Lecture 8: Spectral Line Diagnostics 2

Outline

Radiative Transfer Equation

- radiative transfer equation without ν subscript

$$\mu \frac{\mathrm{d}I}{\mathrm{d}\tau} = I - S$$

- formal solution

$$I_\nu(\tau, \mu) = I(\tau_0, \mu) e^{-(\tau_0 - \tau)/\mu} + \frac{1}{\mu} \int_\tau^{\tau_0} S(\tau') e^{-\frac{\tau' - \tau}{\mu}} \mathrm{d}\tau'$$

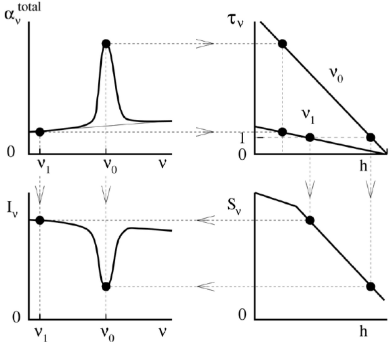
- emergent intensity by integration from $\tau = 0$ to $\tau_0 = \infty$

$$I(\tau = 0, \mu) = \frac{1}{\mu} \int_0^\infty S(\tau) e^{\frac{-\tau}{\mu}} \mathrm{d}\tau$$

- LTE: source function = Planck function
- Eddington-Barbier: $I(\tau = 0, \mu) = S(\tau = \mu)$

Simple Absorption Line

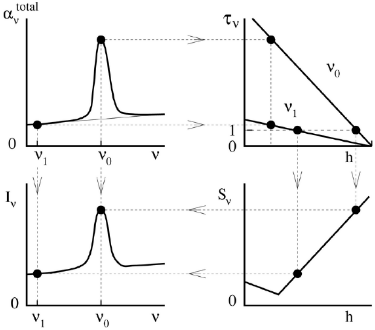
- absorption: transition gives peak in $\kappa = \kappa_C + \kappa_l = (1 + \eta_\nu)\,\kappa_C$
- optical depth: height-invariant $\kappa \Rightarrow$ linear $(1 + \eta_\nu)\,\tau_C$
- source function: same for line and continuum
- intensity: Eddington-Barbier (nearly) exact



Courtesy R.J.Rutten

Simple Emission Line

- extinction: transition process gives peak in $\kappa = \kappa_C + \kappa_l = (1 + \eta_\nu)\,\kappa_C$
- optical depth: height-invariant $\kappa \Rightarrow$ linear $(1 + \eta_\nu)\,\tau_C$
- source function: same for line and continuum
- intensity: Eddington-Barbier (nearly) exact



