Planets and Exoplanets 2010: Exercises to Planetary Atmospheres

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D. M. Stam

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1 Hydrostatic equilibrium

Atmospheres can be assumed to be in hydrostatic equilibrium: there is a balance between the gravity that pulls a parcel downwards and the difference between the pressure below and above a parcel that pushes the parcel upwards:

$$dp = -\rho(z)g(z)\, dz, \tag{1}$$

with p the ambient pressure, ρ the density, g the acceleration of gravity, and ρ the density at altitude z. The minus sign indicates that the direction of the gravity is opposite to that of the pressure.

a. Assume that the gas is ideal, thus that

$$p(z) = \frac{\rho(z)T(z)k}{m(z)}, \tag{2}$$

with T the ambient temperature, k Boltzmann's constant ($1.38 \times 10^{-}23$ m^2 kg s^{-2} K^{-1}), and m the average molecular mass. Assume that m, g, and T are altitude independent and derive the following equation

$$p(z) = p(0)\exp(-zmg/kT). \tag{3}$$

b. The ratio $H = kT/mg$ is called the *scale height* of an atmosphere. Calculate the scale height of the Earth's atmosphere at the surface (use $T = 294$ K) assuming an atmospheric composition of 78 % N_2 and 22 % O_2 and using $g = 9.8$ m s^{-2}.

c. Repeat the calculation under b at an altitude of 50 km, assuming the same atmospheric composition, and
1. $T = 276$ K and the surface value of g
2. $T = 276$ K and the altitude adapted value of g
3. $T = 294$ K and the altitude adapted value of g

Do you think it is valid to assume constant values for T and g across most of the Earth's atmosphere? Explain your answer.

d. Calculate how the scale height of Jupiter's atmosphere would change if the planet moved from its current orbit, where its $T = 120$ K, to an orbit extremely close to the Sun (typical for currently known exoplanets), such that T would increase to 2500 K. How would the atmospheric density and the radius of the planet change (qualitatively)?

2 Equilibrium temperatures

An indication of the temperature on a planet can be obtained by calculating its *equilibrium temperature* T_e:

$$4\pi\sigma\, T_e^4 = (1-a)\,\frac{L}{4d^2}, \tag{4}$$

with σ the constant of Stefan-Boltzmann (5.670×10^{-8} W m^{-2} K^{-4}), a the (wavelength independent and dimensionless) albedo of the planet, L the luminosity of the star (in W), and d the distance between the star and the planet (in m).

a. Write down an expression for the stellar flux that is intercepted by a planet with radius r at a distance d from the star.

b. Write down the expression for the stellar flux that is absorbed by the planet that has an albedo a.

c. Assuming the temperature of the planet is constant, all of the absorbed flux should be reemitted by the planet. According to the law of Stefan-Boltzmann this emitted flux equals σT_e^4 per unit surface of the planet. With this information, derive Eq. 4.

d. Calculate the equilibrium temperatures of Venus ($a = 0.7$) and the Earth ($a = 0.4$). The solar luminosity, $L_{\rm sun}$, is 3.84×10^{26} W.

The planetary system around the red dwarf star Gliese 581 (GJ 581) counts at least six planets. Counting from the star, these planets and the semi-major axes of their orbits are (values from Mayor et al., 2009): GJ 581 e at 0.03 AU, GJ 581 b at 0.04 AU, GJ 581 c at 0.07 AU, GJ 581 g at 0.146 AU, GJ 581 d at 0.218 AU, and GJ 581 e at 0.75 AU (the alphabetical order depends on the order of discovery). The luminosity of GJ 581 is estimated to be only 0.013 $L_{\rm sun}$.

e. Calculate the equilibrium temperatures of planets b, c, g, and d, assuming first a Venus-like albedo (0.7) and then an Earth-like albedo (0.4).

f. GJ 581 g has been in the news recently as a planet that could have liquid water, thought to be essential for life, on its surface. What do you think about this?

3 Bonus exercise: Venus and Earth

Although in some respects the Earth and Venus are 'twin planets', they have very different atmospheres. For example, the surface pressures on Earth and Venus are 1 bar and 95 bars, respectively.

a. Calculate the mass of each atmosphere both in grams and as a fraction of each planet's total mass.

b. Recalculate these values for Earth including Earth's ocaens as part of its 'atmosphere.' (If all of the water above Earth's crust were spread evenly over the planet, this global ocean would be 3 km deep.)

c. Compare the calculations for the two planets and comment on the results.