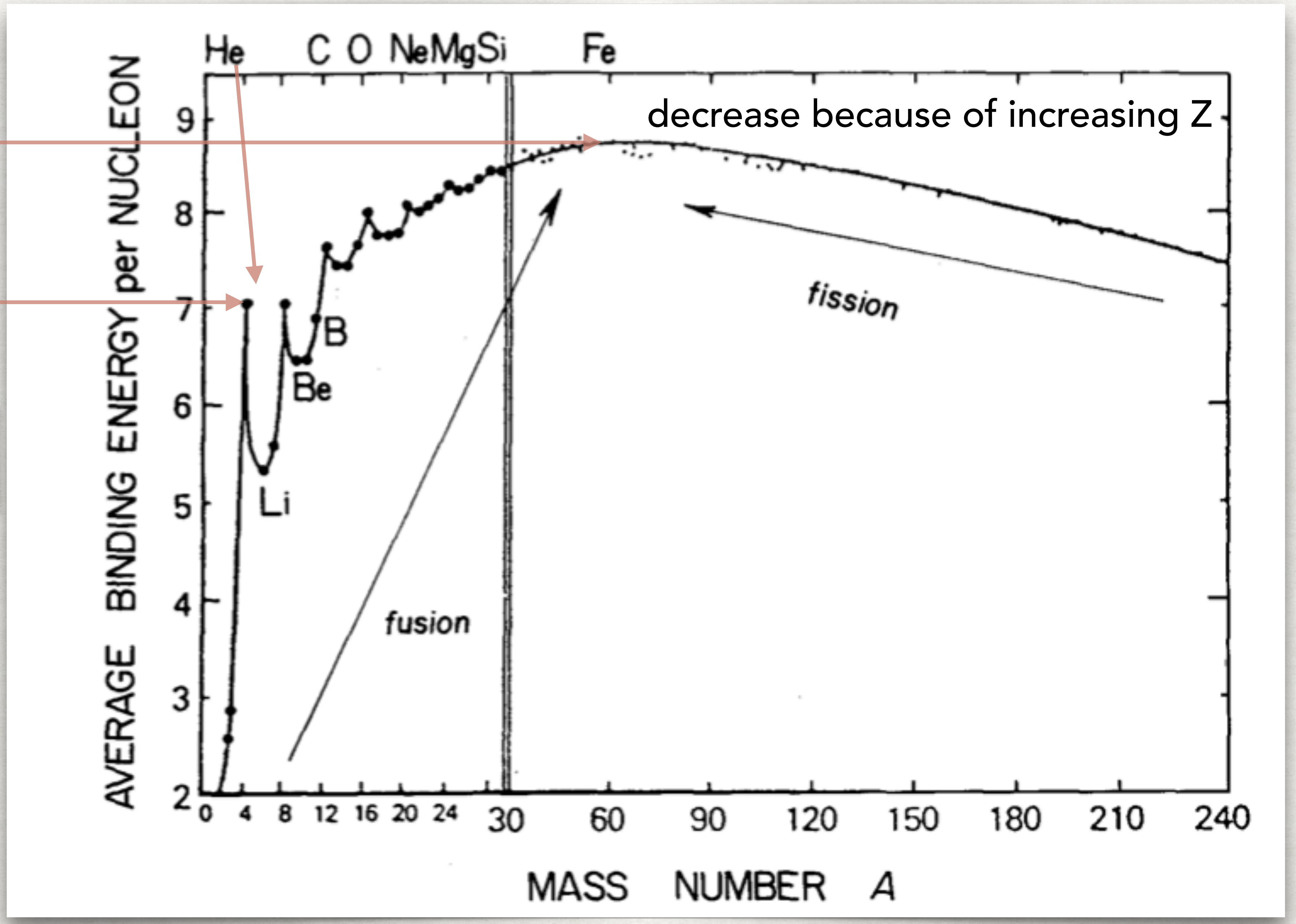
$$E_{B,i} = [(A_i - Z_i)m_\mathrm{n} + Z_i m_\mathrm{p} - m_i]\, c^2,$$

m_p, m_n =
free proton/neutron mass

8.79 MeV

~7 MeV

$$E_\mathrm{b}/A$$



ENERGY RELEASES OR ABSORBED

IN A REACTION TO A MORE BOUND OR LESS BOUND STATE

$$X + a \rightarrow Y + b \qquad \text{or} \qquad X(a, b)Y$$

the energy release is the difference in binding energies, and since A_i and Z_i are conserved:

$$Q = (m_X + m_a - m_Y - m_b)\, c^2.$$

> 0 (exothermic) for fusion ; < 0 (endothermic) for fission

From H to Fe 8.8 MeV are released per nucleon of which 7 are released form H to He

ENERGY GENERATION RATES

Energy generation per unit mass is

$$\epsilon_{ij} = \frac{Q_{ij} r_{ij}}{\rho}.$$

$$Q = (m_X + m_a - m_Y - m_b) c^2.$$

rate

$$\epsilon_{\text{nuc}} = \sum_{i,j} \epsilon_{ij}.$$

<u>It is the total nuclear energy rate we used in the thermal balance.</u>

NUCLEAR REACTION RATES

$$a + b \rightarrow Y$$

- reaction per unit time $\quad r_{\mathrm{a}} = n_{\mathrm{a}} v_{\mathrm{a,b}} \sigma$

the effective area (i.e. "cross section") of particle b is defined and measured in experiments as:

$$\sigma = \frac{\text{number of reactions } X(a,b)Y \text{ per second}}{\text{flux of incident particles } a},$$

