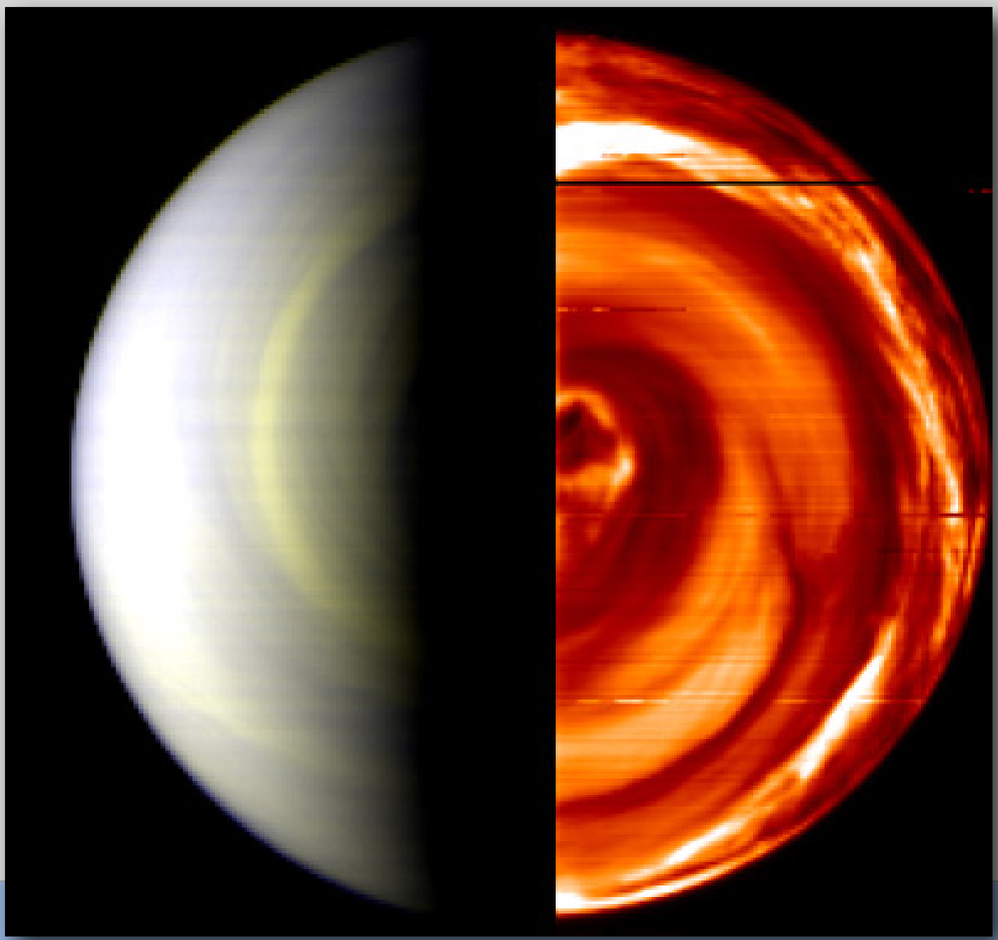
# Radiative transfer

## Planetary Sciences Chapters 3 - 4

Daphne Stam



SRON Netherlands Institute for Space Research

Netherlands Organisation for Scientific Research

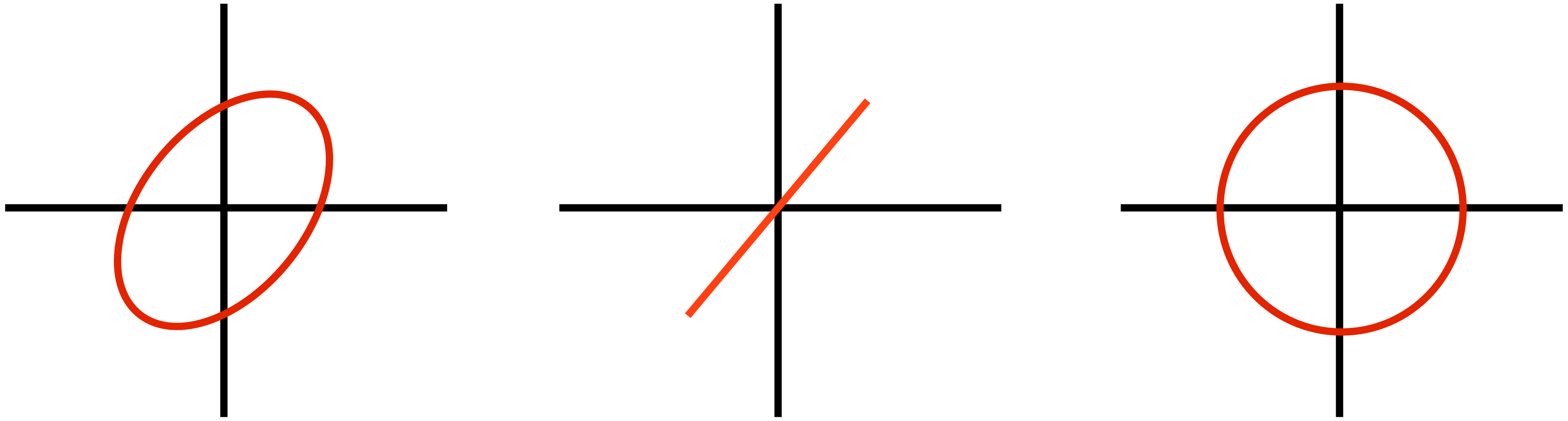
# Outline

- A description of radiation
- Blackbody radiation
- Extinction of radiation
- Optical thicknesses
- Scattering cross-sections
- Colours of the skies
- Absorption cross-sections
- Phase functions and scattering matrices
- Reflection spectra
- Thermal spectra
- Spectra of transiting exoplanets

# A description of radiation

Light consists of many waves that are each completely elliptically polarised: the end-point of their electric vector traces out an ellipse in a plane perpendicular to the direction of propagation of the light.

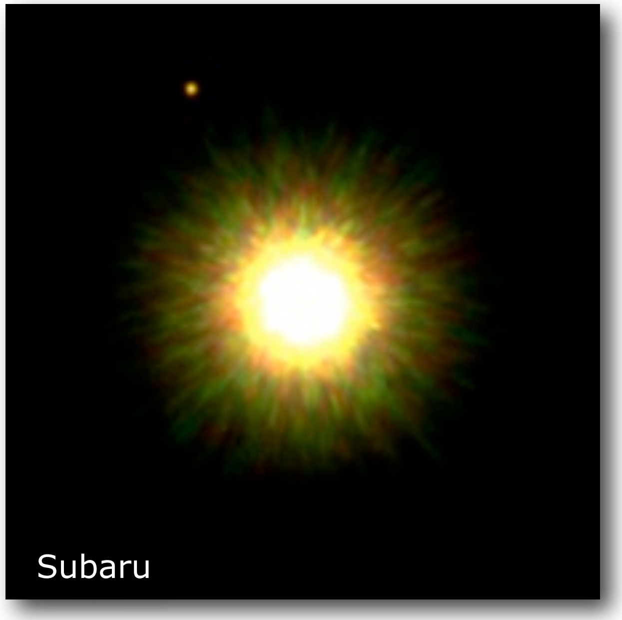
Special cases of the ellipse are a straight line (linear polarisation), and a circle (circular polarisation):



If the waves in a beam of light have a preferential shape and direction of their ellipses, the light is said to be (partially) polarised. The degree of polarisation of this light is defined as:

$$P = I_{\text{polarised}} / I_{\text{total}}$$

# A description of radiation


Subaru

A beam of light can fully be described by a 4-vector:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

$I$ ← flux

$Q$, $U$ ← linearly polarised flux

$V$ ← circularly polarises flux

The degree of polarisation is defined as: $P= \surd(Q^2 + U^2 + V^2)/I$

The degree of linear polarisation as: $P= \surd(Q^2 + U^2)/I$

The degree of circular polarisation as: $P= V/I$

Natural light, e.g. light of solar-type stars, when integrated over the stellar disk, can be assumed to be unpolarised: $P=0$

# Blackbody radiation

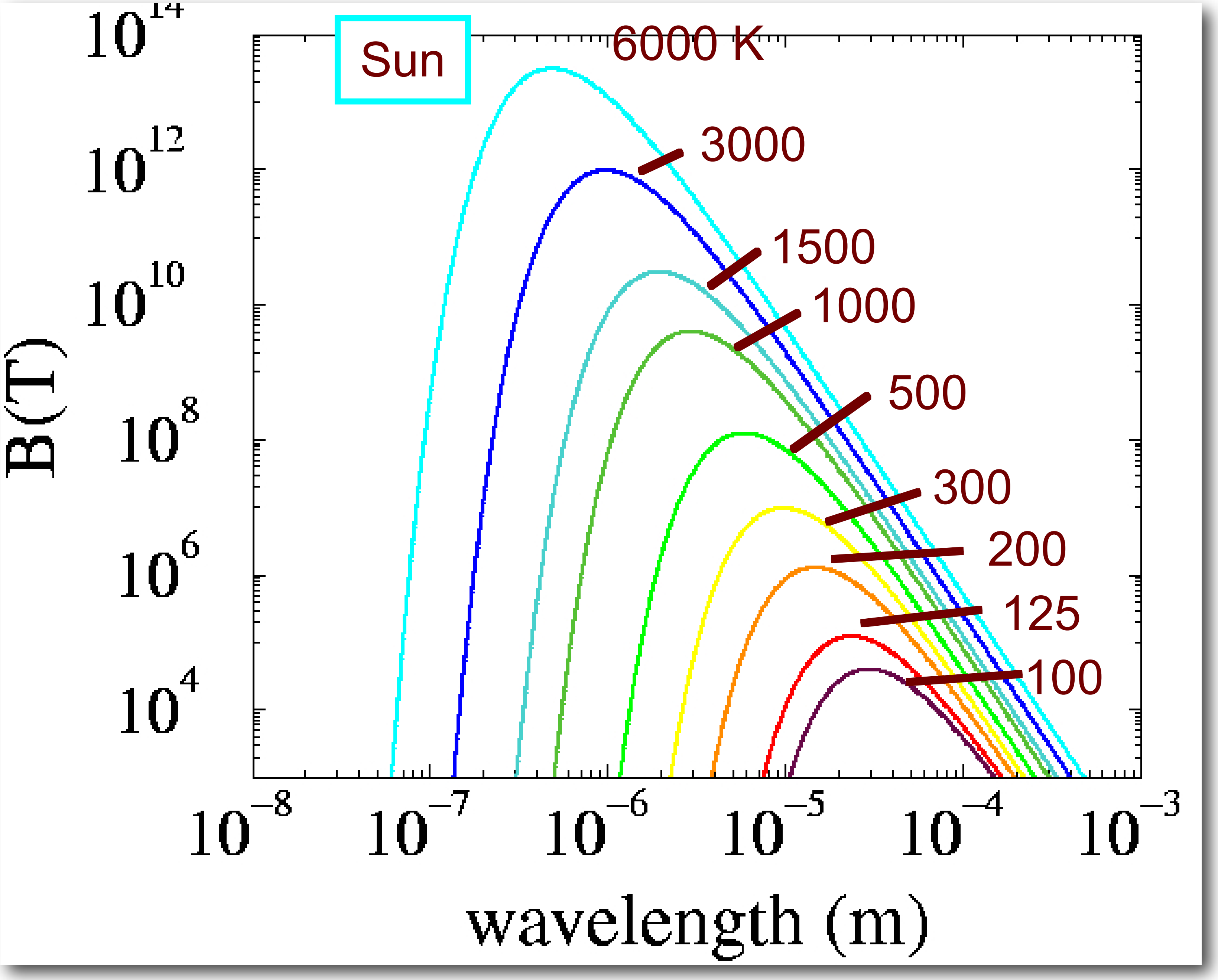
The radiation that is emitted by a blackbody with an effective temperature $T_{\mathrm{eff}}$ can be described by a Planck-curve:

The wavelength $\lambda_{\mathrm{max}}$ (in m) where the flux is maximum is given by Wien's law:

$\lambda_{\mathrm{max}} = 2.898\ 10^{-3}/T_{\mathrm{eff}}$

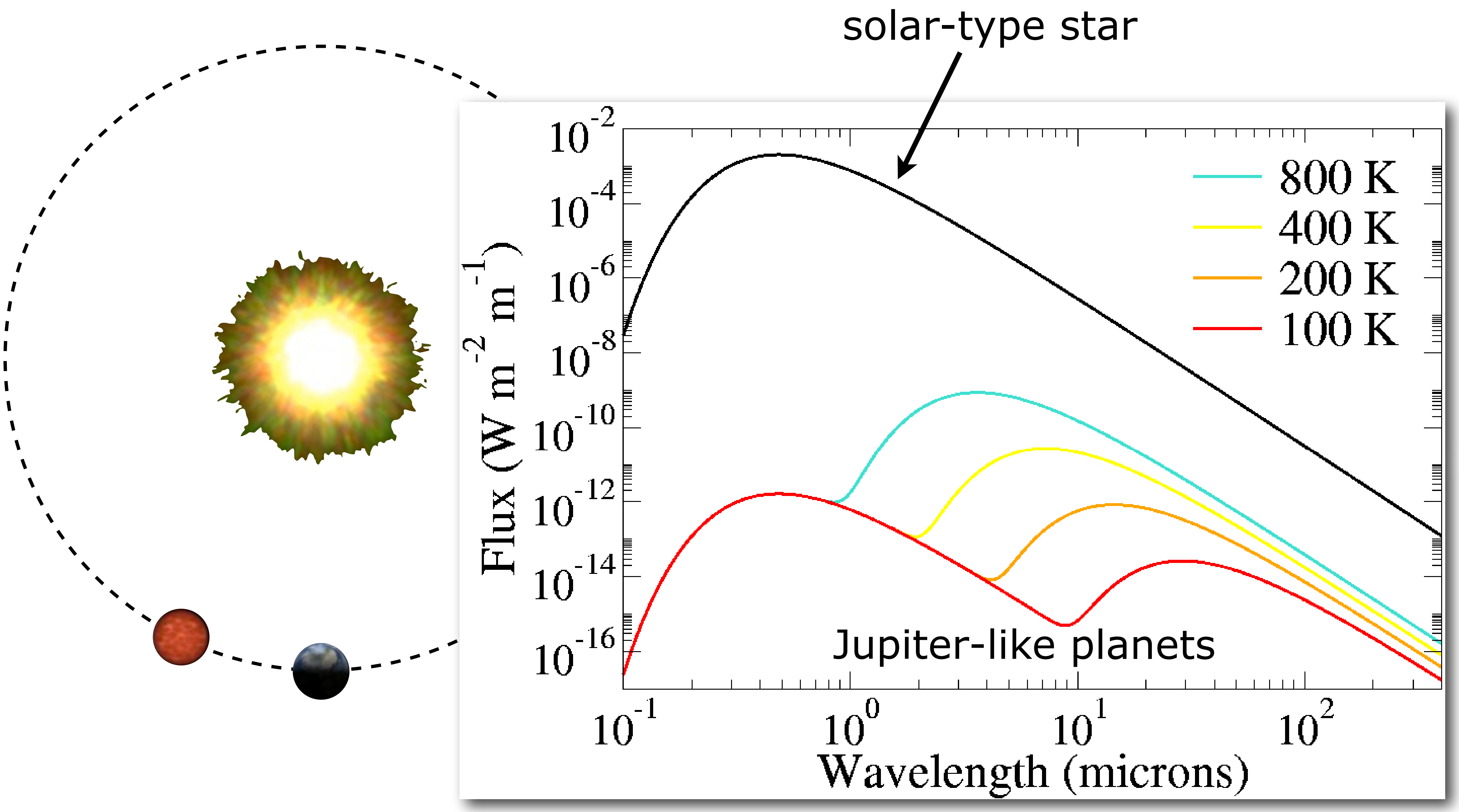
For the Sun ($T_{\mathrm{eff}}$=6000 K):

$\lambda_{\mathrm{max}} = 0.48\ \mu\mathrm{m}$



# Planetary radiation

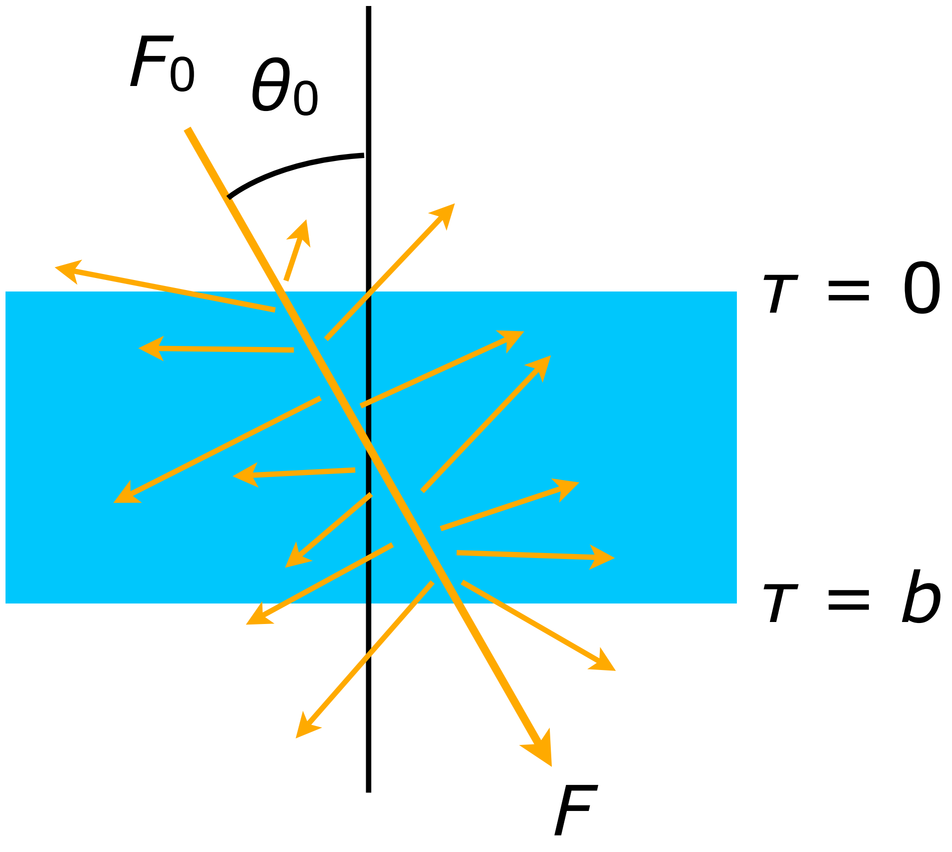
The radiation from a planet consists of thermal radiation (infrared) and reflected starlight (ultraviolet-visible-near infrared):



