Lecture 8: Polarimetry 2

Outline

1. Polarizers and Retarders
2. Polarimeters
3. Scattering Polarization
4. Zeeman Effect

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Polarization Summary

- polarization is an intrinsic property of light
- polarization properties and intensity of light can be described by 4 parameters:
  - *coherent* Jones calculus
  - *incoherent* Stokes/Mueller calculus
- degree of polarization is the fraction of the intensity that is fully polarized
- typical values for degree of polarization:
  - 45 degree reflection off aluminum mirror: 5%
  - clear blue sky: up to 75%
  - 45 degree reflection off glass: 90%
  - LCD screen: 100%
  - solar scattering polarization: 1% to 0.001%
  - exoplanet signal: 0.001%

Polarizers

- polarizer: optical element that produces polarized light from unpolarized input light
- linear, circular, or in general elliptical polarizer, depending on type of transmitted polarization
- linear polarizers by far the most common
- large variety of polarizers

Jones Matrix for Linear Polarizers

- Jones matrix for linear polarizer:

$$\mathrm{J}_p = \begin{pmatrix} p_x & 0 \\ 0 & p_y \end{pmatrix}$$

- $0 \leq p_x \leq 1$ and $0 \leq p_y \leq 1$, real: transmission factors for x, y-components of electric field: $E'_x = p_x E_x, \quad E'_y = p_y E_y$
- $p_x = 1,\ p_y = 0$: linear polarizer in $+Q$ direction
- $p_x = 0,\ p_y = 1$: linear polarizer in $-Q$ direction
- $p_x = p_y$: neutral density filter

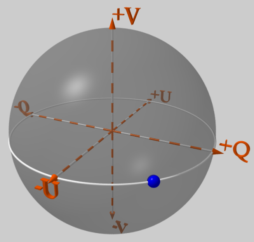
Mueller Matrix for Linear Polarizers

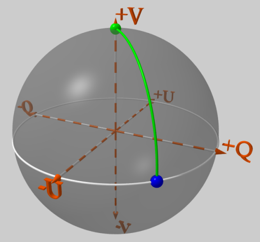
$$\mathrm{M}_p = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

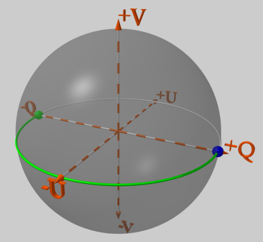
Mueller Matrix for Ideal Linear Polarizer at Angle θ

$$\mathrm{M}_{\mathrm{pol}}(\theta) = \frac{1}{2} \begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Poincare Sphere

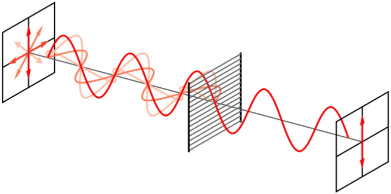


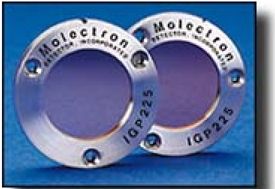




- polarizer is a point on the Poincaré sphere
- transmitted intensity: $\cos^2(l/2)$, l is arch length of great circle between incoming polarization and polarizer on Poincaré sphere

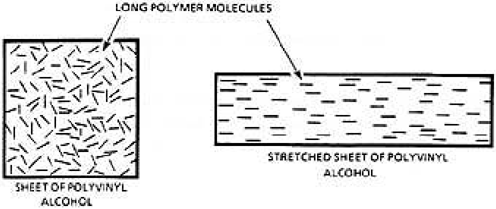
Wire Grid Polarizers





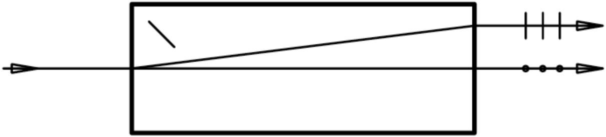
- parallel conducting wires, spacing $d \lesssim \lambda$ act as polarizer
- electric field parallel to wires induces electrical currents in wires
- induced electrical current reflects polarization parallel to wires
- polarization perpendicular to wires is transmitted
- rule of thumb:
  - $d < \lambda/2 \Rightarrow$ strong polarization
  - $d \gg \lambda \Rightarrow$ high transmission of both polarization states (weak polarization)
- mostly used in infrared

