Stokes Inversion

Overview:

How are Stokes profiles connected to the magnetic field?
Radiative transfer for polarized light
Inversion techniques
My research

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Magnetic fields on the sun

- Magnetic fields almost everywhere on the sun, ranging from small nonresolved magnetic elements to huge structures, such as sunspots with diameter up to 50000km and magnetic strength of order 3000 Gauss
- Seen in photosphere, chromosphere and up to the corona
- Most solar phenomena associated with magnetic field
- Need information on magnetic field direction, strength and dynamics
Components are polarized.

Visible polarization depends on observation angle!

Thanks to Zeeman’s discovery of the effect of magnetism on radiation, it appeared that the detection of such a magnetic field should offer no great difficulty, provided it were sufficiently intense. When a luminous vapor is placed between the poles of a powerful magnet, the lines of its spectrum, if observed along the lines of force, appear in most cases as doublets, having components circularly polarized in opposite directions. The distance between the components of a given doublet is directly proportional to the strength of the field. As different lines in the spectrum of the same element are affected in different degree, it follows that in a field of moderate strength many of the lines may be simply widened, while others, which are exceptionally sensitive, may be separated into doublets.

Hale, 1908
Stokes vector – Stokes profile

- Measure the polarization state with the Stokes vector
- Wavelength dependence – Stokes profile

\[ \mathbf{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left – right} \end{pmatrix} \]
Utilizing the Zeeman effect

- V changes sign when changing the direction of B into the opposite
Simple B field reconstruction

Weak field $V \sim$ longitudinal $B$
Magnetogram in $V$
Only approximation

Extract from $V$ signal

Hinode webpage
Full magnetic field vector

Need to do full analysis in order to get
inclination angle $\gamma$
azimuth angle $\phi$
and field strength

Problems:
For azimuth 180 degree ambiguity
Magnetic elements not resolved
SOLIS data!

http://helios.obspm.fr/dasop/index.html

but 180 degree ambiguity!

\[
I = \begin{pmatrix}
Q \\
U
\end{pmatrix}
= \begin{pmatrix}
\text{linear 0° - linear 90°} \\
\text{linear 45° - linear 135°} \\
\text{circular left - right}
\end{pmatrix}
\]
RTE for polarized light

\[ \mu \frac{dl_\nu}{d\tau_{\nu}} = l_\nu - S_\nu \]

Emission produced in LTE

\[ \cos \theta \frac{dl_\nu}{d\tau_{c}} = (1 + \eta_\nu) (l_\nu - B_\nu) \]

For Stokes vector

\[ \cos \theta \frac{d\vec{l}}{d\tau_{c}} = (1 + \eta) (\vec{I} - \vec{B}_\nu) \]

STIX 3.70

Matrix! vector containing Bplanck in first element
RTE for polarized light

\[ \cos \theta \frac{d\vec{I}}{d\tau_c} = (1 + \eta) \left( \vec{I} - \vec{B}_\nu \right) \]

\[ \eta = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \quad \text{STIX 3.68} \]

The absorption matrix contains the absorption (change amplitude) and retardance (change phase) profiles.

\[ \eta_{I,Q,U,V} \text{ and } \rho_{Q,U,V} \text{ depend on } a = (B, \gamma, \theta, v_{\text{LOS}}, T, P_e, v_{\text{mic}}) \]
Retardance and absorption

Decompose electron osz.

Absorption- and Retardance profile
Thermal motions

Gaussian:
Doppler microturbulence

Convolution

Voigt profile Faraday Voigt profile

QM addition: oszillator strength
Also, macroturbulence
RTE for polarized light

Atmosphere, our case magnetized atmosphere

If the atom is in a magnetized atmosphere the energy of each Zeeman sublevel is different, which produces a change of resonance frequency of the transition between sublevels depending on quantum number m.

Associated to each transition there is a absorption profile plus a retardance profile.

\[ \eta, \eta^+, \eta^- \]
Zeeman effect
frequencies shift, absorption at different wavelength

Absorption and retardance for the 3 oscillations
\( \eta, \eta^+, \eta^- \),

\[
\eta = \begin{pmatrix}
\eta_I & \eta_Q & \eta_U & \eta_V \\
\eta_Q & \eta_I & \rho_V & -\rho_U \\
\eta_U & -\rho_V & \eta_I & \rho_Q \\
\eta_V & \rho_U & -\rho_Q & \eta_I \\
\end{pmatrix}
\]

\[
\eta_I = \frac{1}{2} \eta \sin^2 \gamma + \frac{1}{4} (\eta^+ + \eta^-) \left(1 + \cos^2 \gamma\right)
\]
\[
\eta_Q = \left(\frac{1}{2} \eta - \frac{1}{4} (\eta^+ + \eta^-)\right) \sin^2 \gamma \cos 2\phi
\]
\[
\eta_U = \left(\frac{1}{2} \eta - \frac{1}{4} (\eta^+ + \eta^-)\right) \sin^2 \gamma \sin 2\phi
\]
\[
\eta_V = \frac{1}{2} (\eta^+ - \eta^-) \cos \gamma
\]
Stokes Inversion

Any method used to infer the physical conditions of the atmosphere from the interpretation of Stokes profiles

Tools:
- model of the atmosphere
- Radiative transfer
- Directional dependence of Stokes profiles
PCA (Principal Component)