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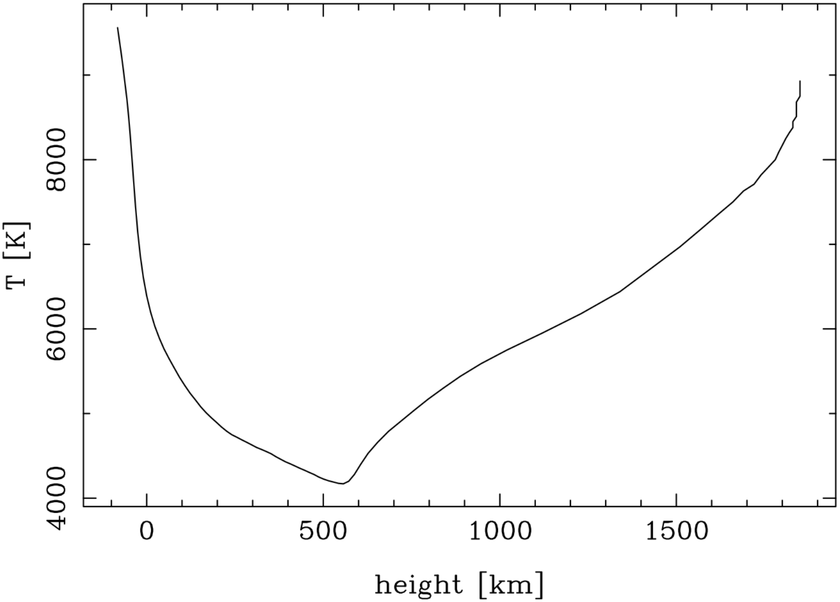
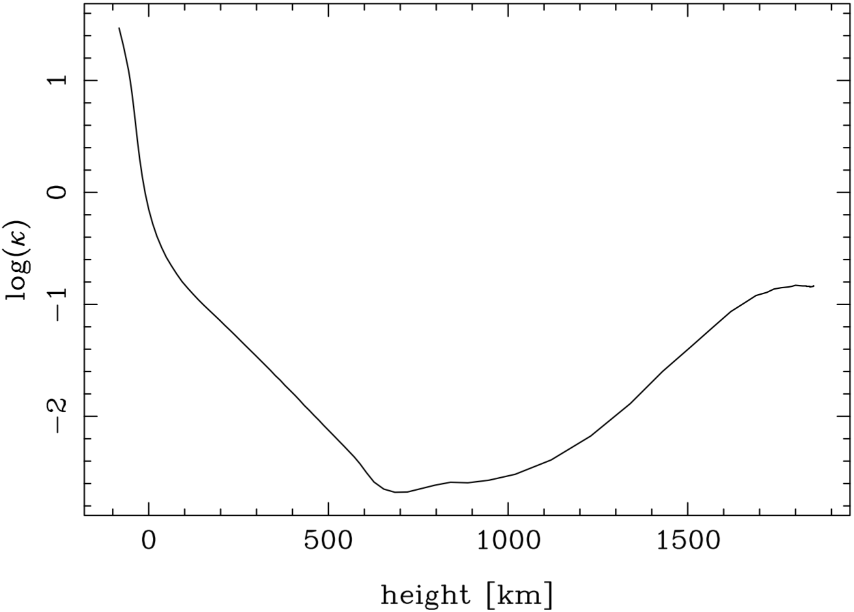
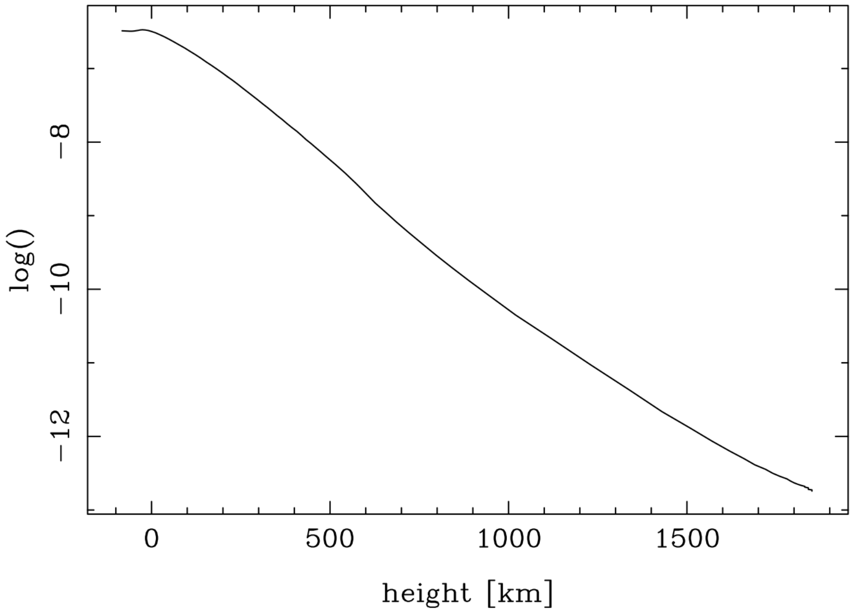
Temperature vs Height in Quiet Solar Atmosphere



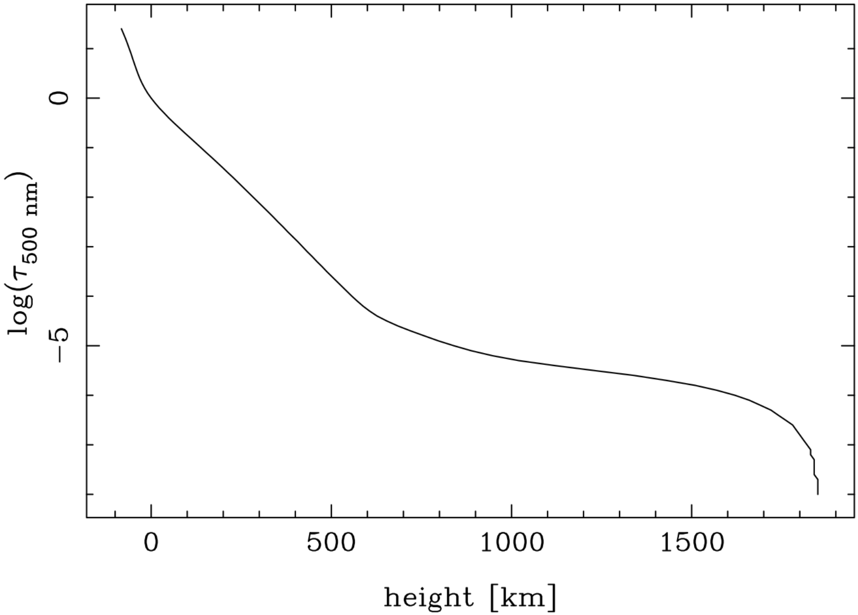
Absorption Coefficient at 500 nm vs Height in Quiet Solar Atmosphere



