Stretched membrane with electrostatic curvature (SMEC): A new technology for ultra-lightweight space telescopes

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ABSTRACT

Very large space telescopes with primary mirrors made of flat segments have been recently proposed.[1] The segments would be extremely lightweight, made like pellicles from stretched, reflective membranes. Here we consider the use of such membrane primary mirrors in which slight concave curvature is induced by electrostatic force, by application of a potential difference between the membrane and a control electrode behind. In this way segmented spherical or paraboloidal primaries of long focal length can be made directly, eliminating the correction optics needed when flat segments are used. The electric potential would be spatially and temporally controlled to obtain uniform curvature despite non-uniformity in membrane tension, to create slight asphericity if needed and to provide active damping of vibrations. We report the operation of a small prototype telescope with a SMEC primary. A design for a 3.2-m space prototype is described, based on the two-mirror anastigmat of Schwartzschild[2] and Couder[3]. A 400 m radius primary is combined with a single concave secondary flying in formation at a distance of 190 m. Together they form a wide-field, well-corrected focus at f/30. A larger telescope with much higher diffraction limited resolution could be made simply by adding more segments of the 27 m diameter parent primary to the formation. The TPF nulling interferomer could be made with similarly sized elements in a 100 m formation, configured as segments of a kilometer focal length parent[4]. A very large SMEC telescope might be made from a continuous 10 x 1000 m membrane, rolled up for launch. Cryogenic cooling for all these configurations would be accomplished by additional spacecraft in the formation to block direct solar illumination of the telescope elements.

Keywords: space telescopes, ultra-lightweight primary mirrors, cryogenic mirrors, formation flying

1. INTRODUCTION

1.1. The value of large telescopes in space

Sharp images are essential to study astronomical objects outside the solar system, with no possibility of increasing angular size by approaching the object. Telescopes in space have the potential for arbitrarily sharp images, limited only by their aperture size. At the limit of resolution set by diffraction, angular resolution (in radians) is given by λ/D, where λ is the wavelength of observation and D is the telescope mirror diameter or interferometer baseline. The Hubble Space telescope is 2.4 m diameter, and is diffraction limited at 0.5 μm wavelength (visible light) for a resolution of 40 milliarcseconds. Similar resolution in the near infrared at 2 μm wavelength is the goal of the 8 m Next Generation Space Telescope (NGST).

A further order of magnitude increase in resolution requires apertures an order of magnitude larger. Already some images sharper than HST's are being obtained from 6 - 10 m class telescopes on the ground, through the use of adaptive correction of atmospheric blurring. In 2004 the Large Binocular Telescope (LBT) with an elongated 23 m wide aperture will provide from the ground images with ten times HST resolution[5]. Even bigger ground-based telescopes of 30 - 100 m diameter are under consideration. There are two fundamental limits, though, to ground based telescope capability. First, at wavelengths beyond two microns, absorption blocks most wavelengths and thermal emission from warm telescope mirrors prevents the detection of faint objects. Second, even after the best possible wavefront correction, residual atmospheric aberration results in a halo of scattered light that will prevent detection of earth-like extra-solar planets[6]. Thus the most valuable characteristics of space telescopes are their potential for cryogenic operation, and for extremely smooth optics that minimize scattered light. Both aspects will be used by the Terrestrial Planet Finder mission to explore Earth-like planets of other stars [7].

1.2. Potential for membrane primaries

The challenge for any very large optical system is to reach diffraction limited resolution: the accumulated rms error in the reflected wavefront must be no more than 1/15 of the wavelength under study, irrespective of telescope aperture. For visible light (λ=550 nm) the surface tolerance is thus 18 nm rms. The HST achieves such accuracy with a mirror made of lightweighted glass but still weighing a good fraction of a ton. The NGST's cryogenic mirror is to be 8 m diameter, larger than available launch fairings, and so must be folded during launch. To date, no mirror segment larger than 0.5 m has been shown by any technology to meet the 15 kg/m^2 density and 40 K operation targeted for NGST. Nevertheless, there is progress, for example, the University of Arizona will test at MSFC a 2-m cryogenic prototype segment made of glass 2 mm thick glass with 166 position actuators[8] this fall.

