Observations in Astrophysics-2

Statistical description of processes

convolution of signal with transfer function, sampling etc

requires the concept of Fourier transforms via the Convolution Theorem & Cross Correlations

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Additional reading

Numerical Recipes

Press et al. 1992

Chapters 12-0,1, 13, 14

www.haoli.org/nr/bookfpdf.html or check : www.nr.com

Observational Astrophysics

Lena, P., Lebrun, F., Mignard, F.

Chapter 2: The observation and analysis of stellar photospheres:

Gray, D.F., C.U.P.,1992

Data reduction and error analysis

Bevington & Robinson 1992

Chapter 1.1, 1.2, 1.3 & 2.1, 2.2 (not 2.2.2), 2.3 OAF-2 & Chapters 3 & 5 of OAF-1

Useful (observational) Astrophysics websites

http://xxx.soton.ac.uk/list/astro-ph/new

http://cdsads.u-strasbg.fr/abstract_service.html

http://simbad.u-strasbg.fr/simbad/sim-fid

http://www.astronomerstelegram.org/

arXiv.org > astro-ph

Search or Article-id (Help | Advanced search)

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  - current month's astro-ph listings
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Measurements in general

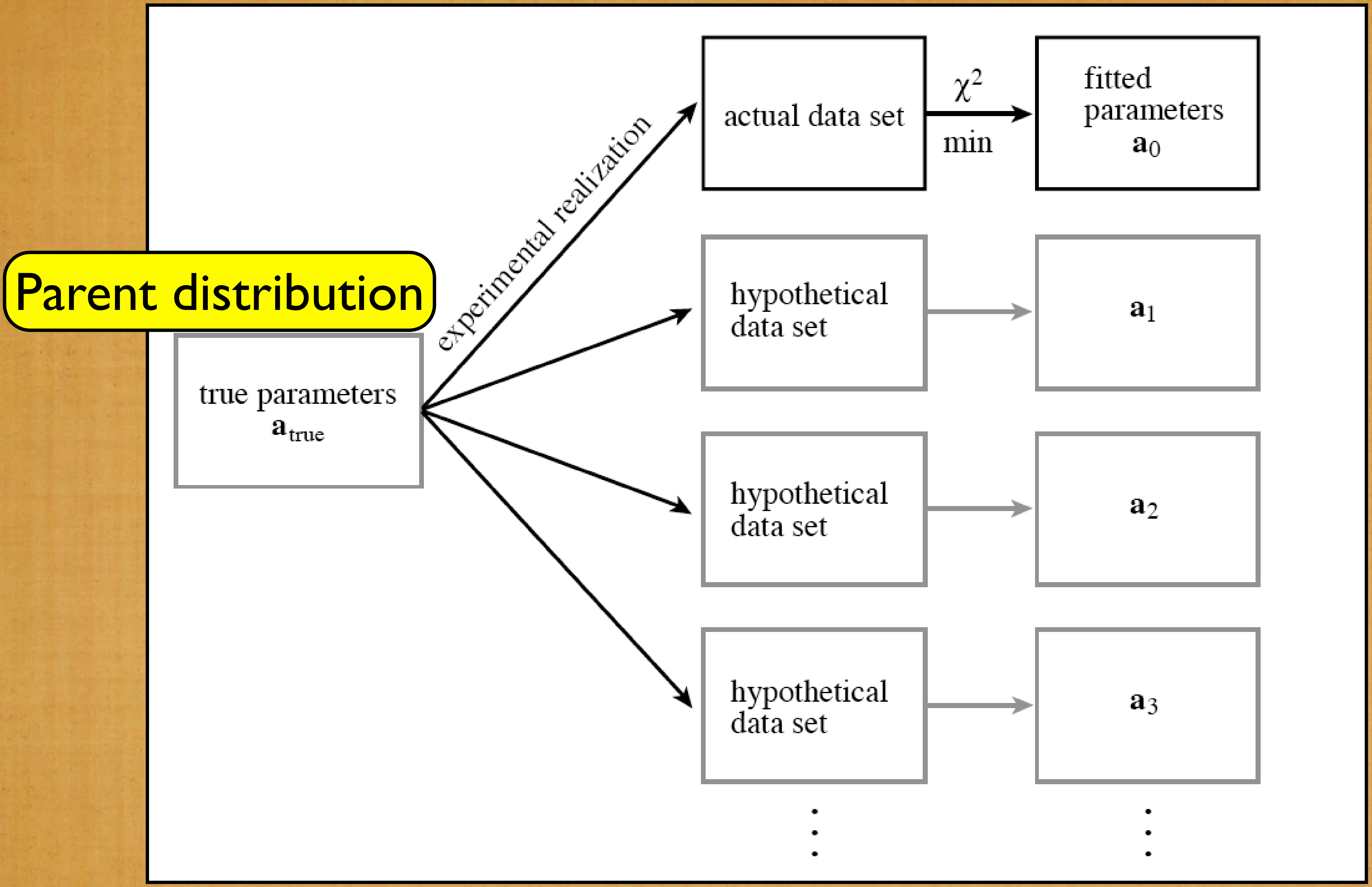
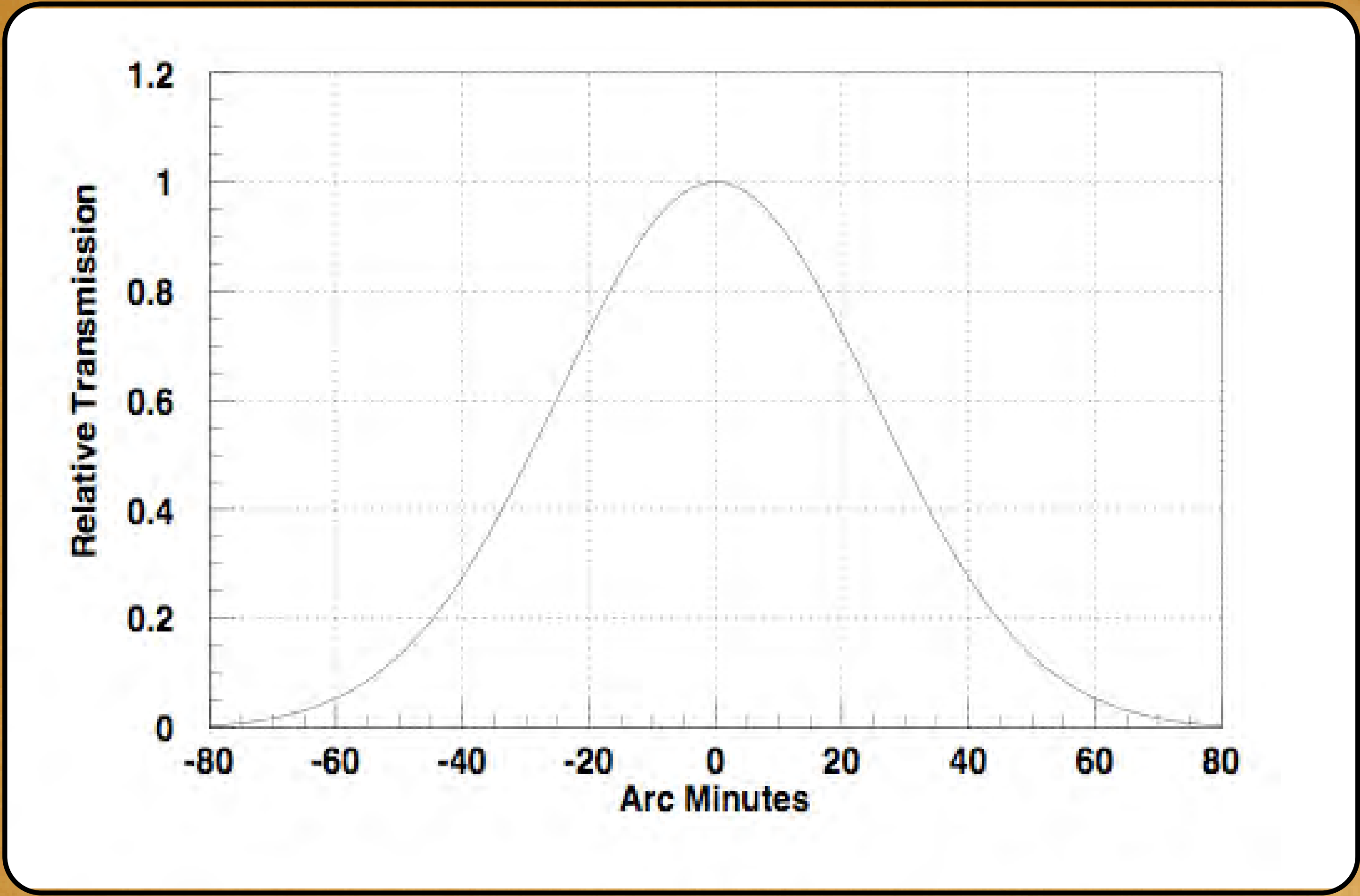