Density vs Height in Quiet Solar Atmosphere

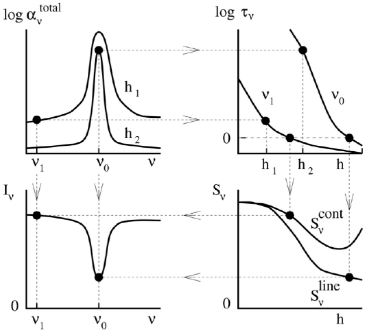


Optical Depth at 500 nm vs Height in Quiet Solar Atmosphere



Realistic, Strong Scattering Absorption Line

- absorbtion: transition peak lower and narrower at larger height
- optical depth: near-log-linear inward increase
- source function: split into line and continuum source functions
- intensity: Eddington-Barbier for $S_\nu^{\mathrm{total}} = (\kappa_C S_C + \kappa_l S_l)/(\kappa_C + \kappa_l) = (S_C + \eta_\nu S_l)/(1 + \eta_\nu)$



Courtesy R.J.Rutten

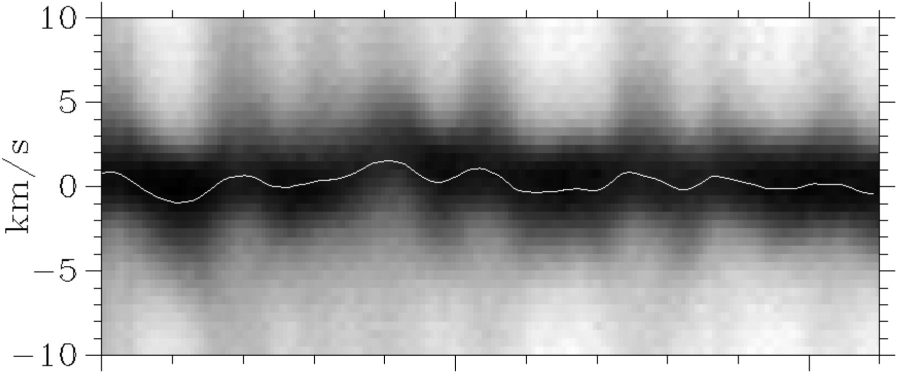
Numerical Absorption Line Calculation in LTE

- solar atmosphere model as function of height: temperature, density, pressure, velocity, (microtuburbulence, magnetic field vector)
- spectral line atomic information: element/molecule, abundance, transition probability/oscillator strength, ionization stage, excitation potential of lower level, upper level, partition function
- compute continuum absorption coefficient κ_C
- integrate κ_C to obtain continuum optical depth τ_C
- calculate continuum intensity from $\int B_\nu e^{-\tau/\mu} d\tau$
- calculate line absorption κ_L for each wavelength
- solve $I_\nu(\tau_\nu = 0, \mu) = \frac{1}{\mu} \int_0^\infty S_\nu(\tau_\nu) e^{\frac{-\tau_\nu}{\mu}} \mathrm{d}\tau_\nu$
- convolve with macroturbulence profile

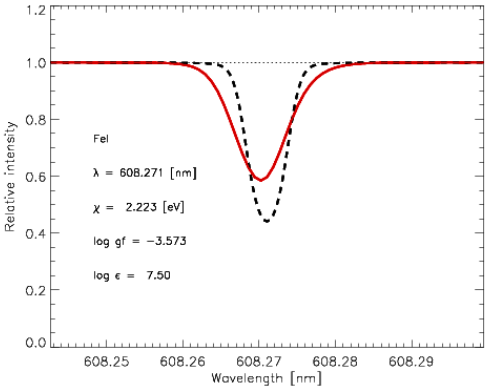
Spectral Line Diagnostics

- velocity from Doppler shift, also as function of height
- microturbulence
- macroturbulence, spectrograph smearing
- abundance
- temperature, density, pressure

