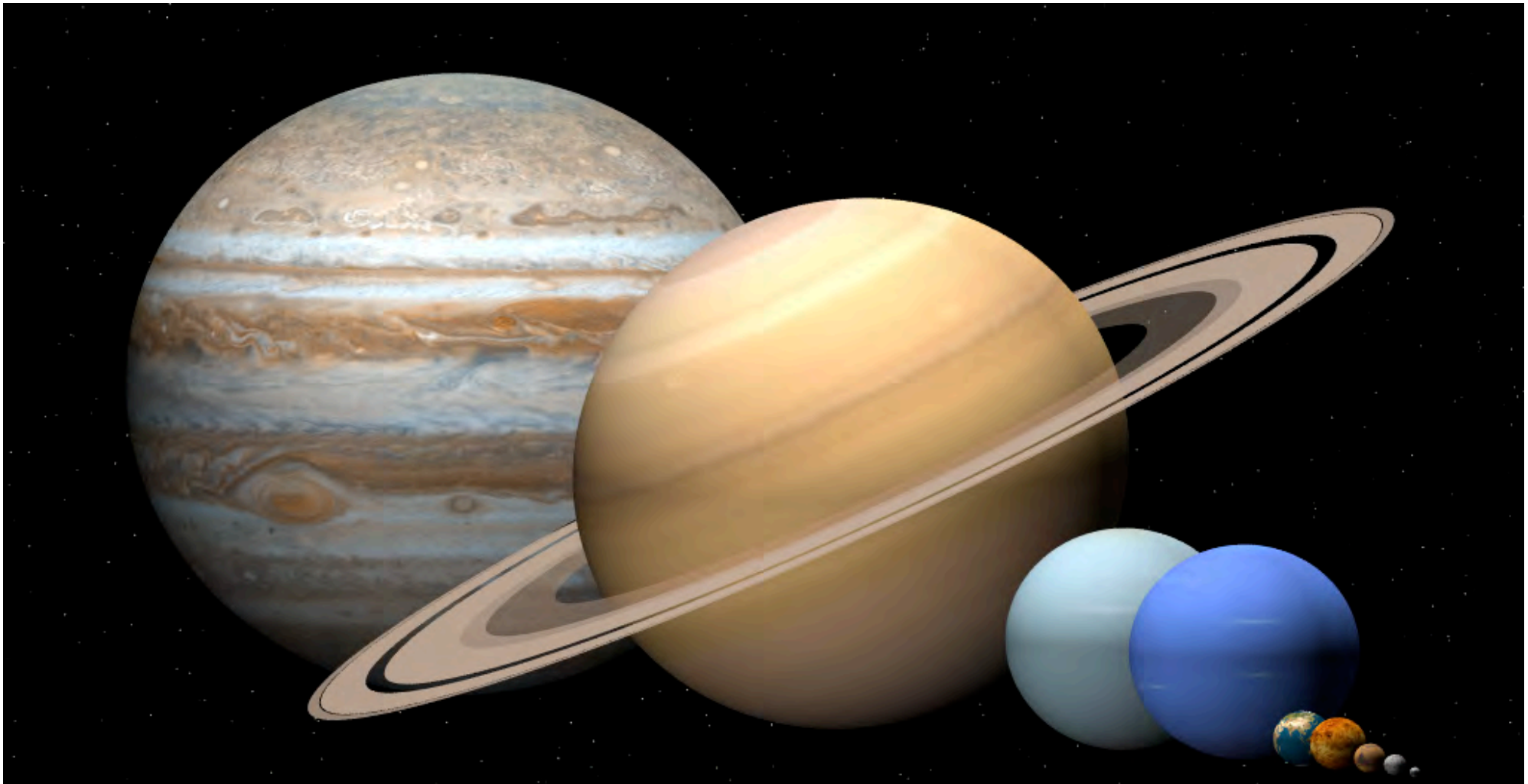


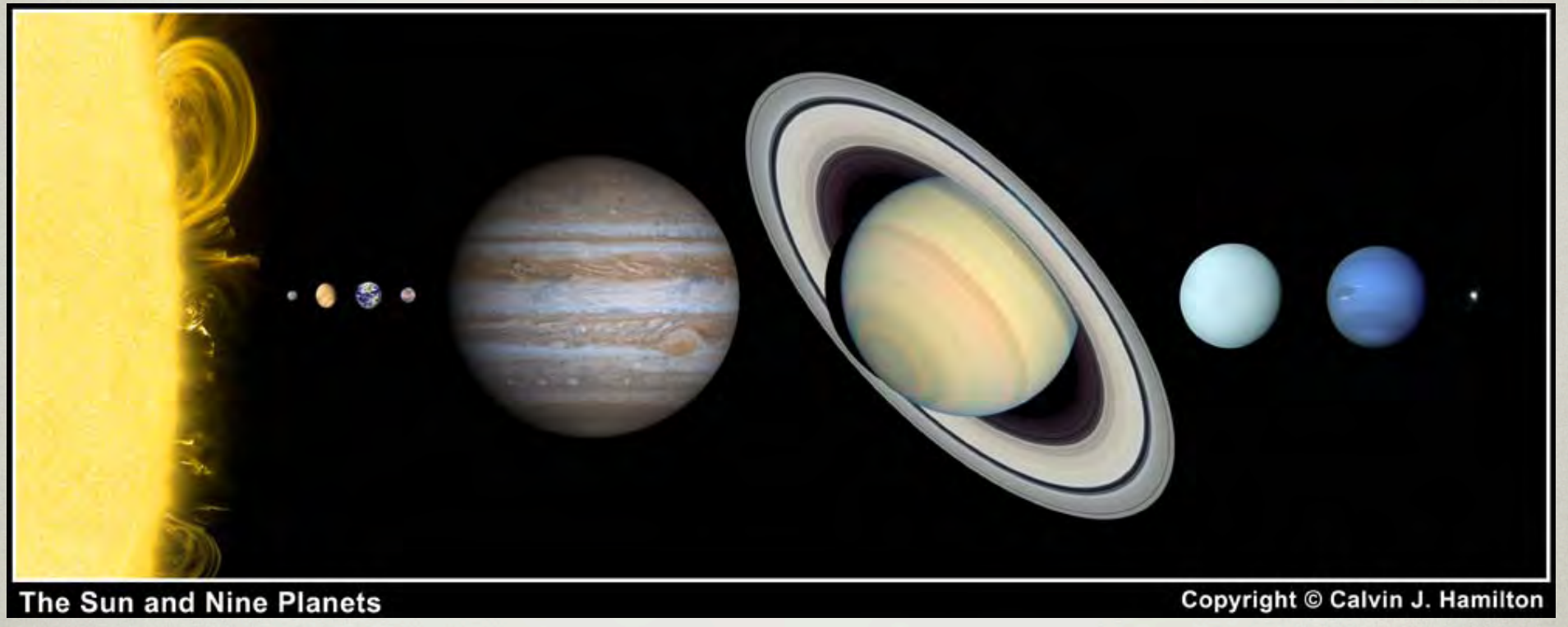
Planets and Exoplanets

Solar System Structure and Orbital Mechanics

Outline

1. Components of the solar system
2. Solar system planet properties
3. Kepler's Laws (Two-body problem)
4. Three-body problem
5. Tides and other forces

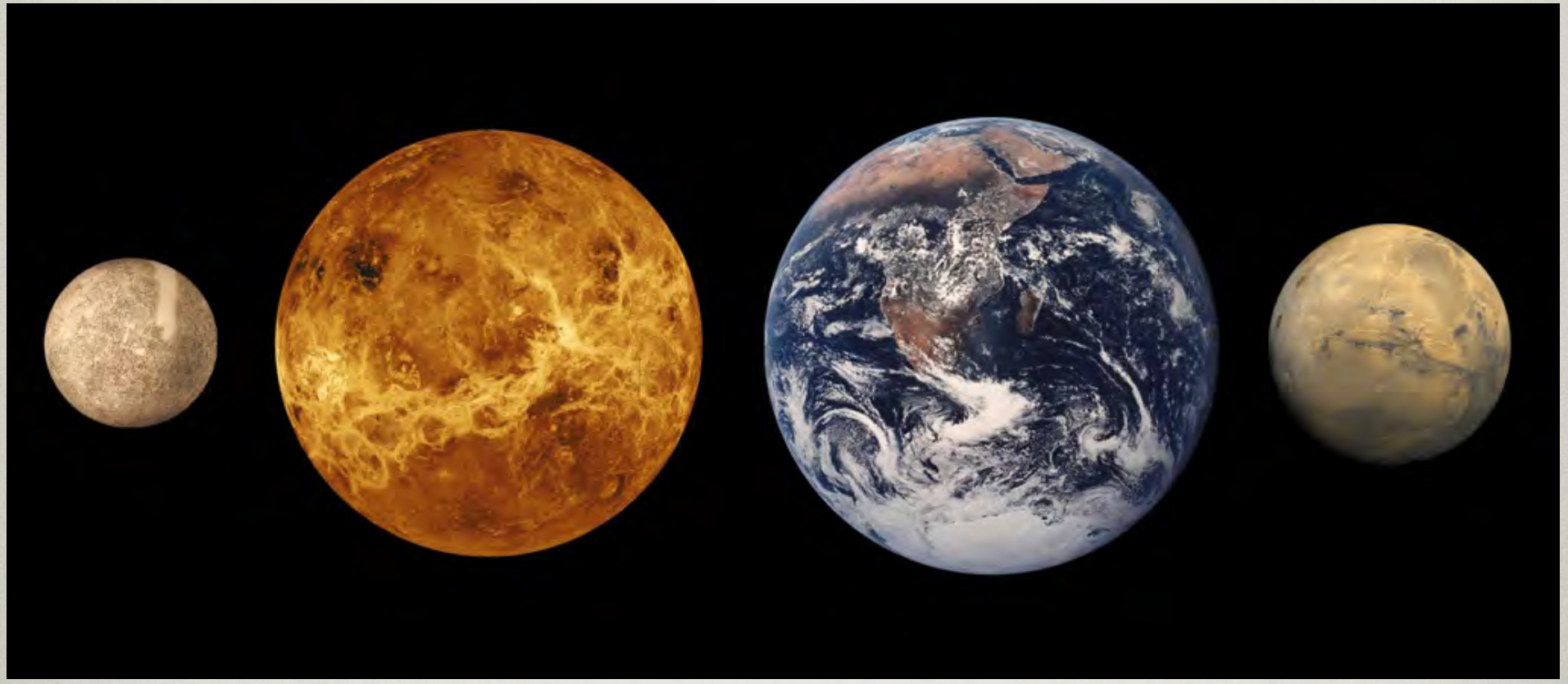
Inventory of the Solar System



www.solarviews.org/cap/misc/solarsystem.htm

- The Sun – our star (see MSc course: Solar Physics)
- $4*10^8$ time more luminous than Jupiter
- 99.8% of mass in solar system
- But 98% of angular momentum in orbiting planets