# Extinction of radiation

The extinction of a parallel beam of radiation through a slab of material can be described as follows:

$$F(\lambda) = F_0(\lambda) \exp(-b(\lambda)/ \cos \theta_0)$$



$F_0$ is the irradiance of the incident beam
$F$ is the irradiance of the transmitted beam
$b$ is the slab's extinction optical thickness
$\tau$ is the optical depth in the slab
$\theta_0$ is the angle of incidence of the beam
$\lambda$ is the wavelength of the radiation

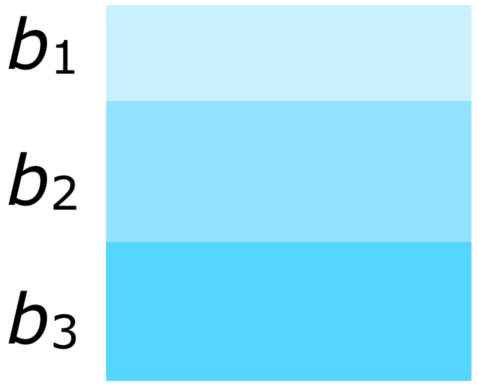
The radiation that is 'lost' to the beam is either absorbed or scattered in all directions. The extinction optical thickness $b$ of an atmospheric layer is thus the sum of the layer's absorption and scattering optical thicknesses:

$$b(\lambda) = b_{abs}(\lambda) + b_{sca}(\lambda)$$

# The optical thickness of an atmospheric layer

A layer's extinction optical thickness $b$ depends on:

- the types of gases, aerosol and cloud particles
- the amounts of gases, aerosol and cloud particles
- the wavelength $\lambda$ of the radiation



$b_{\text{tot}} = b_1 + b_2 + b_3$

A layer's extinction optical thickness is given by:

$$b(\lambda) = \int_{z_1}^{z_2} \sum_{i=1}^{I} \sigma_i(\lambda, z)\, n_i(z)\, \mathrm{d}z$$

With:

$z$ the altitude (in m)

$\sigma_i$ the extinction cross-section of particle type i (in $m^2$)

$n_i$ the number density of particle type i (in $m^{-3}$)

For a homogeneous layer, we find: $b(\lambda) = \sum_{i=1}^{I} \sigma_i(\lambda)\, N_i$

With $N_i$ the column number density of particle type i (in $m^{-2}$)

# Absorption and scattering optical thicknesses

Remember: $b(\lambda) = b_{abs}(\lambda) + b_{sca}(\lambda)$

For a homogeneous layer, we have:

$$b(\lambda) = \sum_{i=1}^{I} (\sigma_{abs\ i}(\lambda) + \sigma_{sca\ i}(\lambda))\, N_i$$

With:

- $\sigma_{abs\ i}$ the *absorption cross-section* of particle type i (in $m^2$)
- $\sigma_{sca\ i}$ the *scattering cross-section* of particle type i (in $m^2$)
- $N_i$ the column number density of particle type i (in $m^{-3}$)

Thus:

$$b_{abs}(\lambda) = \sum_{i=1}^{I} \sigma_{abs\ i}(\lambda)\, N_i$$

$$b_{sca}(\lambda) = \sum_{i=1} \sigma_{sca\ i}(\lambda)\, N_i$$

$N_i$ depends on the ambient pressure, the temperature, and the gas mixing ratio

The single scattering albedo in the layer: $a(\lambda) = b_{sca}(\lambda)/b(\lambda)$

# Scattering cross-sections

Scattering cross-sections $\sigma_{sca}$ of gases, aerosol and cloud particles are usually smooth functions of the wavelength.

Scattering by gases or particles that are very small compared to the wavelength of the radiation ($2\pi r/\lambda \ll 1$) is usually referred to as Rayleigh scattering.

For Rayleigh scattering of Earth-like air, the cross-section is given by:

$$\sigma_{sca}(\lambda) = \frac{24\pi^3}{\lambda^4 N_L^2} \frac{(n^2(\lambda) - 1)^2}{(n^2(\lambda) + 2)^2} \frac{6+3\rho_n(\lambda)}{6-7\rho_n(\lambda)}$$

blue light scatters more efficiently than red light!

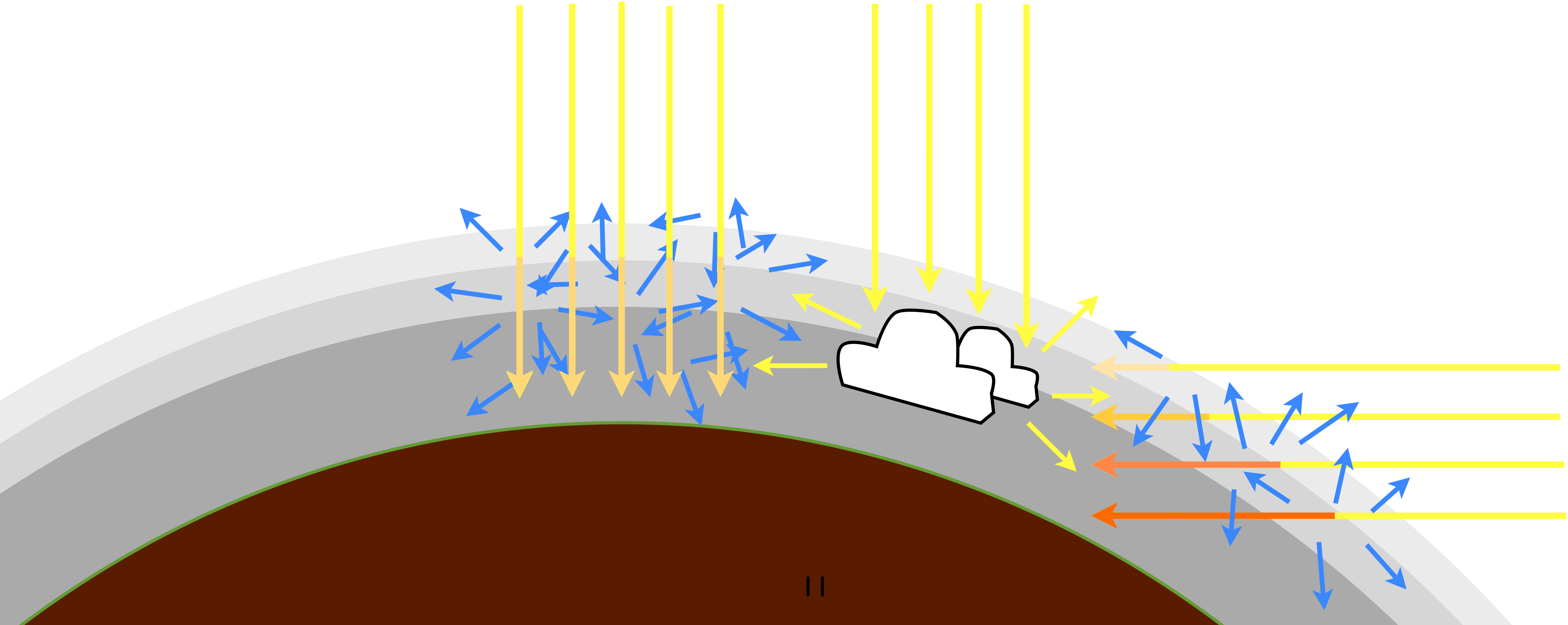
With:

| | |
|---|---|
| $n$ | the refractive index of dry air under standard conditions |
| $N_L$ | Loschmidt's number |
| $\rho_n$ | the depolarisation factor of the air |

# Effects of the scattering cross-section

Why is the clear sky blue during the day?
Why is the clear sky red during twilight?

Particles that are large with respect to the wavelength ($2\pi r \gg \lambda$), such as some types of aerosol and cloud particles, scatter all colours of sunlight equally well, leaving the scattered light whitish or white:



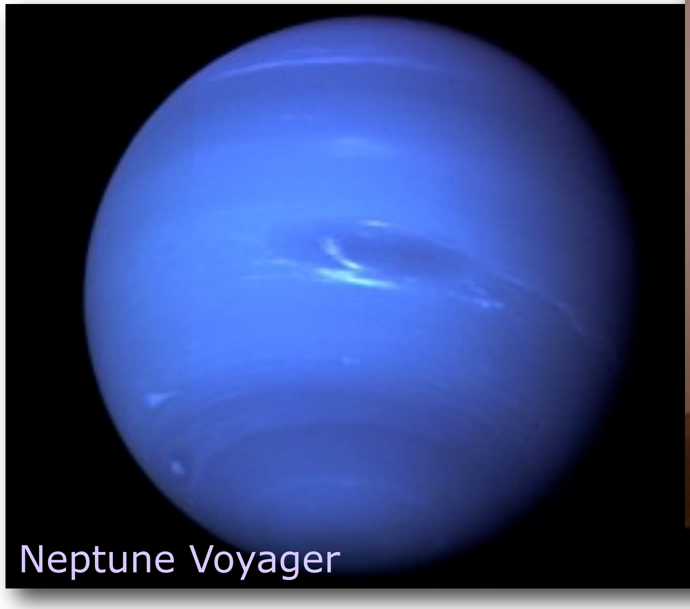
# Colours of the skies:

The colour of a planetary atmosphere depends on its composition:

Earth twilight after the Pinatubo eruption

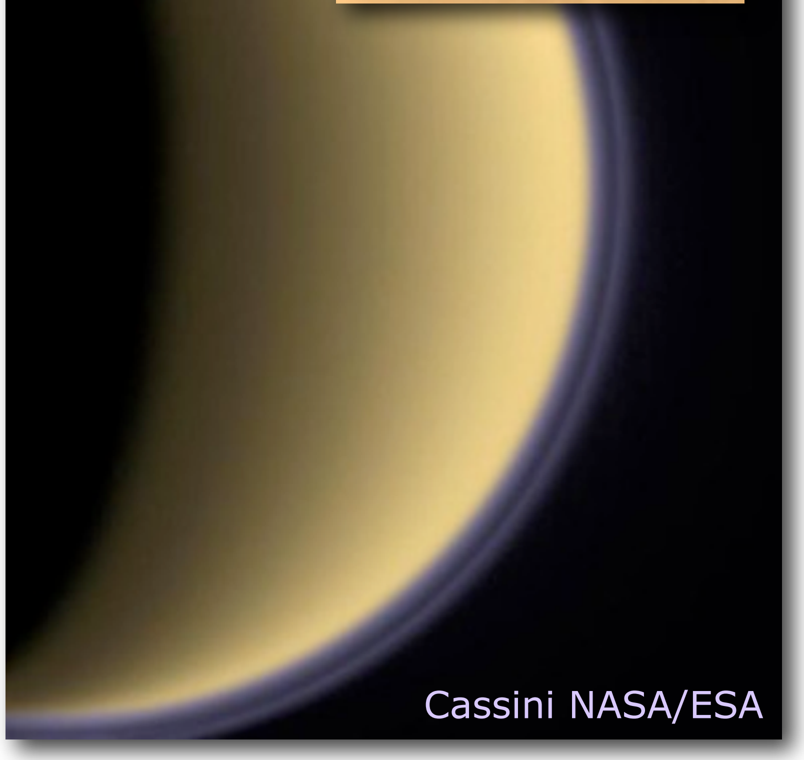
Earth daytime sky

Huygens ESA/NASA



Neptune Voyager

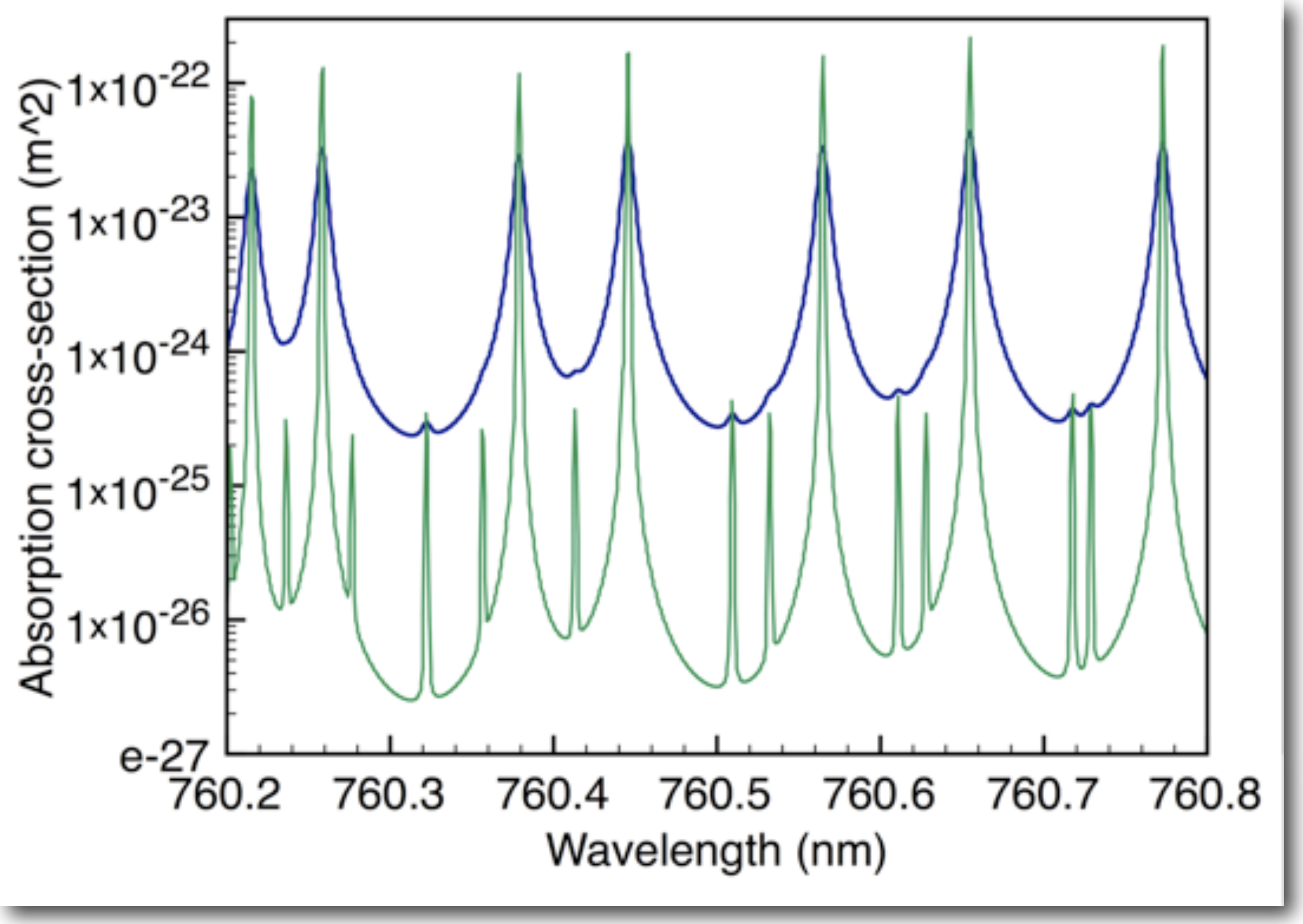
Mars twilight observed by Spirit [NASA]



Cassini NASA/ESA

# Absorption cross-sections

Absorption cross-sections of aerosol and cloud particles are usually smooth functions of the wavelength. Absorption cross-sections of gases are usually strongly varying functions of the wavelength:



Absorption cross-sections of $O_2$ on Earth, at **0 km** (294 K, 1013 hPa) and **30 km** (234 K, 13.20 hPa)

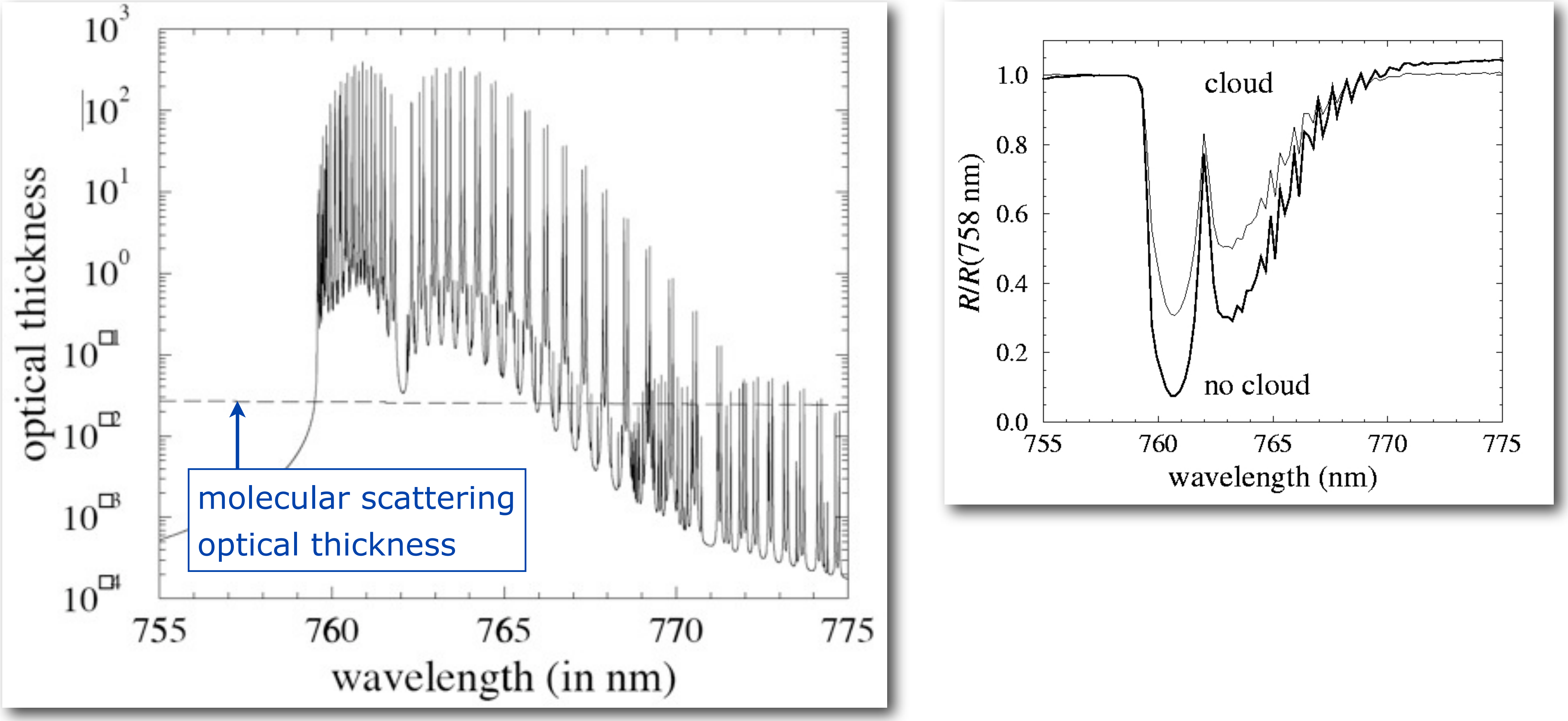
$\sigma_{abs}$ of a gas depends on:

