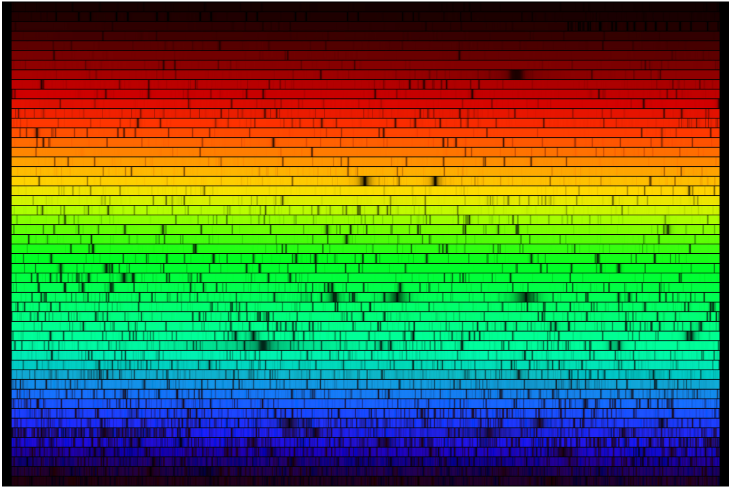
Lecture 7: Spectral Line Diagnostics 1

Outline

Motivation

The Visible Solar Spectrum



N.A.Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF

What can be extracted from spectral lines?

- elemental and molecular compositions (abundances)
- atmospheric properties: temperature, pressure, density
- velocity fields: rotation, convection, turbulence, gravity
- magnetic and electrical fields

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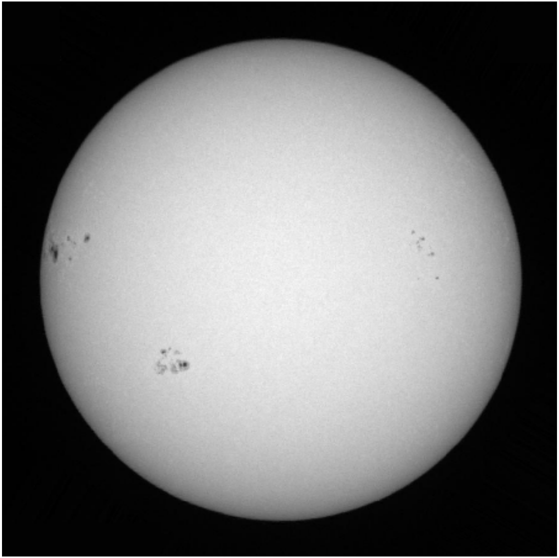
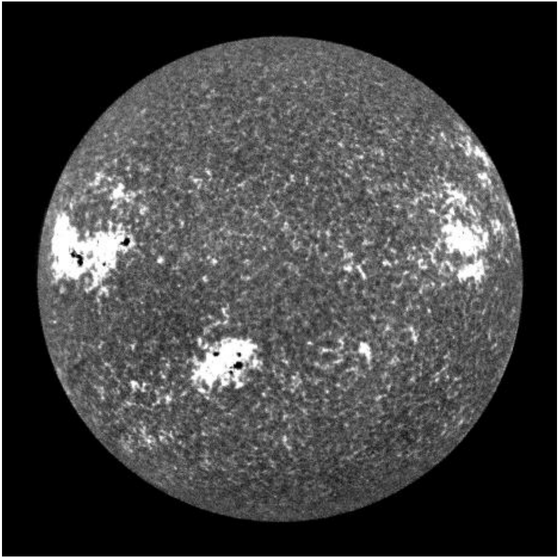
White-Light