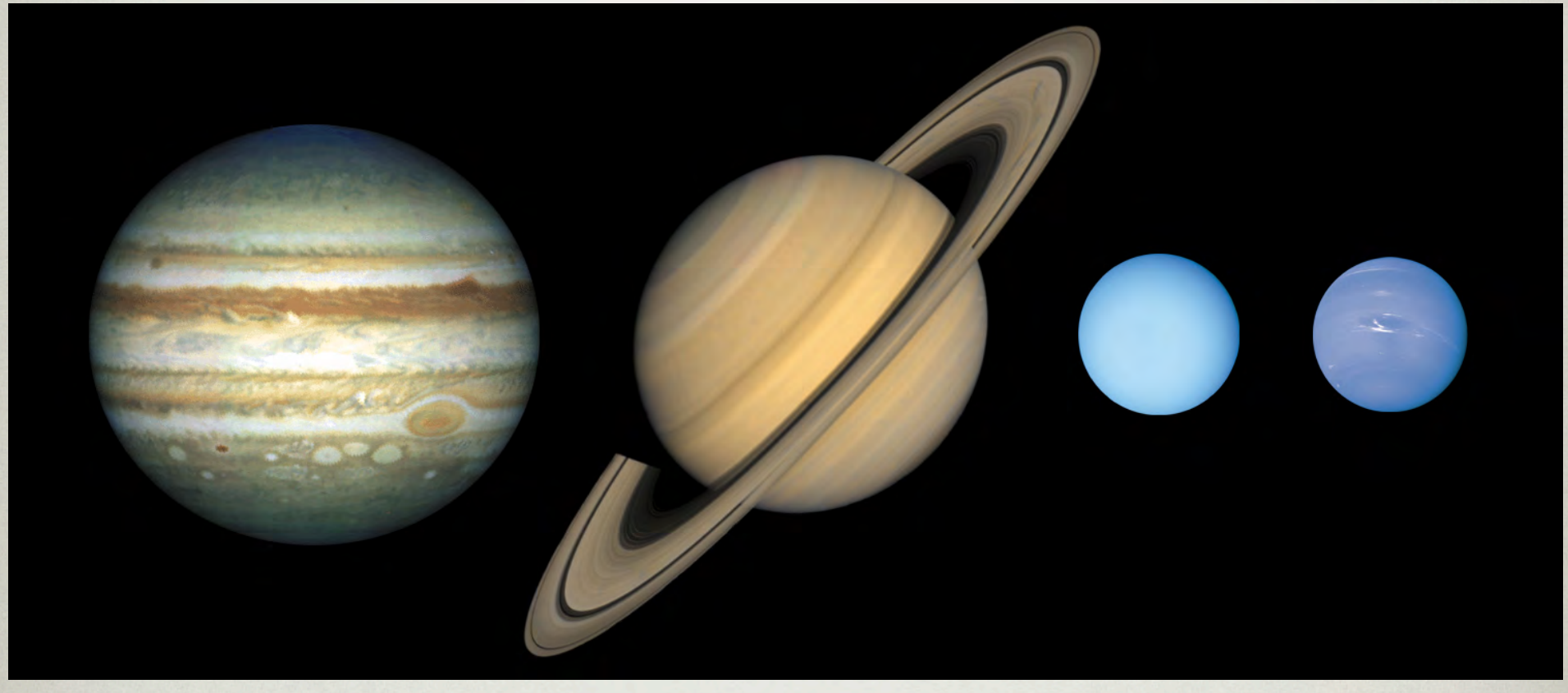
Terrestrial Planets



solarsystem.nasa.gov/multimedia/gallery/terr_sizes.jpg

Mercury Venus Earth Mars

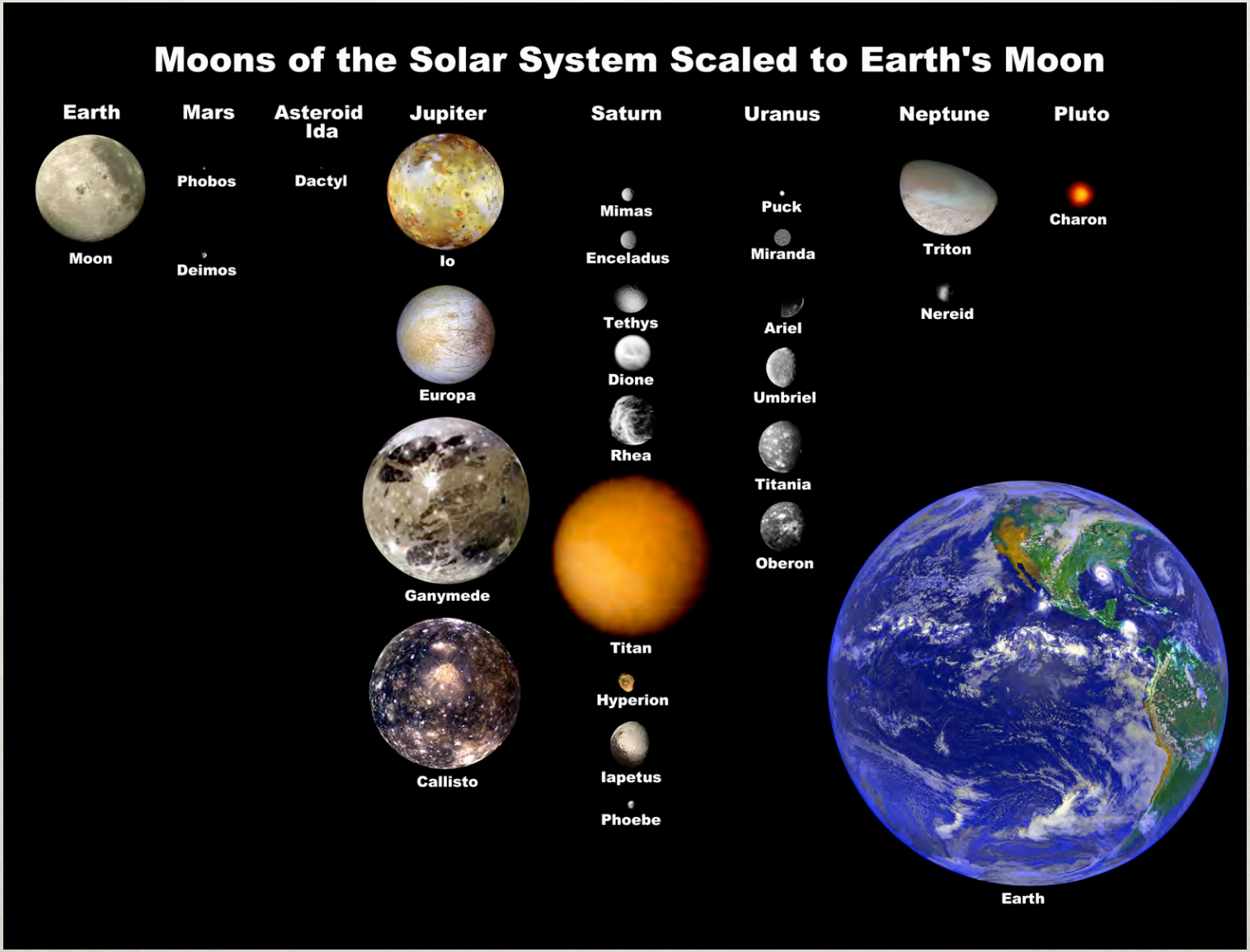
Giant Planets



solarsystem.nasa.gov/multimedia/gallery/gas_sizes.jpg

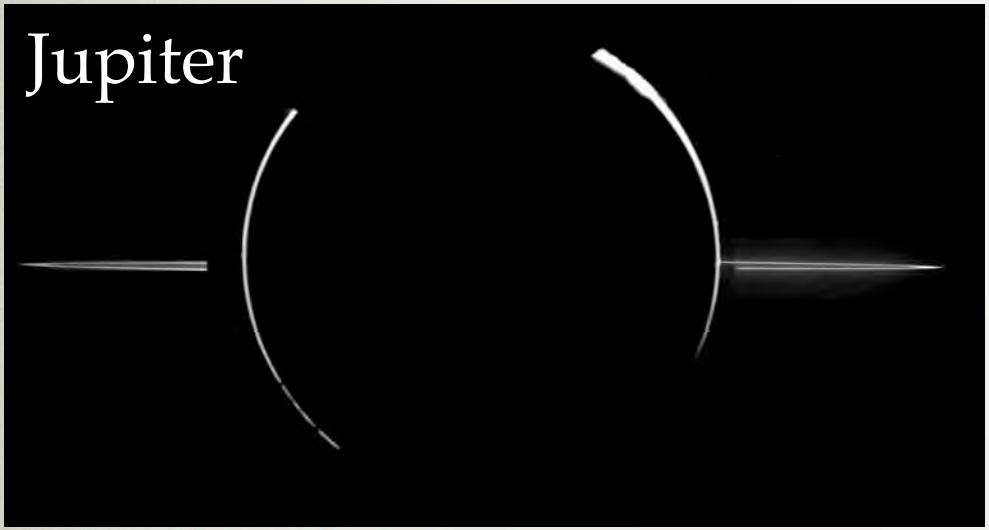
Jupiter Saturn Uranus Neptune

Planetary Satellites (Moons)



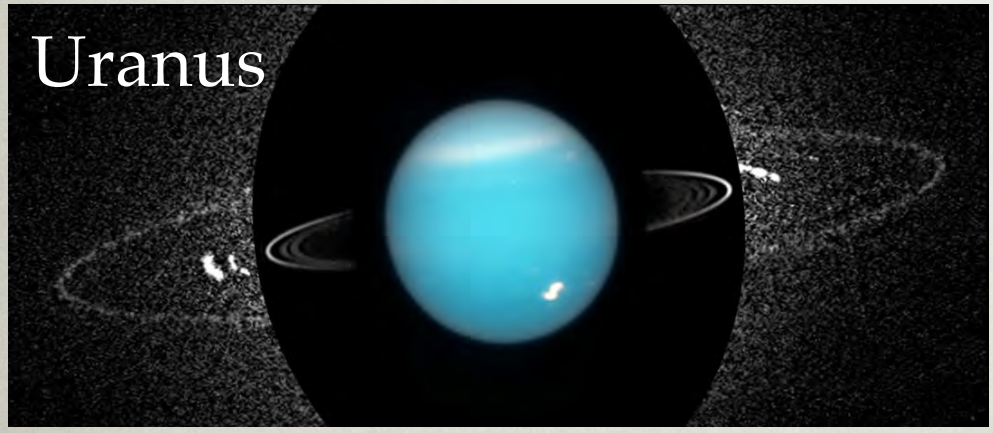
solarsystem.nasa.gov/multimedia/gallery/Many_Moons.jpg

