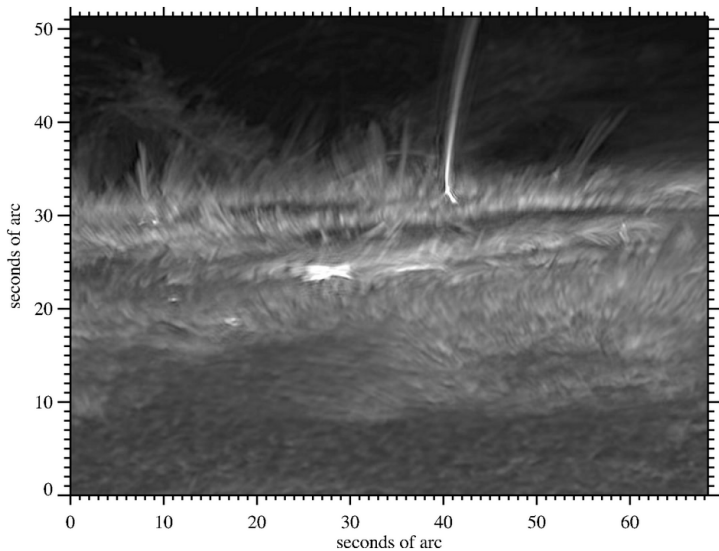


## Outline

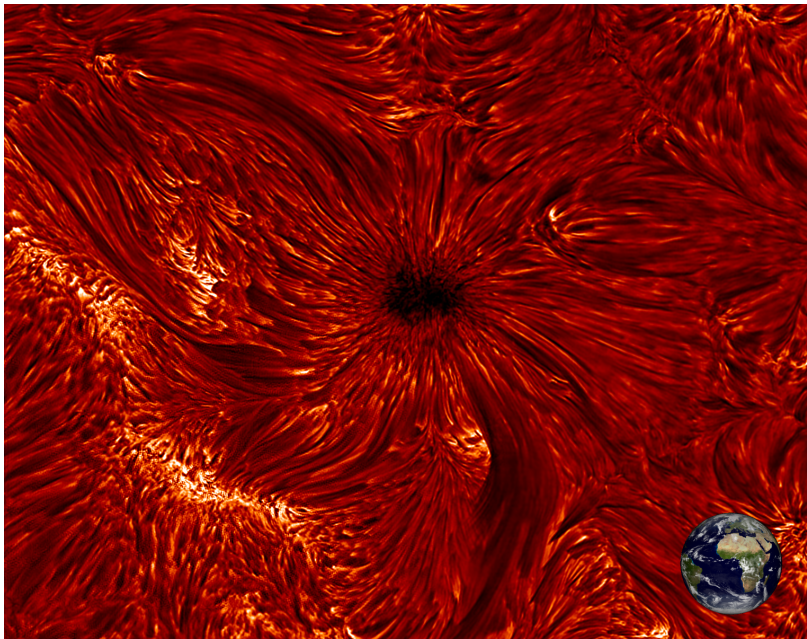
- 1 Evidence for Flux Tubes
- 2 Thin Flux Tube Approximation
- 3 Small-Scale Magnetic Elements
- 4 Faculae

