LECTURES

This page was intentionally left blank

Principles of interferometry

Pierre Lena

Abstract

The lectures will establish the basic physical principles on which astronomical interferometry is based : coherence of light, combination of beams, sampling of image spatial frequency content. The effects of atmospheric perturbations will be analyzed with the limitations they impose on ground based observations, including the role of adaptive optics. Signal detection principles and practical realizations will be presented, with discussion of noise sources, sensitivity limits for various wavelengths and observing conditions (ground and space) of interest. Example of current astronomical results shall be given.

Principles of Interferometry

Pierre Léna, Université Paris VII & Observatoire de Paris

School on Space and Ground- based Optical Interferometry - Leiden, Sept. 18-22, 2000

Bibliography

* Principles of Long Baseline Stellar Interferometry (2000), P.R. Lawson Ed., Course Notes from the Michelson Summer School, Pasadena, August 1999, JPL (http://sim.jpl.nasa.gov/michelson/iss.html)
An excellent tutorial introduction to theory and practice

* Selected Papers on Long Baseline Stellar Interferometry (1997), P.R. Lawson Ed., SPIE Milestone Series, **MS 139**
All the fundamental papers from 1867 to date.

* Diffraction-limited Imaging with Very Large Telescopes (1989), D.M. Alloin & J.-M. Mariotti Ed., NATO ASI Series, **274**, Kluwer
A basic compendium with most of the needed physical and mathematical derivations A

* High Angular Resolution in Astrophysics (1997), A.-M. Lagrange, D. Mourard & P. Léna Ed., NATO ASI Series, **501**, Kluwer
Similar in format with the preceding volume, more recent and complementary

* Adaptive Optics in Astronomy (1999), F. Roddier Ed., Cambridge University Press
Basic concepts and state-of-the art persentation of adaptive optics theory, techniques and impact.

* Observational Astrophysics (1998), Léna, P., Lebrun, F. & Mignard, F., Springer
General introductory book on observation in astronomy, with emphasis on image formation, signal, noise.

* The Fourier Transforms and Its Applications (1965), Bracewell, R., Mc Graw Hill
The basic vademecum on Fourier transforms.

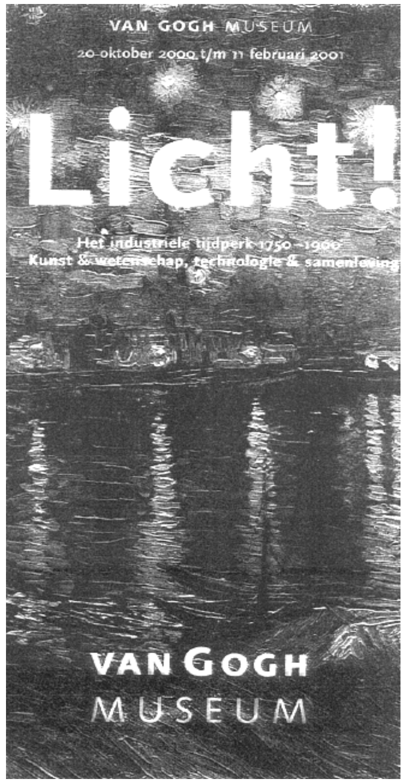
* Principles of Optics (1975), Born, M. & Wolf, E., Pergamon Press
The fundamental book on optics, waves, diffraction, images...

Emerging trends of optical interferometry in astronomy (1999), Saha, S.K., Bull.Astron.Soc.India, **27**, 441-546
A good and recent review

Interferometry in Optical Astronomy (2000), Léna, P. & Quirrenbach, A. Ed., Proceedings of an ESO-SPIE Conference, **4006** (2 vol.), SPIE Publication
A thorough collection of communications and reviews, specialized but up-to-date

Science with the VLT Interferometer (1997), Paresce, F. Ed., ESO Astrophysics Symposia, Springer
A compendium of papers on scientific programs for the VLTI.

* indicates the pedagogical/textbook character of the reference

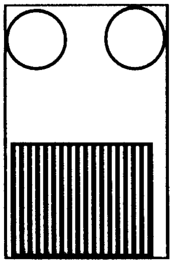


Principles of Interferometry

Pierre Léna, Université Paris VII & Observatoire de Paris

1. The Young experiment
2. From Fizeau to the VLTI
3. Object, Instrument, Image & Fourier spectra
4. Coherence of radiation
5. Measuring coherence with an ideal interferometer
6. Types of interferometers
7. Effects of the Earth's atmosphere
8. Methods of light recombination
9. Signal detection & noise sources, sensitivity

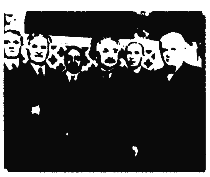
Summer school on Space and Ground-based Optical Interferometry - Leiden, Sept 18-22, 2000 - page 1



1. The Young experiment

- A founding experiment
- Do-it-yourself
- Fringes & spatial structure of the source

Summer school on Space and Ground- based Optical Interferometry - Leiden, Sept 18-22, 2000 - page 2

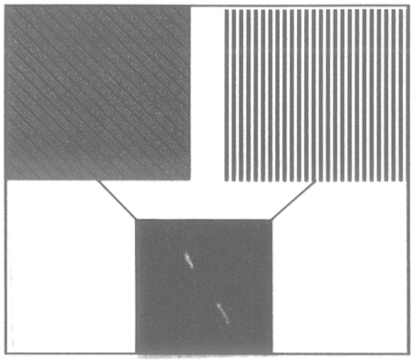


(From left) Albert Michelson, Albert Einstein
[illegible] A. Millikan on the Caltech campus in Pasadena

2. From Fizeau to the VLTI

1802	Fringes and nature of light	Thomas Young, Londres
1868	Concept of interferometry with pupil mask	Hippolyte Fizeau, Paris
1872	Upper limit (0.158”) of stellar diameter	Edouard Stephan, Marseille
1921	First stellar diameter measurement	Albert Michelson, Pasadena
1950	First radio-interfometer	Martin Ryle, Cambridge
1956	First intensity interferometer (visible)	R. Hanbury-Brown & R. Twiss
1970	Speckle interferometry (visible)	Antoine Labeyrie, Paris
1972	First heterodyne fringes (10 µm)	Jean Gay & Alain Journet, Grasse
1973	Deconvolution algorithm	Leon Lucy
1975	Triple correlation (visible)	Gerd Weigelt, Nuremberg
1976	Coupling two independent telescopes	Antoine Labeyrie, Paris
1987	Decision of VLT Interferometer	*Observatoire Européen Austral*
1989	First adaptive optics image (2.2 µm)	Gérard Rousset, Paris
1996	First interferometric image (visible)	James Baldwin, Cambridge
2001	VLTI & Keck Interferometer first light	*C. Paranal, Chili & Mauna Kea*
2001	Adaptive optics on VLT (NAOS)	*Cerro Paranal, Chili*

