Exercise: $\mathrm{ArH^+}$ Chemical Network

$\mathrm{ArH^+}$ (argonium), the first noble gas molecule to be detected in space (Barlow et al. 2013, Science, 342, 1343), is considered a very good tracer of the atomic gas in the ISM (Schilke et al. 2014, A&A, in press). The following is a simple reaction scheme describing the formation and destruction pathways of $\mathrm{ArH^+}$.

$$\mathrm{Ar} + \mathrm{CR} \xrightarrow{k_1} \mathrm{Ar^+} + \mathrm{e^-} \tag{1}$$

$$\mathrm{Ar} + \mathrm{H_2^+} \xrightarrow{k_2} \mathrm{Ar^+} + \mathrm{H_2} \tag{2}$$

$$\mathrm{Ar} + \mathrm{H_3^+} \xrightarrow{k_3} \mathrm{ArH^+} + \mathrm{H_2} \tag{3}$$

$$\mathrm{Ar^+} + \mathrm{e^-} \xrightarrow{k_4} \mathrm{Ar^+} h\nu \tag{4}$$

$$\mathrm{Ar^+} + \mathrm{H_2} \xrightarrow{k_5} \mathrm{ArH^+} + \mathrm{H} \tag{5}$$

$$\mathrm{ArH^+} + \mathrm{H_2} \xrightarrow{k_6} \mathrm{Ar} + \mathrm{H_3^+} \tag{6}$$

$$\mathrm{ArH^+} + \mathrm{CO} \xrightarrow{k_7} \mathrm{Ar} + \mathrm{HCO^+} \tag{7}$$

$$\mathrm{ArH^+} + \mathrm{O} \xrightarrow{k_8} \mathrm{Ar} + \mathrm{OH^+} \tag{8}$$

$$\mathrm{ArH^+} + \mathrm{C} \xrightarrow{k_9} \mathrm{Ar} + \mathrm{CH^+} \tag{9}$$

$$\mathrm{ArH^+} + \mathrm{e^-} \xrightarrow{k_{10}} \mathrm{Ar} + \mathrm{H} \tag{10}$$

$$\mathrm{ArH^+} + h\nu \xrightarrow{k_{11}} \mathrm{Ar^+} + \mathrm{H} \tag{11}$$

1. Classify each type of reaction in the chemical network

2. Write the ordinary differential equation (ODE) which describes the rate of change of abundance of $\mathrm{ArH^+}$ as a function of time

3. Assuming steady state for all species, i.e., $\mathrm{d}n/\mathrm{d}t = 0$, derive the following expression (Schilke et al. 2014, A&A, in press) for the abundance ratio of $\mathrm{ArH^+}$ and Ar in the diffuse ISM,

$$\frac{n(\mathrm{ArH^+})}{n(\mathrm{Ar})} = \frac{\zeta(Ar)}{k_6 n(\mathrm{H_2}) + k_8 n(\mathrm{O})} \quad , \tag{12}$$

where $\zeta(\mathrm{Ar}) = k_1$ is the cosmic-ray ionization rate of Ar. According to Schilke et al. 2014, the following reactions *only* contribute significantly to the formation and destruction of $\mathrm{ArH^+}$,

$$\mathrm{Ar} + \mathrm{CR} \xrightarrow{k_1} \mathrm{Ar}^+ + \mathrm{e}^-$$
$$\mathrm{Ar}^+ + \mathrm{H}_2 \xrightarrow{k_5} \mathrm{ArH}^+ + \mathrm{H}$$
$$\mathrm{ArH}^+ + \mathrm{H}_2 \xrightarrow{k_6} \mathrm{Ar} + \mathrm{H}_3^+$$
$$\mathrm{ArH}^+ + \mathrm{O} \xrightarrow{k_8} \mathrm{Ar} + \mathrm{OH}^+$$
$$\mathrm{ArH}^+ + h\nu \xrightarrow{k_{11}} \mathrm{Ar}^+ + \mathrm{H}$$

Tip: you need the expressions describing the steady-state abundances of both ArH^+ and Ar^+.

4. Consider now the following values for a diffuse molecular cloud:

$$\begin{aligned}
k_1 &= 11.4\,\zeta_p(\mathrm{H})\,\mathrm{s}^{-1} \\
k_6 &= 8.0 \times 10^{-10}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \\
k_8 &= 8.0 \times 10^{-10}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \\
\zeta_p(\mathrm{H}) &= 2.0 \times 10^{-16}\,\mathrm{s}^{-1} \\
n(\mathrm{O}) &= 2.9 \times 10^{-4}\,n_\mathrm{H} \\
n(\mathrm{Ar}) &= 3.2 \times 10^{-6}\,n_\mathrm{H}
\end{aligned}$$

(a) How does the ArH^+ abundance change as a function of the logarithmic molecular fraction $\log_{10}(2n(\mathrm{H}_2)/n_\mathrm{H})$? Evaluate the expression for these values:

$$\begin{aligned}
\log_{10}(2n(\mathrm{H}_2)/n_\mathrm{H}) &= -1.0 \\
\log_{10}(2n(\mathrm{H}_2)/n_\mathrm{H}) &= -2.5 \\
\log_{10}(2n(\mathrm{H}_2)/n_\mathrm{H}) &= -3.5 \\
\log_{10}(2n(\mathrm{H}_2)/n_\mathrm{H}) &= -4.0
\end{aligned}$$

(b) How can we conclude from this that ArH^+ is a good atomic gas tracer?