This lecture:

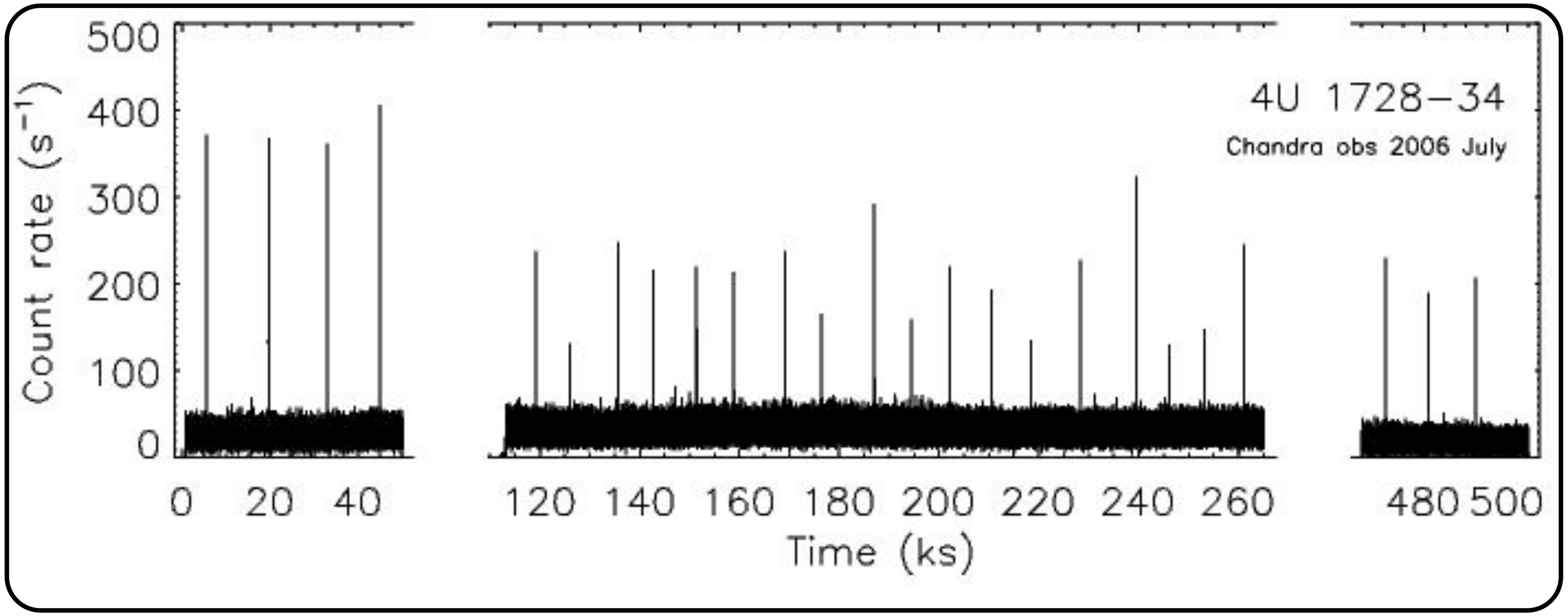
Total overview of a measurement procedure

Dealing with noise, e.g. optimal filtering

Signal-to-noise ratios

Is radiation indeed intrinsically stochastic?

One bin/measurement: average in time domain over box



Type I X-ray bursts

GALLOWAY PRIVATE COMMUNICATION

1 MEASUREMENT OF X(T) IN A TIME T => WINDOWING AND AVERAGING OVER TIME ΔT

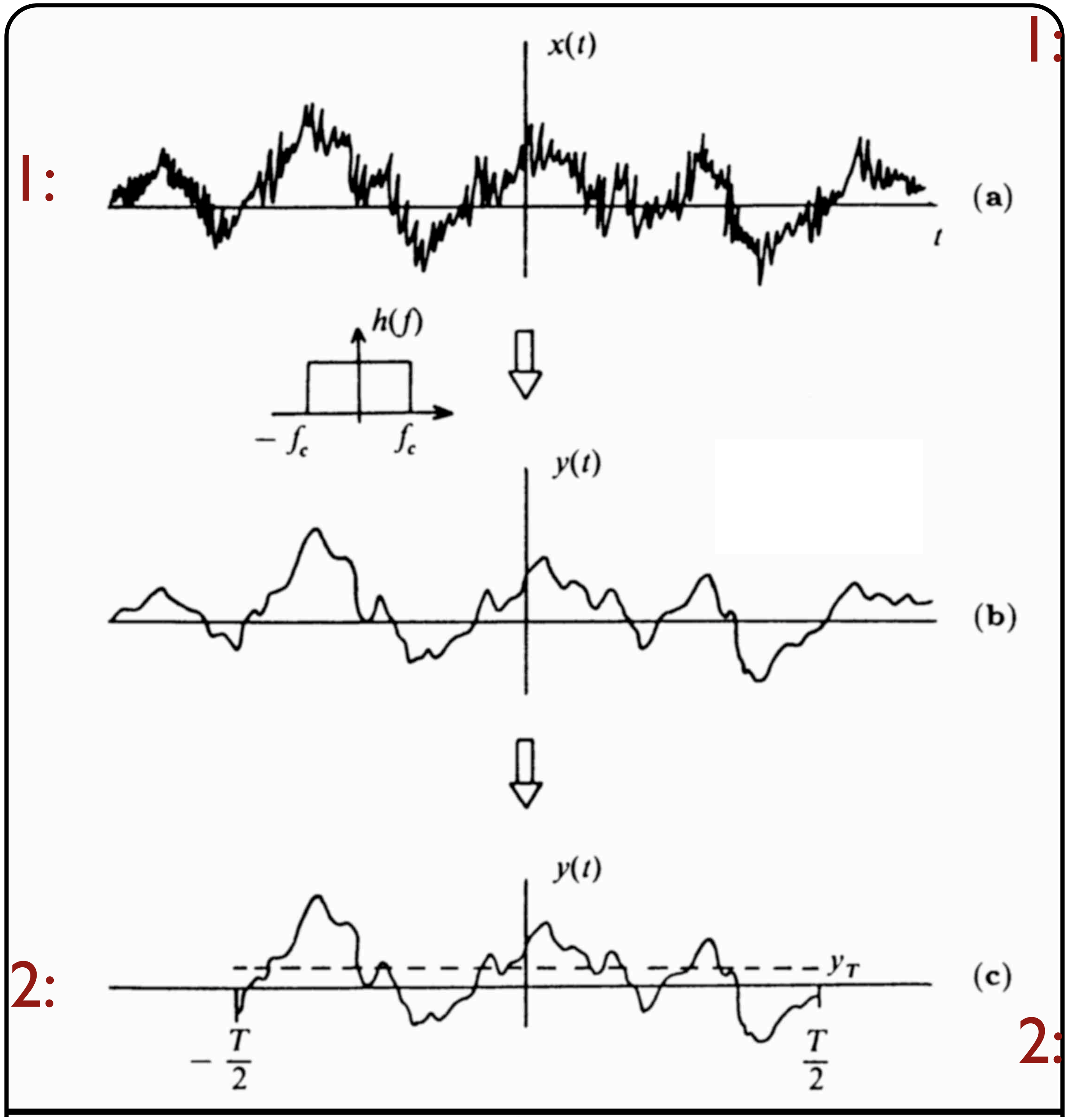
WINDOWING: $y(t) = \Pi(\frac{t}{T})x(t)$

This is a convolution!

