Stokes Inversion

Overview:

How are Stokes profiles connected to the magnetic field?

Radiative transfer for polarized light

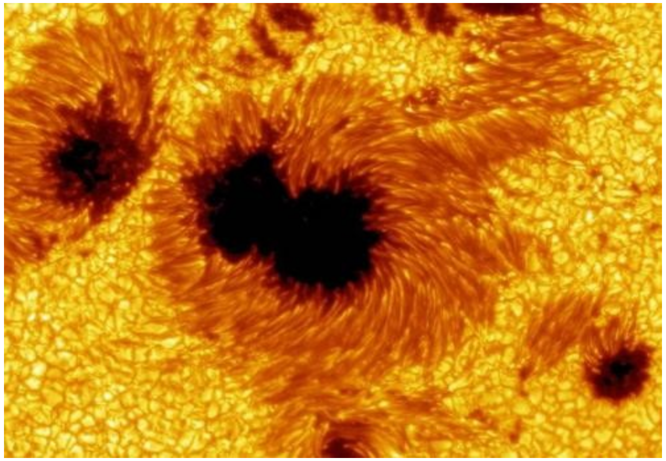
Inversion techniques

My research

Universiteit Utrecht

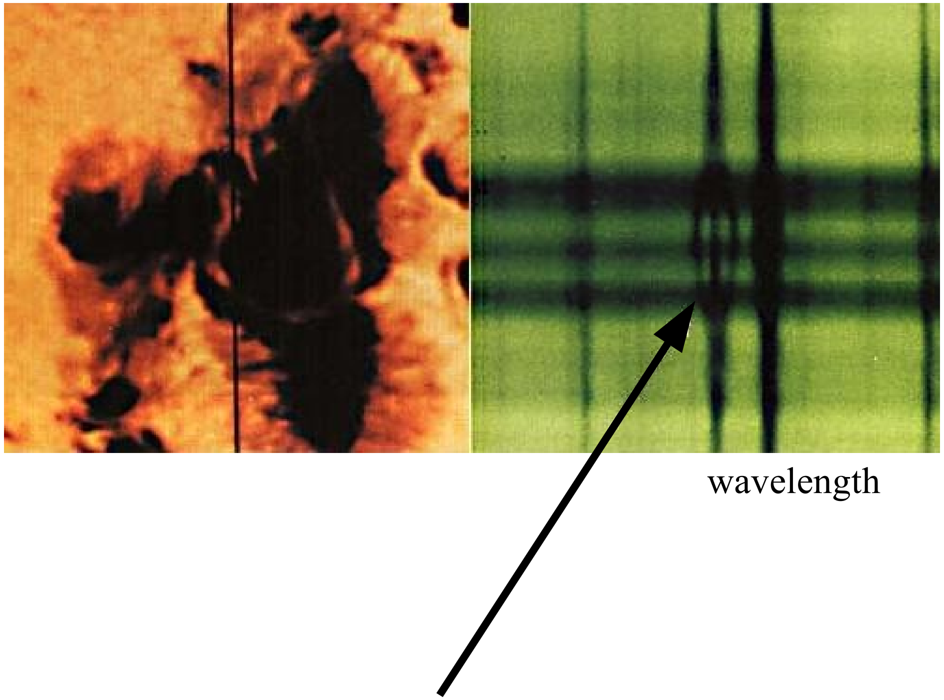
Catherine Fischer

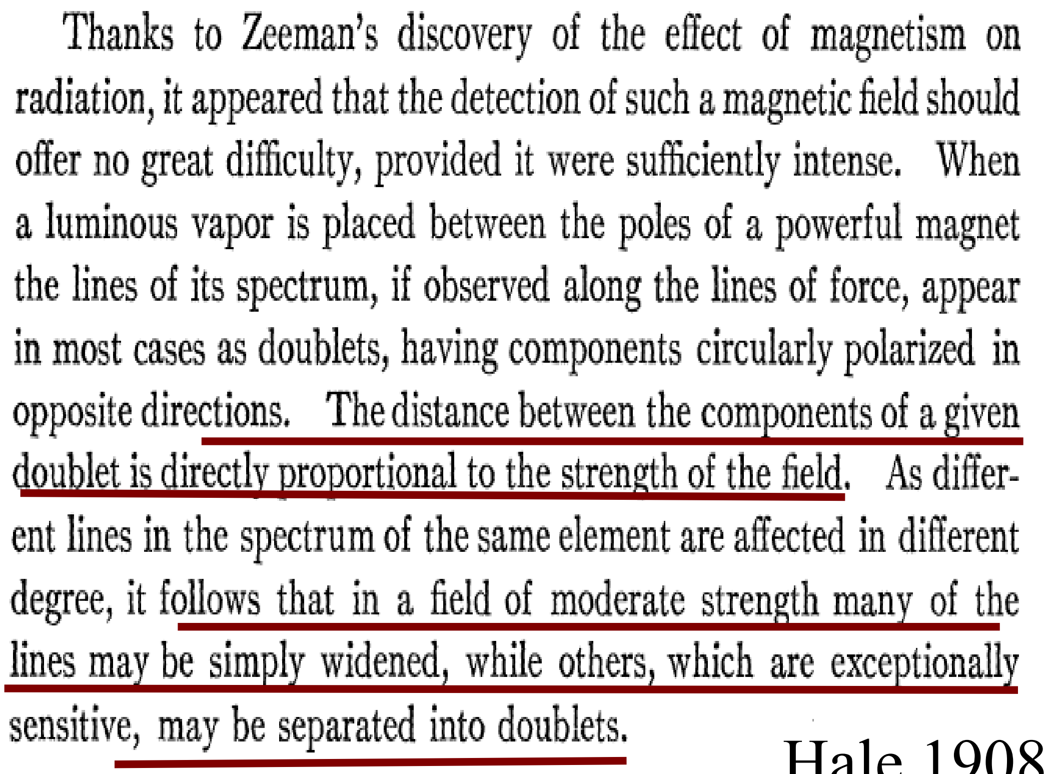
Magnetic fields on the sun



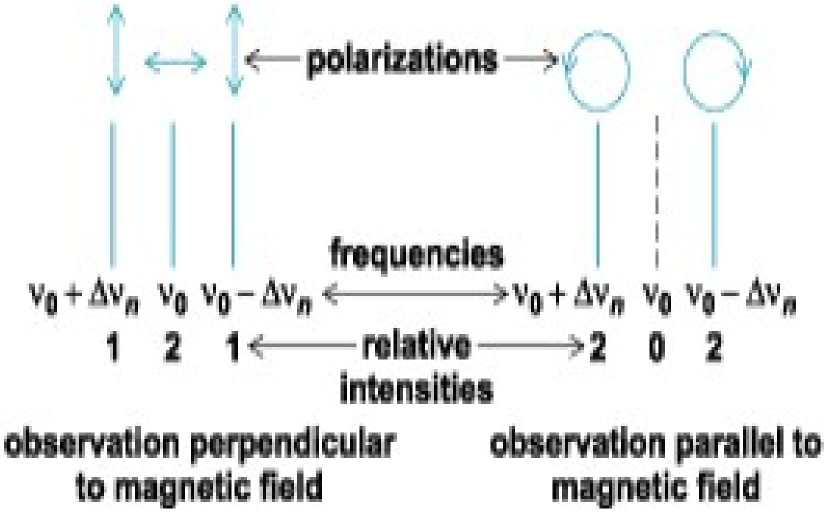
TRACE

- Magnetic fields almost everywhere on the sun, ranging from small nonresolved magnetic elements to huge structures, such as sunspots with diameter up to 50000km and magnetic strength of order 3000 Gauss
- Seen in photosphere, chromosphere and up to the corona
- Most solar phenomena associated with magnetic field
- Need information on magnetic field direction,strength and dynamics



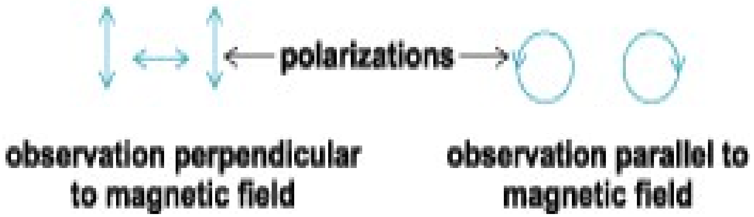

Thanks to Zeeman's discovery of the effect of magnetism on radiation, it appeared that the detection of such a magnetic field should offer no great difficulty, provided it were sufficiently intense. When a luminous vapor is placed between the poles of a powerful magnet the lines of its spectrum, if observed along the lines of force, appear in most cases as doublets, having components circularly polarized in opposite directions. The distance between the components of a given doublet is directly proportional to the strength of the field. As different lines in the spectrum of the same element are affected in different degree, it follows that in a field of moderate strength many of the lines may be simply widened, while others, which are exceptionally sensitive, may be separated into doublets.

Hale,1908



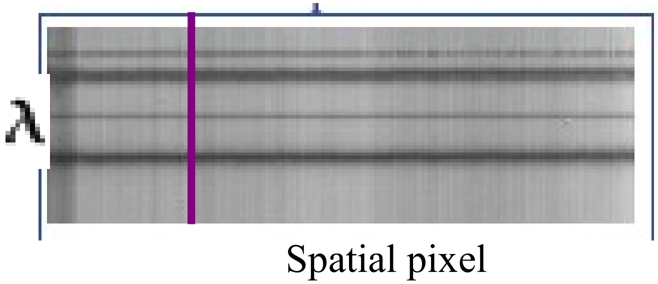
- Components are polarized.
- Visible polarization depends on observation angle!