# Observational Evidence

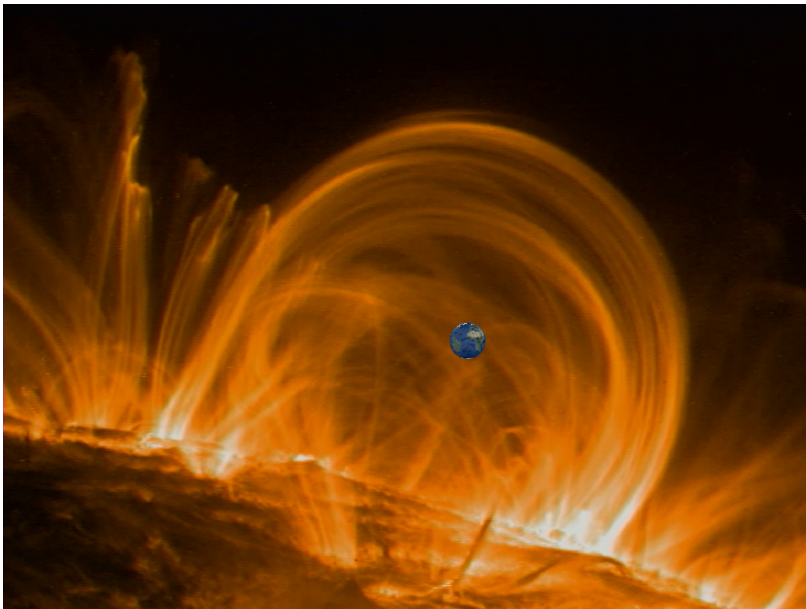
## DOT Call K image close to the limb



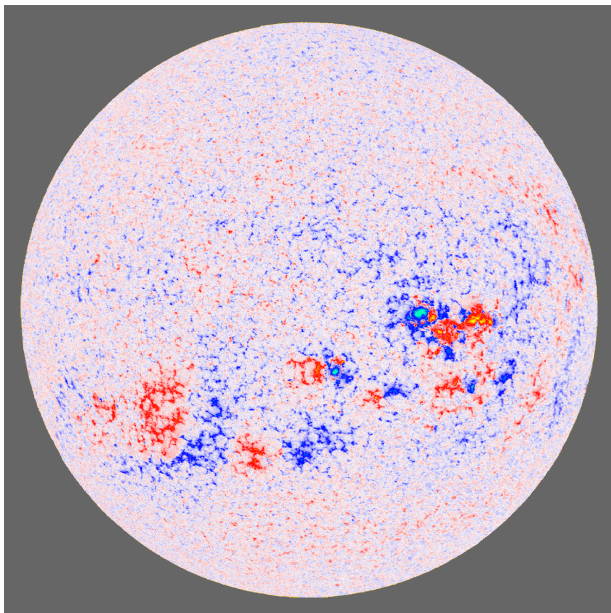
# DOT H $\alpha$ image



## TRACE Loops

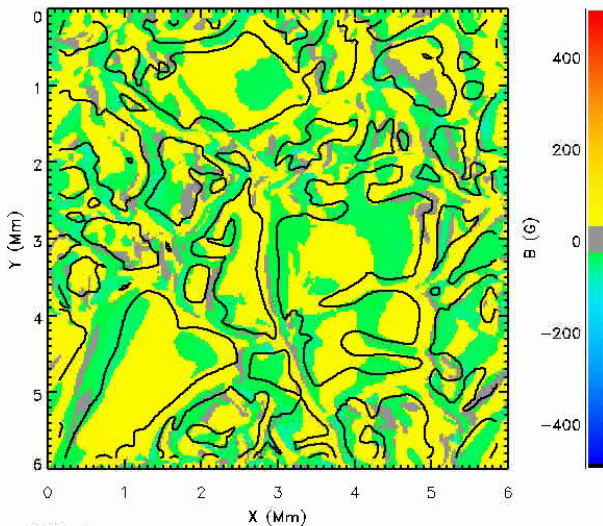


## SOLIS VSM Magnetic Field Distribution



# Evidence from MHD Simulations

## Stein & Nordlund, Quiet Sun



# Thin Flux Tube Approximation

## Force Balance

- all relative length scales are large compared to tube diameter
- neglect diffusion term in induction equation
- equation of motion

$$\rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \vec{j} \times \vec{B} + \vec{F}_{\text{gravity}} + \vec{F}_{\text{viscosity}}$$

- magnetohydrostatic ( $\vec{v} = 0 \Rightarrow F_{\text{viscosity}} = 0$ )
- force balance

$$\nabla p - \vec{j} \times \vec{B} = \vec{F}_{\text{gravity}}$$

- with  $\mu_0 \vec{j} = \nabla \times \vec{B}$  and  $\vec{B} \times (\nabla \times \vec{B}) = \frac{1}{2} \nabla B^2 - (\vec{B} \cdot \nabla) \vec{B}$

$$\nabla \left( p + \frac{B^2}{2\mu_0} \right) - \frac{1}{\mu_0} (\vec{B} \cdot \nabla) \vec{B} = \vec{F}_{\text{gravity}}$$

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- force balance in general coordinate system