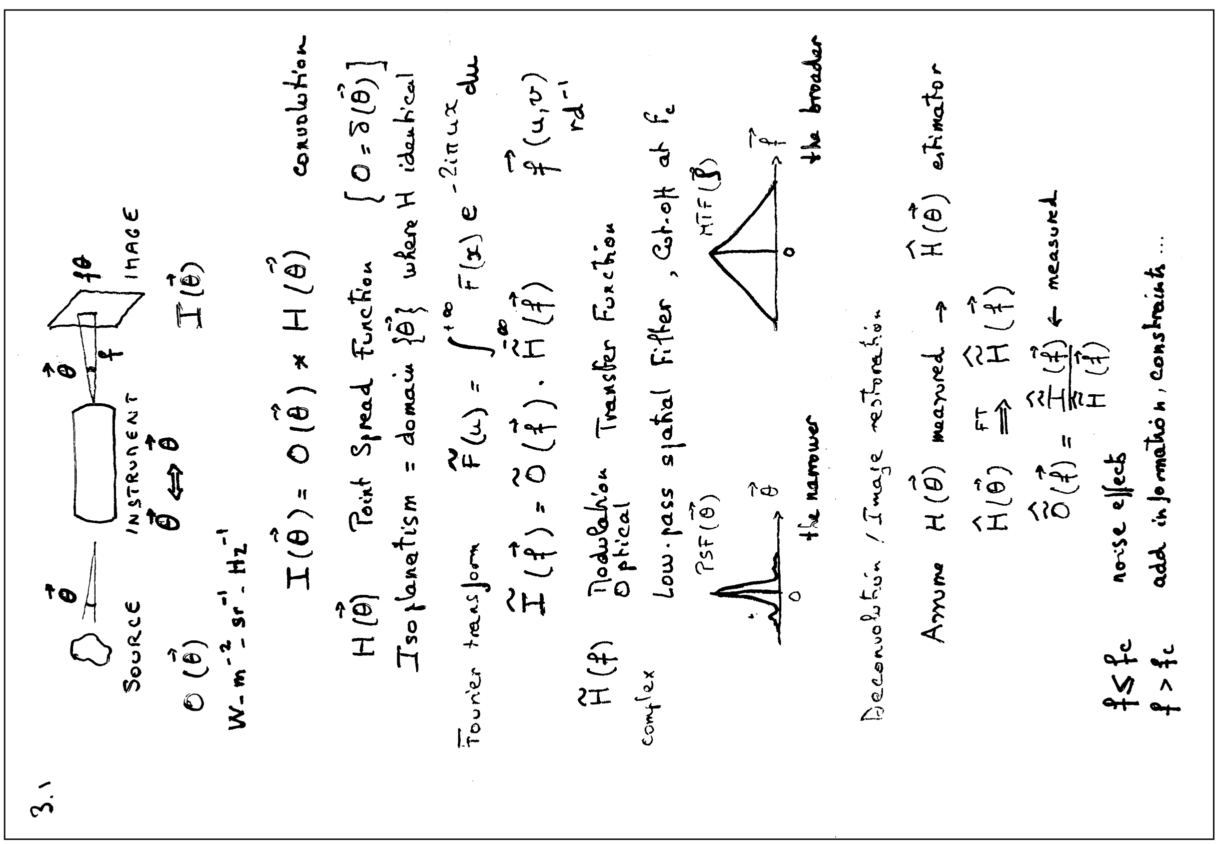
Summer school on Space and Ground- based Optical Interferometry - Leiden, Sept. 18-22, 2000 - page 3

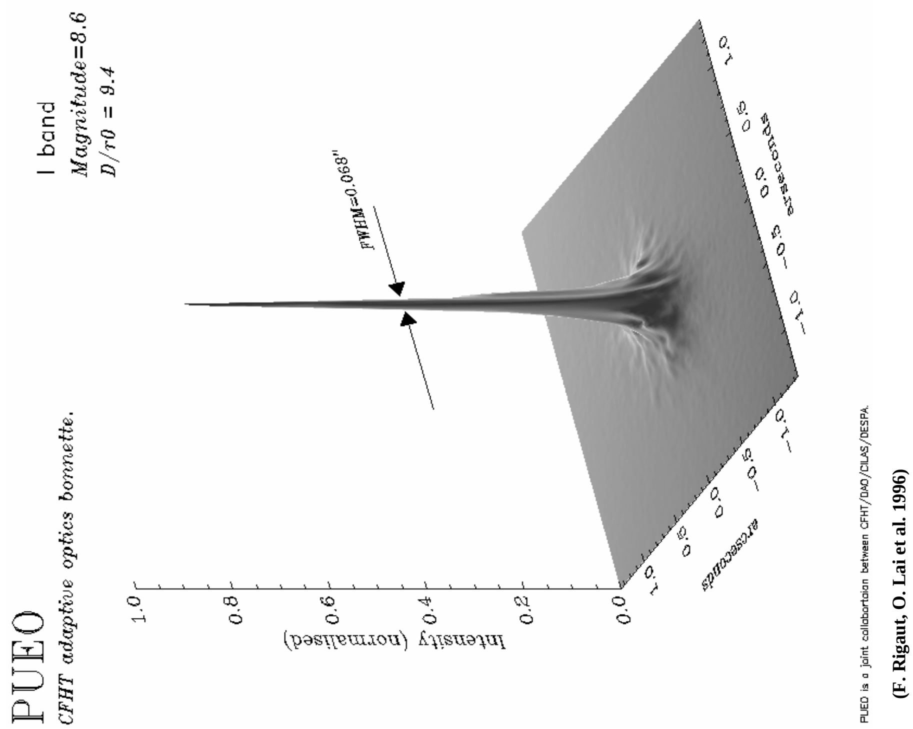


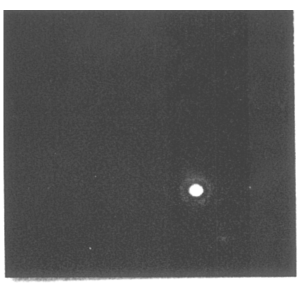
3. Object, Instrument, Image & Fourier spectra

- **Object, instrument, image**
- **Intensity (irradiance) in object/image and its spatial spectrum**
- **Instrument as a spatial filter**
- **Modulation Transfer Function (MTF)**
- **Point Spread Function (PSF)**
- **Isoplanatism**
- **Degraded MTFs : aberrations, atmosphere**