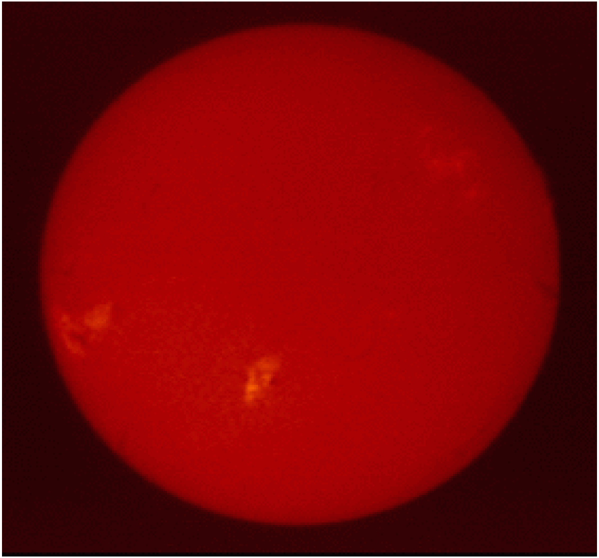
Courtesy of Big Bear Solar Observatory

Calcium II K



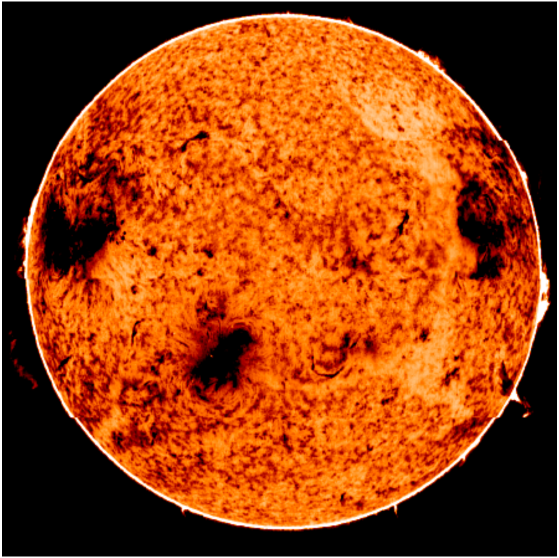
Courtesy of Big Bear Solar Observatory

Hα



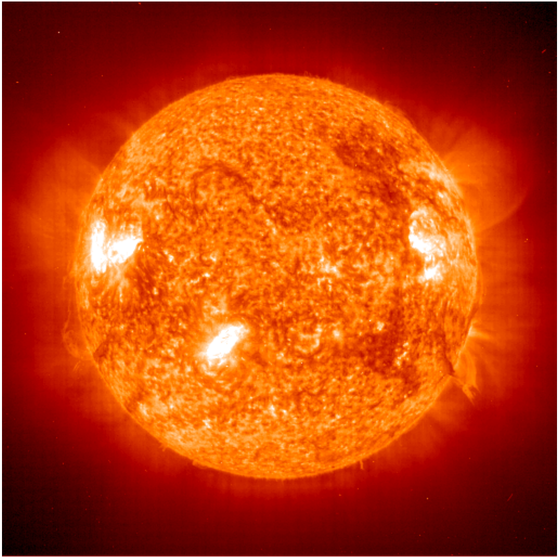
Courtesy of Learmonth Solar Observatory

HeI 1083.0 nm



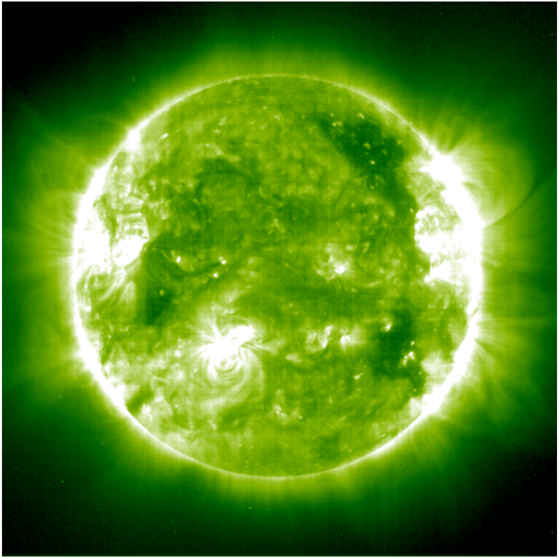
Courtesy of National Solar Observatory

HeII 304 Å



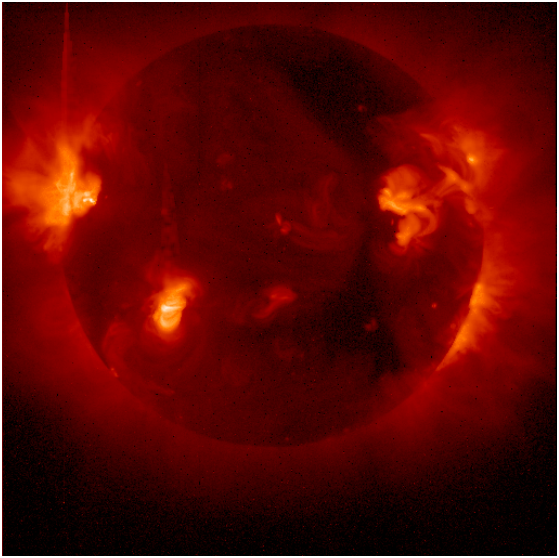
Courtesy of SOHO/EIT consortium

FeXII 195 Å



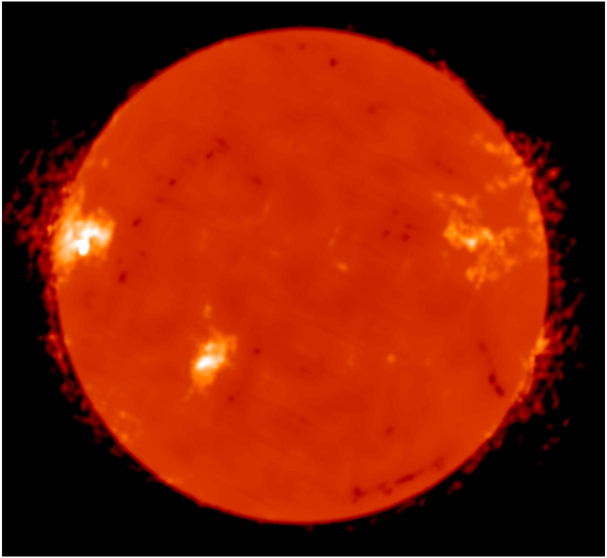
Courtesy of SOHO/EIT consortium

X-ray



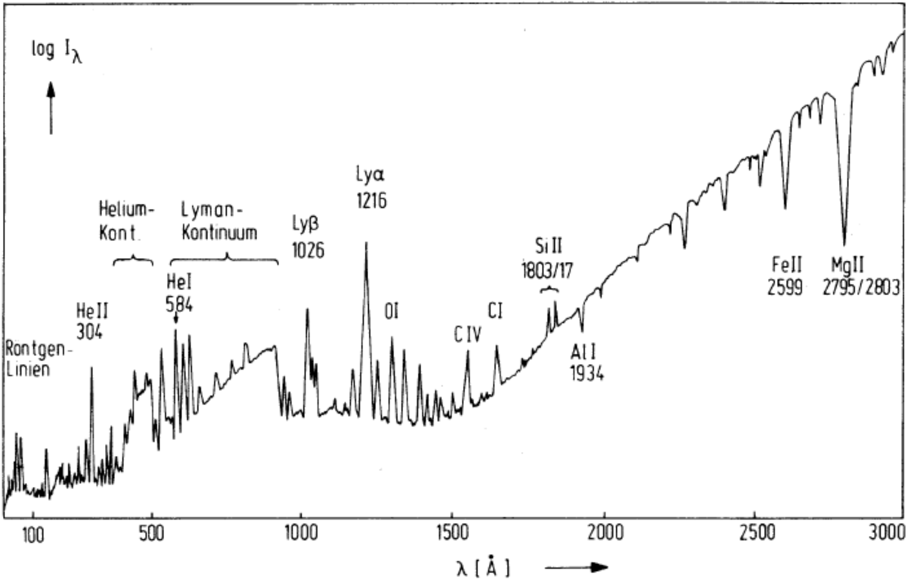
Courtesy of Yohkoh mission

Radio at 1.7 cm



Courtesy of Nobeyama Radio Observatory

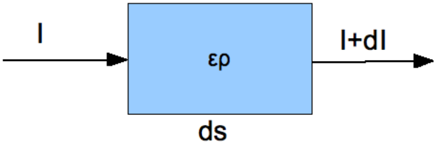
Solar Spectrum from X-rays to UV



Local Emission

- local emission by volume: $\mathrm{d}I_\nu = \epsilon_\nu \mathrm{d}s$
  - ν light frequency
  - I_ν intensity
  - $\mathrm{d}I_\nu$ change in intensity
