Lecture 10: Oscillations

Outline

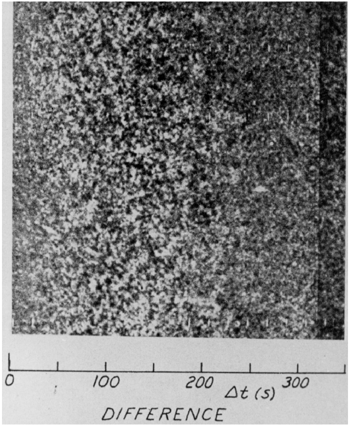
1. Observations
2. Adiabatic Oscillations
3. Helioseismology

Introduction

- Sun rings like a bell, but at many different frequencies
- acoustic waves with pressure as restoring force (*p-modes*)
- frequencies depend on internal structure and motions

Observations

5-Minute Oscillations



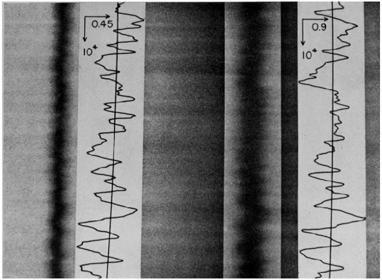
Leighton et al. (1962)

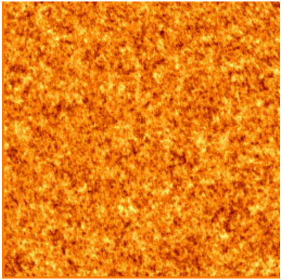
- discovered by R.Leighton in 1960
- spectroheliogram = scanned image at fixed wavelength
- Doppler plate: difference of intensity in blue and red wing:

$$I(\lambda + \Delta\lambda) - I(\lambda - \Delta\lambda) \approx 2\Delta\lambda \frac{\partial I(\lambda)}{\partial \lambda}$$

- Doppler difference plate from forward and backward scans

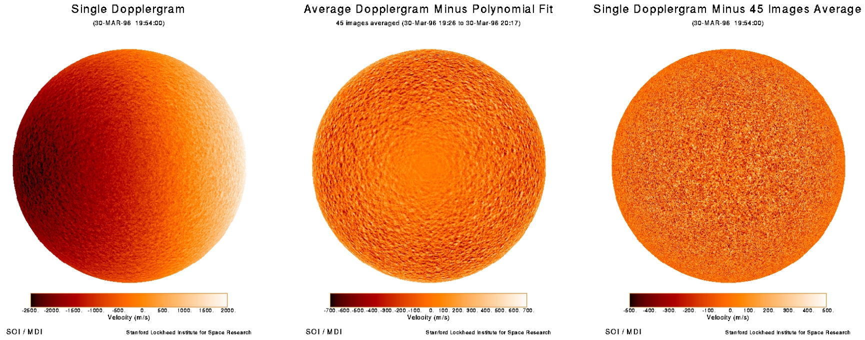
Spectral Observations





- direct measurements of spectral line shifts
- largely vertical oscillations
- amplitudes 0.5-1.0 km/s, increasing with height
- frequencies around 5 minutes dominate in the photosphere, 3 minutes in chromospheric lines
- little phase lag between different heights
- wave numbers from solar diameter to smallest resolvable scales

Solar Oscillations and Supergranulation with SOHO/MDI



Temporal Spectrum of Oscillations

- observations over period T with sampling interval Δt
- temporal frequency resolution $\Delta\omega = 2\pi/T$
- lowest observable temporal frequency is $\Delta\omega$
- highest observable temporable frequency is $\omega_{\mathrm{Ny}} = \pi/\Delta t$
- anti-alias filtering required if frequencies $> \omega_{\mathrm{Ny}}$ exist

Spatial Spectrum of Oscillations

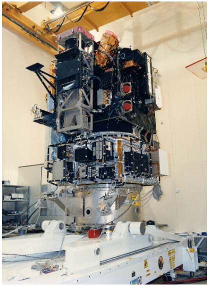
- observations over area L_x with sampling interval Δx
- spatial frequency resolution $\Delta k_x = 2\pi/L_x$
- lowest observable spatial frequency is Δk_x
- highest observable spatial frequency is $k_{\mathrm{Ny}} = \pi/\Delta x$
- anti-alias filtering required if frequencies $> k_{\mathrm{Ny}}$ exist