- reaction per unit time and volume for a particular relative velocity:

$$r_{\mathrm{a}} = n_{\mathrm{b}} n_{\mathrm{a}} v_{\mathrm{a,b}} \sigma$$

- in general for particle i & j:

$$\tilde{r}_{ij} = \frac{1}{1+\delta_{ij}} n_i n_j v \sigma,$$

NUCLEAR REACTION RATE

FOR PARTICLE VELOCITY DISTRIBUTION

In general the cross section is a function of velocity:

$$r_{ij} = \frac{1}{1+\delta_{ij}} n_i n_j \int_0^\infty \phi(v)\, \sigma(v) v \, \mathrm{d}v = \frac{1}{1+\delta_{ij}} n_i n_j \langle \sigma v \rangle.$$

Eg. For a classical gas in LTE, the *relative velocity distribution* is Maxwellian:

depends only on T

$$\phi(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right),$$

$$E = \frac{1}{2}mv^2$$

where the reduced mass in the centre of mass frame is $m = \dfrac{m_i m_j}{m_i + m_j}$. Since: $\phi(v)dv = \phi(E)dE$

depends only on T

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} (kT)^{-3/2} \int_0^\infty \sigma(E)\, E \, \exp\left(-\frac{E}{kT}\right) \mathrm{d}E.$$

A nuclear reaction has a dependence on density and temperature

NUCLEAR CROSS SECTION

DEF=IT IS A MEASURE OF A REACTION TO OCCUR GIVEN THE DENSITIES OF THE REACTANTS

Geometrical cross section:

$$\sigma = \pi\lambda^2$$

$$m = \frac{m_i m_j}{m_i + m_j}.$$

$$E = \frac{1}{2}mv^2$$

$$\lambda = \frac{\hbar}{p} = \frac{\hbar}{(2mE)^{1/2}},$$

De Broglie wavelength associated to their relative momentum

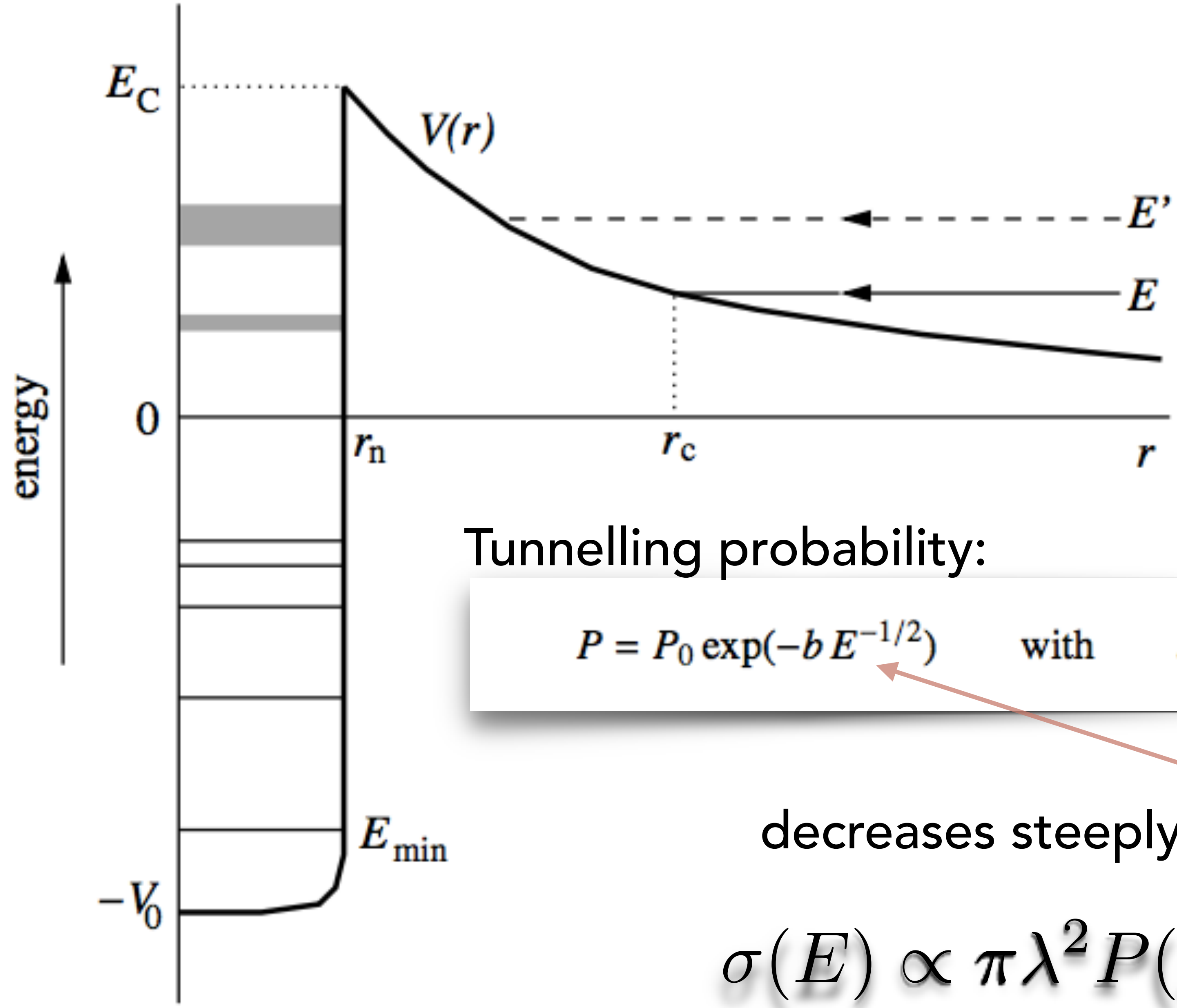
Note, typically: $\lambda > R_i + R_j$

$$R_i \approx R_0 A_i^{1/3} \quad \text{with} \quad R_0 = 1.44 \times 10^{-13}\ \text{cm}.$$

But it is more complicated than that…let's review quickly the physical effects affecting the cross section