AVERAGING:

$$z(t) \equiv y_{\Delta T}(t) = \frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} y(t')dt' = \frac{1}{\Delta T}\int_{-\infty}^{\infty} \Pi(\frac{t-t'}{\Delta T})y(t')dt'$$

= LOW-PASS FILTER, REMEMBER NYQUIST THEOREM



1: RESPONSE

REMOVE HIGH FREQUENCIES DUE TO FINITE RESPONSE OF INSTRUMENT AND DETECTOR

Remember convolution goes from $-\infty$ to ∞

2: 1 SAMPLE → AVERAGING

Average in time domain over length T, i.e. convolve in time domain with box function

Note figure used here different from Lecture notes p25!

$$z(t) = \frac{1}{\Delta T} \prod(\frac{t}{\Delta T}) * y(t)$$

$$z(t) = \frac{1}{\Delta T} \prod(\frac{t}{\Delta T}) * \prod(\frac{t}{T}) x(t)$$

What is the combined effect of finite sample and finite bin size?

Consider example on pages 25-27 of Lecture notes

$$\sigma^2_{x_T} = \sigma^2 / N \text{ for T} >> \tau_0$$

Estimating σ if the probability density function of the s.p. is known

→

Propagation of Errors chapter 5.2

Estimating σ if the probability density function of the s.p. is not known

→

e.g.

bootstrap method

Jackknife method

More on this later

Signal detection involves:

- limited time interval → windowing
- not continuous → sampling
- samples not instantaneous → averaging
- dealing with noise → filtering
- response of the detection system

→

thus the detected signal will only approximate the source signal

Recap

Sampling: high frequencies are filtered out

Window: low frequencies are filtered out

leads to band limited data

Time

measure a process x(t) over interval T assumed zero outside T

$$\equiv y(t) = \Pi(\frac{t}{T})x(t)$$

$$Y(f) = X(f) * T sinc(Tf)$$

all information about frequencies <1/T is lost!

Recap

Sampling: high frequencies are filtered out

Window: low frequencies are filtered out

→ leads to band limited data

→ Frequency

$$Y(f) = X(f)H(f)$$

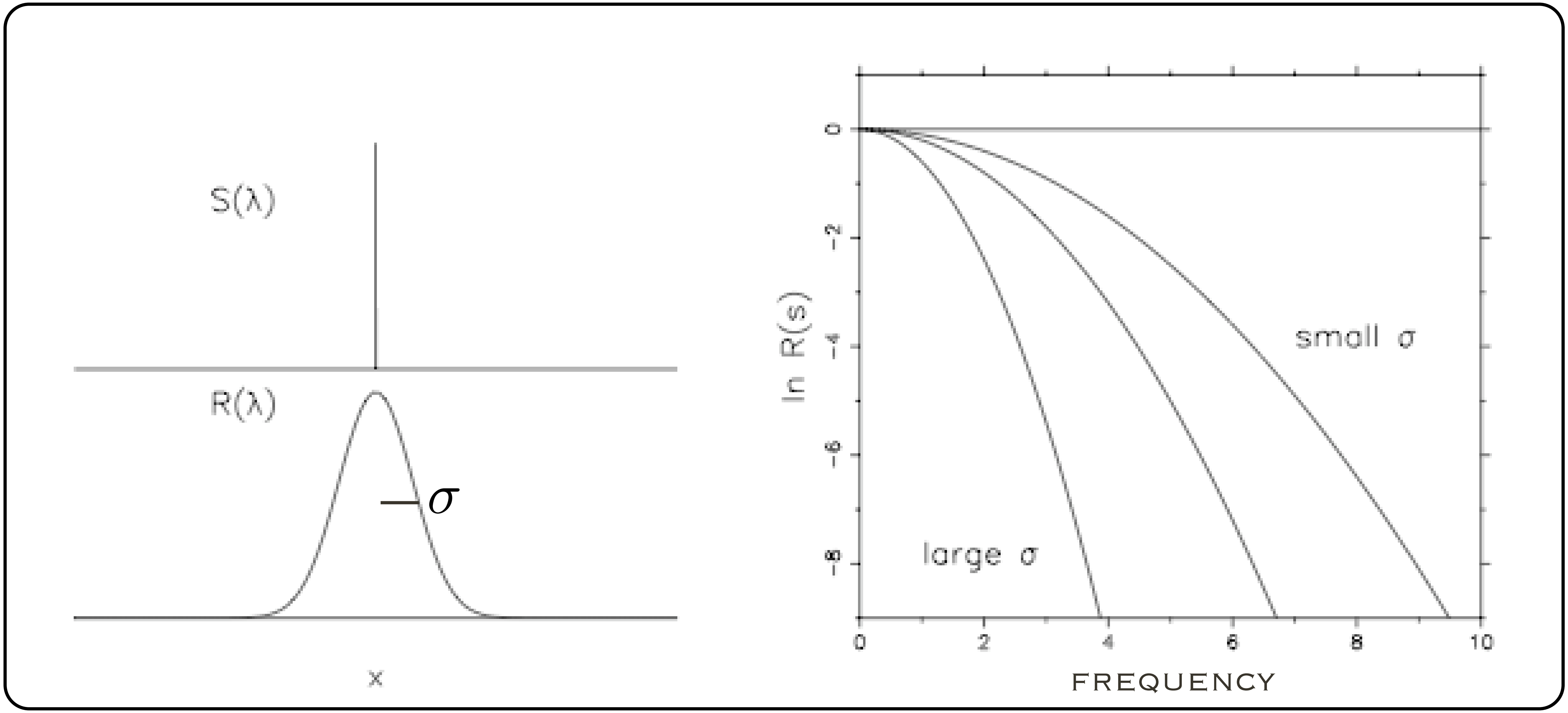
$$y(t) = \int_{-\infty}^{\infty} x(t-\theta)h(\theta)d\theta$$