Polaroid-type Polarizers



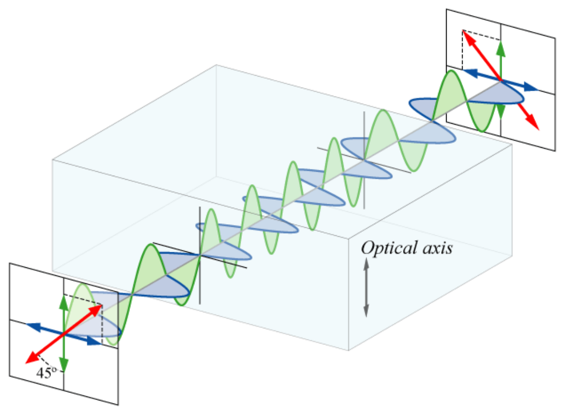
- developed by Edwin Land in 1938 ⇒ Polaroid
- sheet polarizers: stretched polyvynil alcohol (PVA) sheet, laminated to sheet of cellulose acetate butyrate, treated with iodine
- PVA-iodine complex analogous to short, conducting wire
- cheap, can be manufactured in large sizes

Crystal-Based Polarizers



- crystals are basis of highest-quality polarizers
- precise arrangement of atom/molecules and anisotropy of index of refraction separate incoming beam into two beams with precisely orthogonal linear polarization states
- work well over large wavelength range
- many different configurations
- calcite most often used in crystal-based polarizers because of large birefringence, low absorption in visible
- many other suitable materials

Retarders



en.wikipedia.org/wiki/Wave_plate

General Retarders or Wave Plates

- retarder: retards (delays) phase of one electric field component with respect to the orthogonal component

Retarder Properties

- does not change intensity or degree of polarization
- characterized by two (not identical, not trivial) Stokes vectors of incoming light that are not changed by retarder $\Rightarrow$ *eigenvectors* of retarder
- depending on polarization described by eigenvectors, retarder is
  - *linear retarder*
  - *circular retarder*
  - *elliptical retarder*
- linear, circular retarders are special cases of elliptical retarders
- circular retarders sometimes called *rotators* since they rotate the orientation of linearly polarized light
- linear retarders by far the most common type of retarder

Jones Matrix for Linear Retarders

- linear retarder with fast axis at 0° characterized by Jones matrix

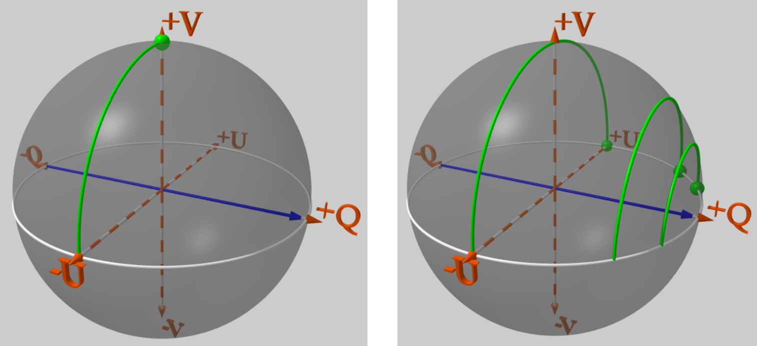
$$\mathsf{J}_r\left(\delta\right) = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathsf{J}_r\left(\delta\right) = \begin{pmatrix} e^{i\frac{\delta}{2}} & 0 \\ 0 & e^{-i\frac{\delta}{2}} \end{pmatrix}$$

