Introduction

- Sun rings like a bell, but at many different frequencies
- Acoustic waves with pressure as restoring force (\textit{p-modes})
- Frequencies depend on internal structure and motions
Observations

5-Minute Oscillations

- discovered by R. Leighton in 1960
- spectroheliogram = scanned image at fixed wavelength
- Doppler plate: difference of intensity in blue and red wing:

\[ I(\lambda + \Delta \lambda) - I(\lambda - \Delta \lambda) \approx 2\Delta \lambda \frac{\partial I(\lambda)}{\partial \lambda} \]

- Doppler difference plate from forward and backward scans

Leighton et al. (1962)
Spectral Observations

- direct measurements of spectral line shifts
- largely vertical oscillations
- amplitudes 0.5-1.0 km/s, increasing with height
- frequencies around 5 minutes dominate in the photosphere, 3 minutes in chromospheric lines
- little phase lag between different heights
- wave numbers from solar diameter to smallest resolvable scales
Solar Oscillations and Supergranulation with SOHO/MDI

Single Dopplergram
20-Mar-96 19:54:32

Average Dopplergram Minus Polynomial Fit
45 images averaged (30-Mar-96 19:26 to 30-Mar-96 20:17)

Single Dopplergram Minus 45 Images Average
20-Mar-96 19:54:32
Temporal Spectrum of Oscillations

- observations over period $T$ with sampling interval $\Delta t$
- temporal frequency resolution $\Delta \omega = \frac{2\pi}{T}$
- lowest observable temporal frequency is $\Delta \omega$
- highest observable temporal frequency is $\omega_{Ny} = \frac{\pi}{\Delta t}$
- anti-alias filtering required if frequencies $> \omega_{Ny}$ exist

Spatial Spectrum of Oscillations

- observations over area $L_x$ with sampling interval $\Delta x$
- spatial frequency resolution $\Delta k_x = \frac{2\pi}{L_x}$
- lowest observable spatial frequency is $\Delta k_x$
- highest observable spatial frequency is $k_{Ny} = \frac{\pi}{\Delta x}$
- anti-alias filtering required if frequencies $> k_{Ny}$ exist
Long-Term Observations

- high temporal frequency resolution requires long observing periods
- day-night cycle $\Rightarrow$ networks around the Earth and satellites
- GONG: Global Oscillation Network Group
- SOHO: GOLF, VIRGO, MDI
- now: Solar Dynamics Observatory
Power Spectrum

- velocity signal as a function of space and time: \( v(x, y, t) \)
- 3-D Fourier transform with respect to \( x, y, t \)

\[
f(k_x, k_y, \omega) = \int v(x, y, t)e^{-i(k_xx + k_yy + \omega t)}\,dx\,dy\,dt
\]

- can also be written as

\[
v(x, y, t) = \int f(k_x, k_y, \omega)e^{i(k_xx + k_yy + \omega t)}\,dk_x\,dk_y\,d\omega
\]

- power spectrum \( P(k_x, k_y, \omega) = f \cdot f^* \)
- if no spatial direction is preferred: \( k_h = \sqrt{k_x^2 + k_y^2} \)

\[
P(k_h, \omega) = \frac{1}{2\pi} \int_0^{2\pi} P(k_h \cos \phi, k_h \sin \phi, \omega)\,d\phi
\]
- power is concentrated into \textit{ridges}
- ridges theoretically predicted by Ulrich in 1970
- first observed by Deubner in 1975
- pressure perturbations $\Rightarrow \textit{p-modes}$
- lowest (fundamental) mode $\Rightarrow \textit{f-mode}$ (surface wave)
Whole-Sun Observations

- spherical coordinate system $r, \theta, \phi$
- velocity field in terms of spherical surface harmonics

$$v(\theta, \phi, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm}(t) Y_{lm}^{m}(\theta, \phi)$$

- $Y_{lm}^{m}(\theta, \phi) = P_{l}^{m|}(\theta)e^{im\phi}$
- $P_{l}^{m}$: associated Legendre function
l and m

