Planets and Exoplanets 2010: Exercises to Interiors

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D.M.Stam

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1 Hydrostatic equilibrium

If a 'parcel' of material is kept in balance by the downward pulling force of the gravity and the upward pushing force of the pressure it is said to be in hydrostatic equilibrium. The relation between the upward pressure p and the downward gravity at a distance r from the centre of a planet in hydrostatic equilibrium can be written as

$$p(r) = -\int_r^R g(r')\,\rho(r')\,dr', \tag{1}$$

with R the radius of the planet, g the acceleration of gravity, and ρ the density at distance r. The minus sign indicates that the direction of the gravity is opposite to that of the pressure.

a. Assume a constant density throughout the planet, and derive starting with Eq. 1 that the pressure $p_{\rm c}$ in the centre of the planet equals:

$$p_{\rm c} = \frac{2\pi G\rho^2 R^2}{3} = \frac{3GM^2}{8\pi R^4}, \tag{2}$$

with G the gravitational constant, and M the total mass of the planet. Hint: express the gravity into a function of the distance r.

b. Calculate $p_{\rm c}$ for the Moon and for the Earth.

c. Compare the pressures you calculated under **b.** with the real pressures in the Moon and the Earth, 0.045 Mbar and 3.6 Mbar, respectively. Explain the differences.

2 Speeding earthquake waves

a. You wake up in the middle of the night by a powerful jolt from below. Exactly 4 seconds later a strong horizontal shaking begins and knocks you off your bed. Explain both types of motion.

b. If the average velocity of the P-waves is 5.8 km/s and that of S-waves 3.4 km/s, calculate your distance from the epicenter of the quake (assume that the epicenter was on the surface).

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c. Explain in words and sketches how many measurements like your own are needed to exactly pinpoint the location of the epicenter.

d. Use Fig. 1 to determine the time that it takes a P-wave to travel from the epicenter of an earthquake to the other side of the Earth, assuming an epicenter at the surface and straight lines of propagation of the wave.

e. Discuss qualitatively how the arrival times of the waves would change if refraction were included.

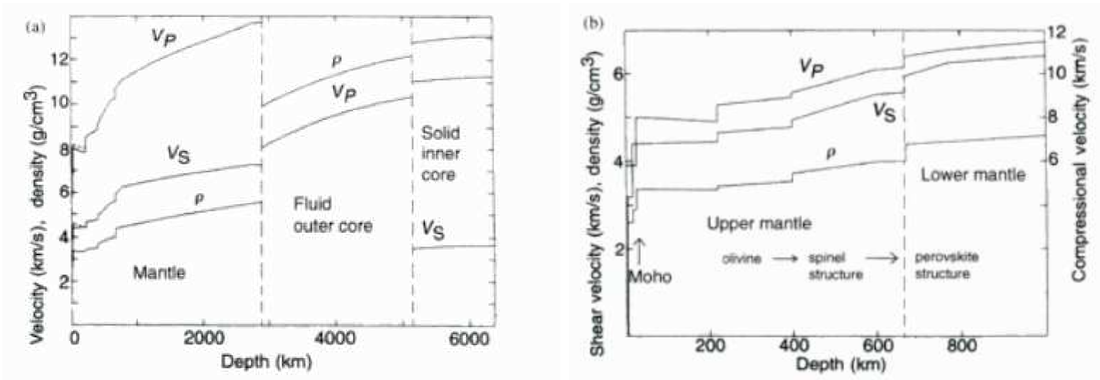

Figure 1: The velocities v_{P} and v_{S} of seismic P- and S-waves through the Earth's mantle and core and the density ρ of the mantle and core. The graph on the right gives the details of the uppermost 1000 km. This is Fig. 6.14 from *Planetary Sciences*, the original is from Pieri and Dziewonski [1999].

3 Mercury's interior

Mercury's mean density $\rho = 5.44$ g/cm^3. If Mercury consists entirely of rock ($\rho = 3.3$ g/cm^3) and iron ($\rho = 7.95$ g/cm^3), calculate the planets' relative abundance of iron by mass (the ratio of the mass of iron and the total mass).

4 The gravity field of a planet

The moment of inertia I of a spherical body with radius R is related to the density ρ via

$$I = \frac{8}{3}\pi \int_0^R \rho(r)\, r^4 \, dr. \tag{3}$$

a. Calculate the moment of inertia in units of MR^2 for a homogeneous planet.

b. Write down an expression for the moment of inertia I of a planet that consists of two homogeneous layers: an inner one with a density ρ_1 and a radius R_1, and an outer one with a density ρ_2, an inner radius R_1 and an outer radius R_2.

c. Calculate I in units of MR^2 for a planet with $\rho_1 = 10\rho_2$ and R_1 equal to $0.1R_2$.

d. Calculate I in units of MR^2 for a planet with $\rho_1 = 0$ and R_1 equal to $0.9R_2$.