PLEASE, ENTER THE DOODLE

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NOTES CH 2

MECHANICAL AND THERMAL EQUILIBRIUM

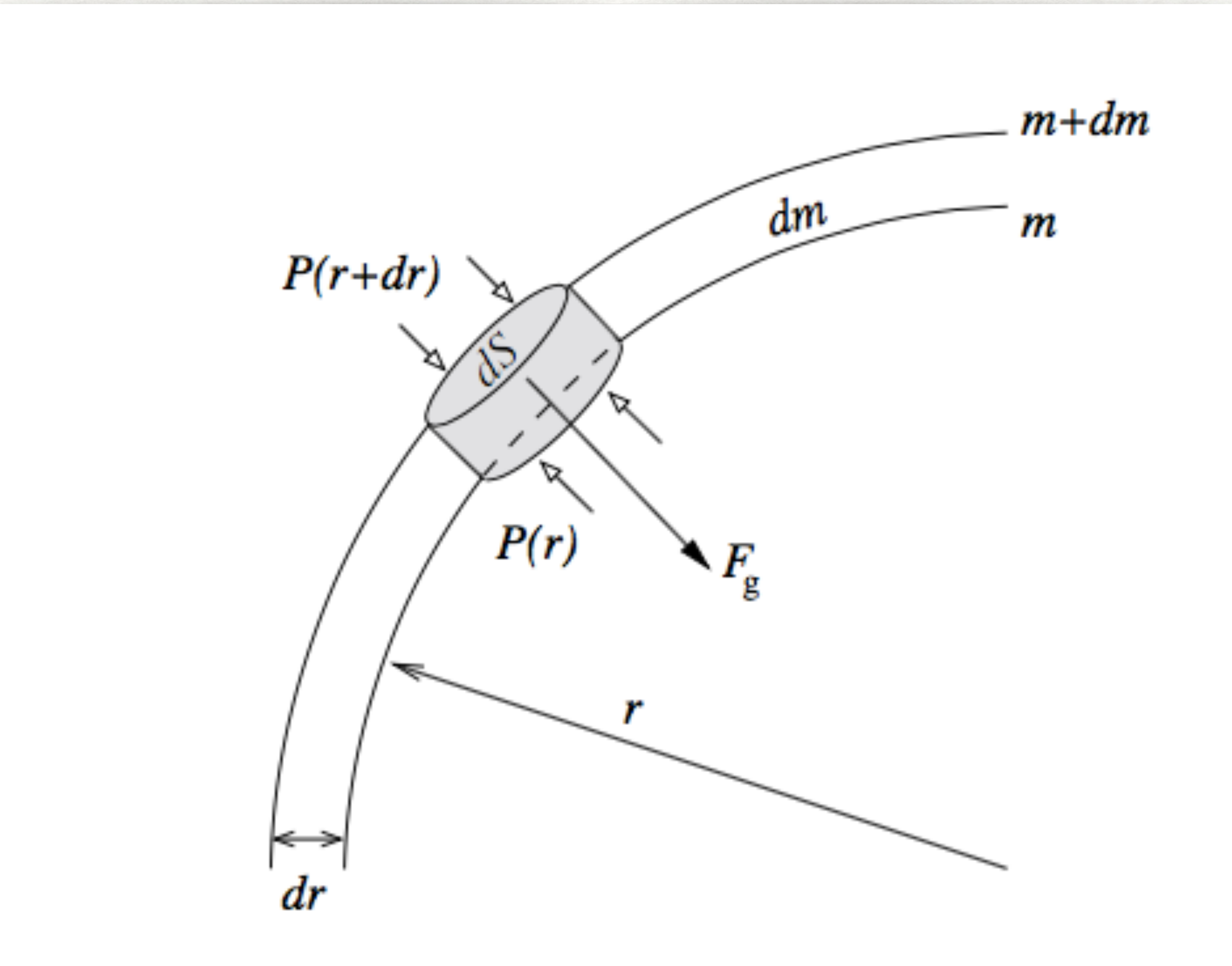
MODEL ASSUMPTIONS

1. **ISOLATION**. So their structure and evolution depends only on native properties (e.g. initial mass and composition). e.g. distance to Proxima Centauri/Solar diameter = $3\ 10^{7}$ ==> both radiation and gravitational fields are suppressed by $\sim 10^{-15}$ (not true in dense stellar clusters)