Planetary Ring Systems

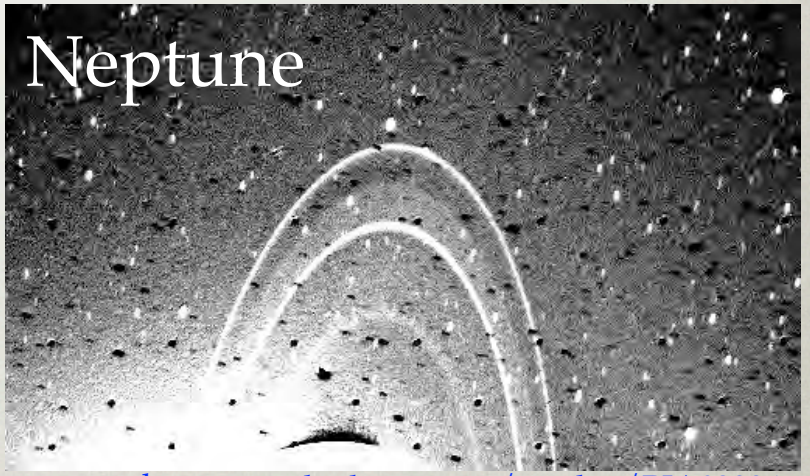


photojournal.jpl.nasa.gov/jpeg/PIA01621.jpg

photojournal.jpl.nasa.gov/jpeg/PIA08388.jpg

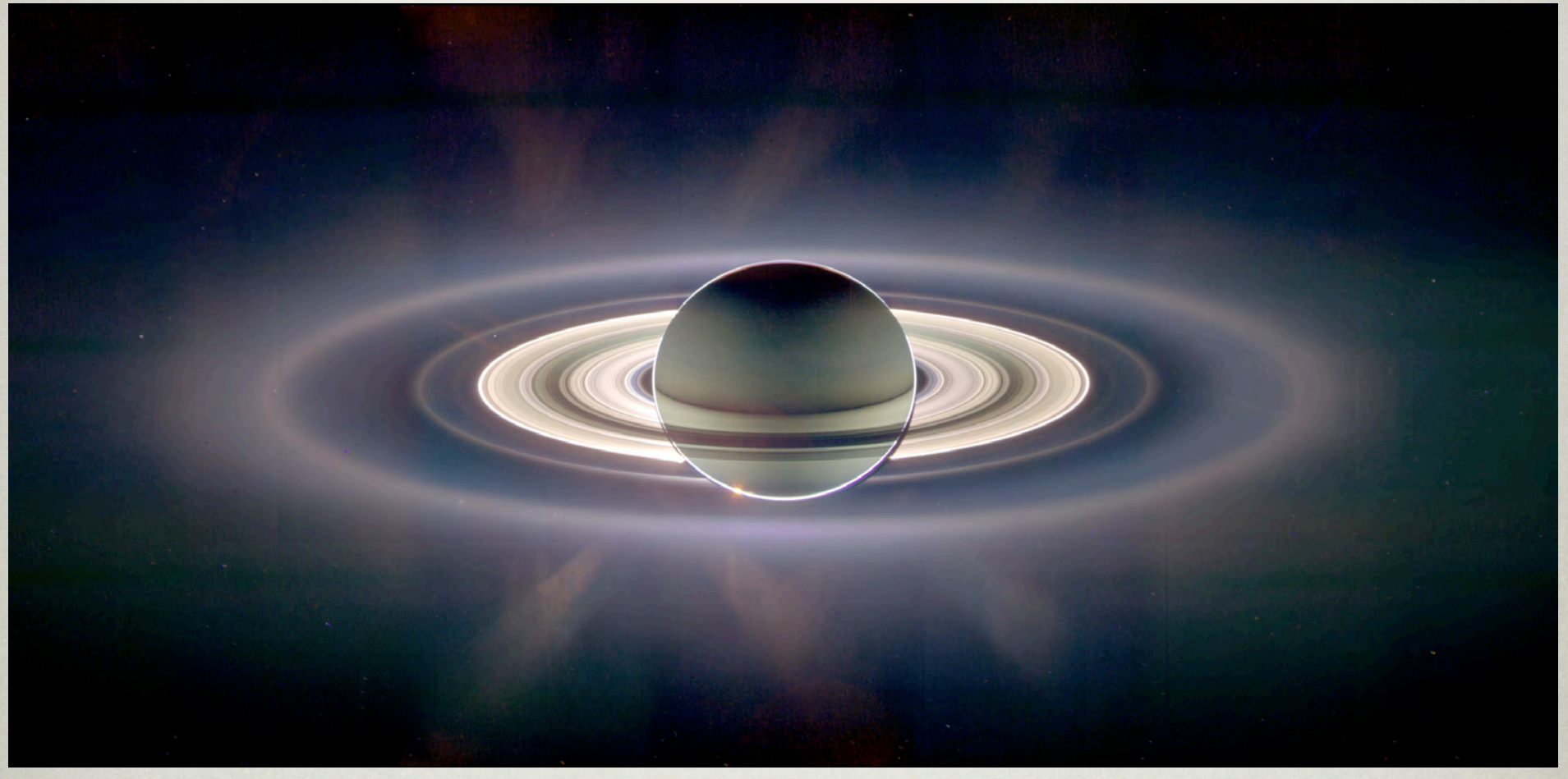


hubblesite.org/newscenter/archive/releases/2005/33/image/c/



photojournal.jpl.nasa.gov/catalog/PIA02224

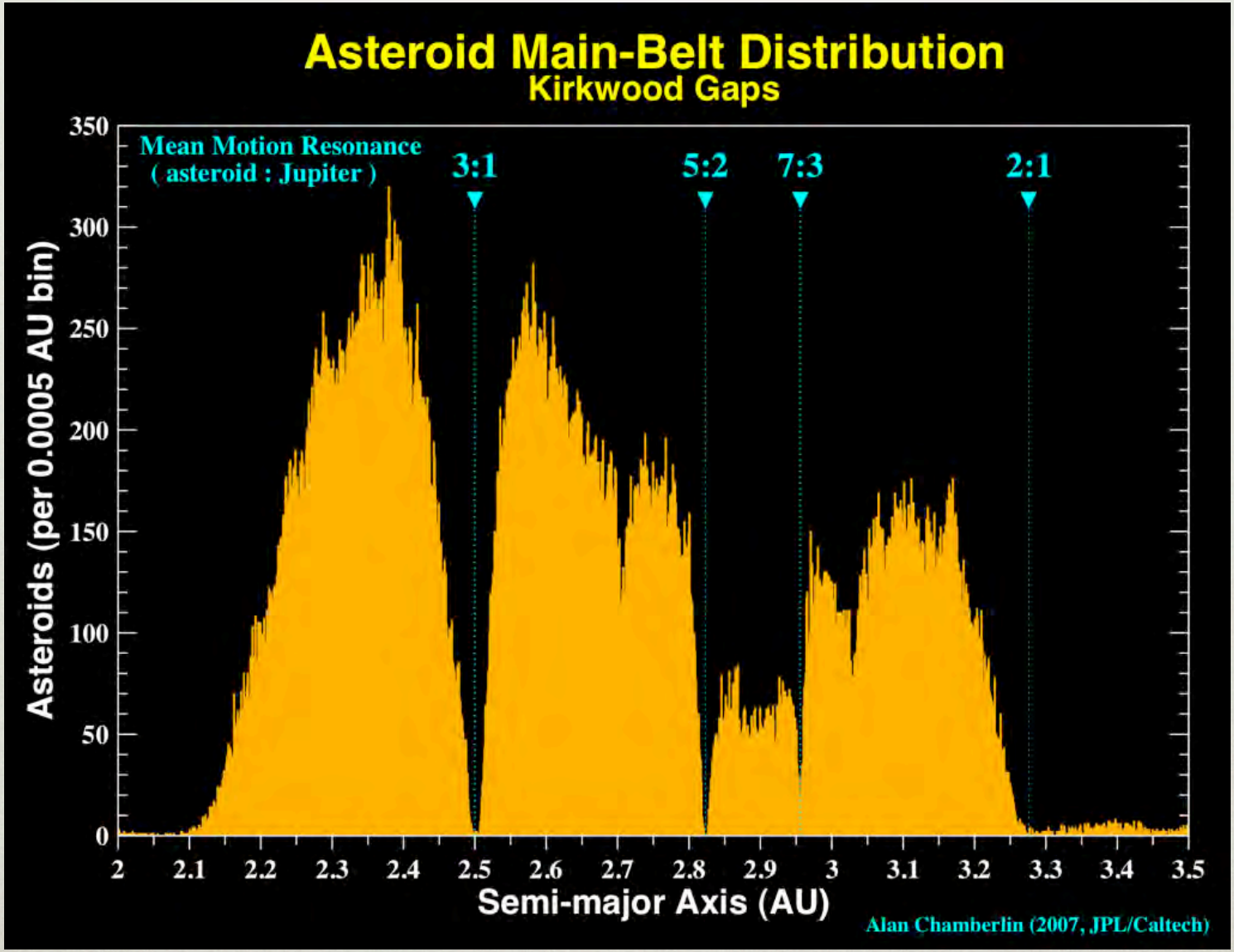
Saturn seen from the Back



photojournal.jpl.nasa.gov/figures/PIA08329_fig2.jpg

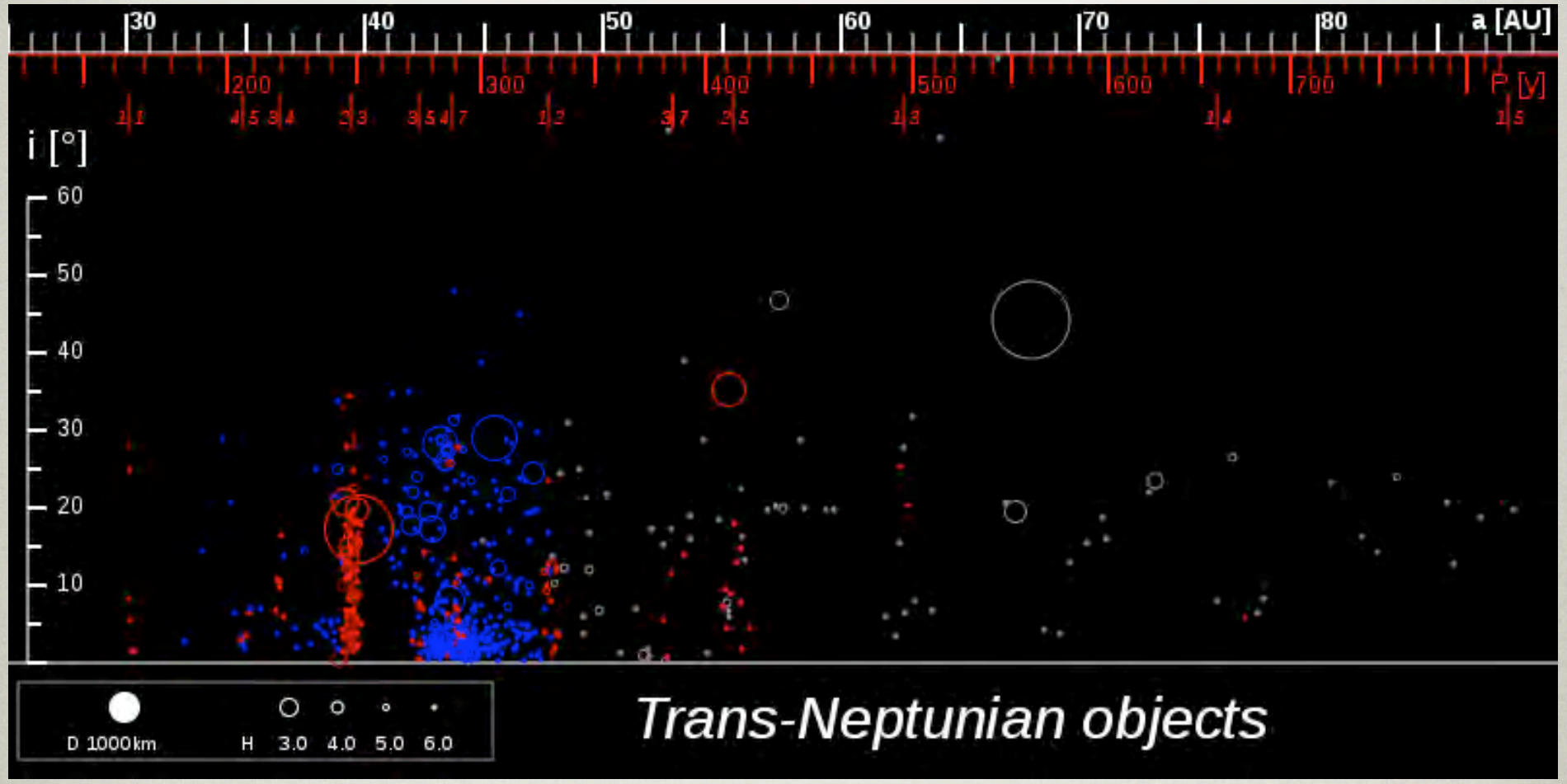
Asteroids

Asteroid Locations



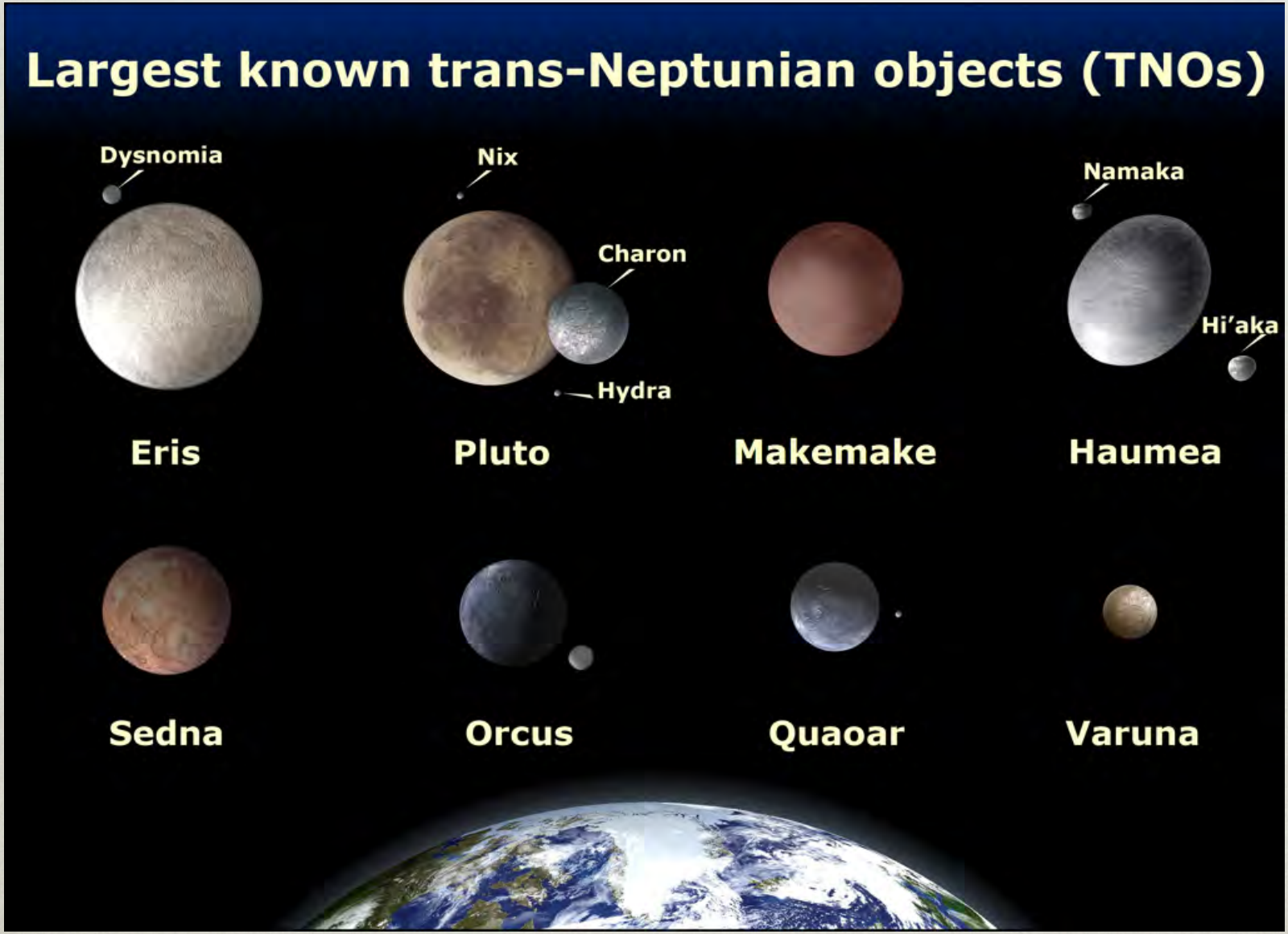
ssd.jpl.nasa.gov/?histo_a_ast

Kuiper Belt