Summer school on Space and Ground- based Optical Interferometry - Leiden, Sept. 18-22, 2000 - page 4







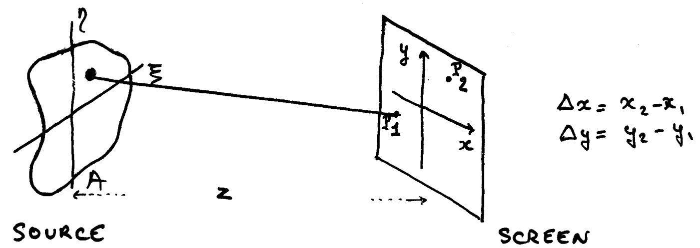
4. Coherence of radiation

- The field radiation $\Psi(\vec{r},t)$ and the source characteristics
- Temporal coherence
- Spatial coherence
- Spatio-temporal coherence $\gamma_{12}(\tau)$
- Quasi-monochromatic case
- Coherence over an illuminated surface : the Zernike-van Cittert theorem
- Area of coherence A_c, étendue $A_c\,\Omega$, volume of coherence

Summer school on Space and Ground- based Optical Interferometry - Leiden, Sept. 18-22, 2000 - page 5

4.2

How to get $\gamma_{12}(0)$?



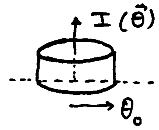
$\Delta x = x_2 - x_1$
$\Delta y = y_2 - y_1$

$$\gamma_{12}(0) = e^{-i\Phi_{12}} \frac{\iint_A I(\xi,\eta)\, e^{i\frac{2\pi}{\lambda_0}\left(\xi\frac{\Delta x}{z} + \frac{\Delta y}{z}\eta\right)} d\xi d\eta}{\iint_A I(\xi,\eta)\, d\xi d\eta}$$

$\Phi_{12} = \frac{\pi}{\lambda_0 z} \overline{P_1P_2}^2$ negligible if $z \gg \frac{2(\rho_2^2 - \rho_1^2)}{\lambda_0}$

Theorem of Zernike-van Cittert

$$\gamma_{12}(0) \simeq \frac{\iint_A I(\vec{\theta})\, e^{2i\pi\, \vec{\theta}\cdot\frac{(\vec{\rho_2}-\vec{\rho_1})}{\lambda_0}} d\vec{\theta}}{\iint_A I(\vec{\theta})\, d\vec{\theta}}$$



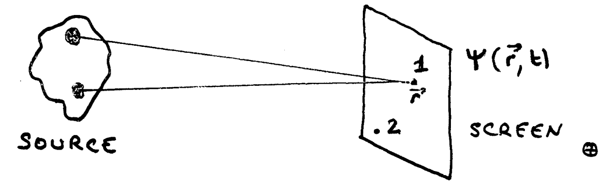
Uniform source of radius θ_0

$$\gamma(\rho,0) = \frac{2 J_1\left(\frac{2\pi\theta_0\rho}{\lambda_0}\right)}{\left(\frac{2\pi\theta_0\rho}{\lambda_0}\right)} = \frac{2 J_1(x)}{x}$$

$x = 2$
$\gamma = 0.577$ $\qquad \rho = \frac{\lambda_0}{\pi\theta_0}$

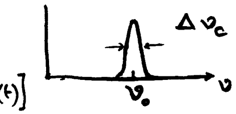
Etendue $\pi\theta_0^2 . \pi\rho^2 = \lambda_0^2$

Volume : $\lambda_0^2 \times c\tau_c$



Monochromatic ν_0

Quasi-monochromatic



$\Delta\nu_0 \ll \nu_0$

$\Psi = \Psi_0\, e^{[2i\pi\nu_0 t + \varphi(t)]}$

Arbitrary spectrum $\quad\Psi$ purely random = incoherent

Compare $\Psi_1(t)$, $\Psi_2(t+\tau)$

$$\gamma_{12}(\tau) = \frac{\langle \Psi_1(\vec{r_1}, t)\, \Psi_2^*(r_2(t+\tau)) \rangle_{time}}{\langle |\Psi_1|^2 |\Psi_2|^2 \rangle^{1/2}}$$

MUTUAL DEGREE OF COHERENCE

$0 \leq |\gamma_{12}(\tau)| \leq 1$

Temporal coherence "1"="2"

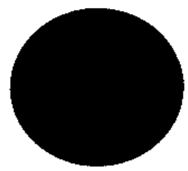
$\gamma(\tau) \quad \Rightarrow \tau_c \quad , \; \ell_c = c\tau_c$

Spatial coherence "1"≠"2"

$$\gamma_{12}(\tau) = \gamma_{12}(0)\, e^{-[2i\pi\nu_0\tau + \alpha(\tau)]}$$

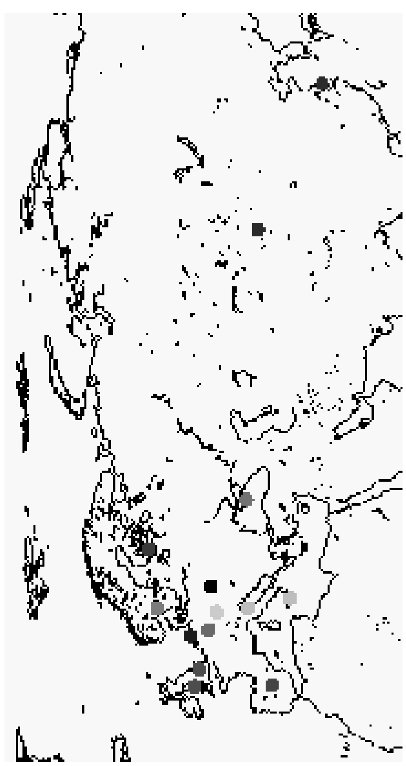
↑ slowly decreasing

Rayon de coherence = $\lambda/\pi\alpha$
α rayon angulaire de la source circulaire



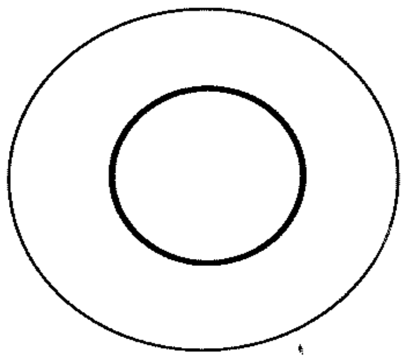
European VLBI Network (EVN)
bleu : quasar 3C273, λ = 3 cm
z=0.16, Ho=75 km s-1 Mpc, ray.source = 30 a.l.