2. **UNIFORM INITIAL COMPOSITION.** A star is born with a given mass and homogenous composition, that depends on time and place of formation in a galaxy. The Sun-like composition is X=0.70, Y=0.28, Z=0.02 *in GAS:* generally we will assume it and explore only the mass dependence

3. **SPHERICAL SYMMETRY:** energy associated to rotation and magnetic field subdominant w.r.t. gravity (see calculations on black board)

CALCULATION SET-UP



CORRELATIONS TO BE EXPLAINED BY THEORY

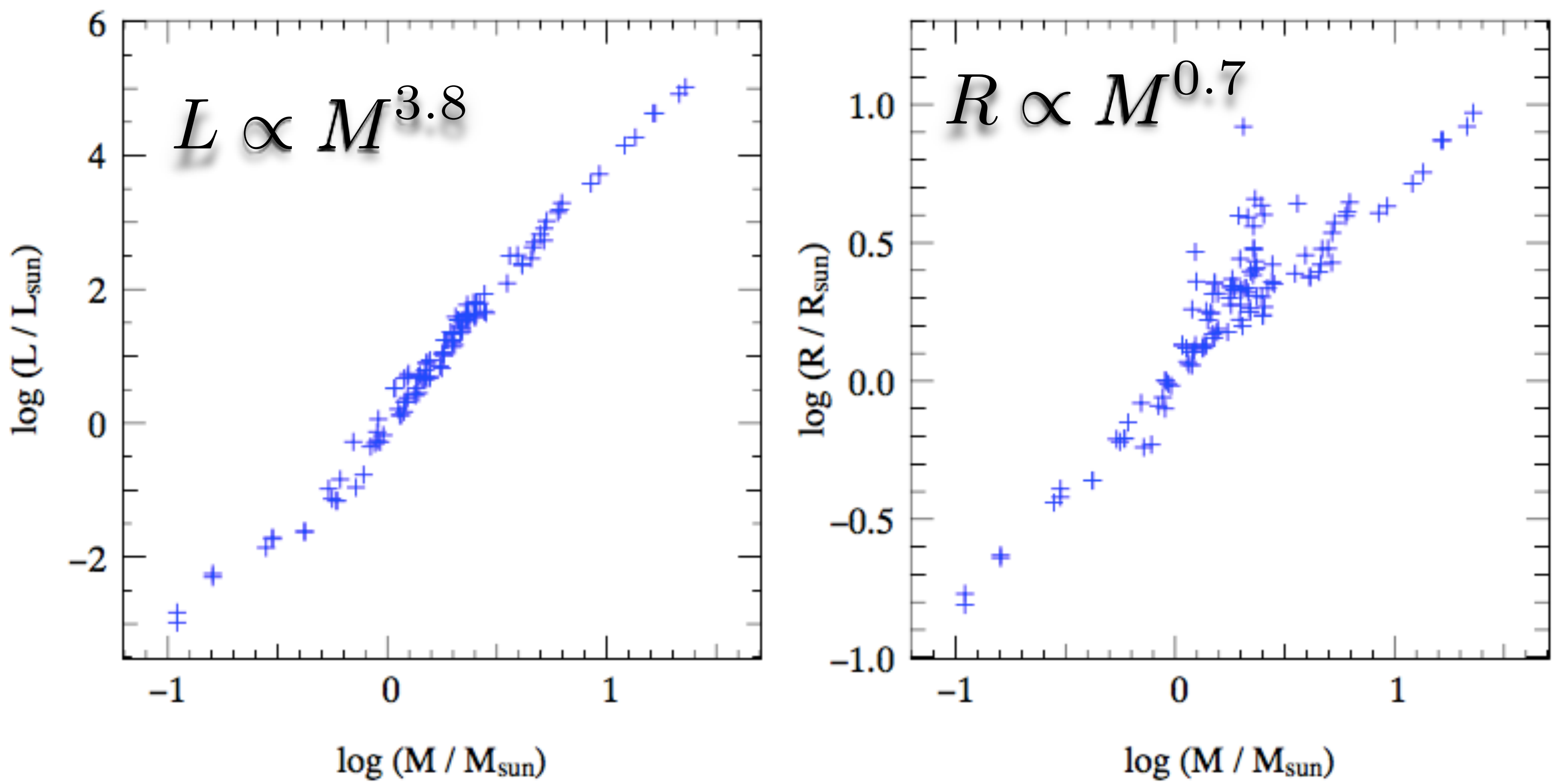


Figure 1.3. Mass-luminosity (left) and mass-radius (right) relations for components of double-lined eclipsing binaries with accurately measured M, R and L.

For main sequence stars with ~1 Solar mass

CONSERVATION LAWS

(neglecting: mass bulk motion, magnetic field, rotation, change in composition)

Summary

Mass:

$$dm = 4\pi r^2 \rho dr \qquad m \equiv m(<r) = \int_0^r \rho 4\pi r^2 dr$$

$$\boxed{\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}}$$

r: Eulerian coordinate; m: Lagrangian (0<m<M)

Momentum:

$$\ddot{r} = -\frac{Gm}{r^2} - 4\pi r^2 \frac{dP(m)}{dm}$$

mechanical (hydrostatic) equilibrium

$$\frac{dP(m)}{dm} = -\frac{Gm}{4\pi r^4}$$

The viral theorem

$$-3\int_0^M \frac{P}{\rho}\,\mathrm{d}m = E_{\mathrm{gr}}.$$

general form

$$< P >= -\frac{1}{3}\frac{E_{\mathrm{gr}}}{V}$$

ideal monoatomic gas

$$\boxed{E_{\mathrm{int}} = -\tfrac{1}{2}E_{\mathrm{gr}}}$$

