Outline

1. Scattering Polarization
2. Zeeman Effect
3. Hanle Effect
4. Polarized Radiative Transfer

Scattering Polarization

Single Particle Scattering

- light is absorbed and re-emitted
- if light has low enough energy, no energy transfered to electron, but photon changes direction $\Rightarrow$ elastic scattering
- for high enough energy, photon transfers energy onto electron $\Rightarrow$ inelastic (Compton) scattering
- Thomson scattering on free electrons
- Rayleigh scattering on bound electrons
- based on very basic physics, scattered light is linearly polarized

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Polarization as a Function of Scattering Angle

- same variation of polarization with scattering angle applies to Thomson and Rayleigh scattering
- scattering angle θ
- projection of amplitudes:
  - 1 for polarization direction perpendicular to scattering plane
  - $\cos\theta$ for linear polarization in scattering plane
- intensities = amplitudes squared
- ratio of $+Q$ to $-Q$ is $\cos^2\theta$ (to 1)
- total scattered intensity (unpolarized = averaged over all polarization states) proportional to $\frac{1}{2}(1 + \cos^2\theta)$

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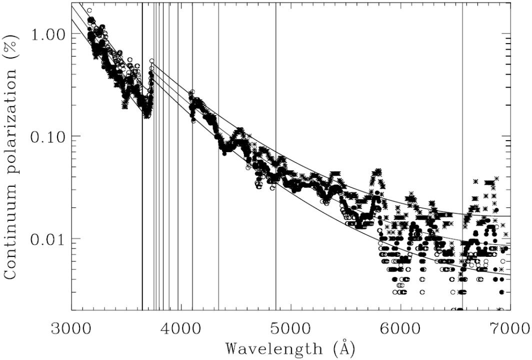
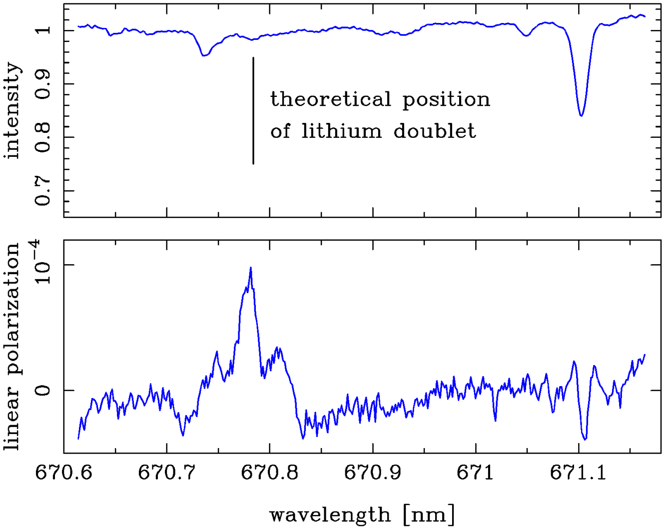
Solar Continuum Scattering Polarization



(from Stenflo 2005)

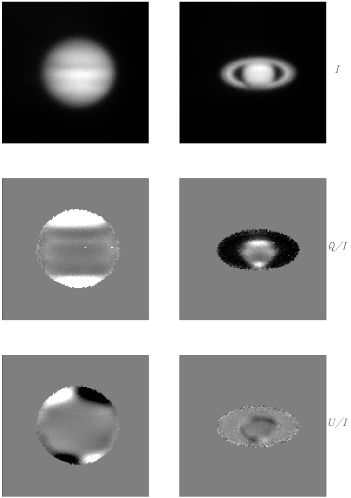
- due to anisotropy of the radiation field
- anisotropy due to limb darkening
- limb darkening due to decreasing temperature with height
- last scattering approximation without radiative transfer

Solar Spectral Line Scattering Polarization



resonance lines exhibit “large” scattering polarization signals

Jupiter and Saturn



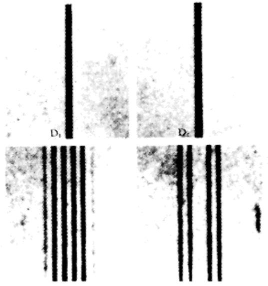
(courtesy H.M.Schmid and D.Gisler)

Planetary Scattered Light

- Jupiter, Saturn show scattering polarization
- multiple scattering changes polarization as compared to single scattering
- much depends on cloud height
- equivalent effect to study extrasolar planetary systems
- NWO/VICI instrument development here in Utrecht

Zeeman Effect

photos.aip.org/



Splitting/Polarization of Spectral Lines

- discovered in 1896 by Dutch physicist Pieter Zeeman
- different spectral lines show different splitting patterns
- splitting proportional to magnetic field
- split components are polarized
- *normal Zeeman effect* with 3 components explained by H.A.Lorentz using classical physics
- splitting of sodium D doublet could not be explained by classical physics (*anomalous Zeeman effect*)
- quantum theory and electron's intrinsic spin led to satisfactory explanation

