NOTES: CH 4 & 5

POLYTROPIC & ENERGY TRANSPORT

SUMMARY PREVIOUS CLASS

EQUATION OF STATE (EOS)FOR STARS
LOCAL THERMAL EQUILIBRIUM + IDEAL GAS

$$P = P(\rho, T, X)$$

relation with internal energy (for both gas & photons)

non relativistic case

$$U = \frac{3}{2}P$$

gas in Main Sequence (MS) stars and protons in WDs

ultra relativistic case

$$U = 3P$$

photons
electrons in massive WD

in general

$$U = \phi P$$

SUMMARY PREVIOUS CLASS

EQUATION OF STATE (EOS)FOR STARS

LOCAL THERMAL EQUILIBRIUM + IDEAL GAS

$$P = P(\rho, T, X)$$

classical limit: NR & R

$$P_{\rm gas} = n k_{\rm b} T = \frac{R}{\mu}\rho T$$

stars on the main sequence

degenerate gas T=0

$$P_{\rm gas,d} = K_{\rm nr}\ \rho^{5/3}$$

$$P_{\rm gas,d} = K_{\rm ur}\ \rho^{4/3}$$

white dwarfs & cores of late stage stars

radiation

$$P_{\rm rad} = \frac{1}{3} a T^4$$

black body spectrum

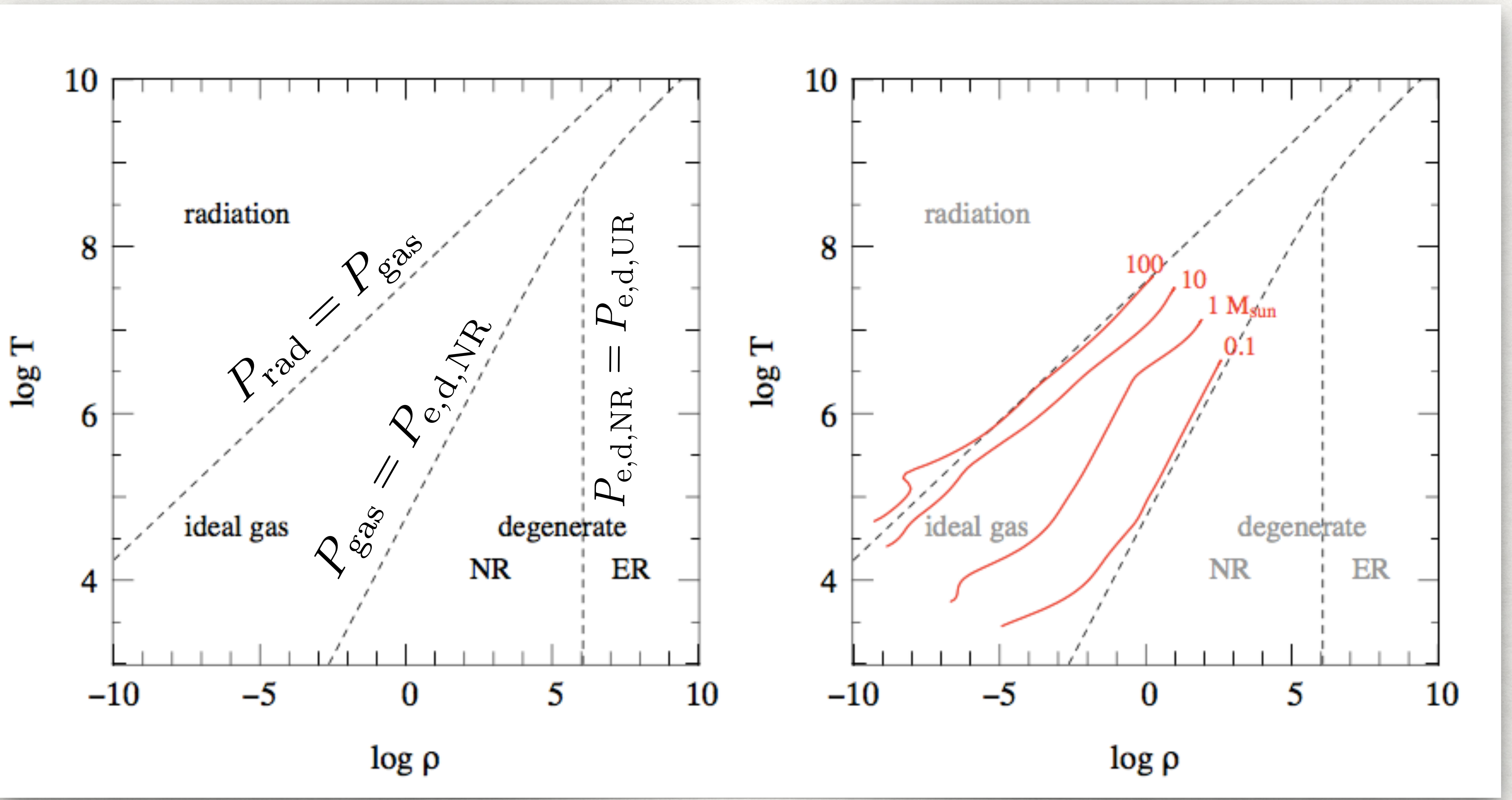
Pressure is additive

$$P = P_{\rm e-} + P_{\rm ion} + P_{\rm neutron} + P_{\rm rad} = P_{\rm gas} + P_{\rm rad}$$

$$P_{\rm gas} = \beta P$$

$$P_{\rm rad} = (1-\beta) P$$

SUMMARY PREVIOUS CLASS



from O.R. Pols

“

POLYTROPIC STARS

Chapter 4

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POLYTROPIC RELATION

CHAPTER 4

polytropic index n:

$$n = \frac{1}{\gamma - 1} \quad \text{or} \quad \gamma = 1 + \frac{1}{n}$$

- $P = K\rho^{\gamma}$ where K and γ are constant

Examples:

1) the pressure of degenerate electrons (see previous class)

$$\boxed{P_e = K_{NR}\left(\frac{\rho}{\mu_e}\right)^{5/3}} \quad \text{with} \quad K_{NR} = \frac{h^2}{20 m_e m_u^{5/3}}\left(\frac{3}{\pi}\right)^{2/3} = 1.0036 \times 10^{13}\ [\text{cgs}].$$

$$\boxed{P_e = K_{ER}\left(\frac{\rho}{\mu_e}\right)^{4/3}} \quad \text{with} \quad K_{ER} = \frac{hc}{8 m_u^{4/3}}\left(\frac{3}{\pi}\right)^{1/3} = 1.2435 \times 10^{15}\ [\text{cgs}].$$

2) the response of pressure to adiabatic compression or expansion (Ch 3.4)

$$\gamma_{ad} = \left(\frac{\partial \log P}{\partial \log \rho}\right)_{ad}$$

$\gamma_{ad} = \text{const.}$

$$P \propto \rho^{\gamma_{ad}}$$

$$\gamma_{ad} = \frac{\phi + 1}{\phi} \quad \text{(for a simple, perfect gas).}$$

$$\begin{cases} \gamma_{ad} = \frac{4}{3}(\phi = 3) & \text{UR} \\ \gamma_{ad} = \frac{5}{3}(\phi = 3/2) & \text{NR} \end{cases}$$

For a mixture of gas and radiation: $\frac{4}{3} < \gamma_{ad} < \frac{5}{3}$

P0LYTROPIC STELLAR MODEL

- for a newtonian star the mechanical structure of a star is fully determined by:

$$\boxed{\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}} \quad + \quad \frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm}{r^2}\rho, \quad + \quad P = K\rho^\gamma \quad \Rightarrow$$

$$\frac{1}{\rho r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 \rho^{\gamma-2}\frac{\mathrm{d}\rho}{\mathrm{d}r}\right) = -\frac{4\pi G}{K\gamma}$$

with

$$\rho(0) = \rho_c \qquad \text{and} \qquad \left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)_{r=0} = 0,$$

LANE-EMDEN EQ.