- the type of gas
- the wavelength
- the ambient temperature
- the ambient pressure
- the types of ambient gases

**Note** energy that is absorbed in the visible is usually emitted again in the infrared!

# Absorption cross-sections

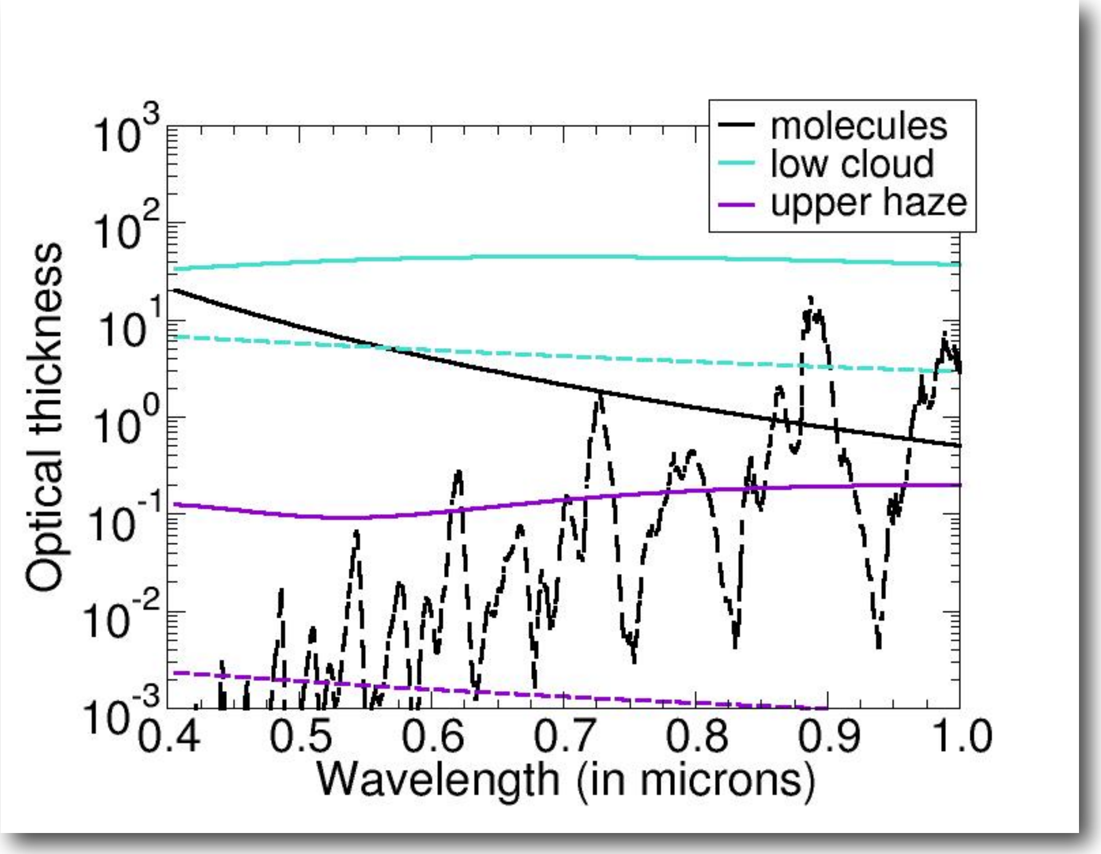
The spectral resolution of observations determines the strength and details that can be seen of an absorption band:

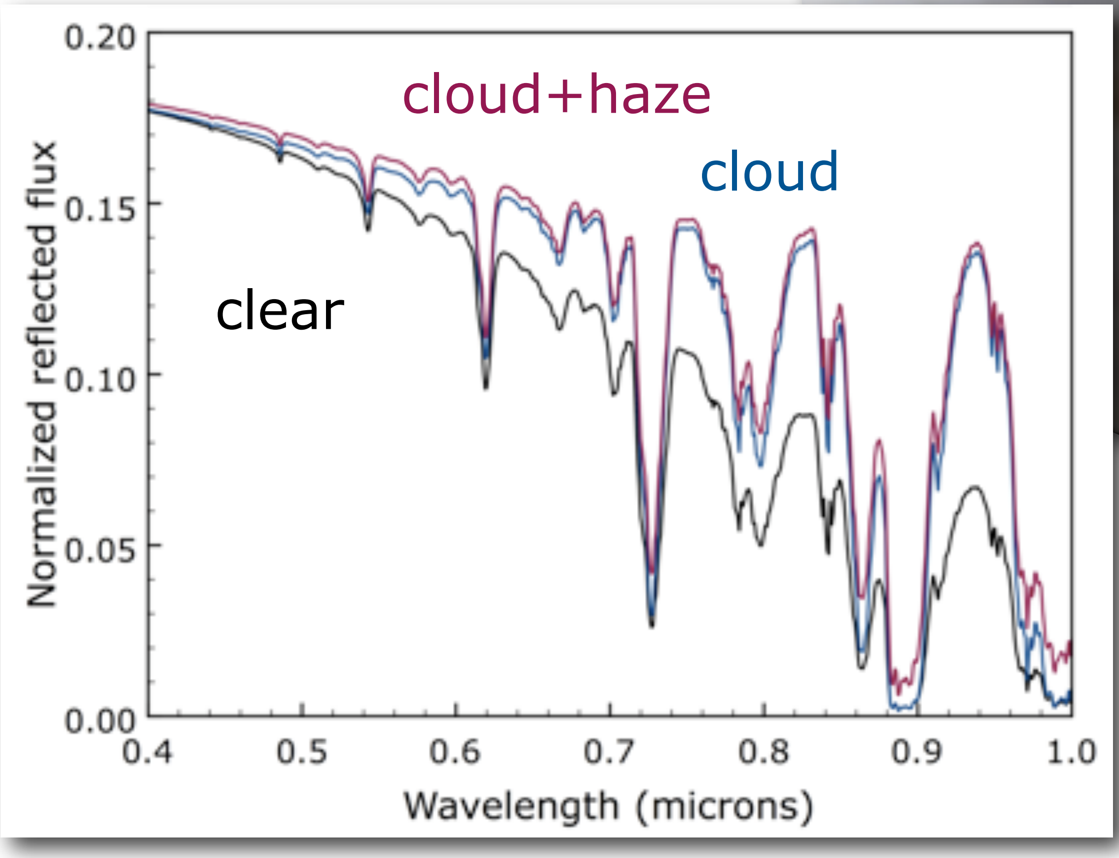


Calculated absorption optical thickness of $O_2$ in the Earth's atmosphere (left), and the same band observed (twice) by the GOME instrument onboard the ERS-2 satellite (right).

# Cross-sections throughout an atmosphere

What a spectrum of a planet looks like depends on the scattering and absorption cross-sections of atmospheric constituents and their spatial distribution.

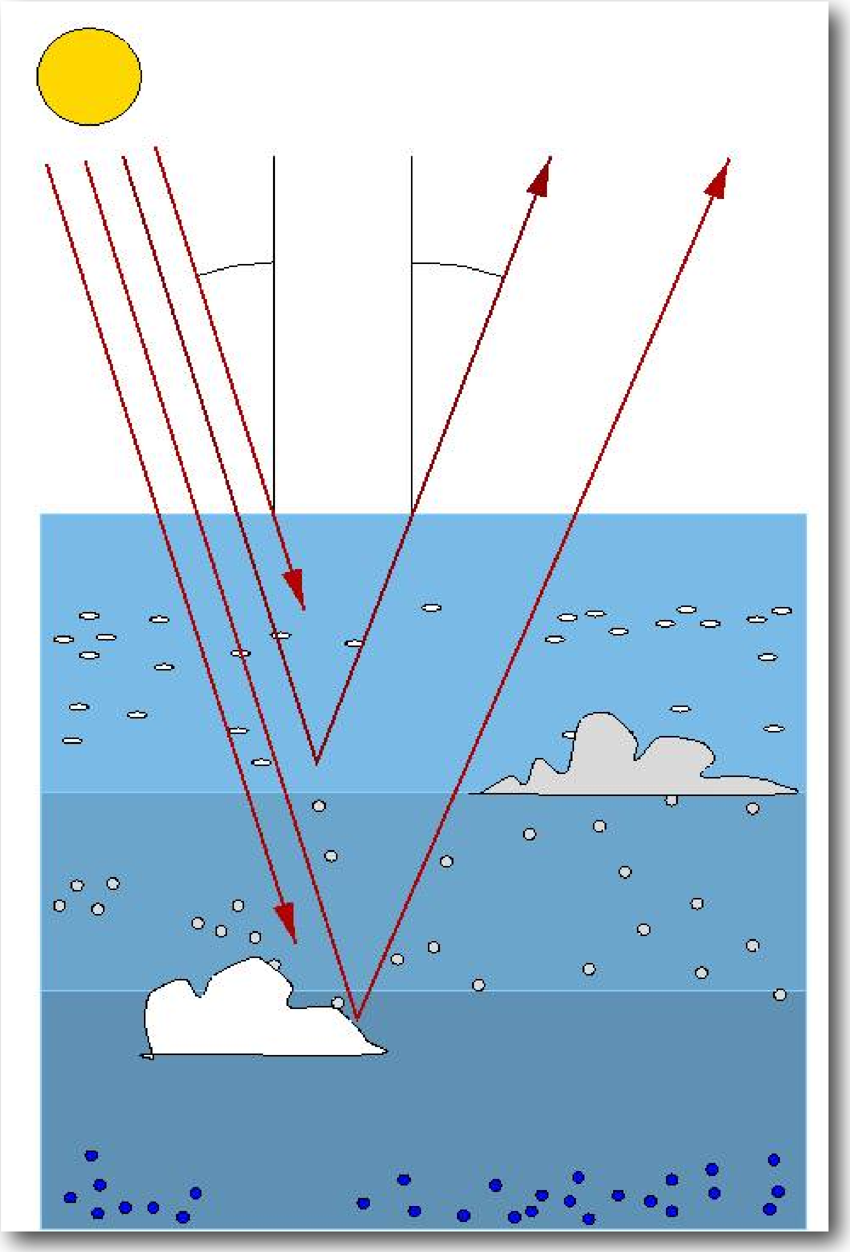




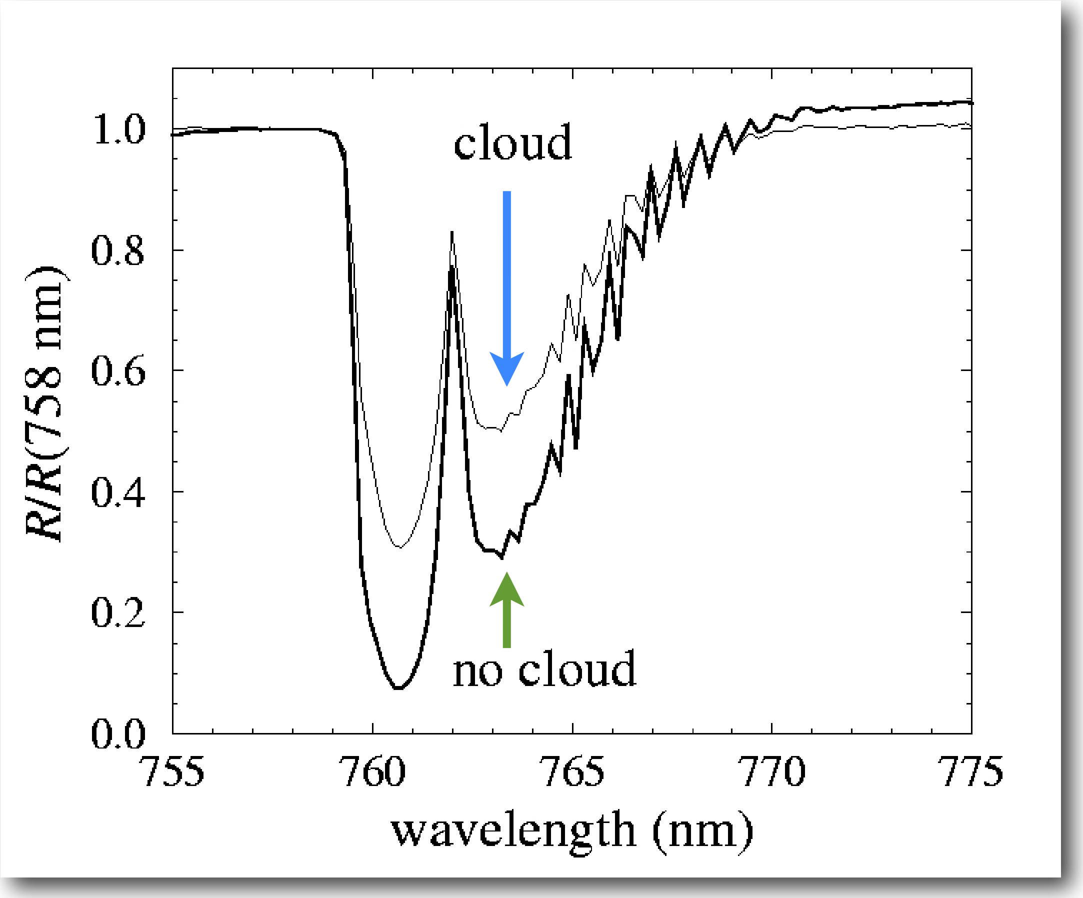
Above left calculated optical thicknesses (solid line: scattering; dashed line: absorption) of components of Jupiter-like atmospheres.
Above right calculated spectra of the planets, seen at 90° phase angles.

# Cloud top altitudes from absorption bands

What a spectrum of a planet looks like depends on the scattering and absorption cross-sections of atmospheric constituents and their spatial distribution:



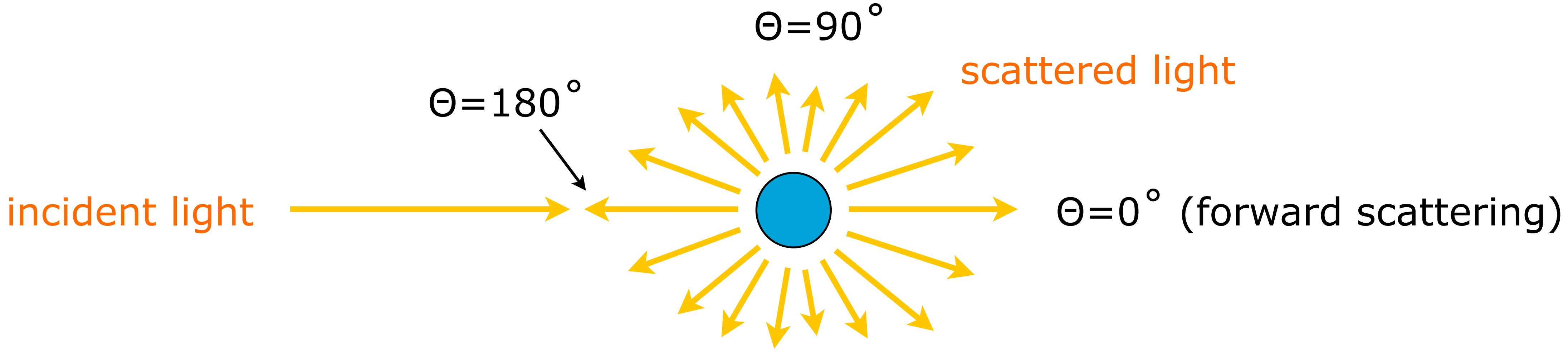
The depth of the $O_2$ *A* band on Earth:



When observing a region on a planet, the cloud coverage will influence the band depth too!