- velocity field in (complex) spherical harmonics

\[ \mathbf{v}(\theta, \phi, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm}(t) P_l^m(\theta)e^{im\phi} \]

- degree \( l \): total number of node circles on sphere
- longitudinal order \( m \): number of node circles through poles
- rotation provides preferred direction
- rotation mostly minor effect \( \Rightarrow m = 0 \) good approximation
Spherical Harmonics and Oscillations

Solar Physics, Lecture 10: Oscillations
$l$ replaces $k_h$, $\nu = \omega / 2\pi$

$\hat{a}_l(\nu)$ is Fourier transform of $a_{l0}(t)$

power in $l-\nu$ diagram given by

$$P(l, \nu) = \hat{a}_l(\nu)\hat{a}_l^*(\nu)$$

see only part of solar surface

$\Rightarrow$ cannot resolve modes in spatial frequency

but different $l$-modes have different frequencies

single mode amplitudes: 30 cm/s or less

interference of $10^7$ modes provides 1 km/s
Low-Degree p Modes

- spatially unresolved Doppler shifts (Sun as a star)
- can only observe the lowest \( l \) modes in velocity from the ground and in intensity from space
- can now also detect this on bright stars

Bison velocity, theoretical frequencies

SOHO/VIRGO
Line Width

- Solar oscillations lines have finite width
- Line width determined by finite mode lifetime due to:
  - Damping mechanism
  - Convective velocity field
- Lorentz profile identical to collisional broadening of spectral lines
- Modes live from hours to months

![Graph showing FWHM Linewidths vs Frequency, mHz](image)
Basic Equations

- assume non-rotating gaseous sphere in hydrostatic equilibrium
- Euler’s field description in fixed coordinate systems
- Lagrange’s particle system in coordinates that flows with gas
- Lagrange (substantial derivative) and Euler descriptions related
- Lagrangian perturbation $\delta$

\[
\frac{d\alpha}{dt} = \left[ \frac{\alpha(t + \Delta t) - \alpha(t)}{\Delta t} \right]_{\vec{r}} = \frac{\partial \alpha}{\partial t} + \vec{v} \cdot \nabla \alpha
\]

- steady flow: $\frac{\partial}{\partial t} = 0$ concept in Euler’s description
- incompressible flow: $\frac{d\rho}{dt} = 0$ concept in Lagrange’s description
Thermodynamics

- first law of thermodynamics

\[
\frac{dq}{dt} = \frac{dE}{dt} + P \frac{dV}{dt}
\]

- \( q \) entropy
- \( E \) energy
- \( P \) pressure
- \( V \) volume

- \( V = 1/\rho \)

- therefore

\[
\rho \frac{dq}{dt} = \rho \frac{dE}{dt} - \frac{P}{\rho} \frac{d\rho}{dt}
\]
Ideal Gas

ideal gas

\[ \delta E = c_v \delta T \quad P = (c_p - c_v) \rho T \quad P = (\gamma - 1) \rho E \quad \gamma = \frac{c_p}{c_v} \]

adiabatic exponent \( \Gamma_1 = \left( \frac{\partial \ln P}{\partial \ln \rho} \right)_{\text{ad}} \)

first law of thermodynamics (from before)

\[ \rho \frac{dq}{dt} = \rho \frac{dE}{dt} - \frac{P}{\rho} \frac{d\rho}{dt} \]

first law of thermodynamics for ideal gas

\[ \frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt} + (\gamma - 1) \rho \frac{dq}{dt} \]
Adiabatic Approximation

- first law of thermodynamics for ideal gas

\[
\frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt} + (\gamma - 1) \rho \frac{dq}{dt}
\]

- adiabatic \((\delta q = 0)\)

\[
\frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt}
\]

- adiabatic approximation implies

\[
\frac{\delta P}{P_0} = \Gamma_1 \frac{\delta \rho}{\rho_0}
\]

- adiabatic exponent related to adiabatic sound velocity

\[
c^2 = \Gamma_1 \frac{P_0}{\rho_0}
\]