TUNNEL EFFECT

- Charge nuclei have a repulsive Coulomb force, weaker than nuclear force but longer range. This "Coulomb barrier" would classically prevent reactions (too little particle if high enough energy) but a quantum-mechanical effect at stellar temperature occurs (discovered by Gamov):



$$E_{\mathrm{C}} = V(r_{\mathrm{n}}) = \frac{Z_i Z_j e^2}{r_{\mathrm{n}}} \approx Z_i Z_j \text{ MeV}$$

Coulomb barrier

Tunnelling probability:

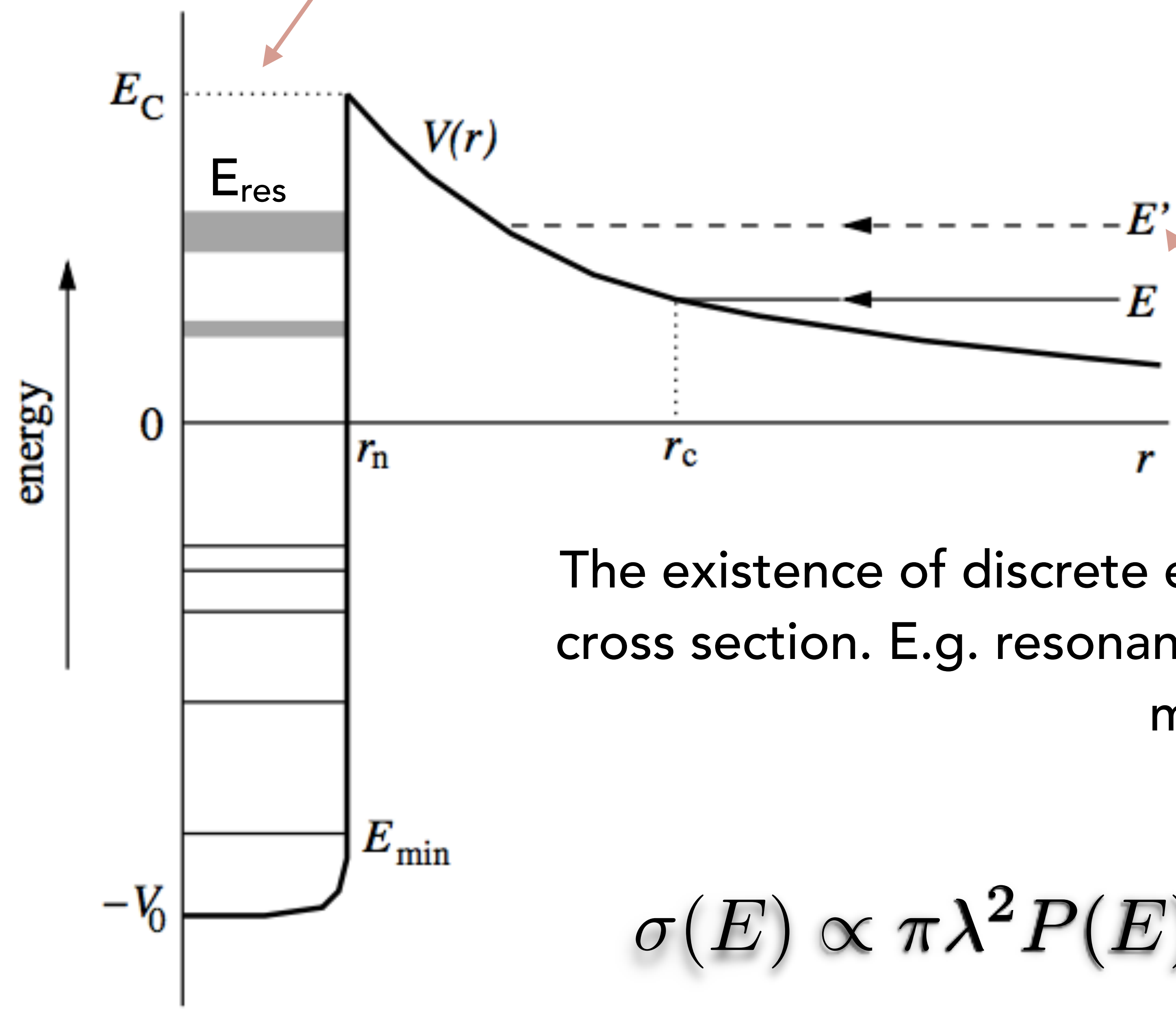
$$P = P_0 \exp(-b\,E^{-1/2}) \qquad \text{with} \qquad b = 2\pi \frac{Z_i Z_j e^2}{\hbar}\left(\frac{m}{2}\right)^{1/2} = 31.29\, Z_i Z_j A^{1/2}\,[\mathrm{keV}]^{1/2}.$$

decreases steeply with E and with ZiZj

$$\sigma(E) \propto \pi \lambda^2 P(E)$$

RESONANCES

- After penetrating the Coulomb barrier, the two nuclei form an excited "compound nucleus" that eventually decays into the reaction products: $X + a \rightarrow C^* \rightarrow Y + b$. (not for beta reactions)



$$\xi(E) \propto \frac{1}{(E - E_{res})^2 + (\Gamma/2)^2}.$$

The existence of discrete energy levels in compound impact cross section. E.g. resonances: if $E' = E_{res}$ the cross section is maximum

$$\sigma(E) \propto \pi \lambda^2 P(E) \xi(E)$$

TEMPERATURE DEPENDENCE

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} (kT)^{-3/2} \int_0^\infty \sigma(E)\, E \, \exp\left(-\frac{E}{kT}\right) \mathrm{d}E. \quad + \quad \sigma(E) \propto \pi \lambda^2 P(E) \xi(E) \quad =$$

$$\langle \sigma v \rangle = (8/\pi m)^{1/2} (kT)^{-3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) \mathrm{d}E.$$