Fig from Numerical Recipes

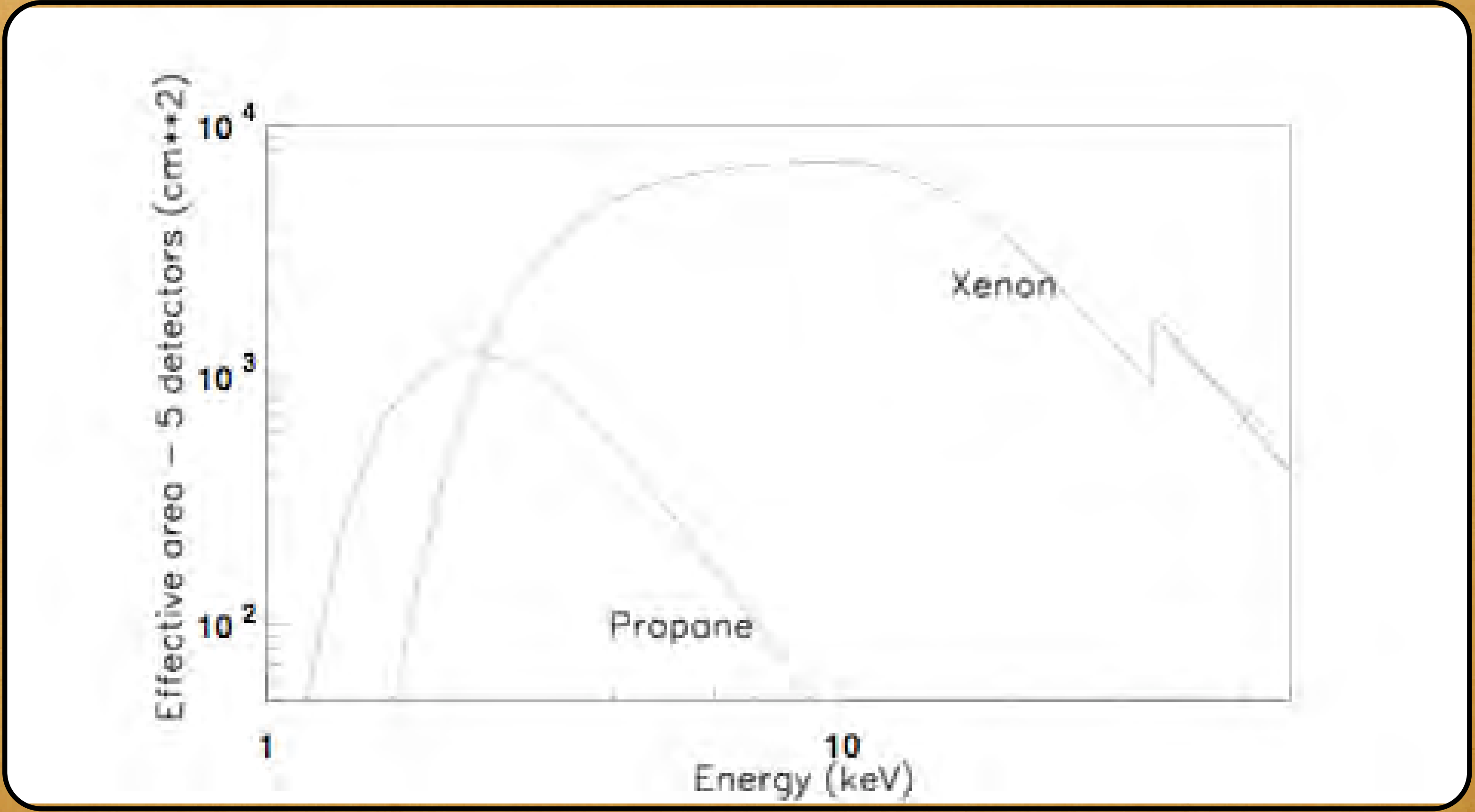
Detection of X-rays with the Rossi X-ray timing Explorer