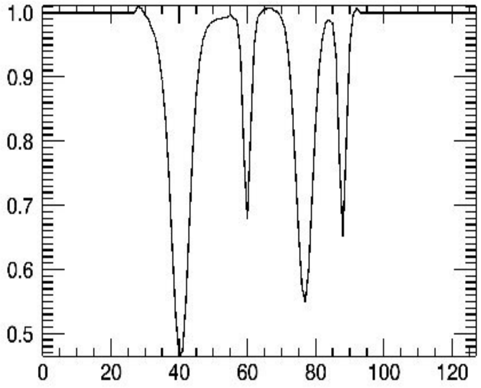
Stokes vector – Stokes profile

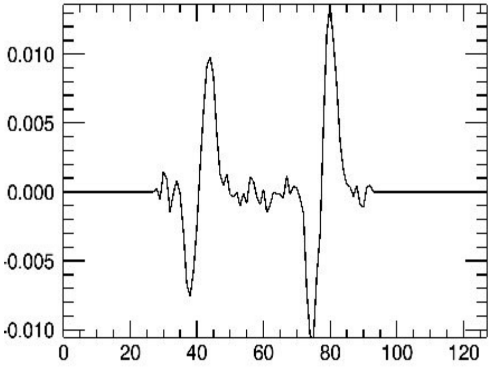


$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

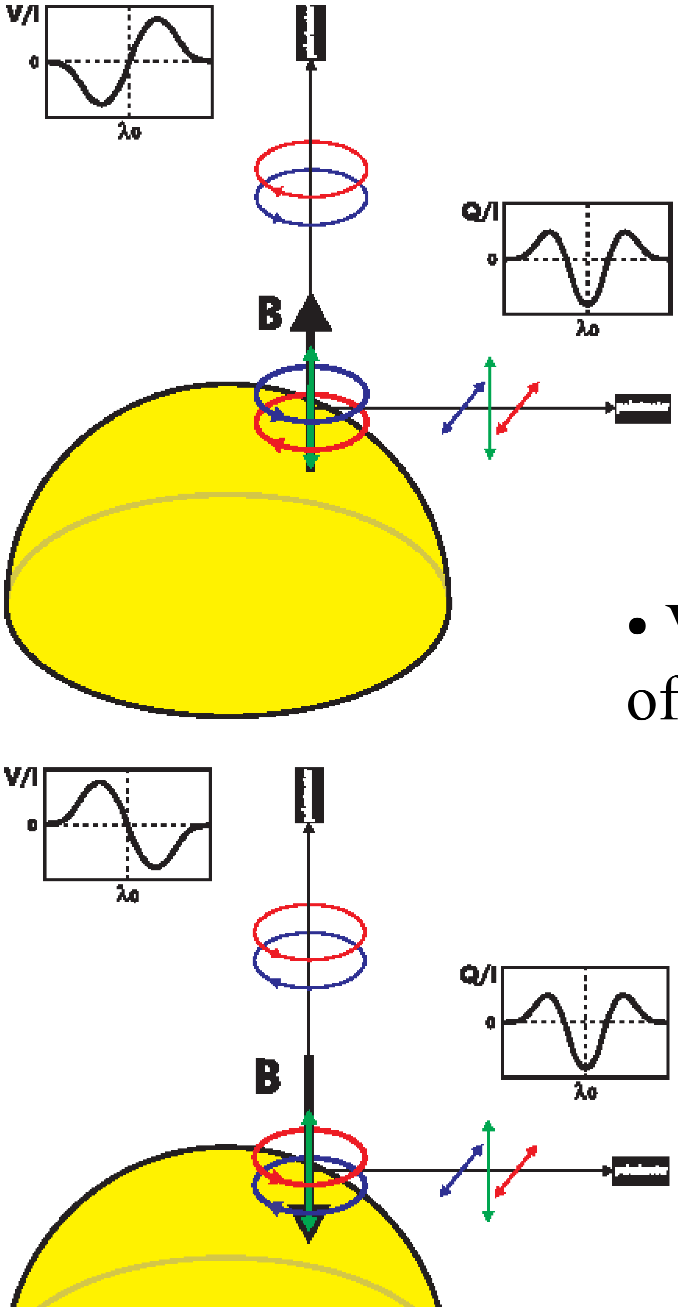
- Measure the polarization state with the Stokes vector
- Wavelength dependence – Stokes profile







Utilizing the Zeeman effect

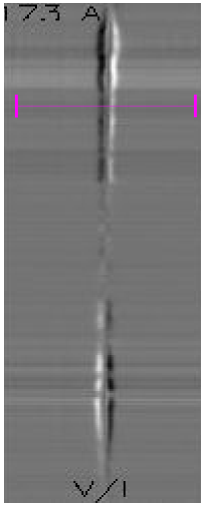


- V changes sign when changing the direction of B into the opposite

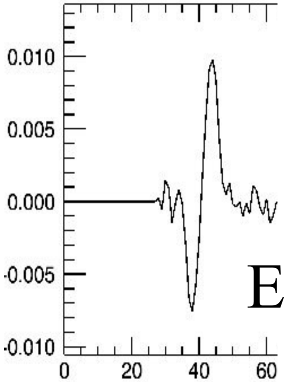
Frans Snik, master thesis

Simple B field reconstruction

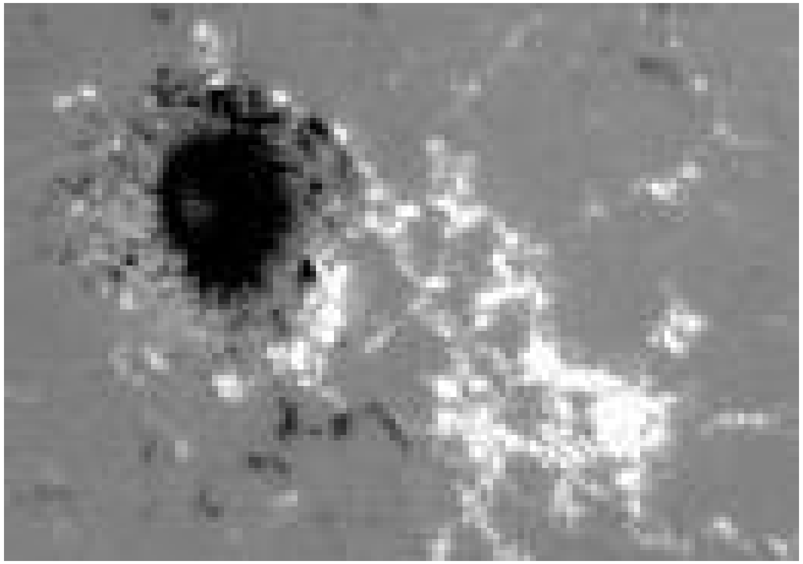
Weak field V ~ longitudinal B
Magnetogram in V
Only approximation



Themis

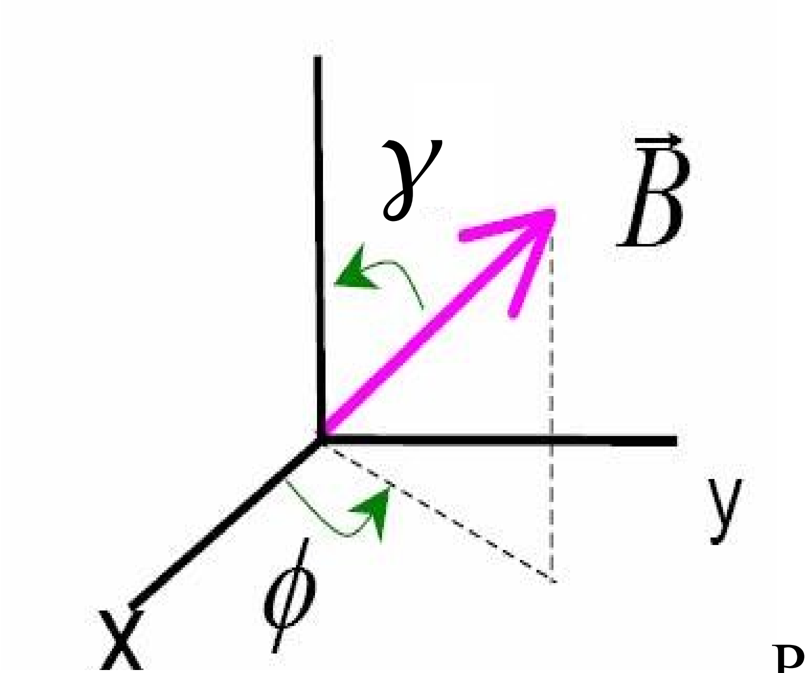


Extract from V signal



Hinode webpage

Full magnetic field vector

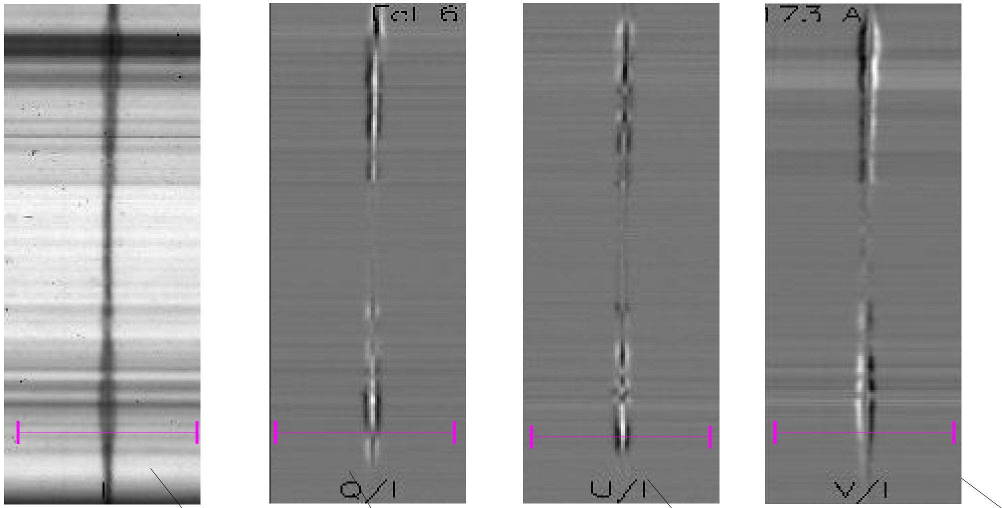


sanchez

Need to do full analysis in order to get
inclination angle γ
azimuth angle ϕ
and field strength

Problems:
For azimuth 180 degree ambiguity
Magnetic elements not resolved

Themis!