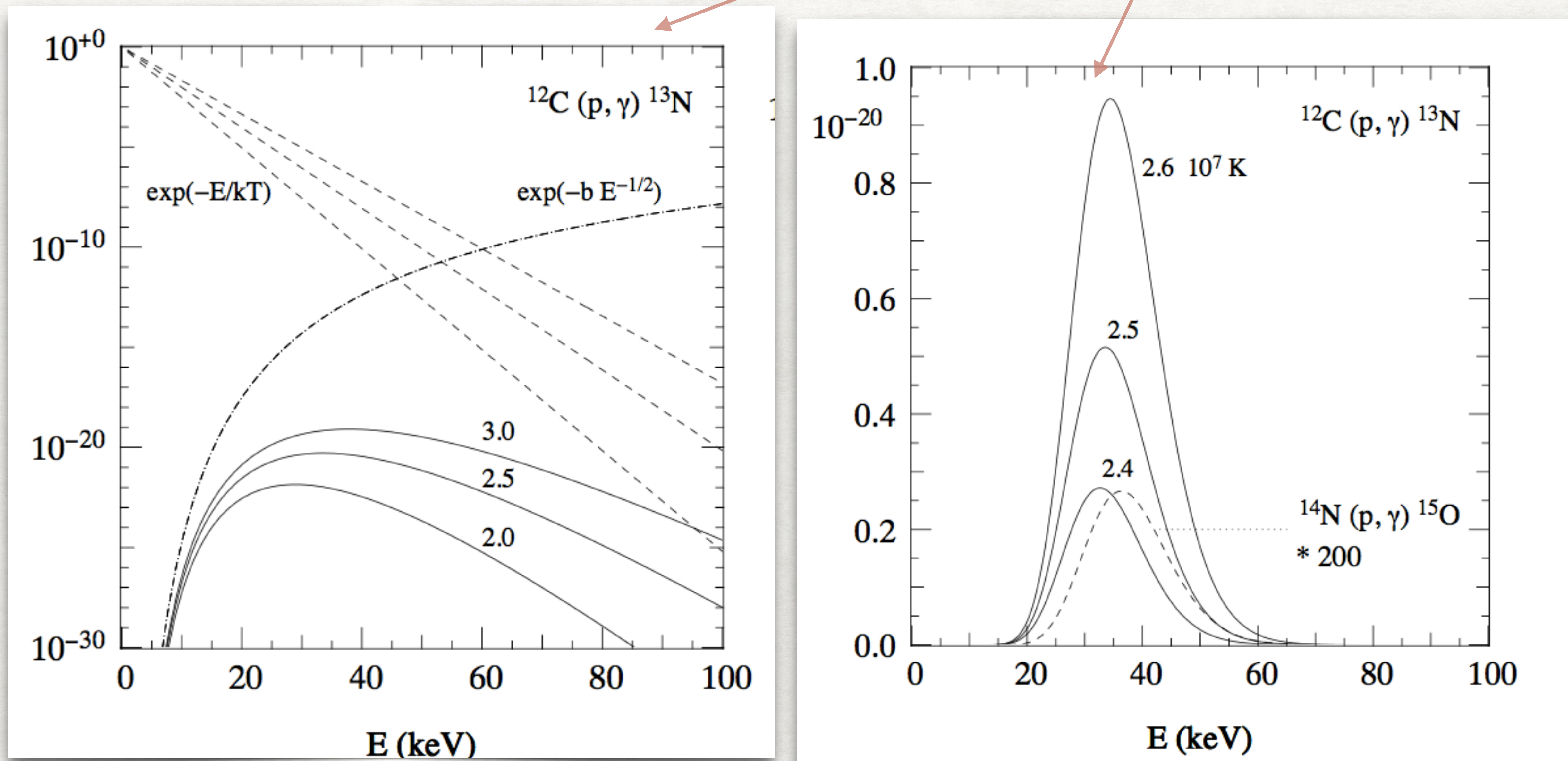
TEMPERATURE DEPENDENCE

$$\langle \sigma v \rangle = (8/\pi m)^{1/2} (kT)^{-3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}} \right) \mathrm{d}E.$$

Astrophysical S-factor containing all effects due to intrinsic nuclear properties, including resonances away from resonates S very slowly with E and we can take it out of integral

GAMOV PEAK

$$\langle \sigma v \rangle = (8/\pi m)^{1/2} (kT)^{-3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}} \right) \mathrm{d}E.$$



Example of the Gamow peak for the $^{12}C(p, \gamma)^{13}N$ reaction.

TEMPERATURE DEPENDENCE

$$\langle \sigma v \rangle = (8/\pi m)^{1/2} (kT)^{-3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}} \right) \mathrm{d}E.$$

To summarize, the properties of the Gamow peak imply that

- the reaction rate $\langle \sigma v \rangle$ increases *very strongly with temperature*.
- $\langle \sigma v \rangle$ decreases strongly with increasing Coulomb barrier.

ANALYTICAL SOLUTION

In a small range of temperature around E_0 / T_0

$$\langle\sigma v\rangle = \langle\sigma v\rangle_0 \left(\frac{T}{T_0}\right)^{\nu} \qquad \text{with} \qquad \nu \equiv \frac{\partial \log\langle\sigma v\rangle}{\partial \log T} = \frac{\tau - 2}{3}.$$

$$\tau = \frac{3E_0}{kT} = 19.72 \left(\frac{Z_i^2 Z_j^2 A}{T_7}\right)^{1/3}.$$

$T = 1.5 \times 10^7\ \mathrm{K}$ $\quad \langle\sigma v\rangle \propto T^{3.9}$ for p+p reaction for H fusion

$T = 1.5 \times 10^7\ \mathrm{K}$ $\quad \langle\sigma v\rangle \propto T^{20}$ for 14N(p,gamma) reaction in CNO