Reflecting membrane optics with still lower density have the potential for a further substantial increase in space telescope size, to 25 m diameter or more, while keeping mass within limits for a single rocket launch. Refractive and diffractive solutions are also possible and Fresnel optics are being explored[9], but reflective membranes have the advantage of being achromatic, with spectral cover from the vacuum ultraviolet to the far infrared, and avoid the limitations of infrared absorption by refractive materials. Membranes are also very lightweight. For example, at 10 μm thickness a 25 m diameter membrane will weigh 5 - 50 kg, depending on material density. Our strategy for keeping down the total mass is to hold the membrane in shape by stretching from its perimeter, with active control to overcome deformations of a very light supporting frame.

In our original concept[1] diffraction limited images would be recovered from a primary made as a mosaic of flat segments by wavefront correction at a reduced scale pupil image, formed on a mirror with scalloped segments. In this paper we explore the use of uniform pressure to induce curvature in stretched membrane surfaces. Gas pressure cannot be easily maintained in space, because of holes created by micro-meteorites. However, small, uniform pressures can be created by configuring the stretched membrane as one plate of a capacitor. In the following sections we explore the optical properties of stretched, charged membranes, examine ways to control the mass of the tensioning and electrode structure, and describe a small prototype telescope and designs for space telescopes.

2. SHAPE OF STRETCHED MEMBRANE MIRRORS

Suppose a membrane is uniform in its thickness and elastic properties, is uniformly tensioned with a force T per unit length and also feels a uniform external pressure p. The deformation is governed by the equation balancing force at each point:

$$T \cdot \nabla^2 z = p. \tag{1}$$

The simplest solution is one in which the boundary is circular. The membrane then takes on uniform spherical curvature, like a soap bubble, with radius a given by

$$R = 2T / p \tag{2}$$

More generally, the same solution holds for any shape of boundary which can be drawn on the surface of a sphere of radius R. If the boundary does not satisfy this condition, the curvature will not be constant.

Suppose that there is still uniform pressure acting and the boundary is a circle of radius a projected on to the r,θ plane, but now with out-of-plane displacement z(a,θ). To determine the displacement across the body of the membrane, z(r,θ), we first express the boundary displacement as a Fourier series, thus

$$z(a,\theta) = \sum A_n \cos(n\theta) + B_n \sin(n\theta) \tag{3}$$

The solution of (1) under this boundary condition is given by a linear superposition of terms

$$z(r,\theta) = \sum A_n (r/a)^n \cos(n\theta) + \sum B_n (r/a)^n \sin(n\theta) + p(r^2 - a^2)/4T \tag{4}$$

We see that the low order Zernike terms of piston, tip, tilt, focus and astigmatism can be induced by pressure and boundary displacement. The capability to induce astigmatism from the perimeter is useful if a paraboloidal surface is to be synthesized from many off-axis segments, as we will explore below. The radial dependence of displacement shown by equation 4 also tells us about the accuracy we need in defining the perimeter of a membrane. Because the nth order term has a radial dependence as r^n, the low order terms have much the strongest influence on the overall shape of the membrane, so it is important to control them accurately. Conversely, we can tolerate significant edge displacement in high order terms, provided the outer edge region of the membrane is masked off. For example, if the perimeter is accurately defined at 20 points around the perimeter, then small displacements between the defining points will fall off as $(r/a)^{10}$ or faster.

When a membrane is made conductive and held at constant potential, it will be attracted to a second electrode held at different potential. The electrostatic pressure is given by

$$p = \varepsilon_0 E^2 \tag{5}$$

where $\varepsilon_0 = 8.85.10^{-12}$ farad/m and E is the electric field in volts/m. We note that since the attractive force increases as the inverse square of the gap, there will be a catastrophic instability if the electrically induced sagittal depth becomes comparable to the gap. In practice, the gap must be kept no less than 5 or ten times the sagittal depth.

