Solar Physics 2005-2006 Practicum
Stokes Inversion

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1 Introduction

This practicum introduces you to the processing of raw polarization spectra to magnetic field vector maps, one of the most important tasks in current solar physics research. The data that are used come from a brand-new instrument that provides the best full-disk magnetic field measurements in the world. While the practicum follows the overall processing strategy, some aspects of the data reduction are simplified as compared to the actual data reduction pipeline.

2 Useful Information

2.1 Brief Instrument Description

The data used here come from the SOLIS VSM instrument (see `solis.nso.edu`). The VSM combines a 50-cm telescope, a polarimeter based on two ferroelectric liquid crystal retarders, and a spectrograph. The grating in the spectrograph has 79 lines per mm and is used in the 36th diffraction order for the 630.2 nm wavelength range. The average angle of the grating for 630.2 nm is 63.69°. The angle between incoming and diffracted beam is 2.8°, and the focal length of the spectrograph is 1120 mm. The spectra are recorded with CMOS Hybrid detectors with 18 μm pixels running at almost 100 frames per second. The liquid crystals switch between four different states that correspond to four linearly independent combinations of Stokes parameters. The spectrograph slit is scanned across the solar disk to measure all four Stokes parameters over the full solar disk. Much of the practicum is based on a single slit location.

2.2 Data

Suitable data for this practicum are available at
`www.astro.uu.nl/~keller/Teaching/SolarPhysics2006/Practicum/Stokes/`. The following table lists the file names, sizes, and content. All files are in the standard FITS file format.

file name	dimensions	contents	comment
`scan.fits`	$2048 \times 128 \times 4$	VSM vector spectrum	2048 spatial points, 128 spectral points, 4 polarization modulation states
`dark.fits`	$2048 \times 256 \times 4$	dark current	use first 128 spectral points, all 4 polarization modulation states
`flat.fits`	$2048 \times 256 \times 4$	flat field	use first 128 spectral points, only the first image contains the flat field
`calib.fits`	2048×33	calibration matrix	2048 spatial points, use first 16 elements for 4×4 matrix

FITS files contain a header and the data. The header is in text format at the beginning of the file. Therefore, a text editor or the `more` command in Unix can be used to look at the information regarding the data in the rest of the file. The data themselves are in binary format and show up as strange characters in a text window.

2.3 Useful Values

The following table lists the wavelengths and effective Landé g-factors for the lines used here.

ion	wavelength [nm]	$g_{\rm eff}$
FeI	630.1509	1.669
O_2	630.2000	—
FeI	630.2502	2.487
O_2	630.2764	—

The angular size of the pixels on the sky is 1.125 arcsec.

2.4 Useful Equations

2.4.1 Grating Equation

Dispersion of light by a grating is described by the *grating equation*

$$m\lambda = d\,(\sin\alpha + \sin\beta) \tag{1}$$

where m is the *diffraction order*, λ is the wavelength of the light, d is the groove spacing, α is the angle between the grating normal and the incident beam, and β is the same for the diffracted beam.

2.4.2 Detector response

The relation between the number of incoming photons, S, and the measured electrical signal, E, delivered by the detector can be approximated with a linear relation

$$E = S * F + D \tag{2}$$

where D is called the *dark current*, and F is called the *flat field*. As can be seen from the above equation, if no light falls onto the detector, the measured signal corresponds to D. If the illumination has no spatial structure (it is flat), the measured signal, corrected for the dark current, corresponds to the flat field F.

2.4.3 Signal matrix

We combine the measured intensities into a signal vector $\vec{S}$, which is related to the incoming Stokes vector, $\vec{I}$ by the signal matrix X, where

$$\vec{S} = \mathsf{X}\vec{I}\,. \tag{3}$$

X is a 4 by 4 matrix. X is a function of the polarimeter design. Since the polarimeter optics can be described by Mueller matrices, each row of X corresponds to the first row of the Mueller matrix describing the particular intensity measurement.

