Outline

Fundamentals of Polarized Light

Electromagnetic Waves in Matter

- electromagnetic waves are a direct consequence of *Maxwell's equations*
- optics: interaction of electromagnetic waves with matter as described by *material equations*
- polarization properties of electromagnetic waves are integral part of optics

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Maxwell's Equations in Matter

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{j}$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \cdot \vec{B} = 0$$

Symbols

- $\vec{D}$ *electric displacement*
- ρ *electric charge density*
- $\vec{H}$ *magnetic field vector*
- c *speed of light in vacuum*
- $\vec{j}$ *electric current density*
- $\vec{E}$ *electric field vector*
- $\vec{B}$ *magnetic induction*
- t time

Linear Material Equations

$$\vec{D} = \epsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$
$$\vec{j} = \sigma \vec{E}$$

Symbols

- ϵ *dielectric constant*
- μ *magnetic permeability*
- σ *electrical conductivity*

Isotropic and Anisotropic Media

- ϵ and μ are scalars for isotropic media
- ϵ and μ tensors of rank 2 for anisotropic media
- isotropy of medium can be broken by
  - anisotropy of material itself (e.g. crystals)
  - external fields (e.g. Kerr effect)

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Wave Equation in Matter

- static, homogeneous medium with no net charges ($\rho = 0$)
- for most materials $\mu = 1$
- combination of Maxwell's and material equations leads to differential equations for a damped (vector) wave

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{E}}{\partial t} = 0$$

$$\nabla^2 \vec{H} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{H}}{\partial t} = 0$$

- $\vec{E}$ and $\vec{H}$ are equivalent
- interaction with matter almost always through $\vec{E}$
- but: at interfaces, boundary conditions for $\vec{H}$ are crucial
- damping controlled by conductivity σ

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Plane-Wave Solutions

Plane Vector Wave ansatz

$$\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

- $\vec{k}$ spatially and temporally constant *wave vector*
- $\vec{k}$ normal to surfaces of constant phase
- $|\vec{k}|$ *wave number*
- $\vec{x}$ spatial location
- ω *angular frequency* ($2\pi\times$ frequency)
- t time
- $\vec{E}_0$ a (generally complex) vector independent of time and space

- damping if $\vec{k}$ is complex
- real electric field vector given by real part of $\vec{E}$

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Complex index of refraction

- after doing temporal derivatives ⇒ Helmholtz-equation

$$\nabla^2\vec{E} + \frac{\omega^2\mu}{c^2}\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)\vec{E} = 0,$$

- *dispersion relation* between $\vec{k}$ and ω

$$\vec{k}\cdot\vec{k} = \frac{\omega^2\mu}{c^2}\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)$$

- *complex index of refraction*

$$\tilde{n}^2 = \mu\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right), \quad \vec{k}\cdot\vec{k} = \frac{\omega^2}{c^2}\tilde{n}^2$$

- split index into purely real and imaginary parts:

$$\tilde{n} = n + ik$$

n: (real) index of refraction, k: extinction coefficient

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Transverse Waves

- plane-wave solution must also fulfill Maxwell's equations

$$\vec{E}_0 \cdot \vec{k} = 0, \;\; \vec{H}_0 \cdot \vec{k} = 0$$
$$\vec{H}_0 = \frac{\tilde{n}}{\mu} \frac{\vec{k}}{|\vec{k}|} \times \vec{E}_0$$

- isotropic media: electric, magnetic field vectors normal to wave vector $\Rightarrow$ transverse waves
- $\vec{E}_0$, $\vec{H}_0$, and $\vec{k}$ orthogonal to each other, right-handed vector-triple
- conductive medium $\Rightarrow$ complex $\tilde{n}$, $\vec{E}_0$ and $\vec{H}_0$ out of phase
- $\vec{E}_0$ and $\vec{H}_0$ have constant relationship $\Rightarrow$ consider only one of two fields

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Energy Propagation

- *Poynting vector*

$$\vec{S} = \frac{c}{4\pi}\left(\vec{E} \times \vec{H}\right)$$

- $|\vec{S}|$: energy through unit area perpendicular to $\vec{S}$ per unit time
- direction of $\vec{S}$ is direction of energy flow
- time-averaged Poynting vector given by

$$\left\langle \vec{S} \right\rangle = \frac{c}{8\pi}\mathrm{Re}\left(\vec{E}_0 \times \vec{H}_0^*\right) ,$$

Re real part of complex expression
$*$ complex conjugate
$\langle . \rangle$ time average

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Polarization

- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_1$ and $\vec{e}_2$, both perpendicular to $\vec{k}$

$$\vec{E}_0 = E_1\vec{e}_1 + E_2\vec{e}_2.$$

  E_1, E_2: arbitrary complex scalars
- damped plane-wave solution with given ω, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called *polarization*
- many ways to represent these four quantities
- if E_1 and E_2 have identical phases, $\vec{E}$ oscillates in fixed plane