$$y(t) = x(t) * h(t)$$

Filtering of process x with filter h

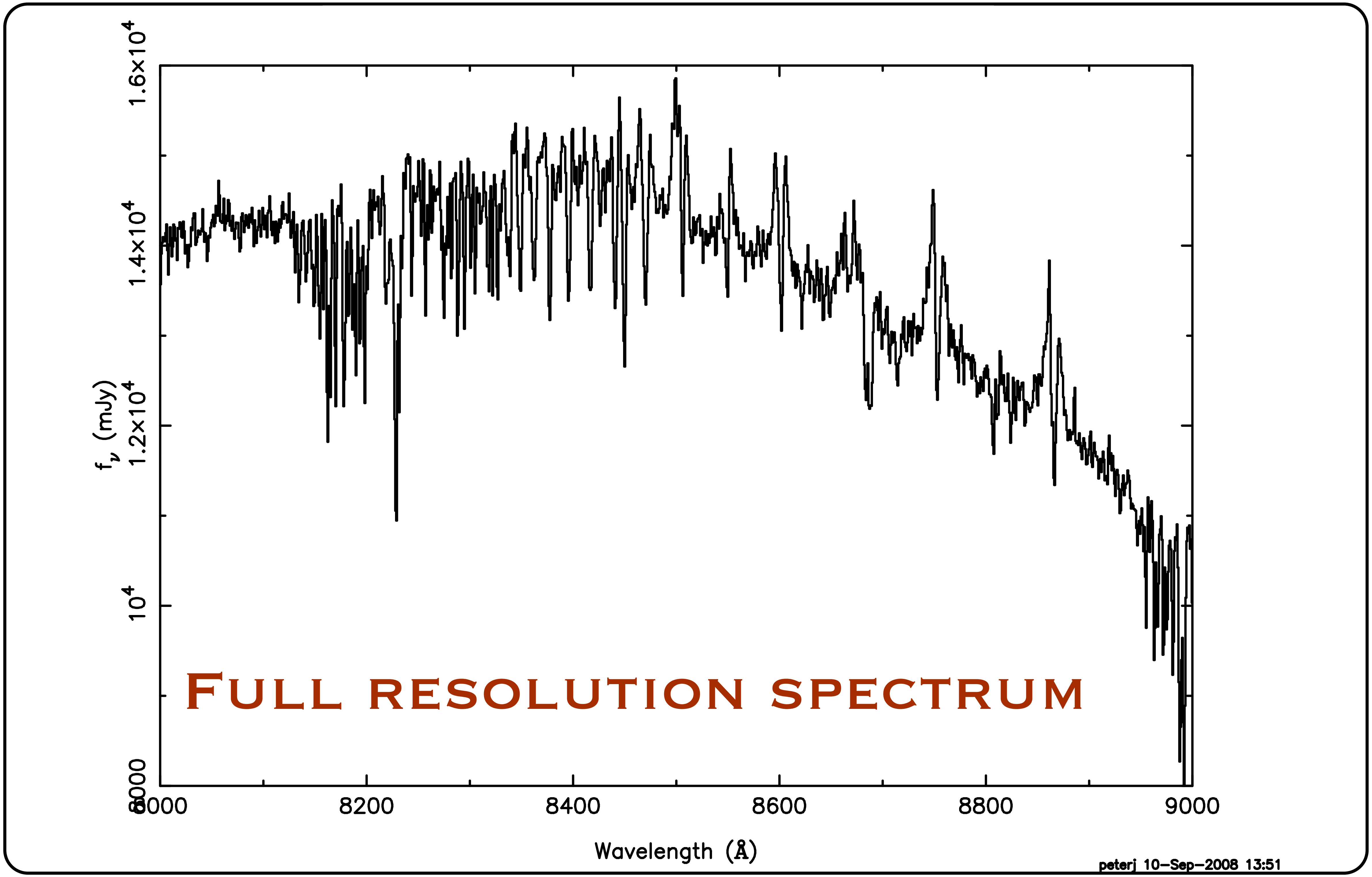
Recap: detector/instrument system response