Three instruments: an ASM, the PCA, and Hexte

Collimator response

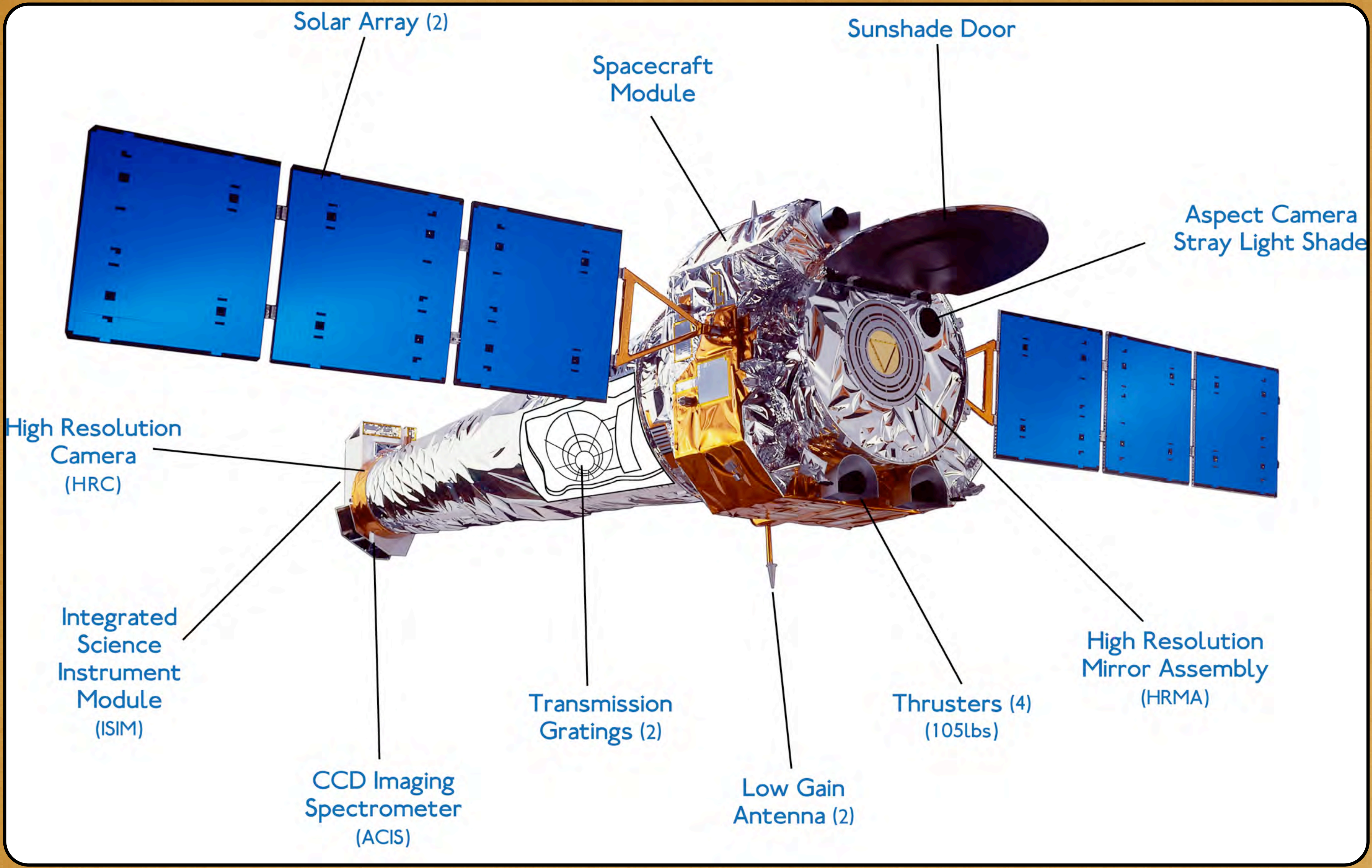


Effective Area

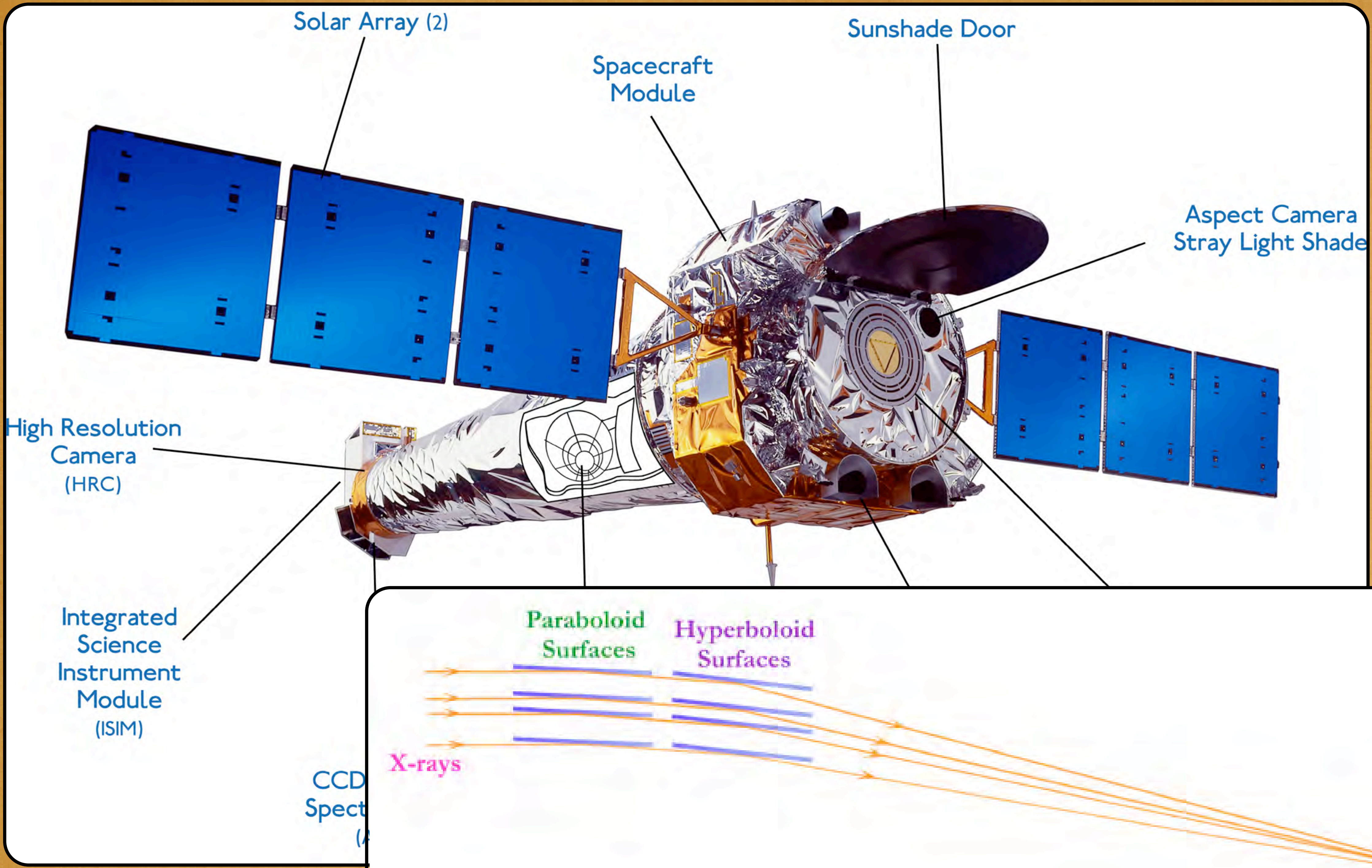


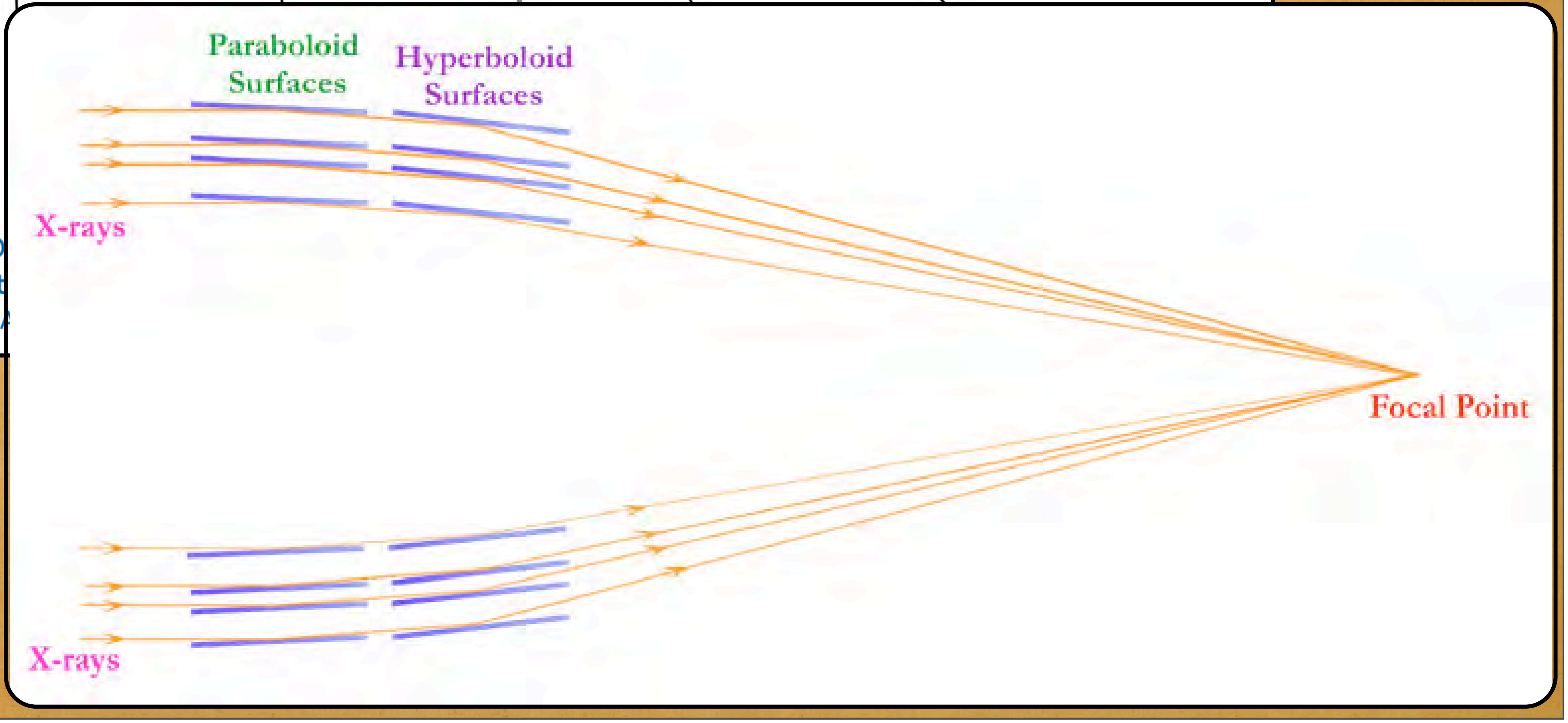