Long-Term Observations

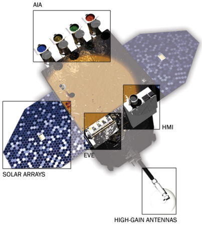
- high temporal frequency resolution requires long observing periods
- day-night cycle $\Rightarrow$ networks around the Earth and satellites
- GONG: Global Oscillation Network Group
- SOHO: GOLF, VIRGO, MDI
- now: Solar Dynamics Observatory

GONG

SOHO/MDI



SDO



Power Spectrum

- velocity signal as a function of space and time: $v(x, y, t)$
- 3-D Fourier transform with respect to x, y, t

$$f(k_x, k_y, \omega) = \int v(x, y, t) \mathrm{e}^{-i(k_x x + k_y y + \omega t)} dx\, dy\, dt$$

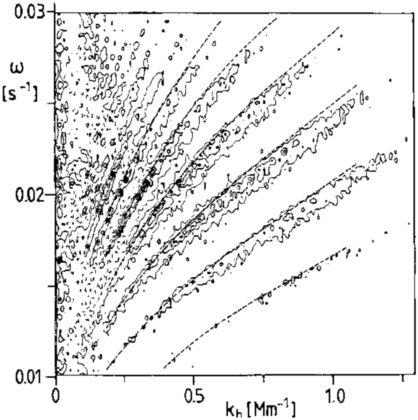
- can also be written as

$$v(x, y, t) = \int f(k_x, k_y, \omega) \mathrm{e}^{i(k_x x + k_y y + \omega t)} dk_x\, dk_y\, d\omega$$

- power spectrum $P(k_x, k_y, \omega) = f \cdot f^*$
- if no spatial direction is preferred: $k_h = \sqrt{k_x^2 + k_y^2}$

$$P(k_h, \omega) = \frac{1}{2\pi} \int_0^{2\pi} P(k_h \cos\phi, k_h \sin\phi, \omega) d\phi$$

k-ω Diagram



- power is concentrated into *ridges*
- ridges theoretically predicted by Ulrich in 1970
- first observed by Deubner in 1975
- pressure perturbations $\Rightarrow$ *p-modes*
- lowest (fundamental) mode $\Rightarrow$ *f-mode* (surface wave)

Whole-Sun Observations

- spherical coordinate system r, θ, ϕ
- velocity field in terms of spherical surface harmonics

$$v(\theta,\phi,t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm}(t) Y_l^m(\theta,\phi)$$

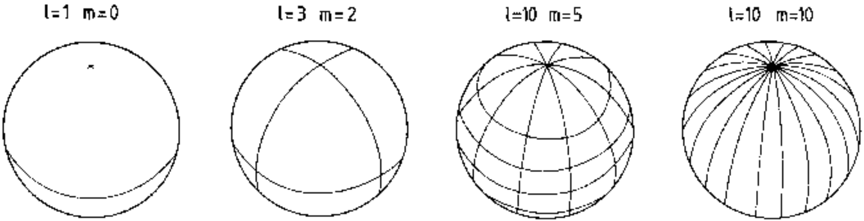
- $Y_l^m(\theta,\phi) = P_l^{|m|}(\theta) \mathrm{e}^{im\phi}$
- P_l^m: associated Legendre function

l and m

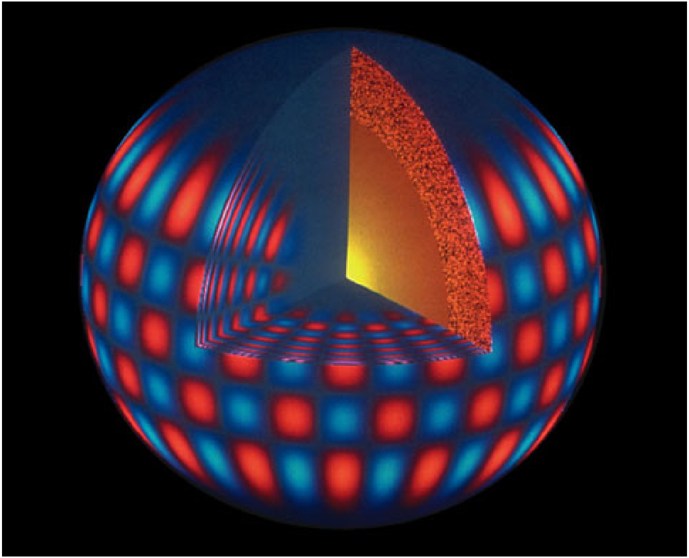
- velocity field in (complex) spherical harmonics

$$v(\theta,\phi,t)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}a_{lm}(t)P_l^{|m|}(\theta)\mathrm{e}^{im\phi}$$