http://helios.obspm.fr/dasop/index.html

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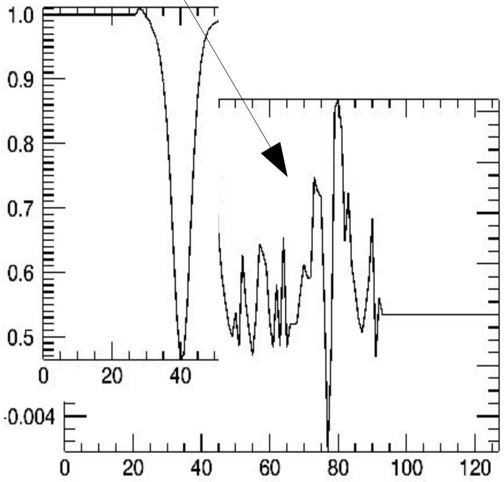
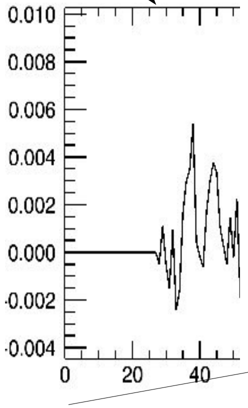
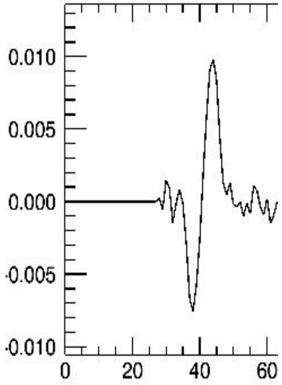
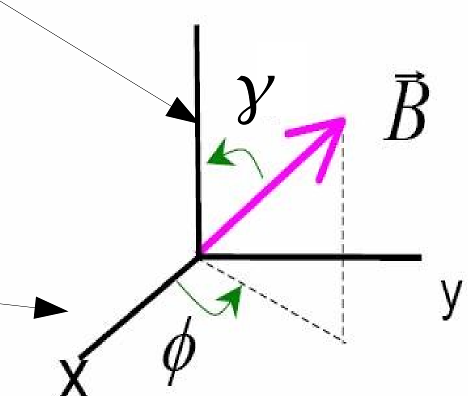






SOLIS data!



but 180 degree
ambiguity!

RTE for polarized light

$$\mu\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu$$

Emission produced in LTE

$$\cos\theta\frac{dI_\nu}{d\tau_c} = \left(1+\eta_\nu\right)\left(I_\nu - B_\nu\right)$$

For Stokes vector

$$\cos\theta\frac{d\vec{I}}{d\tau_c} = \left(1+\eta\right)\left(\vec{I} - \vec{B}_\nu\right)$$

STIX 3.70

Matrix!

vector containing
Bplanck in first element

RTE for polarized light

$$\cos\theta \frac{d\vec{I}}{d\tau_c} = (1+\eta)\left(\vec{I} - \vec{B}_\nu\right)$$

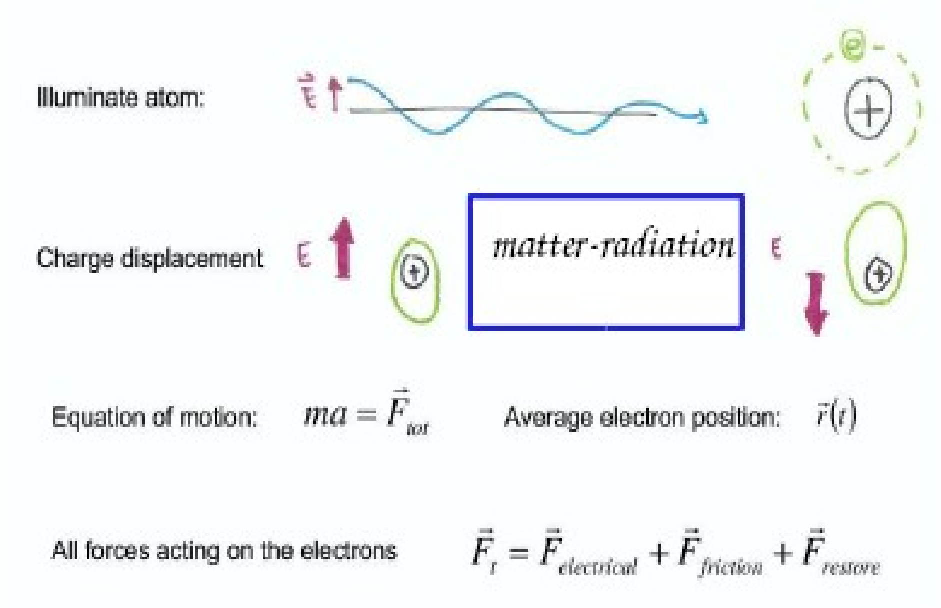
$$\eta = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_v & \eta_I & \rho_Q \\ \eta V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}$$

STIX 3.68

The absorption matrix contains the absorption (change amplitude) and retardance (change phase) profiles.

$\eta_{I,Q,U,V}$ and $\rho_{Q,U,V}$ depend on $\mathbf{a} \equiv (B, \gamma, \theta, v_{LOS}, T, P_e, v_{mic})$

Retardance and absorption



Illuminate atom:

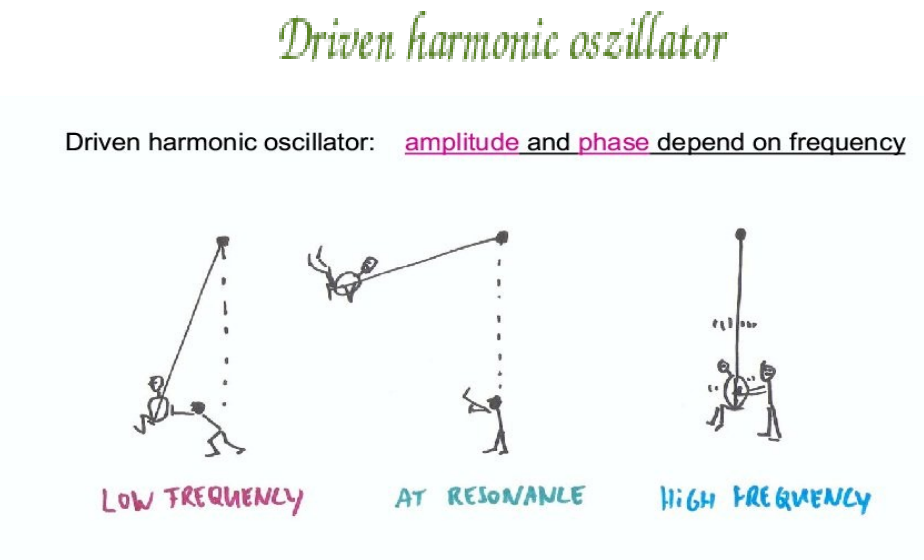
Charge displacement

matter-radiation

Equation of motion: $ma = \vec{F}_{tot}$

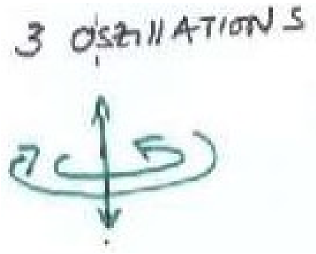
Average electron position: $\vec{r}(t)$

All forces acting on the electrons $\vec{F}_t = \vec{F}_{electrical} + \vec{F}_{friction} + \vec{F}_{restore}$



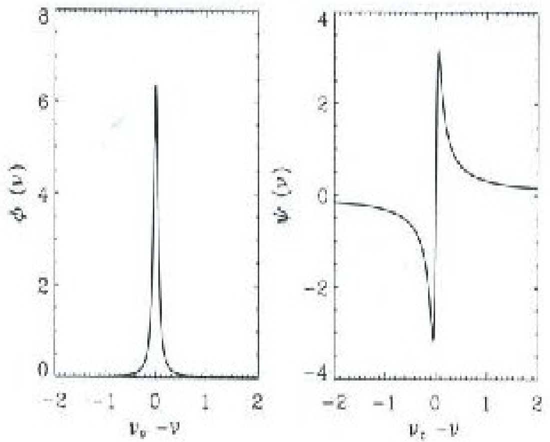
Driven harmonic oszillator

Driven harmonic oscillator: amplitude and phase depend on frequency

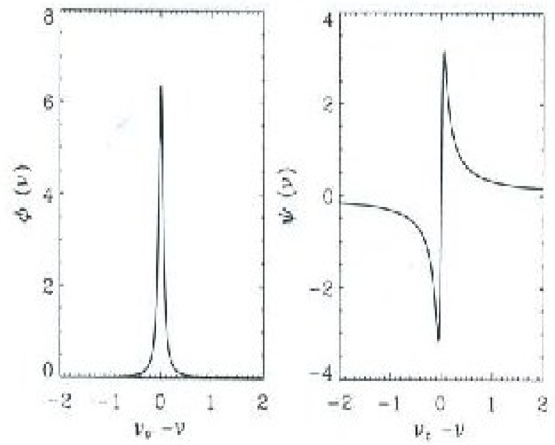


Decompose electron osz.

Absorption- and Retardanceprofile



Thermal motions

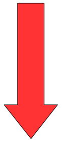


Convolution

⊗

Gaussian:

Doppler
microturbulence

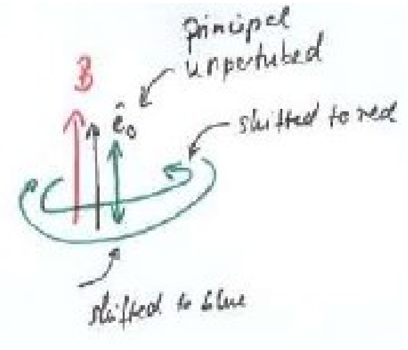


Voigt profile FaradayVoigt profile

QM addition: oszillator strength
Also, macroturbulence

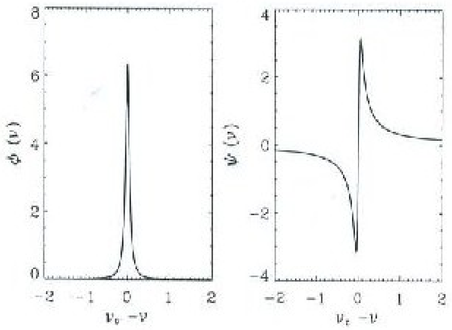
RTE for polarized light

Atmosphere, our case magnetized atmosphere



If the atom is in a magnetized atmosphere the energy of each Zeeman sublevel is different, which produces a change of resonance frequency of the transition between sublevels depending on quantum number m.

Associated to each transition there is a absorption profile plus a retardance profile.