ENERGY GENERATION RATES

$$\epsilon_{ij} = \frac{Q_{ij} r_{ij}}{\rho}.$$

Energy

$$Q = (m_X + m_a - m_Y - m_b)\, c^2.$$

Rate

$$r_{\mathrm{i,j}} = \frac{1}{1+\delta_{\mathrm{ij}}} n_i n_j \langle \sigma v \rangle$$

$$\langle \sigma v \rangle = \langle \sigma v \rangle_0 \left(\frac{T}{T_0} \right)^{\nu}$$

Energy generation per unit mass is

$$\epsilon_{ij} = \epsilon_{0,ij} X_i X_j \rho T^{\nu}.$$

$$\epsilon_{\mathrm{nuc}} = \sum_{i,j} \epsilon_{ij}.$$

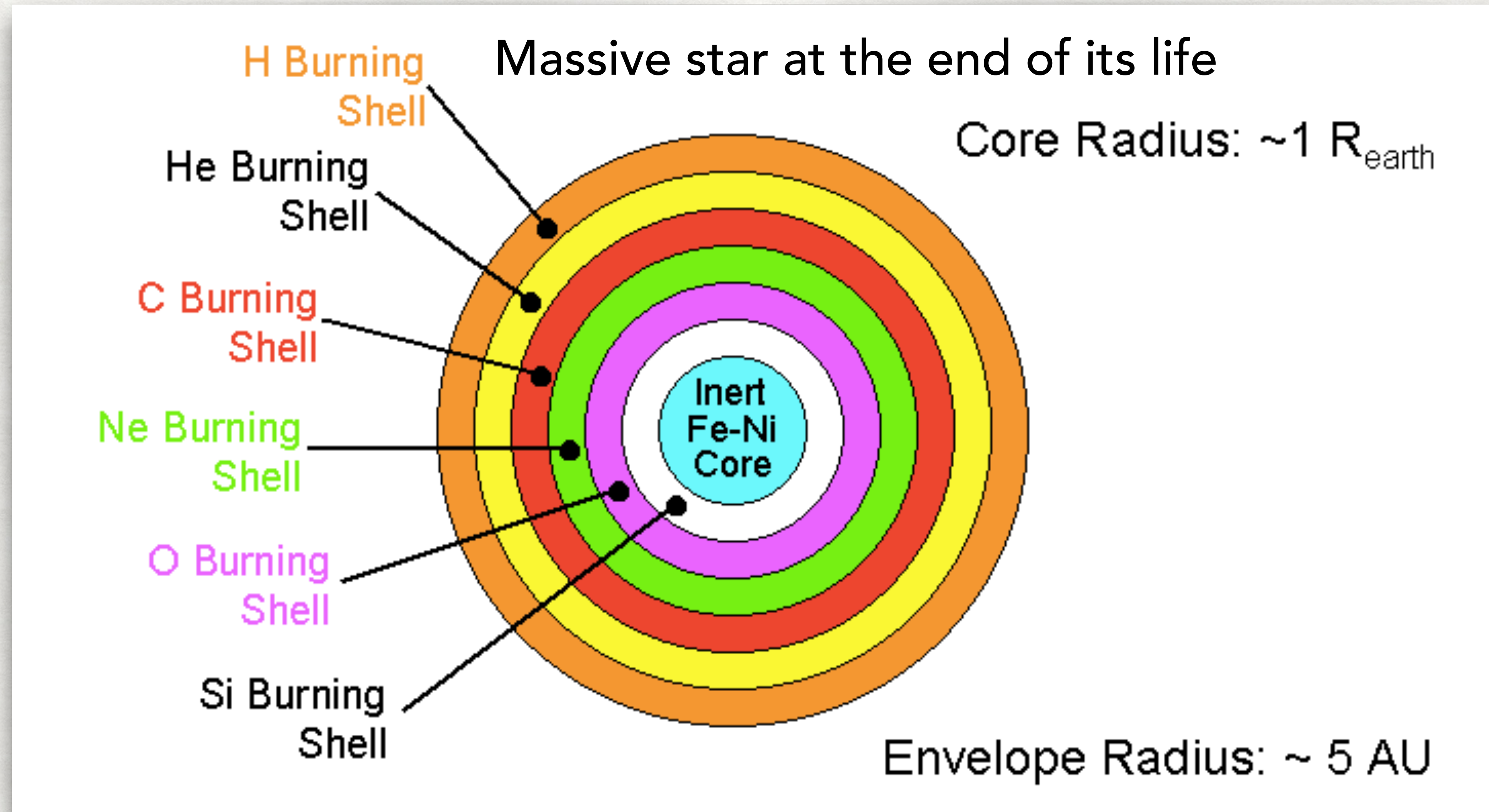
COMPOSITION CHANGE

OF COURSE IN ANY SHELL WHERE NUCLEAR REACTION OCCUR, THERE IS A COMPOSITION CHANGE AT A RATE EQUAL THE REACTION RATES

$$\frac{\mathrm{d}X_i}{\mathrm{d}t} = A_i \frac{m_\mathrm{u}}{\rho} \left(-\sum_j (1+\delta_{ij})\, r_{ij} + \sum_{k,l} r_{kl,i} \right)$$

THE MAIN NUCLEAR BURNING CYCLES

All elements up to iron are made in stars



Because of strong dependence on temperature and ZiZj (coulomb barrier), a star is

Like a tree: evolution of a star proceeds through several distinct nuclear burning cycles that generate layers with different composition