- with a dimensional analysis it is possible to build a length-scale from G, K, and $\boldsymbol{\rho}_c$

$$r_c = \sqrt{\frac{n+1}{4\pi} \frac{\kappa \rho_c^{\frac{1}{n}-1}}{G}}$$

$$x = \frac{r}{r_c} \quad \text{et} \quad y = \left(\frac{\rho}{\rho_c}\right)^{\frac{1}{n}}$$

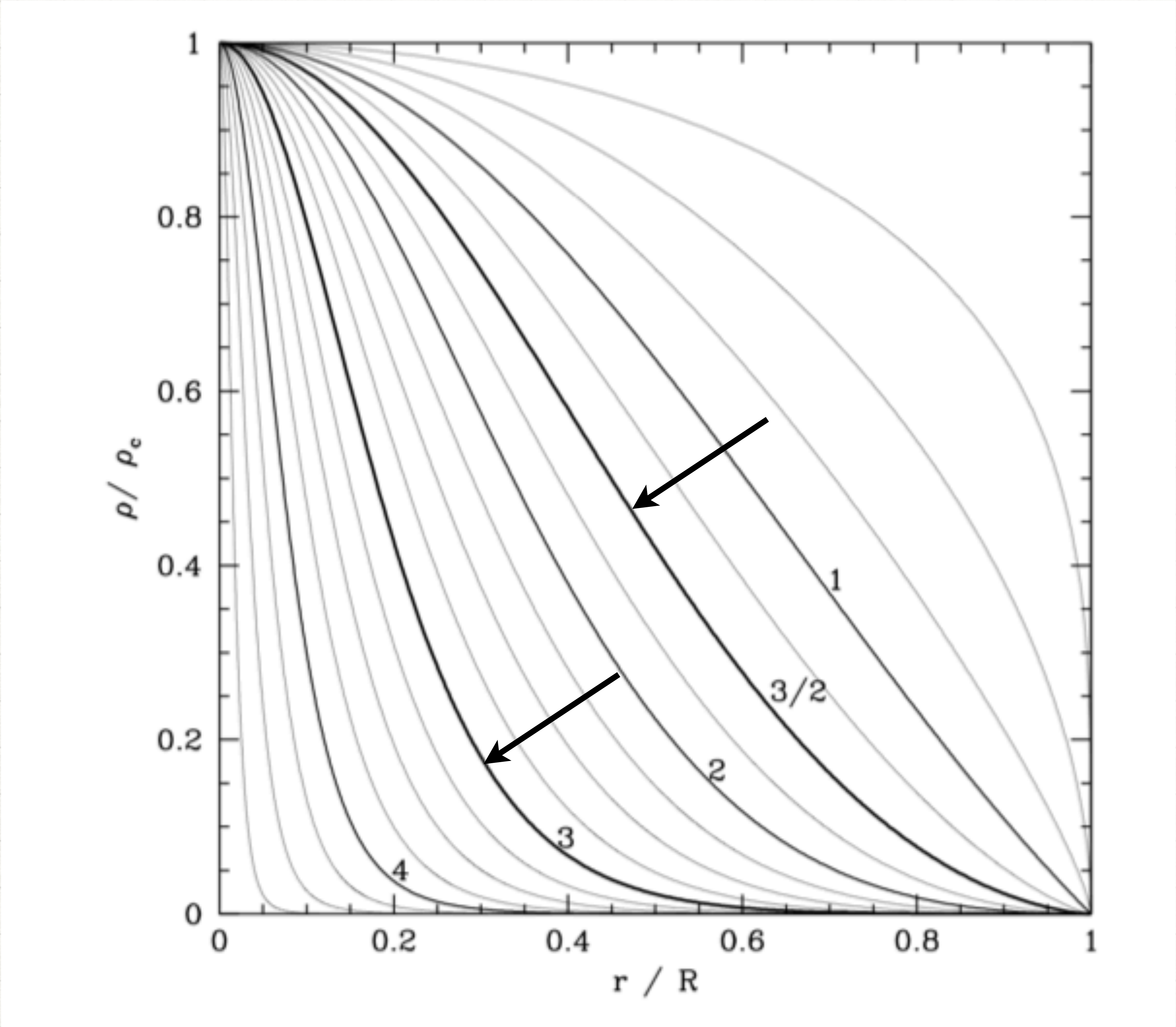
normalised variables

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2 \frac{dy}{dx}\right) = -y^n$$

$$y(0) = 1 \quad ; \quad y'(0) = 0$$

only n< 5 have a finite size (at x=x_{max}) and it is physically acceptable

LANE-EMDEN SOLUTIONS



in evidence solutions for n=1, 2/3, 2,3,4

LANE-EMDEN EQ.

It is possible to find relation between R, M, central density

$$
\begin{aligned}
R &= \left((n+1)\frac{\kappa}{4\pi G}\right)^{\frac{1}{2}} \rho_c^{\frac{1-n}{2n}}\, x_{\max} \\
\rho_c &= \left((n+1)\frac{\kappa}{4\pi G}\right)^{\frac{n}{n-1}} R^{\frac{2n}{1-n}}\, x_{\max}^{\frac{2n}{n-1}} \\
M &= 4\pi\left((n+1)\frac{\kappa}{4\pi G}\right)^{\frac{3}{2}} \rho_c^{\frac{3-n}{2n}} \left(-x^2\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x_{\max}} \\
M &= 4\pi\left((n+1)\frac{\kappa}{4\pi G}\right)^{\frac{n}{n-1}} R^{\frac{3-n}{1-n}} \left(-x^{\frac{5-3n}{1-n}}\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x_{\max}}
\end{aligned}
$$