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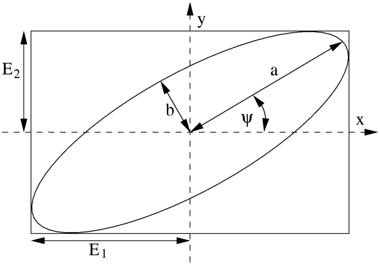
Description of Polarized Light

Polarization Ellipse



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$$\vec{E}_0 = E_1 e^{i\delta_1}\vec{e}_x + E_2 e^{i\delta_2}\vec{e}_y$$

- wave vector in z-direction
- $\vec{e}_x$, $\vec{e}_y$: unit vectors in x, y directions
- E_1, E_2: (real) amplitudes
- $\delta_{1,2}$: (real) phases

Polarization Description

- 2 complex scalars not the most useful description
- at given $\vec{x}$, time evolution of $\vec{E}$ described by *polarization ellipse*
- ellipse described by axes a, b, orientation ψ

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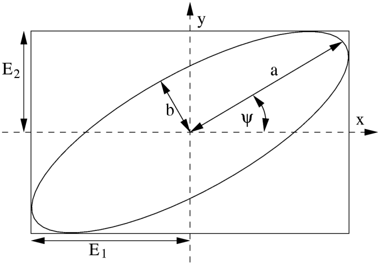

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Jones Formalism

Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

- beam in z-direction
- $\vec{e}_x$, $\vec{e}_y$ unit vectors in x, y-direction
- complex scalars $E_{x,y}$
- Jones vector

$$\vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- phase difference between E_x, E_y multiple of π, electric field vector oscillates in a fixed plane $\Rightarrow$ *linear polarization*
- phase difference $\pm\frac{\pi}{2}$ $\Rightarrow$ *circular polarization*

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Summing and Measuring Jones Vectors

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- Maxwell's equations linear $\Rightarrow$ sum of two solutions again a solution
- Jones vector of sum of two waves $=$ sum of Jones vectors of individual waves if wave vectors $\vec{k}$ the same
- addition of Jones vectors: *coherent* superposition of waves
- elements of Jones vectors are not observed directly
- observables always depend on products of elements of Jones vectors, i.e. intensity

$$I = \vec{e} \cdot \vec{e}^* = e_x e_x^* + e_y e_y^*,$$

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Jones matrices

- influence of medium on polarization described by 2×2 complex *Jones matrix* J

$$\vec{e}' = \mathsf{J}\vec{e} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \vec{e} \ .$$

- assumes that medium not affected by polarization state
- different media 1 to N in order of wave direction, combined influence described

$$\mathsf{J} = \mathsf{J}_N \mathsf{J}_{N-1} \cdots \mathsf{J}_2 \mathsf{J}_1$$

- order of matrices in product is crucial
- Jones calculus describes coherent superposition of polarized light

Quasi-Monochromatic Light

- monochromatic light: purely theoretical concept
- monochromatic light wave always fully polarized
- real life: light includes range of wavelengths $\Rightarrow$ *quasi-monochromatic light*
- quasi-monochromatic: superposition of mutually incoherent monochromatic light beams whose wavelengths vary in narrow range $\delta\lambda$ around central wavelength λ_0

$$\frac{\delta\lambda}{\lambda} \ll 1$$

- measurement of quasi-monochromatic light: integral over measurement time t_m
- amplitude, phase (slow) functions of time for given spatial location
- *slow*: variations occur on time scales much longer than the mean period of the wave

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Polarization of Quasi-Monochromatic Light

- electric field vector for quasi-monochromatic plane wave is sum of electric field vectors of all monochromatic beams

$$\vec{E}(t) = \vec{E}_0(t)\, e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)} \; .$$

- can write this way because $\delta\lambda \ll \lambda_0$
- measured intensity of quasi-monochromatic beam

$$\left\langle \vec{E}_x \vec{E}_x^* \right\rangle + \left\langle \vec{E}_y \vec{E}_y^* \right\rangle = \lim_{t_m -> \infty} \frac{1}{t_m} \int_{-t_m/2}^{t_m/2} \vec{E}_x(t)\vec{E}_x^*(t) + \vec{E}_y(t)\vec{E}_y^*(t) dt \; ,$$

  $\langle \cdots \rangle$: averaging over measurement time t_m.
- measured intensity independent of time
- Quasi-monochromatic: frequency-dependent material properties (e.g. index of refraction) are constant within $\Delta\lambda$

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Stokes and Mueller Formalisms