The European VLBI Network

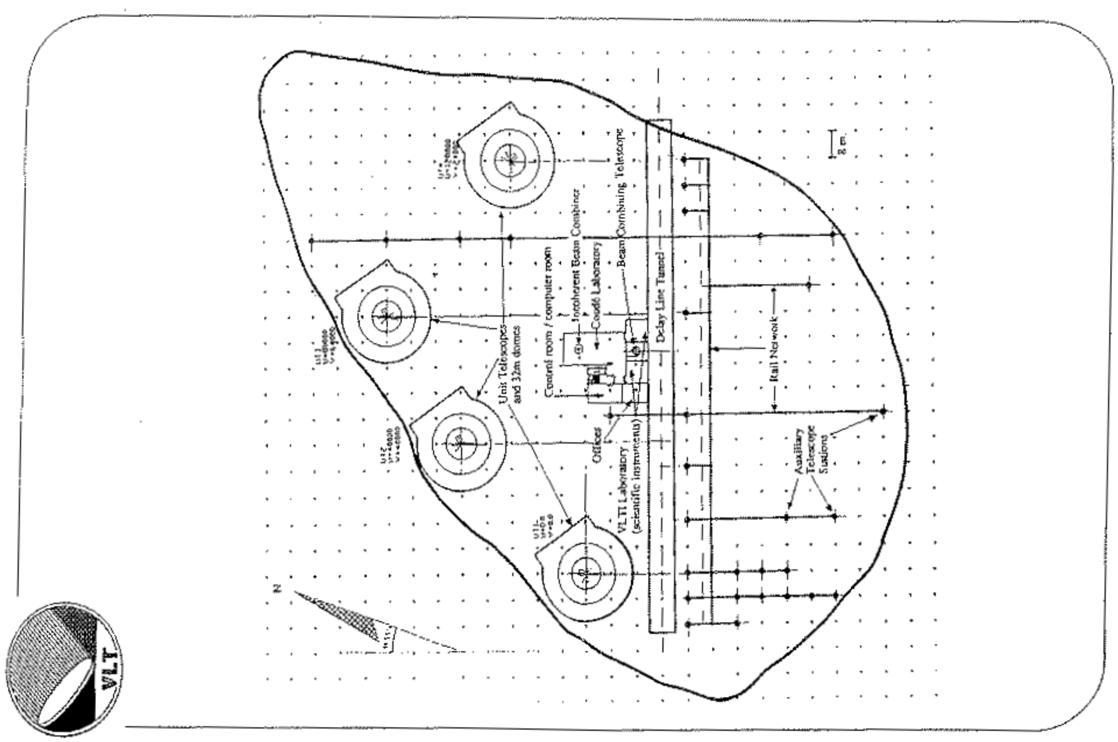


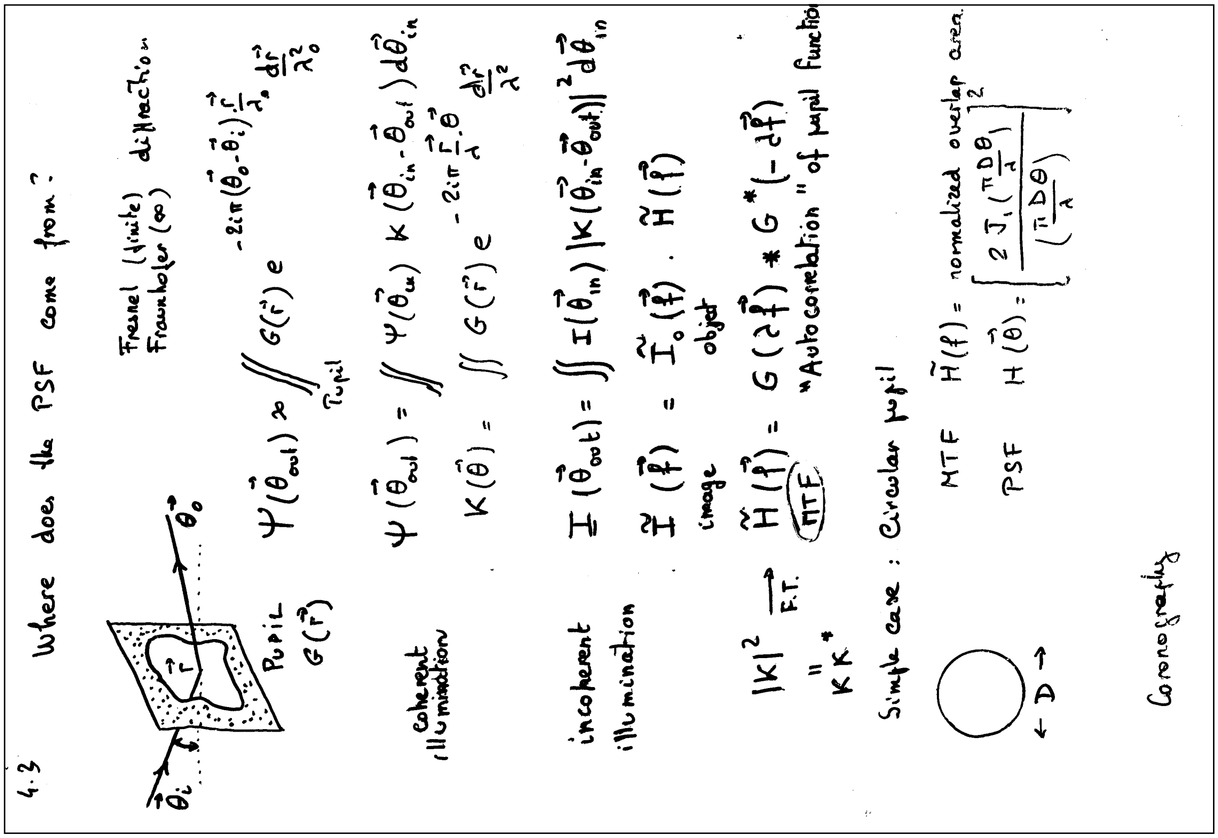
- MPIfR, Bonn Correlator and Effelsberg 100-m telescope
- NRAL, Jodrell Bank (76-m Lovell and MkII) and Cambridge 32-m
- JIVE Correlator (Dwingeloo) and NFRA Westerbork Array
- Onsala Space Observatory 60 and 85ft telescopes
- OAN, Yebes; DSN Robledo, Madrid.
- IRA, Medicina and Noto 32-m
- IfAG, Wettzell
- Simeiz 22-m
- TCfA, Torun 32-m
- Metsahovi Radio Research Station
- Shanghai Observatory
- Urumqi Observatory

