Solar Physics 2010: Exercises to Lecture 7
Due Date: 25 May 2010 at 09:00

C.U.Keller

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1 Exercises from Stix

Problem 4.1. Assume that there are neither absorbers nor emitters between two points of a radiation field. Show that the intensity, according to its definition in Sect. 1.5.1, is the same at these two points; convince yourself that this is consistent with the equation of transfer, (4.2).

$$H(a,v) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(v-y)^2 + a^2} \, dy \; . \tag{4.13}$$

The Voigt function is normalized such that for small a its value at the line center is 1, i.e.,

$$H(a,0) = 1 + O(a) \; , \tag{4.14}$$

while $\phi(\nu)$ is normalized so that the integral over all frequencies is 1.

For solar spectral lines we generally have $a \ll 1$. Inspection of (4.13) shows that in this case the Doppler profile dominates near the line center ν_0. On the other hand, the Gaussian rapidly drops with distance from ν_0, and hence the line wings are shaped according to the collisional, or "damping", part of the profile. An example where the two parts of the line profile can clearly be distinguished is shown in Fig. 4.1 (although a strong line such as Mg b_2 cannot very accurately be treated under the assumption of LTE!).

Problem 4.2. Show that the integral over v of $H(a,v)$ is $\sqrt{\pi}$. Prove (4.14).

Problem 4.9. The Eddington–Barbier approximation. Expand the source function $S(\tau_\lambda)$ around some optical depth τ_λ^*. Find the intensity $I_\lambda(0, \mu)$ and show that, up to terms of second order in $\tau_\lambda - \tau_\lambda^*$, we have $S_\lambda(\tau_\lambda^*) = I_\lambda(0, \mu)$ for $\tau_\lambda^* = \mu$.