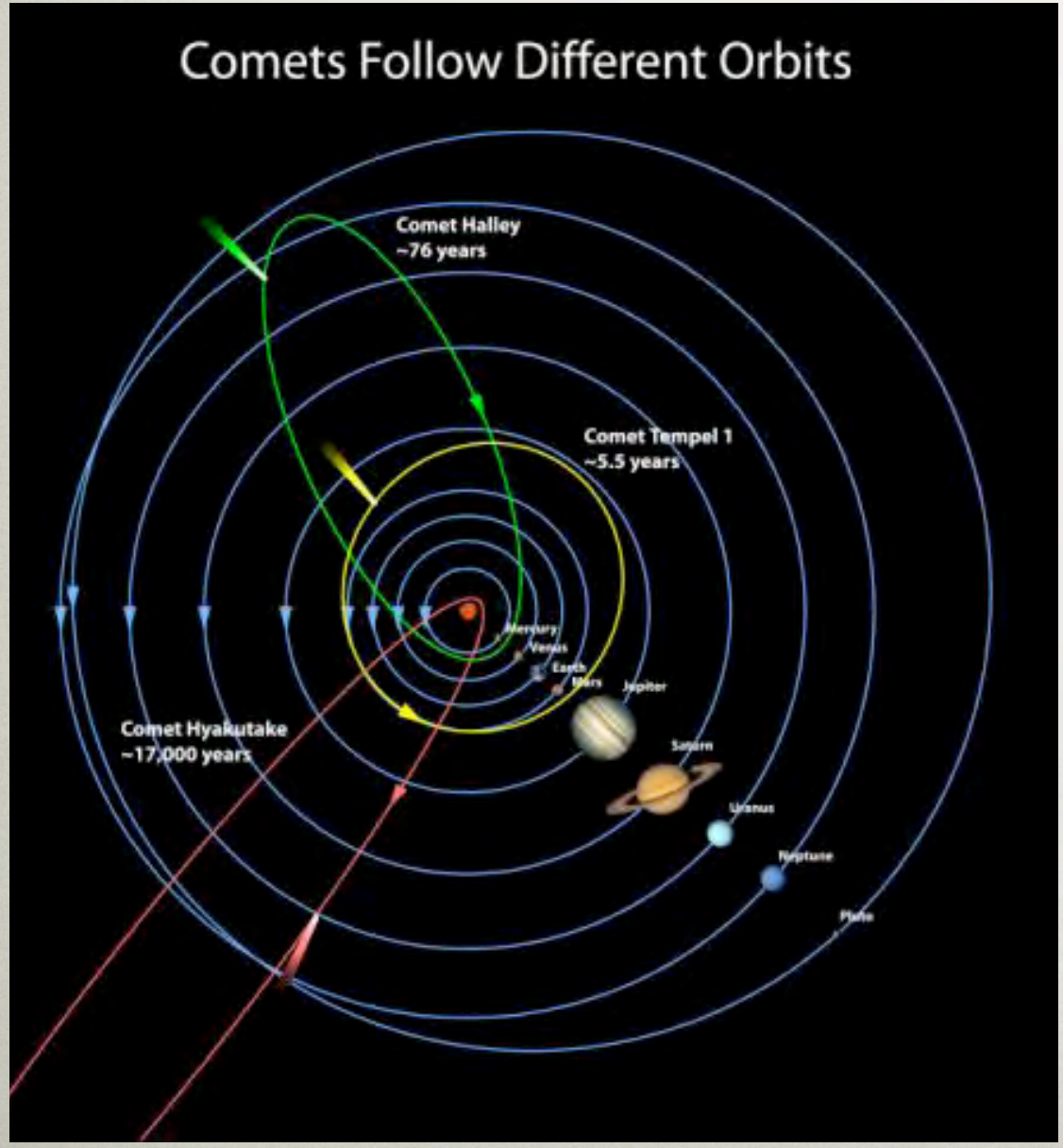
en.wikipedia.org/wiki/File:TheKuiperBelt_75AU_All.svg

Kuiper-Belt Objects



en.wikipedia.org/wiki/File:EightTNOs.png

Comets



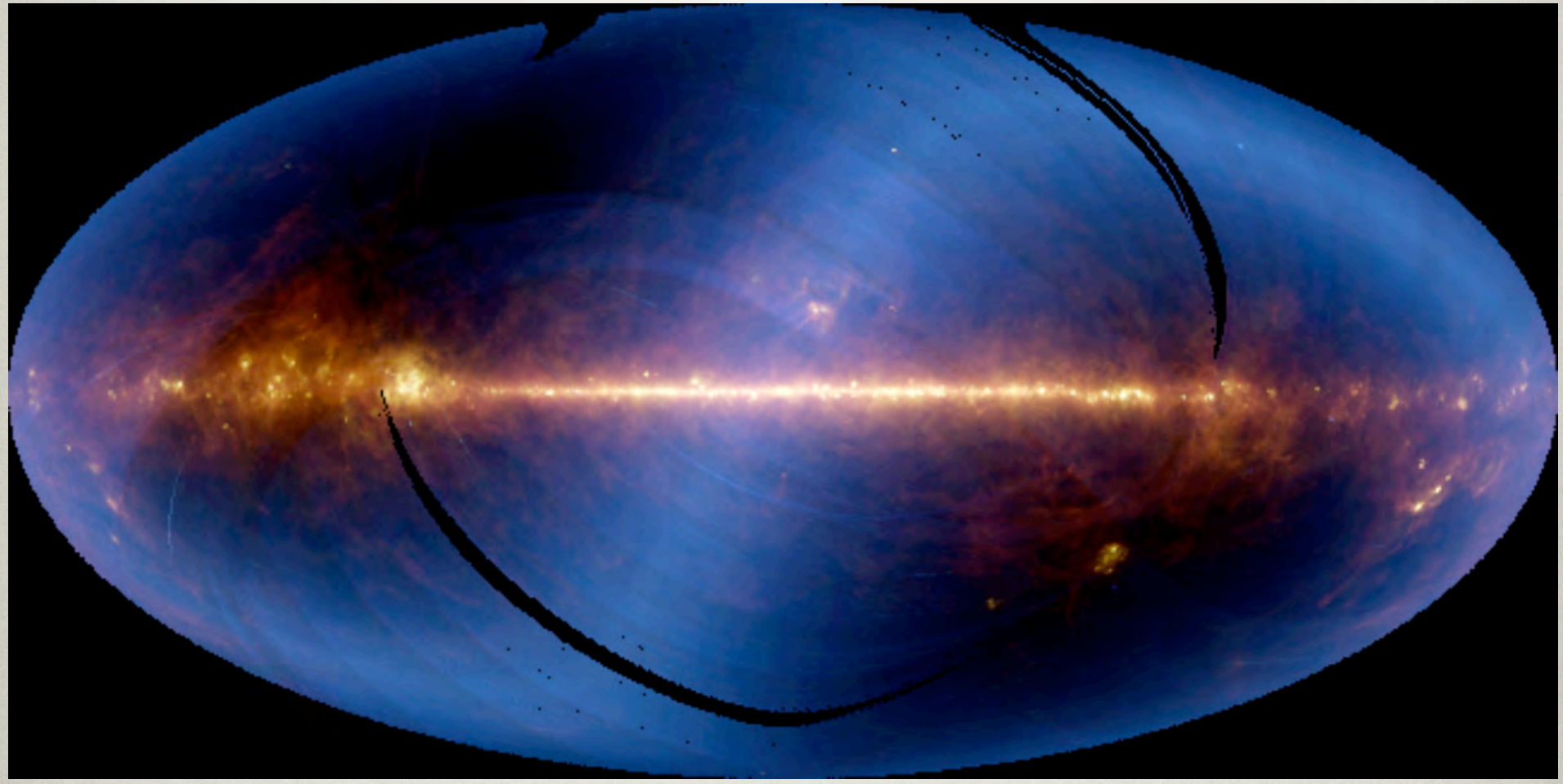
deepimpact.umd.edu/gallery/comet_orbits.html

Zodiacal Dust

- Dust particles in solar system
- Near plane of solar system
- Very low density
- Particles slowly spiral into Sun
- Comets and collisions between asteroids maintain dust cloud
- Structures associated with debris from particular asteroid families and comet trails
- Zodiacal light: sunlight scattered of zodiacal dust