Quantum-Mechanical Hamiltionian

- classical interaction of magnetic dipol moment $\vec{\mu}$ and magnetic field given by magnetic potential energy

$$U = -\vec{\mu} \cdot \vec{B}$$

  $\vec{\mu}$ the magnetic moment and $\vec{B}$ the magnetic field vector
- magnetic moment of electron due to orbit and spin
- Hamiltonian for quantum mechanics

$$H = H_0 + H_1 = H_0 + \frac{e}{2mc}\left(\vec{L} + 2\vec{S}\right)\vec{B}$$

H_0 Hamiltonian of atom without magnetic field
H_1 Hamiltonian component due to magnetic field
e charge of electron
m electron rest mass
$\vec{L}$ the orbital angular momentum operator
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Energy States in a Magnetic Field

- energy state $\langle E_{NLSJ}|$ characterized by
  - main quantum number N of energy state
  - $L(L+1)$, the eigenvalue of $\vec{L}^2$
  - $S(S+1)$, the eigenvalue of $\vec{S}^2$
  - $J(J+1)$, the eigenvalue of $\vec{J}^2$, $\vec{J} = \vec{L} + \vec{S}$ being the total angular momentum
  - M, the eigenvalue of J_z in the state $\langle NLSJM|$
- for the magnetic field in the z-direction, the change in energy is given by

$$\Delta E_{NLSJ}(M) = \langle NLSJM|H_1|NLSJM\rangle$$

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The Landé g Factor

- based on pure mathematics (group theory, Wiegner Eckart theorem), one obtains

$$\Delta E_{NLSJ}(M) = \mu_0 g_L B M$$

with $\mu_0 = \frac{e\hbar}{2m}$ the Bohr magneton, and g_L the Landé g-factor

- in LS coupling where B sufficiently small compared to spin-orbit splitting field

$$g_L = 1 + \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)}$$

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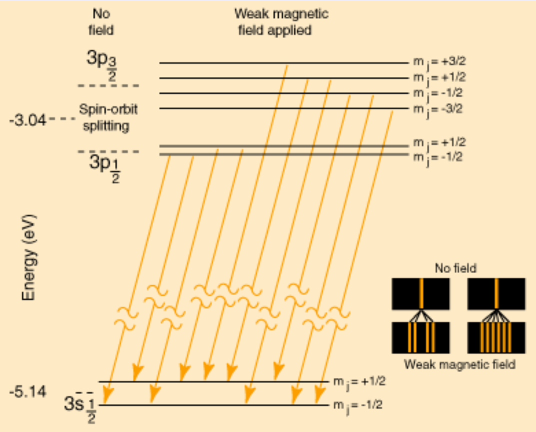


hyperphysics.phy-astr.gsu.edu/hbase/quantum/sodzee.html

Spectral Lines - Transitions between Energy States

- spectral lines are due to transitions between energy states:

  lower level with $2J_l + 1$ sublevels M_l

  upper level with $2J_u + 1$ sublevels M_u

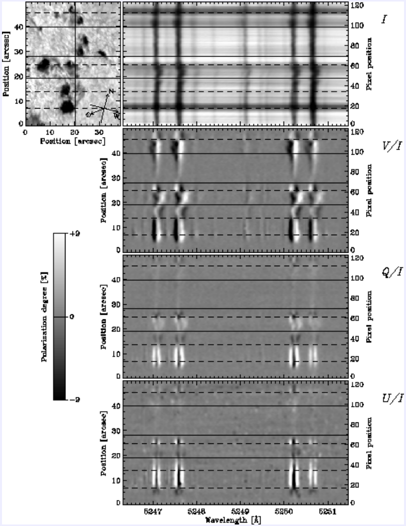
- not all transitions occur

Selection rule

- not all transitions between two levels are allowed
- assuming dipole radiation, quantum mechanics gives us the *selection rules*:
  - $L_u - L_l = \Delta L = \pm 1$
  - $M_u - M_l = \Delta M = 0, \pm 1$
  - $M_u = 0$ to $M_l = 0$ is forbidden for $J_u - J_l = 0$
- total angular momentum conservation: photon always carries $J_{\rm photon} = 1$
- *normal Zeeman effect*: line splits into three components because
  - Landé g-factors of upper and lower levels are identical
  - $J_u = 1$ to $J_l = 0$ transition
- *anomalous Zeeman effect* in all other cases

Effective Landé Factor and Polarized Components

- each component can be assigned an effective Landé g-factor, corresponding to how much the component shifts in wavelength for a given field strength
- components are also grouped according to the linear polarization direction for a magnetic field perpendicular to the line of sight
  - π components are polarized parallel to the magnetic field (**p**i for *parallel*)
  - σ components are polarized perpendicular to the magnetic field (**s**igma for German *senkrecht*)
- for a field parallel to the line of sight, the π-components are not visible, and the σ components are circularly polarized