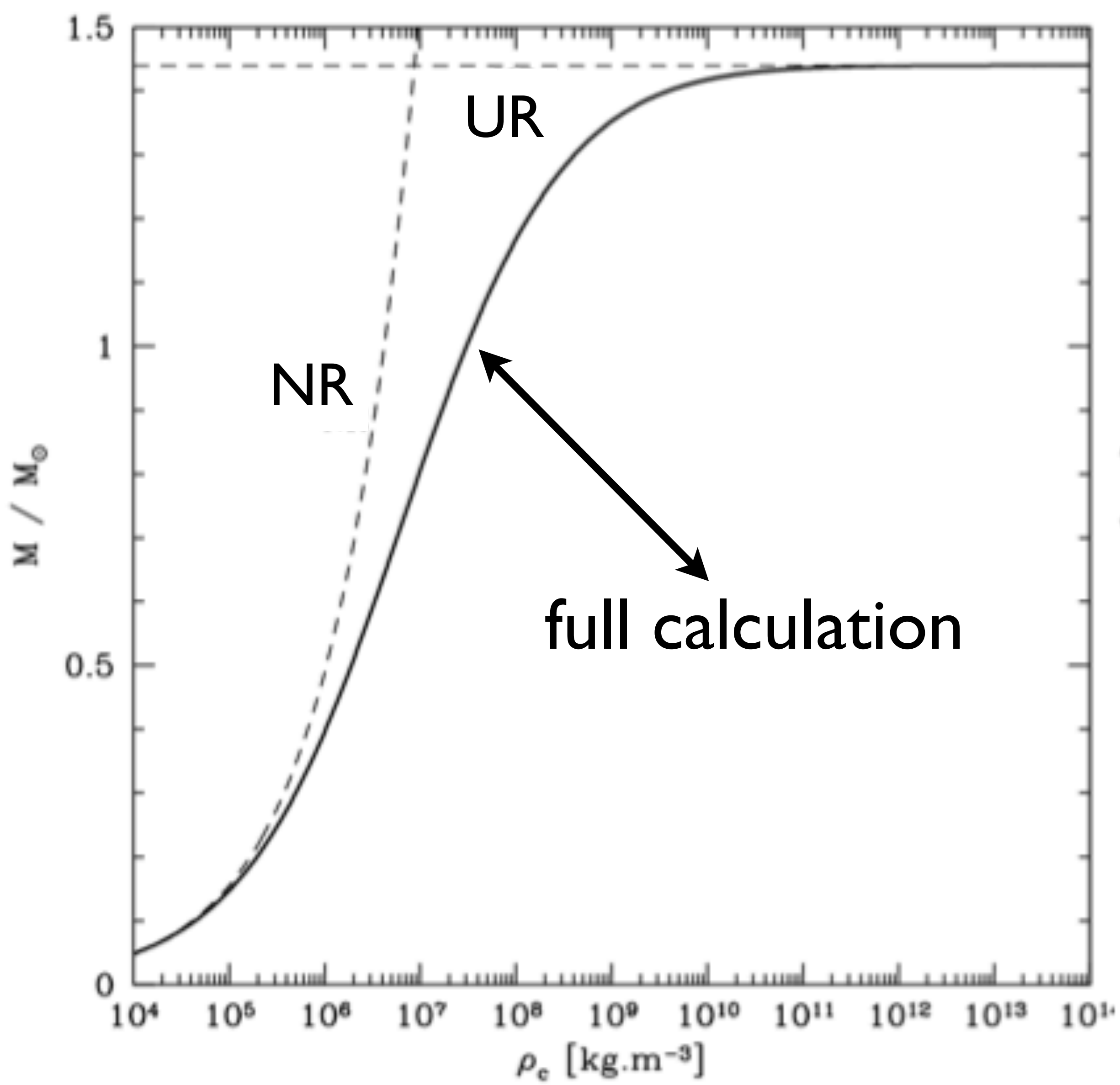
mass & radius for n=3/2 n=3

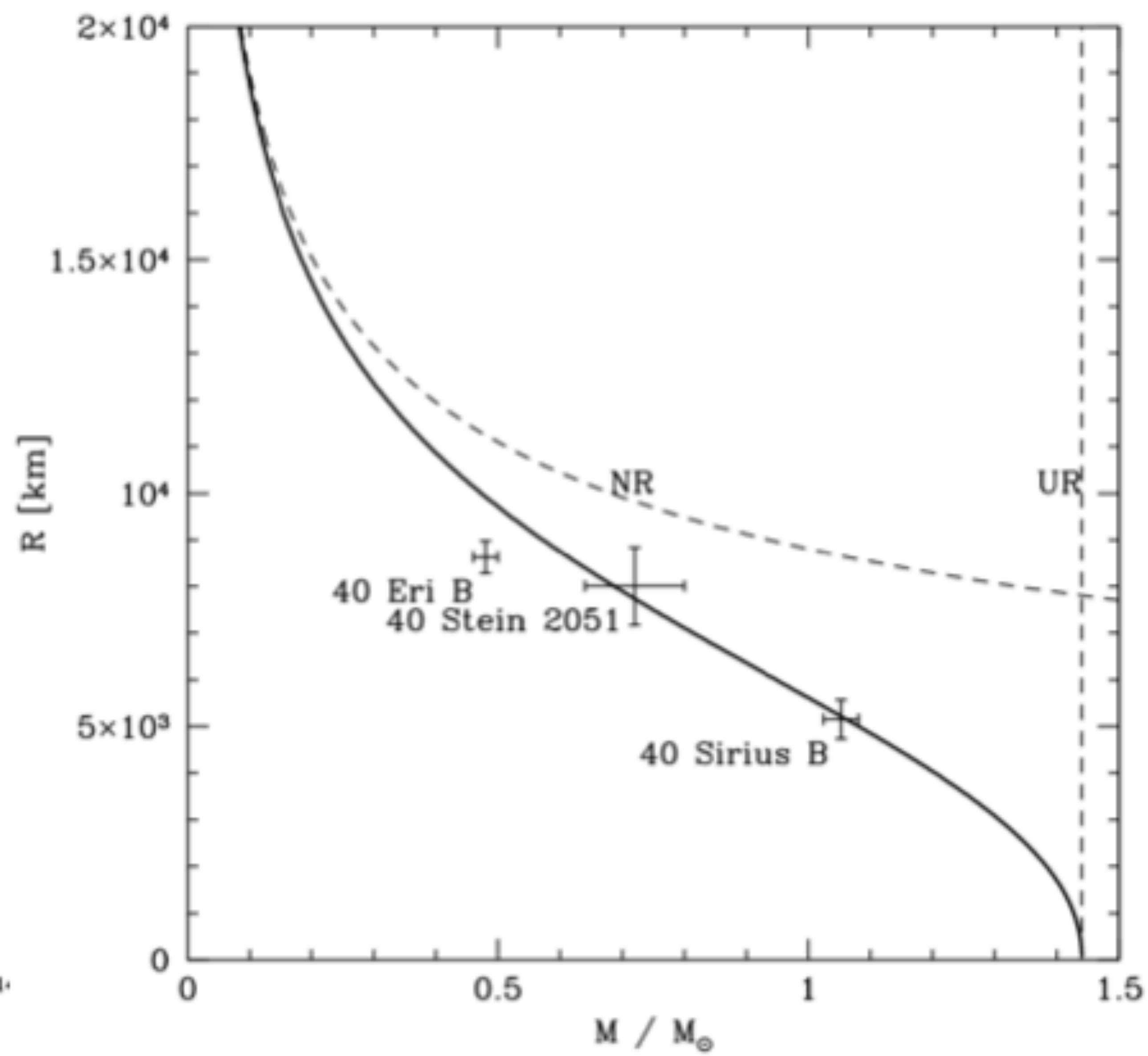
γ	n	$x_{\max}$	$-x_{\max}^2 y'(x_{\max})$	Mass & Radius
5/3	3/2	3.65375	2.71406	$R = 11\,220\,\mathrm{km}\left(\frac{\rho_c}{10^9\,\mathrm{kg.m^{-3}}}\right)^{-\frac{1}{6}}\left(\frac{Y_e}{0.5}\right)^{5/6}$ $M = 0.4964\,M_\odot\left(\frac{\rho_c}{10^9\,\mathrm{kg.m^{-3}}}\right)^{\frac{1}{2}}\left(\frac{Y_e}{0.5}\right)^{5/2}$ $= 0.7011\,M_\odot\left(\frac{R}{10^4\,\mathrm{km}}\right)^{-3}\left(\frac{Y_e}{0.5}\right)^{5}$
4/3	3	6.89685	2.01824	$R = 33\,470\,\mathrm{km}\left(\frac{\rho_c}{10^9\,\mathrm{kg.m^{-3}}}\right)^{-\frac{1}{3}}\left(\frac{Y_e}{0.5}\right)^{2/3}$ $M = 1.457\,M_\odot\left(\frac{Y_e}{0.5}\right)^{2}$

$\gamma = 5/3$ $M \propto R^{-3}$: the more massive, the denser, the more relativistic…

$\gamma = 4/3$ $M = \mathrm{const.}$ with a mass limit given by UR limit!