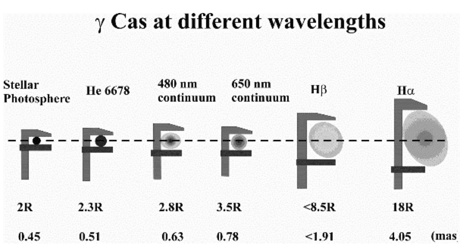
Rayon de coherence = λ/πα
α rayon angulaire de la source circulaire



VLTI (échelle du plan du site)
noir : **Soleil à 5 pc, λ = 0.5 μm**
rouge : **Soleil à 10 pc, λ = 0.5 μm**
: **Soleil à 5 pc, λ = 1.0 μm**



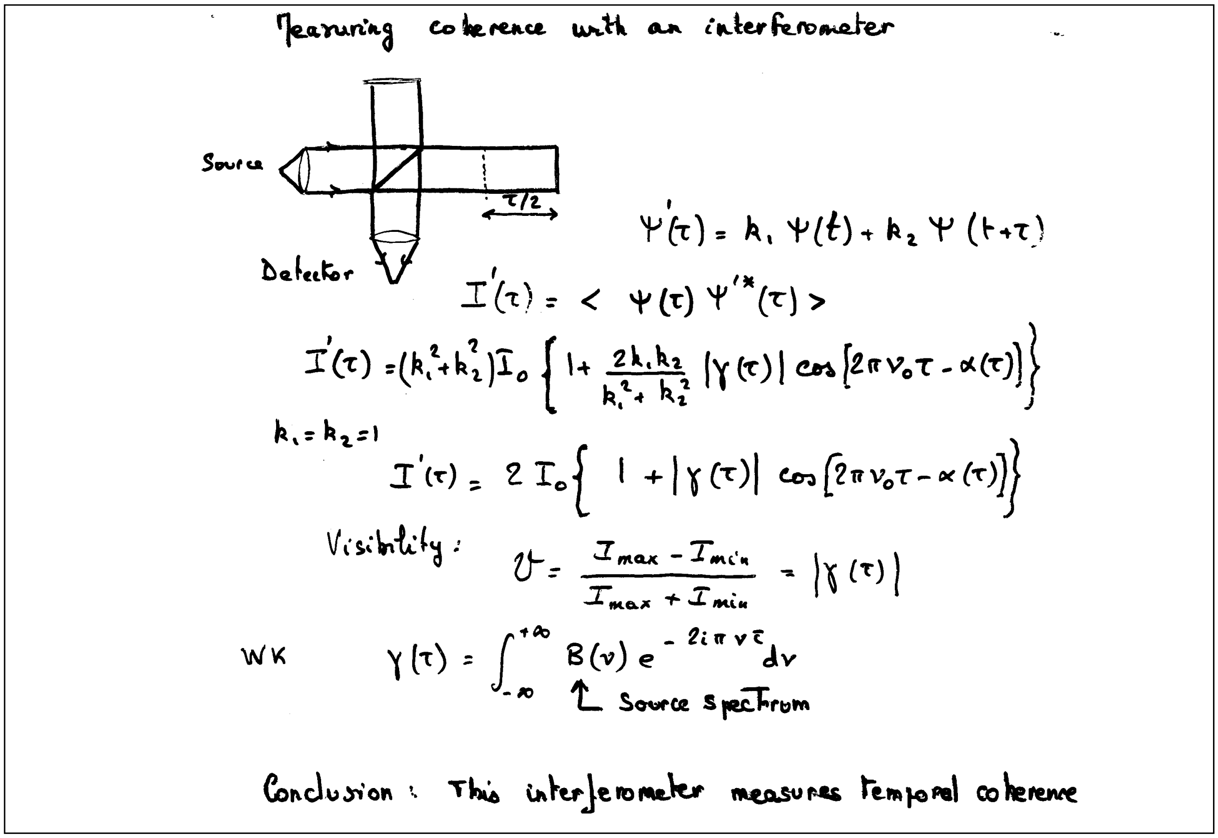


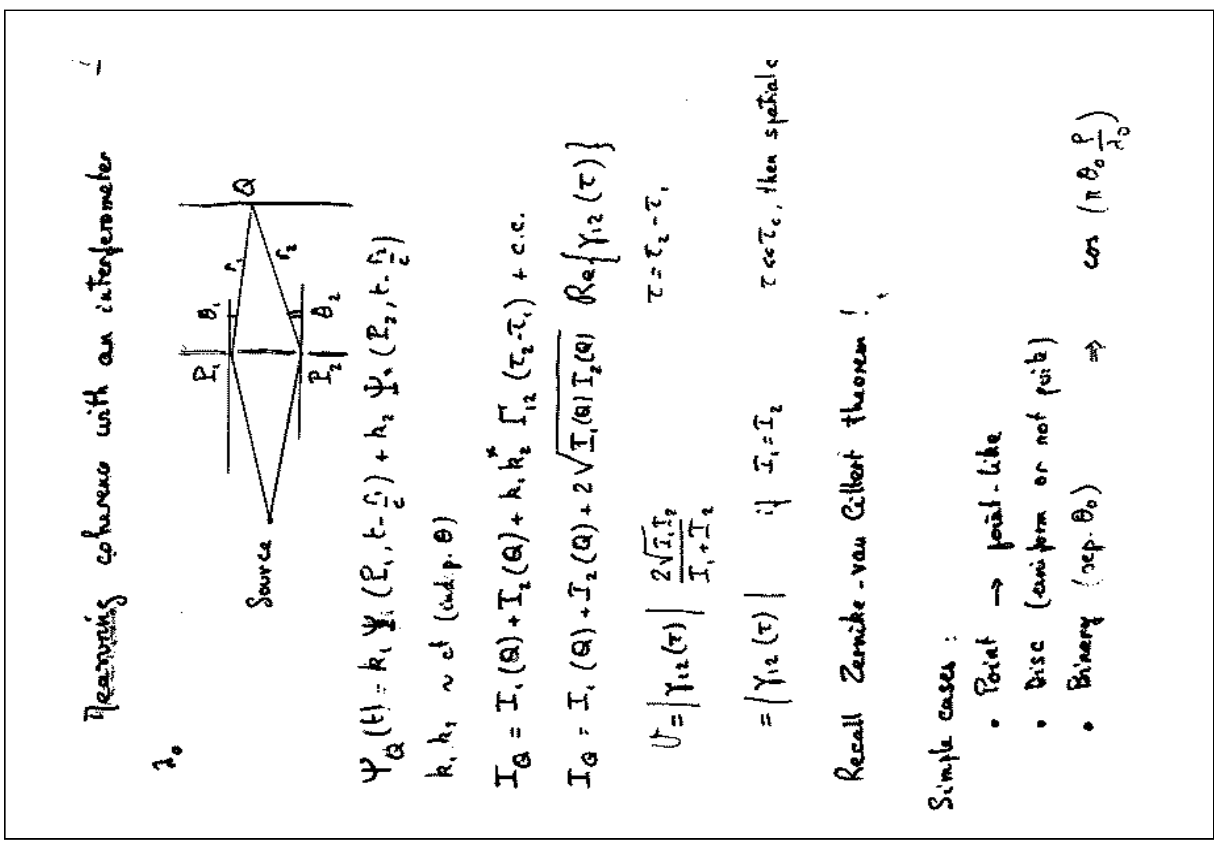


5. Measuring coherence with an ideal interferometer

- Principles of coherence (correlation) measurement
- Fringes, complex visibility & source spatial spectrum
- Some simple sources
  - point-like
  - uniform disk
  - binary star
- Wavefront structure
  - loss of coherence : diffraction, scattering, atmospheric propagation
  - Zernike polynomials
  - Maréchal & Rayleigh criteria
- From coherence measurement to an image. Aperture synthesis
- Interferometry and imaging : is it the same ?

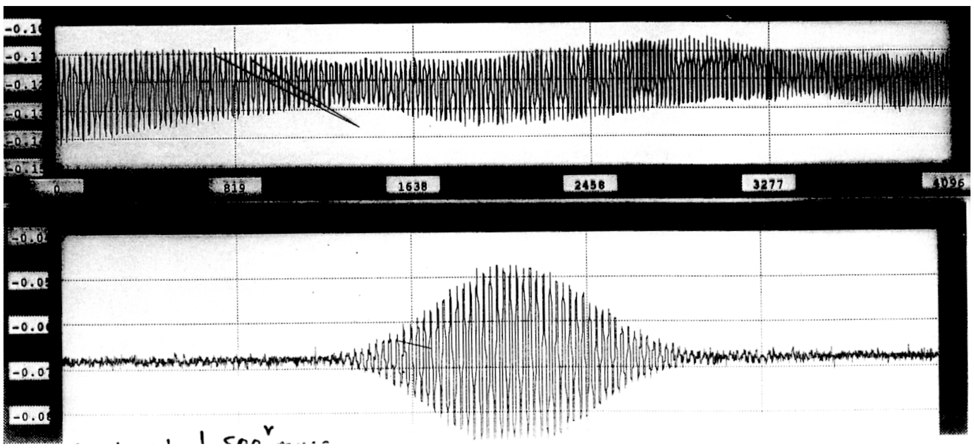
Summer school on Space and Ground- based Optical Interferometry - Leiden, Sept 18-22 2000 - page 6


Measuring coherence with an interferometer
Source
Detector
τ/2
$\Psi'(\tau) = k_1 \Psi(t) + k_2 \Psi(t+\tau)$
$I'(\tau) = < \Psi(\tau) \Psi'^*(\tau) >$
$I'(\tau) = (k_1^2+k_2^2) I_0 \left\{ 1 + \frac{2k_1k_2}{k_1^2+k_2^2} |\gamma(\tau)| \cos[2\pi\nu_0\tau - \alpha(\tau)] \right\}$