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- in cylindrical coordinates, radial component

$$\frac{\partial}{\partial r} \left( p + \frac{B^2}{2\mu_0} \right) - \frac{1}{\mu_0} \left( B_r \frac{\partial B_r}{\partial r} + B_z \frac{\partial B_r}{\partial z} - \frac{B_\phi^2}{r} \right) = 0$$

- with  $B_{r,\phi} = 0$

$$\frac{\partial}{\partial r} \left( p + \frac{B^2}{2\mu_0} \right) = 0$$

- and therefore *horizontal pressure balance*

$$p_{\text{inside}} + \frac{B^2}{2\mu_0} = p_{\text{outside}}$$

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## Plasma Beta

$$\beta = \frac{2\mu_0 p}{B^2}$$

- *Plasma*  $\beta$ : ratio of gas pressure to magnetic pressure
- determines whether gas pressure or magnetic field 'pressure' is more important

## Vertical Force Balance

- in the z-direction (along the field lines)

$$\frac{\partial p}{\partial z} = -\rho g$$

- with ideal gas law  $\rho = \frac{\mu p}{kT}$

$$\frac{\partial p}{\partial z} = -\frac{\mu g}{kT} p$$

- pressure as a function of height z

$$p(z) = p(z_0) \exp\left(-\int_{z_0}^z \frac{1}{H(z')} dz'\right)$$

- with the *pressure scale height*

$$H(z) = \frac{kT}{\mu g}$$

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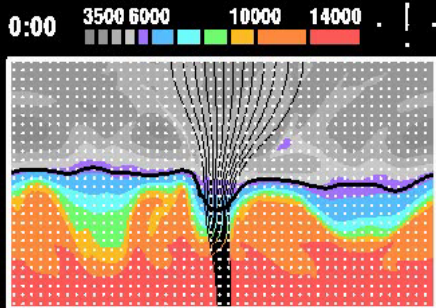
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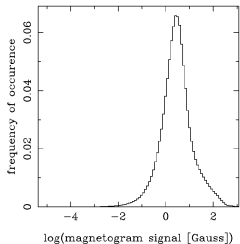
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## 2-D Simulations by Oskar Steiner

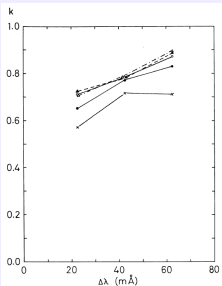


# Small-Scale Magnetic Elements

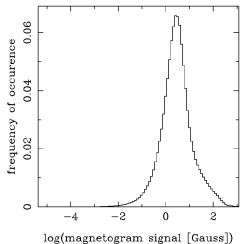


## A Little History

- much of measured magnetogram signal 0-100 Gauss
- ratio of magnetograms in 2 spectral lines with "only" different Landé g-factors (line ratio technique)
- indicates high field strengths of 1000-2000 Gauss
- indicates that magnetic field is not space-filling
- *filling factor* describes fraction of resolution element filled with magnetic field

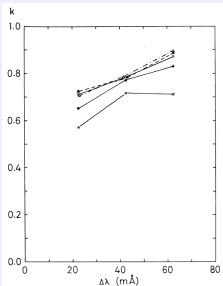


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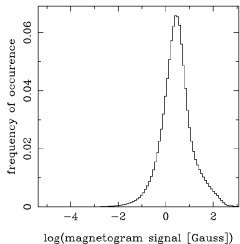


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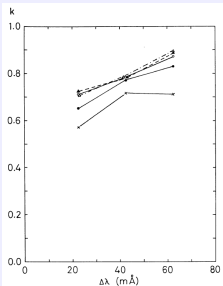