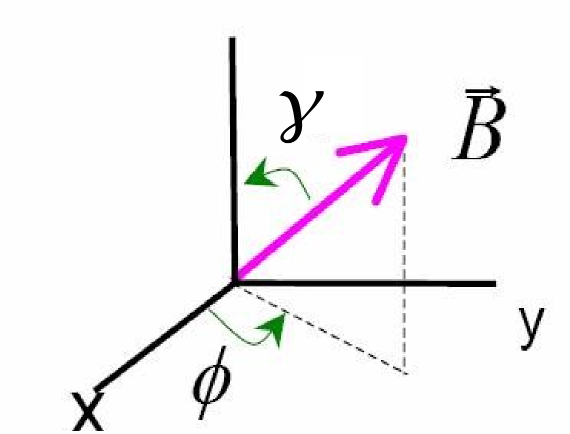
$\eta, \eta+, \eta-,$

Zeeman effect
frequencies shift, absorbtion at different wavelength

Absorption and retardance for the 3 Oszillations

$\eta, \eta+, \eta-,$

$$\eta = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}$$



$$\begin{aligned} \eta_I &= \frac{1}{2}\eta \sin^2\gamma + \frac{1}{4}\left(\eta^+ + \eta^-\right)\left(1+\cos^2\gamma\right) \\ \eta_Q &= \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma\cos 2\phi \\ \eta_U &= \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma\sin 2\phi \\ \eta_V &= \frac{1}{2}\left(\eta^+ - \eta^-\right)\cos\gamma \end{aligned}$$

Stokes Inversion

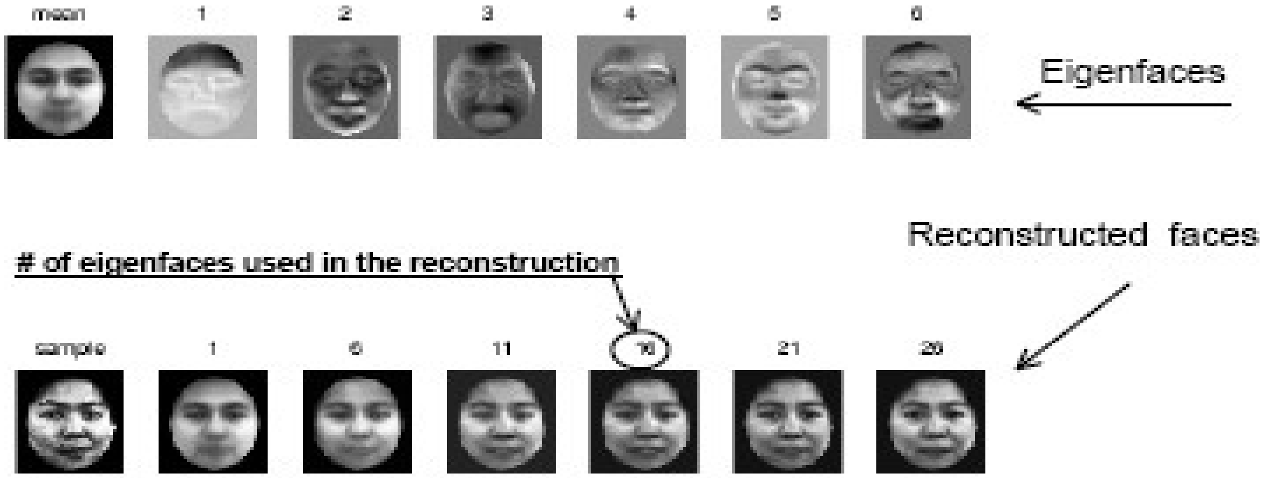
Any method used to infer the physical conditions of the atmosphere from the interpretation of Stokes profiles

Tools:

- model of the atmosphere
- Radiative transfer
- Directional dependance of Stokes profiles

PCA (Principal Component)

- Not Taylor made
- Vast cataloge of Stokes profiles
- Observed profiles thought of as combinations of these readymade profiles.

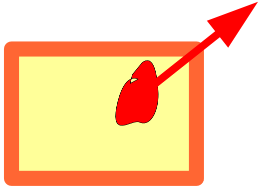


$$\text{face} = \sum_i \text{eigen value}_i \times \text{eigenface}_i$$

Rees et al., 2000

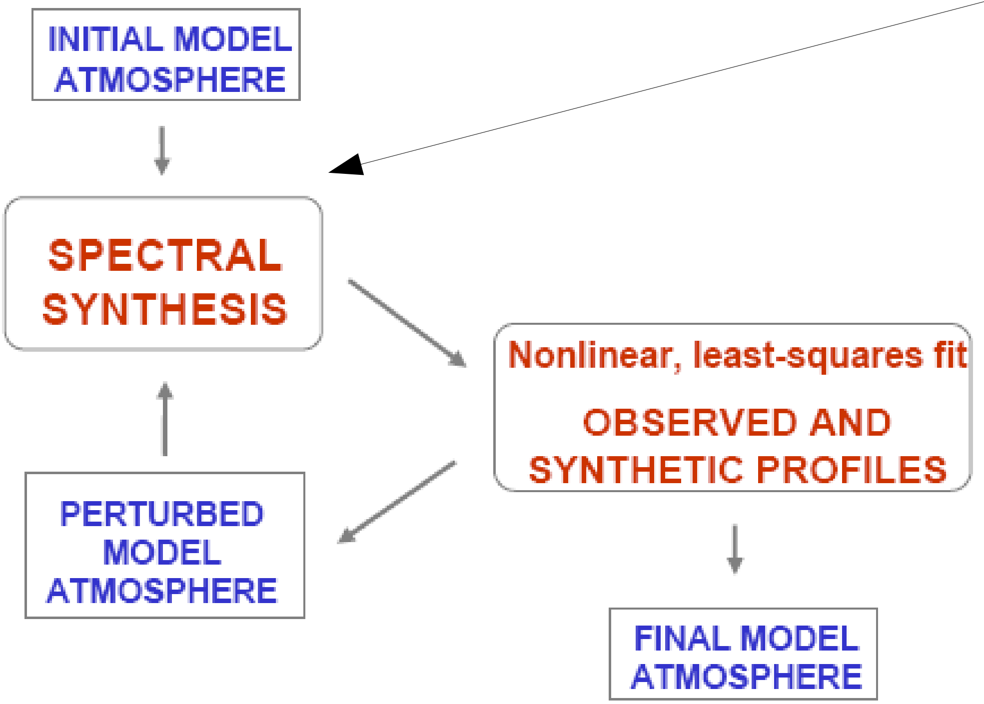
Least square inversion

$$\mathbf{a} \equiv (B, \gamma, \theta, v_{LOS}, T, P_e, v_{mic})$$



Filling factor,

I=(1-f)I(mag) + f I(no mag)



INITIAL MODEL ATMOSPHERE

SPECTRAL SYNTHESIS

Nonlinear, least-squares fit

OBSERVED AND SYNTHETIC PROFILES

PERTURBED MODEL ATMOSPHERE

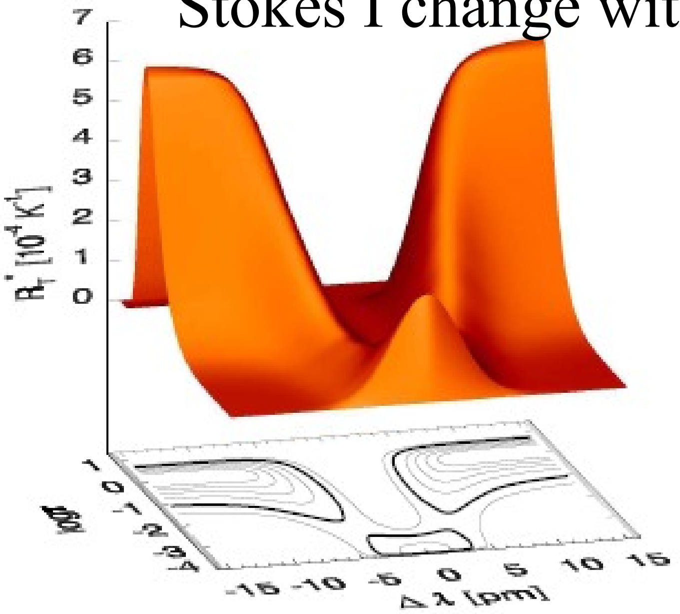
FINAL MODEL ATMOSPHERE

$$\chi^2(\mathbf{a}) = \sum [I_{obs}(\lambda_i) - I_{syn}(\lambda_i, \mathbf{a})]^2$$

The inversion process minimizing Chi squared

$$\chi^2(\mathbf{a}) = \sum\left[I_{obs}(\lambda_i) - I_{syn}(\lambda_i, \mathbf{a})\right]^2$$

Need to minimize Chi squared!
Use Levenberg-Marquardt algorithm.



Stokes I change with T

Derivative can be expressed in terms of response functions.
Response functions tell us about how the observed spectrum responds
to modifications in the physical parameters of the model.

STIX 3.68

Milne eddington inversion

Assumptions : Source function linear with optical depth
Absorption matrix does not vary with optical depth