en.wikipedia.org/wiki/Zodiacal_light

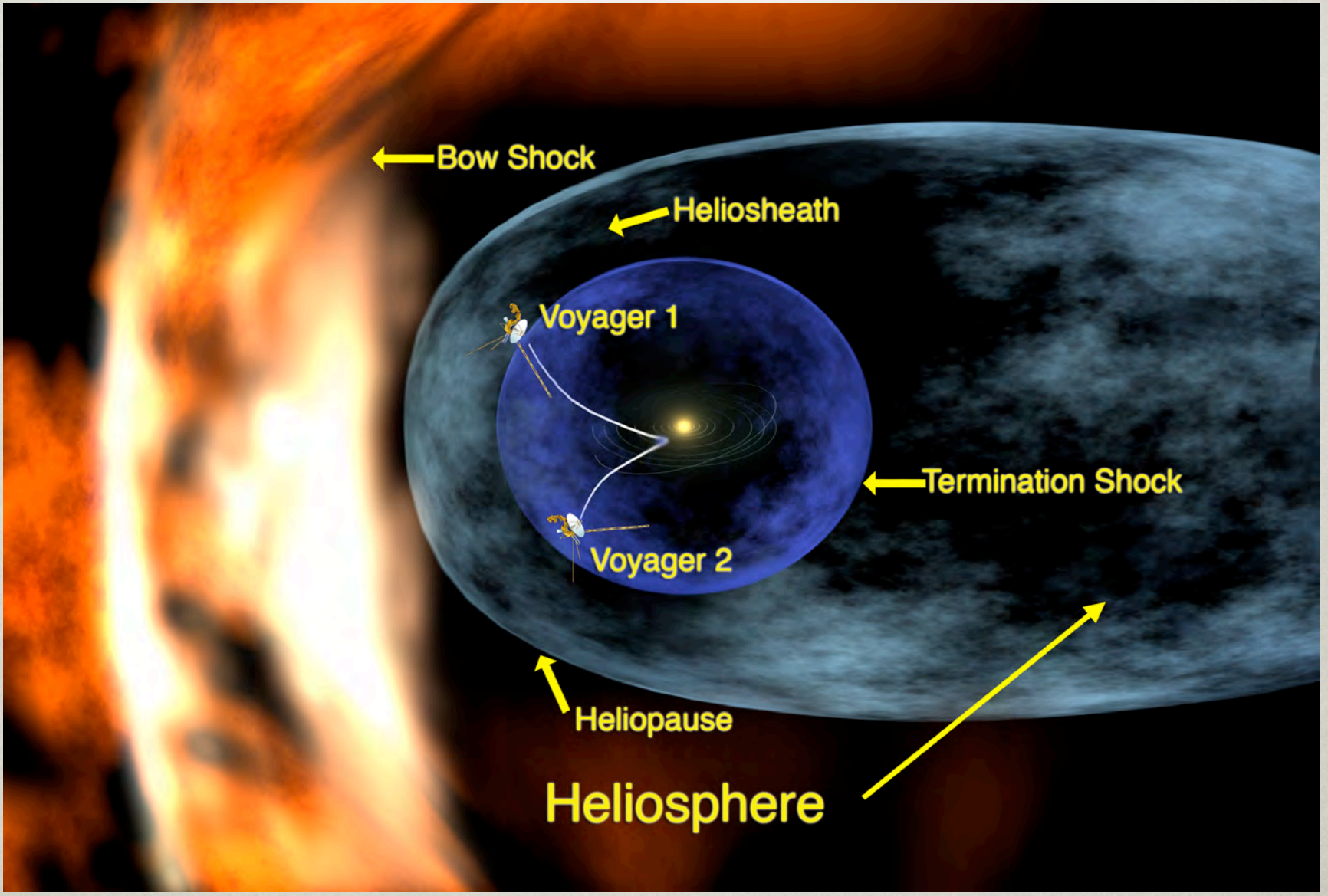
IRAS Infrared All-Sky Image



www.ipac.caltech.edu/Outreach/Gallery/IRAS/allsky.html

- White-blue 'S' is due to zodiacal dust

Heliosphere

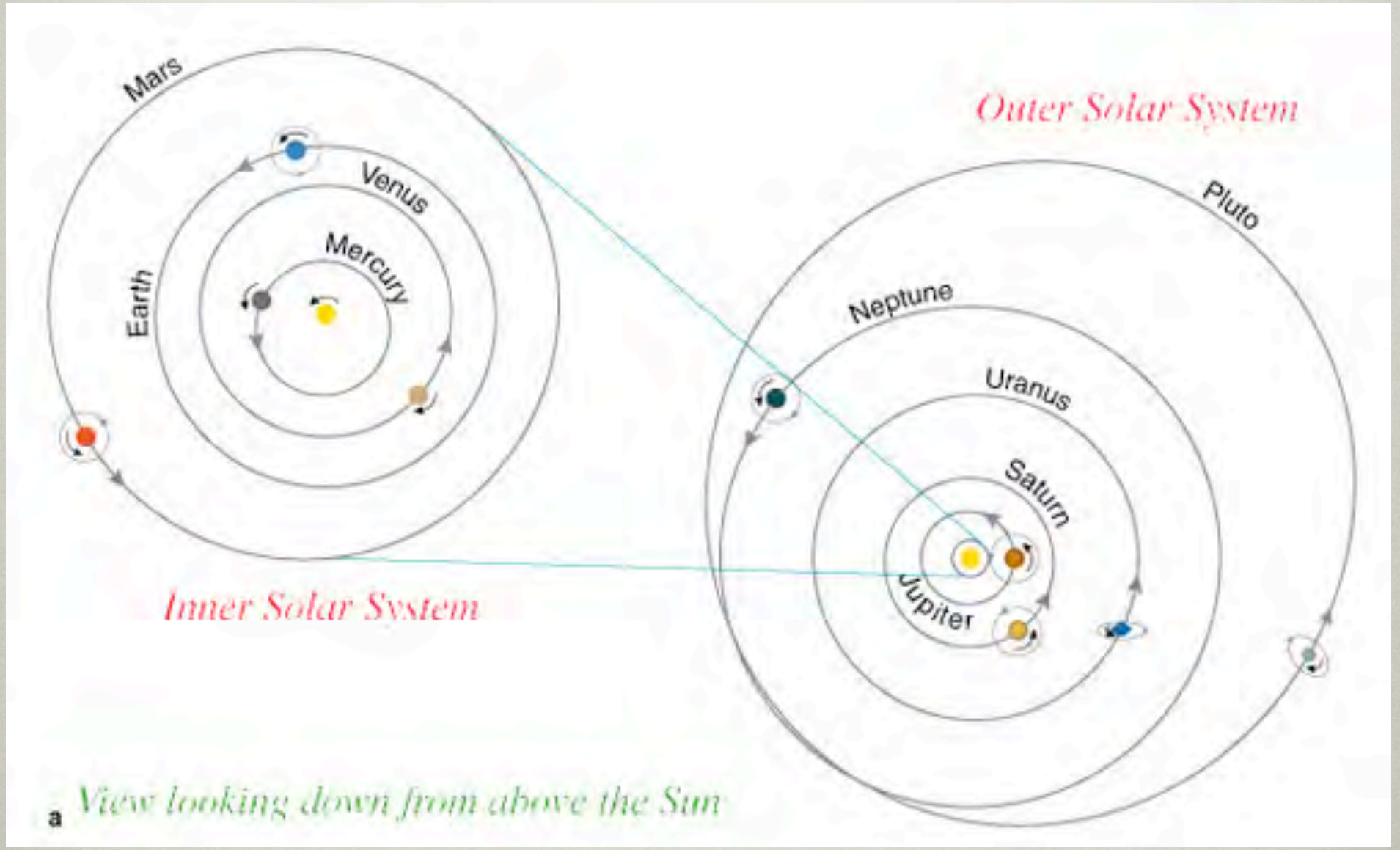


www.nasa.gov/centers/goddard/news/topstory/2007/dragon_fire.html

Planet Properties

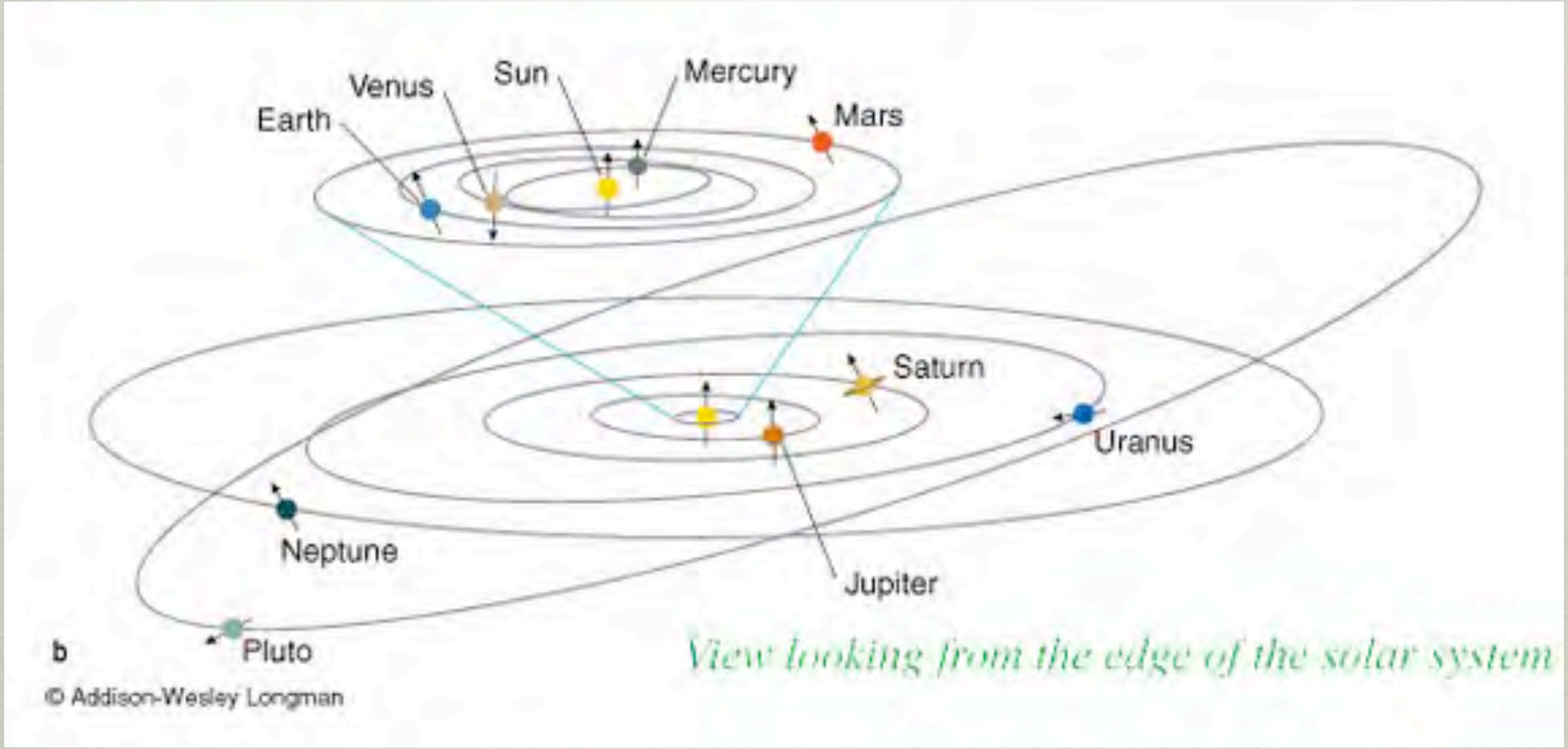
- Orbit (elliptical)
- Mass, distribution of mass
- Size
- Rotation rate and direction
- Shape
- Temperature
- Magnetic Field
- Surface Composition
- Surface structure
- Atmospheric structure and composition

Solar System Orbits from Above



www.ditrianum.org/English/Articles/Numerology/0006.htm

Solar System Orbits from Edge



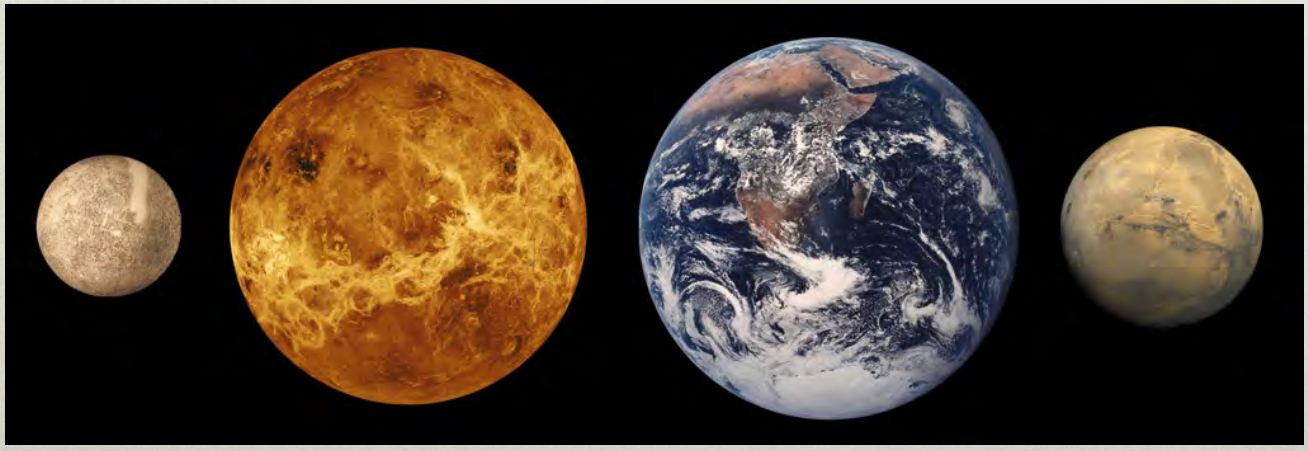
www.ditrianum.org/English/Articles/Numerology/0006.htm

Mean Planetary Orbits

	a (AU)	e (rad)	I (deg)	Period (years)
Mercury	0.38709927	0.20563593	7.00497902	0.241
Venus	0.72333566	0.00677672	3.39467605	0.615
Earth/Moon	1.0000026	0.01671123	-0.0000153	1.000
Mars	1.52371034	0.09339410	1.84969142	1.881
Jupiter	5.20288700	0.04838624	1.30439695	11.87
Saturn	9.53667594	0.05386179	2.48599187	29.45
Uranus	19.18916464	0.04725744	0.77263783	84.07
Neptune	30.06992276	0.00859048	1.77004347	164.9
Pluto	39.48211675	0.24882730	17.14001206	248.1

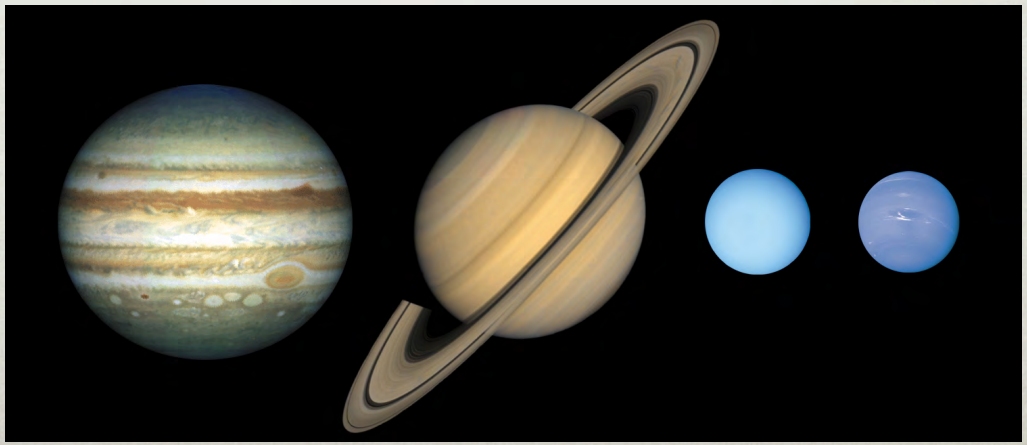
Geophysics Terrestrial Planets

	Mercury	Venus	Earth	Mars
Radius (km)	2440	6052	6371	3390
Mass (10^{23} kg)	3.302	48.685	59.736	6.4185
Density (g/cm^3)	5.427	5.204	5.515	3.933
Sidereal rotation (days)	58.6462	-243.02	0.99725792	1.02595675
Equatorial gravity (m/s^2)	3.701	8.870	9.789327	3.690
Obliquity (deg)	0.1	177.3	23.45	25.19
Escape velocity (km/s)	4.435	10.361	11.186	5.027
Geometric albedo	0.106	0.65	0.367	0.150