  - $\mathrm{d}s$ infinitesimal path length
  - s *geometrical path length* along the beam
  - ϵ_ν *emission coefficient* (by volume)
- local emission by mass: $\mathrm{d}I_\nu = \epsilon_\nu \rho \mathrm{d}s$
  - ρ density
  - ϵ_ν *emission coefficient* (by mass)

Local Emission



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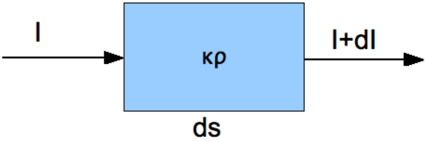
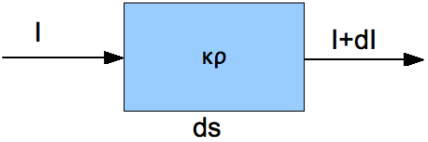
Local Absorption



- local absorption by volume: $\mathrm{d}I_\nu = -\sigma_\nu n I_\nu \mathrm{d}s = -\alpha_\nu I_\nu \mathrm{d}s$
  - $\mathrm{d}I_\nu$ change in intensity
  - I_ν intensity
  - $\mathrm{d}s$ infinitesimal path length
  - σ_ν *cross-section* per particle
  - n *absorber density* in particles per volume
  - $\alpha_\nu = \sigma_\nu n$ *extinction coefficient*
- local absorption by mass: $\mathrm{d}I_\nu = -\kappa_\nu \rho I_\nu \mathrm{d}s$
  - κ_ν *absorption coefficient*
  - ρ density

Local Absorption