Data sampling & Data binning

Chandra satellite

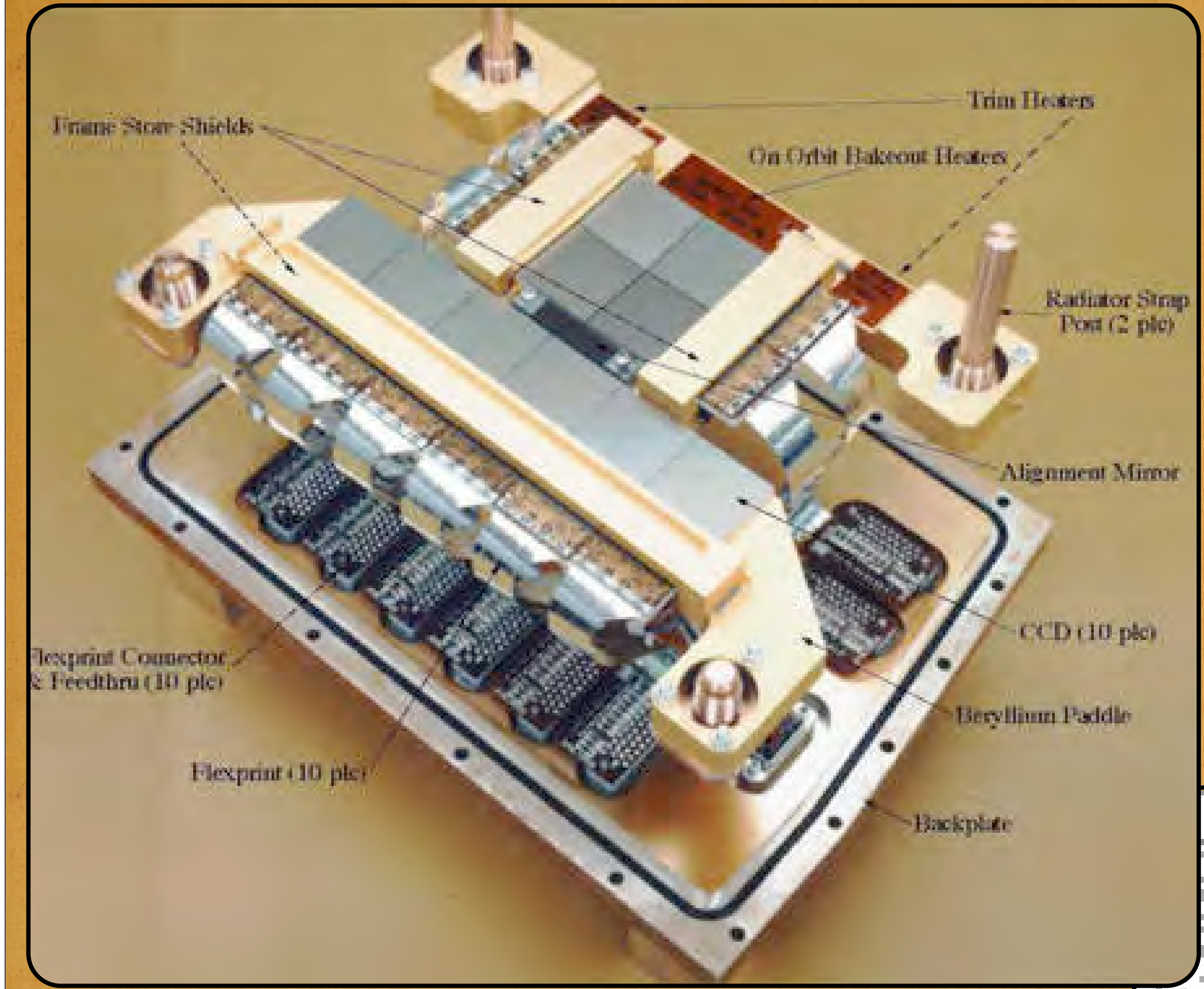


Chandra satellite

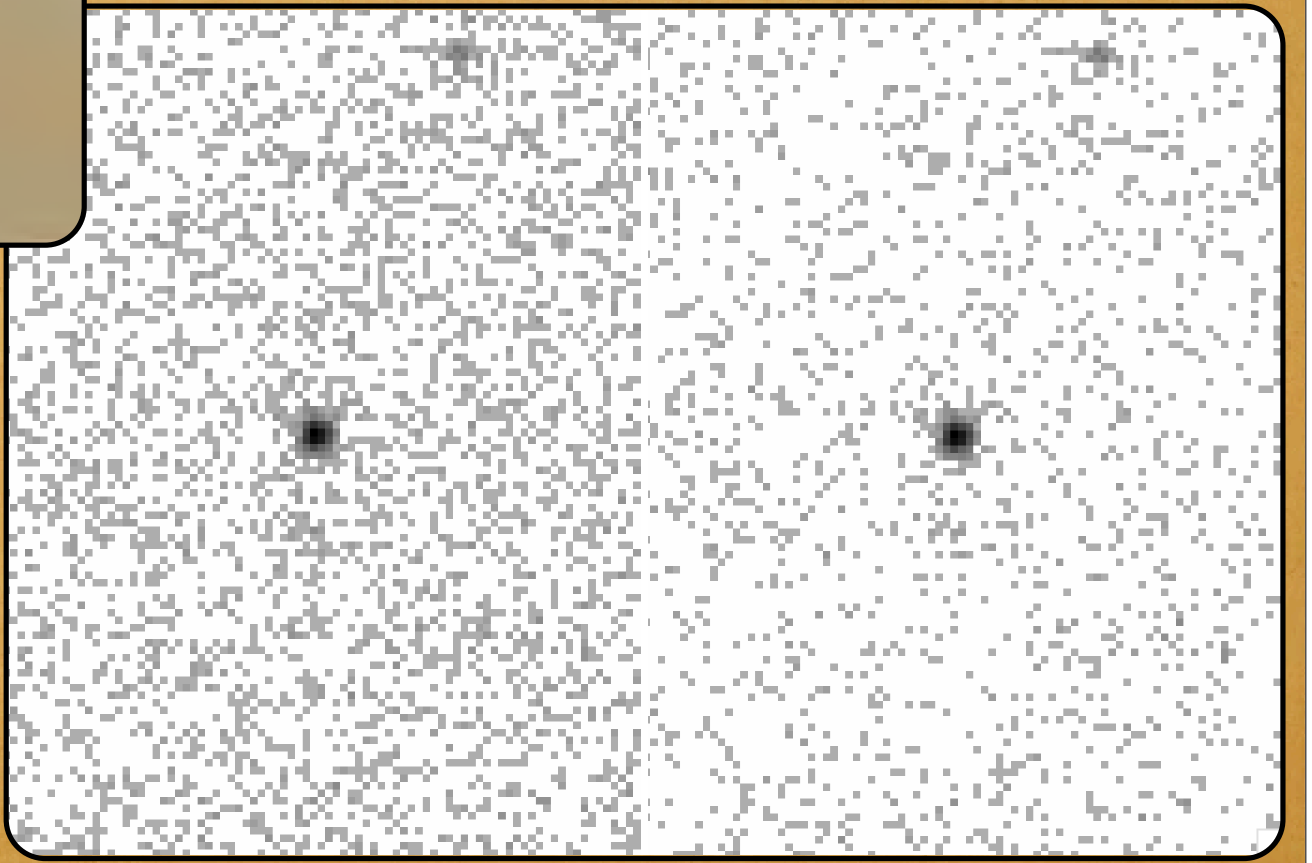




The ACIS CCDs