To determine the Stokes vector $\vec{I}$ from the measurements $\vec{S}$, X needs to be inverted, i.e.

$$\mathsf{Y} = \mathsf{X}^{-1}\,, \tag{4}$$

The calibration file contains the inverted matrix, Y, as a function of the position along the spectrograph slit.

2.4.4 Zeeman splitting

The change in wavelength of a spectral line due to the normal Zeeman effect is given by

$$\Delta\lambda = 4.7 \cdot 10^{-13} \lambda^2 g B \tag{5}$$

The separation of the σ-components is therefore given by $2 \cdot \Delta\lambda$. $\Delta\lambda$ and the wavelength of the spectral line λ are expressed in Ångström 1 Å$= 10^{-10}$ m), the magnetic field strength B in Gauss.

2.4.5 Polarized Zeeman Profiles

From the equations of polarized radiative transfer and the absorption coefficients for the normal Zeeman effect, we have

$$\eta_I = \frac{1}{2}\eta \sin^2\gamma + \frac{1}{4}\left(\eta^+ + \eta^-\right)\left(1 + \cos^2\gamma\right) \tag{6}$$

$$\eta_Q = \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma\cos 2\phi \tag{7}$$

$$\eta_U = \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma\sin 2\phi \tag{8}$$

$$\eta_V = \frac{1}{2}\left(\eta^+ - \eta^-\right)\cos\gamma \tag{9}$$

with magnetic field inclination γ and azimuth ϕ.

Neglecting the radiative transfer effects, we can use these relations to model the polarized line profiles by assuming that the absorption coefficients correspond to line profiles and that

$$\eta^{\pm} = \eta\left(\lambda \pm \delta\lambda\right) \tag{10}$$

where λ is the center wavelength of the spectral line without magnetic fields, and $\Delta\lambda$ is the Zeeman splitting, and η is the spectral line intensity profile without a magnetic field.

2.5 Chi Squared

To automatically fit artificial Stokes profiles to an observed set of Stokes profiles, one needs to describe the 'fitting' in terms of equations. The classical approach to do this is to minimize a *merit function* such as the sum of differences squared between observed data and model predictions.

The merit function is then defined as

$$\chi^2 = \sum_i^m \left(y_i - Y_i\right)^2 \tag{11}$$

where m is the number of measured points, y_i are the observed points, and Y_i are the artificial data points as calculated with a model. The model has n free parameters a_j.

For the best fit, we obviously require

$$\frac{\partial\chi^2}{\partial a_j} = 0 \tag{12}$$

2.6 Gradient Search

If we write the free parameters of the model a_j as a vector $\vec{a}$, then we try to determine $\vec{\delta a}$ such that

$$\chi^2\left(\vec{a} + \vec{\delta a}\right) < \chi^2\left(\vec{a}\right) \tag{13}$$

A simple way to achieve this is to determine the gradient of χ^2 with respect to the free parameters a_j. The gradient is defined as

$$\nabla\chi^2 = \sum_{j=1}^{m} \frac{\partial\chi^2}{\partial a_j}\vec{e}_j \tag{14}$$

where the $\vec{e}_j$ are unit vectors in the direction of a_j. The components of this gradient vector are then given by

$$\left(\nabla\chi^2\right)_j = \frac{\partial\chi^2}{\partial a_j} \approx \frac{\chi^2\left(a_j + f\Delta a_j\right) - \chi^2\left(a_j\right)}{f\Delta a_j} \tag{15}$$

where we have used a discretized version of a derivative. The Δa_j are the step sizes that we want to take, and f is on the order of 0.1 to obtain an reasonably accurate approximation of the derivative.

The gradient has both magnitude and units, and the units for the different parameters do not have to be the same (e.g. Gauss for B, and degrees for γ and ϕ. We therefore define a unit-less gradient vector $\vec{g}$ with magnitude unity that has the components

$$g_i = \frac{\frac{\partial\chi^2}{\partial a_j}\Delta a_j}{\sqrt{\sum_{k=1}^{n}\left(\frac{\partial\chi^2}{\partial a_k}\Delta a_k\right)^2}} \tag{16}$$

and the final step to be taken is given by

$$\delta a_j = -g_j\Delta a_j \tag{17}$$

Note that you define the step sizes Δa_j, and these values should be commensurate with the magnitudes of a_j. For instance, since the field strength is on the order of 1000 Gauss, we might choose a step size of about 10 Gauss.