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Optical Depth

- local absorption by mass: $\mathrm{d}I_\nu(s) = -\kappa_\nu(s)\rho(s)I_\nu(s)\mathrm{d}s$
- dividing by intensity $I_\nu(s)$

$$\frac{\mathrm{d}I_\nu(s)}{I_\nu(s)} = \mathrm{d}\left(\ln I_\nu(s)\right) = -\kappa_\nu(s)\rho(s)\mathrm{d}s = -\mathrm{d}\tau_\nu$$

- *optical depth*

$$\tau_\nu(s) = \int_0^s \kappa_\nu(s')\rho(s')\mathrm{d}s'$$

- integration of both sides from 0 to s_0 of $\mathrm{d}\left(\ln I_\nu(s)\right) = -\mathrm{d}\tau_\nu$ gives

$$\ln I_\nu(s) - \ln I_\nu(0) = \ln \frac{I_\nu(s)}{I_\nu(0)} = -\tau_\nu(s)$$

- intensity as a function of optical depth

$$I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)}$$

Radiative Transfer Equation

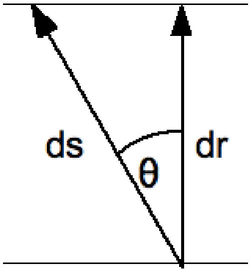
- local emission and absorption by mass:

$$\begin{aligned} dI_\nu(s) &= \epsilon_\nu(s)\rho(s)ds \\ dI_\nu(s) &= -\kappa_\nu(s)\rho(s)I_\nu(s)ds \end{aligned}$$

