Planets and Exoplanets 2009: Exercises to Lecture 4
Due: 13 October 2009 at 11:15

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1 Hydrostatic equilibrium

If a 'parcel' of material is kept in balance by the downward pulling force of the gravity and the upward pushing force of the pressure it is said to be in hydrostatic equilibrium. The relation between the upward pressure p and the downward gravity at a distance r from the centre of a planet in hydrostatic equilibrium can be written as

$$p(r) = -\int_r^R g(r')\, \rho(r')\, dr', \tag{1}$$

with R the radius of the planet, g the acceleration of gravity, and ρ the density at distance r. The minus sign indicates that the direction of the gravity is opposite to that of the pressure.

a. Assume a constant density throughout the planet, and derive that the pressure $p_{\rm c}$ in the centre of the planet equals:

$$p_{\rm c} = \frac{2\pi G\rho^2 R^2}{3} = \frac{3GM^2}{8\pi R^4}, \tag{2}$$

with G the gravitational constant, and M the total mass of the planet. Hint: express the gravity into a function of the distance r.

b. Calculate $p_{\rm c}$ for the Moon and for the Earth.

c. Compare the pressures you calculated under **b.** with the real pressures in the Moon and the Earth, 0.045 Mbar and 3.6 Mbar, respectively. Explain the differences.

2 Speeding earthquake waves

a. You wake up in the middle of the night by a powerful jolt from below. Exactly 4 seconds later a strong horizontal shaking begins and knocks you off your bed. Explain both types of motion.

b. If the average velocity of the P-waves is 5.8 km/s and that of S-waves 3.4 km/s, calculate your distance from the epicenter of the quake (assume that the epicenter was on the surface).

c. Explain in words and sketches how many measurements like your own are needed to exactly pinpoint the location of the epicenter.
Three measurements.

d. Use Fig. 1 to determine the time that it takes a P-wave to travel from the epicenter of an earthquake to the other side of the Earth.

e. *Estimate* the time it takes P- and S-waves to travel to a point 60° distant from the epicenter. Ignore the refraction of the waves, thus assume the waves follow a straight line.

f. Discuss qualitatively how the arrival times of the waves would change if refraction were included.

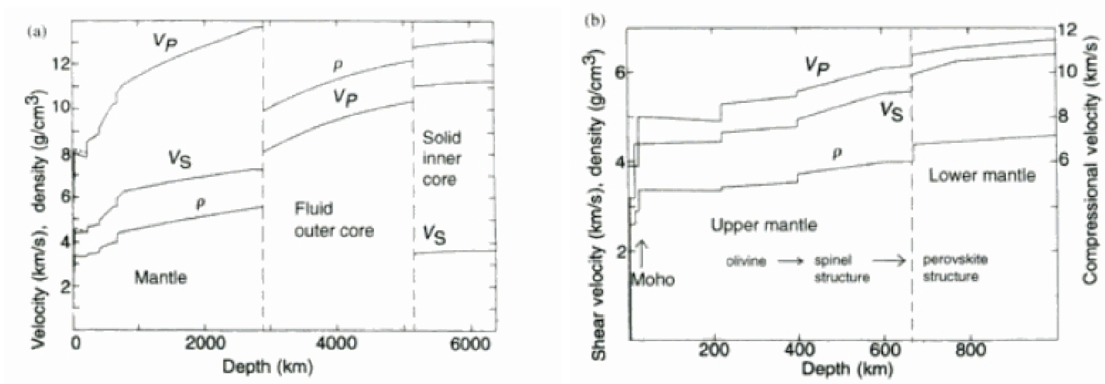

Figure 1: The velocities v_P and v_S of seismic P- and S-waves through the Earth's mantle and core and the density ρ of the mantle and core. The graph on the right gives the details of the uppermost 1000 km. This is Fig. 6.14 from *Planetary Sciences*, the original is from Pieri and Dziewonski [1999].

3 Mercury's interior

Mercury's mean density $\rho = 5.44$ g/cm^3. If Mercury consists entirely of rock ($\rho = 3.3$ g/cm^3) and iron ($\rho = 7.95$ g/cm^3), calculate the planets' relative abundance of iron by mass.

4 Bonus exercise: The gravity field of a planet

a. Compute the gravitational potential ϕ_g of a uniform sphere of radius R and density ρ.

b. Compute the gravitational potential ϕ_g of a sphere with an equal mass of that in **a**, but whose mass is distributed so that the core, with radius $R/2$, has twice the density of the mantle.

c. The moment of inertia I of a spherical body is related to the density via

$$I = \frac{8}{3}\pi \int_0^R \rho(r)\, r^4\, dr. \tag{3}$$

Calculate the moment of inertia in units of MR^2 for the planets for which the density distributons are given in **a** and **b**.