THE MAIN NUCLEAR BURNING CYCLES

Facts that simplify the description of a complex nuclear network of reactions

1. Evolution of a star proceeds through several distinct nuclear burning cycles
2. Per burning cycle, only a few reactions matters for energy production and/or composition changes
3. In a chain of reactions, the slowest determines the rate of the whole chain

hydrogen burning

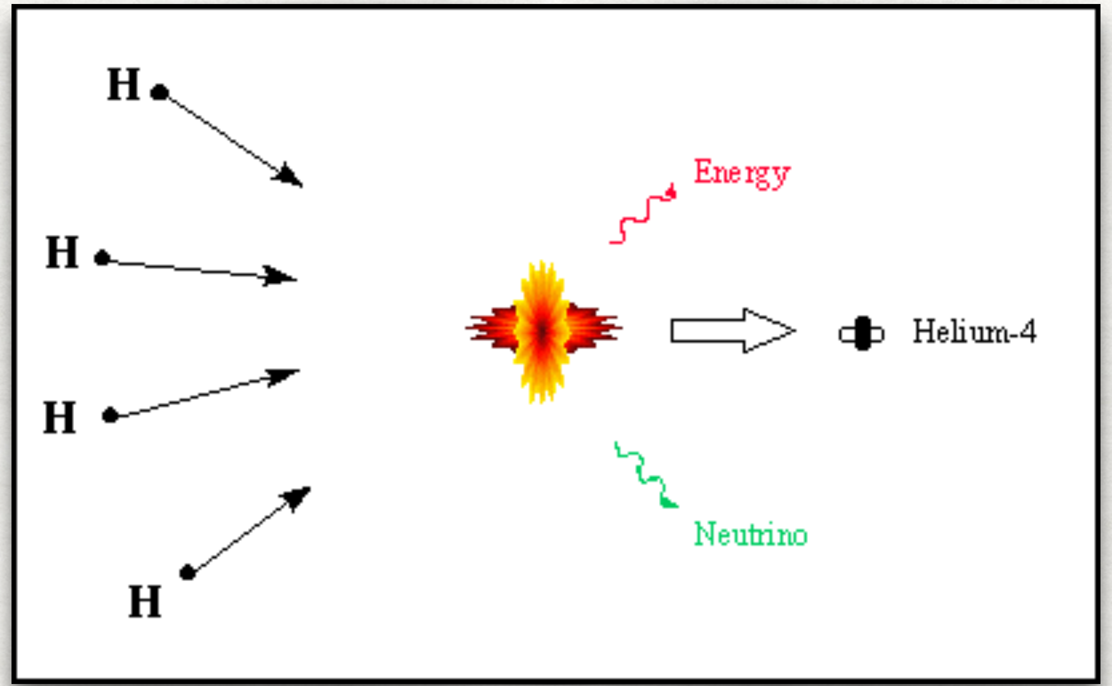
main sequence

HYDROGEN BURNING

DURING MAIN SEQUENCE LIFETIME FOR ALL STARS

net result: $4\,^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2\,\mathrm{e}^+ + 2\,\nu$

$$8 \times 10^6 \text{ K and } 5 \times 10^7 \text{ K}$$



$$Q = 26.734 \text{ MeV}$$

Two protons need to be converted into neutrons

$$p \rightarrow n + e^+ + \nu$$

weak interaction
beta-decay

Energy is transferred to stellar gas by radiation, radiation from pair annihilation and kinetic energy of nuclei (neutrinos leave the star without interaction)

HYDROGEN BURNING

net result: $4\,{}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2\,e^+ + 2\,\nu$

two possible chains

The p-p chains:

direct fusion of protons

It starts with a simultaneous strong interaction+ beta decay that form Deuterium: quite rare (10-20 a strong interaction only)