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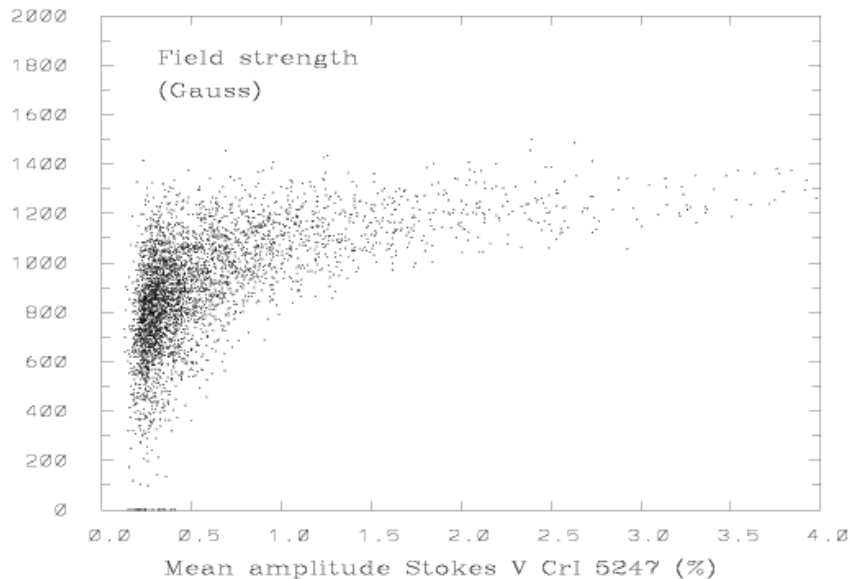


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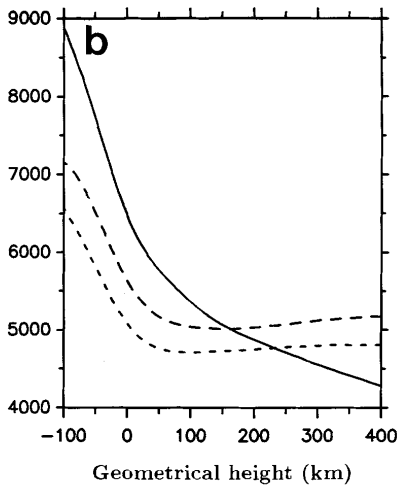
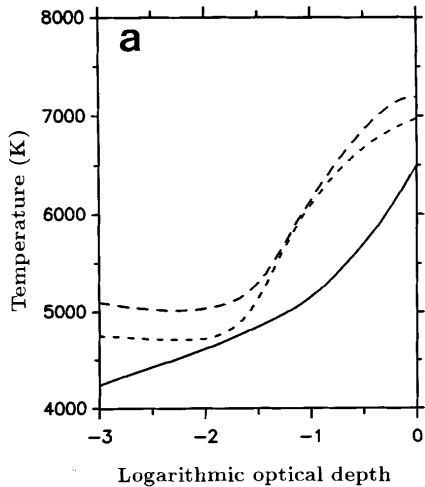
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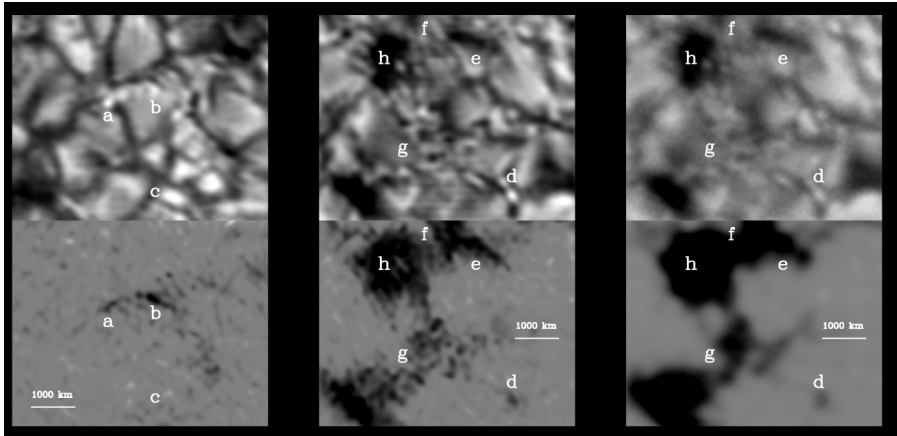
## Flux vs. Field Strength