Chandrasekhar mass





classical WD, observations

EDDINGTON ``STANDARD MODEL"

Let's assume $\beta = \text{const.}$ (for radiative stars this is roughly true)

$$1 - \beta = \frac{P_{\rm rad}}{P} = \frac{aT^4}{3P}$$

$$P_{\rm gas} = \beta P = \frac{R}{\mu}\rho T$$

$$P = \frac{aT^4}{3(1-\beta)}$$

$$T = \frac{\beta P \mu}{\rho R}$$

$$P = \left(\frac{3\mathcal{R}^4}{a\mu^4}\,\frac{1-\beta}{\beta^4}\right)^{1/3} \rho^{4/3} = K\rho^{4/3}$$

Polytropic relation with n=3

$$M = 4\pi\,\Theta_3 \left(\frac{K}{\pi G}\right)^{3/2}$$

Unique relation between beta and Mass:
the more massive the star, the more radiation dominated

ENERGY TRANSPORT IN STELLAR INTERIOR

Chapter 5

LOCAL ENERGY CONSERVATION

- first law of thermodynamics [changes per unit mass in a time ∂t]

$$\delta u = \delta q + \frac{P}{\rho^2}\,\delta\rho.$$

work compression adds energy $(\delta\rho\ >\ 0)$

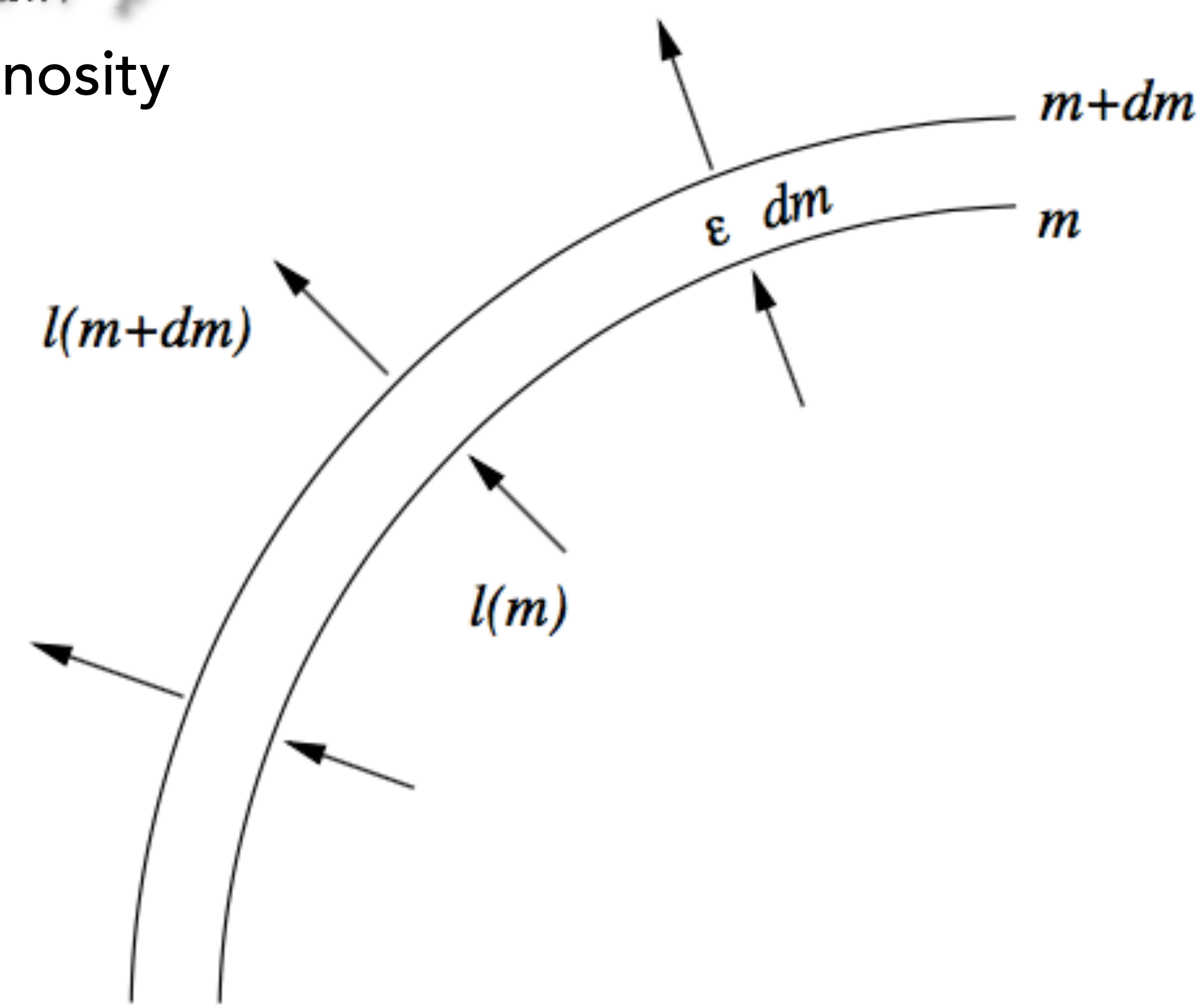
heat

$$\partial q = \left(\epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial l}{\partial m}\right)\partial t$$

$$\boxed{\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}}$$

$l = a\pi r^2 F$

luminosity



<u>LOCAL</u> ENERGY CONSERVATION

$$\boxed{\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu - \frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}}$$

+

second law of thermodynamics
(s =entropy)

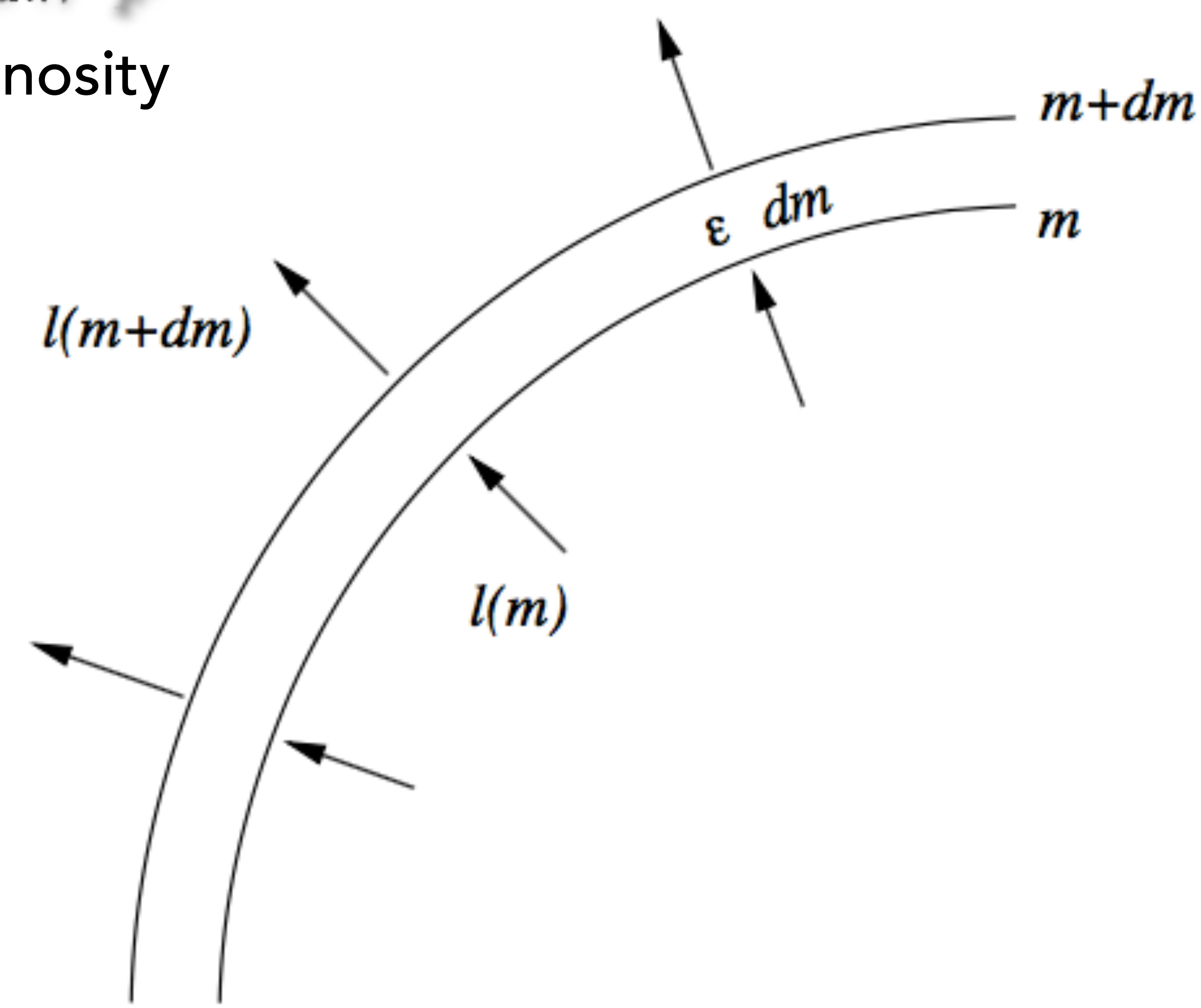
$$dq = Tds = du + PdV = du - \frac{P}{\rho^2}d\rho$$