Stokes Vector

- need formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i \left(E_x E_y^* - E_y E_x^* \right) \end{pmatrix} = \begin{pmatrix} E_1^2 + E_2^2 \\ E_1^2 - E_2^2 \\ 2E_1 E_2 \cos\delta \\ 2E_1 E_2 \sin\delta \end{pmatrix}$$

  Jones vector elements $E_{x,y}$, real amplitudes $E_{1,2}$, phase difference $\delta = \delta_2 - \delta_1$

- $$I^2 \geq Q^2 + U^2 + V^2 .$$

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Stokes Vector Interpretation

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

- *degree of polarization*

$$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

  1 for fully polarized light, 0 for unpolarized light
- summing of Stokes vectors = *incoherent* adding of quasi-monochromatic light waves

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Mueller Matrices

- 4×4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

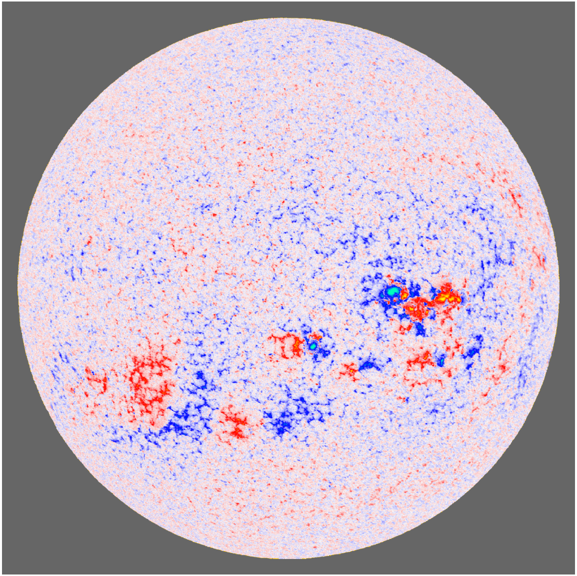
$$\vec{I}' = \mathrm{M}\vec{I}\,,$$

$$\mathrm{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

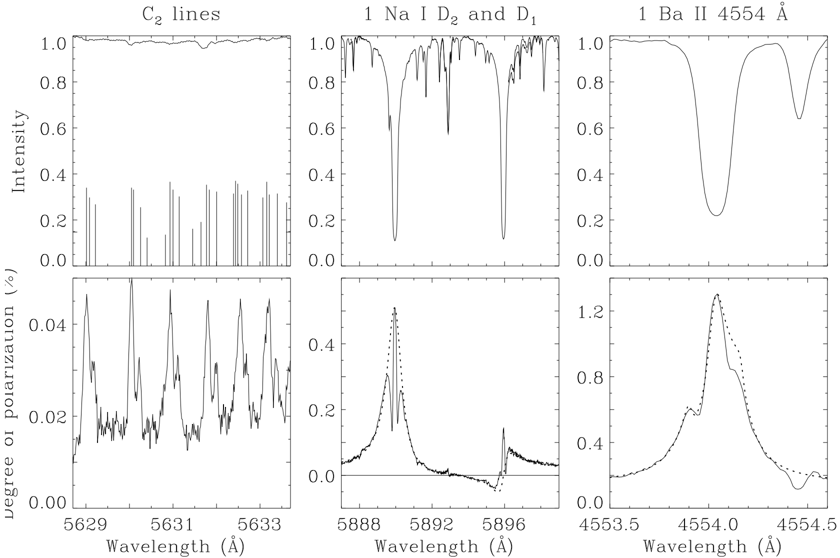
- N optical elements, combined Mueller matrix is

$$\mathrm{M}' = \mathrm{M}_N \mathrm{M}_{N-1} \cdots \mathrm{M}_2 \mathrm{M}_1$$

Magnetic Field Maps from Longitudinal Zeeman Effect



Second Solar Spectrum from Scattering Polarization



SOLIS Vector-Spectro-Magnetograph (VSM)

Science Goals

Provide unique observations to understand

- the **solar activity cycle**
- **sudden energy releases** in the solar atmosphere (flares, coronal mass ejections)
- solar **irradiance changes** and relationship to global change

Magnetic field

- Line-of-sight component of photospheric magnetic field: Averaged over 2 Mm^2, sensitivity = 1 gauss, zero point stable to 0.1 gauss, time for a full disk map = 15 minutes
- Transverse component of the photospheric magnetic field: Same parameters as line-of-sight component except sensitivity $\geq$ 20 gauss.