## Temperature Structure



## Direct Detection of Concentrated Fields



## A Description

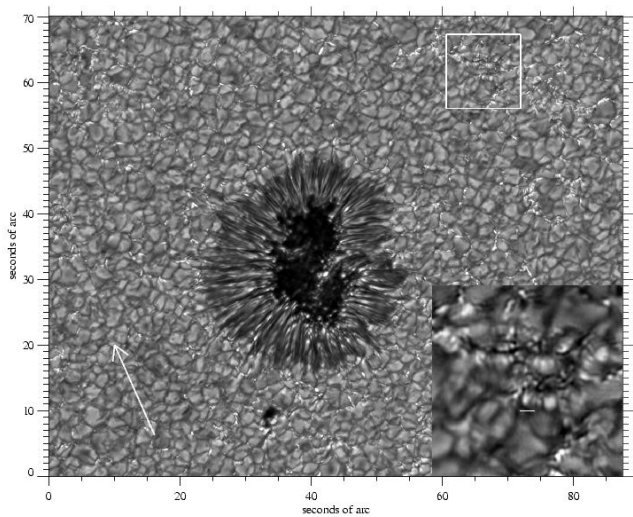
- kinetic equipartition field strength

$$\frac{B^2}{2\mu_0} = \frac{\rho}{2}v^2$$

- typical values in the photosphere: 400 G
- magnetic field inhibits convection
- reduced heating leads to lower temperature
- correspondingly higher density makes material sink