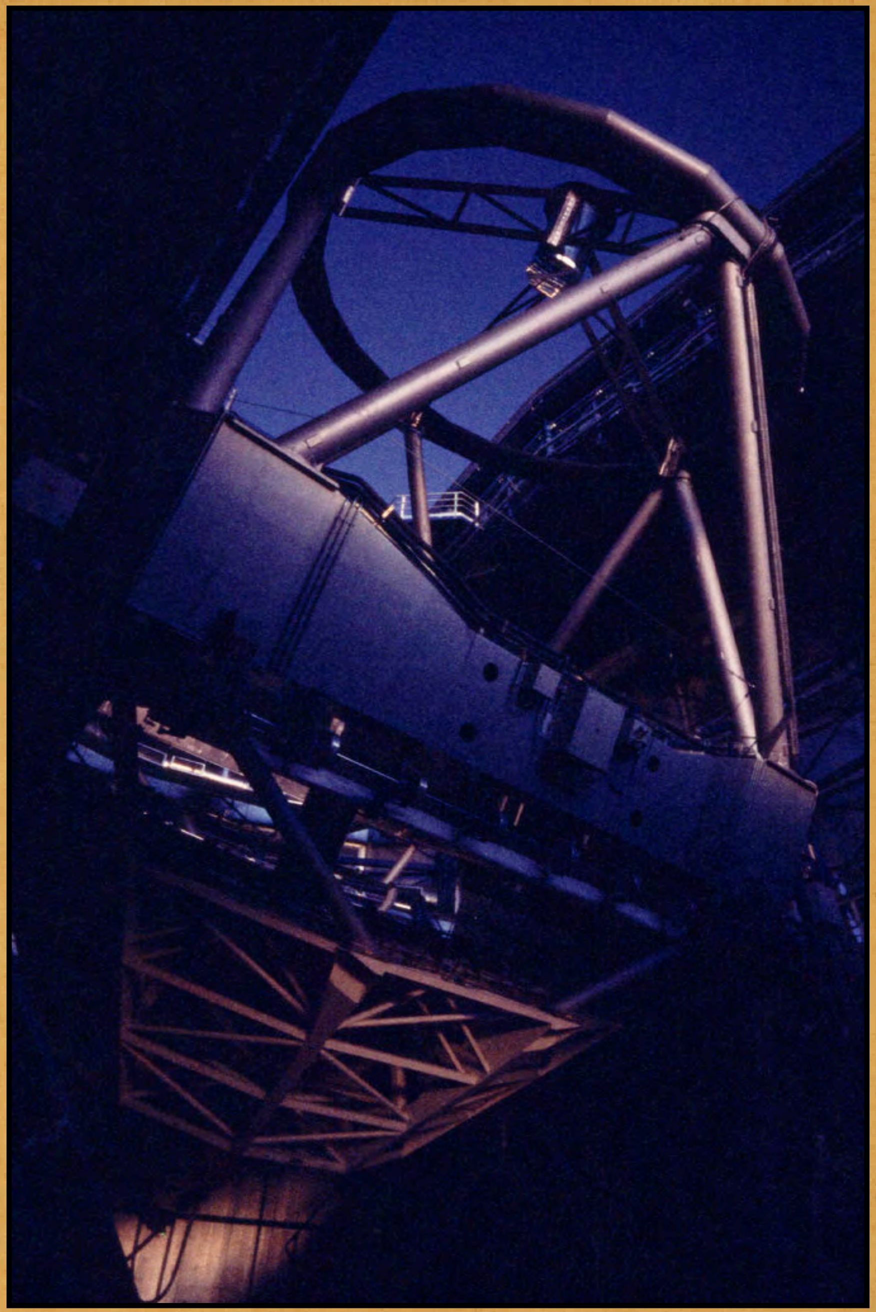
Close-up of an observation



Discrete pixels & filtering

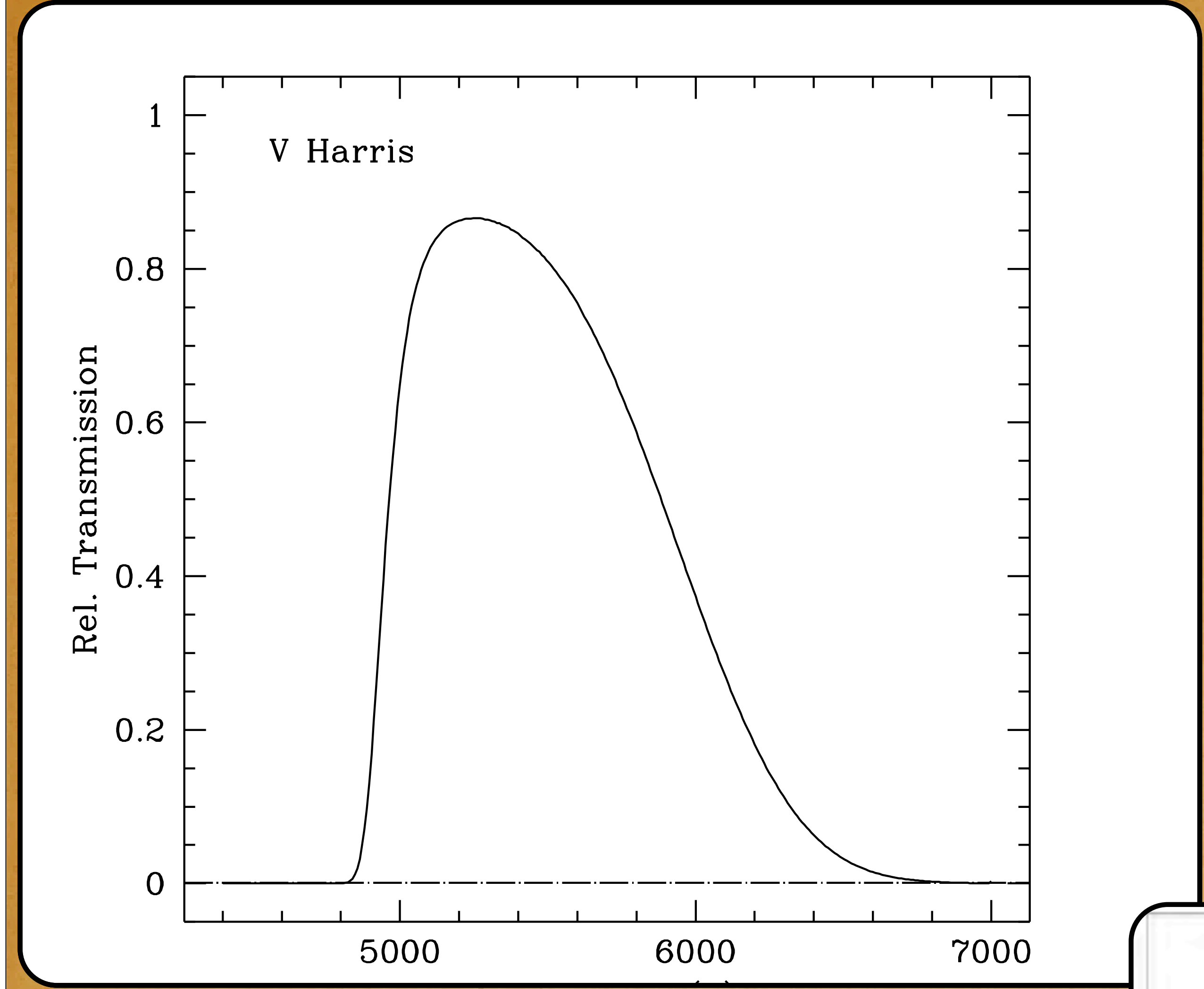
Examples: detection of optical light via a telescope