1st p-p reaction : slowest reaction

$${}^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu$$

$${}^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$$

$${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H}$$

pp1

dominates at T< 1.5 10^7 K

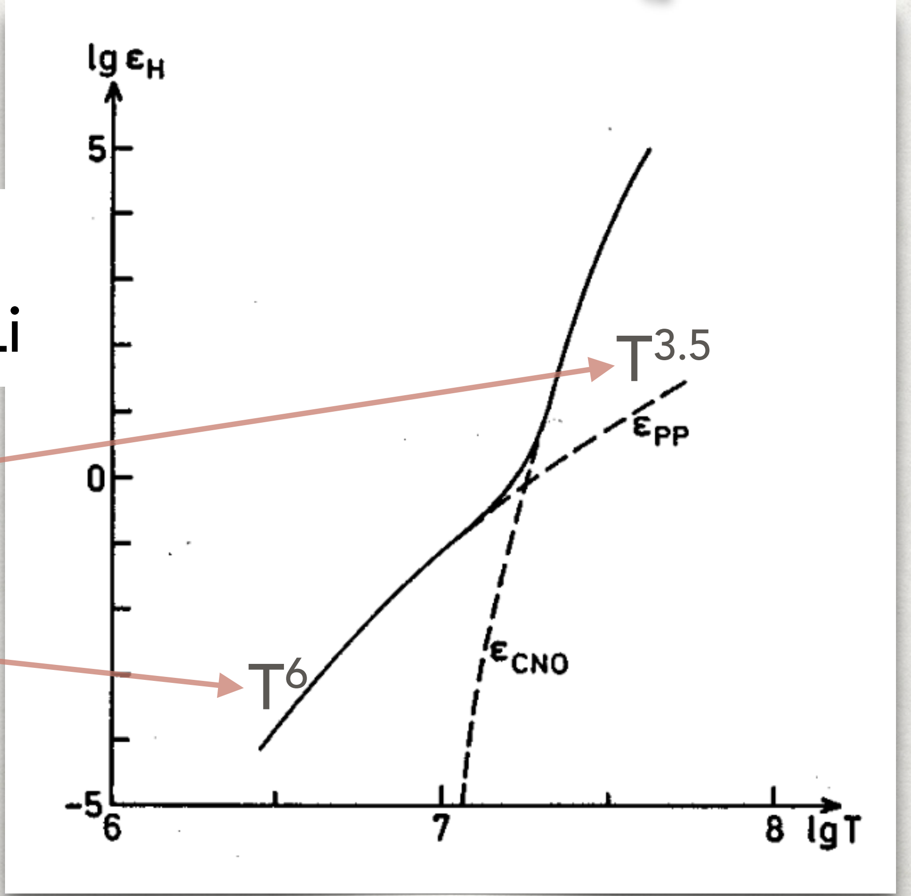
<u>main energy source for Sun</u>

pp2 pp3

involving short lived Be, Li

dominate at T> 1.5 10^7 K

$$\epsilon_{\mathrm{pp}} \propto X^2 \frac{\rho}{m_{\mathrm{u}}} T^4$$



HYDROGEN BURNING

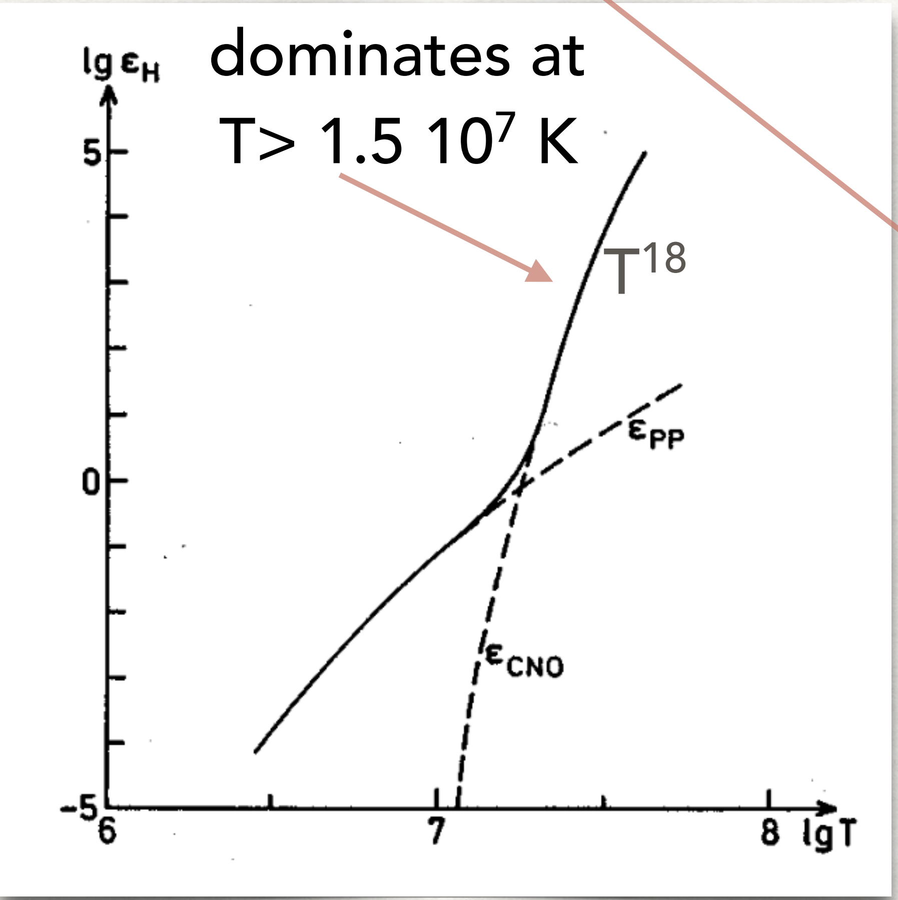
net result: $4\,^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + 2\,\mathrm{e}^{+} + 2\,\nu$