- *optical depth* at frequency ν

$$d\tau_\nu = -\kappa_\nu \rho dr$$

- $ds = dr/\mu$ with $\mu = \cos\theta$
- radiative transfer equation



$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{\epsilon_\nu}{\kappa_\nu} = I_\nu - S_\nu$$

S_ν *source function*

Emergent Intensity

- radiative transfer equation

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu$$

- formal solution

$$I_\nu(\tau_\nu, \mu) = I_\nu(\tau_{0\nu}, \mu) e^{-(\tau_{0\nu} - \tau_\nu)/\mu} + \frac{1}{\mu} \int_{\tau_\nu}^{\tau_{0\nu}} S_\nu(\tau'_\nu) e^{-\frac{\tau'_\nu - \tau_\nu}{\mu}} \mathrm{d}\tau'_\nu$$

- emergent intensity by integration from $\tau_\nu = 0$ to $\tau_{0\nu} = \infty$

$$I_\nu(\tau_\nu = 0, \mu) = \frac{1}{\mu} \int_0^\infty S_\nu(\tau_\nu) e^{\frac{-\tau_\nu}{\mu}} \mathrm{d}\tau_\nu$$

- calculate emergent intensity from model atmosphere
- derive source function from $I_\nu(\mu)$

Solution for Constant Source Function

- radiative transfer equation for $\mu = 1$, leaving out subscript ν

$$\frac{\mathrm{d}I}{\mathrm{d}\tau} = I - S$$

- with S constant along path and $I(\tau = 0) = I_0$, forml solution simplifies to

$$I = I_0 e^{-\tau} + S\left(1 - e^{-\tau}\right)$$

- with no incoming light, i.e. $I_0 = 0$

$$I = S\left(1 - e^{-\tau}\right)$$

Optically Thick and Thin

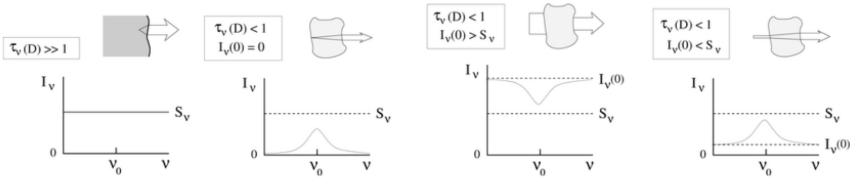
- intensity for constant source function: $I = S(1 - e^{-\tau})$
- $\tau \ll 1$: *optically thin* ($e^x = 1 + x - x^2/2 + ...$)

$$I = \tau S$$

- $\tau \gg 1$: *optically (very) thick*

$$I = S$$

black body radiation in LTE independent of κ_ν



Courtesy R.J.Rutten

Eddington-Barbier Relation

- emergent intensity

$$I_\nu(\tau_\nu = 0, \mu) = \frac{1}{\mu}\int_0^\infty S_\nu(\tau_\nu) e^{\frac{-\tau_\nu}{\mu}} \mathrm{d}\tau_\nu$$

- assume $S_\nu(\tau_\nu) = a_\nu + b_\nu \tau_\nu$
- emergent intensity

$$I_\nu(\tau_\nu = 0, \mu) = a_\nu + b_\nu \mu = S_\nu(\tau_\nu = \mu)$$

- emergent flux through integration over solid angle

$$\pi F_\nu = \pi(a_\nu + \frac{2}{3} b_\nu) = \pi S_\nu(\tau_\nu = \frac{2}{3})$$

Equilibria

Thermodynamic Equilibrium

- *thermal equilibrium*: single temperature T describes thermodynamic state everywhere
- ionization according to Saha equations for same T
- excitation according to Boltzmann equations for same T
- radiation field is homogeneous, isotropic black-body according to Kirchhoff-Planck equation for same T