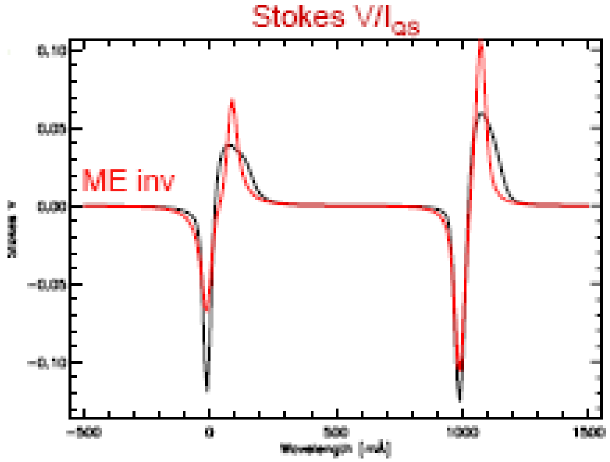
Flat atmosphere

Solve RTE analyticaly

Fast and SIMPLE treatment of the RTE

BUT

Can not account for asymmetric Stokes profiles



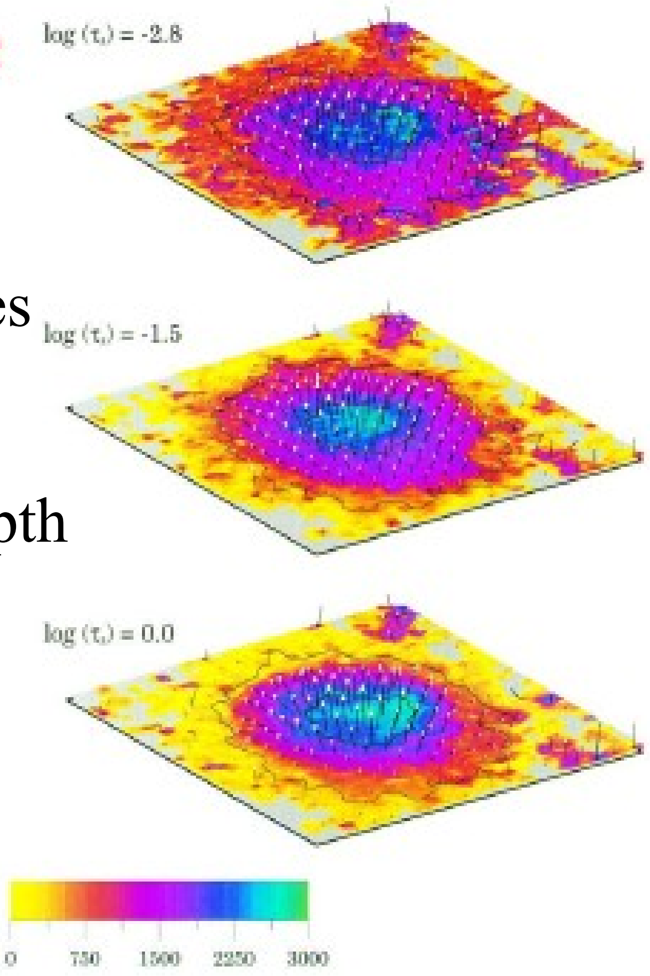
Inversion height dependence

Gradients of the physical parameters along LOS can cause asymmetric profiles

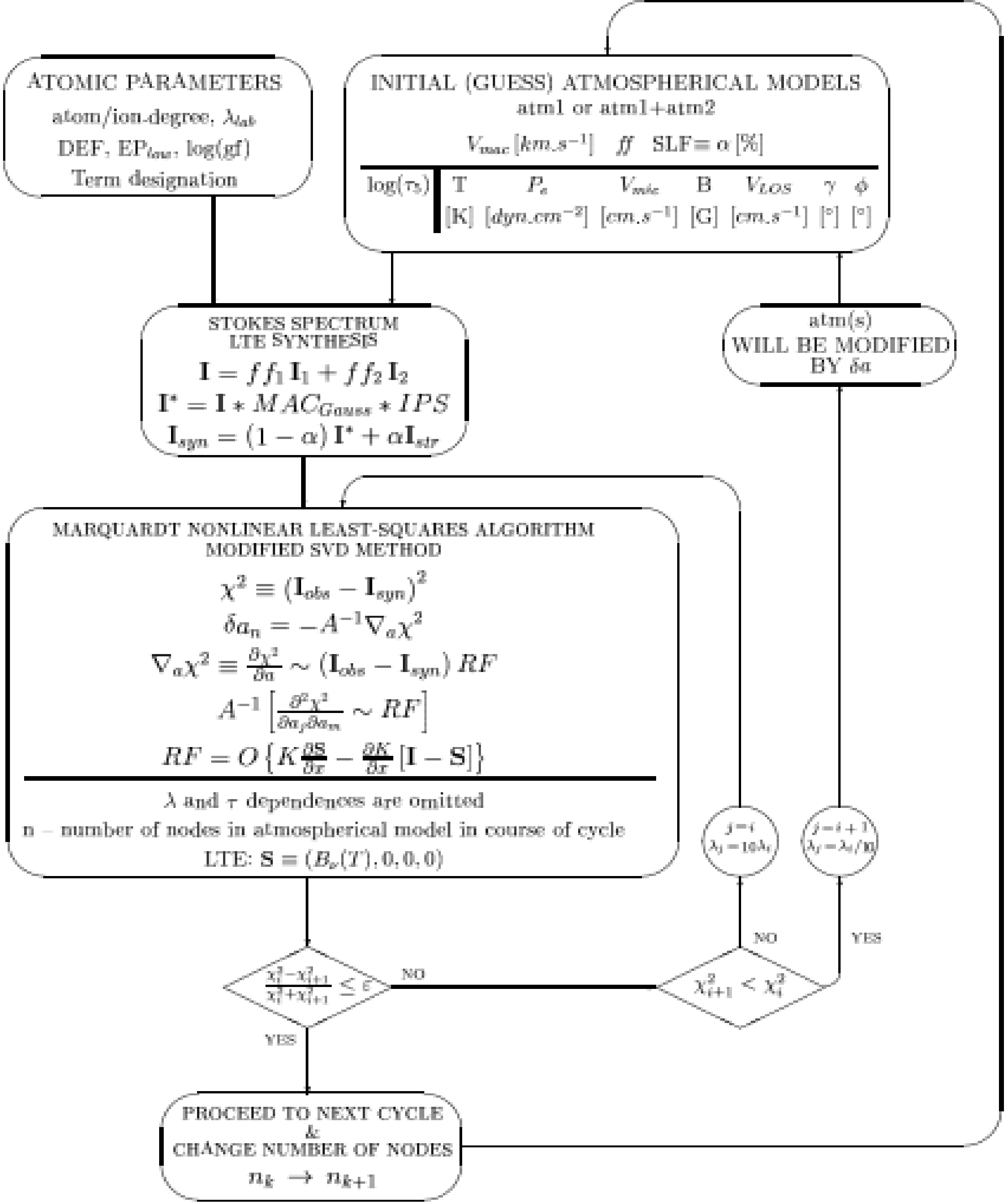
Solve RTE numericaly!

Inversion codes capable of dealing with asymmetries
-are based on numerical solution of RTE
-Provide reliable thermal information
-Infer stratifications of physical parameters with depth

Examples:
SIR Cobo, del Toro Iniesta
LILIA H.Navarro



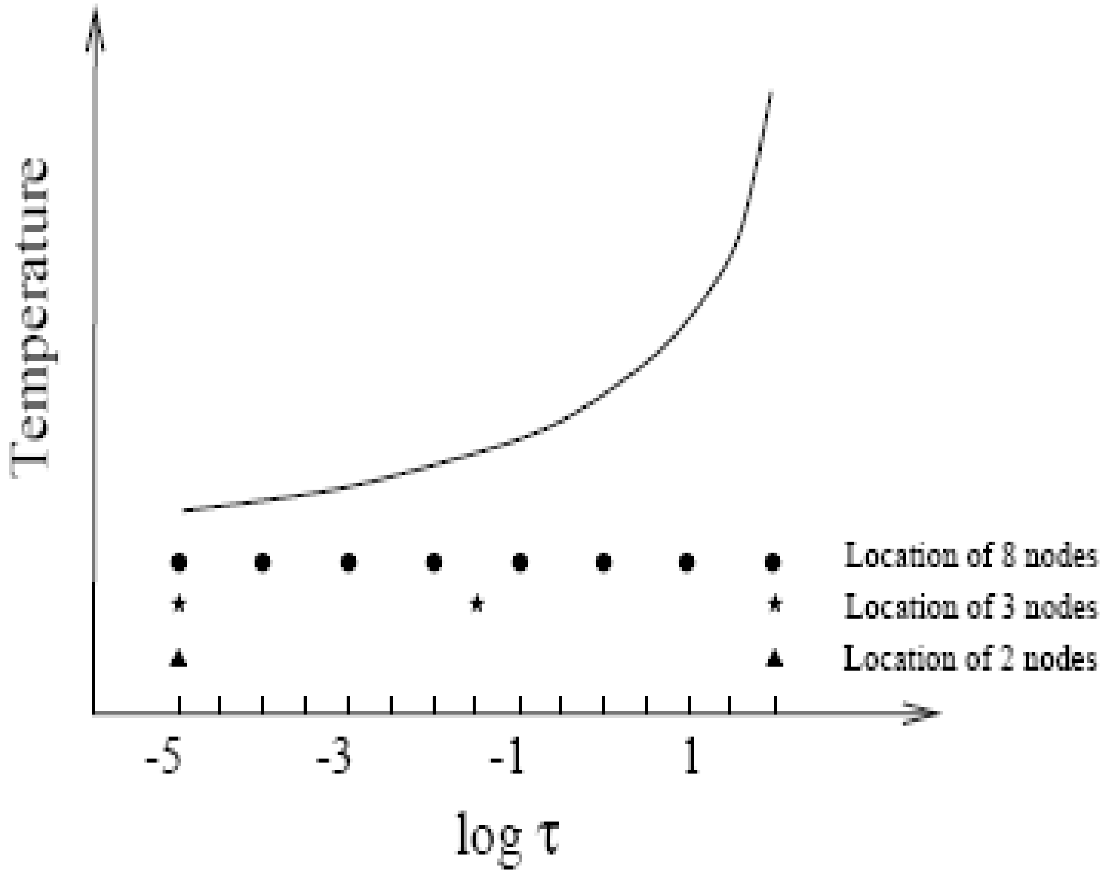
Westendorp Plaza et al. 2001, Astrophysical Journal 547, 1130


ATOMIC PARAMETERS
atom/ion-degree, λ_{lab}
DEF, EP$_{low}$, log(gf)
Term designation
INITIAL (GUESS) ATMOSPHERICAL MODELS
atm1 or atm1+atm2
$V_{mac}\,[km.s^{-1}]$ ff SLF$\equiv\alpha$ [%]
log(τ_5) T [K] P_e [$dyn.cm^{-2}$] V_{mic} [$cm.s^{-1}$] B [G] V_{LOS} [$cm.s^{-1}$] γ [°] ϕ [°]
STOKES SPECTRUM
LTE SYNTHESIS
$\mathbf{I} = ff_1\,\mathbf{I}_1 + ff_2\,\mathbf{I}_2$
$\mathbf{I}^* = \mathbf{I} * MAC_{Gauss} * IPS$
$\mathbf{I}_{syn} = (1-\alpha)\,\mathbf{I}^* + \alpha\mathbf{I}_{str}$
atm(s)
WILL BE MODIFIED
BY δa
MARQUARDT NONLINEAR LEAST-SQUARES ALGORITHM
MODIFIED SVD METHOD
$\chi^2 \equiv (\mathbf{I}_{obs} - \mathbf{I}_{syn})^2$
$\delta a_n = -A^{-1}\nabla_a\chi^2$
$\nabla_a\chi^2 \equiv \frac{\delta\chi^2}{\delta a} \sim (\mathbf{I}_{obs} - \mathbf{I}_{syn})\,RF$
$A^{-1}\left[\frac{\delta^2\chi^2}{\delta a_j \delta a_m} \sim RF\right]$
$RF = O\left\{K\frac{\delta\mathbf{S}}{\delta x} - \frac{\delta K}{\delta x}[\mathbf{I}-\mathbf{S}]\right\}$
λ and τ dependences are omitted
n – number of nodes in atmospherical model in course of cycle
LTE: $\mathbf{S} \equiv (B_\nu(T), 0, 0, 0)$
$j = i$
$\lambda_j = 10\lambda_i$
$j = i + 1$
$\lambda_j = \lambda_i/10$
$\frac{\chi_i^2 - \chi_{i+1}^2}{\chi_i^2 + \chi_{i+1}^2} \leq \varepsilon$
NO
YES
$\chi_{i+1}^2 < \chi_i^2$
NO
YES
PROCEED TO NEXT CYCLE
&
CHANGE NUMBER OF NODES
$n_k \rightarrow n_{k+1}$