# Phase functions

The angular distribution of the flux of the light that is singly scattered by molecules, aerosol or cloud particles is described by the phase function:

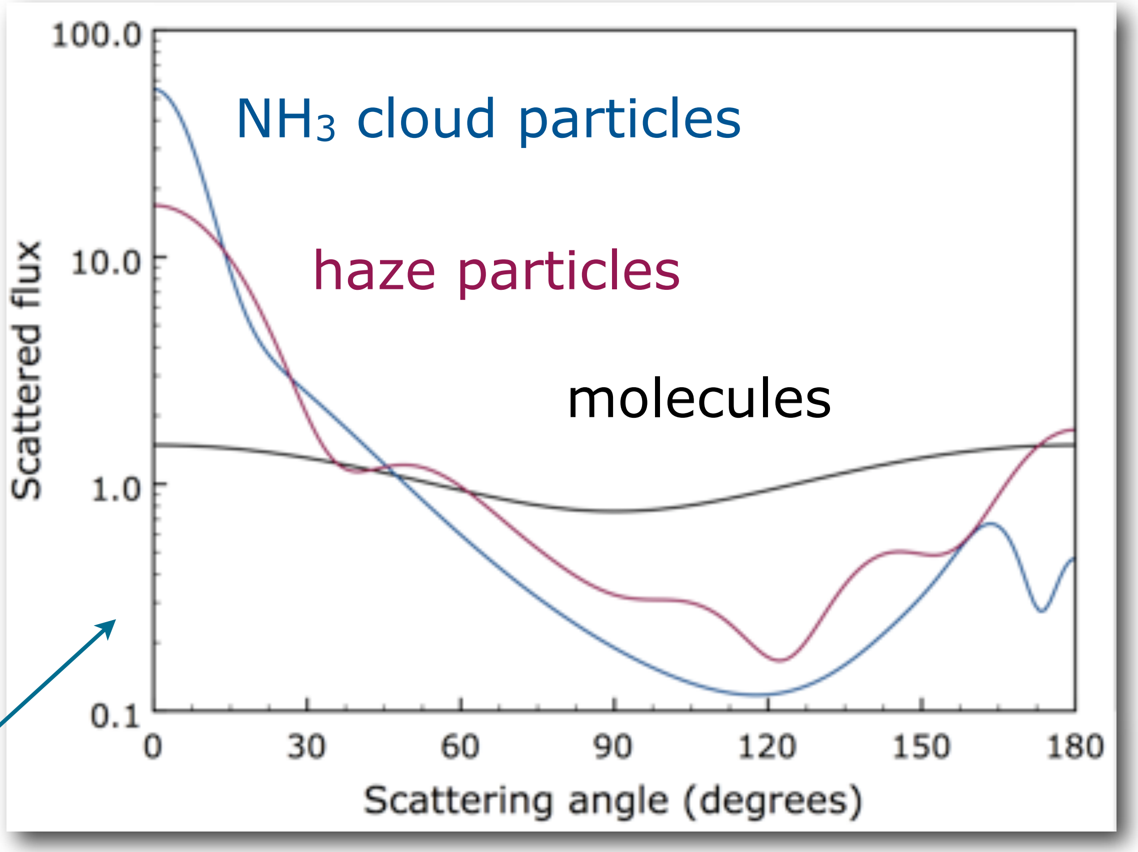


The phase function of molecules depends on:

- the type of gas
- the wavelength

The phase function of particles depends on:

- their size distribution
- their composition
- their shape
- the wavelength



These phase functions are normalised such that their average over all directions equals 1

# Scattering matrices

The angular distribution of the flux and state of polarisation of the light that is singly scattered by molecules, aerosol or cloud particles* is described by the scattering matrix:

$$\underset{\textit{scattered light}}{\begin{bmatrix} F \\ Q \\ U \\ V \end{bmatrix}} = \underset{\textit{scattering matrix}}{\begin{bmatrix} \overset{\substack{\text{phase function} \\ \swarrow}}{P_{11}} & P_{21} & 0 & 0 \\ P_{21} & P_{22} & 0 & 0 \\ 0 & 0 & P_{33} & -P_{43} \\ 0 & 0 & P_{43} & P_{44} \end{bmatrix}} \underset{\textit{incident light}}{\begin{bmatrix} F_0 \\ Q_0 \\ U_0 \\ V_0 \end{bmatrix}}$$

For incident unpolarised light:

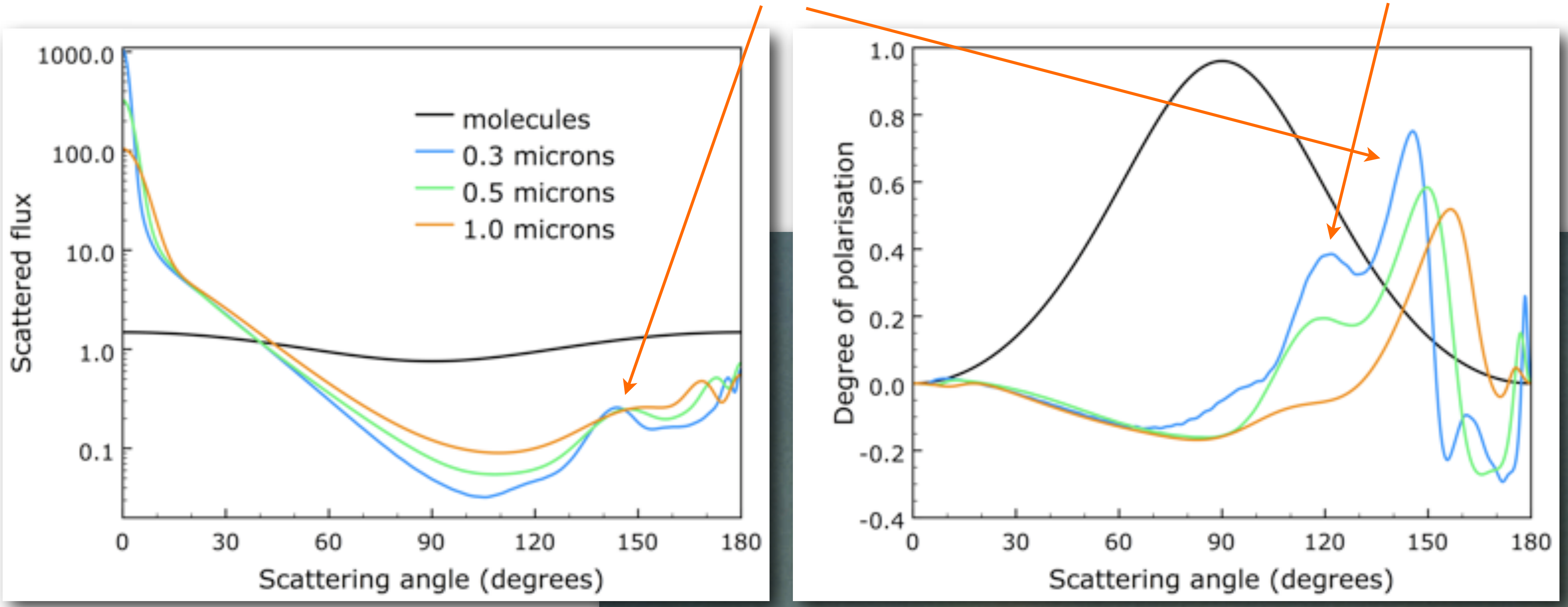
Flux $F(\Theta) = P_{11}(\Theta)\, F_0$

Degree of linear polarisation $P(\Theta) = -\, P_{21}(\Theta)/P_{11}(\Theta)$

* if non-spherical, these have to be randomly oriented

# Scattering matrices of water droplets

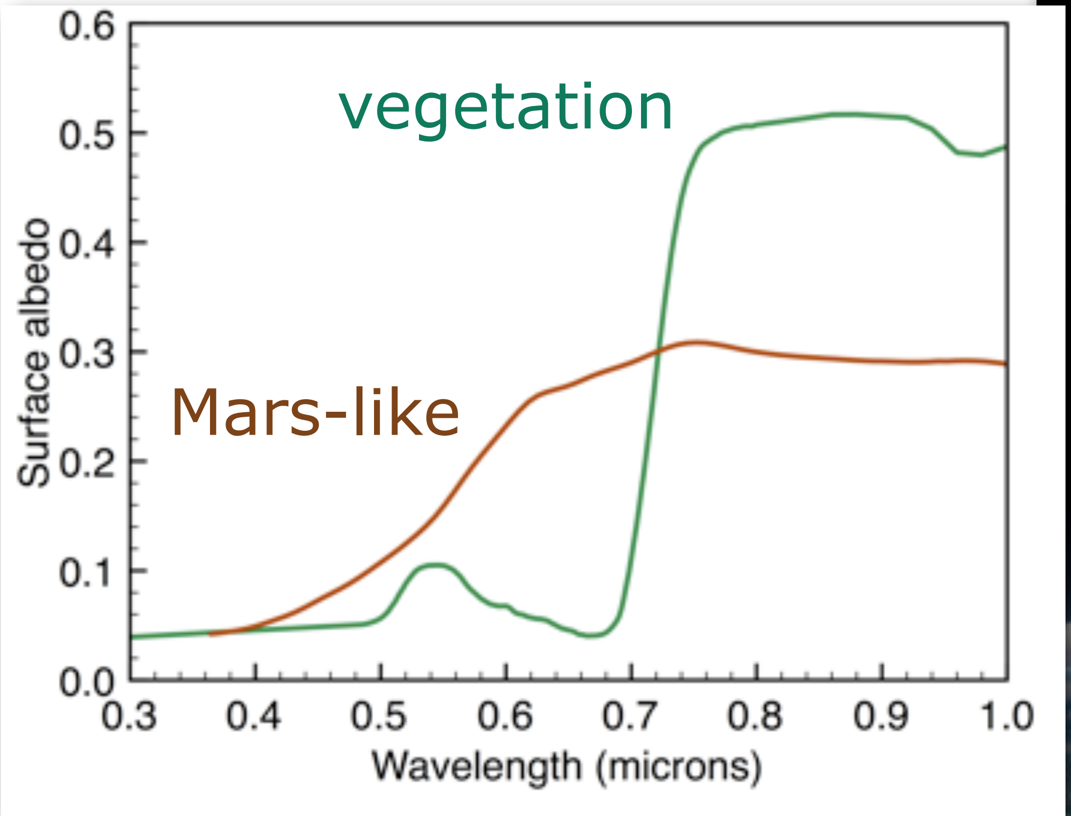
The flux (left) and degree of linear polarisation (right) of unpolarised incident light of different wavelengths that is singly scattered by small water droplets:


primary rainbow

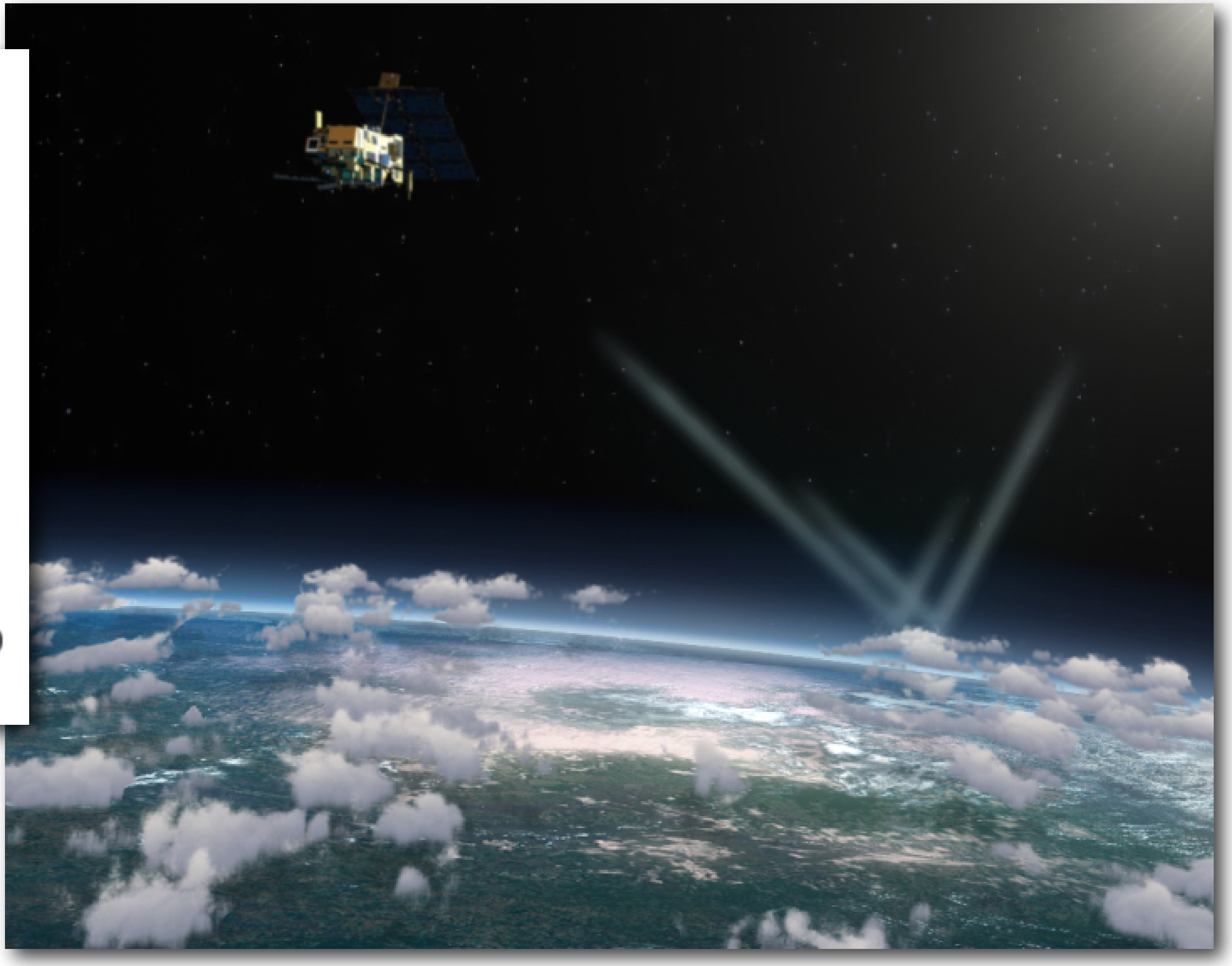
secondary rainbow

# Reflection spectra: $b \approx 1$

When observing rocky planets with optically thin atmospheres, you will not only receive sunlight that has been singly or multiply scattered by the molecules, aerosol and cloud particles within the atmosphere, but also light that has been reflected by the surface:

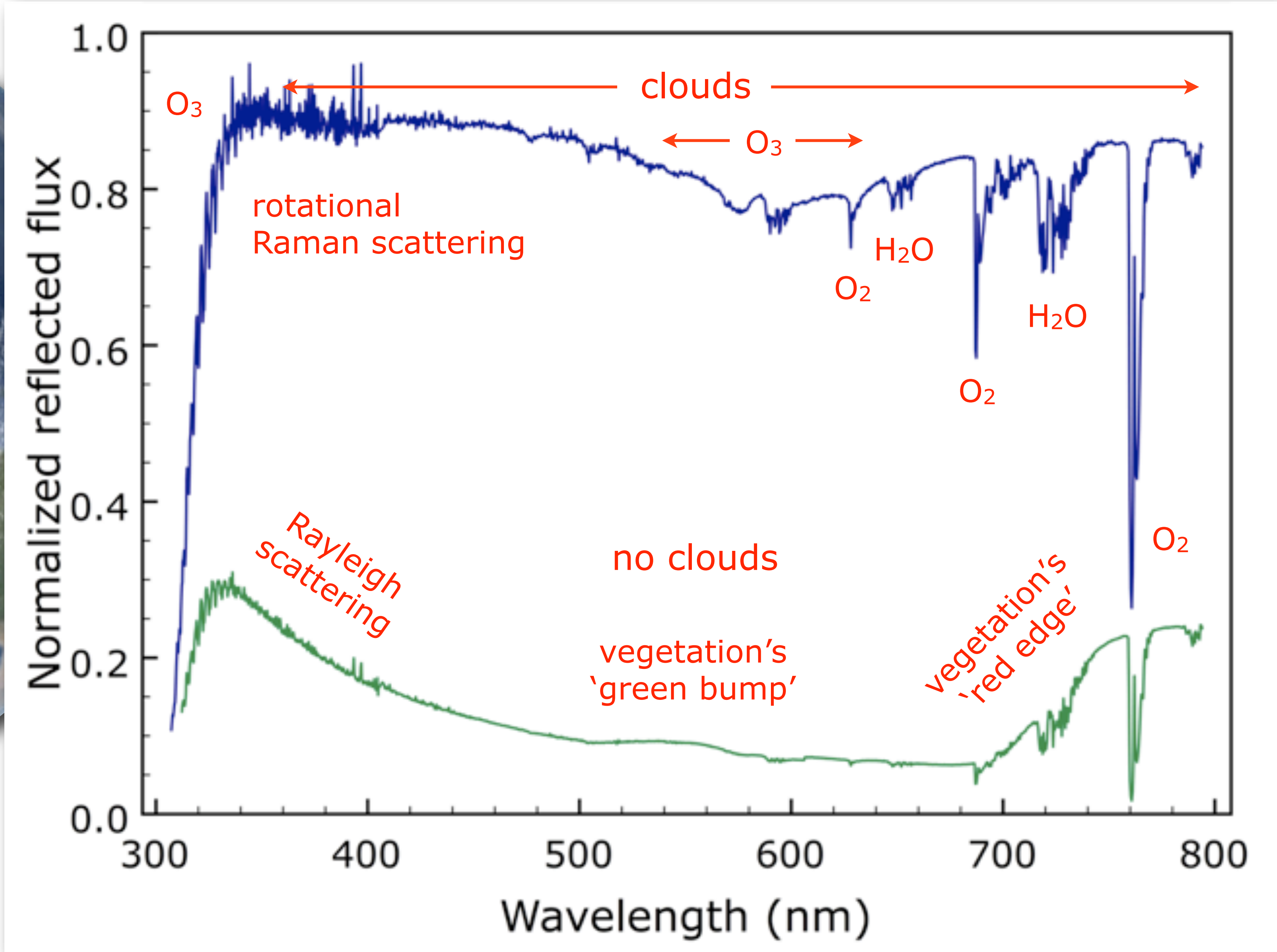


Examples of surface albedo’s



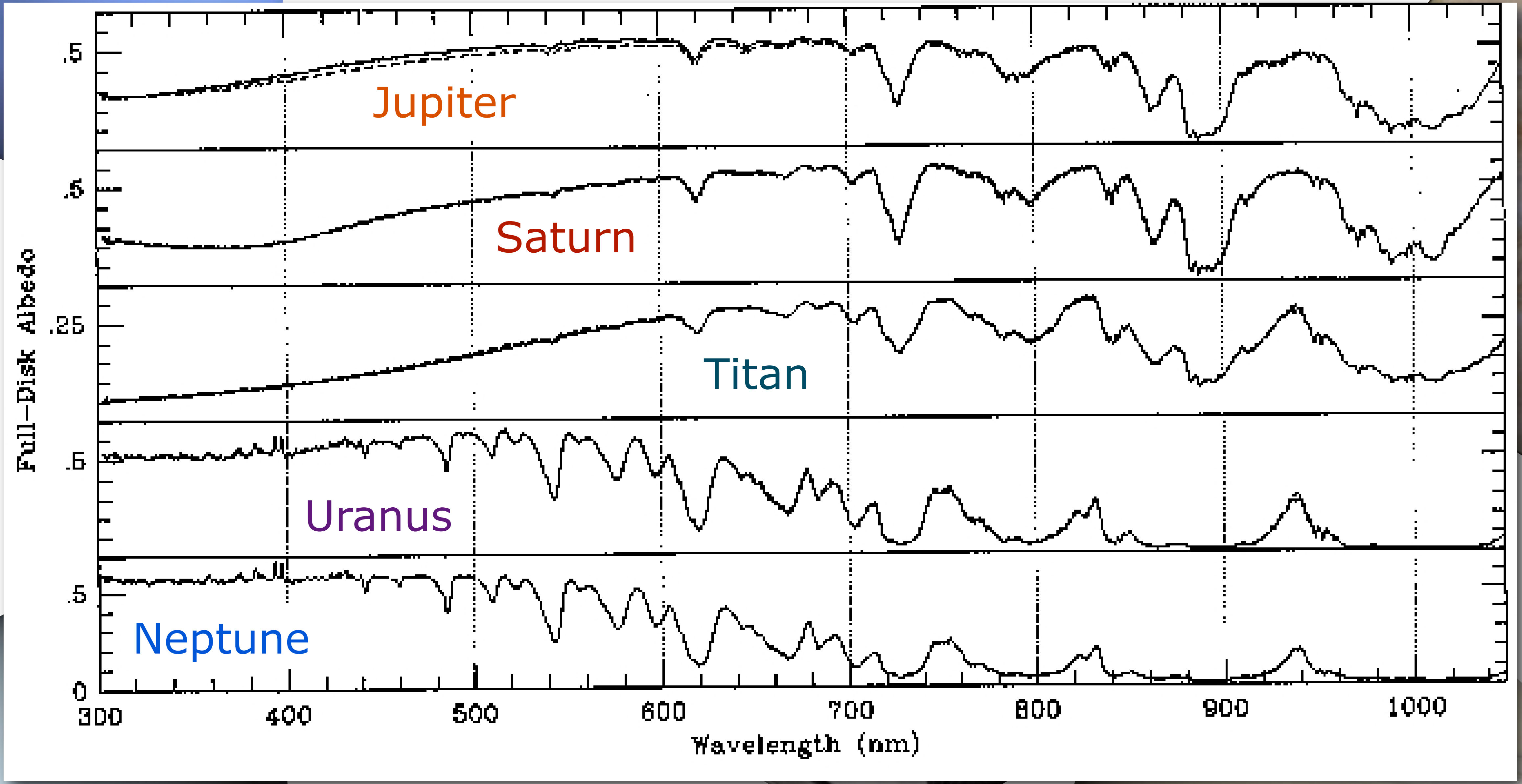
# Reflected sunlight: Earth

Reflected flux of a region on Earth, once clear and once completely covered by clouds, as measured by GOME on the ERS-2 satellite, for nadir viewing angles and solar zenith angles of 34°:



# Reflected starlight: observations

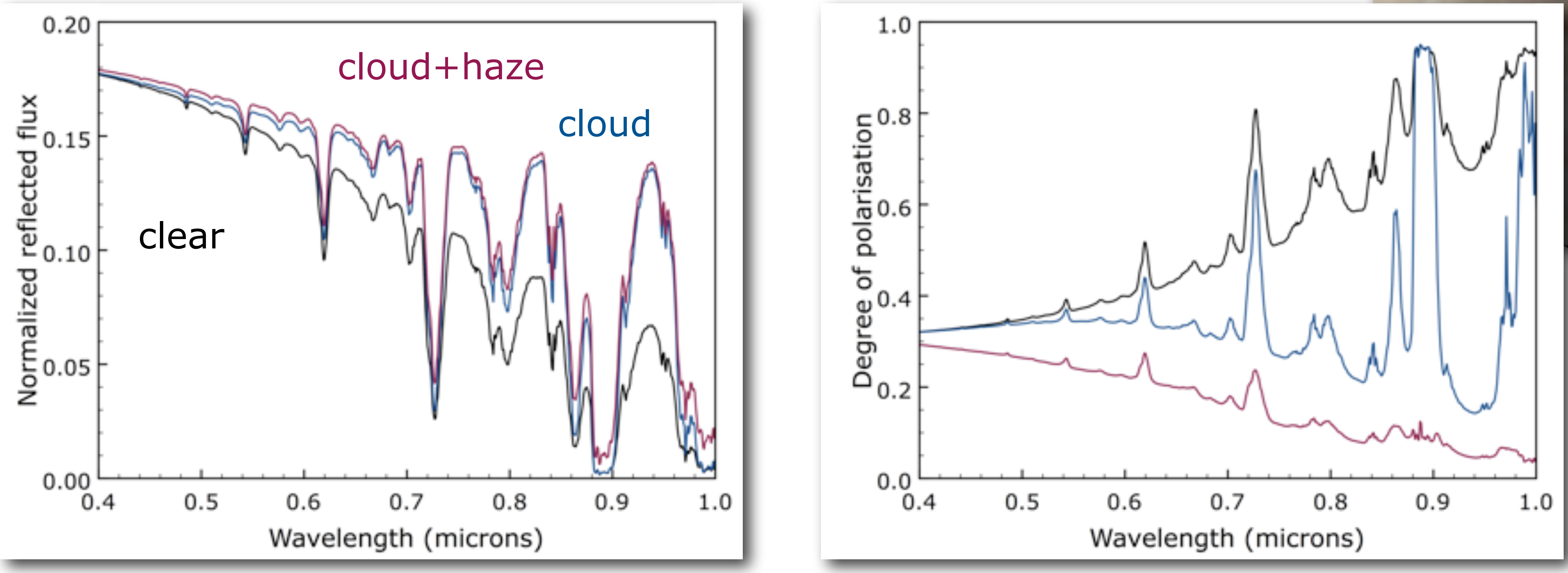
Disk-integrated observations of sunlight reflected by the Solar System's giant planets and Titan. Almost all absorption bands are due to $CH_4$:



Credit: Karkoschka [1994]

# Reflected starlight: simulations

The flux and degree of polarisation of starlight that is reflected by exoplanets can be calculated. Such simulations are required for instrument design, optimising the observation strategy, and the interpretation of observations.

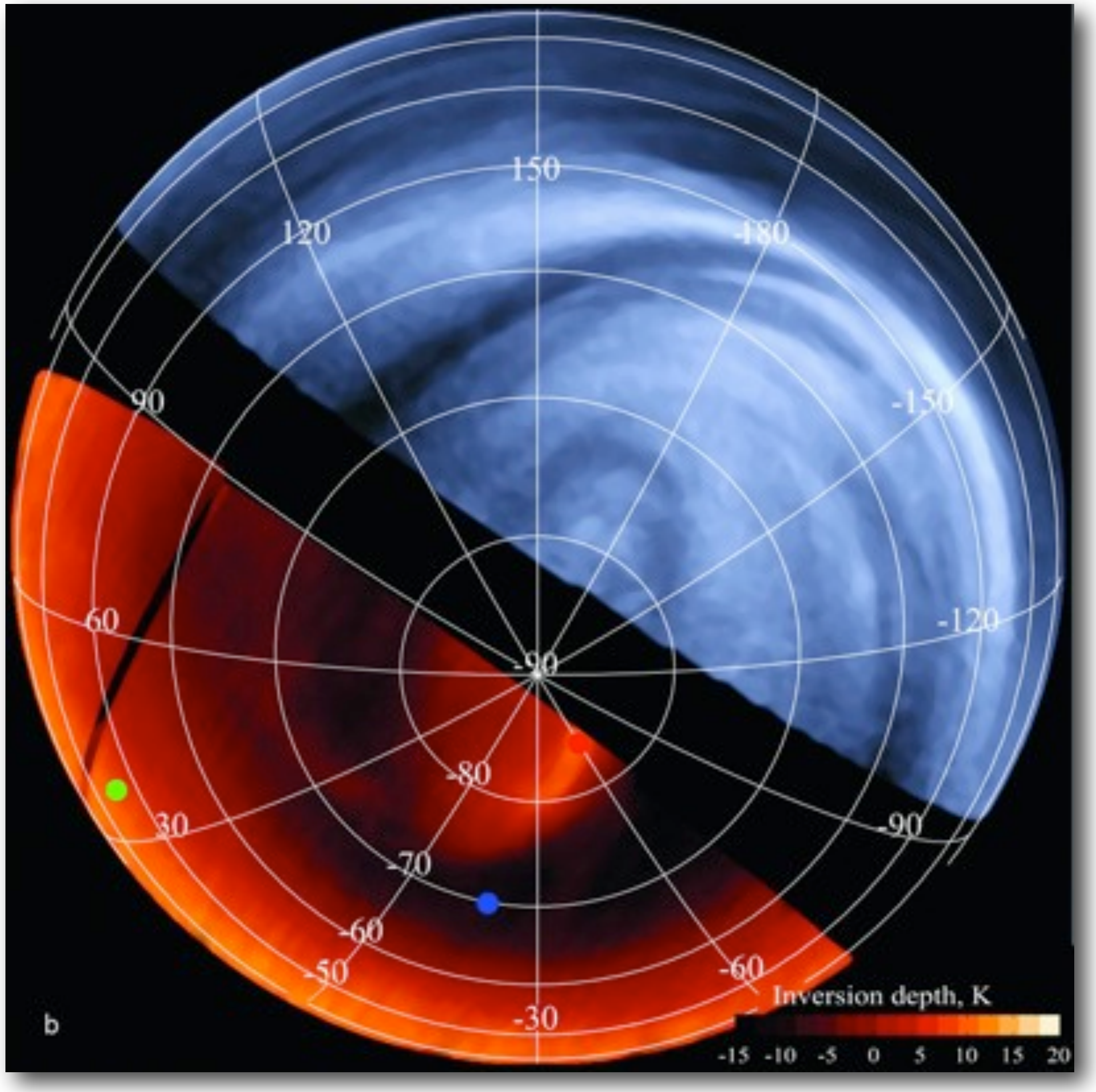


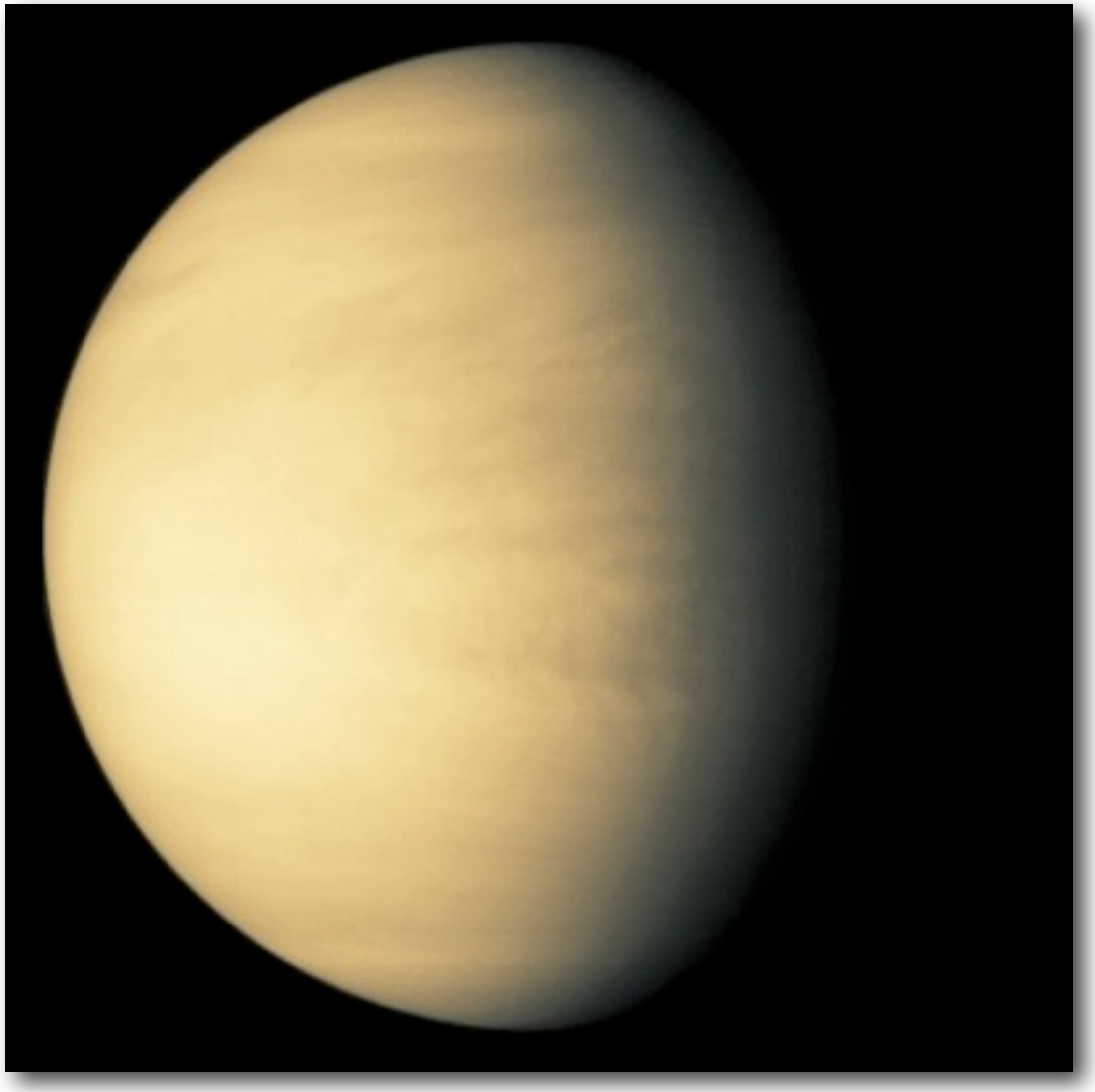
Above left calculated fluxes of starlight that is reflected by Jupiter-like exoplanets. Above right calculated degree of polarisation of the reflected starlight. The planets are seen at 90° phase angles.

# Ultraviolet, visible & thermal radiation: Venus

Observations at different wavelengths reveal different compositions of the atmosphere and/or show different altitude regions.

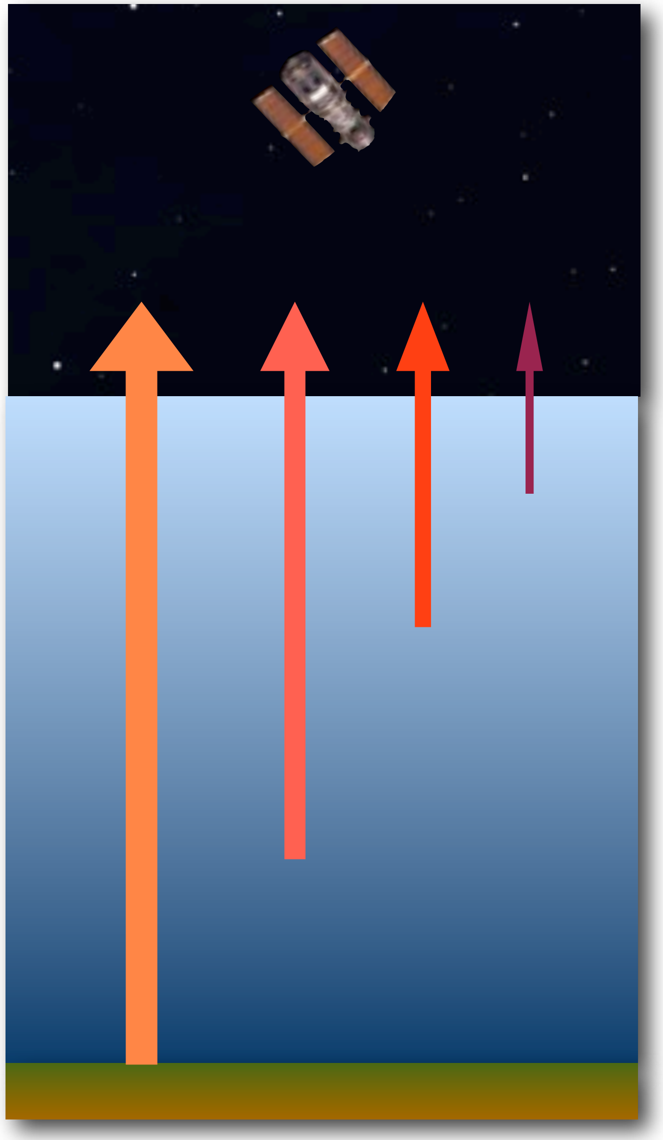
Left: Venus in UV reflected light and IR radiation as seen by Venus Express.
Right: Venus in VIS reflected light as seen by the Galileo mission.