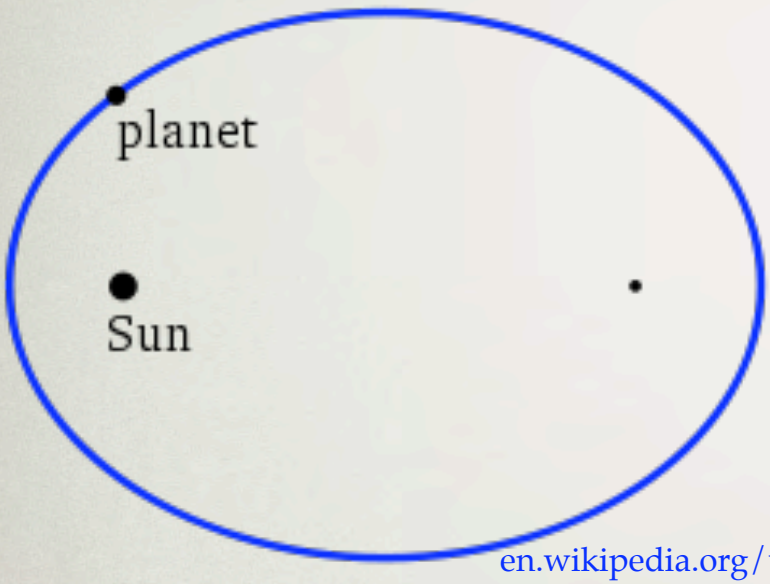


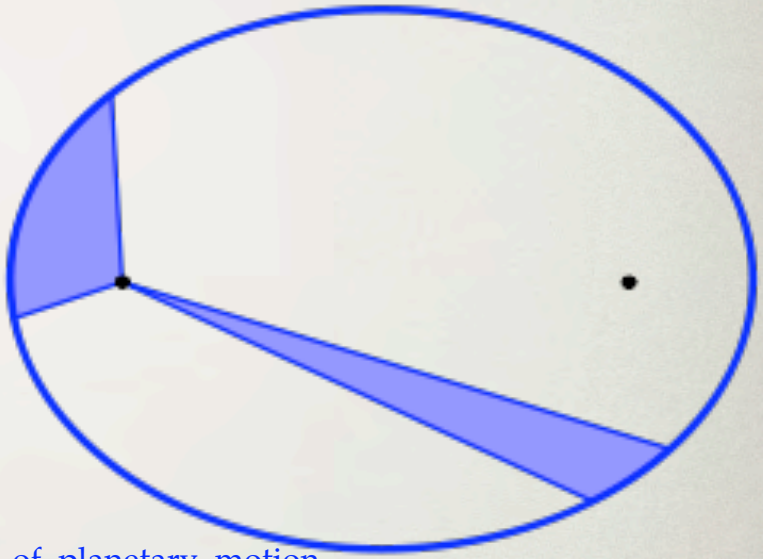
Physical Data Giant Planets

	Jupiter	Saturn	Uranus	Neptune
Radius (km)	71492	60268	25559	24766
Mass (10^{23} kg)	18986	5685	868	1024
Density (g/cm^3)	1.326	0.6873	1.318	1.683
Sidereal rotation (hours)	9.925	10.655	17.24	16.11
Equatorial gravity (m/s^2)	23.12	8.96	8.69	11.00
Obliquity (deg)	3.12	26.73	97.86	29.56
Escape velocity (km/s)	59.5	35.5	21.3	23.5
Geometric albedo	0.52	0.47	0.51	0.41



Kepler's Laws

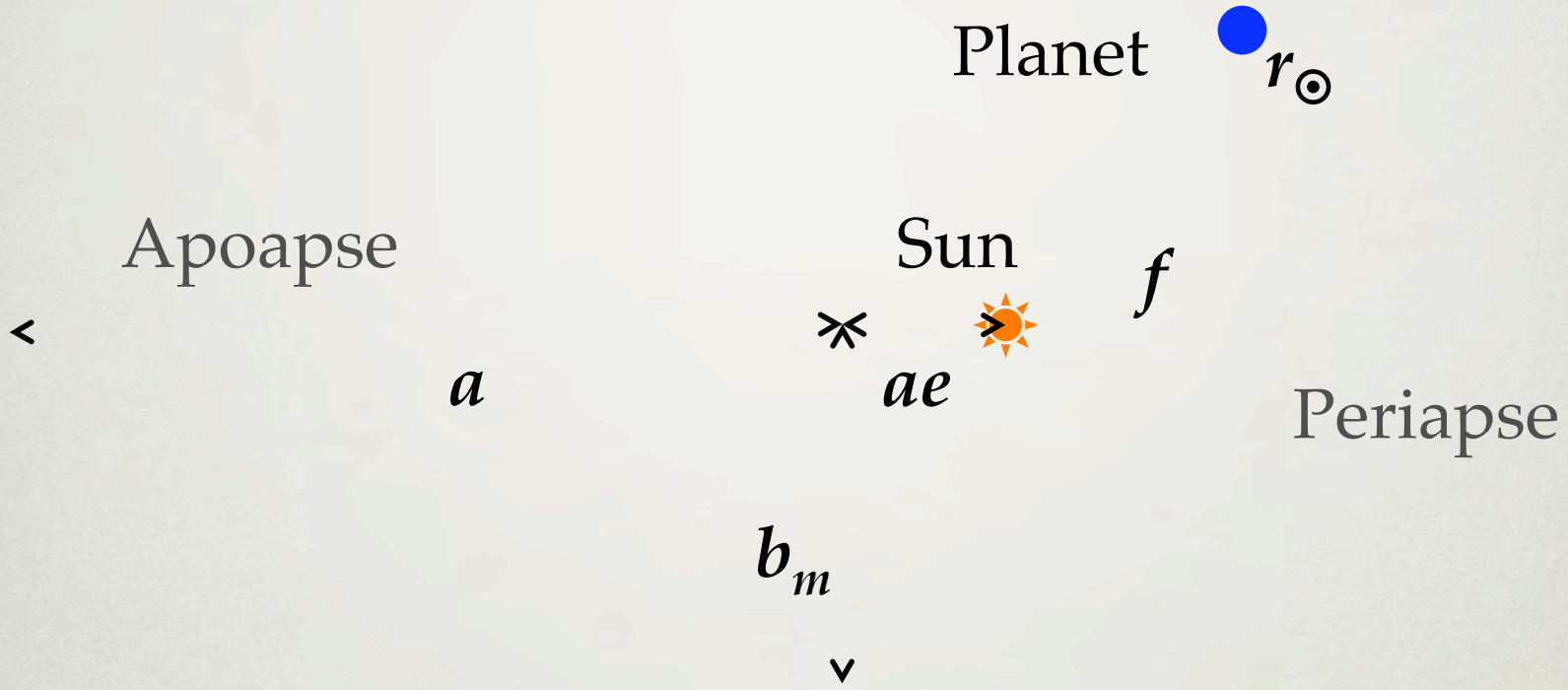




en.wikipedia.org/wiki/Kepler's_laws_of_planetary_motion

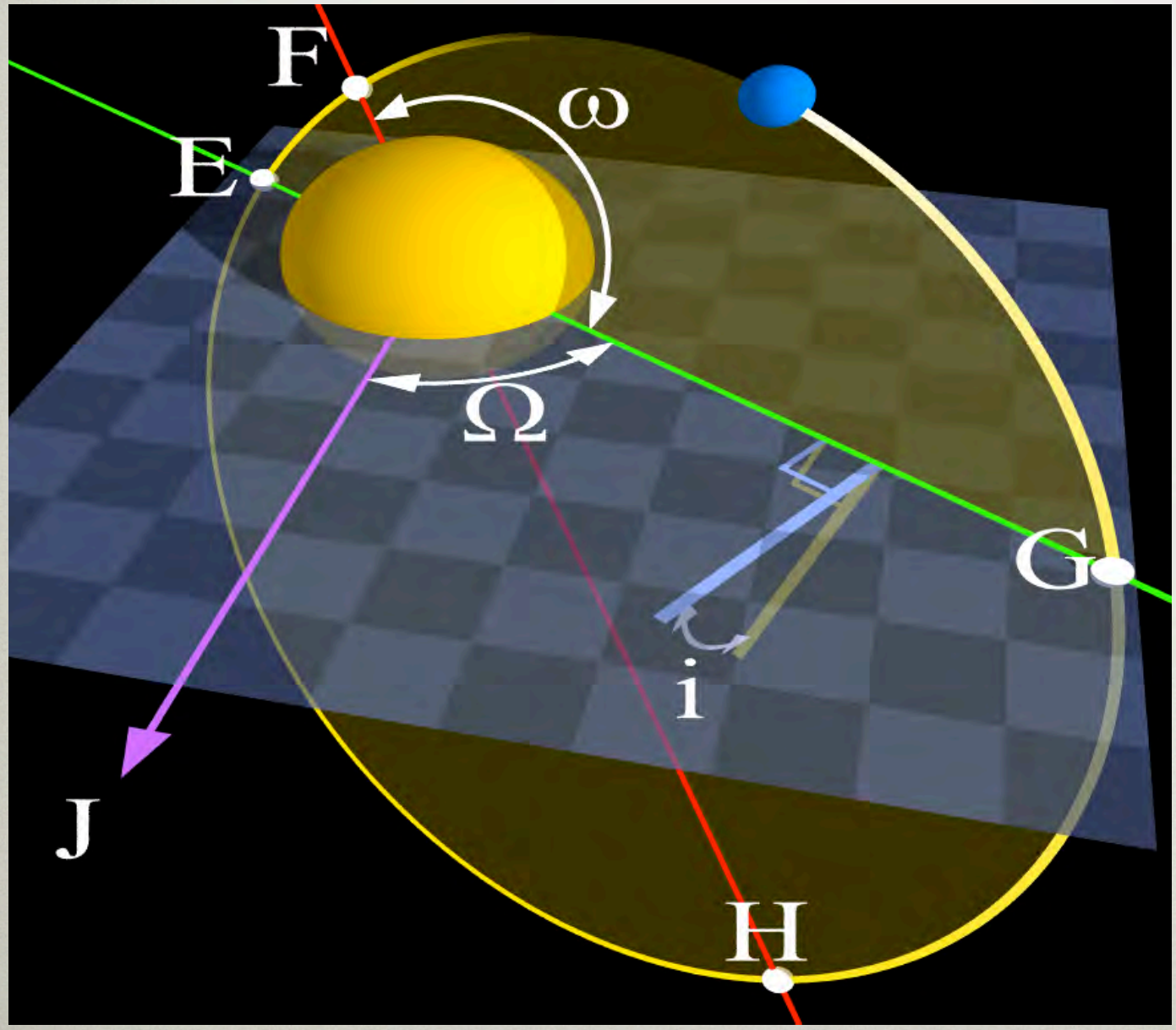
1. Orbit of planet is ellipse with Sun in one focus
2. Line joining planet and sun sweeps over equal areas during equal intervals of time
3. Square of orbital period of planet is proportional to cube of semi-major axis of its orbit

Elliptical Orbit



- $r_\odot$ instantaneous heliocentric location
- a semimajor axis of ellipse
- b_m semiminor axis of ellipse
- f true anomaly, angle between periapse and location

3-D Elliptical Orbit



E – Descending node
F – Periapsis
G – Ascending node
H – Apoapsis
i – Inclination
J – Reference direction
Ω – Longitude of the ascending node
ω – Argument of the periapsis

red - line of apsides; coincides with major axis
green - node line; line where reference plane intersects orbital plane

Based on code at en.wikipedia.org/wiki/Apsis

Planetary Orbits and Location

Determined by 6 orbital elements:

1. Semimajor axis, a
2. Eccentricty, e
3. Inclination, I
4. Argument of peripapse, ω
5. Longitude of ascending node, Ω
6. True anomaly, f

Motion and Gravitation

- Equation of motion:

$$\frac{d}{dt}(m_1 \mathbf{v}_1) = \mathbf{F}_1$$

- Action and reaction:

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

- Gravitation:

$$\mathbf{F}_{g12} = -\frac{G m_1 m_2}{r^2}\hat{\mathbf{r}}$$

- gravitational constant G
- $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \qquad \hat{\mathbf{r}} = \mathbf{r}/r$

Equation of relative Motion

- Relative motion given by (see exercises)

$$\mu_r \frac{d^2}{dt^2}\mathbf{r} = -\frac{G\mu_r M}{r^2}\hat{\mathbf{r}}$$

- Reduced mass: $\mu_r = m_1 m_2/(m_1+m_2)$
- Total mass: $M = m_1+m_2$
- Equivalent to particle of reduce mass μ_r orbiting a fixed mass M

Regarding Kepler's First Law

- Kepler's laws assume that planets have negligible mass
- Cannot neglect planet's mass in some cases
- Derive equivalent laws with finite planet mass

1. Two bodies move along elliptical paths with one focus of each ellipse located at the center of mass $\mathbf{r}_{cm}$ of the system

$$\mathbf{r}_{\rm cm} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/M$$

Regarding Kepler's Second Law

- A line connecting two bodies and a line from each body to the center of mass sweeps out an area at a constant rate
- Consequence of conservation of angular momentum (vector) $\mathbf{L}$

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$$

$$\frac{d}{dt}\mathbf{L} = 0$$

Regarding Kepler's Third Law

- Orbital period P_{orb} of pair of bodies about their mutual center of mass:

$$P_{\mathrm{orb}}^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}$$

- Orbital period only depends on semimajor axis and masses
- If $m_1 >> m_2$, period is independent of m_2

Elliptical Motion

- Sun contains 99.8% of solar system mass
- To first approximation: consider planetary motion around a fixed point in space
- Eccentricity

$$e = \sqrt{1 - b_m^2/a^2}$$

- Elliptic orbit relation

$$r_\odot = \frac{a(1-e^2)}{1 + e\cos f}$$

General Orbits

- Centripetal force of mass μ_r on circular orbit of radius r with speed v_c is

$$\mathbf{F}_c = \frac{\mu_r v_c^2}{r}\hat{\mathbf{r}}$$

- Has to be equal to gravitational force ➔ circular velocity v_c

$$v_c = \sqrt{\frac{GM}{r}}$$

- Total energy E of system is sum of kinetic and potential energy of system

$$E = \frac{1}{2}\mu_r v^2 - \frac{GM\mu_r}{r} = \frac{1}{2}\mu_r \frac{GM}{r} - \frac{GM\mu_r}{r} = -\frac{GM\mu_r}{2a}$$