Gaussian response function

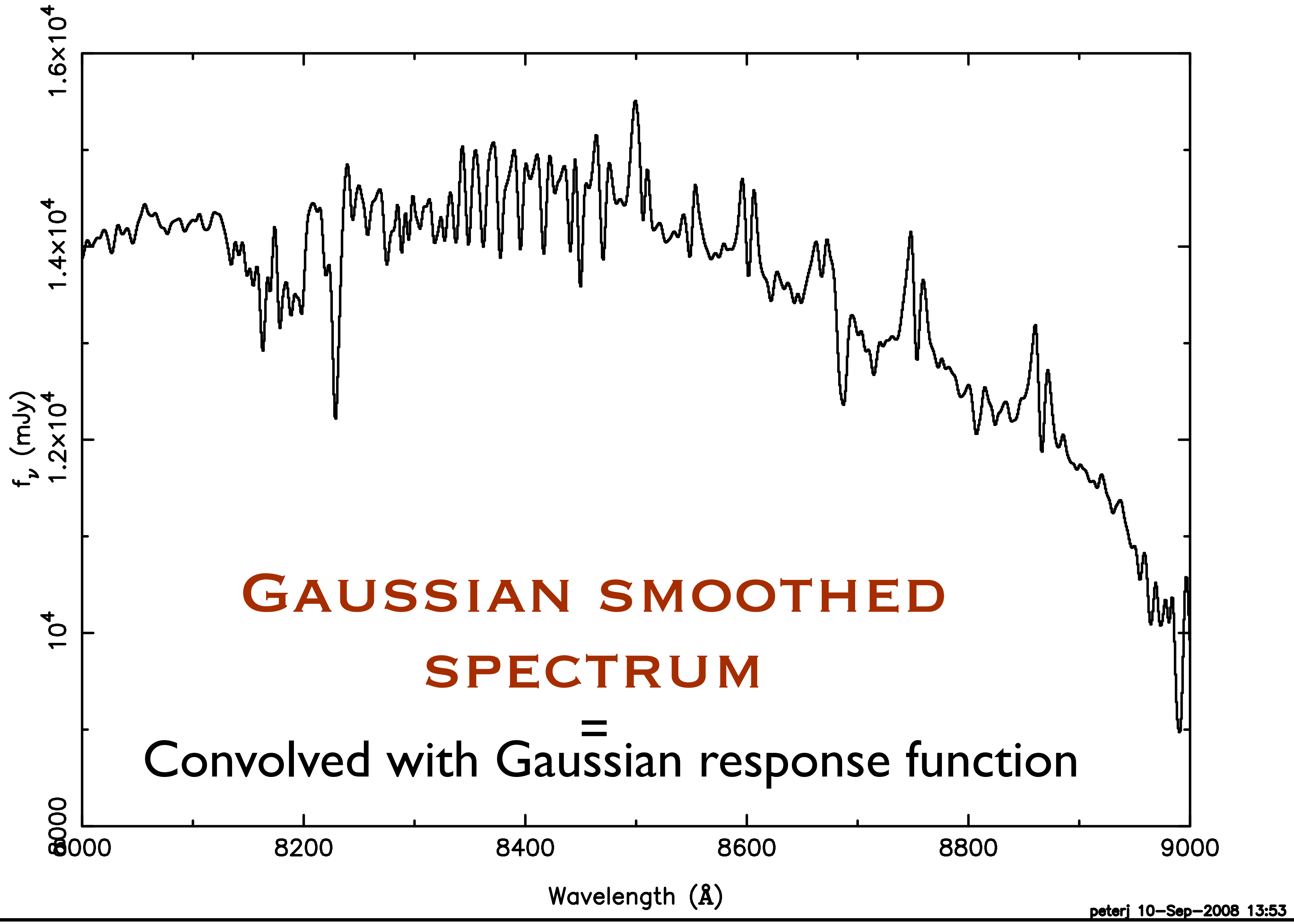


$$R(\lambda)\frac{1}{\sqrt(2\pi)\sigma}exp-(\frac{\lambda^2}{2\sigma^2})$$

Gaussian response function



Gaussian response function



Gaussian smoothed spectrum

=

Convolved with Gaussian response function

Mazur: Peer Instruction

Identify in a real astrophysical measurement all the steps discussed above

Response instrument/detector
Finite duration measurement
Sampling
Average of 1 sample
Noise contributions?

Recap

Data sampling: ideal case Nyquist criterium is fulfilled → sampling does not lead to loss of information RV completely described by the samples

Conditions either:

→ Band-limited response of the detector removes highest noise powers and the sampling is fast enough to cover the band limit of the detector

or:

→ Signal is band-limited and

$$\nu_{\text{sampling}} > \nu_{\text{max,detector}} > \nu_{\text{max,signal}}$$

however, noise is often white and thus contains high frequencies!

Noise removal by optimal filtering

Num. Res 13.3

$$c(t) = s(t) + n(t)$$

$s(t)$ is the smeared signal i.e. true $\ast$ response

$$s(t) = u(t) * r(t)$$

Design an optimal filter ϕ(t) that produces a signal ũ(t) as close as possible to u(t) or $U(f)$

$$\widetilde{U(f)} = \frac{C(f)\phi(f)}{R(f)}$$

Close in least square sense

$$\int_{-\infty}^{\infty} |\widetilde{U(f)} - U(f)|^2 df$$ is minimised

Noise removal by optimal filtering

$$\int_{-\infty}^{\infty} \left| \frac{[S(f)+N(f)]\phi(f)}{R(f)} - \frac{S(f)}{R(f)} \right|^2 df \ = 0$$

$\int S(f)N(f)df$ terms are zero since noise and signal are uncorrelated

$$\int_{-\infty}^{\infty} |R(f)|^{-2} \underbrace{\{|S(f)|^2|1-\phi(f)|^2 + |N(f)|^2|\phi(f)|^2\}}_{\Theta} df \ = 0$$

Θ minimised with respect to ϕ

Noise removal by optimal filtering

$$\frac{d\theta}{d\phi} = 0$$

$$-2S^2(1-\phi) + 2N^2\phi = 0$$

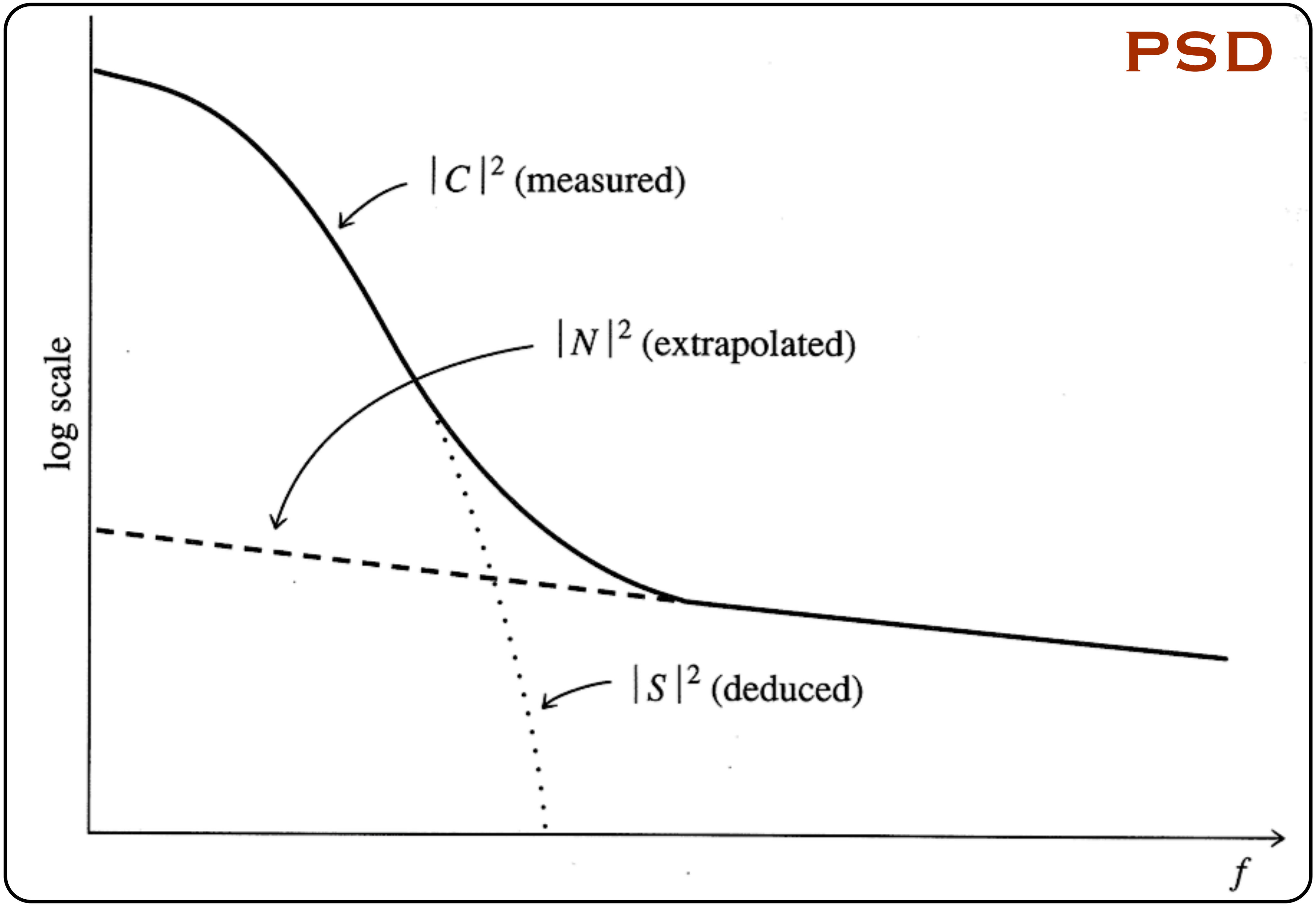
Optimal filter $$\phi = \frac{S^2}{S^2 + N^2}$$