$k_1 = k_2 = 1$
$I'(\tau) = 2 I_0 \left\{ 1 + |\gamma(\tau)| \cos[2\pi\nu_0\tau - \alpha(\tau)] \right\}$
Visibility : $\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = |\gamma(\tau)|$
WK $\gamma(\tau) = \int_{-\infty}^{+\infty} B(\nu) e^{-2i\pi\nu\tau} d\nu$
Source spectrum
Conclusion : This interferometer measures temporal coherence


Measuring coherence with an interferometer
Source
P_1 P_2 θ_1 θ_2 r_1 r_2 Q
$\Psi_Q(t) = k_1 \Psi_1(P_1, t - \frac{r_1}{c}) + k_2 \Psi_2(P_2, t - \frac{r_2}{c})$
$k_1, k_2 \sim$ ct (ind. p. θ)
$I_Q = I_1(Q) + I_2(Q) + k_1 k_2^* \Gamma_{12}(\tau_2 - \tau_1) + c.c.$
$I_Q = I_1(Q) + I_2(Q) + 2\sqrt{I_1(Q) I_2(Q)}\, Re\{\gamma_{12}(\tau)\}$
$\tau = \tau_2 - \tau_1$
$\mathcal{V} = |\gamma_{12}(\tau)| \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$
$= |\gamma_{12}(\tau)|$ if $I_1 = I_2$
$\tau \ll \tau_c$, then spatial c
Recall Zernike - van Cittert theorem !
Simple cases :
• Point → point-like
• Disc (uniform or not point)
• Binary (sep. θ_0) ⇒ $\cos(\pi \theta_0 \frac{\rho}{\lambda_0})$

All-fiber Michelson FTS

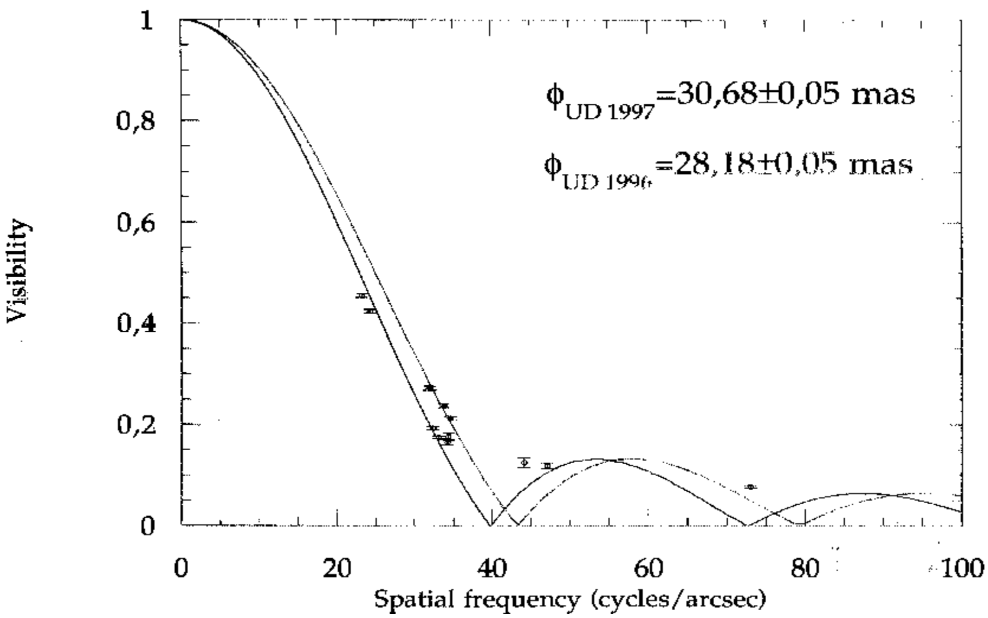
Total length of fibers per arm: ≳ 10 meters
Path modulation through a PZT drum: 500 V, 18 turns
(next Delay line: Δ multiplied by 10)