$$\bar{T} = \frac{\alpha}{3}\frac{\mu m_{\mathrm{u}}}{k}\frac{GM}{R}. \qquad \phi = 3/2$$

$$E_{\mathrm{int}} = -\tfrac{1}{3}\phi E_{\mathrm{gr}}$$

non ideal gas with

$$u = \phi\frac{P}{\rho}.$$

Summary

TOTAL ENERGY

(for ideal non-relativistic gas)

$$E_{\text{tot}} = E_{\text{int}} + E_{\text{gr}} \quad \Rightarrow \quad E_{\text{tot}} = -E_{\text{int}} \quad \text{and} \quad E_{\text{tot}} = \frac{1}{2} E_{\text{gr}}$$

w Virial Theorem

where

$$E_{\text{gr}} = -\int_0^M \frac{Gm}{r} dm = -\alpha \frac{GM^2}{R} \qquad E_{\text{in}} = -\int_0^M u dm = \frac{3}{2} \frac{k_{\text{b}}}{\mu m_{\text{u}}} \langle T \rangle$$

THERMAL EQUILIBRIUM

$$\dot{E}_{\text{tot}} = L_{\text{nuc}} - L = 0 \quad \Rightarrow \quad \begin{cases} \dot{E}_{\text{tot}} = \frac{1}{2} \dot{E}_{\text{gr}} = 0 \\ \dot{E}_{\text{tot}} = -\dot{E}_{\text{int}} = 0 \end{cases}$$

both conserved!

w Virial Theorem

$L_{\text{nun}} = 0$ $\quad \dot{E}\text{gr} = -2L < 0$ contraction! $\quad \dot{E}_{\text{int}} = L > 0$ hotter!

IMPORTANT TIMESCALES

- **H fusion timescale = star lifetime.** Note: energy produced per unit rest mass: $0.007c^2$ and fractional material burnt: 0.1 M

$$\tau_{\rm nuc} = \frac{E_{\rm nuc}}{L} = \epsilon f_{\rm nuc} \frac{Mc^2}{L} \approx 10^{10}\,{\rm yr}$$

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- **Dynamical timescale.** From ME

$$\tau_{\mathrm{dyn}} = \frac{1}{2}\left(G\langle\rho\rangle\right)^{-1/2} \approx 50\ \mathrm{min}$$