- Not Taylor made
- Vast cataloge of Stokes profiles
- Observed profiles thought of as combinations of these readymade profiles.

\[
\text{face} = \sum_i \text{eigen value}_i \times \text{eigenface}_i
\]

Rees et al., 2000
Least square inversion

\[ a \equiv (B, \gamma, \theta, v_{\text{LOS}}, T, P_e, v_{\text{mic}}) \]

Filling factor,
\[ I = (1-f)I(\text{mag}) + f I(\text{no mag}) \]

\[ \chi^2(a) = \sum \left[ I_{\text{obs}}(\lambda_i) - I_{\text{syn}}(\lambda_i, a) \right]^2 \]
The inversion process minimizing Chi squared

\[ \chi^2(a) = \sum [I_{\text{obs}}(\lambda_i) - I_{\text{sym}}(\lambda_i, a)]^2 \]

Need to minimize Chi squared! Use Levenberg-Marquardt algorithm.

Derivative can be expressed in terms of response functions. Response functions tell us about how the observed spectrum responds to modifications in the physical parameters of the model.

STIX 3.68
Milne eddington inversion

Assumptions: Source function linear with optical depth
Absorption matrix does not vary with optical depth

Flat atmosphere
Solve RTE analytically

Fast and SIMPLE treatment of the RTE
BUT
Can not account for asymmetric Stokes profiles
Inversion height dependence

Gradients of the physical parameters along LOS can cause asymmetric profiles

Solve RTE numerically!
Inversion codes capable of dealing with asymmetries
- are based on numerical solution of RTE
- Provide reliable thermal information
- Infer stratifications of physical parameters with depth

Examples:
SIR, Cobo, del Toro Iniesta
LILIA, H. Navarro

Westendorp Plaza et al. 2001, Astrophysical Journal 547, 1130
Nodes concept

Compututional time!

Temperature vs. \( \log \tau \) for different locations of nodes.
What happens if magnetic elements with opposite polarity in one resolution element?
SOLIS VSM data

- high sensitivity
- high temporal resolution,
- moderate spatial resolution

less than 20 min. for full disk
1.13 arcsec resolution
Input parameters

Mode: Inversion! Mode of operation (Synthesis or Inversion)

! Fields required for both modes, synthesis and inversion:

Wavelength grid: HINODE.grid
Input Model: hsra.mod
Synthetic Profiles: invertprofiles.pro ! Output profiles
Heliocentric Angle: 1.0 ! Cos(mu)

! Fields required only for the inversion mode:

Observed Profiles: observed.pro
Output Model: modelout.mod

! Other misc optional fields

Formal solution method: 0 ! (0=auto, 1=Hermite, 2=WPM)
Stray light file: straylight.pro ! Optional
Printout detail: 1 ! (default=1)
noise: 0.000630795

LILIA by Hector Socas-Navarro
- atmospheric model from best fit
- $V_{\text{mac}}$
- Chisq.
- staylight/filling factor
13 September 2005

18 time steps
1.13 arcsec per pixel
5 minutes per areaskan
Figure 2. The figure shows the magnetic field strength in Gauss (here selected at optical depth $\tau_{500\text{m}} = -1.0$) as the result of the inversion. The inverted area is 644 arcsec times 339 arcsec large. The stripes in the center of the images are due to noise in the original spectra, which was caused by a failing polarization modulator.

- 3 diff. model atmosphere
- condition for Chisq.
- straylight was 10 quiet
- sun surrounding pixels
- 1 timestep 2 d on 30 Nodes
• Still have to resolve 180 degree ambiguity!
• Nonpotential field calculation (NPFC) Code from M. Georgoulis

FINAL RESULT
Problem: highly dynamic, low Signal to noise
Currently working with Helena Becher in order to improve the signal extraction.

Hinode quiet sun

We observed short lived variations in linear polarization with the SOLIS spectropolarimeter.

We think that these variations are due to rapid changes in the magnetic field vector inclination angle.

SOLIS VSM data lines Fe $630.15$ nm and $630.25$ nm
Problem: highly dynamic, low Signal to noise

Currently working with Helena Becher on techniques in order to improve the signal extraction.
Summary

• The radiative transfer equation is extended to deal with the Stokes vector. One obtains an absorption matrix.

• Stokes Inversions try to infer physical properties of the atmosphere by interpreting the Stokes profiles.

• Inversion techniques rely on several assumptions and require a starting atmospheric model.

• Only height dependent models can take care of asymmetries in the profiles.

• Parameters such as the filling factor, stray light and velocities have to be taken into account.
Exercises

Part of Practicum:

We will write a simple inversion code in the practicum using actual SOLIS VSM data ...
\[ \rho_Q = \frac{k_i}{2} \left( \rho_\pi - \frac{\rho_{\sigma^+} + \rho_{\sigma^-}}{2} \right) \sin^2 \theta \cos 2\phi \]
\[ \rho_U = \frac{k_i}{2} \left( \rho_\pi - \frac{\rho_{\sigma^+} + \rho_{\sigma^-}}{2} \right) \sin^2 \theta \sin 2\phi \]
\[ \rho_V = \frac{k_i}{2} \left( \frac{\rho_{\sigma^+} - \rho_{\sigma^-}}{2} \right) \]