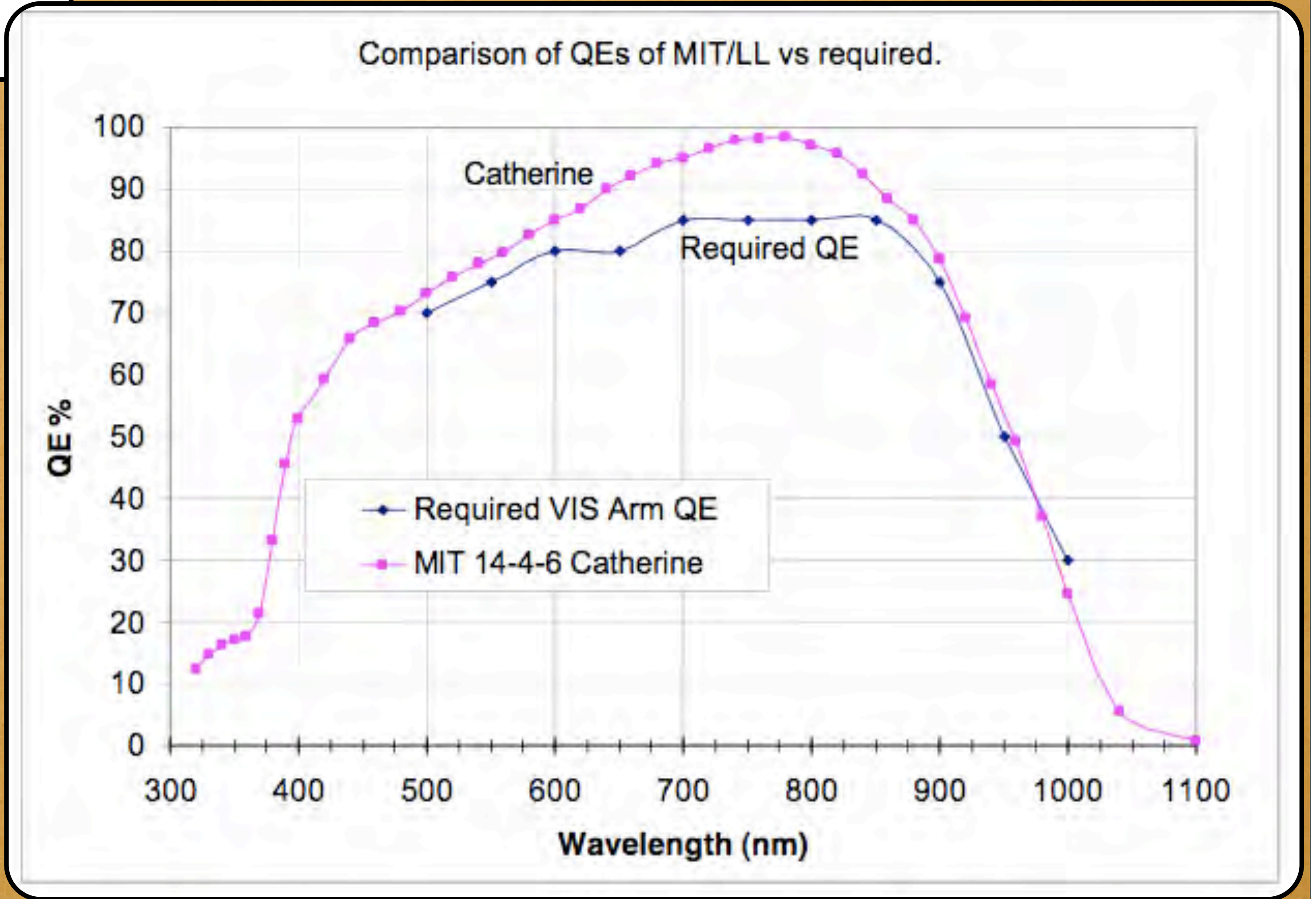
ESO's 4 Very large telescopes



Inner workings

Broadband filter





Efficiency detector

Characterization of the instrumental response

- An instrument is a measuring system/device, designed such that measurements or observations can yield answers to some scientifically compelling questions (driven by requirements from scientific community)

- Its reponse can be described by a multi-dimensional parameter space in which each parameter has an important impact on the quality of the measurement

- Each instrument has different system optimizations driven by the scientific goals. In general not all parameters can be optimized simultaneously due to various boundary conditions e.g. financial, size, mass of instrument, technology not available, etc.

Important system parameters are:

a) Bandwidth : interval in energy/frequency or wavelength over which the
($\varepsilon\varepsilon$, $\lambda\lambda$, $\nu\nu$) the instrument has a good detection efficiency → effective/sensitive area A_{eff} (E_γ)

b) Field of view : Solid angle subtended on the sky by telescope
(FOV) configuration. Wide (many sources simultaneously) / narrow (one/couple of sources) field imaging.

- Intermezzo: difference precision/resolution

a) Precision: accuracy with which the exact value of a quantity can be established

b) Resolution: capability of measuring the separation between two closely space features

The precision can be much larger than the resolution!