2.7 Useful IDL routines and commands

The following IDL routines may be useful for this practicum.

IDL routine	brief description	usage
readfits()	reads FITS formatted files	`Result = READFITS(Filename)`

3 Observed Stokes Profiles

In this section of the practicum, you will carry out the basic data reduction steps to obtain spectra of Stokes parameters and use them to do a simple field strength estimate. You will also learn why this simple estimate is very misleading.

1. Check out the 4 spectra in the data file, which correspond to 4 linearly independent combinations of Stokes vectors. Make an image showing all four at the same time.

2. Correct the measured signal for dark current and flat field. Figure out a way to deal with bad pixels.

3. Apply the calibration matrix to obtain 4 spectra corresponding to the four Stokes parameters I, Q, U, and V. make an image showing all four Stokes spectra.

4. Figure out the wavelength scale, i.e. the wavelength of each pixel in the spectral direction. The wavelength interval per pixel is called the *linear dispersion*. Compare it to what you would expect based on the grating equation and the focal length of the spectrograph.

5. Measure the wavelength of the deepest point of one of the iron lines at several locations along the slit. Note that the absolute wavelength scale can change along the slit. Explain the result.

6. Select one spatial location that contains interesting Stokes profiles and plot all four Stokes parameters as a function of wavelength.

7. Measure the splitting at several locations in V, Q, and U and use it to estimate the field strength.

8. Discuss the field strength estimates and explain why you probably did not measure the field strength. Hint: write Stokes V in terms of a difference of two shifted profiles and simplify it with a Taylor expansion in $\Delta\lambda$.

4 Artificial Stokes Profiles

To understand the observed Stokes profiles, we will play with simple artificial Stokes profiles. We will use them to better understand the relation between magnetic field vector and Stokes profiles.

1. Write an IDL routine based on the simplified equations for the polarized line profiles above that calculates spectral line profiles in all four Stokes parameters as a function of magnetic field strength B, inclination γ, azimuth χ, as well as line depth, line width, and Doppler shift of the non-magnetic intensity profile. Use a Gaussian to approximate the line profile.

2. Write an IDL routine that shows observed profiles along with artificial Stokes profiles so that you can easily compare them by eye.

3. For $\gamma = 45°$, $\chi = 0$, plot the maximum V and Q signals as a function of field strength from 0 to 4000 Gauss. What is the approximate relation between field strength and maximum signals, and how does it depend on the Landé g-factors of the lines?

4. Play with the magnetic field parameters to approximate the observed Stokes profiles at the spatial position you selected above. You will probably have to assume that not all of the spatial resolution element is filled with magnetic field and therefore have to model the observations with

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_{\rm obs} = (1-f) \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix}_{B=0} + f \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_{B\neq 0} \tag{18}$$

   where f is called the filling factor.

5 Inversion

You will most likely notice that it is not easy to choose the parameters such that the observed Stokes profiles are well matched. This is because of the inherently non-linear relation between the free parameters and the resulting artificial Stokes profiles. In this section, you will develop a simple, automatic procedure to find the parameters that result in the best match between observed and artificial Stokes profiles. This process is sometimes call *inversion*.

1. Write an IDL routine that calculates χ^2 for given observed Stokes profiles and a set of magnetic field parameters.

2. Write an IDL routine that calculates the derivative of χ^2 with respect to the magnetic field parameters. Use this routine a few times to improve a set of parameters.

3. Write an IDL routine that iteratively minimizes χ^2. Define a stopping criterium so that the code stops once a good fit has been achieved.

4. Start the gradient search with different start values. Do you get the same final results? If not, what could be a reason for that?

5. Use the built-in IDL routine `curvefit` to optimize the parameters. Compare the speed and quality of the fit with your gradient routine.