Elliptic, Parabolic, Hyperbolic Orbits

- $E<0$: potential energy larger than kinetic energy, system is bound, elliptical orbits
- $E>0$: kinetic energy larger than potential energy, system is unbound, hyperbolic orbit
- $E=0$: kinetic and potential energies are equal, parabolic orbit
- Escape velocity ($E=0$): $v_e = \sqrt{\frac{2GM}{r}} = \sqrt{2} v_c$
- Ellipse, parabola, hyperbola are "conic sections" with constants e and ζ

$$r = \frac{\zeta}{1 + e \cos f}$$

Gravitational Potential

- Gravitational force can be described as gradient of a potential

$$\phi_g(\mathbf{r}) = -\int_{\infty}^{\mathbf{r}} \frac{\mathbf{F}_g(\mathbf{r}')}{m} \cdot d\mathbf{r}'$$

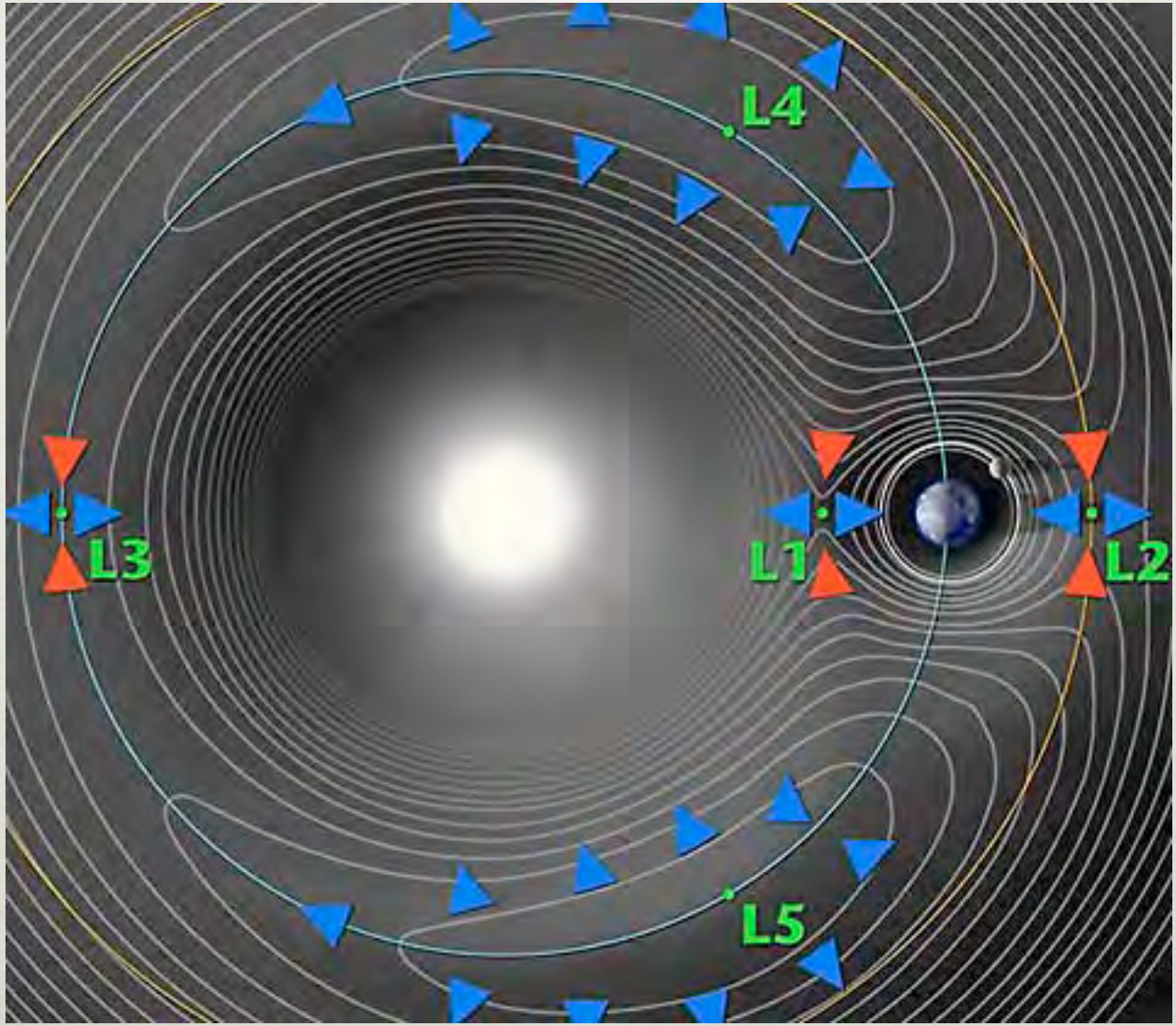
- Equation of motion becomes

$$\frac{d^2}{dt^2}\hat{\mathbf{r}} = -\nabla\phi_g$$

Three-Body Problem

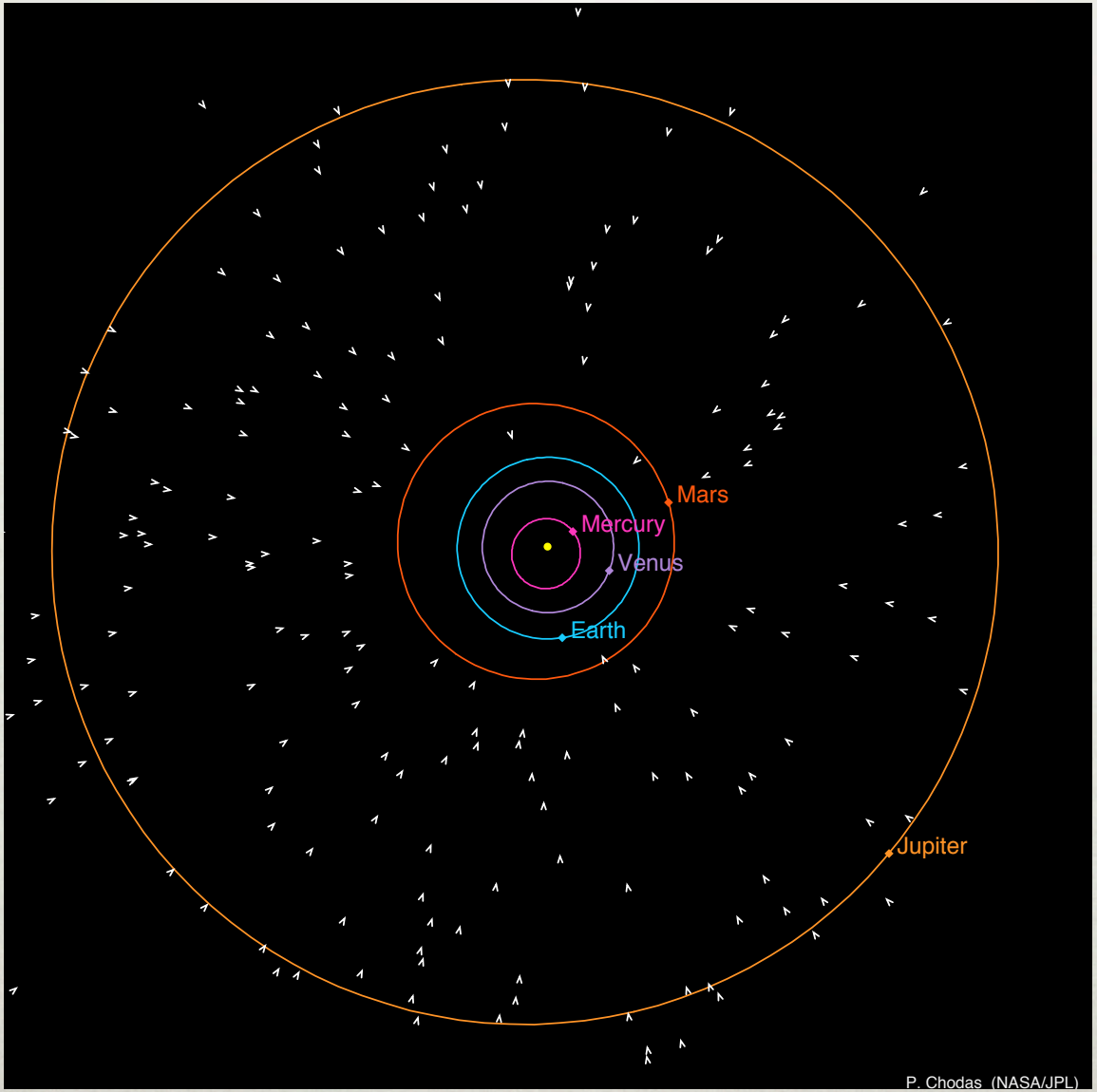
- Two-body problem can be solved analytically
- Two-body problem is a first-order approximation for the solar system
- Trajectories of 3 interacting bodies can, in general, not be solved analytically
- If one body has small mass compared to the other two (asteroid, ring particle, moon) ➔ restricted 3-body problem
- If two massive bodies are on circular orbits ➔ circular restricted 3-body problem

Lagrangian Points



map.gsfc.nasa.gov/mission/observatory_l2.html

Asteroids and Jupiter

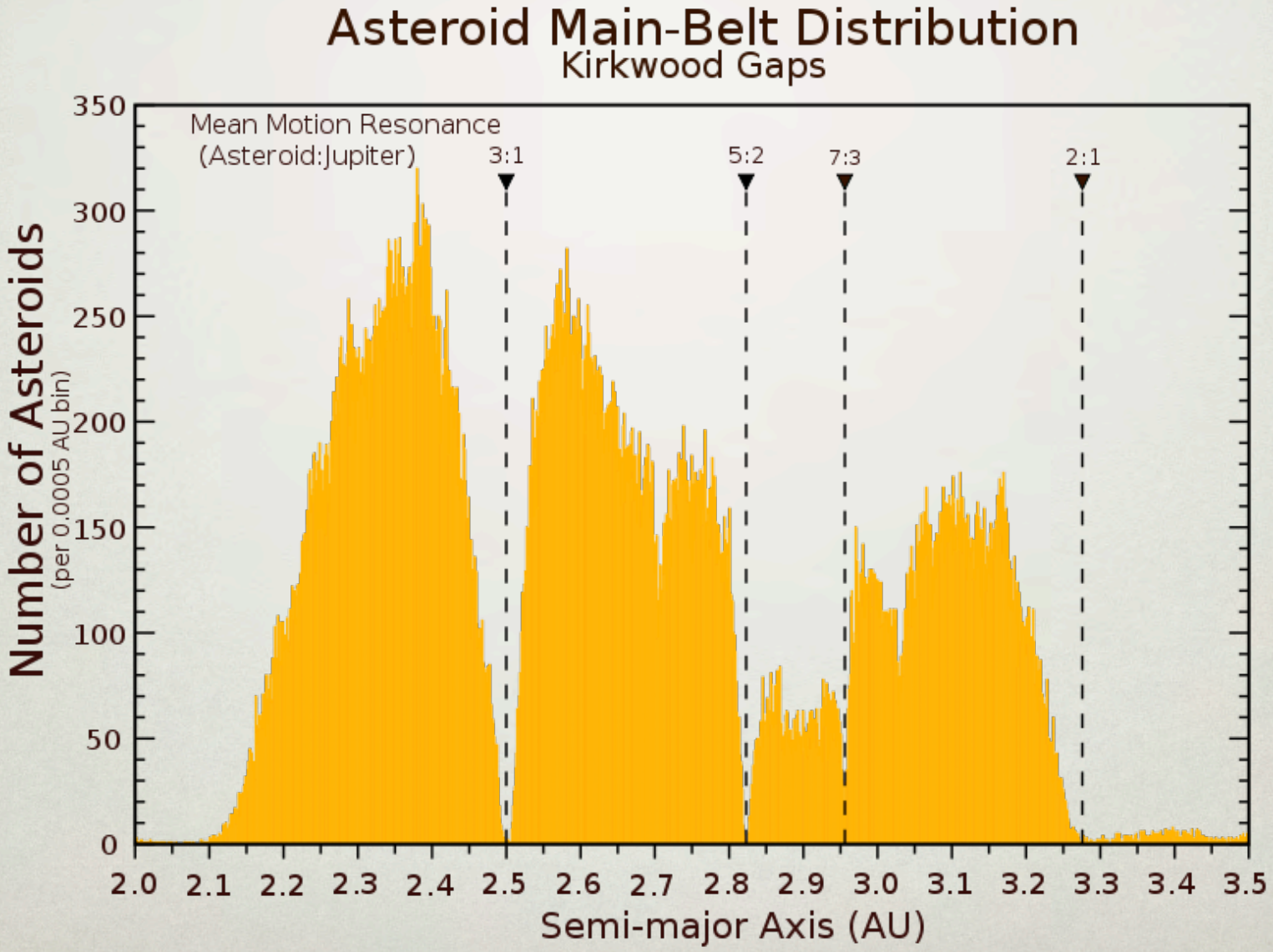


ssd.jpl.nasa.gov/?ss_inner

Planetary Perturbations

- For very accurate planet position calculations, all masses in the solar system have to be included ➔ n-body problem
- Asteroids and comets are strongly influenced by Jupiter
- Kuiper-belt objects are strongly influenced by Neptune
- Neptune was discovered with the help of perturbations of Uranus' orbit
- General relativity needs to replace Newton's law of gravity