SRON

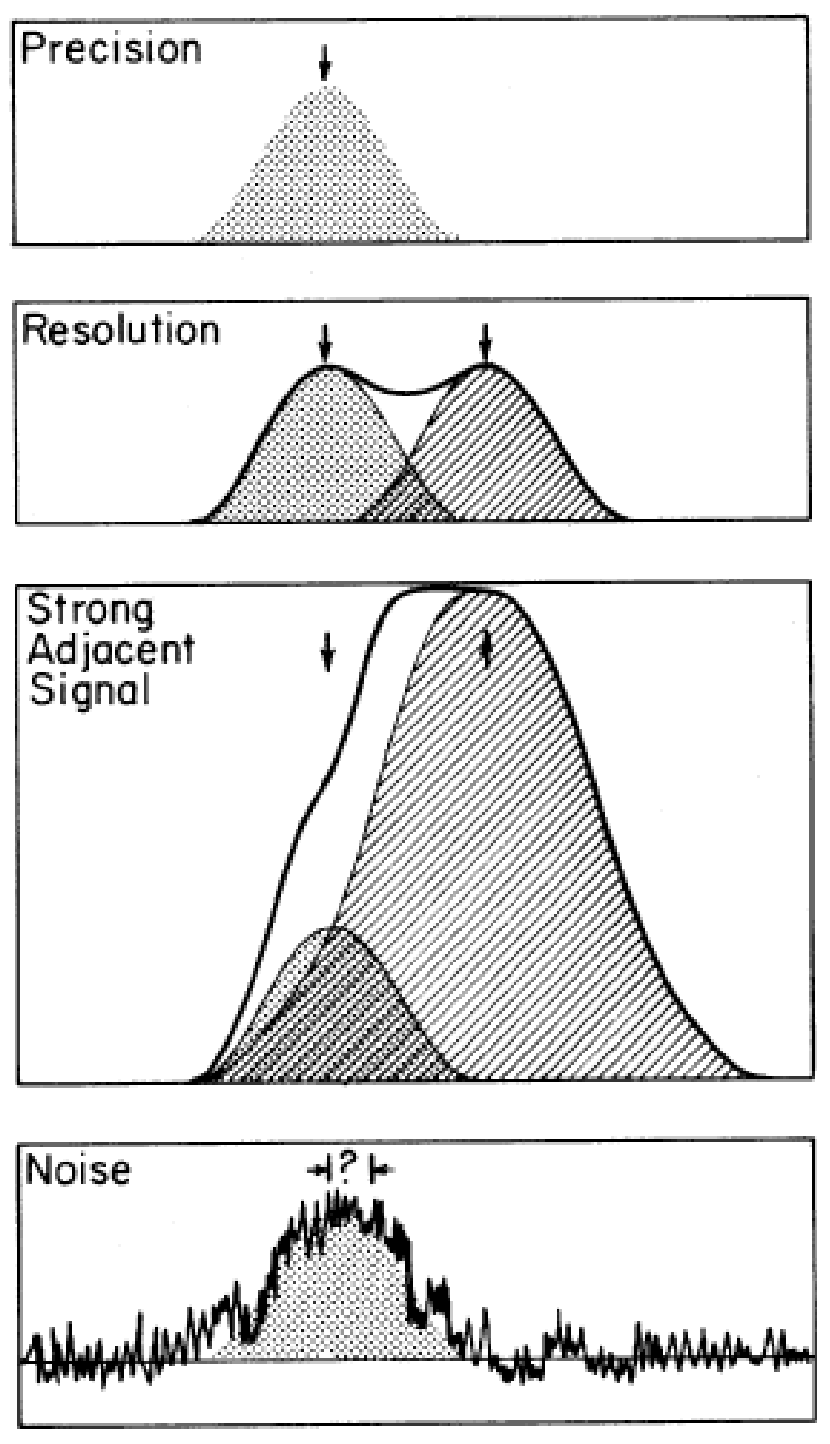


Figure 5.1: *Precision and resolution. The top panel shows that precision is the accuracy with which the centroid of a point (line) spread function can be determined. The panel below shows that the resolution is the interval which two signals of equal strength should be apart to be recognised separately. It is harder to resolve a weak signal adjacent to a strong one (panel below). The bottom panel shows the deterioration of precision in the presence of noise. Figure taken from Harwitt (1984).*

Within the bandwidth and FOV of the instrument the following parameters play an important role:

a) Angular resolution: minimum angular separation between
   ($\Delta\theta$) two equally bright point-sources

Resolving power $R_\theta = 1 / \Delta\theta$

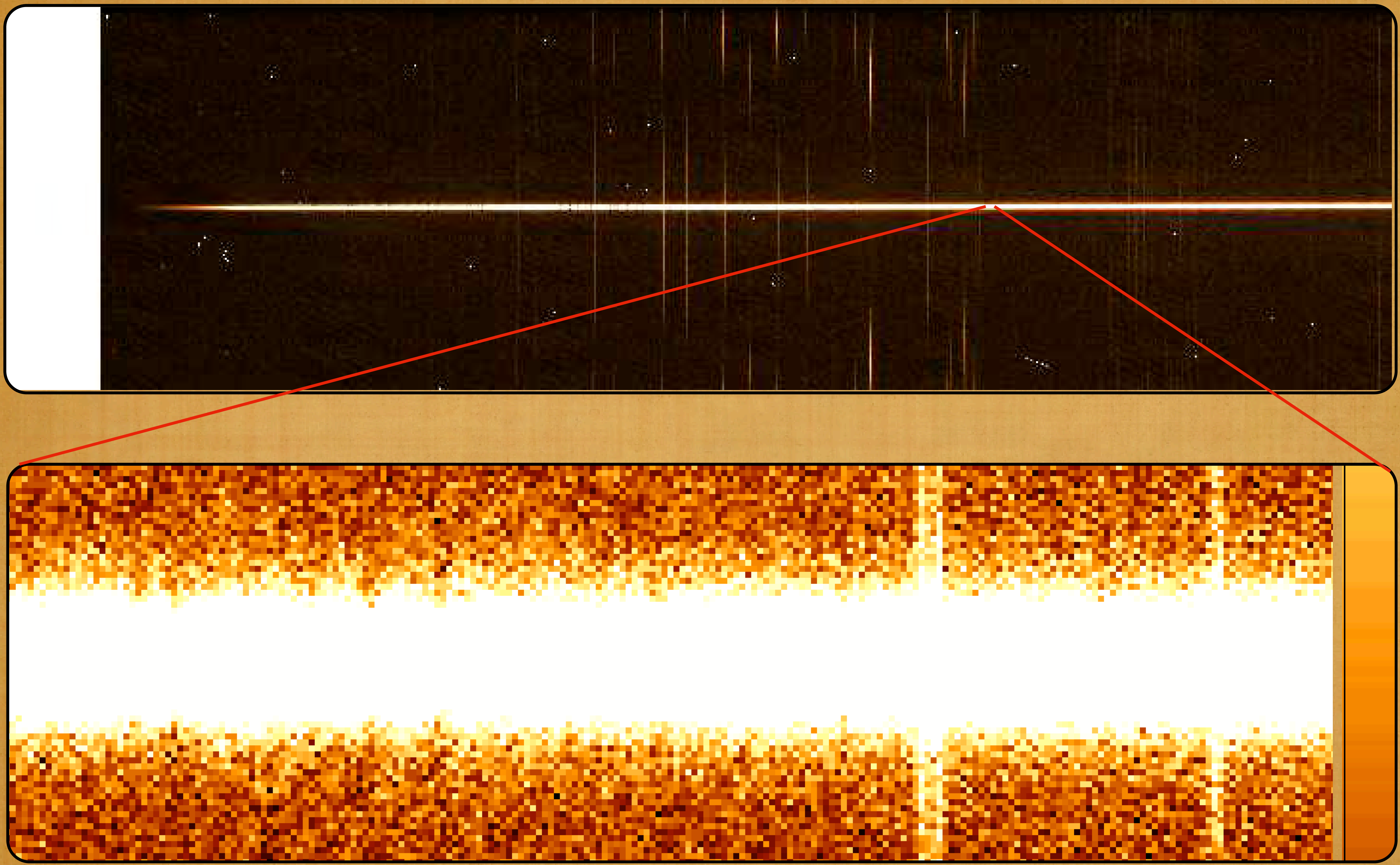
b) Spectral/energy resolution: minimum separation in (photon)
   ($\Delta\lambda, \Delta\varepsilon, \Delta\nu$) energy to resolve two equally strong spectral lines

Spectral resolving power $R_S = \varepsilon / \Delta\varepsilon = \nu / \Delta\nu = \lambda / \Delta\lambda$

c) Time resolution: Minimum time interval between to consecutive
   (ΔT) uncorrelated/independent events (processes)

SRON

Data is discretely sampled



Optical spectrum recorded with a CCD camera

cf. Horne and Harris papers for presentation

- Data is filtered before detection
- Data is filtered during detection
- Data is filtered/processed after detection
- Data is discretely sampled
- Data is stochastic in nature

Stochastic

Wikipedia: "A stochastic process is one whose behaviour is non-deterministic in that a system's subsequent state is determined both by predictable actions and by a random element

A random phenomenon: the outcome is not predictable in a deterministic sense, but it has a smooth distribution of outcomes if the experiment is repeated many times

In the limit of infinite measurements this distribution tends to the parent distribution

These phenomena are addressed by statistics: " A mathematical science pertaining to the collection, analysis, interpretation or explanation and presentation of data

$x(\zeta)$ describes the relation between the possible outcomes ζ and the random variable x

E.g.