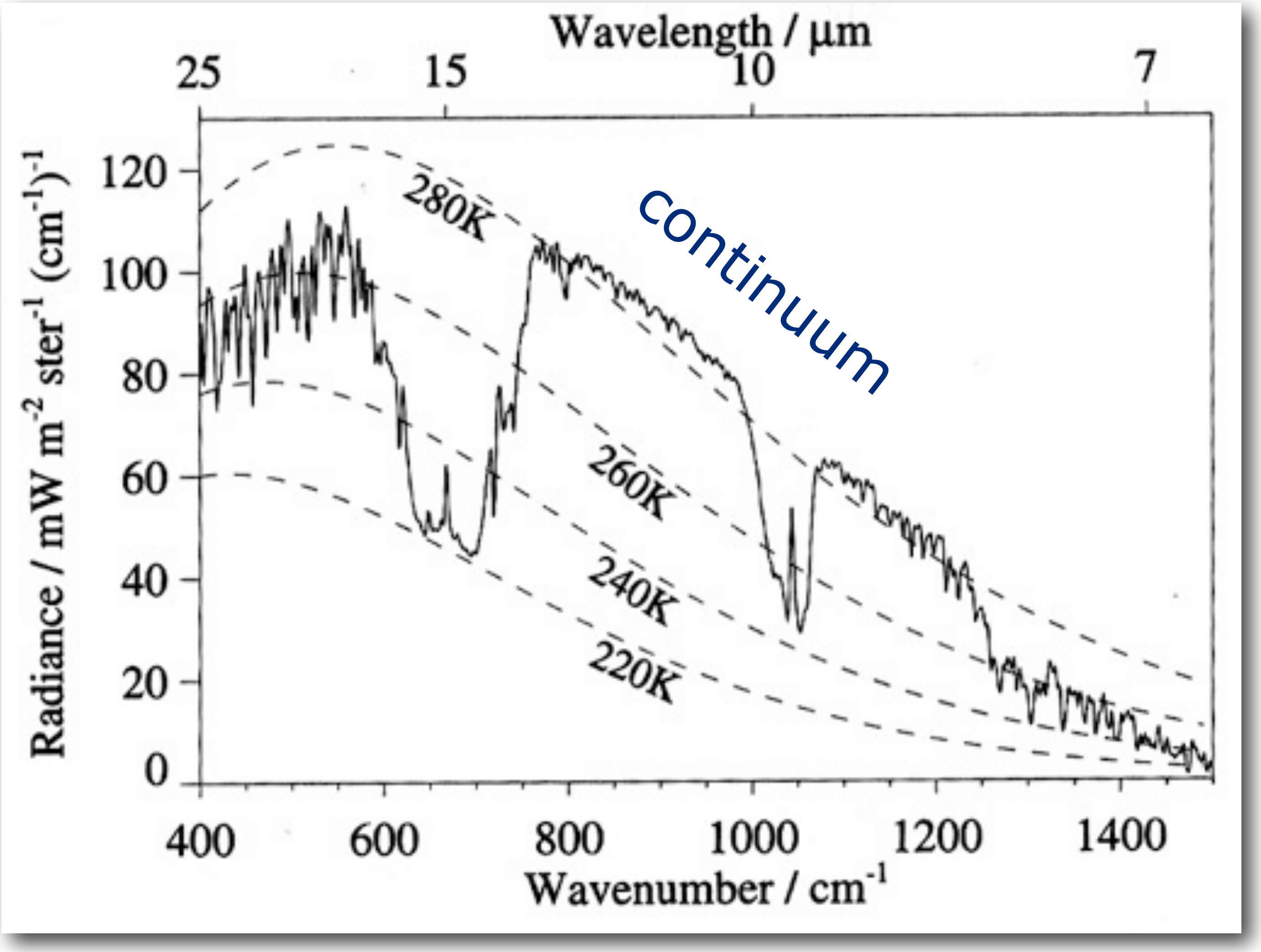




# Thermal spectra: b ≈ 1

At wavelengths where the atmosphere is **optically thin**, most of the thermal radiation will come from the surface and the lowest, densest regions of the atmosphere, and is not absorbed before escaping to space:

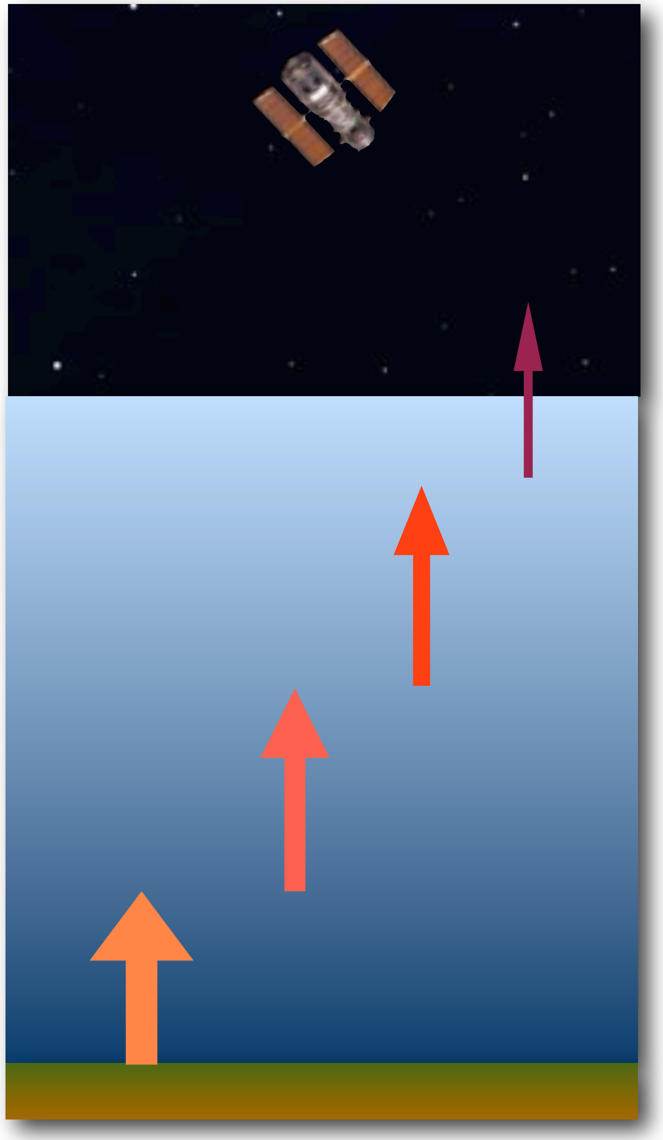


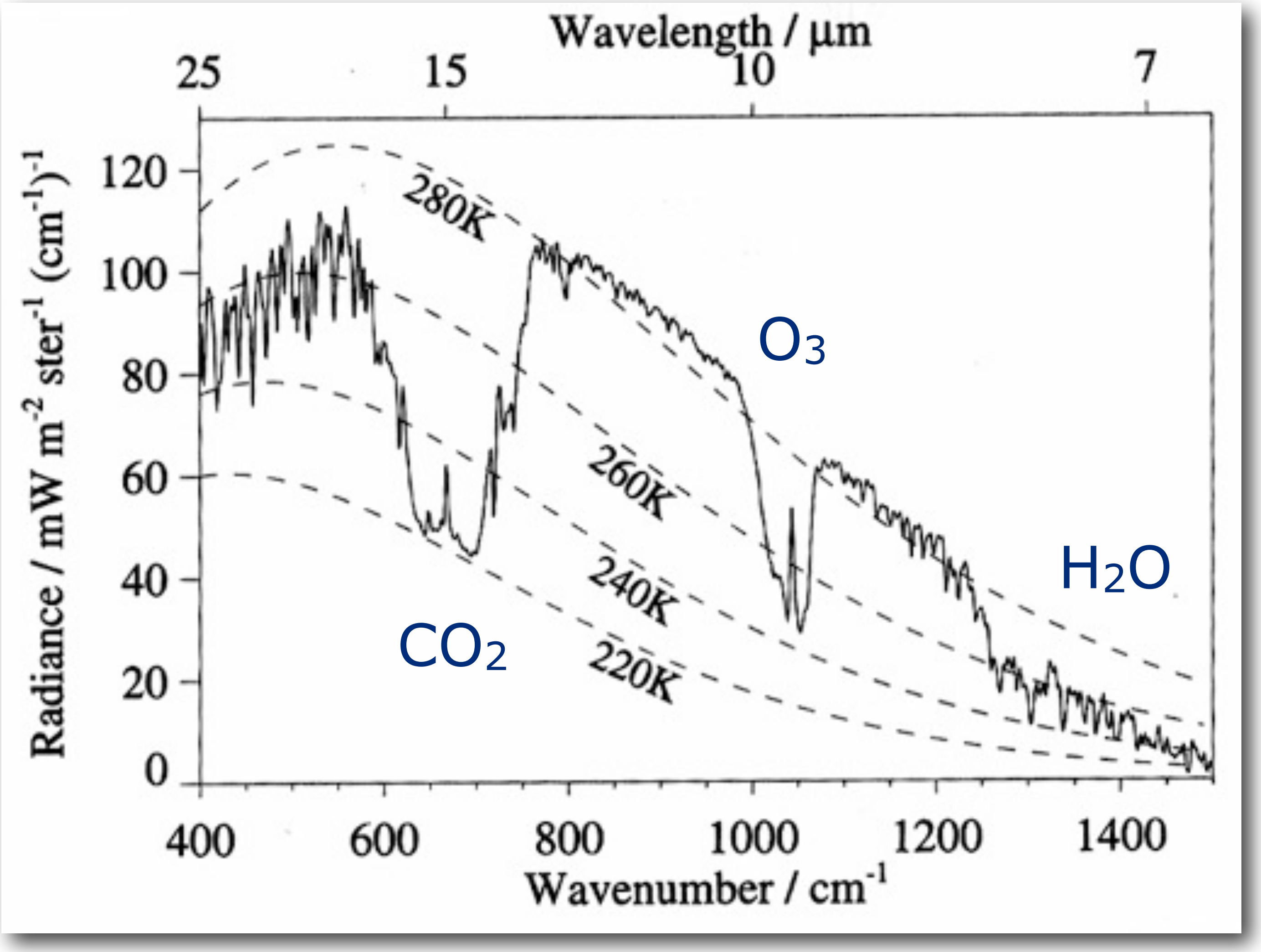


Observed radiances of the Earth

# Thermal spectra: $b \gg 1$

At wavelengths where the atmosphere is **optically thick**, most of the thermal radiation observed from space will come from the highest regions of the atmosphere:



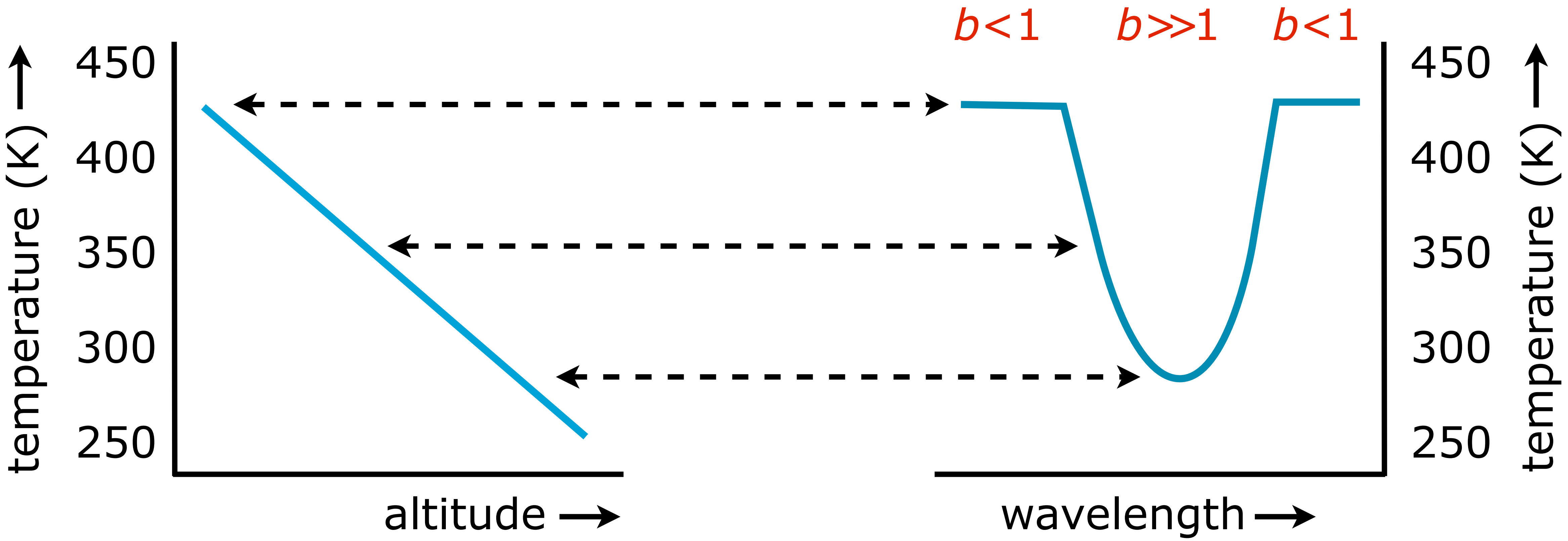


Observed radiances of the Earth

# Thermal spectra: absorption features

The **shape** of molecular absorption bands in thermal emission spectra depends on the atmospheric temperature profile:

**If *T* decreases with altitude, the band is seen in absorption**



# Thermal spectra: absorption features

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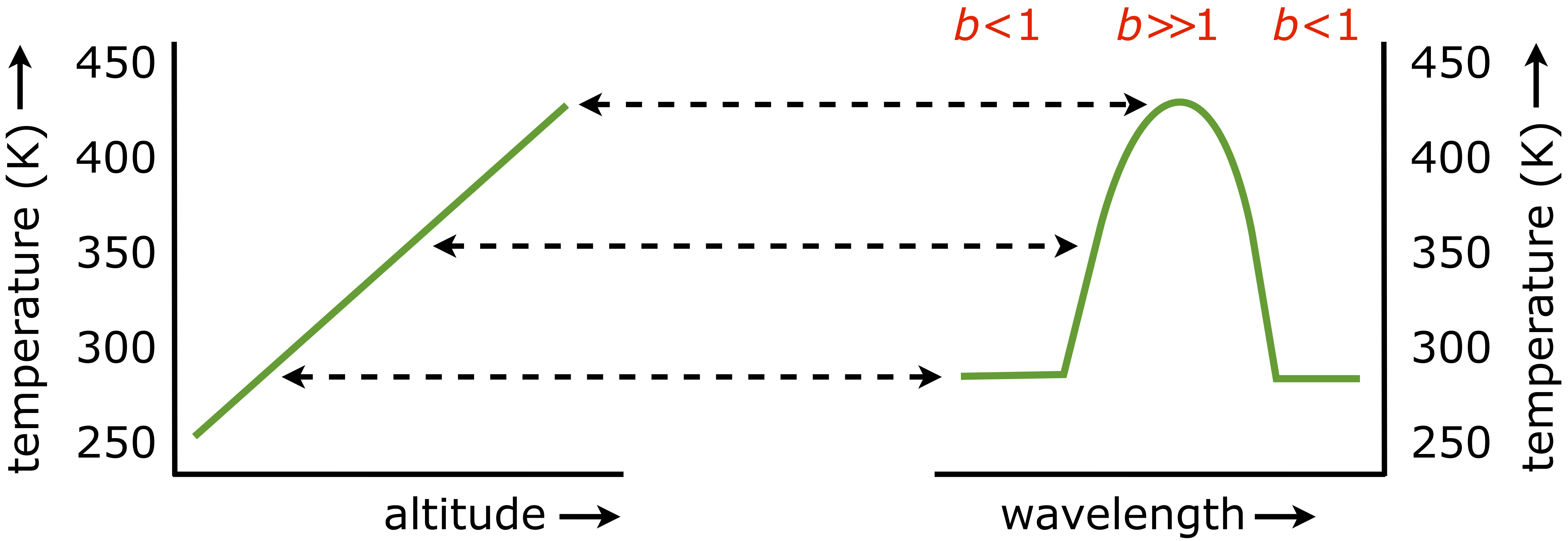
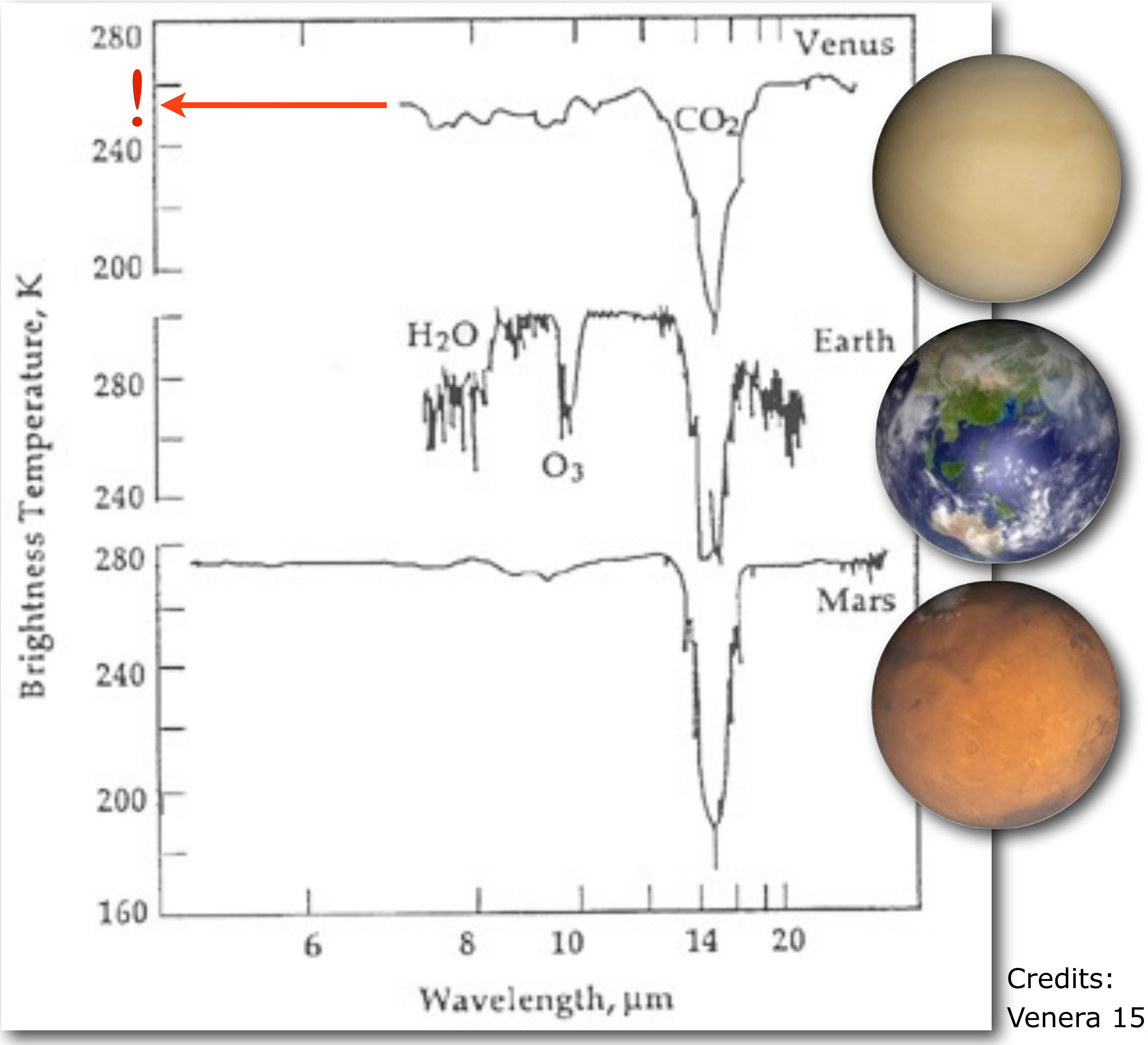


