Solar Physics 2005-2006: Exercises to Lecture 6

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1 Polarization Ellipse

Show that at a given point $\vec{x}$, the time evolution of the electric field vector of an electromagnetic wave in an isotropic medium is described by an ellipse. Hint: Use the plane-wave ansatz,

$$\vec{E}\,(t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}\ , \tag{1}$$

with the polarization described as

$$\vec{E}_0 = E_1 e^{i\delta_1}\vec{e}_x + E_2 e^{i\delta_2}\vec{e}_y. \tag{2}$$

$\vec{e}_x$ and $\vec{e}_y$ are unit vectors in the x and y directions, respectively. The beam propagates along the z-axis. The coefficients E_1 and E_2 are the (real) amplitudes and $\delta_{1,2}$ are the phases.

2 Photons

Think of quasi-monochromatic, partially polarized light in terms of photons. Photons have spin 1, and since they have no mass, their spin needs to be parallel to the propagation direction. This means that a photon can be considered to be either left- or right-circularly polarized. Describe four independent properties of these photons that completely characterize the polarization properties.

3 Mueller Matrix

The most general Jones matrix describing the interaction of monochromatic light with matter has eight independent parameters. How many independent parameters does a Mueller matrix have that describes the same interaction of a polarized beam with matter?

4 The rotating mirror problem

Rotation of elements described by Mueller matrices are given by

$$\mathsf{M}' = \mathsf{R}(-\alpha)\mathsf{M}\mathsf{R}(\alpha) \ , \tag{3}$$

where α is the rotation angle and the rotation matrix R is given by

$$\mathsf{R}(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\alpha & \sin 2\alpha & 0 \\ 0 & -\sin 2\alpha & \cos 2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{4}$$

The Mueller matrix for an ideal mirror at normal incidence is given by

$$\mathsf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} , \tag{5}$$

Calculate the Mueller matrix of a mirror as a function of the rotation angle α around its normal. What's wrong and why?

5 Poincaré Sphere

Since a Stokes vector for a fully polarized beam obeys the following relationsship

$$I^2 = Q^2 + U^2 + V^2 \ , \tag{6}$$

we can think of this as the equation describing a sphere in cartesian coordinates labeled Q, U, and V, the *Poincaré Sphere*. The plane defined by the Q and U axes defines the equator, the poles correspond to circularly polarized light.

Describe what a linear retarder with a retardation of 180° (half-wave retarder) and a fast axis orientation of α acting on a linearly polarized beam with orientation β does on the Poincaré sphere.

Can you find other properties that are particularly easy to do with the Poincaré sphere?