3. IMAGING WITH A SMALL PROTOTYPE SMEC TELESCOPE

A small demonstration telescope with an electrostatically curved membrane primary has been built. It makes use of a membrane mirror made commercially for adaptive optics systems by OKO[10], with a silicon nitride membrane 0.6 μm thick coated with 0.1 μm of aluminum. It is made by chemical vapor deposition of the nitride on a nominally flat substrate of silicon which is then etched away to leave the membrane free-standing over a 15 mm diameter. The membrane is spaced 70 μm above a flat printed circuit board with a 10 mm diameter array of electrodes. We found that a uniform potential difference of 133 V induces an approximately concave spherical surface with 6 m radius of curvature. The sagittal depth is thus about 5 μm, much less than the gap. From the equations above, the operating parameters are: E=2 x 10^6 V/m, p=35 Pa, T=100 N/m, membrane stress = 150 Mpa (20,000 psi).

Figure 1. The moon imaged at Steward Observatory with the first telescope to use a primary mirror of Stretched Membrane with Electrostatic Curvature (SMEC). The silicon nitride membrane was 0.7 μm thick and curved to a 3 m focal length by a field of 2 MV/m. A 6 mm stop was used to obtain diffraction limited image quality, as shown by the Airy pattern of the star image inserted bottom left.

Optical tests showed that the silicon membrane boundary had an irregular displacement of order 1 μm. peak to peak. To obtain diffraction limited images, we stopped down the clear aperture to 6 mm diameter, leaving a residual aberration of mostly astigmatism, in accordance with eqn (4). This was compensated by adjustment of the spatial distribution of electrode potentials by 10% (20% in E^2).

A telescope was made by placing a 1024 x 1024 pixel CCD camera at the prime focus of the electrostatic mirror and tuning the applied voltage to bring stars into focus. A filter was used to obtain a wavelength band 450 – 650 nm, for diffraction limited images with FWHM λ/D = 90 μR = 19 arcsec. The images formed at a plate scale of 15 μm/arcsec were considerably oversampled by the 25 μm CCD pixels subtending 8 μR or 1.7 arcsec. An image of the moon recorded on March 23, 2000 is shown in Figure 1. To our knowledge, this is the first astronomical image ever made with an electrostatically curved primary mirror. It is often pointed out that the working part of a telescope mirror is just the 100 nm skin depth of the reflective surface, the rest is just supporting mechanical structure. For this mirror, the substrate support was not much thicker than the reflecting layer, weighing less than a milligram.

4. CONCEPT FOR A 4 M SMEC SPACE TELESCOPE FOR THE INFRARED

The practicality of SMEC mirrors in space could be proven relatively soon with a 4m class infrared telescope made as two elements flying in formation. Polyimide membranes 10 μm thick are suitable, since they are already being made commercially for many uses on the ground and in space. The best films currently available are uniform in thickness to a few hundred nm rms, and could thus be used to make mirrors diffraction limited down to ~ 5 μm wavelength. At 4 m diameter there would be no requirement for space deployment, a stretched mirror could be completely assembled and checked out before launch. The four elements of the telescope formation, including the mirror, the detector and two sun-blocker spacecraft would be launched together in a single vehicle. As we show below, free-flying sun shields allow for passive cooling to very low temperatures, as low as 10 K in an orbit well away from earth's thermal radiation. The telescope would thus be ideal to explore the mid-infrared sky with much higher resolution and sensitivity than SIRTF. In an earth-trailing orbit it would reach the limiting sensitivity set by zodiacal background at wavelengths as long as 50 μm.

4.1. Defining the membrane boundary

The key to making a space membrane mirror is to control the mass of the stretching frame. Existing small membrane mirrors such as pellicles and the electrostatic deformable mirror described above have supports which are hundreds or thousands of times heavier than the membrane. For space membranes it is not desirable or necessary to use a continuous rigid perimeter. A lightweight frame attached to the membrane through a discrete number of edge nodes can be used. The inevitable flexibility and deformation of the light structure would be compensated by active control of the position of the defining nodes, based on wavefront measurements. As we have seen from the Fourier analysis above, active control of the low order terms is all that is necessary, because only these propagate far from the boundary; high order errors are greatly attenuated. The spacing of the active nodes must be chosen so that errors from the uncontrolled perimeter between them are negligible. We envisage the perimeter of the membrane attached to evenly spaced sections of a broken hoop, with each section applying a controlled radial tension as well as active adjustment of the surface height.