Observed Doppler Shifts in Quiet Solar Atmosphere



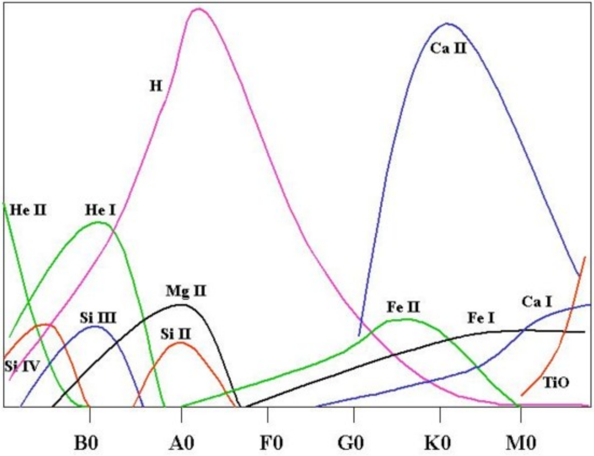
Line Broadening due to Convection in Quiet Solar Atmosphere



Predicted spatially and temporally averaged 3D LTE solar line profile (solid line) compared with calculation when ignoring all Doppler shifts due to photospheric velocity field (dashed line)

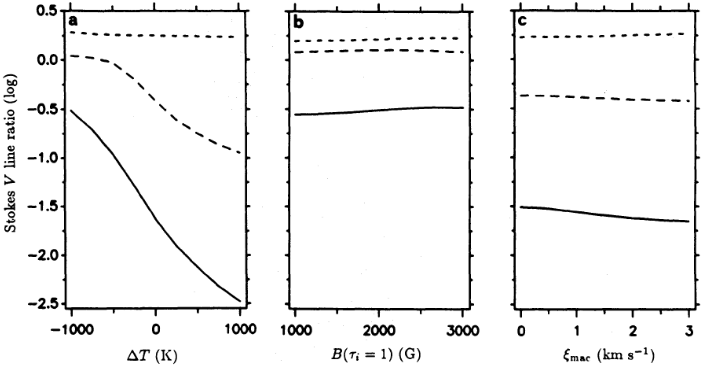
From http://solarphysics.livingreviews.org/Articles/lrsp-2009-2

Ionization Equilibria as Function of Temperature



http://www.astrogeo.va.it/astronom/spettri/teoriaen.htm

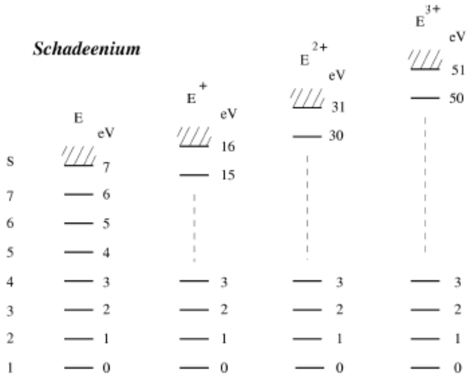
Temperature in Magnetic Fields



Keller et al. (1990)

Model Atom

- given species and ionization states
- *N* discrete *bound states*
- *bound-bound* transitions between:
  - lower state *L* with energy E_L
  - upper state *U* with energy E_U
- *bound-free and free-bound transitions* between states and continuum
- *free-free transitions between continuum and continuum*



Courtesy R.J.Rutten

Spontaneous Emission

- $n_U A_{UL}$ photons are spontaneously emitted per unit time and volume
- states are not necessarily sharp, Doppler broadening $\Rightarrow$ frequency distribution $\chi(\nu)$
- emission in arbitrary direction:

$$n_U A_{UL} \chi(\nu)/4\pi$$

- A_{UL}: *Einstein coefficient* for *spontaneous emission*
- inverse of lifetime in upper state against spontaneous emission
- typical values for lifetime: 10^{-8} s
- forbidden transitions have lifetimes of up to 10^{-2} s

Induced (or Stimulated) Emission

- atom in upper state
- radiation with frequency $\nu = (E_U - E_L)/h$
- n_U atoms in upper level
- finite width of emitted line: $\psi(\nu)$
- induced emission in the same direction as inducing radiation:

$$n_U B_{UL} I_\nu \psi(\nu)/4\pi$$

- B_{UL}: *Einstein coefficient* of *induced emission*

Absorption (or Radiative Excitation)