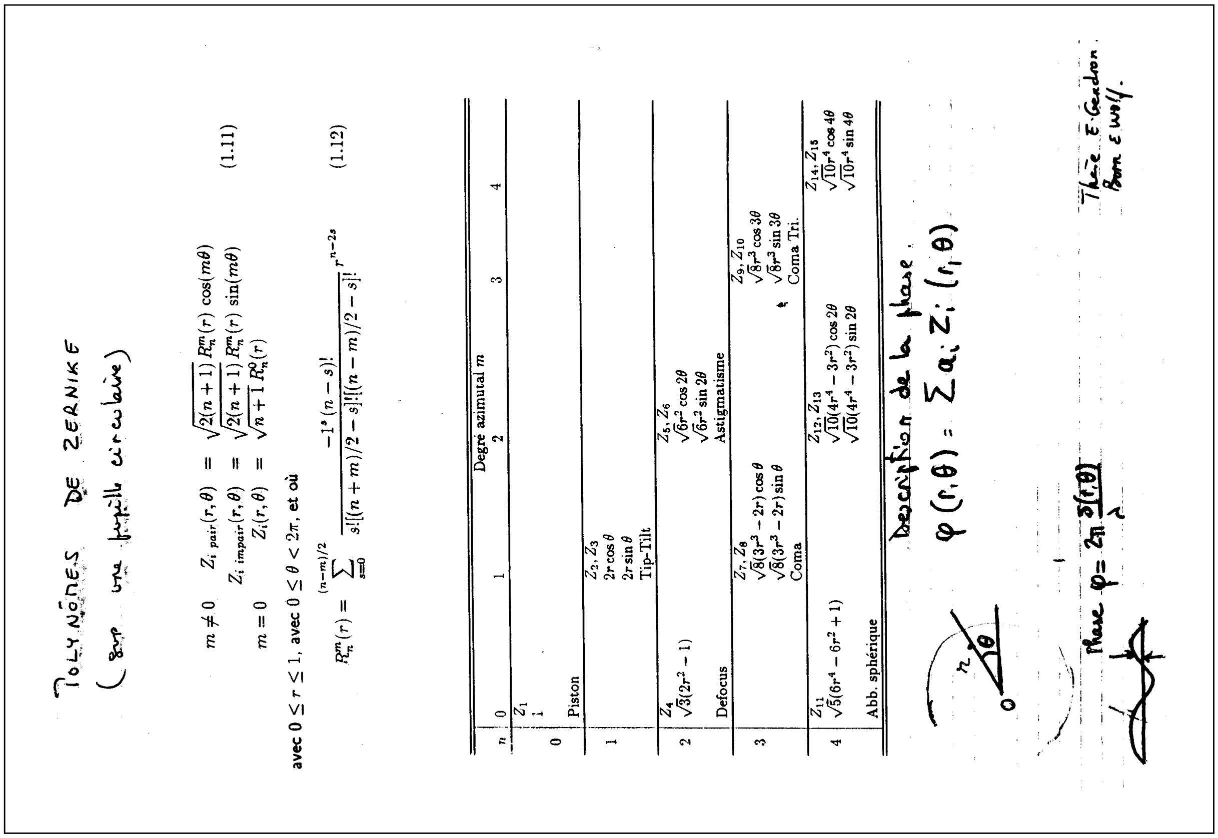
Phase reference & control width λ=1.52μm

Interferogram of a thermal source $\lambda/\Delta\lambda$=20 K band

R Leonis
M8 III Mira type star



FLUOR/IOTA, Mt Hopkins, Arizona I Perrin et al., 1999



POLYNÔMES DE ZERNIKE (sur une pupille circulaire)

$$m \neq 0 \quad Z_{i\ pair}(r,\theta) = \sqrt{2(n+1)}\, R_n^m(r)\cos(m\theta)$$
$$Z_{i\ impair}(r,\theta) = \sqrt{2(n+1)}\, R_n^m(r)\sin(m\theta) \qquad (1.11)$$
$$m = 0 \quad Z_i(r,\theta) = \sqrt{n+1}\, R_n^0(r)$$

avec $0 \le r \le 1$, avec $0 \le \theta < 2\pi$, et où

$$R_n^m(r) = \sum_{s=0}^{(n-m)/2} \frac{-1^s (n-s)!}{s![(n+m)/2-s]![(n-m)/2-s]!} r^{n-2s} \qquad (1.12)$$