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

- temperature gradients are not allowed!
- unrealistic for stellar atmosphere

Local Thermodynamic Equilibrium

- concept of *local thermodynamic equilibrium* (LTE) where single temperature T is sufficient to locally describe gas and radiation field
- as a consequence of the Kirchhoff law:

$$S_\nu = B_\nu(T)$$

- LTE: thermalization length must be smaller than length scale of temperature change
- thermalization: particle/photon looses its identity in distribution
- assumption of LTE depends on spectral lines
- rule of thumb: continuum in visible and infrared, weak lines, and wings of stronger lines are formed in LTE, but not line cores and strong spectral lines
- LTE: absorption in a single line $\Rightarrow$ black-body emission

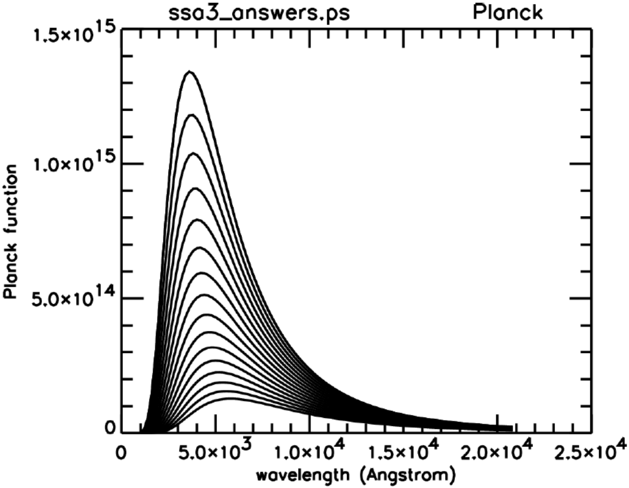
non-LTE

- non-LTE (NLTE) often when radiative processes are rare, i.e. photons travel large distances from areas where temperature is different
- single temperature is inadequate to describe radiation field, ionization stages, and atomic levels
- in most cases electrons are still Maxwell-distributed with *electron temperature* T_e because of frequent collisions
- but population of atomic levels depends on radiative processes, which may be rare; levels described by *statistical equations*

Black-Body Radiation

Planck:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$



Courtesy R.J.Rutten

Black-Body Approximations

- Planck:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

- Wien Approximation:

$$e^{h\nu/kT} \gg 1 : B_\nu(T) \approx \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$$

- Rayleigh-Jeans Approximation:

$$e^{h\nu/kT} \ll 1 : B_\nu(T) \approx \frac{2\nu^2 kT}{c^2}$$

Absorption Lines in LTE

- total optical depth given by continuum and line absorption coefficients

$$\mathrm{d}\tau_\nu = \mathrm{d}\tau_C + \mathrm{d}\tau_l = (1+\eta_\nu)\mathrm{d}\tau_C$$

- with

$$\eta_\nu = \frac{\kappa_l(\nu)}{\kappa_C}$$

- emergent intensity from before

$$I_\nu(\tau_\nu = 0, \mu) = \frac{1}{\mu}\int_0^\infty S_\nu(\tau_\nu) e^{\frac{-\tau_\nu}{\mu}} \mathrm{d}\tau_\nu$$

- emergent intensity at disk center ($\mu = 1$) under LTE

$$I_\nu(\tau = 0, \mu = 1) = \int_0^\infty (1+\eta_\nu) B_\nu e^{\left(-\int_0^\tau (1+\eta_\nu)\mathrm{d}\tau'\right)} \mathrm{d}\tau$$