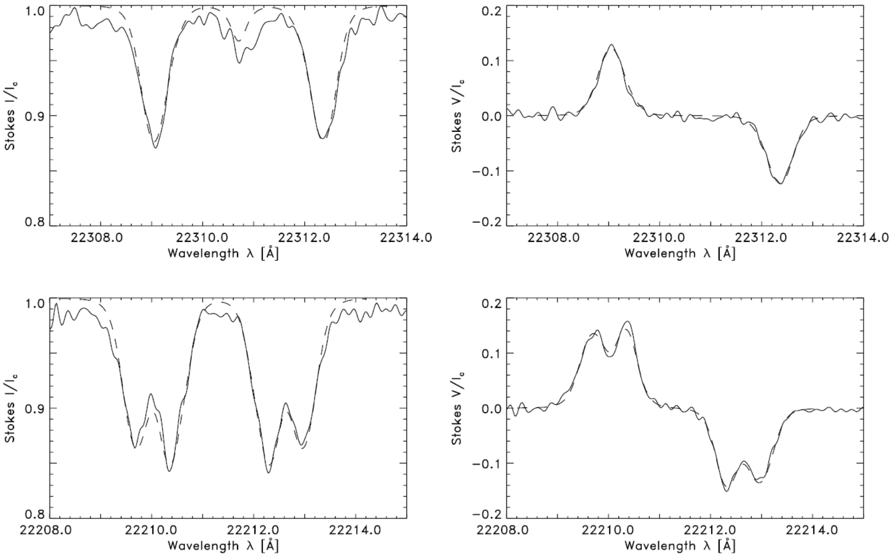


Bernasconi et al. 1998

Zeeman Effect in Solar Physics

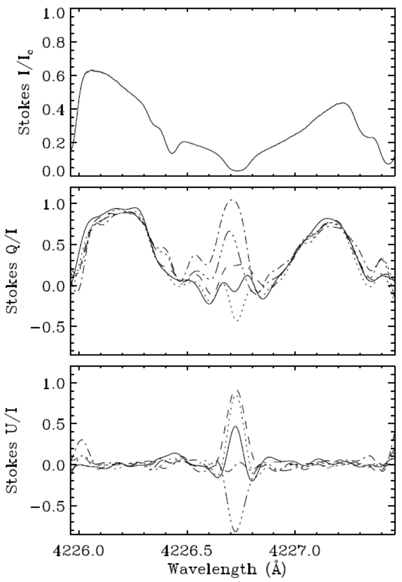
- discovered in sunspots by G.E.Hale in 1908
- splitting small except for in sunspots
- much of intensity profile due to non-magnetic area $\Rightarrow$ filling factor
- a lot of strong fields outside of sunspots
- full Stokes polarization measurements are key to determine solar magnetic fields
- 180 degree ambiguity

Fully Split Titanium Lines at 2.2μm



Rüedi et al. 1998

Hanle Effect



Bianda et al. 1998

Depolarization and Rotation

- scattering polarization modified by magnetic field
- precession around magnetic field depolarizes and rotates polarization
- sensitive $\sim 10^3$ times smaller field strengths that Zeeman effect
- measureable effects even for isotropic field vector orientations

Polarized Radiative Transfer

Intensity Radiative Transfer in LTE

radiative transfer equation for intensity

$$\cos\theta \frac{dI_\nu}{d\tau_c} = (1 + \eta_\nu)(I_\nu - B_\nu)$$

with τ_c the continuum optical depth, η the ratio of spectral line absorption coefficient to continuum absorption coefficient.

Polarized Radiative Transfer in LTE

radiative transfer equation for Stokes vector

$$\cos\theta \frac{d\vec{I}}{d\tau_c} = (1 + \eta)\left(\vec{I} - \vec{B}_\nu\right)$$

with $\vec{B}_\nu^T = (B_\nu, 0, 0, 0)$ and absorption matrix η.

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Absorption Matrix for Zeeman Effect

$$\eta = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_v & \eta_I & \rho_Q \\ \eta V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}$$

with

$$\begin{aligned} \eta_I &= \frac{1}{2}\eta \sin^2\gamma + \frac{1}{4}\left(\eta^+ + \eta^-\right)\left(1 + \cos^2\gamma\right) \\ \eta_Q &= \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma \cos 2\phi \\ \eta_U &= \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma \sin 2\phi \\ \eta_V &= \frac{1}{2}\left(\eta^+ - \eta^-\right)\cos\gamma \end{aligned}$$

with magnetic field inclination γ and azimuth ϕ