- opposite of induced emission
- n_L atoms in lower state
- radiation with frequency $\nu = (E_U - E_L)/h$
- absorption profile $\phi(\nu)$
- absorption of incoming radiation

$$n_L B_{LU} I_\nu \phi(\nu)/4\pi$$

- B_{LU} *Einstein coefficient* of *radiative excitation*

Continuum Radiation

Photoionization

- transitions from bound states to continuum (*bound-free*)
- energy of radiation larger than energy between bound states and continuum
- proportional to incident intensity I_ν
- from level j photo-ionization cross-section $\alpha_j(\nu)$
- $\alpha_j(\nu) I_\nu$ is absorbed energy per time, frequency, target atom, and solid angle
- number density of atoms n_j in state j
- number of photoionizations from level j per time, volume, frequency interval, solid angle

$$n_j \alpha_j(\nu) I_\nu / h\nu$$

- $\alpha_j(\nu)$ typically from measurements

Radiative Recombination

- inverse of photoionization
- induced component proportional to intensity and spontaneous component
- both components proportional to number of atoms n_C in continuum state C
- number of radiative recombinations per time, volume, frequency interval, solid angle

$$n_C \left(\gamma_J(\nu) + \beta_J(\nu) I_\nu\right) / h\nu$$

- frequency interval corresponds to certain velocity interval of electron distribution

Collisions

Collisional Transition Rates

- collisional transitions between levels of atoms
- collisions have no direct influence on radiation field
- but indirectly by affecting population of atomic levels
- C_{ij} *collisional transition rates* from level i to j
- collisions between (fast) electrons and (slow) atoms
- ratio of collision rates C_{ij}/C_{ji} can only depend on atomic properties and particle distribution
- if velocity distribution is in LTE

$$\frac{C_{ij}}{C_{ji}} = \frac{n_j^*}{n_l^*}$$

Bound-Bound and Bound-Free Collisional Transitions Rates

- transitions between upper and lower state

$$C_{UL} = \frac{g_L}{g_U} e^{\frac{E_U - E_L}{kT}} C_{LU}$$

- transitions to and from the continuum

$$C_{Cj} = \left(\frac{h^2}{2\pi m_e kT} \right)^{\frac{3}{2}} \frac{n_e g_j}{2u_C} e^{\frac{E_C - E_j}{kT}} C_{jC}$$

Overview

- departure from local thermal equilibrium, i.e. upper atmosphere
- both continuum and line absorption in NLTE because lower levels do not obey Boltzmann's law
- assume Maxwellian velocity distribution because of frequent collisions
- in corona and solar wind non-Maxwellian velocity distributions
- assume that all particles have Maxwellian velocity distribution with the same temperature
- collisions with electrons play dominant role for atomic processes $\Rightarrow$ assume $T = T_e$

NLTE Line Source Function

- differentiate between line and continuum

$$\kappa = \kappa_l + \kappa_C \quad \epsilon = \epsilon_l + \epsilon_C$$

- source function for spectral line radiation

$$S_l = \frac{n_U A_{UL} \chi}{n_L B_{LU} \phi - n_U B_{UL} \psi}$$

- *Complete Redistribution (CRD)* under special circumstances

$$\chi(\nu) = \phi(\nu) = \psi(\nu)$$

- line source function independent of frequency

$$S_l = \frac{n_U A_{UL}}{n_L B_{LU} - n_U B_{UL}}$$

- General case: *Partial Redistribution (PRD)*

NLTE Source Function

- total source function is weighted sum of individual source functions

$$S = \frac{\kappa_l S_l + \kappa_C S_C}{\kappa_l + \kappa_C}$$

Statistical Equilibrium

- in statistical equilibrium, number of transitions from and to a level must be the same
- 2-level atom with states L and U only

$$n_L\left(B_{LU}\bar{J}_{LU}+C_{LU}\right)=n_U\left(A_{UL}+B_{UL}\bar{J}_{UL}+C_{UL}\right)$$

- intensity averaged over frequency and solid angle assuming CRD

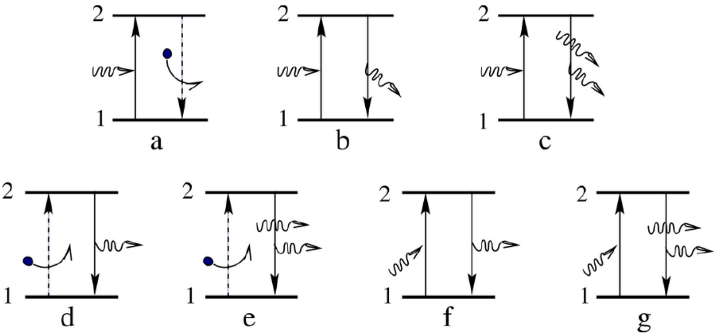
$$\bar{J}_{LU}=\bar{J}_{UL}=\frac{1}{4\pi}\int I_\nu\phi(\nu)\mathrm{d}\nu\mathrm{d}\Omega$$