- radiative exchange in solar atmosphere is fast \(\Rightarrow\) non-adiabatic
Linear Perturbations

- **Linear perturbations**

\[ P = P_0 + P_1 \quad \rho = \rho_0 + \rho_1 \quad \vec{v} = \vec{v}_0 + \vec{v}_1 = \vec{v}_1 \]

\[ P_1 \ll P_0 \quad \rho_1 \ll \rho_0 \quad \vec{v} \ll c_s \]

- **Lagrangian perturbations** (S 5.15 with displacement \( \delta \vec{r} = \xi \))

\[ \delta P = P_1 + \delta \vec{r} \cdot \nabla P_0 \quad \rho = \rho_1 + \delta \vec{r} \cdot \nabla \rho_0 \quad \vec{v} = \frac{\partial \delta \vec{r}}{\partial t} \]

- **Continuity** (S 5.13)

\[ \frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \vec{v}) = 0 \quad \rho_1 + \nabla \cdot (\rho_0 \delta \vec{r}) = 0 \]

- **Momentum** (S 5.14)

\[ \rho_0 \frac{\partial^2 \delta \vec{r}}{\partial t^2} = \rho_0 \frac{\partial \vec{v}}{\partial t} = -\nabla P_1 + \rho_0 \vec{g}_1 + \rho_1 \vec{g}_0 = -\nabla P_1 - \rho_0 \nabla \Phi_1 + \frac{\rho_1}{\rho_0} \nabla P_0 \]
Linear Perturbations (continued)

- Cowling approximation (S 5.2.3, 5.29): waves ⇒ many radial sign changes ⇒ average out
  \[ \nabla^2 \Phi_1 = 4\pi G \rho_1 \quad \Phi_1 = -G \int \frac{\rho_1(r')}{|r - r'|} dr' \approx 0 \n\]
  - adiabatic energy (S 5.10)
  \[ \frac{P_1}{P_0} = \gamma \frac{\rho_1}{\rho_0} \quad \frac{\delta P}{P_0} = \gamma \frac{\delta \rho}{\rho_0} \]
Isothermal Atmosphere

- Coefficients except for $\rho_0$ and $P_0$ are constant
- $\rho_0$ and $P_0$ have exponential stratification
- Cowling approximation
- Assume vertical wavelength small compared to solar radius $r$
- Define $S^2_l = \frac{l(l+1)}{r^2} c^2$
- Oscillations of the form

$$\xi_r \sim \frac{1}{\sqrt{\rho_0}} e^{ik_r r}$$

$$P_1 \sim \sqrt{\rho_0} e^{ik_r r}$$

- $\sqrt{\rho_0}$ terms take care of variable $\rho_0$
Density Scale Height

- Brunt-Väisälä frequency

\[ N^2 = g \left( \frac{1}{\Gamma_1 P_0} - \frac{1}{\rho_0} \frac{d\rho_0}{dr} \right) \]

- density scale height \( H \) is a constant

\[ H = -\rho_0/(d\rho_0/dr) = \left( \frac{g}{c^2} + \frac{N^2}{g} \right)^{-1} \]

- dispersion relation

\[ k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_l^2 \frac{N^2 - \omega^2}{c^2 \omega^2} \]

- acoustic cutoff frequency \( \omega_A = c/2H \)
Diagnostic Diagram

- dispersion relation

\[ k_r^2 = \frac{\omega^2 - \omega_A^2}{c^2} + S_f^2 \frac{N^2 - \omega^2}{c^2 \omega^2} \]

- oscillatory solutions require real \( k_r \)
- right-hand side has to be positive
- calculate curves of \( k_r^2 = 0 \) in k-\( \omega \) diagram
- three areas
Overview

- Frequencies can be inverted to derive sound speed profile as a function of location and time inside the Sun.
- Global helioseismology derives results that are independent of longitude such as internal rotation.
- Local helioseismology derives results as a function of longitude, latitude, and radius.