- δ: phase shift between two linear polarization components (in radians)
- absolute phase does not matter

Mueller Matrix for Linear Retarder

$$\mathsf{M}_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

Retarders on the Poincaré Sphere



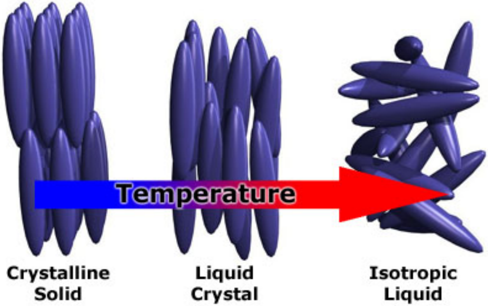
- retarder eigenvector (fast axis) in Poincaré sphere
- points on sphere are rotated around retarder axis by amount of retardation

Variable Retarders

Introduction

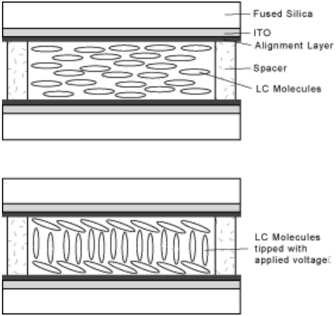
- sensitive polarimeters requires retarders whose properties (retardance, fast axis orientation) can be varied quickly (*modulated*)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of *c*-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

Liquid Crystals



- liquid crystals: fluids with elongated molecules
- at high temperatures: liquid crystal is isotropic
- at lower temperature: molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

Liquid Crystal Retarders



- dielectric constant anisotropy often large $\Rightarrow$ very responsive to changes in applied electric field
- birefringence δn can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few μm thick
- birefringence shows strong temperature dependence

Rotating Mueller Matrices

- optical element with Mueller matrix M
- Mueller matrix of the same element rotated by θ around the beam given by

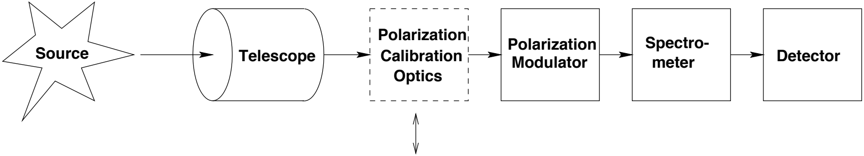
$$\mathrm{M}(\theta) = \mathrm{R}(-\theta)\mathrm{M}\mathrm{R}(\theta)$$

with

$$\mathrm{R}\left(\theta\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

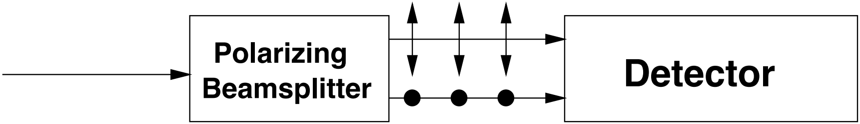
Temporal and Spatial Modulation

General Polarimeters



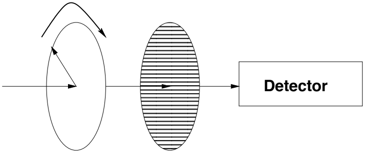
- polarimeters: optical elements (e.g. retarders, polarizers) that change polarization state of incoming light in controlled way
- detectors always measure only intensities
- intensity measurements combined to retrieve polarization state of incoming light
- polarimeters vary by polarization modulation scheme
- polarimeter should also include polarization calibration optics

Polarizing Beam-Splitter Polarimeter



- simple linear polarimeter: polarizing beam-splitter producing 2 beams corresponding to 2 orthogonal linear polarization states
- full linear polarization information from rotating assembly
- *spatial modulation*: simultaneous measurements of two (or more) Stokes parameters