- abundance ϵ relative to hydrogen with number density n_H

$$n_L+n_U=\epsilon n_H$$

- source function S depends on radiation field $\bar{J}$

Equilibria: LTE

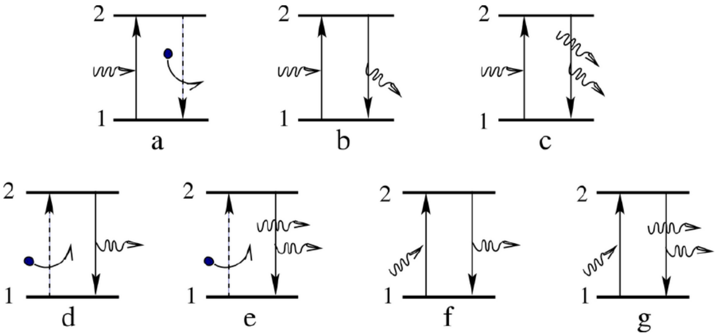


Courtesy R.J.Rutten

LTE = large collision frequency – interior, low photosphere

- up: mostly collisional = thermal creation (d + e)
- down: mostly collisional = large destruction probability (a)
- photon travel: "honorary gas particles" or negligible leak

Equilibria: NLTE

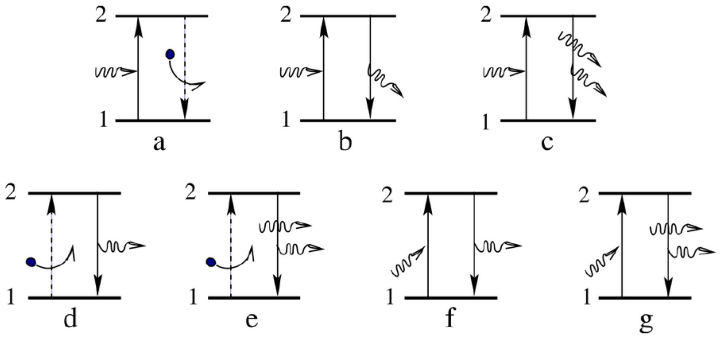


Courtesy R.J.Rutten

NLTE = statistical equilibrium – chromosphere, TR

- photon travel: non-local impinging (pumping), loss (suction)
- two-level scattering: coherent/complete/partial redistribution
- multi-level: photon conversion, sensitivity transcription

Equilibria: Coronal Equilibrium



Courtesy R.J.Rutten

coronal equilibrium = *hot tenuous* – coronal EUV

- up: only collisional = thermal creation (only d)
- down: only spontaneous (only d)
- photon travel: escape / drown / scatter bf H I, He I, He II

Relations between Einstein Coefficients

- monochromatic bound-bound rates expressed in Einstein coefficients (intensity units)

$$
\begin{aligned}
n_u A_{ul} \chi(\nu)/4\pi \\
n_u B_{ul} I_\nu \psi(\nu)/4\pi \\
n_l B_{lu} I_\nu \phi(\nu)/4\pi \\
n_u C_{ul} \ , n_l C_{lu}
\end{aligned}
$$

- Einstein relations (LTE and NLTE)

$$
\begin{aligned}
g_u B_{ul} &= g_l B_{lu} \\
(g_u/g_l) A_{ul} &= (2h\nu^3/c^2)\, B_{lu} \\
C_{ul}/C_{lu} &= (g_l/g_u) \exp(E_{ul}/kT)
\end{aligned}
$$

Statistical Equilibrium Equations

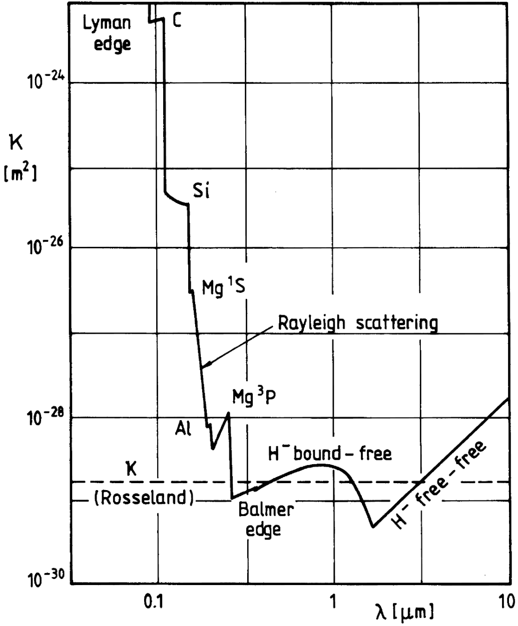
- statistical equilibrium equations for level j