We shall assume a 4 m diameter membrane stretched from 32 nodes at 0.4 m intervals around the perimeter, with a goal of realizing good optical figure over the inner 3.2 m diameter. The amplitude of uncorrectable edge errors with $n \geq 16$, the highest order than can be actively corrected, will be reduced at 1.6 m radius by a factor $\leq 0.8^{16}$, i.e. ≤ 0.028. We imagine that the radial tension at each node is transmitted into the membrane between nodes by a lightweight clamp some 30 cm long, with 10 cm unsupported between clamps. The 4 m diameter stretching frame itself could be made from carbon fiber and would include the back electrode and a further grounded screen to control the potentials behind. It could be built along the lines used by Composite Optics Inc to build radio dishes at density 1 kg/m^2. Even at twice this density, the 4 m frame would weigh only 24 kg. With the membrane and tensioning and actuation hardware, the total mirror mass should be $\leq$ 40 kg, i.e. having a density of 3.5 kg/m^2.

4.2. Electrostatic pressure

The balance of tension and electric field to produce given focal length has not yet been properly optimized. We give here some representative values, for a feeling for what might be used. We choose first a radius of curvature R = 400 m, for a focal length of 200 m. This radius allows the use of electric fields considerably less than those of the silicon nitride prototype with R = 6 m. It corresponds to a sagittal depth of 5 mm over the full 4-m membrane diameter, and we assume an electrode gap of 40 mm to avoid instability. We suppose also that the radial tensioning force applied by each clamp is 40 N, for an average tension in the body of the membrane of 100 N/m. The average stress in a 10 μm thick membrane would be quite modest at 10 MPa, or 1500 psi.

For 100 N/m tension, the electrostatic pressure corresponding to 400 m radius, from eqn (2), is 0.5 Pa. This requires an electric field of 240 kV/m, i.e 9.5 kV over the 40 mm gap. Such fields and potential differences should be safely realizable in the hard vacuum of space without risk of breakdown. The desired shape of the membranes, very close to spherical, requires uniform field strength. This can be achieved even if the gap is not constant by setting the potential difference to the control electrodes behind will in proportion to gap thickness. An additional important use of the electrodes will be to sense (through capacitance variation) and actively suppress membrane vibration.

4.3. Optical design

The long focal length of a SMEC mirror gives an inconveniently large plate scale for use directly at prime focus. Fortunately, the powerful two mirror telescope design by Schwartzschild[2] and Couder[3] can be used to obtain a wide field at a faster focal ratio. The second mirror is concave, and is placed before the prime focus. Figure 2 shows a design with a 3.2 m primary of 200 m focal length, f/62. It is taken 12 m off-axis from the 27.2 m diameter primary of an axisymmetric parent system. The prescription is given in Table 1. The 78 cm diameter secondary mirror is separated by 192 m, and forms an image at f/30 spaced by 4 m. The plate scale is 467 μm/arcsec, and the images are diffraction limited down to optical wavelengths over the 0.2 degree diameter field shown (336 mm). The astigmatism to be introduced at the 4 m diameter perimeter of the off-axis primary is 6 μm peak-to-valley. The curvature of 5 mm peak-to-valley is induced by electrostatic deflection.

Table 1. Prescription for 3.2 m off-axis telescope (dimensions in meters). The primary is centered 12 m off-axis.