Resonances

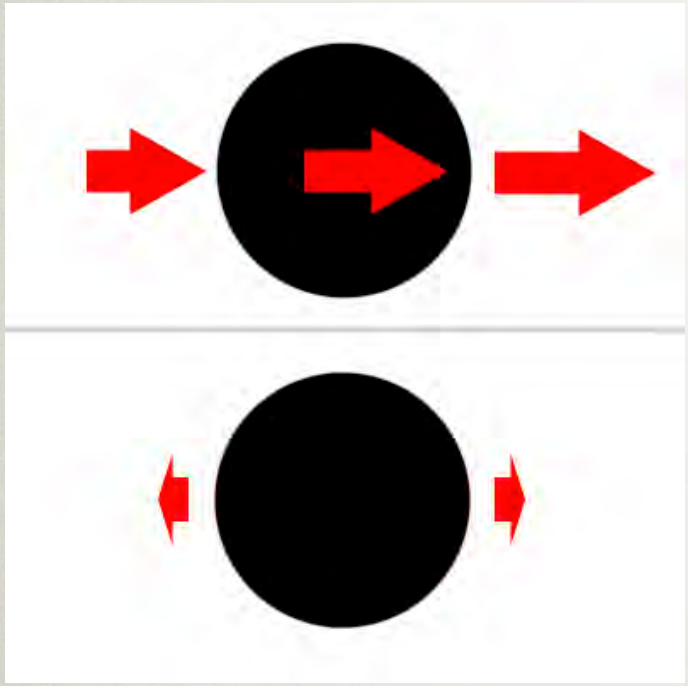


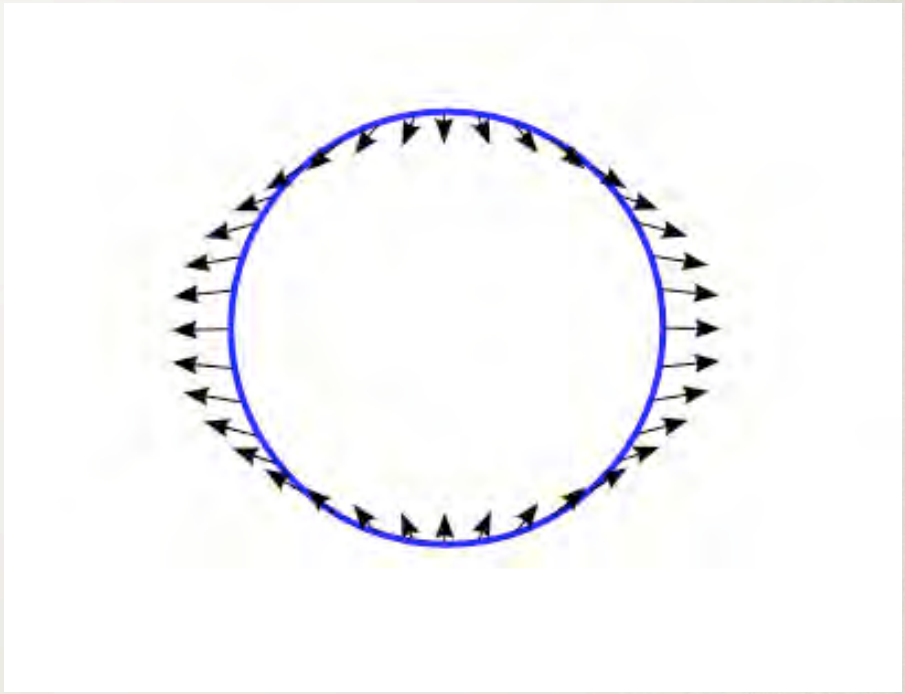
en.wikipedia.org/wiki/Kirkwood_gap

Long-term Stability

- Orbits of planets are chaotic over longer time scales
- Position and orbits impossible to predict on very long time scales
- Orbits may change dramatically (asteroids, comets)
- Unknowns include:
  - Current orbit measurements
  - Asteroids including impacts
  - Solar mass loss (radiation, particles)
  - Drag of solar wind on planetary magnetospheres
  - Galactic tidal forces, passing stars

Tides





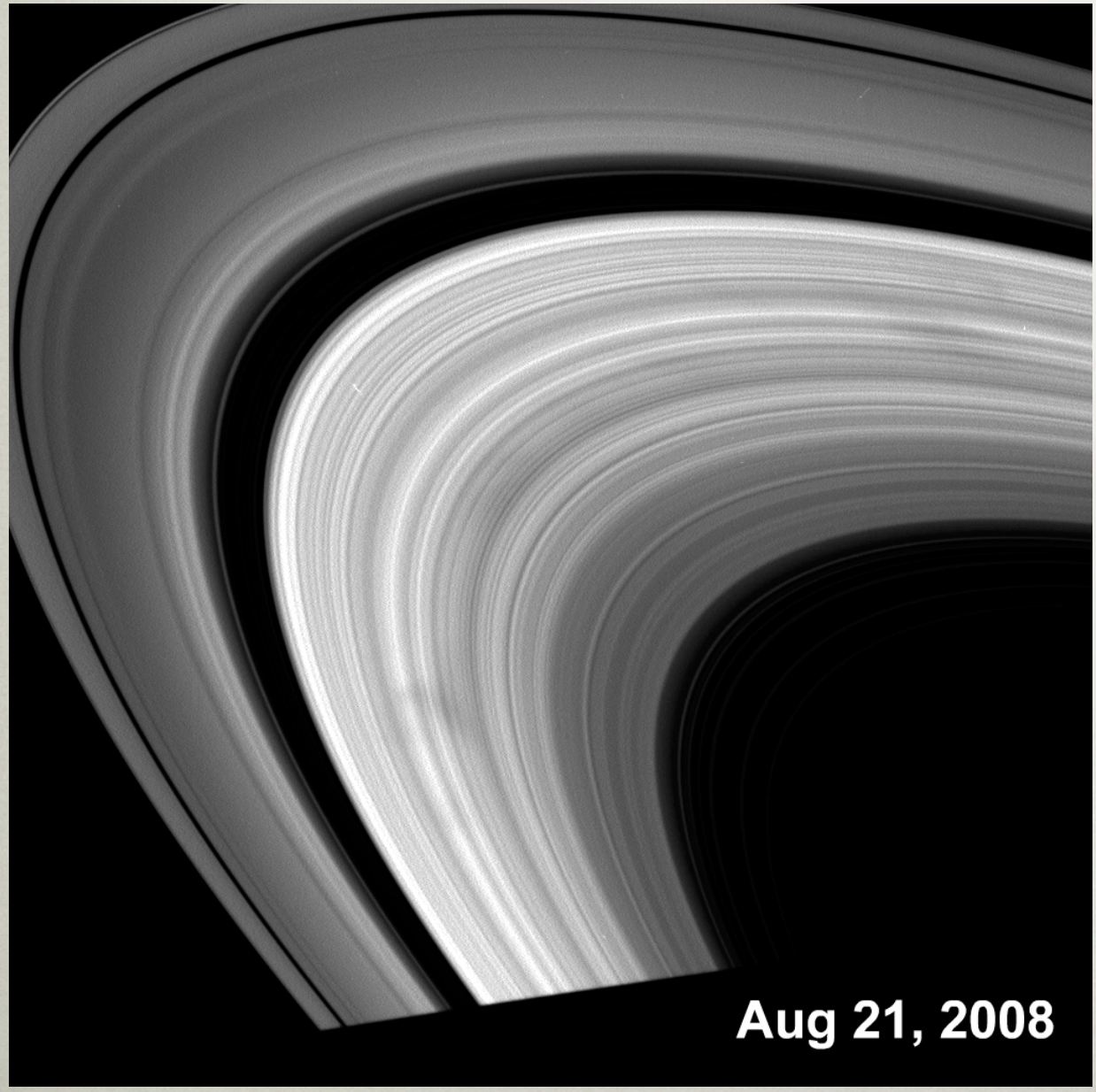
en.wikipedia.org/wiki/Tidal_force

- Tidal forces are due to decrease of gravitational force with distance
- Bulges towards and away from gravitational source
- Slows rotation of moons, prevents formation of moons in Saturn's rings

Beyond Gravity

- Radiation pressure: Pushes micron and submicron particles away from Sun
- Poynting-Robertson drag: cm-size particles spiral towards Sun due to anisotropy in radiation absorption and emission
- Yarkowski effect: changes orbits of meter to kilometer size objects due to uneven surface temperature distribution at their surfaces (diurnal and seasonal
- Corpuscular drag: sub-micron particles are dragged by solar wind
- Gas drag: protoplanetary disks, ring particles

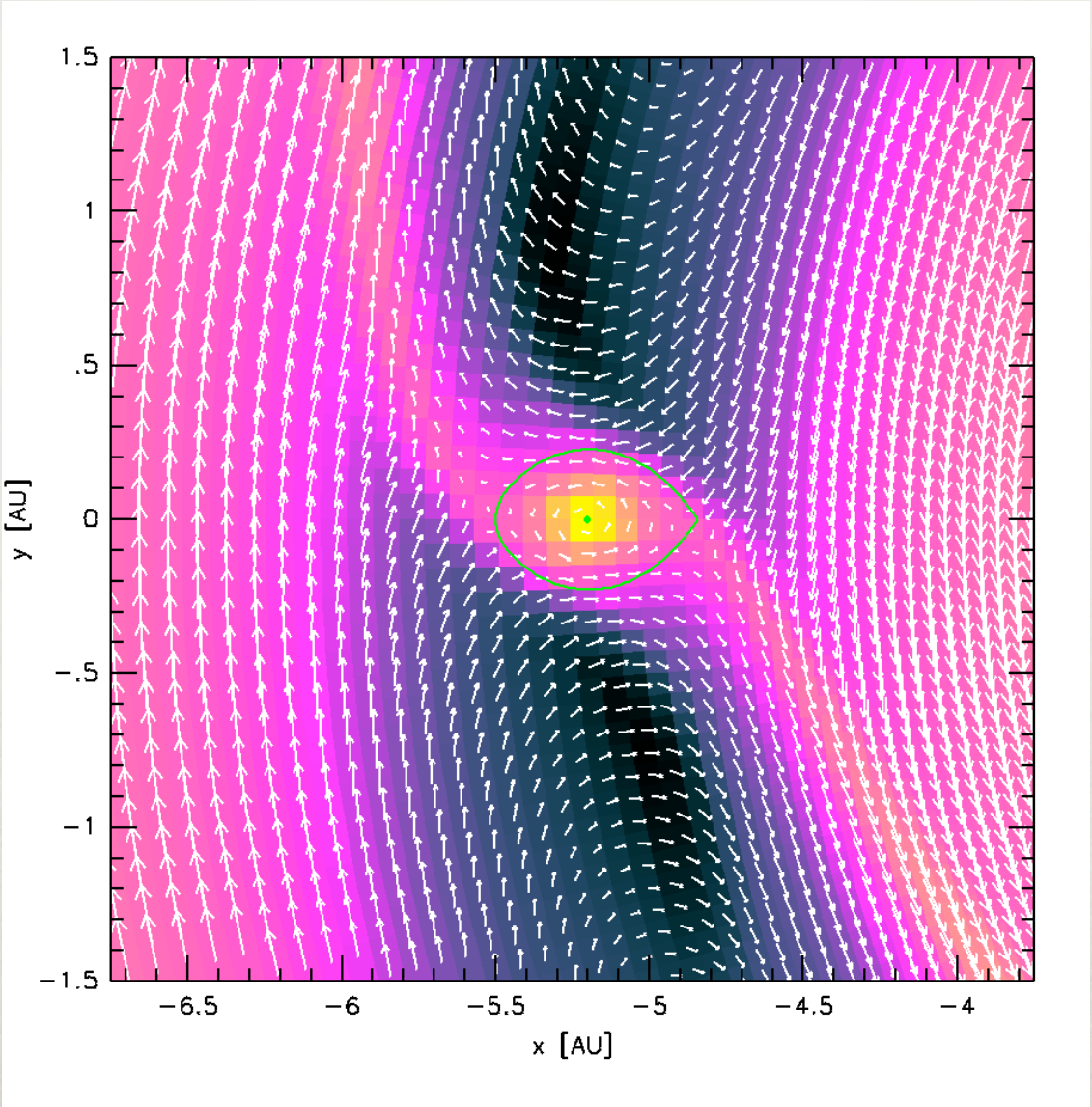
Spokes in Saturn's Rings



- Non-Keplerian motion
- Electrostatic repulsion from main ring, rotation synchronous with magneto-sphere

photojournal.jpl.nasa.gov/archive/PIA11144.mov

Prograde Planet Rotation



www.tat.physik.uni-tuebingen.de/~kley/research/planets/flow1.png