# Thermal spectra: observations

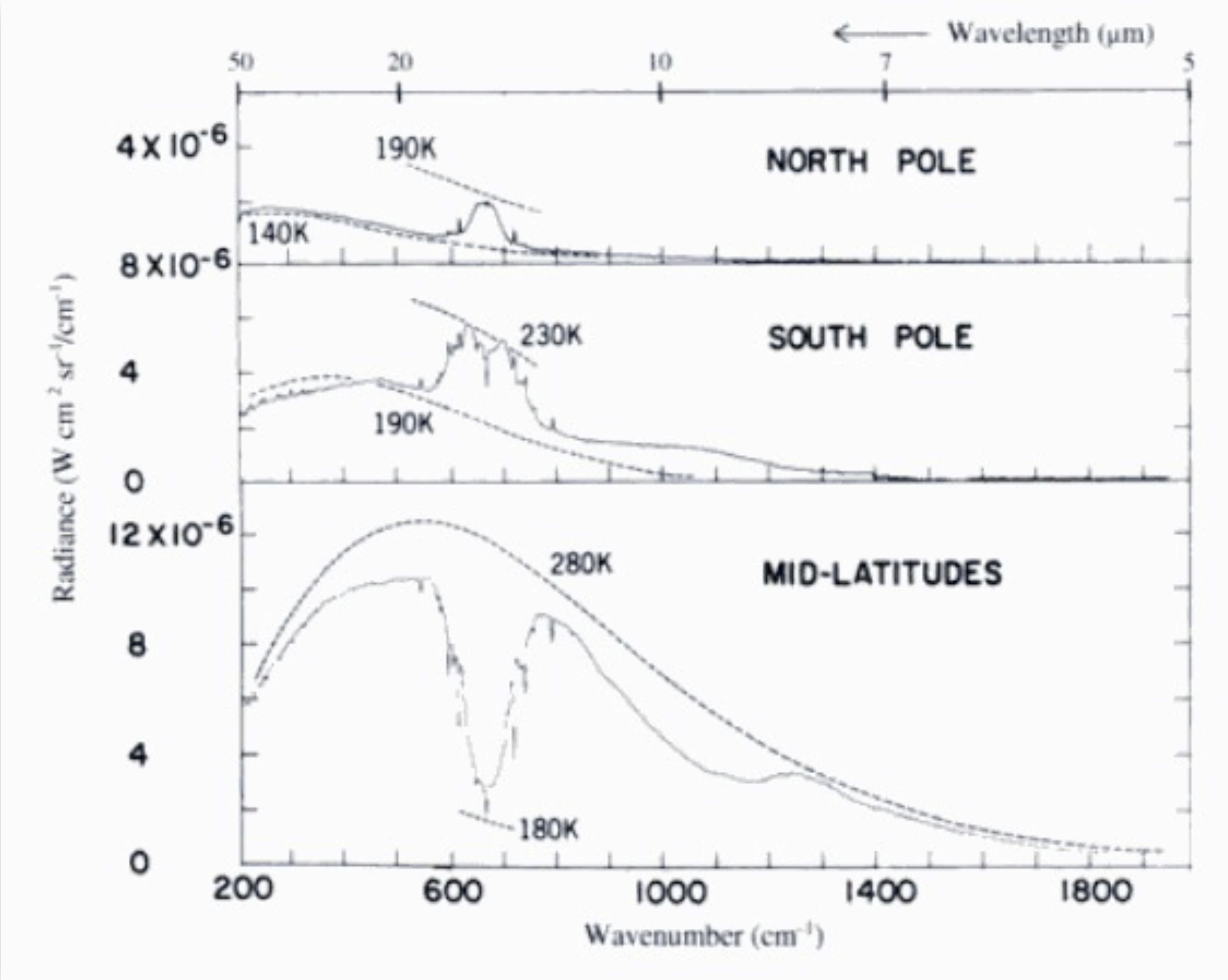
The brightness temperatures of Venus, the Earth, and Mars:



Credits:
Venera 15, Nimbus 4, and Mariner 9

# Thermal spectra: observations

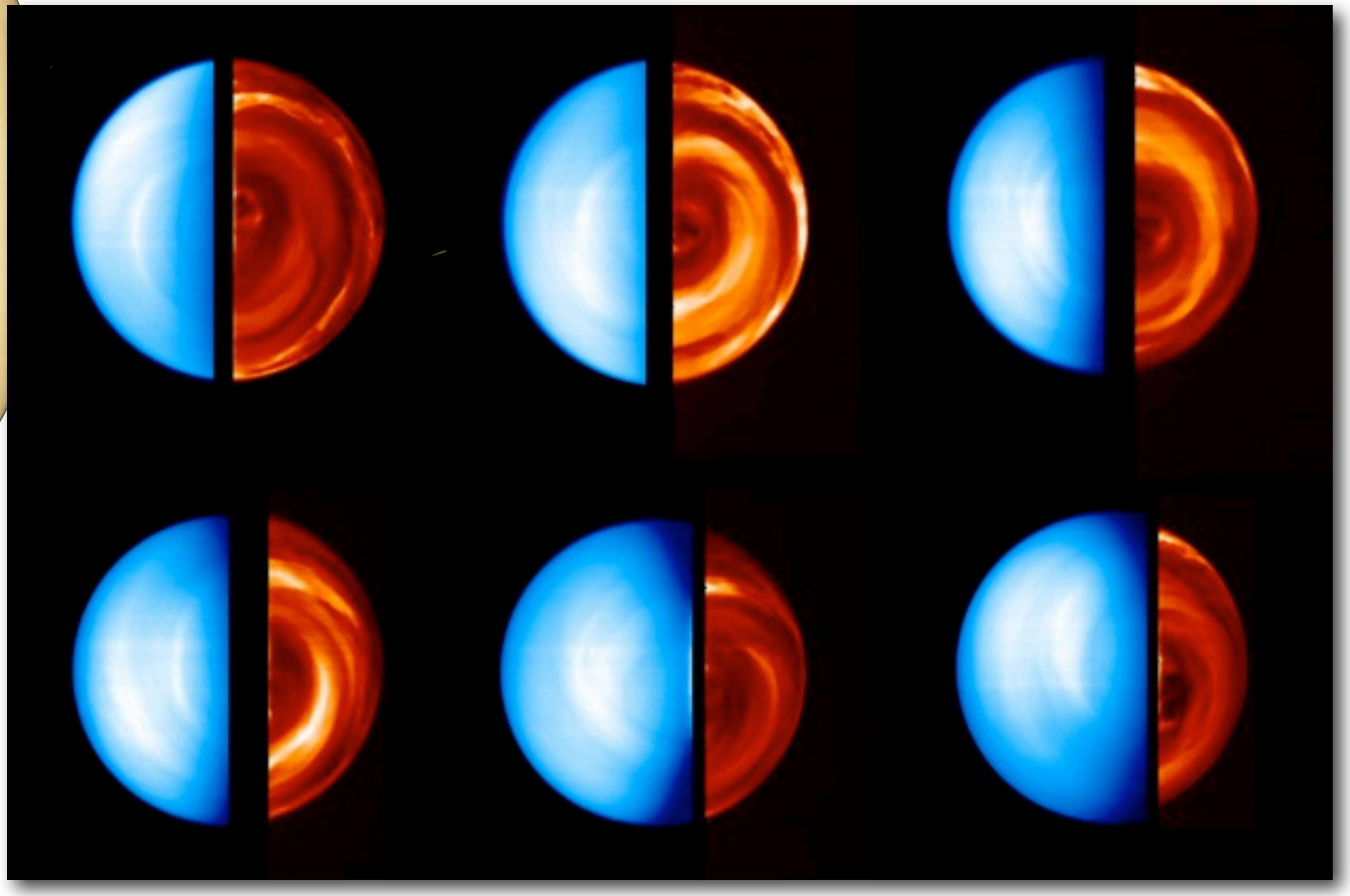
Emission spectra can vary across a planet, depending on the local temperatures:



Credit: MGS/NASA

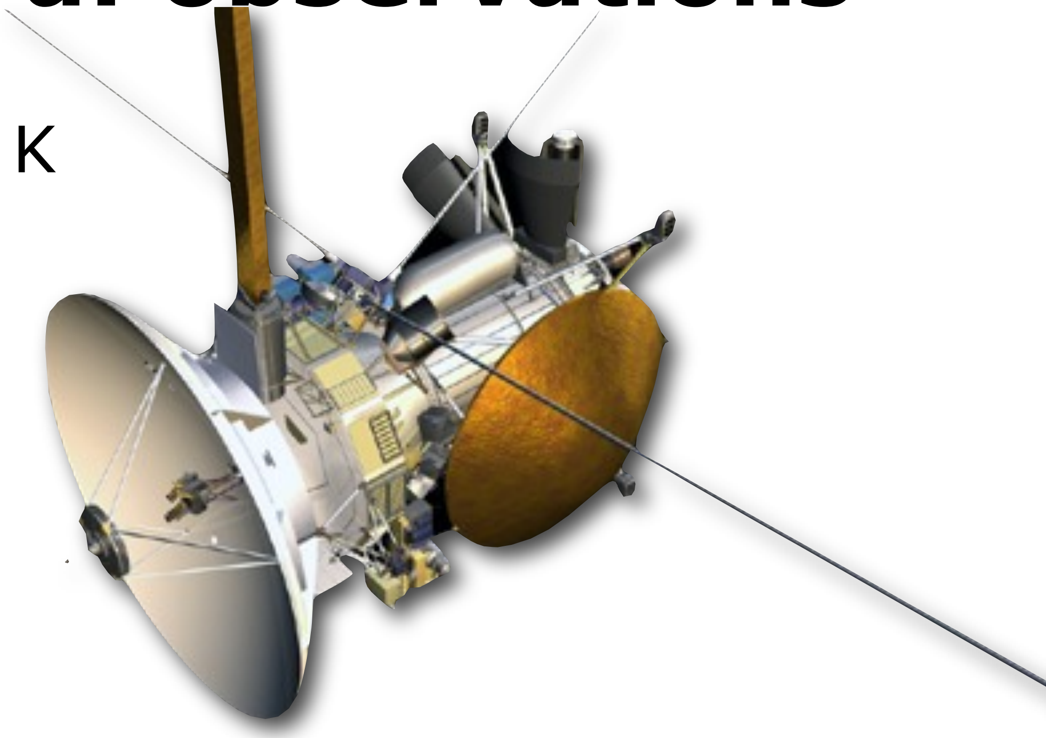
# Thermal spectra: observations

Venus’ south pole observed by VIRTIS on the Venus Express mission over 7 consecutive Earth days. Left side: reflected light; right side: thermal emission

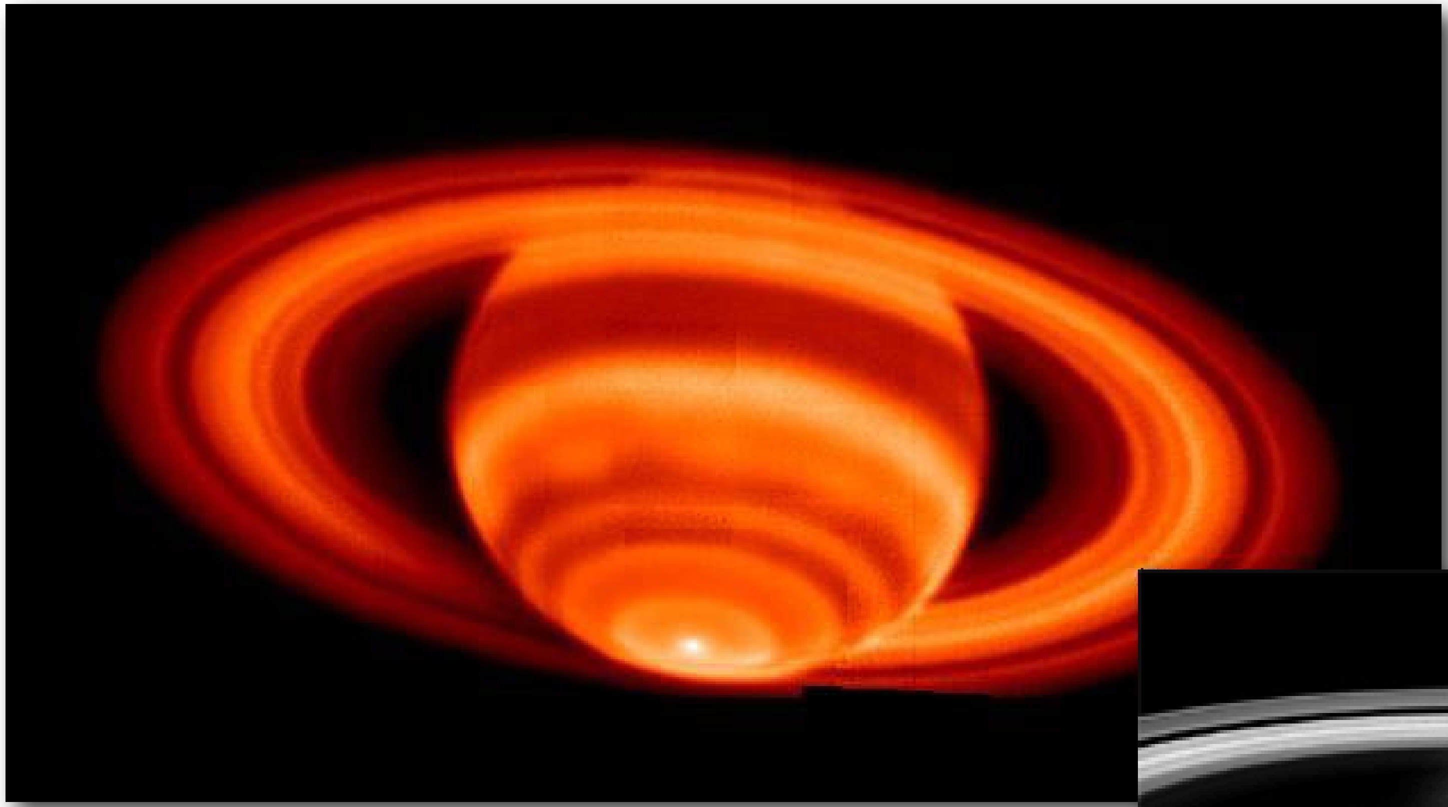


Venus’ south polar vortex: bright at λ=1.7 μm: low, warm clouds, dark at λ=1.7 μm: upper, cold clouds

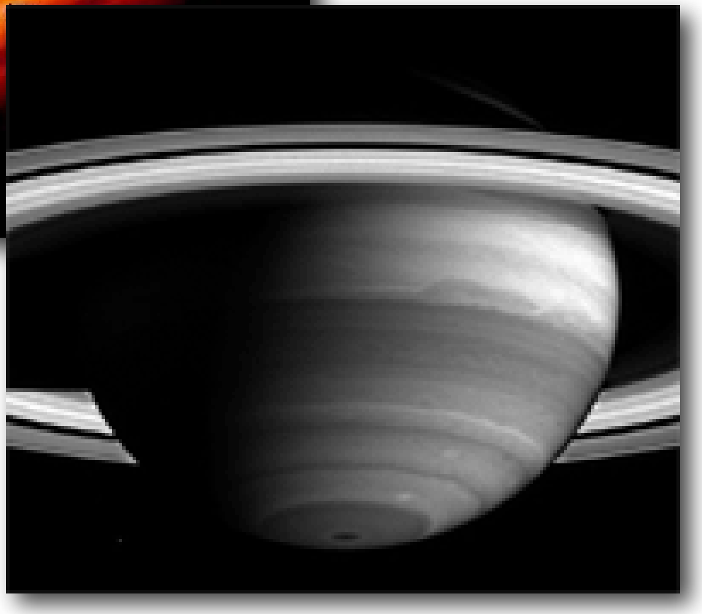
# Reflected and thermal spectra: observations