	Radius	**Spacing**	**Diameter**	**Conic const**	**6th order coeffic**
Primary	400	-191.693	3.2	-1.372	3.77 E-14
Secondary	15.430	4	0.78	-.978	-4.94 E-7
Focal plane	7.445		0.34		

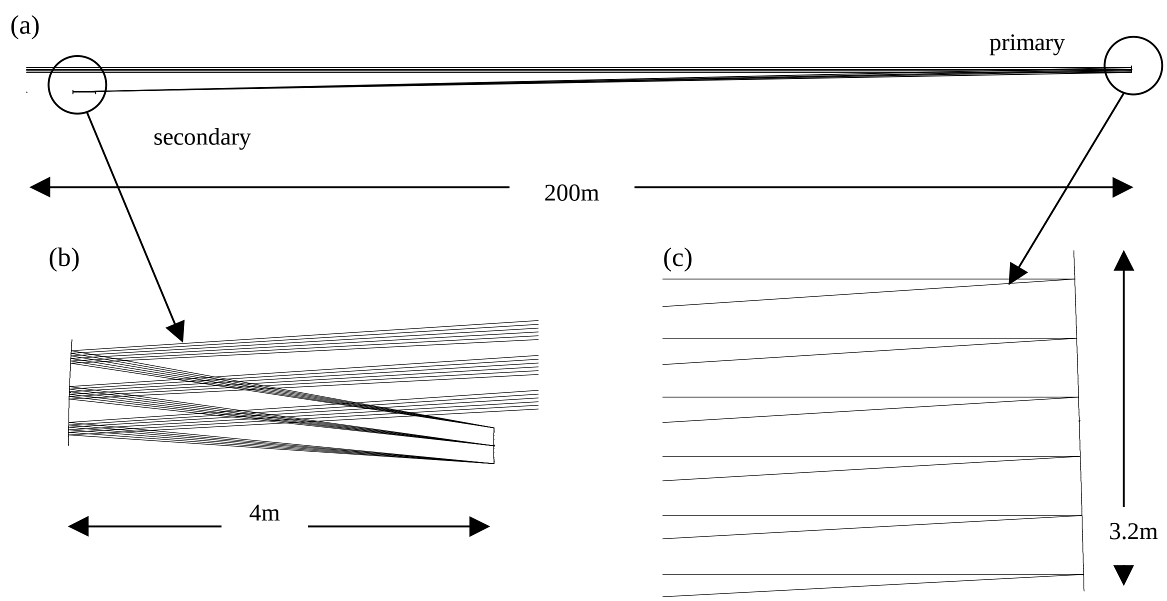

Figure 2. Optical diagram of a formation SMEC telescope. a) gives the overall system, to scale. b) shows a blow-up of the secondary and image plane, which would be in a 4-m long spacecraft. The primary, shown at the same scale, is in figure c)

4.4. Active controls and passive cooling by sun-blockers in the telescope formation

The two main spacecraft making up the formation are as shown above. Each will be equipped with thrusters to control relative alignment and absolute orientation. Wavefront aberrations of starlight will be sensed at the focal plane, and used to develop corrections to be applied to the membrane perimeter actuators and electrode potentials. The information on alignment and orientation will be derived from both starlight and laser metrology made between the spacecraft. Two additional spacecraft will be flown about 1 km away so as to put both primary and secondary units into permanent eclipse. Each will block sunlight with a screen about 12 m across.

Such free-flying sunshields offer an opportunity to passively create extremely low temperature spacecraft appropriate for infrared and radio astronomy detectors without further cooling. The ultimate limit that would be obtained by complete elimination of radiation from the sun, earth and moon is set by zodiacal dust. At 1AU this presents a radiation temperature of ~ 266K with a dilution of ~6.5×10^{-8}. The corresponding black sphere temperature is 4.1K. There is also heating by the cosmic microwave background at 2.7K.

This limit can be approached by a telescope formation in a fall-away orbit with Earth and Moon far away. We imagine that the sun-blocker is coated with titanium dioxide or solar cells on the sun-facing side, and gold on the back, and is far enough away so as to subtend 1/100 radian, little larger than the sun. The radiation coming towards the spacecraft would heat a black object as if it were exposed to 270K radiation with dilution 10^{-6}. But if also the shieldward side of the spacecraft is coated with gold, to reflect away the blocker radiation, then the dilution of solar heating would be improved to 2×10^{-8} , when it would be less than the dust emission, and comparable with the CMB. Thus the spacecraft could in principal come to a temperature ~5K.