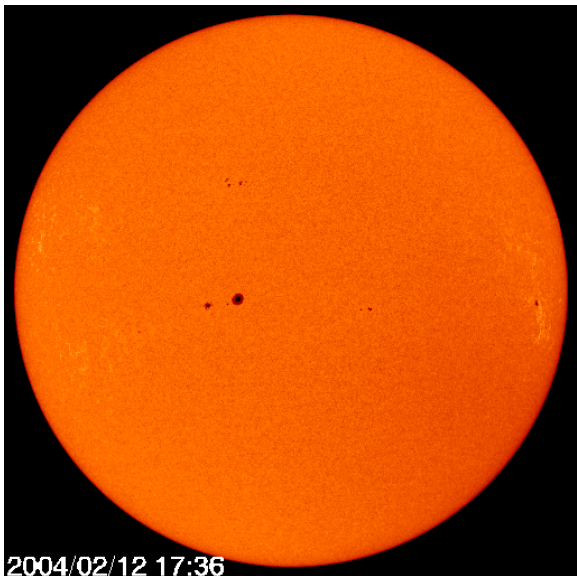


## G-Band Bright Points



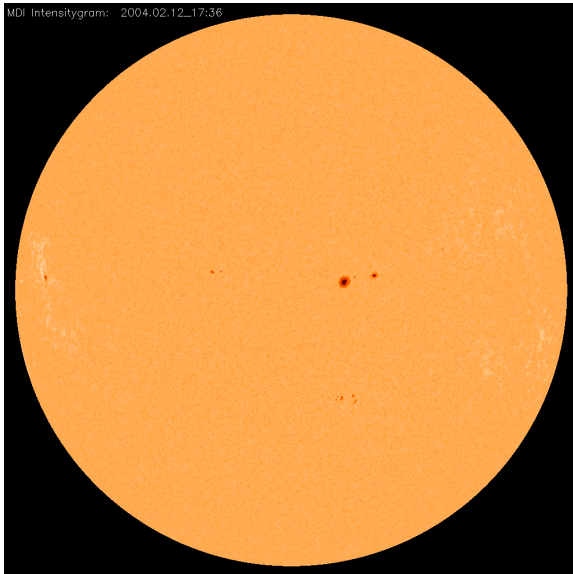
# Faculae

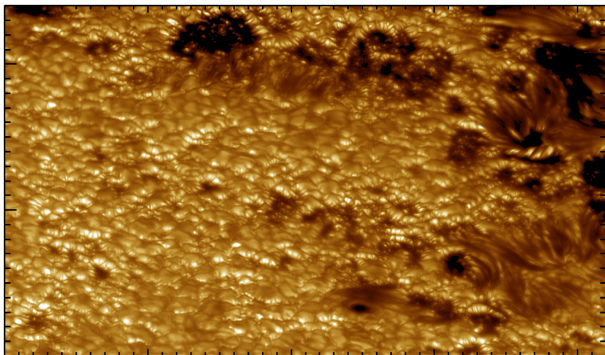
## The Sun in White Light



2004/02/12 17:36

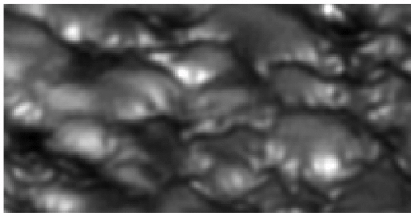
## The Sun without Limb Darkening





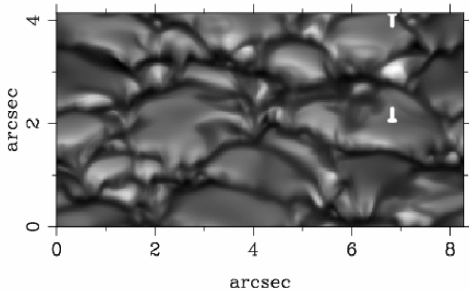
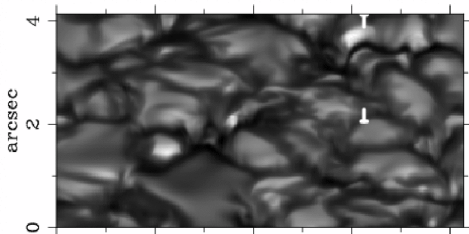
- 3-D impression when looking at images
- Faculae appear predominantly in plages
- Facular brightenings on disk-center side of granules
- Brightening can extend over about 0.5 arcsec
- Narrow, dark lanes centerward of the facular brightening

Observations by Lites et al. (2004)