$\tau = \tau_C$: continuum optical depth

Line Absorption Coefficient

- line broadening mechanisms:
  - natural line width (finite lifetime of upper state)
  - Doppler broadening (random thermal motion)
  - collisional broadening
  - Stark effect (H only)
  - microturbulent velocity
- convolution of Lorentz and Gaussian distributions

$$\phi(\nu) = \frac{1}{\sqrt{\pi}\Delta\nu_D} H(a, \nu)$$

with Voigt function

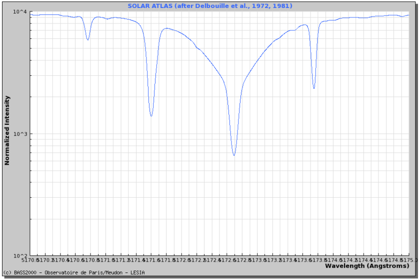
$$H(a, \nu) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(\nu - y)^2 + a^2} dy$$

Voigt Function

- Voigt function

$$H(a,\nu) = \frac{a}{\pi}\int_{-\infty}^{\infty} \frac{e^{-y^2}}{(\nu - y)^2 + a^2} dy$$

- special case: $H(a \ll 1, 0) \approx 1$
- normalized profile: $\int_0^\infty \phi(\nu)\mathrm{d}\nu = 1$
- Gaussian dominates in cores, Lorentzian in wings



Microturbulence and Macroturbulence

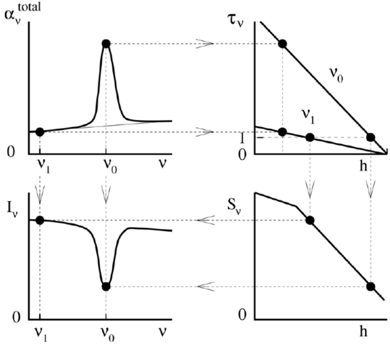
- convective motions in solar atmospheres on spatial scales smaller than range of optical depth over which spectral line is formed
- add microturbulent fudge factor to Doppler broadening

$$\Delta\nu_{\mathrm{D}} \equiv \frac{\nu_0}{c}\sqrt{\frac{2RT}{A} + \xi_{\mathrm{t}}^2}$$

- convective motions on scales larger than formation range of spectral lines
- convolve complete line profile with Gaussian profile
- both macro- and micro-turbulence are not needed anymore in realistic 3D atmosphere models

Simple Absorption Line

- absorption: transition gives peak in $\kappa = \kappa_C + \kappa_l = (1 + \eta_\nu)\, \kappa_C$
- optical depth: height-invariant $\kappa \Rightarrow$ linear $(1 + \eta_\nu)\, \tau_C$
- source function: same for line and continuum
- intensity: Eddington-Barbier (nearly) exact



Courtesy R.J.Rutten

Simple Emission Line

- extinction: transition process gives peak in $\kappa = \kappa_C + \kappa_l = (1 + \eta_\nu)\,\kappa_C$
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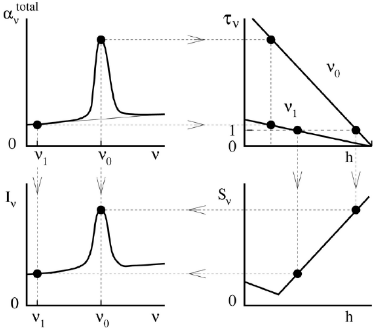
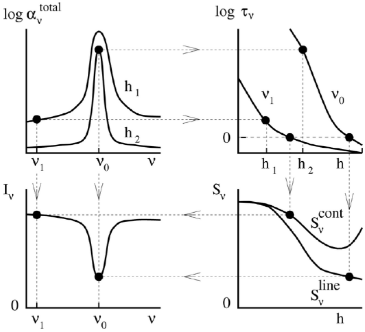
- source function: same for line and continuum
- intensity: Eddington-Barbier (nearly) exact



Courtesy R.J.Rutten

Realistic Absorption Line

- extinction: transition peak lower and narrower at larger height
- optical depth: near-log-linear inward increase
- source function: split for line and continuum
- intensity: Eddington-Barbier for $S_\nu^{\text{total}} = (\kappa_C S_C + \kappa_l S_l)/(\kappa_C + \kappa_l) = (S_C + \eta_\nu S_l)/(1 + \eta_\nu)$



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