Telescope power would have to be minimized to realize this limit, arriving carefully beamed from the sunshield, and with controlled radiation of the waste. It certainly seems possible to create temperatures low enough for the passive operation of infrared detectors, and for sensitivity than is limited by the self emission of the telescope for wavelengths out to ~40µm.

5. BIG MEMBRANE TELESCOPES

5.1. Filling the aperture of a Schwartzschild telescope

Membrane optics lead us naturally to much larger telescopes. For example, the design given above could be directly extended to a bigger system simply by adding more off axis elements of the 27.2 m parent system to the formation. A few identical primary elements could be added to the formation as elements of a 27.2 m baseline interferometer, for an increase in resolution of nearly an order of magnitude. A full 27.2 m annular primary could be deployed or assembled from 4 m segments to achieve the highest sensitivity to faint objects. The design given in Table 1 is in fact diffraction limited to optical wavelengths even for the full aperture, for resolution of 3.8 milliarcsec over the 720 arcsec field.

5.2. TPF made with SMEC elements

TPF is currently conceived as a 4 element nulling interferometer with 4 m elements spaced over 80 m. The individual elements could be very similar to the 4 m primary described above. They would be shaped to form parts of an 80 m parent, probably with longer focal length of ~ 1 km.

5.3. Telescopes by the yard

The objective of the NAIC funded study that supports this work is to develop concepts for 100 m space telescopes. One configuration would be as an elongated SMEC mirror with a single membrane 10 m wide by 1 km long. This geometry respects the natural possibilities for membrane production and deployment. Rigid materials come in lumps, and it has been natural to make roughly circular mirrors from such pieces. However, most plastic film is made in continuous strips, often aluminized, and rolled up for transportation. Currently material is produced in typical widths of 2m. However 10m wide material would seem possible, and a roll as long as a kilometer for a single piece primary mirror would be easily carried in currently available rocket shrouds. A 10 µm polyimide strip 10x1000 m would weigh 100 kg. The mechanical links that tension the membrane across its width would have to move on curved tracks along the two long edges, to maintain tension everywhere normal to the boundary while allowing for translation caused by membrane stretching.

As we have seen in section 2, if the perimeter is made to conform to a spherical surface, and appropriate electrostatic pressure is applied, the entire membrane surface will be naturally spherical. Suppose the primary mirror is made with focal ratio of f/10, (focal length 10 km). The sagittal depth across the width is then 600 µm, so the electric field for even 1000 N/m tension is only 100,000 V/m. A potential difference of 500 V across a 5 mm gap is all that is required.

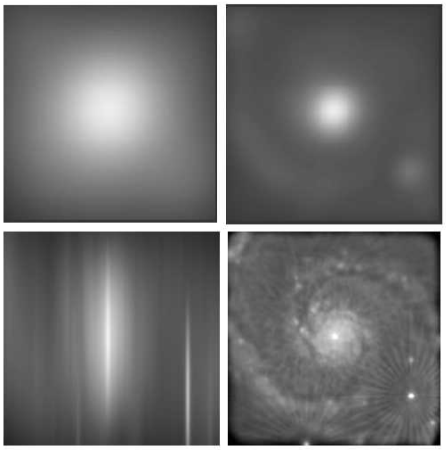

Figure 3. Improvement in image resolution obtained with an elongated telescope. On top are two images of a galaxy as it would appear at the diffraction limited resolution of the 2.4 m HST (left) and 8 m NGST (right). Lower left is the elongated image that would be obtained with a 2 x 100 m telescope. Lower right is an image deconvolved by the method described by Hege et al[11] from 18 such elongated images at 10 degree rotation intervals.

The images formed by the elongated telescope are themselves elongated, for example at 10 μm wavelength the 10x1000 m primary would yield images 200 x 2 milliarcsec. Images with the full resolution in two dimensions can be reconstructed from a series of elongated images taken at different position angles. Figure 3 show such a reconstruction derived from 18 elongated images computed for different rotation angles of a smaller 2 x 100 m aperture (for which polyimide material is already available).

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