$$n_j \sum_{j \neq i}^{N} R_{ji} = \sum_{j \neq i}^{N} n_j R_{ij}$$

$$R_{ji} = A_{ji} + B_{ji}\overline{J_{ji}} + C_{ji}$$

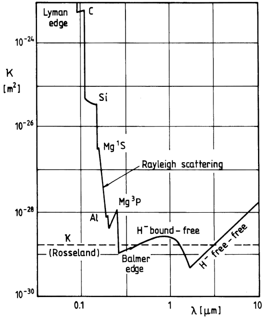
$$\overline{J_{ji}} \equiv \frac{1}{4\pi} \int_0^{4\pi} \int_0^{\infty} I_\nu \, \phi(\nu) d\nu d\Omega$$

Continuum Opacity in Solar Atmosphere



H$^-$ Opacity

- hydrogen atom with additional electron
- requires neutral hydrogen and free electrons
- binding energy: 0.754 eV (1.64μm)
- opacity minima at 0.36μm and 1.6μm
- major opacity in solar-like stellar photospheres
- Balmer jump at 3646 Å: electrons in second energy level are ionized
- bound-free in visible, free-free in infrared and radio



Radiative Processes in Solar Atmosphere

- *bound-bound* – S_ν, κ_ν NLTE? PRD?
  - neutral atom transitions
  - ion transitions
  - molecule transitions
- *bound-free* – S_ν, κ_ν NLTE? always CRD
  - H^- optical, near-infrared
  - HI Balmer, Lyman; HeI, HeII
  - FeI, SiI, MgI, AlI electron donors
- *free-free* – S_ν always LTE, κ_ν NLTE
  - H^- infrared, sub-mm
  - HI radio

Radiative Processes in Solar Atmosphere (continued)

- *electron scattering* – always NLTE, Doppler?
  - Thomson scattering
  - Rayleigh scattering
- *collective* – p.m.
  - cyclotron, synchrotron radiation
  - plasma radiation

Radio Continuum

- free-free/thermal bremsstrahlung: individual electrons being accelerated by Coulomb field of ions
- main emission/absorption at radio wavelengths
- incoherent radiation from optically thick source
- intensity only depends on temperature of emitting electrons
- intensity expressed as brightness temperature T_b (in Rayleigh-Jeans approximation)

$$T_b = \frac{I_\nu c^2}{2k\nu^2} ,$$

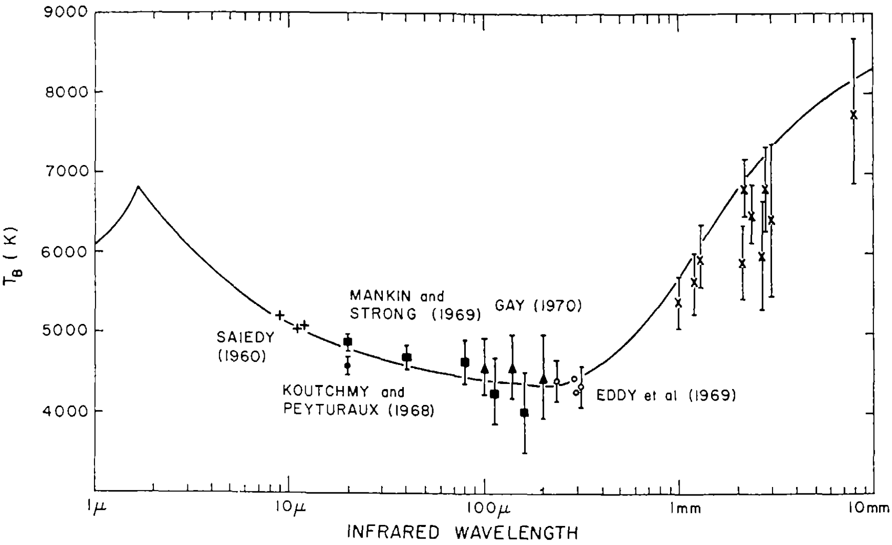
Radio Continuum (continued)