$$\epsilon_{\rm gr} = -\frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} = -T\frac{\partial s}{\partial t}$$

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu + \epsilon_{\rm gr}$$

$l = a\pi r^2 F$

luminosity



THERMAL EQUILIBRIUM

$$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu + \cancel{\epsilon_{\rm gr}}$$

$$\frac{\mathrm{d}l}{\mathrm{d}m} = \epsilon_{\rm nuc} - \epsilon_\nu .$$

equilibrium = no changes in time

At equilibrium over whole star

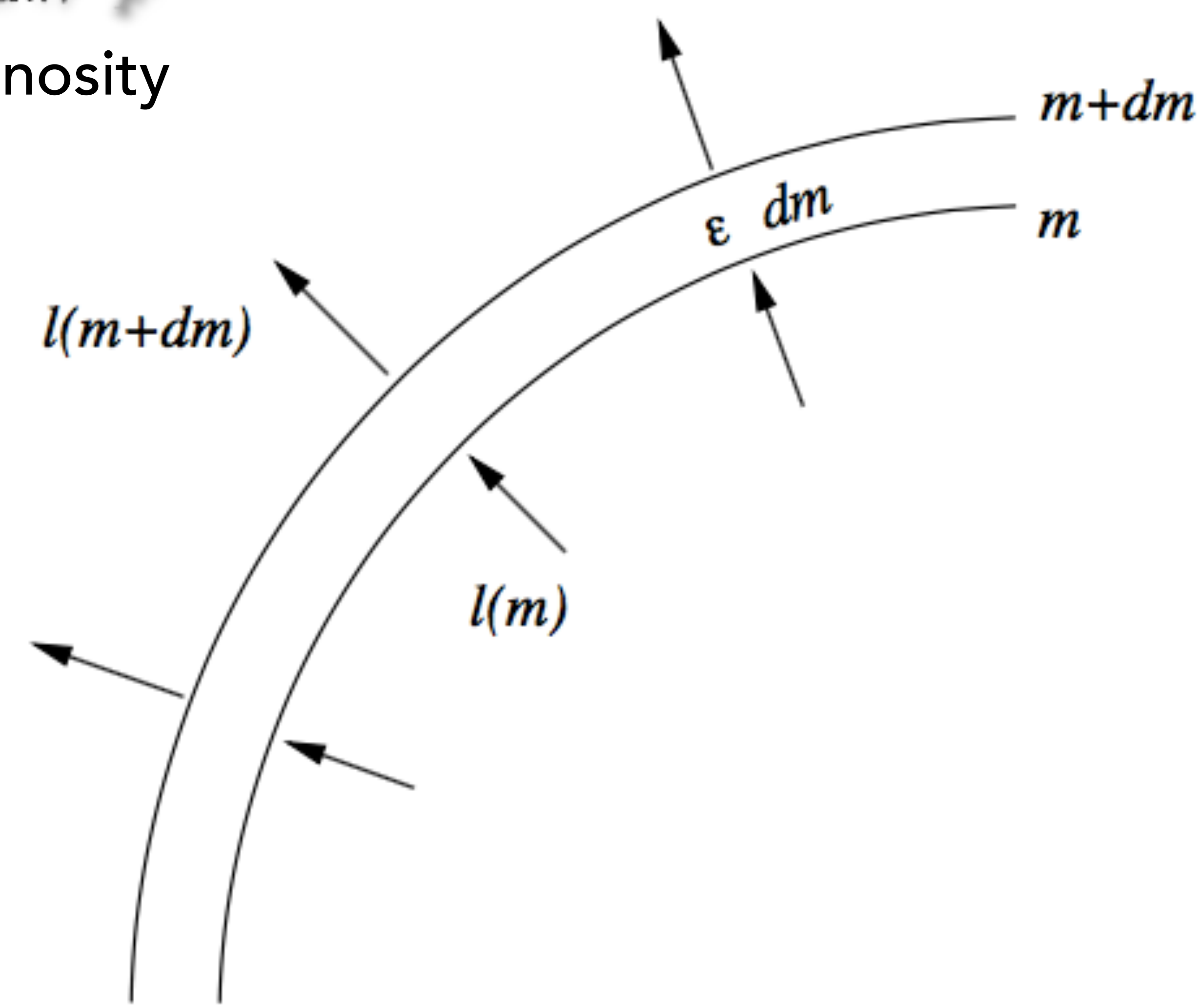
nuclear reaction luminosity

$$L = \int_0^M \epsilon_{\rm nuc}\,\mathrm{d}m - \int_0^M \epsilon_\nu\,\mathrm{d}m \equiv L_{\rm nuc} - L_\nu$$