Rotating Waveplate Polarimeter



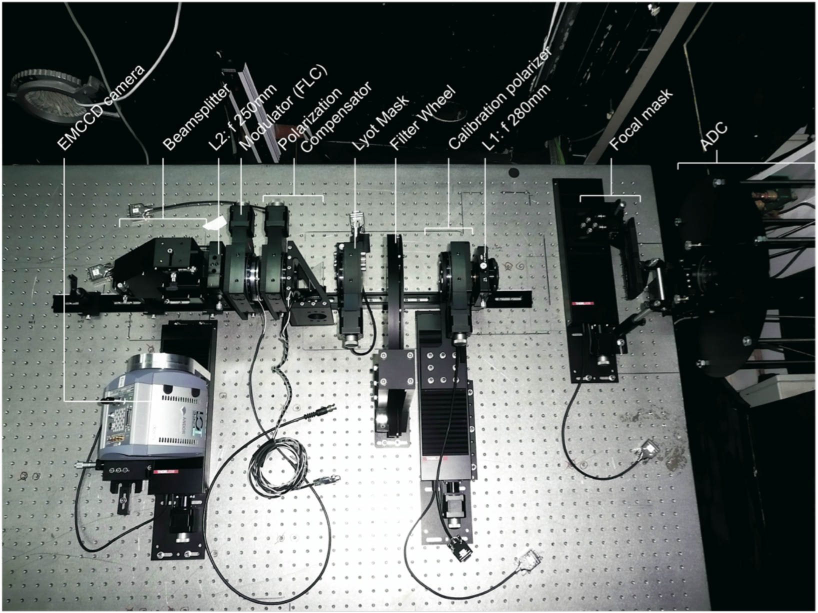
- rotating retarder, fixed linear polarizer
- measured intensity is function of retardance δ, position angle θ
- only terms in θ lead to modulated signal
- *temporal modulation*: sequential measurements of $I\pm$ one or more Stokes parameters

Comparison of Temporal and Spatial Modulation Schemes

Modulation	Advantages	Disadvantages
temporal	negligible effects of flat field and optical aberrations	influence of seeing if modulation is slow
	potentially high polarimetric sensitivity	limited read-out rate of array detectors
spatial	off-the-shelf array detectors	requires up to four times larger sensor
	high photon collection efficiency	influence of flat field
	allows post-facto reconstruction	influence of differential aberrations

schemes rather complementary ⇒ modern, sensitive polarimeters use both to combine advantages and minimize disadvantages

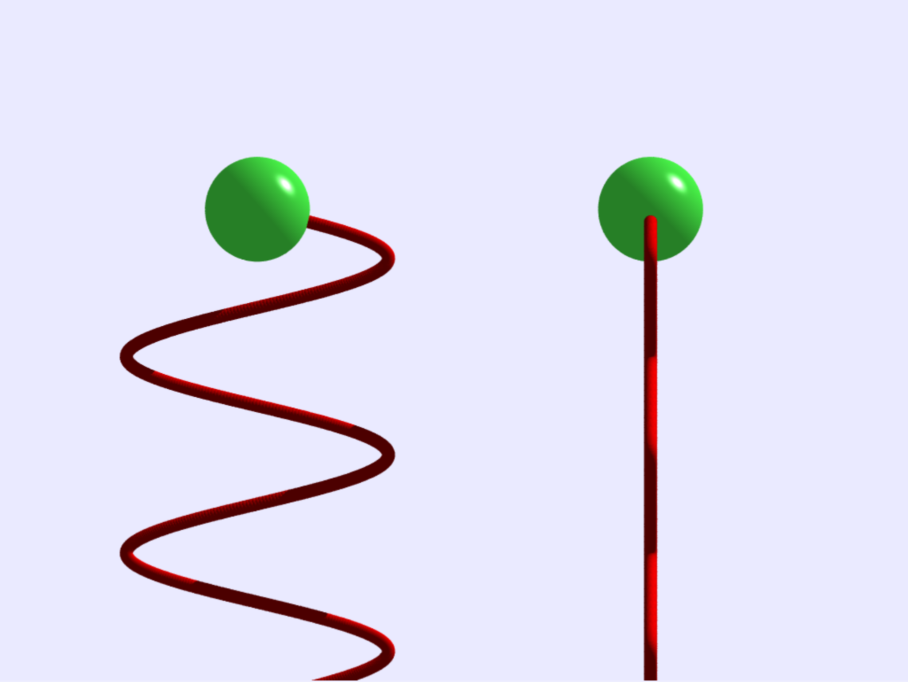
Example: ExPo (Extreme Polarimeter)