- for given atmospheric stratification

$$T_b = \int_0^{\tau_{max}} T e^{-\tau} d\tau \ ,$$

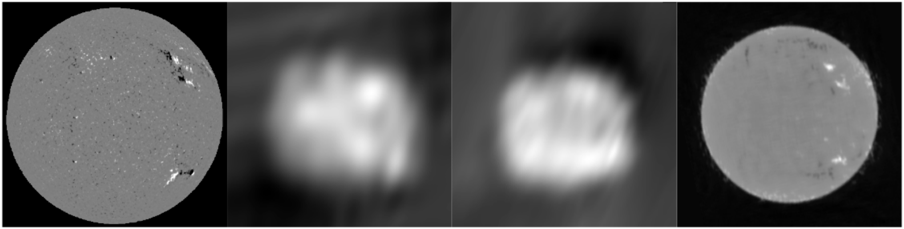
- T is temperature, τ is optical depth, $\tau_{max} >> 1$ is suitably large optical depth
- can be used to test models of quiet sun temperature stratification with T_b as a function of wavelength and center-to-limb variation
- problem 1: no information about optical depth
- problem 2: solar atmosphere is inhomogeneous in space and time
- problem 3: intensity only depends on square of temperature
- problem 4: optically thin corona contributes, too

Full-Disk Solar Radio Observations



Harvard Smithsonian Reference Atmosphere, Gingerich et al. (1971)

Full-Disk Solar Radio Observations

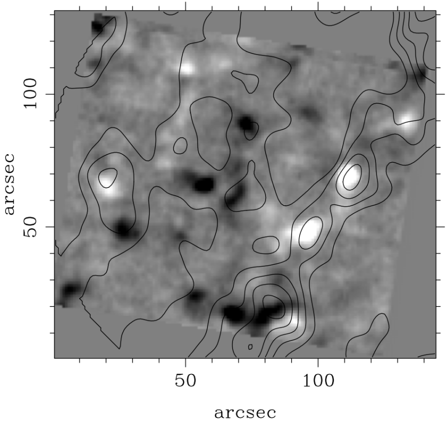


Qu

Sun on Feb. 1, 1998 (from left to right)

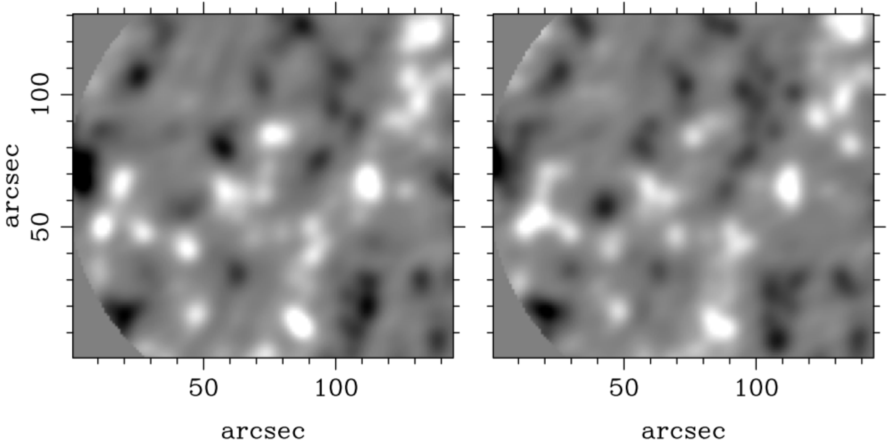
1. Kitt Peak magnetogram
2. Nançay radioheliograph at 164 MHz
3. Nançay radioheliograph at 327 MHz
4. Nobeyama radioheliograph at 17 GHz

Quiet Sun at 2 cm



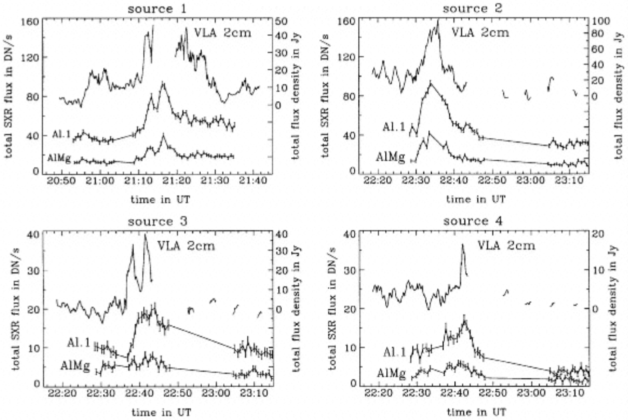
Magnetogram of quiet Sun close to disk center with contour lines showing 2-cm radio emission intensity as observed with VLA

Rapid Variation of 2-cm Radio Signal



Two radio maps of the quiet Sun obtained with the VLA at 2 cm spaced 2 minutes apart

Connection between Radio and X-Ray Emission



Comparison of network flares in soft X-ray and radio waves at 2 cm (from Krucker et al. 1997)