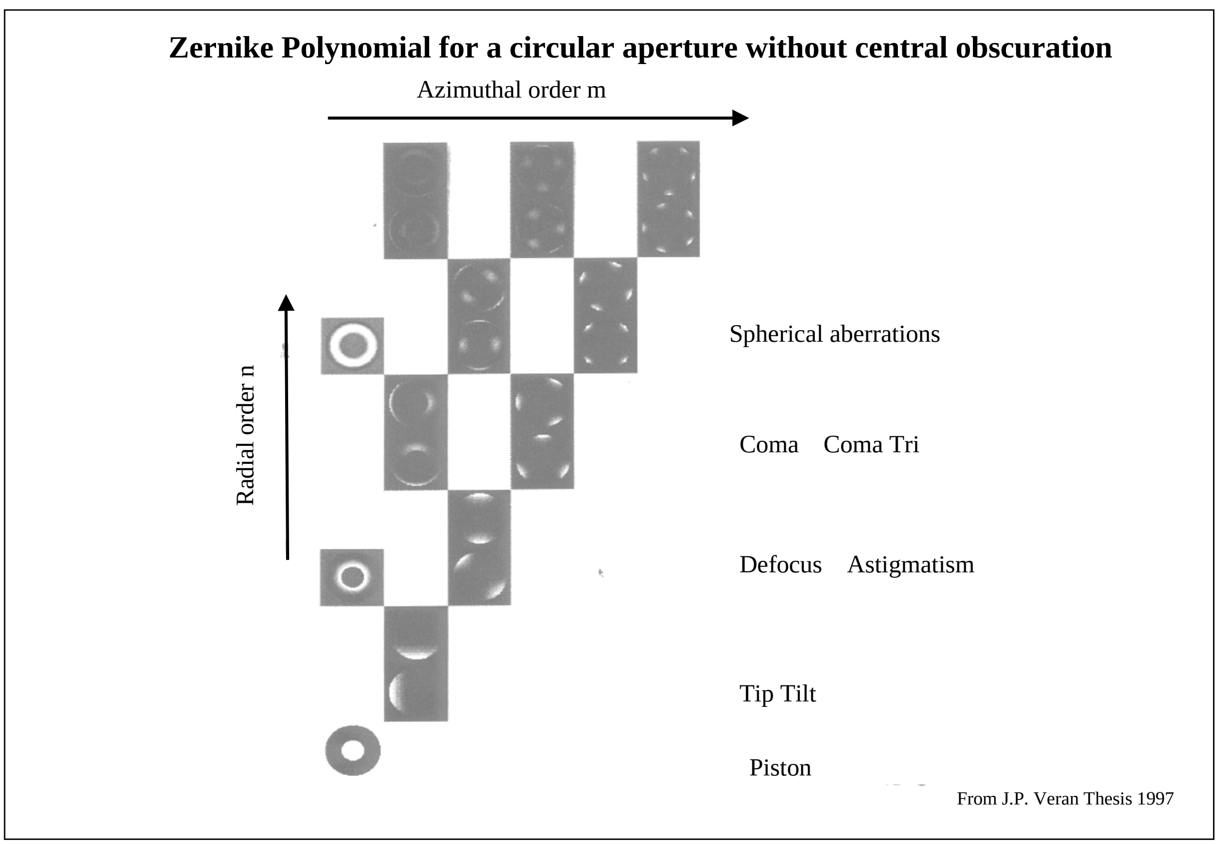
n	Degré azimutal m: 0	1	2	3	4
0	Z_1 1 Piston				
1		Z_2, Z_3 $2r\cos\theta$ $2r\sin\theta$ Tip-Tilt			
2	Z_4 $\sqrt{3}(2r^2-1)$ Defocus		Z_5, Z_6 $\sqrt{6}r^2\cos 2\theta$ $\sqrt{6}r^2\sin 2\theta$ Astigmatisme		
3		Z_7, Z_8 $\sqrt{8}(3r^3-2r)\cos\theta$ $\sqrt{8}(3r^3-2r)\sin\theta$ Coma		Z_9, Z_{10} $\sqrt{8}r^3\cos 3\theta$ $\sqrt{8}r^3\sin 3\theta$ Coma Tri.	
4	Z_{11} $\sqrt{5}(6r^4-6r^2+1)$ Abb. sphérique		Z_{12}, Z_{13} $\sqrt{10}(4r^4-3r^2)\cos 2\theta$ $\sqrt{10}(4r^4-3r^2)\sin 2\theta$		Z_{14}, Z_{15} $\sqrt{10}r^4\cos 4\theta$ $\sqrt{10}r^4\sin 4\theta$

Description de la phase.

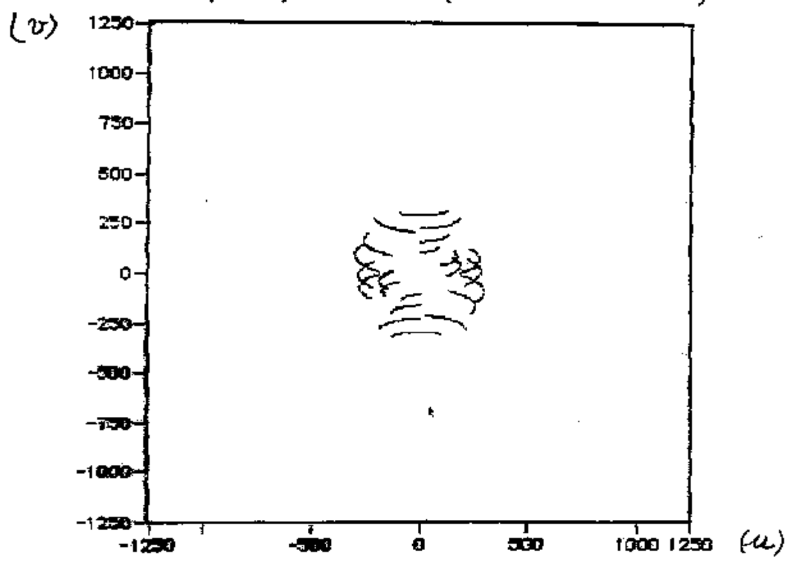
$$\varphi(r,\theta) = \sum a_i Z_i(r,\theta)$$

Phase $\varphi = 2\pi \frac{\delta(r,\theta)}{\lambda}$

Thèse E. Gendron. Born & Wolf.



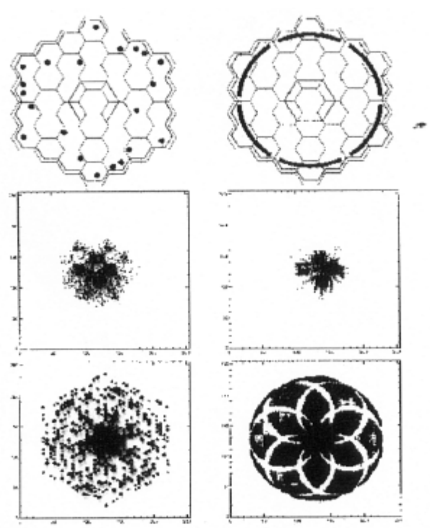
VLTI (u,v) traces (11 baselines)



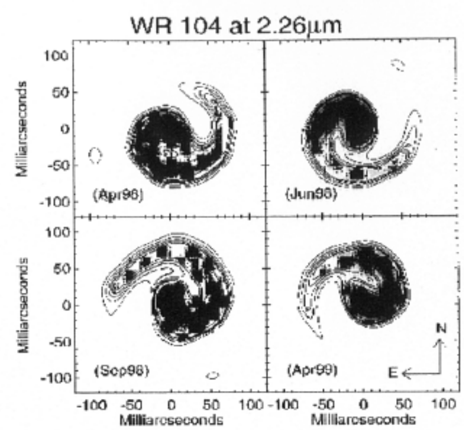
Unités $\frac{D_m}{\lambda_{\mu m}}$

$D \leq 200$ m

$\lambda = 0.5\ \mu m$



21 hole Golay Annulus pupil
on Keck Telescope
Middle : single exp. interferograms
Bottom : 100 exposures



Contour map of WR 104 at 2.26 µm

Tuthill et al., SPIE 2000, Vol. 4006