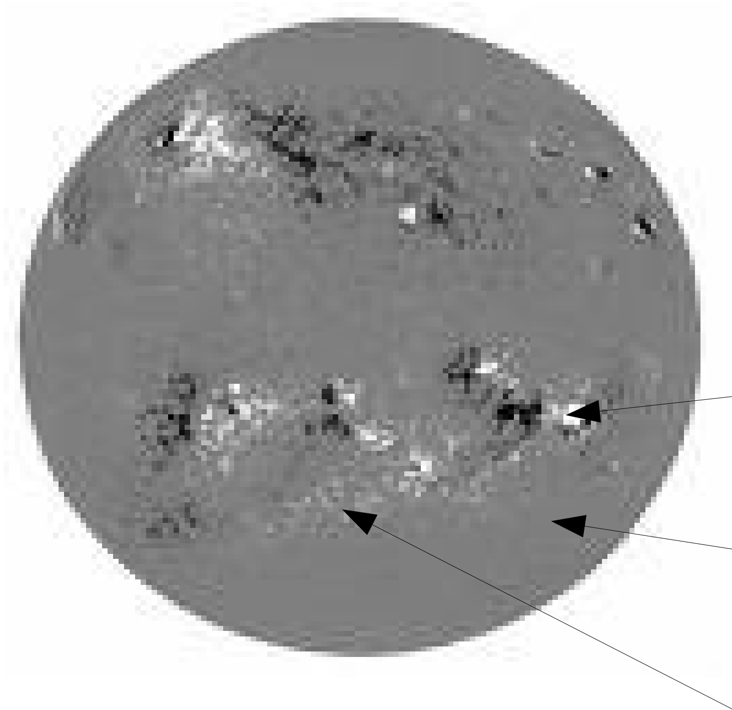
SIR scheme

Nodes concept

Computional time!



Problems



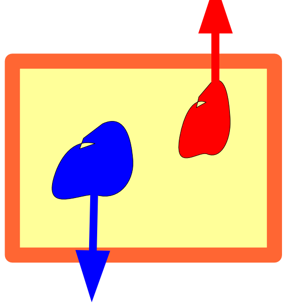
Sensitivity

$\frac{V}{I}$ Sunspot about 30%

Network 1%

Plage 10 %

Cancellation



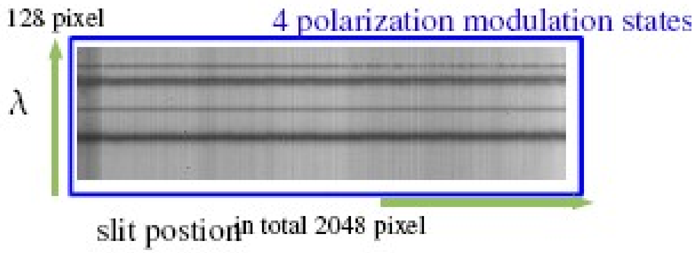
What happens if magnetic elements with opposite polarity in one resolution element?

SOLIS VSM data

- high sensitivity
- high temporal resolution,
- moderate spatial resolution

less than 20 min. for full disk
1.13 arcsec resolution

Fe I 630.1509 nm
Fe I 630.2502 nm



Input parameters

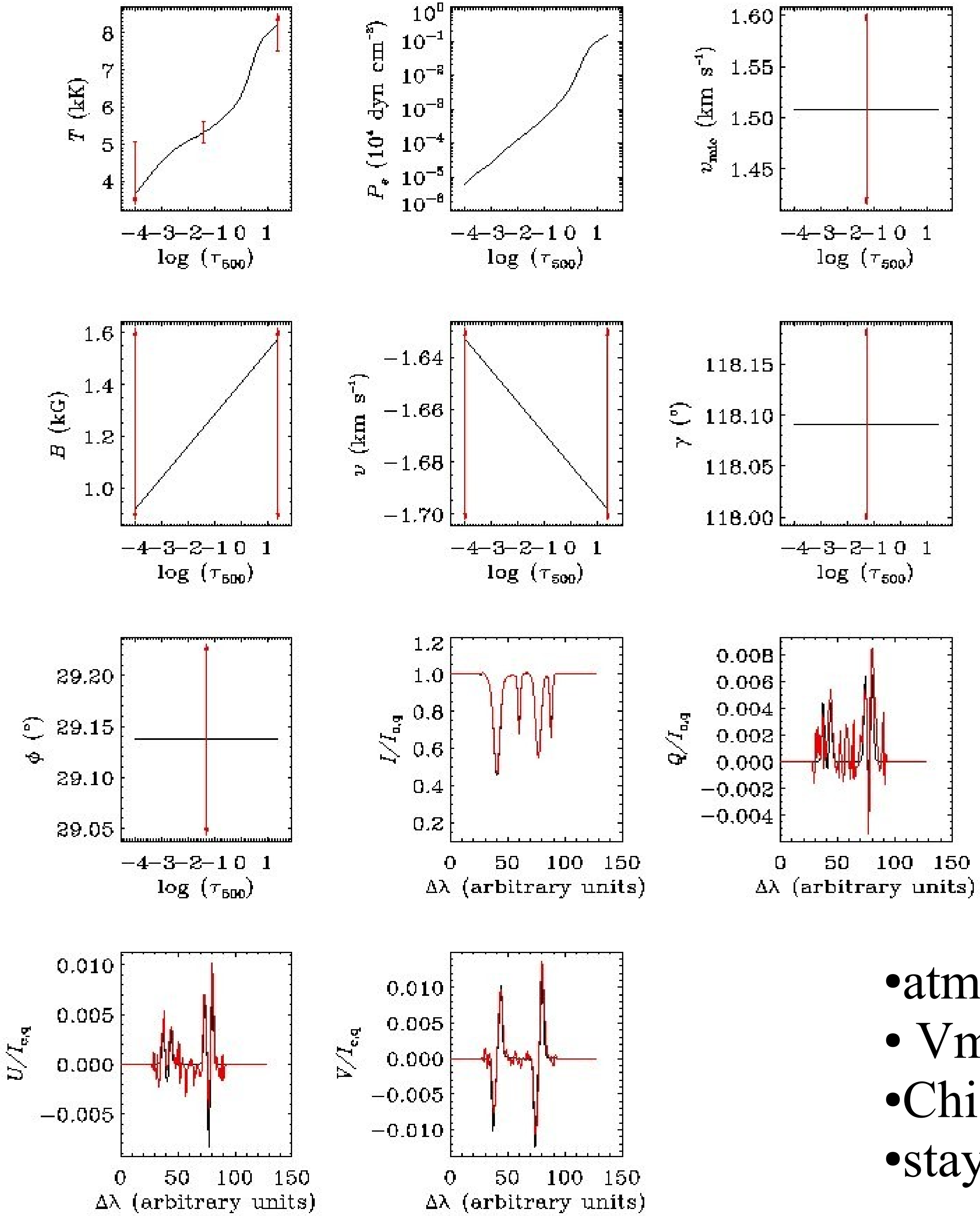
```
Mode: Inversion ! Mode of operation (Synthesis or Inversion)
!
! Fields required for both modes, synthesis and inversion:
!
Wavelength grid: HINODE.grid
Input Model: hsra.mod
Synthetic Profiles: invertprofiles.pro ! Output profiles
Heliocentric Angle: 1.0 ! Cos(mu)
!
! Fields required only for the inversion mode:
!
Observed Profiles: observed.pro
Output Model: modelout.mod
!
! Other misc optional fields
!
Formal solution method: 0 ! (0=auto, 1=Hermite, 2=WPM)
Stray light file: straylight.pro   ! Optional
Printout detail: 1 ! (default=1)
noise:0.000630795
```

guess model

position on sun

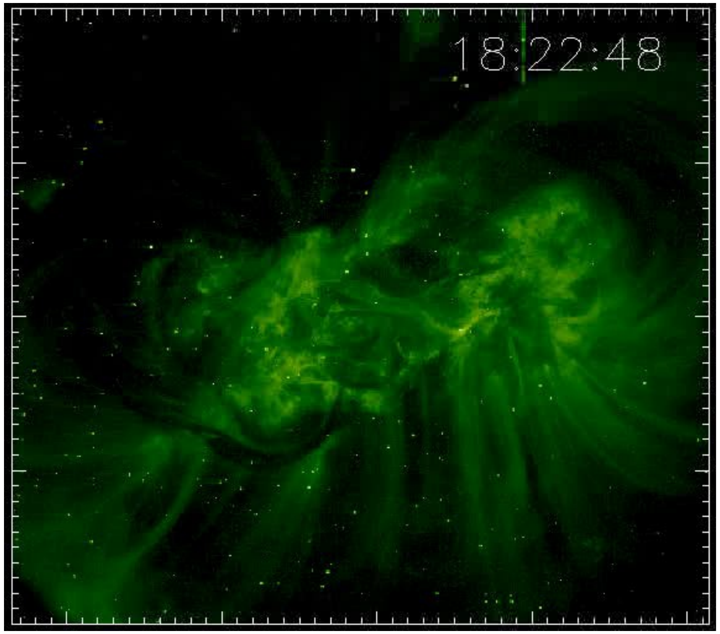
stray light can be used
as filling factor