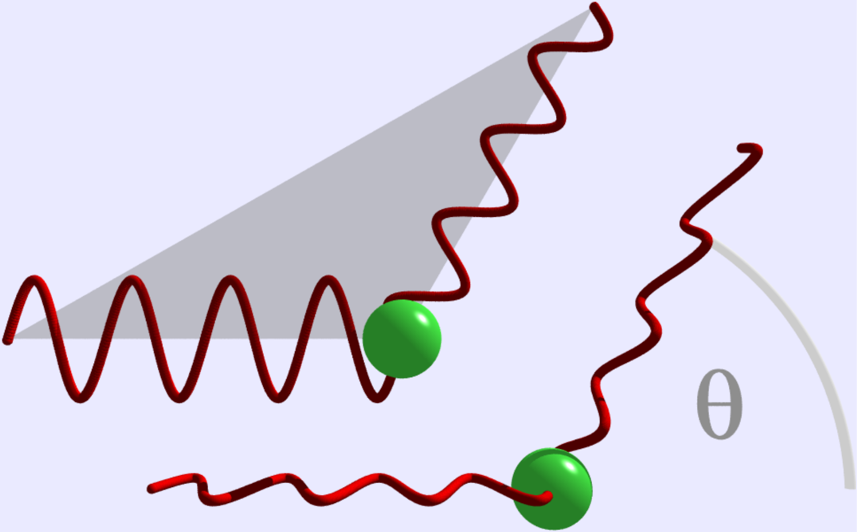


Scattering Polarization

Single Particle Scattering

- light is absorbed and re-emitted
- if light has low enough energy, no energy transfered to electron, but photon changes direction ⇒ elastic scattering
- for high enough energy, photon transfers energy onto electron ⇒ inelastic (Compton) scattering
- Thomson scattering on free electrons
- Rayleigh scattering on bound electrons
- based on very basic physics, scattered light is linearly polarized




θ

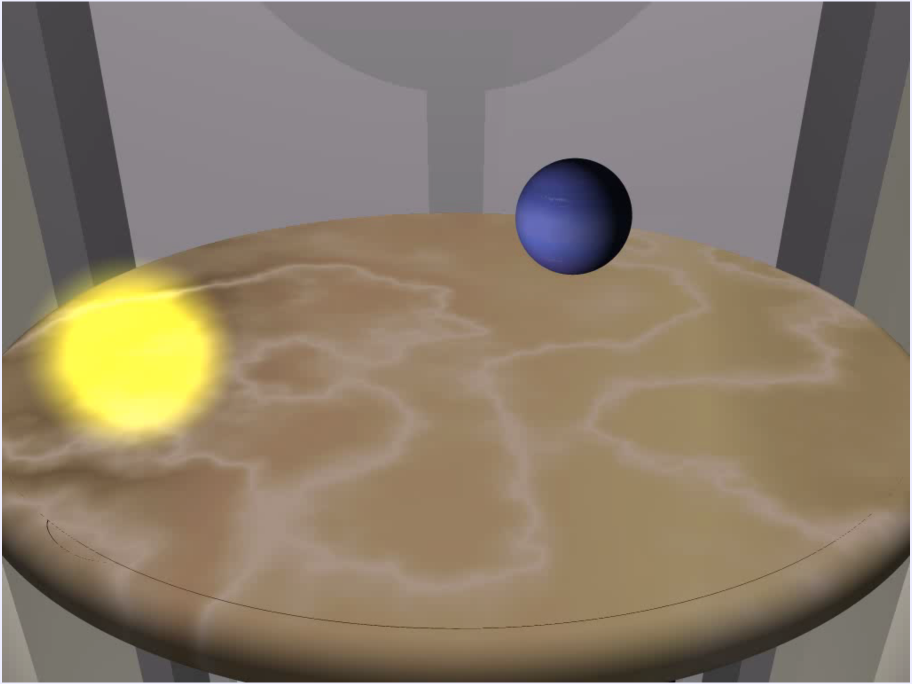
Polarization as a Function of Scattering Angle