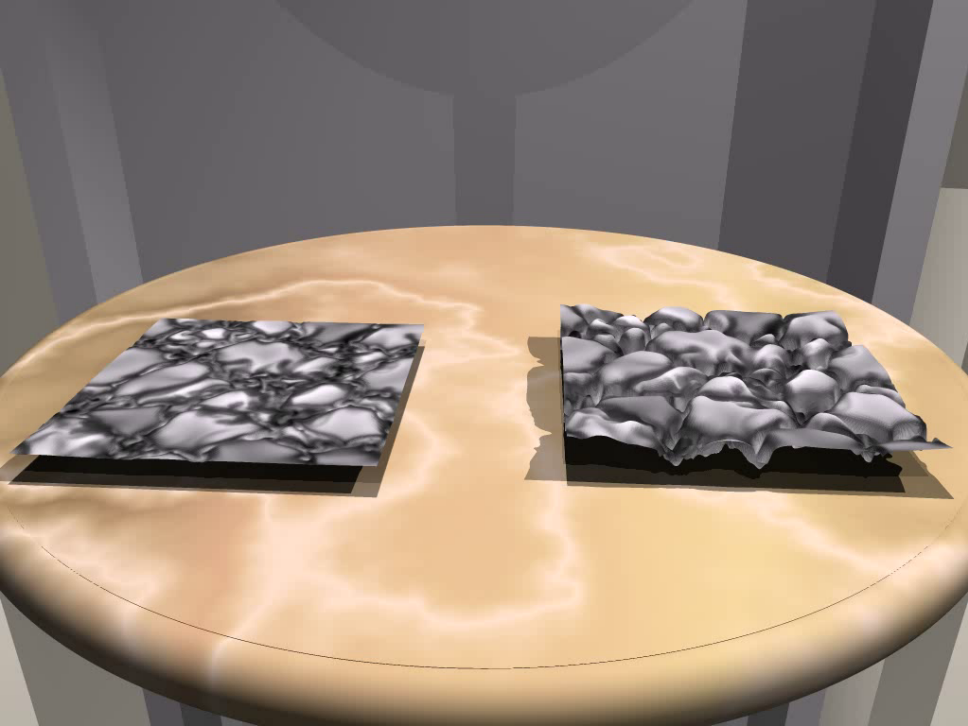


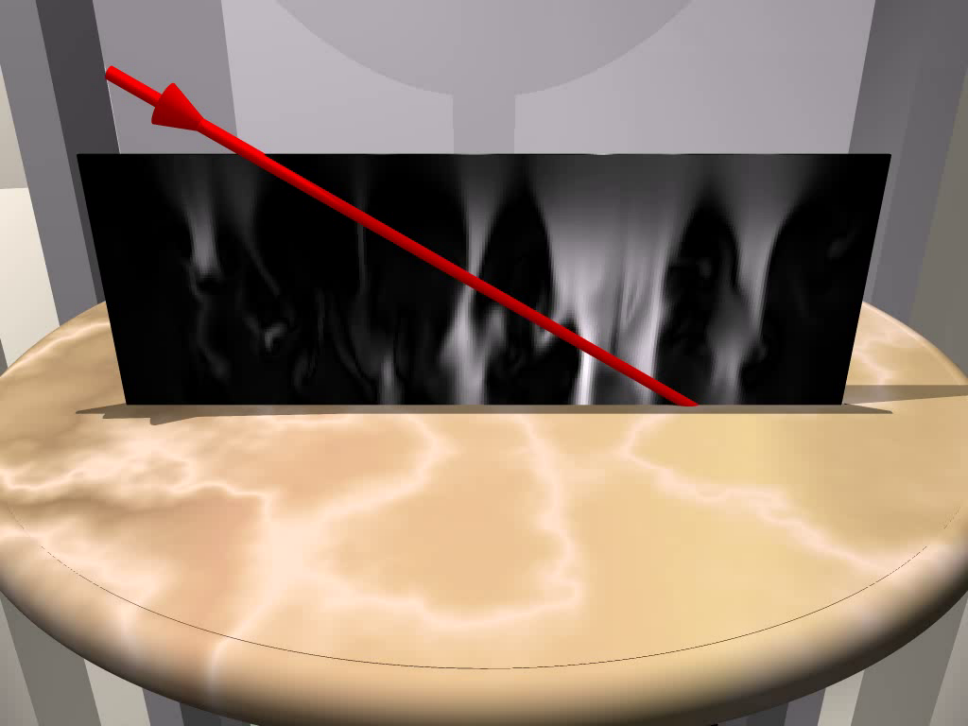
Simulations by Keller,  
Schüssler, Vögler, Zakharov,  
ApJL 607, L59 (2004 May 20)

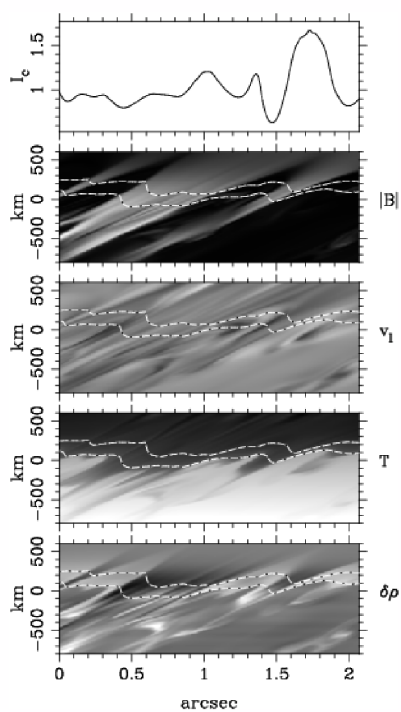
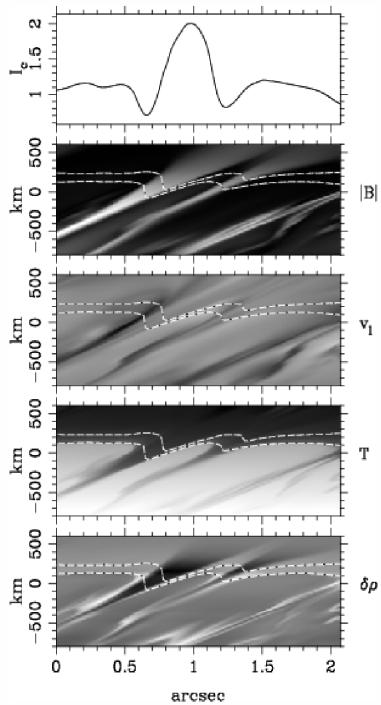
Simulation, 200 G



Simulation, 400 G









- Largely consistent with 'bright wall' model of Spruit (1976)
- Qualitatively similar features (dark lane followed by more extended brightening) already apparent in 2D flux sheet models (e.g., Deinzer et al. 1984, Knölker & Schüssler 1988, Knölker et al. 1991)
- Expansion of flux concentrations with height and 3D geometry of granules lead to limbward extension of facular brightenings significantly in excess of the Wilson depression
- The same simple geometric effects lead to the formation of the narrow, dark lanes centerward of the facular brightenings