Does not contain true signal directly!

$$U(f)$$

$$|S(f)|^2 + |N(f)|^2 = PDS(f) = |C(f)|^2$$

Noise removal by optimal filtering



Page 542 Num Res

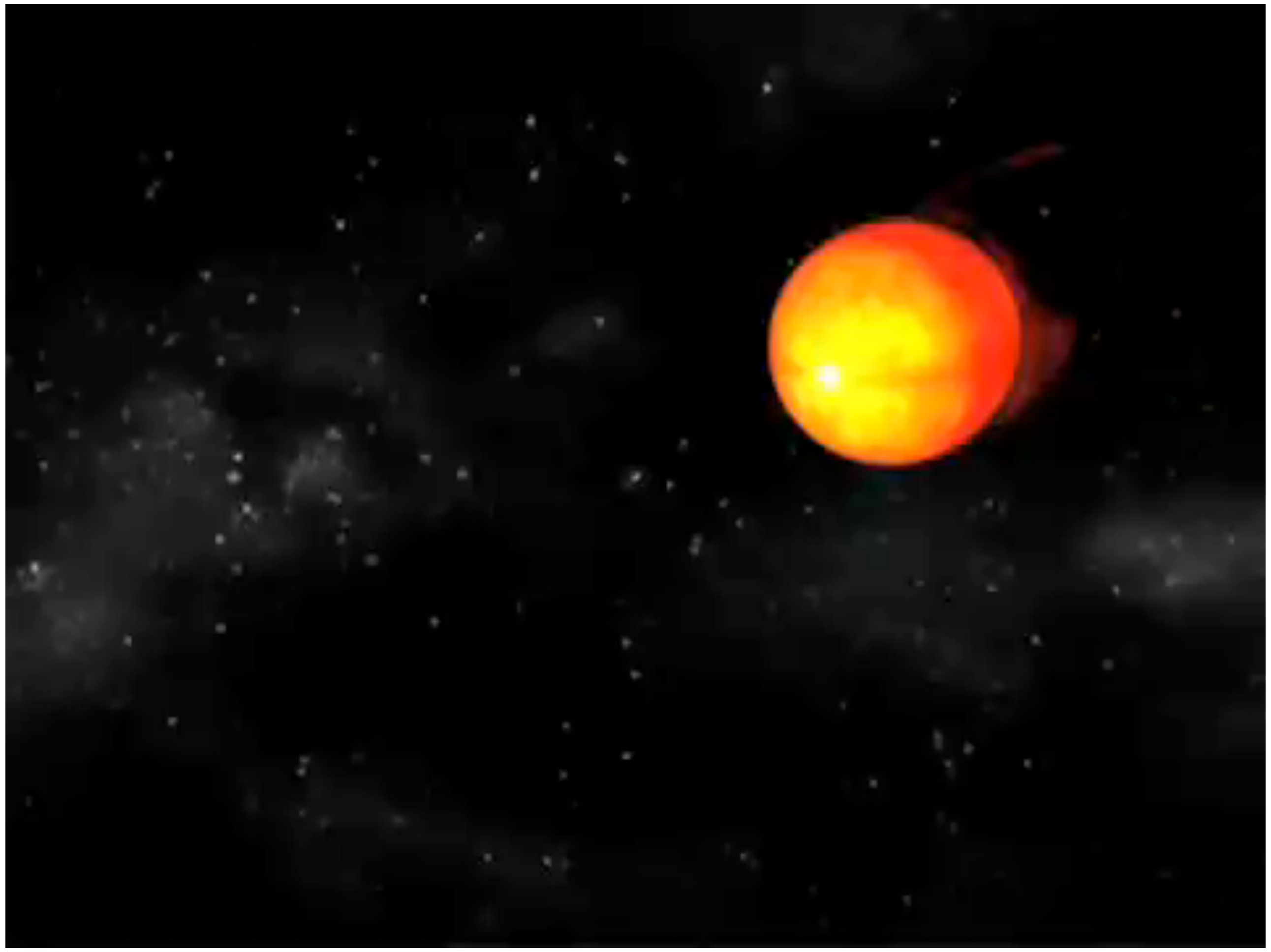
Some Applications of filtering

On optimal detection of point sources in CMB maps
Vio et al, 2002, A&A, 391, 789

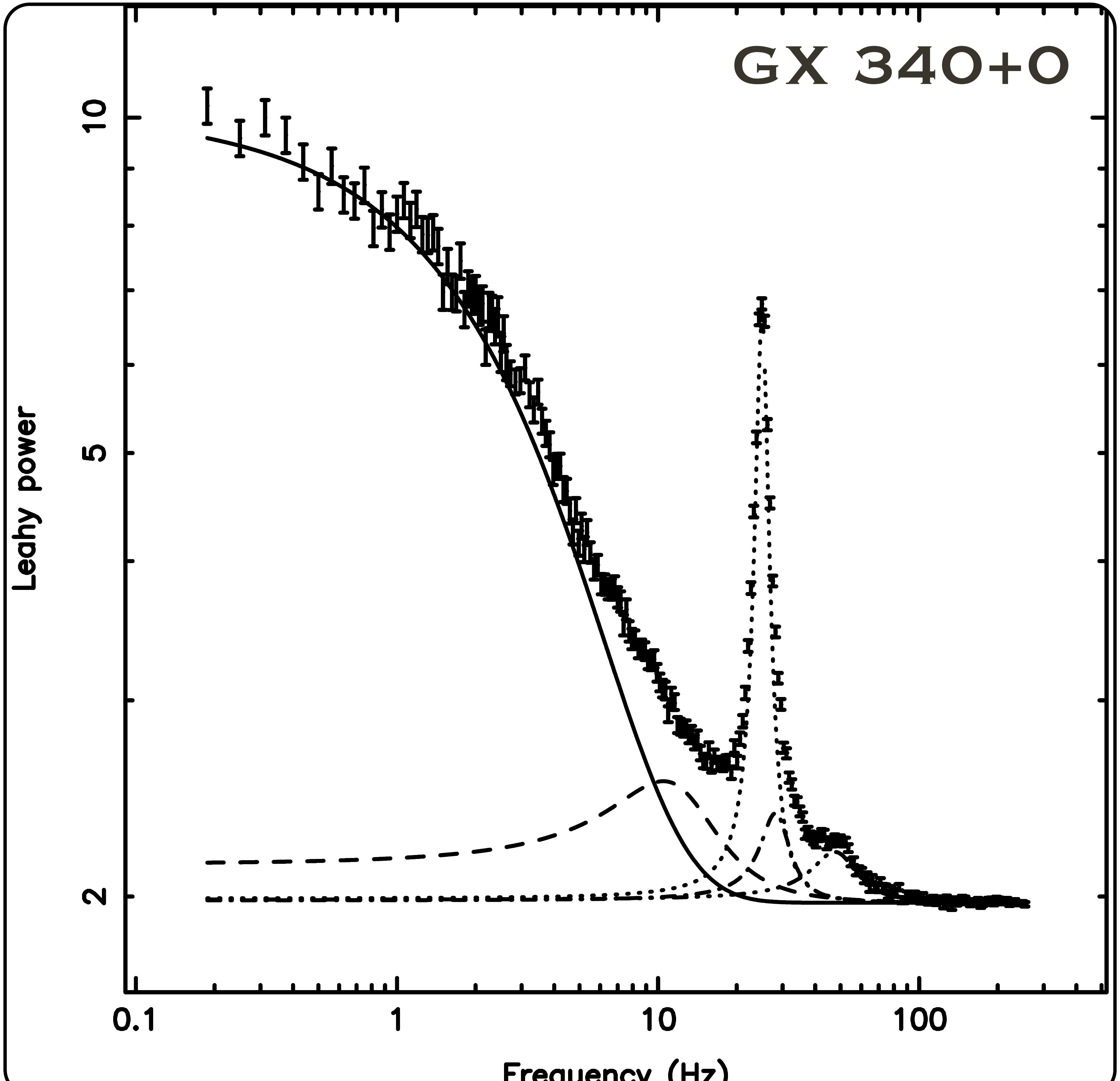