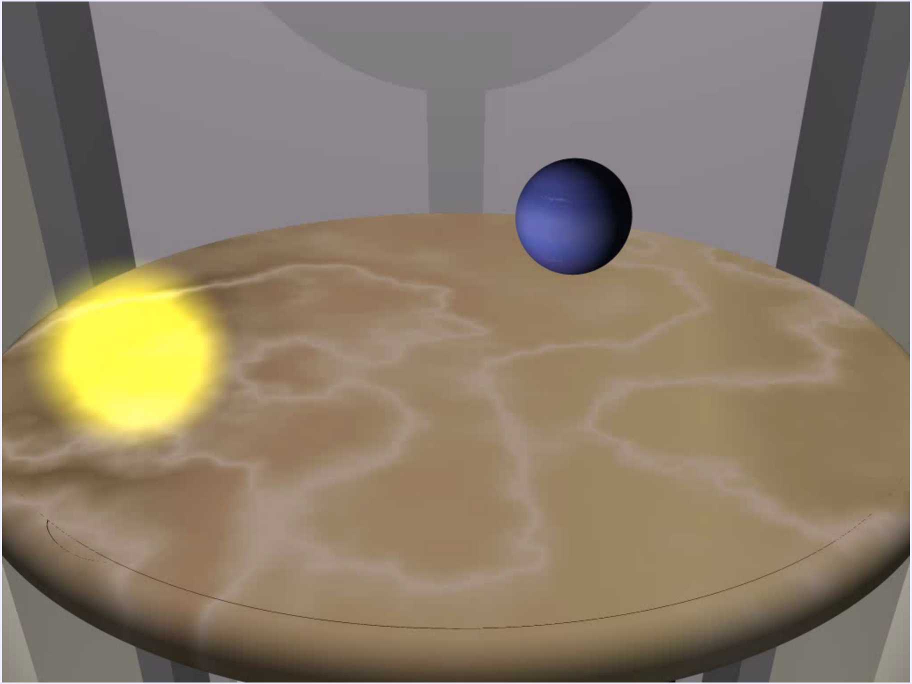
- same variation of polarization with scattering angle applies to Thomson and Rayleigh scattering
- scattering angle θ
- projection of amplitudes:
  - 1 for polarization direction perpendicular to scattering plane
  - $\cos\theta$ for linear polarization in scattering plane
- intensities = amplitudes squared
- ratio of $+Q$ to $-Q$ is $\cos^2\theta$ (to 1)
- total scattered intensity (unpolarized = averaged over all polarization states) proportional to $\frac{1}{2}\left(1+\cos^2\theta\right)$