Science Requirements

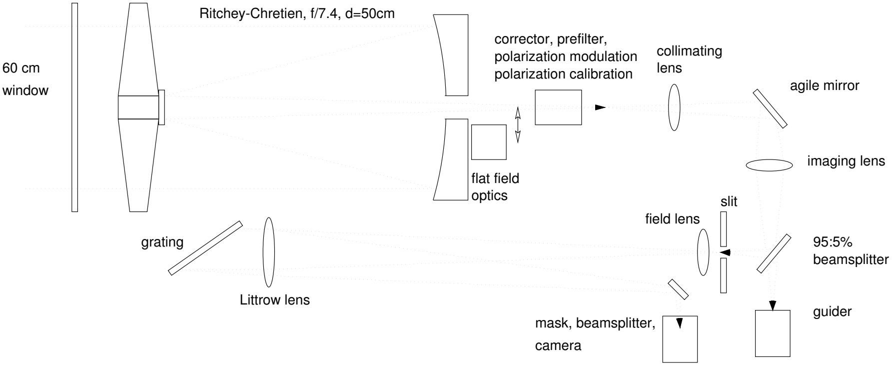
Parameter	Specification
angular element	$1\farcs125$ by $1\farcs125$
angular coverage	$2048''$ by $2048''$
geometric accuracy	$<0\farcs5$ rms after remapping
motion in RA	$\pm 0.25^\circ$ for flat-fielding
scan rate in Dec	0.2-5.0 s/$''$
timing accuracy	better than 1 ms
spectral resolution	200,000
wavelength ranges	630.2 $\pm$0.1 nm
polarimetry	630.2 nm: I,Q,U,V
polarimetric sensitivity	0.0002 per pixel in 0.5 s
polarimetric accuracy	0.001
image stabilization	$>$40 Hz to improve spatial resolution

Design Challenges

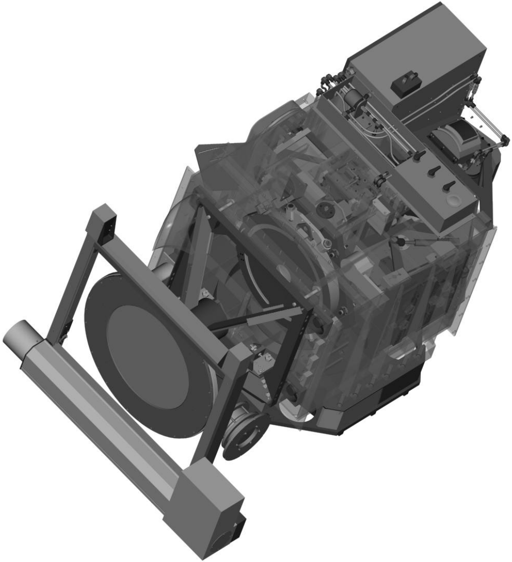
- compact instrument no longer than 2.5 m
- athermal optical design that is stable at varying ambient temperatures
- high guiding accuracy of better than $0\overset{\prime\prime}{.}5$ rms
- low instrumental polarization of less than $1 \cdot 10^{-3}$
- large wavelength range (630 to 1090 nm) with constant magnification
- high spectral resolution of 200,000
- highest possible throughput
- high energy densities of up to 20 W/cm^2
- high data rate of up to 300 MByte/s

Concept in Proposal

Vector Spectromagnetograph

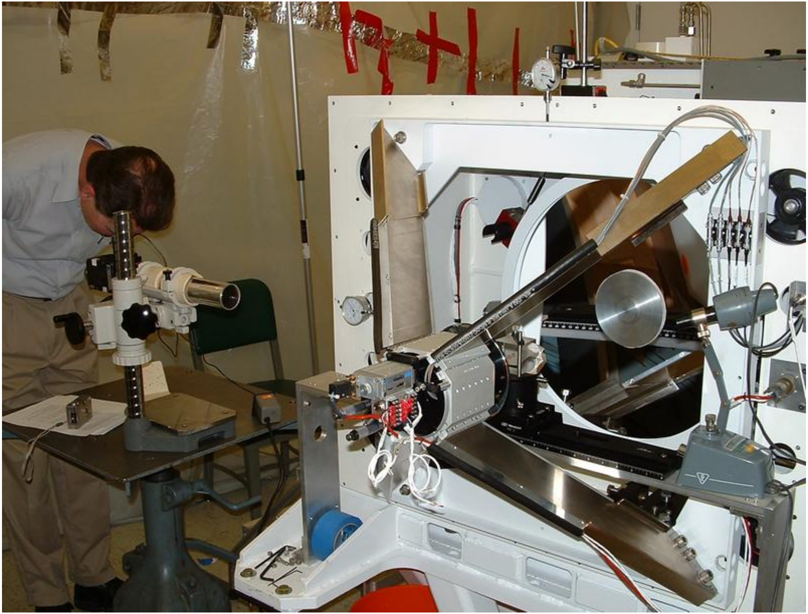


On the Computer



From the Welders

Aligning the Optics



Ready for Science