two possible chains

$$\epsilon_{\mathrm{pp}} \propto X X_{14} \frac{\rho}{m_{\mathrm{u}}} T^{18}$$

dominates at

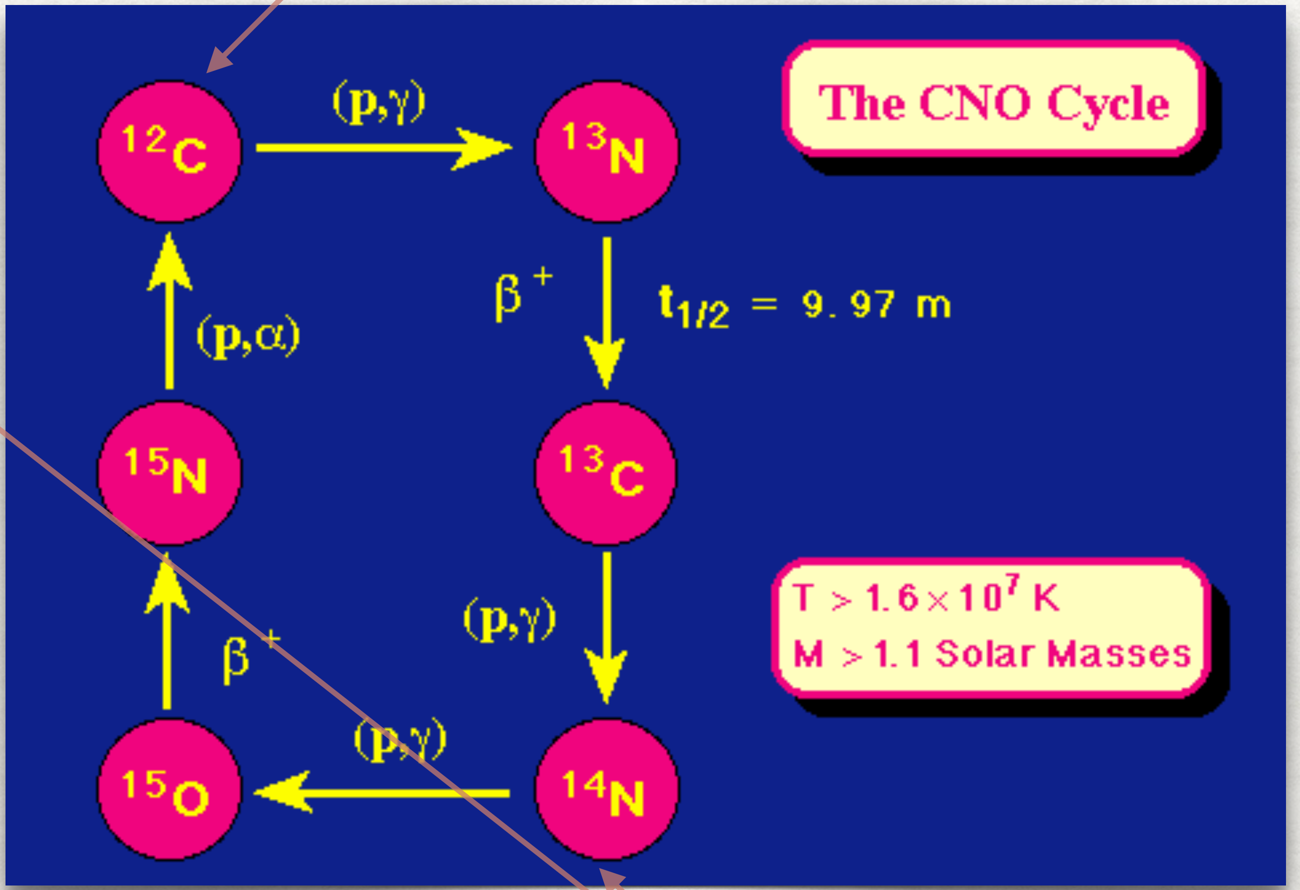
T> 1.5 10^7 K



The CNO cycle:

if some C N O present

Starts with p captured by 12C



slowest reaction

AFTER THAT AS T INCREASES...

post main sequence stages

Helium burning: ``triple alpha reaction” $3\,^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma,$ produce ^{12}C & ^{16}O

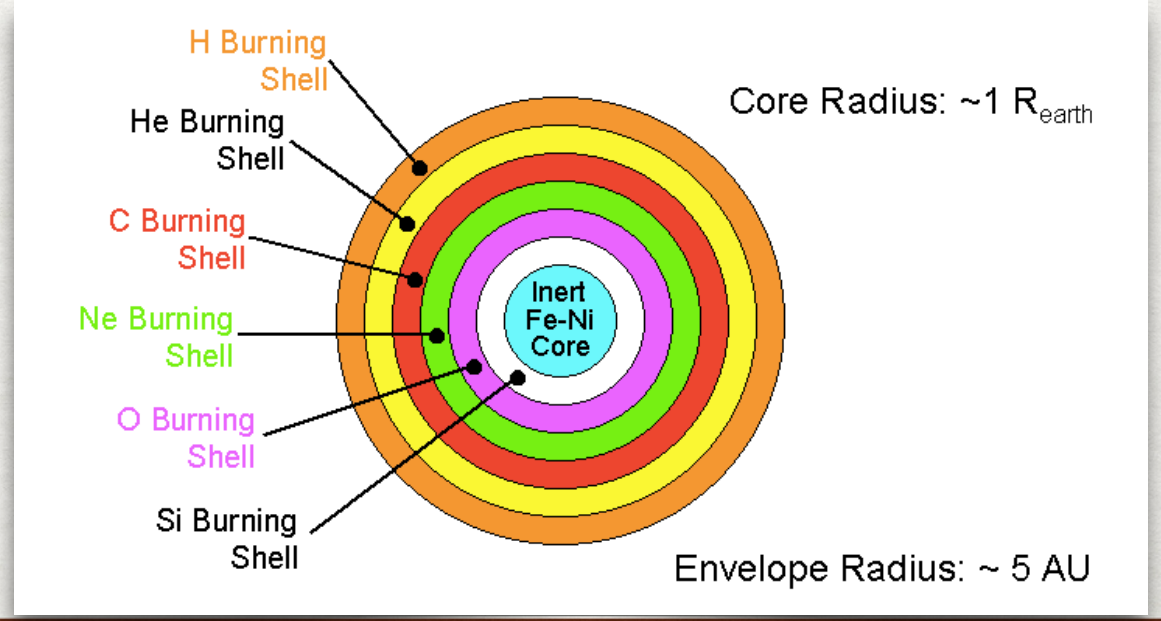
$$\epsilon_{3\alpha} \propto Y^3 T^{40} \qquad T = 10^8 \text{ K}$$

Carbon (^{12}C) burning: T> 5 10^8 K, leaves mostly ^{16}O ^{20}Ne ^{24}Mg

Neon (^{20}Ne) burning (photodisitegration for O): T>1.5 10^9 K leaves mostly ^{16}O ^{24}Mg

Oxygen (^{16}O) burning: T>2 10^9 K leaves mostly ^{28}Si ^{32}S

Silicon (^{28}Si) burning: T>3 10^9 K leaves mostly ^{56}Fe



“

SUMMARY AND STELLAR STABILITY

—Ch 7 :but only 7.1, 7.3, 7.4 (but not the derivations),7.5

”

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{7.1}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t} \tag{7.2}$$

$$\frac{\partial l}{\partial m} = \epsilon_{\mathrm{nuc}} - \epsilon_\nu - T\frac{\partial s}{\partial t} \tag{7.3}$$

$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla \qquad \text{with} \qquad \nabla = \begin{cases} \nabla_{\mathrm{rad}} = \dfrac{3\kappa}{16\pi acG}\dfrac{lP}{mT^4} & \text{if } \nabla_{\mathrm{rad}} \leq \nabla_{\mathrm{ad}} \\ \nabla_{\mathrm{ad}} + \Delta\nabla & \text{if } \nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} \end{cases} \tag{7.4}$$

$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\sum_j (1+\delta_{ij})\, r_{ij} + \sum_{k,l} r_{kl,i}\right) \quad [\text{+ mixing terms}] \qquad i = 1 \ldots N \tag{7.5}$$