Extrasolar Planetary Systems in Polarized Light

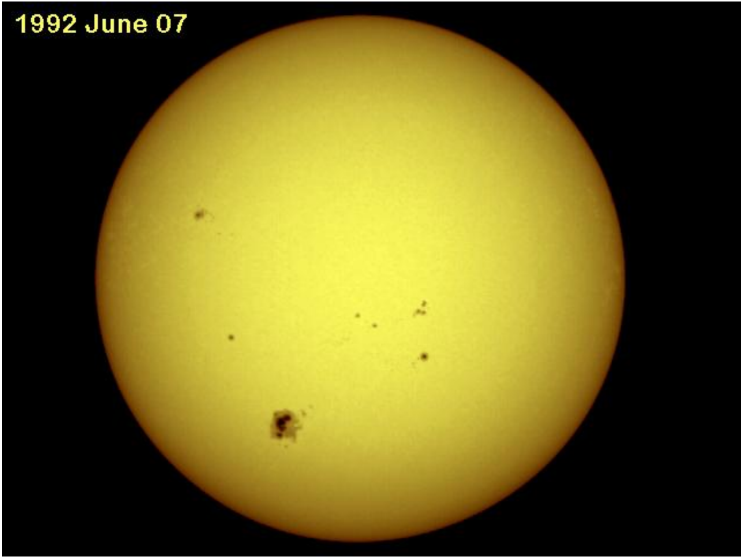
- *Past and Present:* Detecting planetary systems with indirect methods
- *The Future:* Understanding formation and evolution of planetary systems by direct characterization
- *The Problem:* Scattered light from central star dominates
  - Disks: about 10^4 times fainter than central star
  - Jupiter: 10^9 times fainter than Sun

Hubble Space Telescope image of M4 Star Cluster

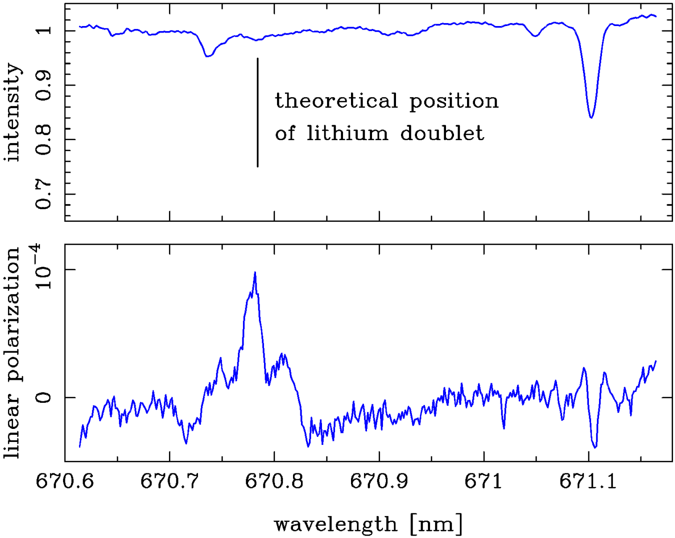




Limb Darkening



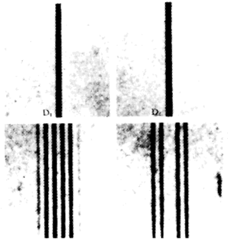
Solar Spectral Line Scattering Polarization