neutrino luminosity

$l = a\pi r^2 F$

luminosity



neglecting neutrino losses, we re-obtain

$$L = L_{\rm nun}$$

$$\dot{E}_{\rm tot} = L_{\rm nun} - L = 0$$

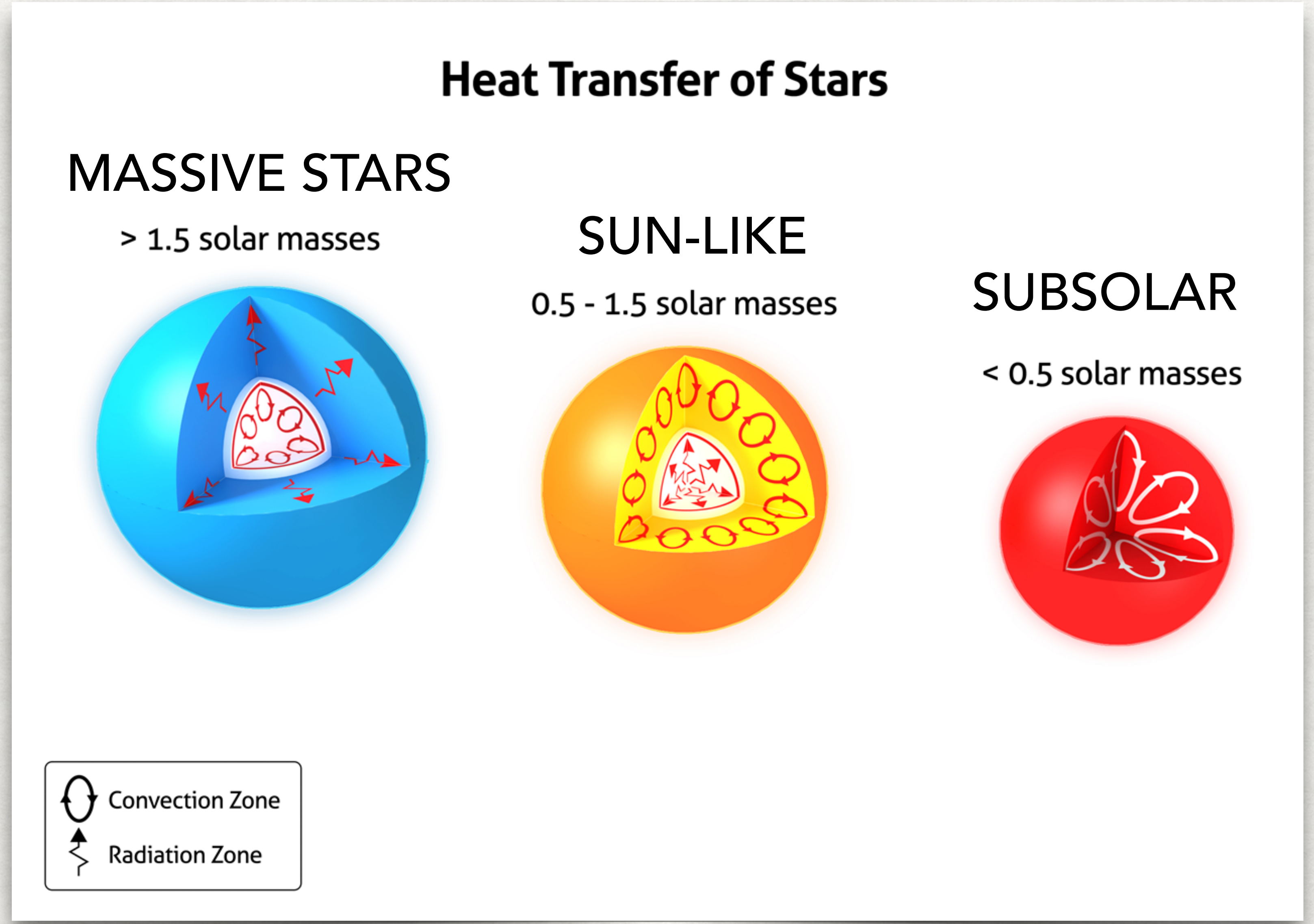
MECHANISMS TO TRANSPORT ENERGY FROM HOT STELLAR INTERIOR TO THE COOLER ATMOSPHERE

- **HEAT DIFFUSION**
  - **radiative diffusion** by motion of radiation
  - **heat conduction** by motion of gas particles (e-)

  <u>random</u> thermal motion of particles

- **CONVECTION**

  <u>bulk</u> motion of particles



RELATIVE IMPORTANCE: <u>MASS DEPENDENT</u>

MECHANISMS TO TRANSPORT ENERGY FROM HOT STELLAR INTERIOR TO THE COOLER ATMOSPHERE

- HEAT DIFFUSION

  random *thermal*
  motion of particles

  - **radiative diffusion** by motion of radiation
  - **heat conduction** by motion of gas particles (e-)

✓ **Equation of heat conduction**, valid for all particles in LTE (gas & photons)

$$F = -K\nabla T$$

with conductivity

$$K = \frac{1}{3}\langle v\rangle\, l\, C_{\rm v}$$

mean free path

mean velocity

specific heat per unit volume

RADIATIVE DIFFUSION

FACTS ALREADY KNOWN TO US

- ✓ If nuclear source is suddenly quenched (L_{nuc} =0) it takes a Kelvin-Helmholtz timescale (~10^7 yr) for the star to realise it. this is also the time for photons to diffuse outwards => stars very opaque.
- ✓ locally the radiation field is <u>close</u> to black body => $U = u\rho = aT^4$

  there is some anisotropy in the field that gives rise to heat diffusion

RADIATIVE DIFFUSION

conductivity $K = \frac{1}{3} \langle v \rangle \, l \, C_{\mathrm{v}}$

PHOTONS: $C_{\mathrm{v}} \equiv \frac{dU}{dT} = 4aT^3$ and $\langle v \rangle = c$

and mean free path ?

equation of radiative transfer without absorption

optical depth: $d\tau_\nu = \rho\kappa_\nu$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu \rho \, I_\nu,$$

$$I_\nu(s) = I_\nu(s_0) e^{-\int_{s_0}^{s} \rho\kappa_\nu ds'} = I_\nu(s_0) e^{-\rho\kappa_\nu \Delta s}$$

homogeneous medium

mass absorption coefficient or opacity $\mathrm{cm}^2\,\mathrm{g}^{-1}$

$$\Delta s = \frac{1}{\rho\kappa_\nu} \equiv l_\nu \quad \text{then} \quad \frac{I_\nu(s)}{I_\nu(s_0)} = e^{-1}$$

mean free path: length over which the intensity is suppressed by ``e”

Thomson scattering

A free electron has a cross section to radiation independent of frequency given by

$$\sigma_\mathrm{T} = 6.7 \times 10^{-25}\ \mathrm{cm}^2$$

$$\kappa_\mathrm{T} = \frac{n_\mathrm{e}}{\rho}\sigma_\mathrm{T} = \frac{\sigma_\mathrm{T}}{\mu_\mathrm{e} m_\mathrm{u}} = 0.4\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

for pure hydrogen

$m_\mathrm{u} = 1.66 \times 10^{-24}\ \mathrm{g}$ and $\dfrac{1}{\mu_\mathrm{e}} \approx \dfrac{2}{1+X} = 1$

$$l_\mathrm{T} = \frac{1}{\rho\kappa_\mathrm{T}} \approx 1.7\ \mathrm{cm}$$

RADIATIVE TEMPERATURE GRADIENT

PUTTING ALL TOGETHER:

$$\boxed{K_{\mathrm{rad}} = \frac{4}{3}\frac{acT^3}{\kappa\rho}}$$

opacity averaged over frequency (see later)

$$\boldsymbol{F}_{\mathrm{rad}} = -K_{\mathrm{rad}}\nabla T = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\nabla T.$$

working in spherical symmetry and solving for grad T:

$$\frac{\partial T}{\partial r} = -\frac{3\kappa\rho}{16\pi acT^3}\frac{l}{r^2}$$

r—>m

$$\boxed{\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}}$$

This is the temperature gradient needed in a star to transport the luminosity "l"

- when this happens a star or region within a star is said to be **radiative**
- temperature decreases from the core towards the atmosphere (thus pressure)

RADIATIVE TEMPERATURE GRADIENT

$$\boxed{\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 ac}\frac{\kappa l}{r^4 T^3}} \quad + \quad \boxed{\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}} \quad =$$

$$\boxed{\nabla_{\rm rad} = \left(\frac{d\log T}{d\log P}\right)_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa l P}{mT^4}}$$

logarithmic temperature gradient as a function of depth (Pressure) for a star in hydrostatic equilibrium, where energy is transported only radiatively