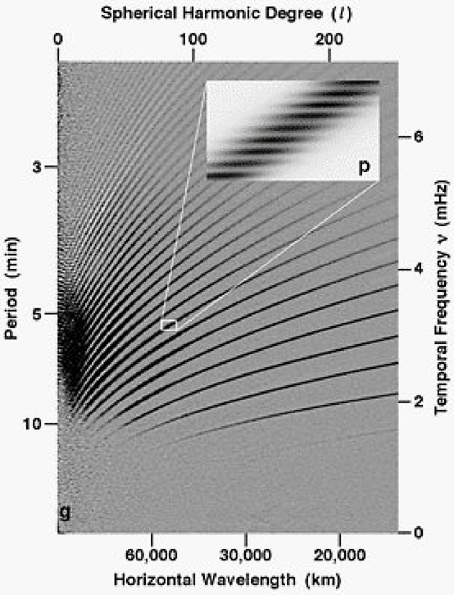
- degree l: total number of node circles on sphere
- longitudinal order m: number of node circles through poles
- rotation provides preferred direction
- rotation mostly minor effect $\Rightarrow m=0$ good approximation



Spherical Harmonics and Oscillations



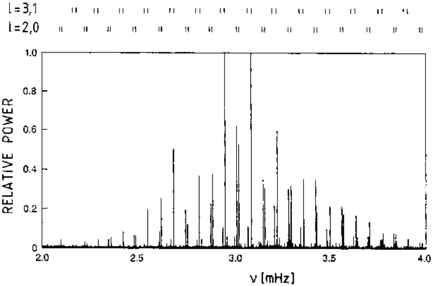
Spherical Power Spectrum



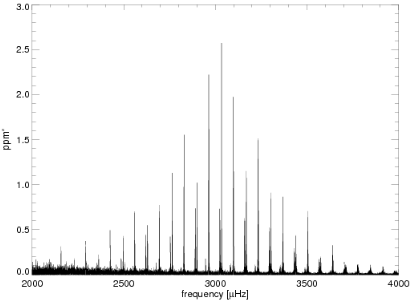
- l replaces k_h, $\nu = \omega/2\pi$ replaces ω
- $\hat{a}_l(\nu)$ is Fourier transform of $a_{l0}(t)$
- power in l-ν diagram given by $P(l,\nu) = \hat{a}_l(\nu)\hat{a}_l^*(\nu)$
- see only part of solar surface $\Rightarrow$ cannot resolve modes in spatial frequency
- but different l-modes have different frequencies
- single mode amplitudes: 30 cm/s or less
- interference of 10^7 modes provides 1 km/s

Low-Degree p Modes

- spatially unresolved Doppler shifts (Sun as a star)
- can only observe the lowest l modes in velocity from the ground and in intensity from space
- can now also detect this on bright stars



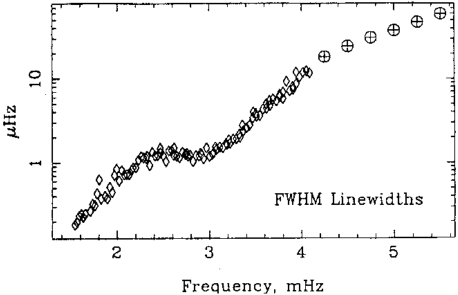
Bison velocity, theoretical frequencies



SOHO/VIRGO

Line Width

- solar oscillations lines have finite width
- line width determined by finite mode life time due to
  - damping mechanism
  - convective velocity field
- Lorentz profile identical to collisional broadening of spectral lines
- modes live from hours to months



Linear Adiabatic Oscillations

Basic Equations

- assume non-rotating gaseous sphere in hydrostatic equilibrium
- Euler's field description in fixed coordinate systems
- Lagrange's particle system in coordinates that flows with gas
- Lagrange (substantial derivative) and Euler descriptions related
- Lagrangian perturbation δ

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \left[\frac{\alpha(t+\Delta t) - \alpha(t)}{\Delta t}\right]_{\vec{\delta r}} = \frac{\partial\alpha}{\partial t} + \vec{v}\cdot\nabla\,\alpha$$