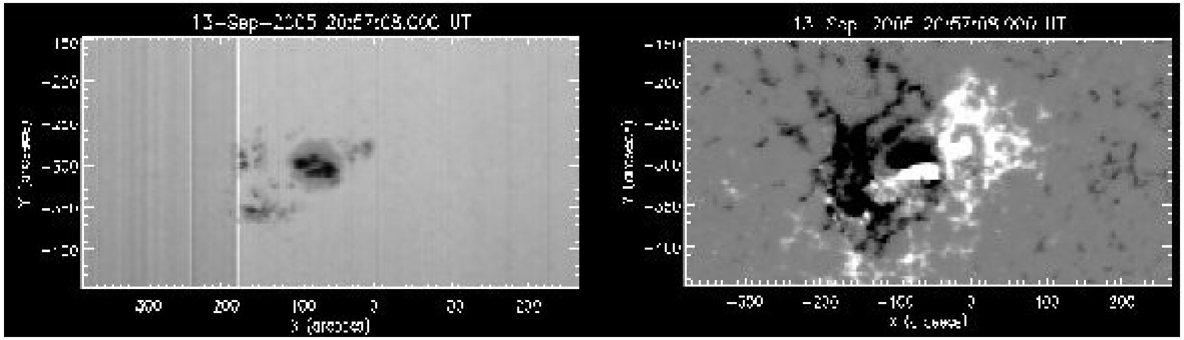
hydrostatic equilibrium
local thermodynamic equilibrium



- •atmospheric model from best fit
- • Vmac
- •Chisq.
- •staylight/filling factor

13 September 2005





covered by SOLIS

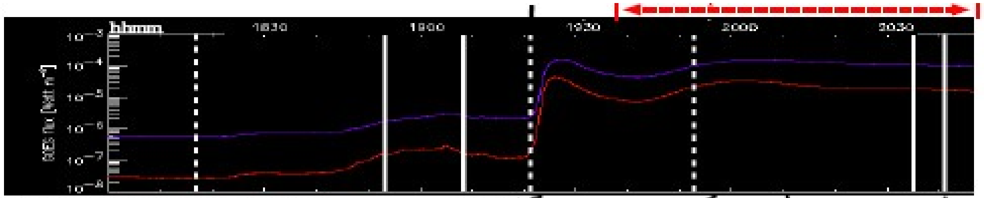
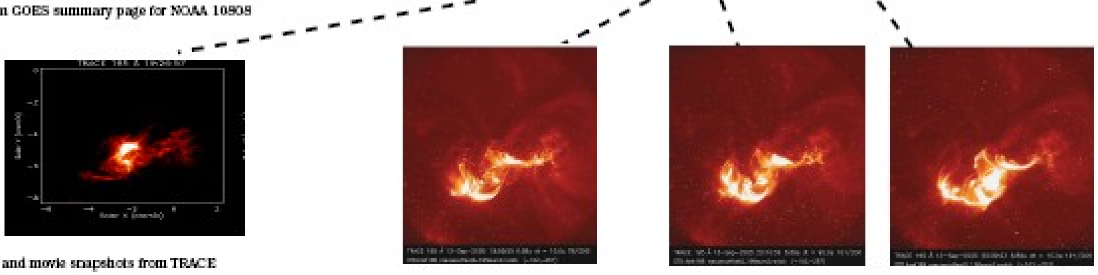


Image from GOES summary page for NOAA 10808



Images and movie snapshots from TRACE
at http://trace.lmsal.com

18 time steps
1.13 arcsec per pixel
5 minutes per areascan

Magnetic field strength in Gauss

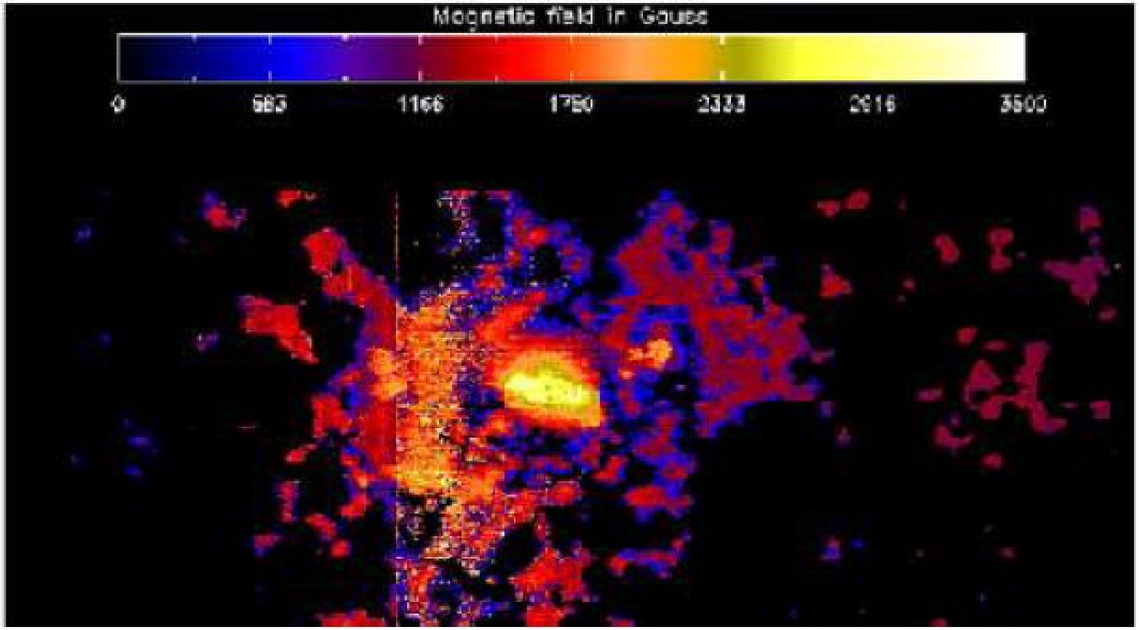
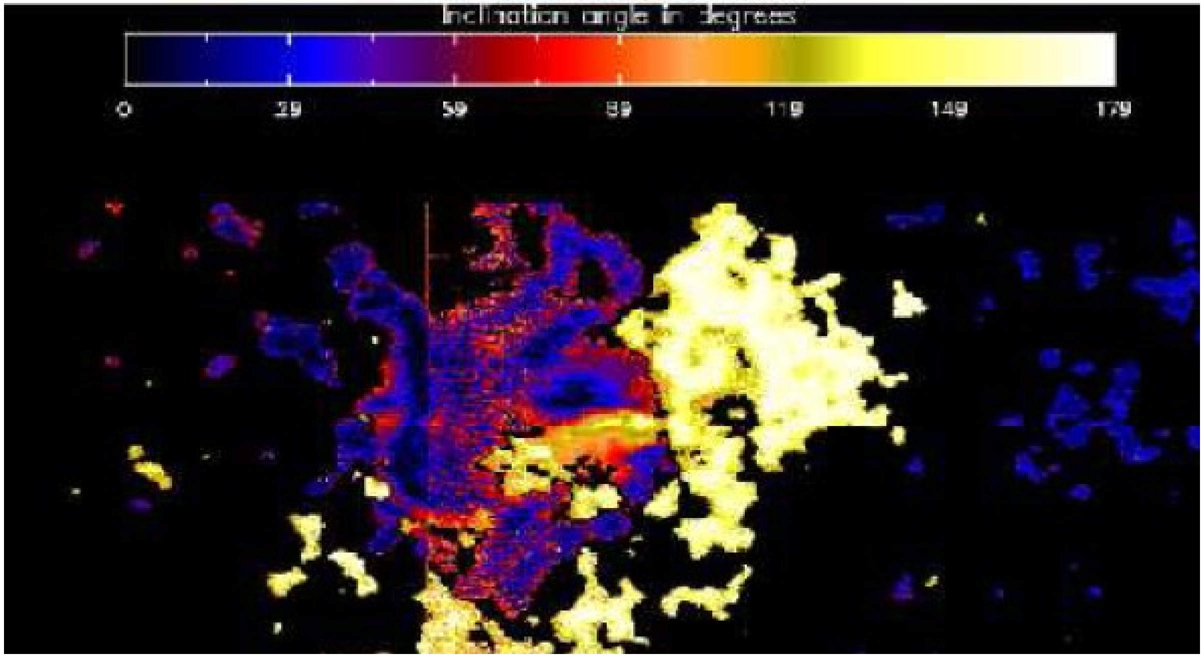


Figure 2. The figure shows the magnetic field strength in Gauss (here selected at optical depth $\tau_{500nm} = -1.0$) as the result of the inversion. The inverted area is 644 arsec times 339 arsec large. The stripes in the center of the images are due to noise in the original spectra, which was caused by a failing polarization modulator.