ROSSELAND MEAN OPACITY

$$\boldsymbol{F}_{\mathrm{rad}} = -K_{\mathrm{rad}}\,\boldsymbol{\nabla} T = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\boldsymbol{\nabla} T.$$

Derivation:

$$\boldsymbol{F}_\nu = -D_\nu \boldsymbol{\nabla} U_\nu = -D_\nu \frac{\partial U_\nu}{\partial T}\boldsymbol{\nabla} T$$

where

$$D_\nu = \tfrac{1}{3}c\ell_\nu = \frac{c}{3\kappa_\nu\rho}.$$

Energy density

$$U_\nu = h\nu\; n(\nu)$$

$$F_{\mathrm{rad}} = \int_0^\infty F_\nu d\nu = -\left[\frac{c}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial T}d\nu\right]\nabla T$$

$$F_{\mathrm{rad}} = -\frac{c}{3\rho}\left(\int_0^\infty \frac{\partial U_\nu}{\partial \nu}d\nu\right)\boxed{\frac{\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial U_\nu}{\partial \nu}d\nu}{\int_0^\infty (\frac{\partial U_\nu}{\partial \nu})d\nu}}\nabla T$$

$$\left(\int_0^\infty \frac{\partial U_\nu}{\partial \nu}d\nu\right) = 4aT^3$$

$$= \frac{1}{k}$$

harmonic mean of opacity = transparency:
i.e. frequency with smaller opacity are weighted more

LOCAL EDDINGTON LIMIT

in radiative zones, the radiative temperature implies a Pressure gradient:

$$\left.\begin{array}{l} \dfrac{dT}{dr} = -\dfrac{3\rho\kappa}{16\pi a c T^3}\dfrac{l}{r^2} \\ P_{\rm rad} = \dfrac{1}{3}aT^4 \end{array}\right\} \quad \frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}r} = -\frac{4}{3}aT^3\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{\kappa\rho}{4\pi c}\frac{l}{r^2}.$$

In hydrostatic equilibrium this should be smaller than the inward gravitational force

$$\frac{1}{\rho}\frac{dP_{\rm rad}}{dr} \leq \frac{Gm}{r^2} \quad \Rightarrow \quad \frac{\kappa\rho}{4\pi c}\frac{l}{r^2} \leq \frac{Gm}{r^2}$$

$$l \leq \frac{Gm4\pi c}{\kappa\rho} = 3.8\times 10^4\left(\frac{m}{M_{\rm sun}}\right)\left(\frac{0.34}{\kappa}\right)L_{\rm sun} \equiv L_{\rm edd}$$

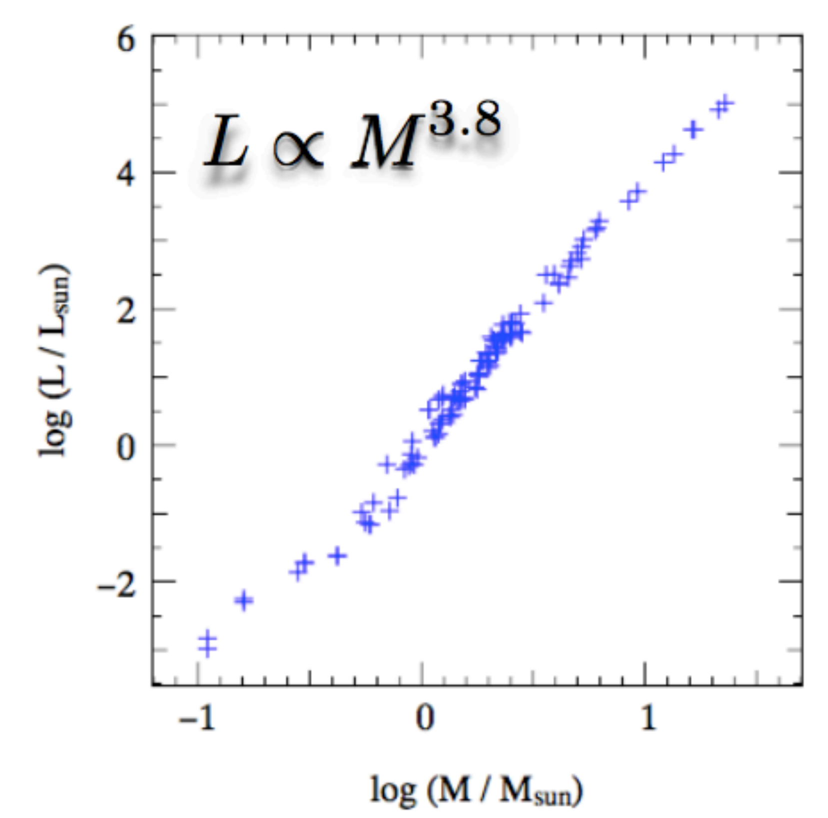
Thomson scattering X=0.7

LOCAL EDDINGTON LIMIT

$$l \leq \frac{Gm4\pi c}{\kappa\rho} = 3.8 \times 10^4 \left(\frac{m}{M_{\rm sun}}\right)\left(\frac{0.34}{\kappa}\right) L_{\rm sun} \equiv L_{\rm edd}$$

- $l = L_{edd}$ when $P \sim P_{rad}$: i.e. for radiation dominated stars=> for massive stars
- Since $L \sim M^x$ $x > 1$ eventually $L >= L_{edd}$ as M increases
- $l > L_{edd}$ in zones of large opacity (low T) like outer layers of Sun

When l > Ledd convection must take over to ensure hydrostatic equilibrium



HEAT CONDUCTION

$$F = -K\nabla T$$

with conductivity $$K = \frac{1}{3}\langle v\rangle\, l\, C_{\rm v}$$

mean free path

specific heat per unit volume

mean velocity

IDEAL GAS:

$$\langle v\rangle = \sqrt{\frac{3kT}{m}} \ll c$$

electron collision:

$$\sigma_{\rm c} = 10^{-18} - 10^{-20}\ {\rm cm}^2$$

$$\Rightarrow \frac{l_{\rm c}}{l_{\rm ph}} \approx \frac{\sigma_{\rm T}}{\sigma_{\rm c}} = 10^{-7} - 10^{-5}$$

$$K_{\rm rad} \gg K_{\rm cd}$$

$l_{\rm ph}$ is mean free path for $\simeq l_{\rm T}$

note: $\sigma_{\rm T} = 6.7 \times 10^{-25}\ {\rm cm}^2$

- Where ideal gas conditions of temperature and density hold, **heat conduction is unimportant relative to radiative transport**
- **main sequence stars**

HEAT CONDUCTION

$$F = -K \nabla T$$

mean free path

with conductivity $$K = \frac{1}{3} \langle v \rangle \, l \, C_{\mathrm{v}}$$

specific heat per unit volume

mean velocity

degenerate electron gas:

$$\frac{p_{\mathrm{F}}}{m_{\mathrm{e}} c} \approx 0.8 \quad \langle v \rangle \to c$$

degenerate electron collision:

$$\frac{l_{\mathrm{c}}}{l_{\mathrm{ph}}} \gg 1$$

$$K_{\mathrm{rad}} \ll K_{\mathrm{cd}}$$

- When conditions for degenerate electrons hold, **heat conduction can dominate radiative transport:**
- **degenerate core of evolved stars**
- **white dwarfs**