- steady flow: $\frac{\partial}{\partial t} = 0$ concept in Euler's description
- incompressible flow: $\frac{\mathrm{d}\rho}{\mathrm{d}t} = 0$ concept in Lagrange's description

Thermodynamics

- first law of thermodynamics

$$\frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t} + P\frac{\mathrm{d}V}{\mathrm{d}t}$$

q entropy
E energy
P pressure
V volume

- $V = 1/\rho$
- therefore

$$\rho\frac{\mathrm{d}q}{\mathrm{d}t} = \rho\frac{\mathrm{d}E}{\mathrm{d}t} - \frac{P}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}$$

Ideal Gas

- ideal gas

$$\delta E = c_v \, \delta T \qquad P = (c_p - c_v) \, \rho \, T \qquad P = (\gamma - 1) \, \rho \, E \quad \gamma = \frac{c_p}{c_v}$$

- adiabatic exponent $\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_{\mathrm{ad}}$
- first law of thermodynamics (from before)

$$\rho \frac{\mathrm{d}q}{\mathrm{d}t} = \rho \frac{\mathrm{d}E}{\mathrm{d}t} - \frac{P}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}t}$$

- first law of therodynamics for ideal gas

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\gamma P}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}t} + (\gamma - 1) \, \rho \frac{\mathrm{d}q}{\mathrm{d}t}$$

Adabatic Approximation

- first law of therodynamics for ideal gas

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\gamma P}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} + (\gamma - 1)\,\rho\,\frac{\mathrm{d}q}{\mathrm{d}t}$$

- adiabatic ($\delta q = 0$)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\gamma P}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}$$

- adiabatic approximation implies

$$\frac{\delta P}{P_0} = \Gamma_1 \frac{\delta\rho}{\rho_0}$$

- adiabatic exponent related to adiabatic sound velocity

$$c^2 = \Gamma_1 \frac{P_0}{\rho_0}$$

- radiative exchange in solar atmosphere is fast $\Rightarrow$ non-adiabatic

Linear Perturbations

- linear perturbations

$$P = P_0 + P_1 \quad \rho = \rho_0 + \rho_1 \quad \vec{v} = \vec{v}_0 + \vec{v}_1 = \vec{v}_1$$

$$P_1 \ll P_0 \quad \rho_1 \ll \rho_0 \quad \vec{v} \ll c_s$$

- Lagrangian perturbations (S 5.15 with displacement $\vec{\delta r} = \boldsymbol{\xi}$)

$$\delta P = P_1 + \vec{\delta r} \cdot \nabla P_0 \quad \rho = \rho_1 + \vec{\delta r} \cdot \nabla \rho_0 \quad \vec{v} = \frac{\partial \vec{\delta r}}{\partial t}$$

- continuity (S 5.13)

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \, \vec{v}) = 0 \qquad \rho_1 + \nabla \cdot (\rho_0 \, \vec{\delta r}) = 0$$

- momentum (S 5.14)

$$\rho_0 \frac{\partial^2 \vec{\delta r}}{\partial t^2} = \rho_0 \frac{\partial \vec{v}}{\partial t} = -\nabla P_1 + \rho_0 \, \vec{g}_1 + \rho_1 \, \vec{g}_0 = -\nabla P_1 - \rho_0 \, \nabla \Phi_1 + \frac{\rho_1}{\rho_0} \nabla P_0$$

Linear Perturbations (continued)

- Cowling approximation (S 5.2.3, 5.29): waves ⇒ many radial sign changes ⇒ average out

$$\nabla^2\Phi_1 = 4\pi G\rho_1 \qquad \Phi_1 = -G\int \frac{\rho_1(r')}{|r-r'|}\mathrm{d}r' \approx 0$$

- adiabatic energy (S 5.10)

$$\frac{P_1}{P_0} = \gamma\,\frac{\rho_1}{\rho_0} \qquad \frac{\delta P}{P_0} = \gamma\,\frac{\delta\rho}{\rho_0}$$

Isothermal Atmosphere

- coefficients except for ρ_0 and P_0 are constant
- ρ_0 and P_0 have exponential stratification
- Cowling approximation
- assume vertical wavelength small compared to solar radius r
- define $S_l^2 = \frac{l(l+1)}{r^2} c^2$
- oscillations of the form

$$\begin{aligned} \xi_r &\sim \frac{1}{\sqrt{\rho_0}} \mathrm{e}^{ik_r r} \\ P_1 &\sim \sqrt{\rho_0} \mathrm{e}^{ik_r r} \end{aligned}$$