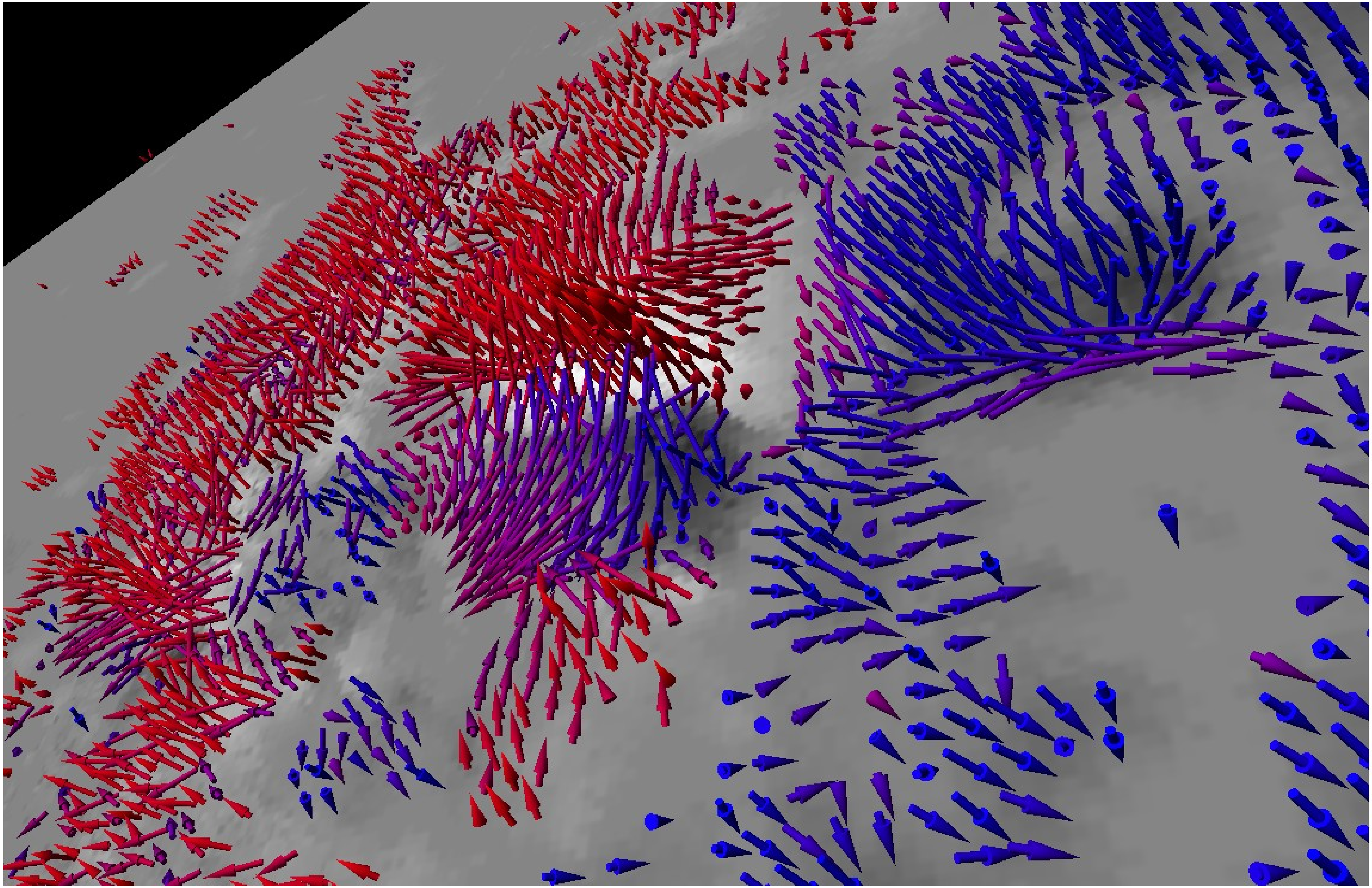
- 3 diff. model atmosphere
- condition for Chisq.
- straylight was 10 quiet
- sun surrounding pixels
- 1 timestep 2 d on 30 Nodes

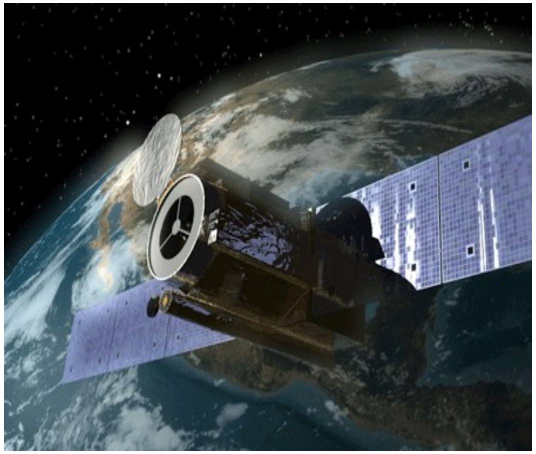
Inclination angle in degree



- Still have to resolve 180 degree ambiguity!
- Nonpotential field calculation (NPFC) Code from M.Georgoulis

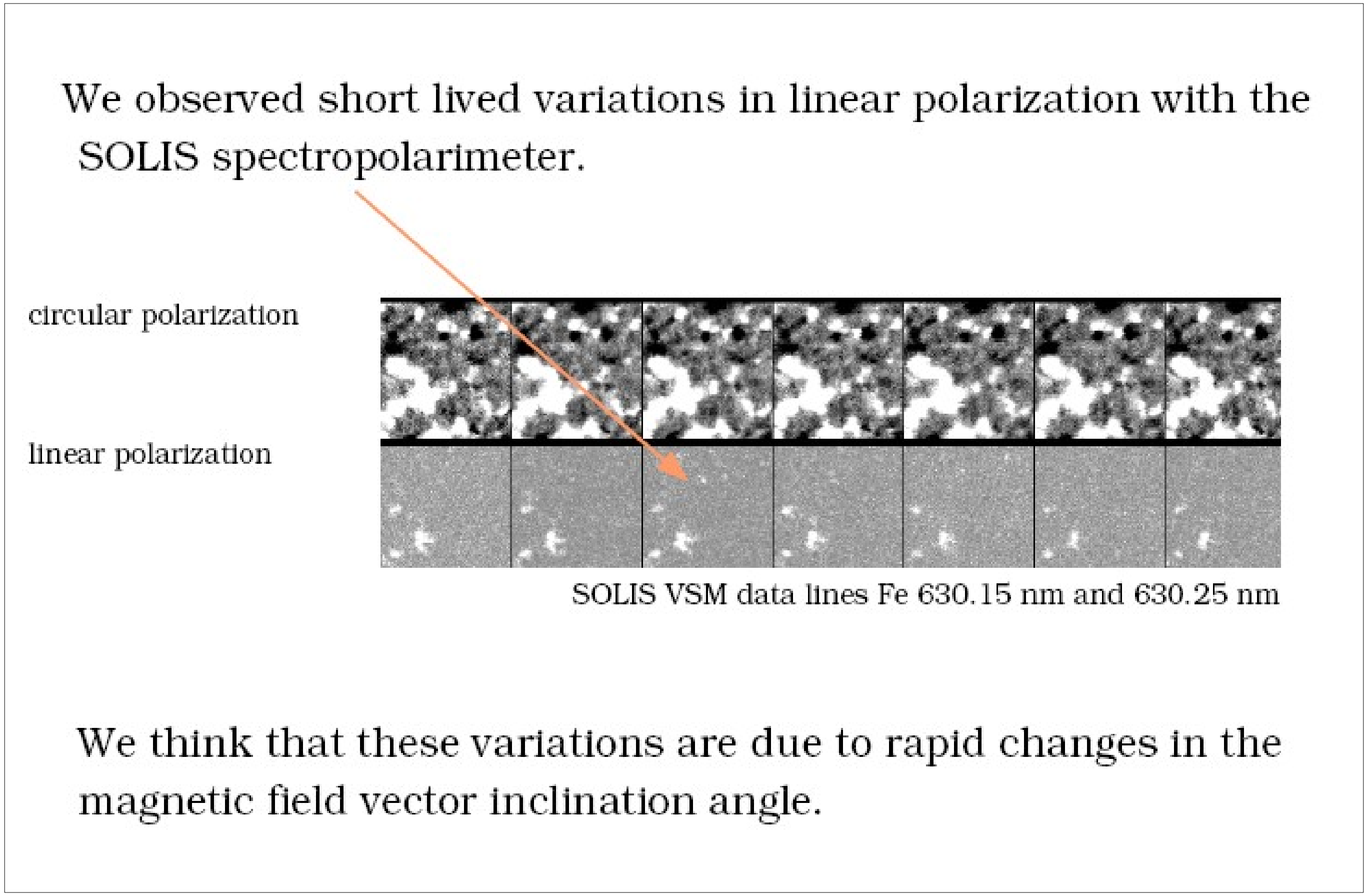
FINAL RESULT





Hinode quiet sun

Project



We observed short lived variations in linear polarization with the SOLIS spectropolarimeter.

circular polarization

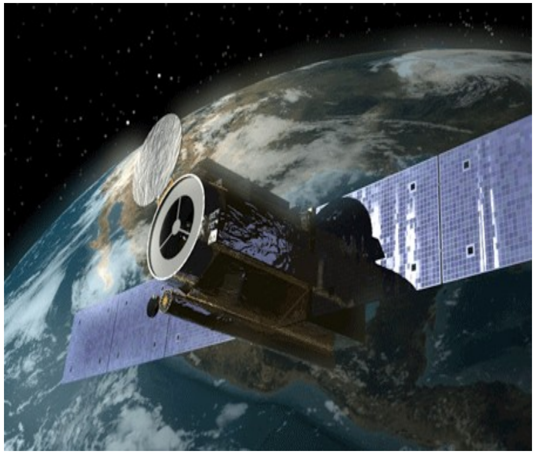
linear polarization

SOLIS VSM data lines Fe 630.15 nm and 630.25 nm

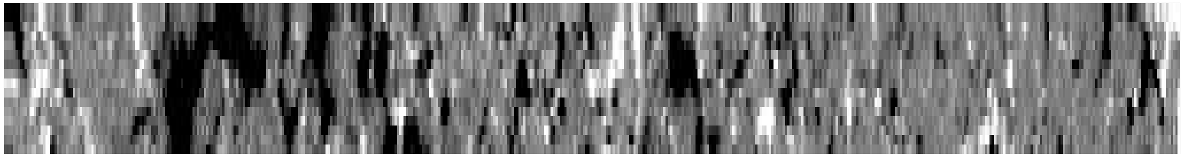
We think that these variations are due to rapid changes in the magnetic field vector inclination angle.

Problem: highly dynamic, low Signal to noise

Currently working with Helena Becher in order to improve the signal extraction.



Hinode quiet sun



Problem: highly dynamic,
low Signal to noise

Currently working with Helena Becher on
techniques in order to improve the signal
extraction.

Summary

- The radiative transfer equation is extended to deal with the Stokes vector. One obtains an absorption matrix.

- Stokes Inversions try to infer physical properties of the atmosphere by interpreting the Stokes profiles

- Inversion techniques rely on several assumptions and require a starting atmospheric model

- Only height dependent models can take care of asymmetries in the profiles

- Parameters such as the filling factor, stray light and velocities have to be taken into account

Exercises

Part of Practicum:

We will write a simple inversion code in the practicum using actual SOLIS VSM data ...

$$\rho_Q = \frac{k_l}{2}\left(\rho_\pi - \frac{\rho_{\sigma+} + \rho_{\sigma-}}{2}\right)\sin^2\theta\cos 2\phi$$

$$\rho_U = \frac{k_l}{2}\left(\rho_\pi - \frac{\rho_{\sigma+} + \rho_{\sigma-}}{2}\right)\sin^2\theta\sin 2\phi$$

$$\rho_V = \frac{k_l}{2}\left(\frac{\rho_{\sigma+} - \rho_{\sigma-}}{2}\right)$$