resonance lines exhibit “large” scattering polarization signals

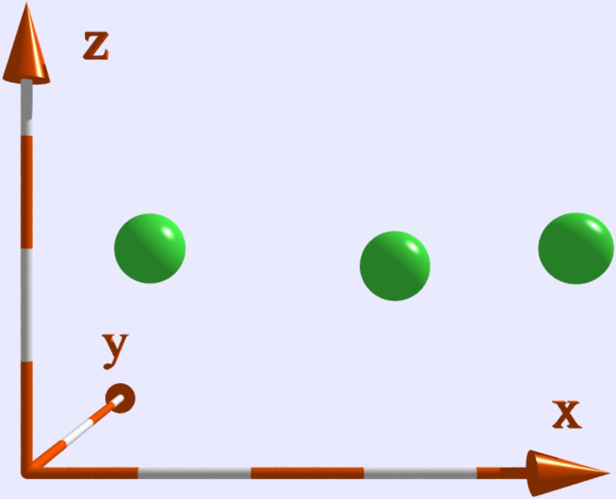
Zeeman Effect

photos.aip.org/



Splitting/Polarization of Spectral Lines

- discovered in 1896 by Dutch physicist Pieter Zeeman
- different spectral lines show different splitting patterns
- splitting proportional to magnetic field
- split components are polarized
- *normal Zeeman effect* with 3 components explained by H.A.Lorentz using classical physics
- splitting of sodium D doublet could not be explained by classical physics (*anomalous Zeeman effect*)
- quantum theory and electron's intrinsic spin led to satisfactory explanation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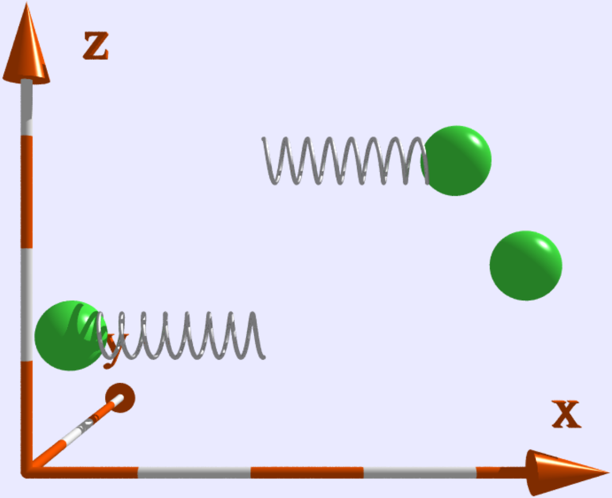
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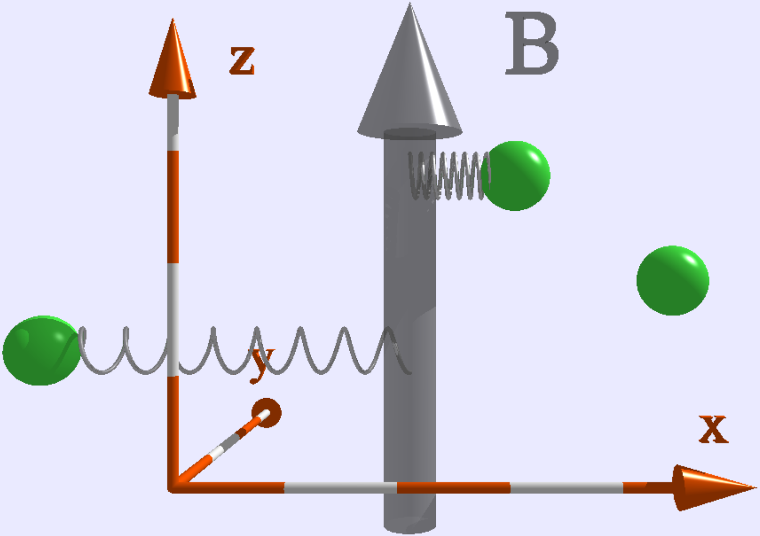
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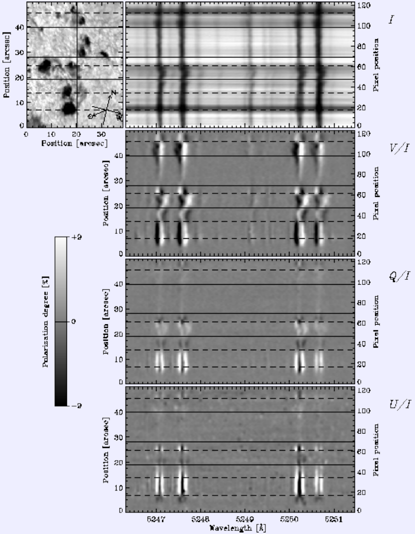

Bernasconi et al. 1998

Zeeman Effect in Solar Physics

- discovered in sunspots by G.E.Hale in 1908
- splitting small except for in sunspots
- much of intensity profile due to non-magnetic area ⇒ filling factor
- a lot of strong fields outside of sunspots
- full Stokes polarization measurements are key to determine solar magnetic fields
- 180 degree ambiguity