- $\sqrt{\rho_0}$ terms take care of variable ρ_0

Density Scale Height

- Brunt-Väisälä frequency

$$N^2 = g\left(\frac{1}{\Gamma_1 P_0} - \frac{1}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}r}\right)$$

- density scale height H is a constant

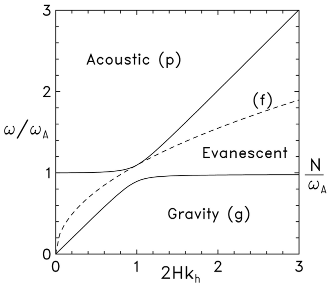
$$H = -\rho_0/(d\rho_0/dr) = \left(\frac{g}{c^2} + \frac{N^2}{g}\right)^{-1}$$

- dispersion relation

$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2\frac{N^2 - \omega^2}{c^2\omega^2}$$

- acoustic cutoff frequency $\omega_A = c/2H$

Diagnostic Diagram

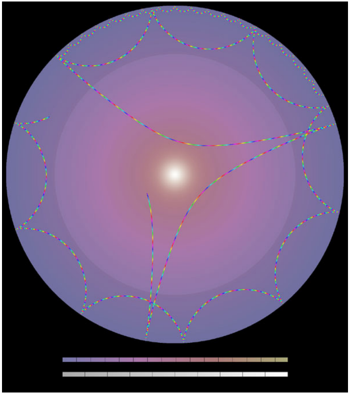


- dispersion relation

$$k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2\omega^2}$$

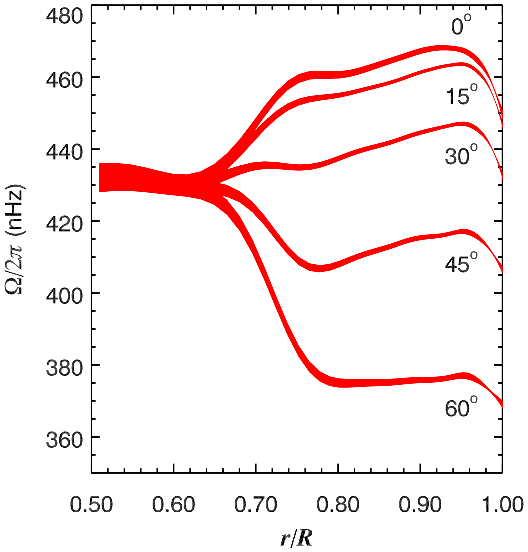
- oscillatory solutions require real k_r
- right-hand side has to be positive
- calculate curves of $k_r^2 = 0$ in k-ω diagram
- three areas

Overview

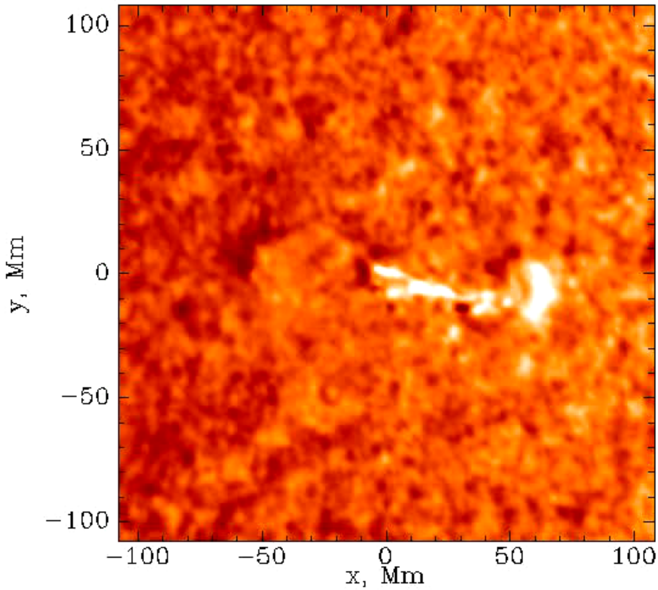


- frequencies can be inverted to derive sound speed profile as a function of location and time inside the Sun
- global helioseismology derives results that are independent of longitude such as internal rotation
- *local helioseismology* derives results as a function of longitude, latitude, and radius

Internal Rotation



Sun Quake



Farside Imaging