An optimal filter for the detection of galaxy clusters through weak lensing
Maturi, et al. 2005, A&A, 442, 851

The largest scale perturbations: A window on the physics of the beginning
Wandelt, New Astronomy Review, 2006, 11, 900

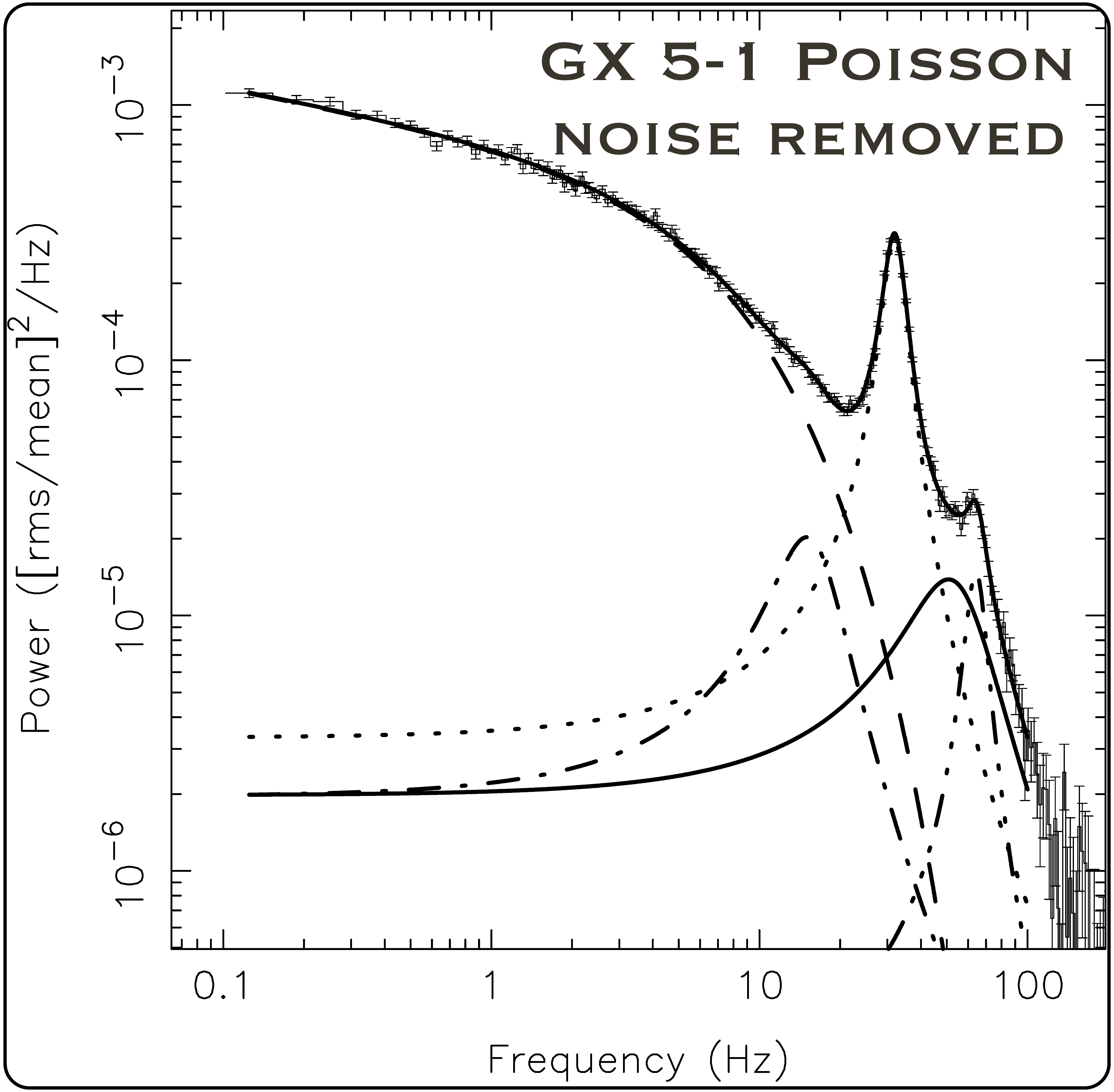
Application to timing experiments in Low-mass X-ray binaries