Saturn’s South pole is unexpectedly hot: > 150 K



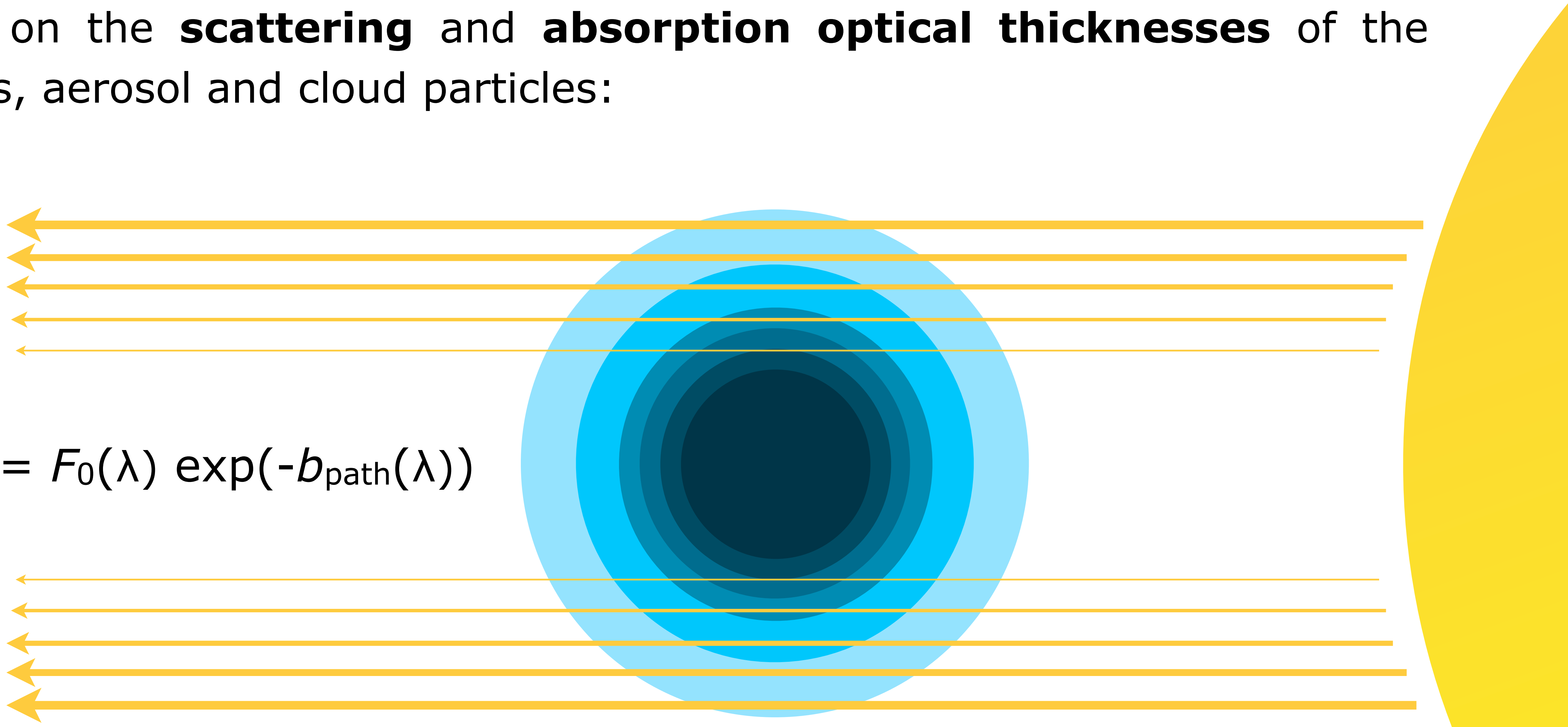
Keck-image at 17.65 microns



Cassini-image in the visible

# Planetary transit spectroscopy: principle

During the primary transit, starlight filters through the outer edges of the planetary atmosphere. How much flux at wavelength λ is transmitted, depends on the atmospheric extinction optical thickness at wavelength λ, thus on the **scattering** and **absorption optical thicknesses** of the gases, aerosol and cloud particles:

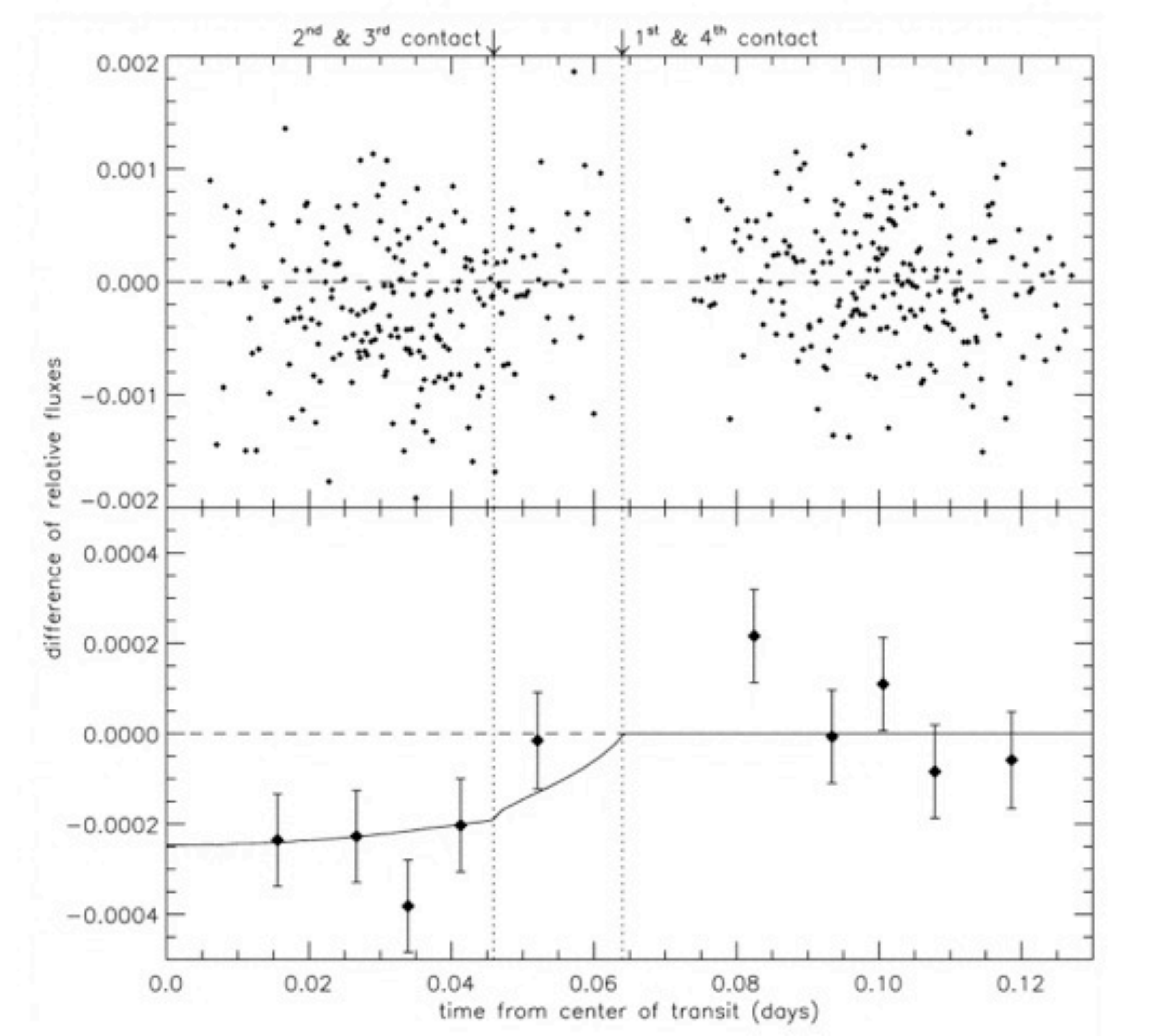


$$F(\lambda)= F_0(\lambda)\ \exp(-b_{\mathrm{path}}(\lambda))$$

Side-view of a primary transit (not to scale)

# Planetary transit spectroscopy: observations

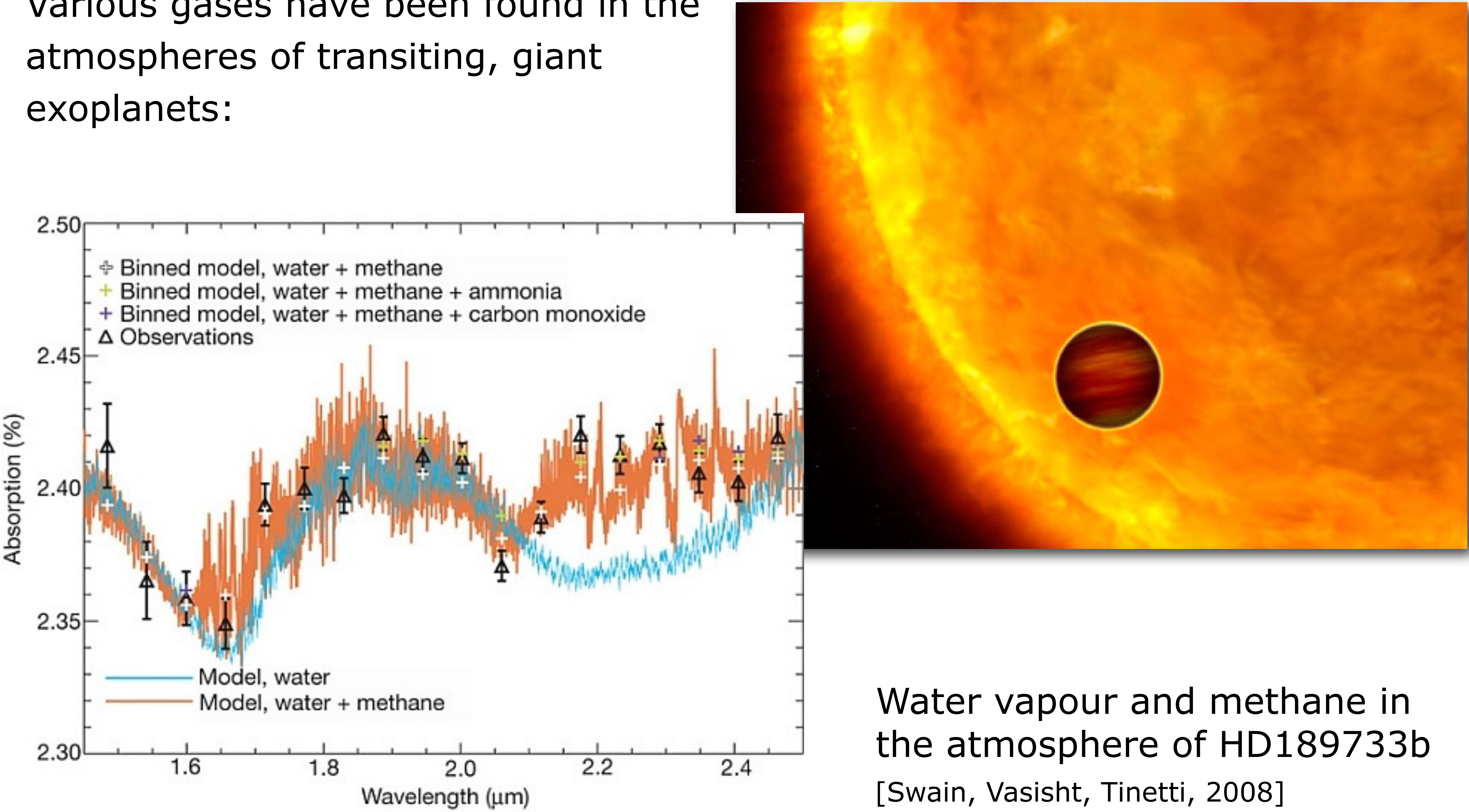
The first indications of an exoplanetary atmosphere were obtained from a primary transit observation of HD209458b (Charbonneau et al., 2001):



Observations of the depth of the Na D absorption lines (589.3 nm) with STIS/HST: less sodium detected than expected from models (high clouds?)

# Planetary transit spectroscopy: observations

Various gases have been found in the atmospheres of transiting, giant exoplanets:

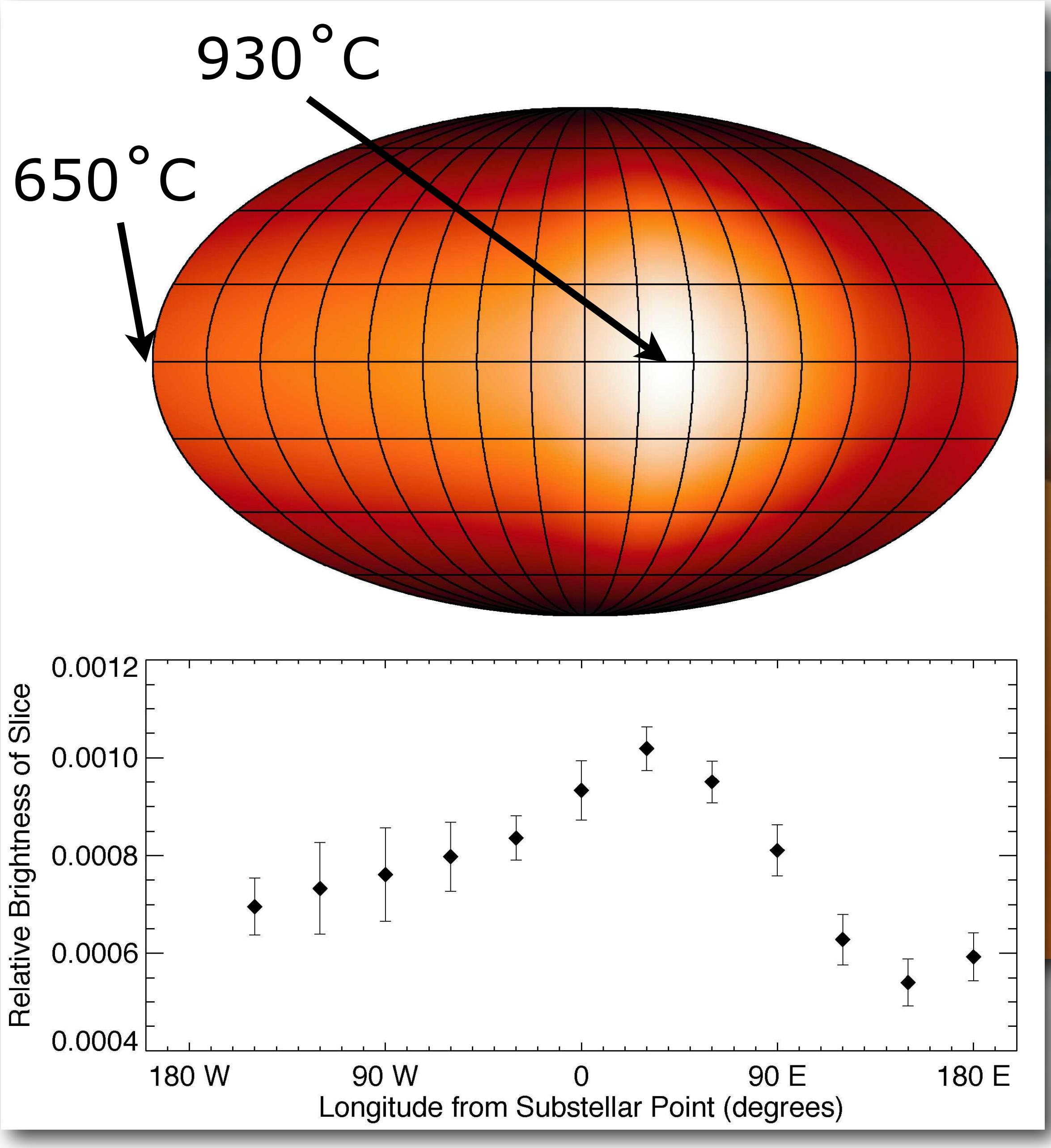


Water vapour and methane in the atmosphere of HD189733b

[Swain, Vasisht, Tinetti, 2008]

# Transiting planets: a thermal map

Below is the first thermal map of an exoplanet derived from observations at 8 microns (Knutson et al., 2007). HD189733b is a hot-Jupiter that orbits its star in 2.2 days, it is tidally locked to its star:



Artist's impression of the planet

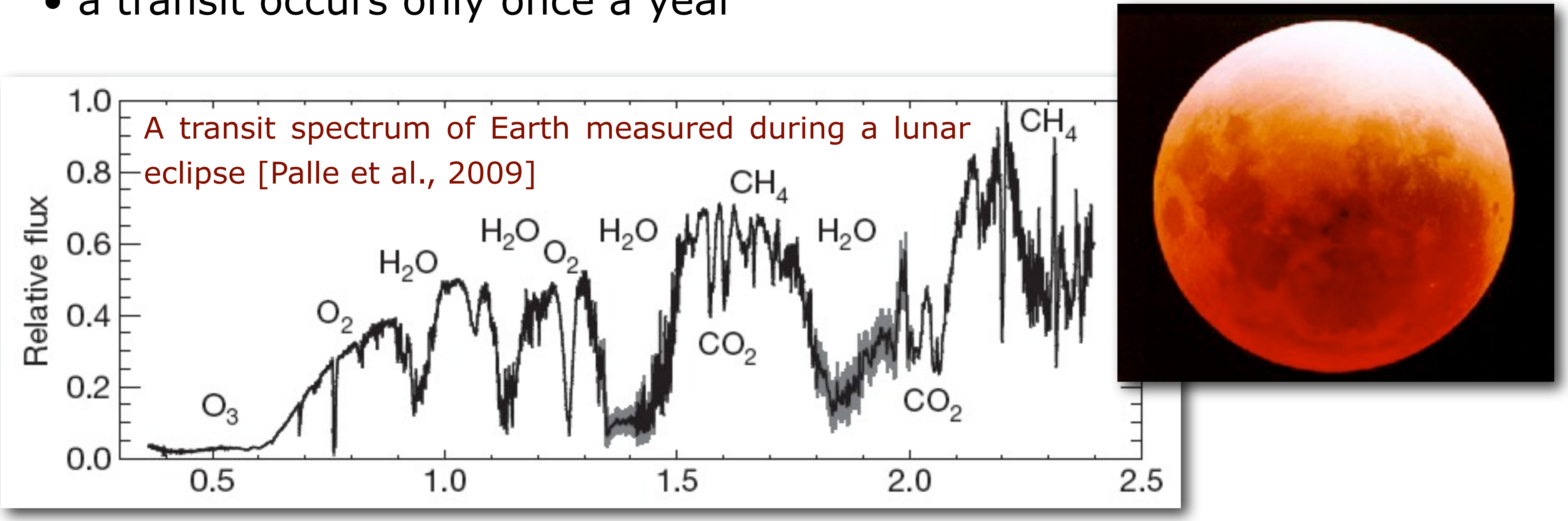
# Planetary transit spectroscopy: exo-Earths?

Using transit spectroscopy to detecting gases in the atmospheres of terrestrial exoplanets appears to be very difficult:

- the atmospheres are *very* thin compared to the solid planet
- the variability/noise of the starlight will overwhelm the planet’s signal

If the terrestrial exoplanet is in the habitable zone of a solar-type star, there are additional problems:

- a transit takes only a few hours (for Earth ~ 13 hours)
- a transit occurs only once a year


A transit spectrum of Earth measured during a lunar eclipse [Palle et al., 2009]