HEAT CONDUCTION

In general

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{rad}} + \boldsymbol{F}_{\mathrm{cd}} = -(K_{\mathrm{rad}} + K_{\mathrm{cd}})\,\boldsymbol{\nabla} T.$$

$$\boldsymbol{F} = -\frac{4acT^3}{3\kappa\rho}\boldsymbol{\nabla} T \qquad \text{with} \qquad \frac{1}{\kappa} = \frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cd}}}$$

conductive opacity

the mechanism that dominates energy flux is that with the smallest opacity (highest transparency)

the opacity determined how large the temperature gradient should be in order to carry a given luminosity

OPACITY

- **conductive opacity**: important only in degenerate electron gas

$$\kappa_{\rm cd} \approx 4.4 \times 10^{-3} \, \frac{\sum_i Z_i^{5/3} X_i / A_i}{(1+X)^2} \frac{(T/10^7\,{\rm K})^2}{(\rho/10^5\,{\rm g/cm^3})^2} \;\; {\rm cm^2/g}.$$

At high density and low temperature it becomes very small and flux of energy important

OPACITY (Rosseland mean over frequency)

- **Thomson scattering** : elastic scattering of photons with free electrons

$$\kappa_{\mathrm{T}} = \frac{\sigma_{\mathrm{T}}}{\mu_{\mathrm{e}} m_{\mathrm{u}}} = 0.2\ (1+X)\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

$h\nu \ll m_{\mathrm{e}}c^2$ fully ionised $10^4\ K \leq T \leq 10^8\ K$

degree of ionisation drops drastically for T < 10^4 K ==> too few electrons, opacity strongly reduced

- **Free-free absorption** : inverse process of bremsstrahlung (ion + free e-)

$$\kappa_{\mathrm{ff}} \approx 3.8 \times 10^{22}\ (1+X)\rho T^{-7/2}\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

Kramer opacity

- **Bound-free absorption** : absorption of photons by bound electrons

$$\kappa_{\mathrm{bf}} \approx 4.3 \times 10^{25}\,(1+X)Z\,\rho T^{-7/2}\ \ \mathrm{cm}^2/\mathrm{g}.$$

for T > 10^4 K , below photons not energetic enough to ionise electrons

Z= metallicity

$\kappa_{\mathrm{bf}} \approx 10^3 Z \times \kappa_{\mathrm{ff}}$ ===>bound- free dominates over free-free for Z > 10^{-3}

- **Bound bound absorption** : photon induced electron transitions T< 10^6 K.
  cross section maybe large because the lines are broaden by motion

OPACITY

- **The negative hydrogen ion**: bound-free absorption of a photon by H^-

Important in cool stars and atmosphere (e.g. Sun's atmosphere)

Neutral H can form a bound state with another electron—>H^-
with a small ionisation potential (0.75 eV) so it is easily ionised with T~3-6 10^3 K
Free electrons come from single ionised atom such as Na, K, Ca…so **it is sensitive to metallicity and temperature**

$T \sim (3-6) \times 10^3$ K, $\rho \sim (10^{-10} - 10^{-5})$ g/cm^3 and $0.001 < Z < 0.02$ i

$$\kappa_{H-} \approx 2.5 \times 10^{-31} \left(\frac{Z}{0.02}\right) \rho^{1/2} T^9 \ \ \mathrm{cm}^2/\mathrm{g}$$

- at lower temperature becomes negligible
- at T >10^4 K H- disappears and Kramer dominates

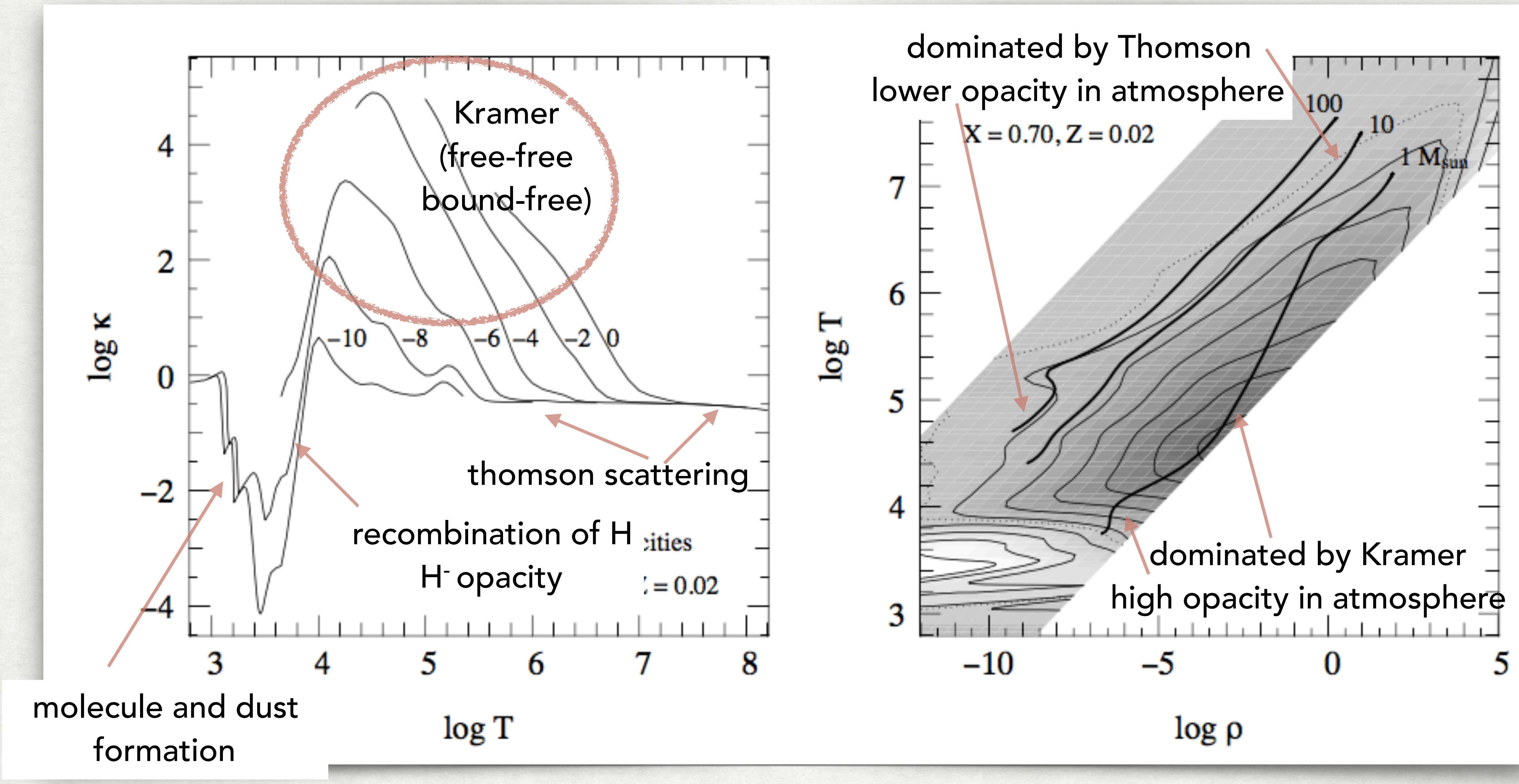
- **Molecules:** dominant in T < 4 10^3 K
- **Dust**: dust formation and opacity important at T < 1.5 10^3 K

T

OPACITY $\kappa = \kappa(\rho, T, X)$

IT IS A COMPLICATED FUNCTION AND OPACITY NEEDS TO BE ADD FREQUENCY BY FREQUENCY AND THEN AVERAGED OVER

SO STELLAR STRUCTURE CODE INTERPOLATE PRE-CALCULATED TABLES



note: these are Rosseland mean opacities