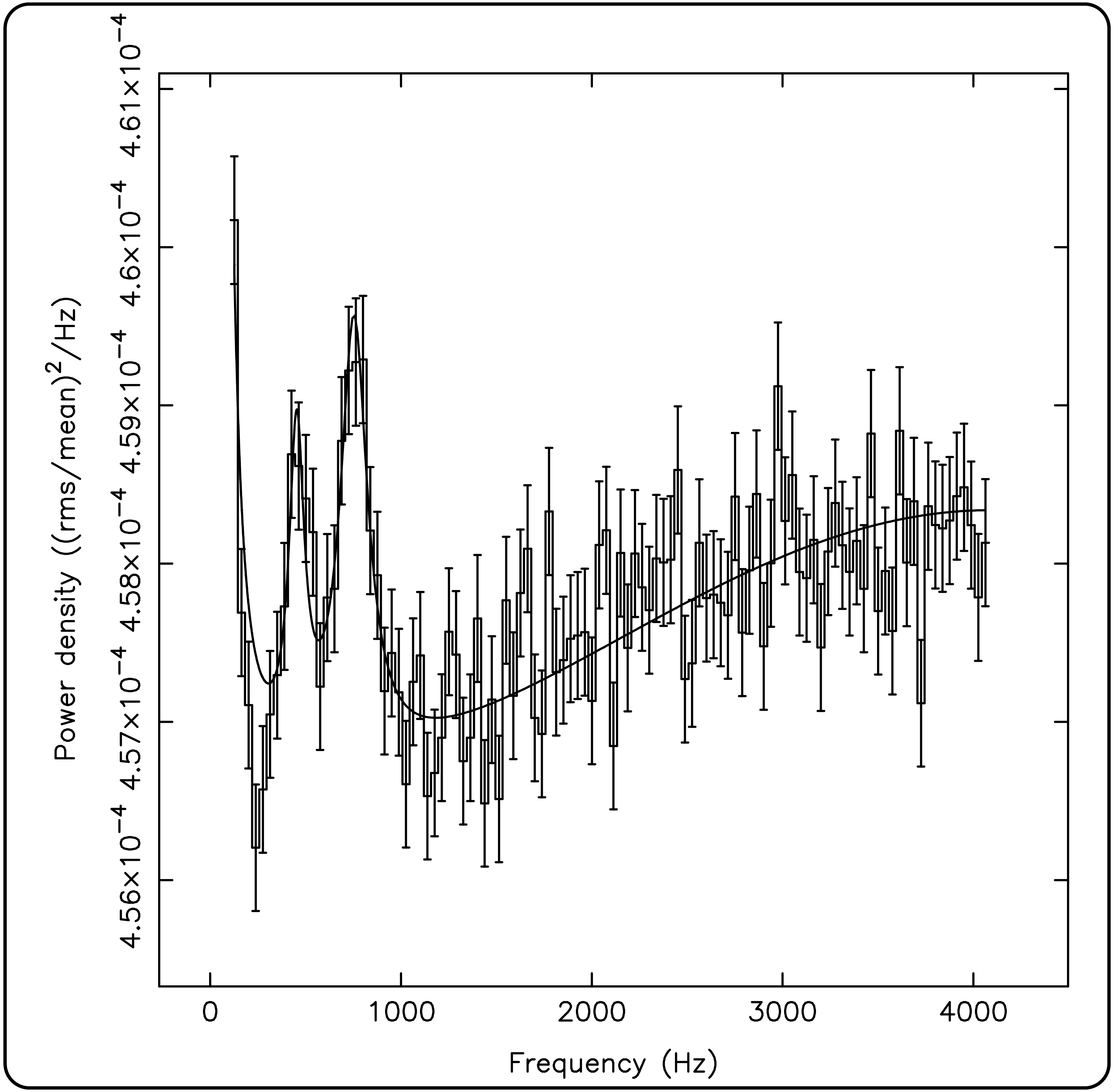
C.f. X-ray Timing experiments



X-ray Timing experiments



X-ray Timing experiments



Recap Filtering:

one can design an optimal filter such that the filtered measured data-set is as close as possible (in least-square sense) to the uncorrupted signal

The Signal-to-Noise Ratio & Limiting Sensitivity

1. The level of noise determines the detectability of a weak (source) signal

2. Various noise contributions from :

   a) other sources in FoV (point-source / diffuse) → sky-noise
   b) operational environment of instrument (radiation belts, scatter light from Earth, Sun, Moon, atmosphere etc.)
   c) constituting parts of instrument e.g. noise from amplifiers in electronics, induced radio-activity → instrumental noise

3. Quality of an observation is determined by the signal-to-noise ratio **SNR**

$$SNR = SNR(T_{exp}, \Delta E, A_{eff}, B)$$

4. For photon energies $\gtrsim$ eV (i.e. optical, UV, X–rays, γ–rays) quantum characterisation applicable: statistical treatment of the accumulated quanta (Poissonian statistics)

SRON

- Consider point source embedded in uniform sky background noise with photon flux n_s *(ph/m^{2}s)* i.e. the source-region

- Sky background noise n_{bg} (ph/m^{2}s sr)

- Instrumental noise n_{det} (ph/m^{2}s)

Number of counts registered in source region within observation time T_{obs}:

$$N_1 = ((n_s + n_{bg}\,\Delta\Omega)\,\bar{A}_{eff} + n_{det}\,A_{pix})\,T_{obs}$$

$\Delta\Omega$ - solid angle subtended by source region
$\bar{A}_{eff}$ - effective area (averaged over bandwidth ΔE: $\int A_{eff}(E)\,dE / \Delta E$)
A_{pix} - pixel surface

- Similar expression holds for a region excluding the source: the background region

$$N_2 = (n_{bg}\,\Delta\Omega\,\bar{A}_{eff} + n_{det}\,A_{pix})\,T_{obs}$$

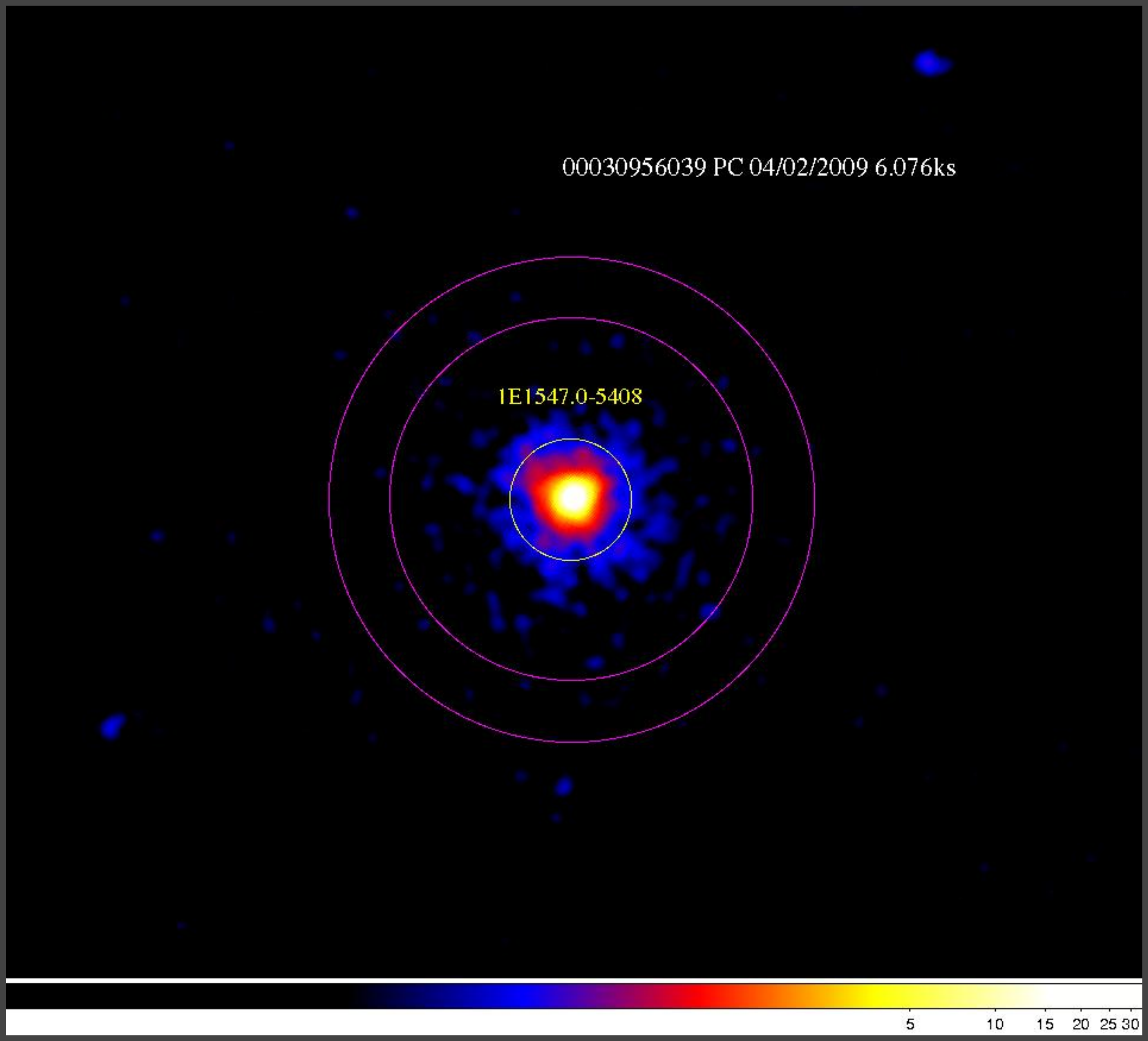
collected during same integration time simultaneously

- Fluctuations in N1 and N2 are governed by Poissonian stat. : $\sqrt{N_1}$, $\sqrt{N_2}$

SRON

Example:

Swift-XRT observation of anomalous X-ray pulsar 1E1547.0-5408 during monitoring campaign after its Jan. 2009 outburst (in PC-mode)



Inner (yellow) ring: 60″ source region
Purple annulus : 180″-240″ bakground region

Note: Background region 7x larger than source region

SNR can be defined by:

$$SNR = (N_1 - N_2) / \sqrt{(N_1+N_2)}$$

Signal strength is expressed in terms of the fluctuation of the noise term(s)!

We speak of a 3σ measurement, if $SNR=k_{min}=3$ (k_{min} is minimum confidence set by observer)

5σ measurement, if $SNR=k_{min}=5$

SRON

Limiting sensitivity

We consider now 2 limiting cases:

a) Source signal dominates the noise: $N_1 \gg N_2$

$SNR \sim \sqrt{N_1} = \sqrt{(n_s \bar{A}_{eff} T_{obs})}$

demanding a minimum SNR of k_{min} (typically 3 / 5 to speak of a detection) →

$n_{smin} = k^2_{min} / (\bar{A}_{eff} T_{obs})$ ⇨ the detection is *signal-photon-noise* limited

b) Source signal drowned in the noise: $N_1 \approx N_2$

$SNR = k_{min} \sim (n_s \bar{A}_{eff} T_{obs}) / \sqrt{(2N_2)}$ ⇒

$n_{smin} = k_{min}\sqrt{(2N_2)}/(\bar{A}_{eff} T_{obs}) \sim k_{min}\sqrt{(2\bar{A}_{eff}T_{obs}[n_{bg}\Delta\Omega + n_{det}\delta])}/\bar{A}_{eff}T_{obs} \propto (\bar{A}_{eff}T_{obs})^{-\frac{1}{2}}$

$(\delta = A_{pix}/\bar{A}_{eff})$

n_{smin} improves only with the square root of the observation time and effective area…

If $n_{det}\,\delta \ll n_{bg}\,\Delta\Omega$ (i.e. detector noise negligible) ⇨

the detection is *background-photon-noise* limited

In summary:

a) *signal-photon-noise* limited observation: $n_{s_{min}} \sim (\bar{A}_{eff}\, T_{obs})^{-1}$

b) *background-photon-noise* limited obs. : $n_{s_{min}} \sim (\bar{A}_{eff}\, T_{obs})^{-½}$

Note: bandwidth ΔE is implicitely incorporated $n_s = \int_{\Delta E} n_s(E)dE = \check{n}_s\, \Delta E$

Similar expressions for n_{bg} and n_{det} :

signal-photon-noise limited: $\check{n}_{s_{min}} \sim (\bar{A}_{eff}\, T_{obs}\, \Delta E)^{-1}$

background-photon-noise limited: $\check{n}_{s_{min}} \sim (\bar{A}_{eff}\, T_{obs}\, \Delta E)^{-½}$