die throwing: ζ_1 outcome is face 1 of die

$$x(\zeta_1)$$

is for instance the gain in a game of dice

$$x(\zeta_1) = 0€$$
$$x(\zeta_2) = x(\zeta_3) = 10€$$
$$x(\zeta_4) = x(\zeta_5) = 100€$$
$$x(\zeta_6) = 1000€$$

example from book of Lena, appendix B

Another example: the number of ADUs* measured by a CCD camera behind 5 telescopes for a source of magnitude $M_V=15$

$$x(\zeta_1) = 1001.3$$

$$x(\zeta_2) = 1045.3$$

$$x(\zeta_3) = 1099.1334$$

$$x(\zeta_4) = 953.2$$

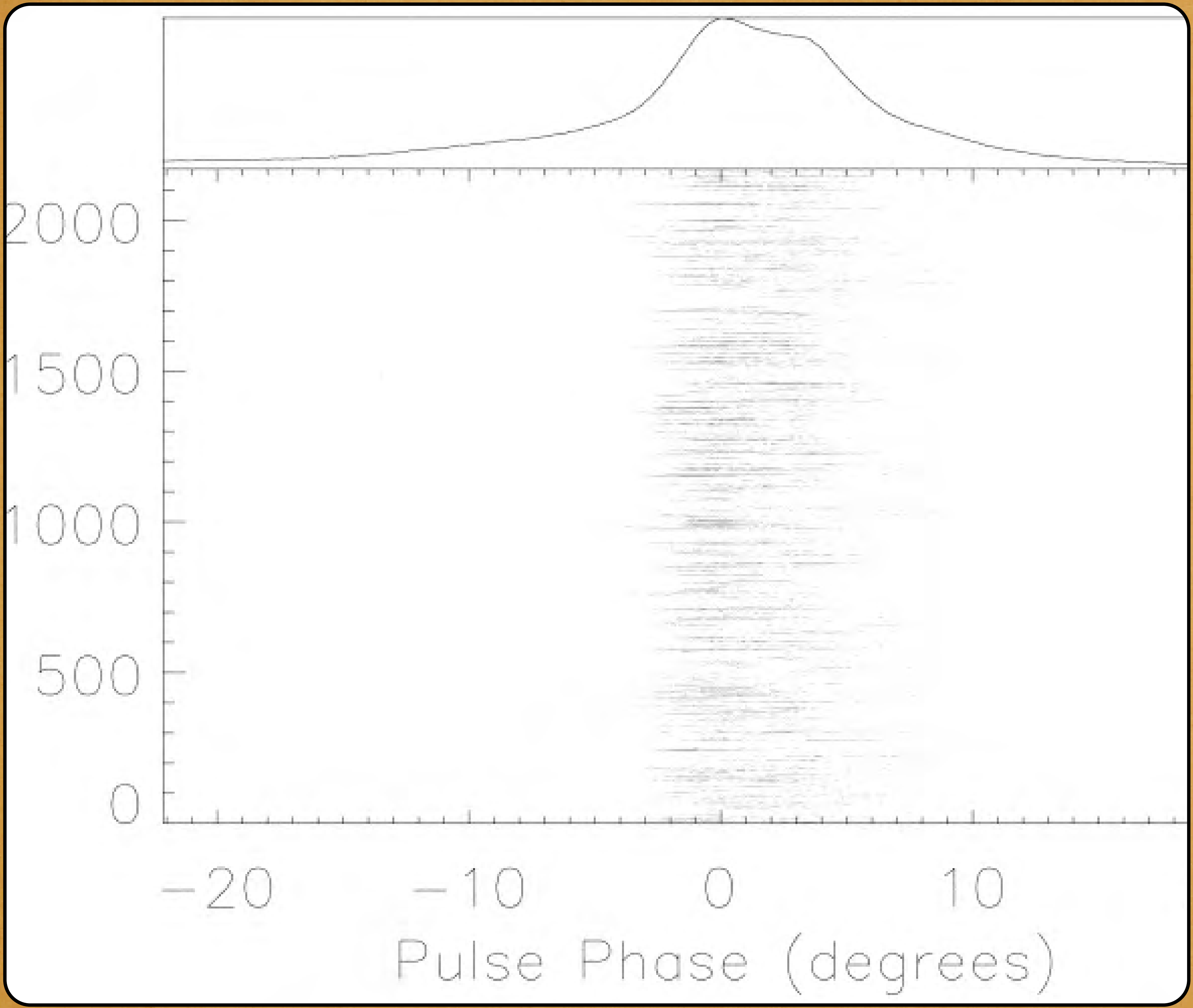
$$x(\zeta_5) = 988.55$$

*what are these?

Mazur (prof at Harvard): Peer Instruction

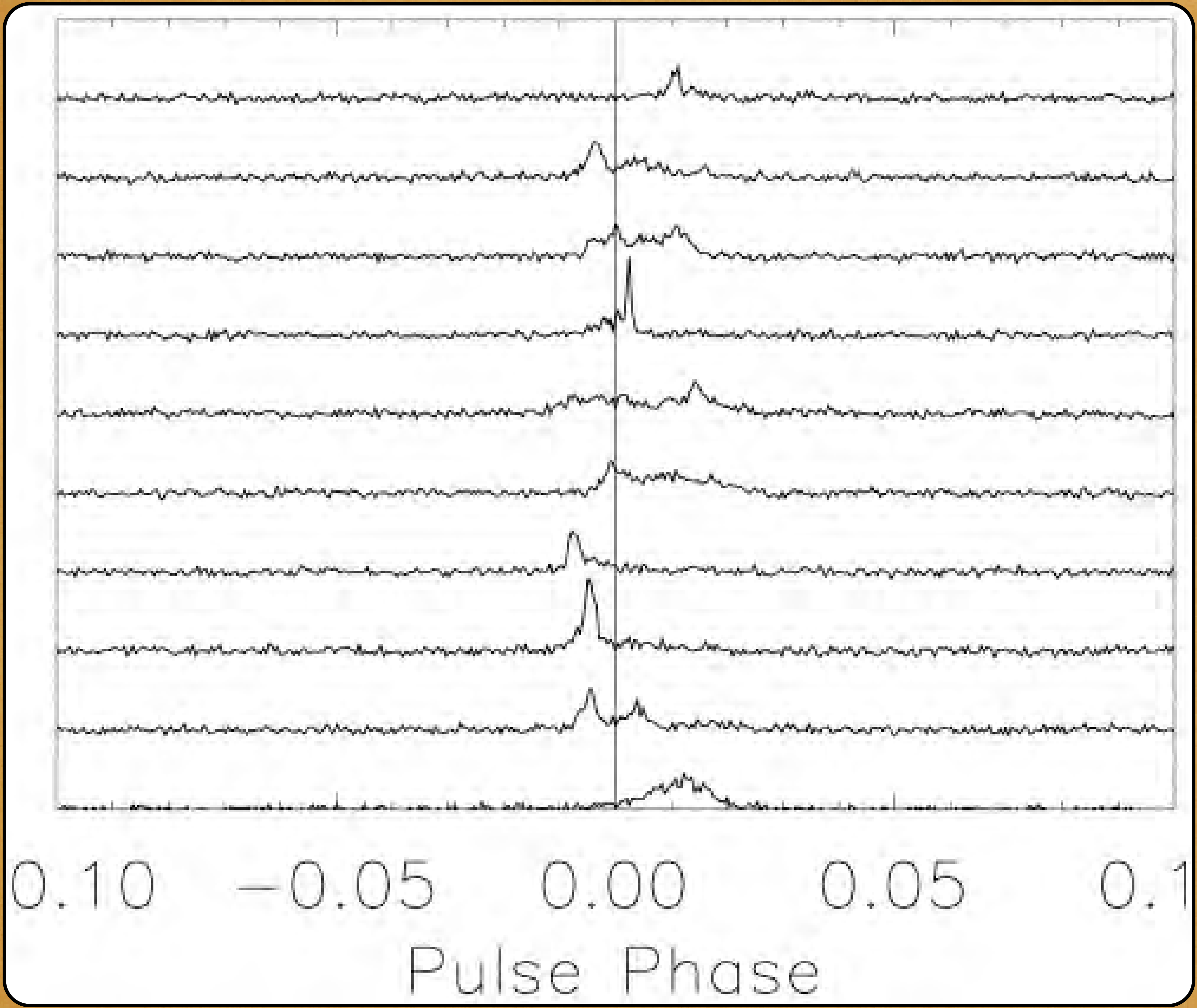
Provide to your neighbor one example of a stochastic process in astronomy

Average pulse profile & individual pulses



PSR J0437-4715 Janet et al. 1998

Average pulse profile & individual pulses



PSR J0437-4715 Janet et al. 1998

Cumulative distribution function

$$F(x) = P\{x \leq y\}$$

The probability that a set of outcomes of the r.v. has a value $\leq$ y

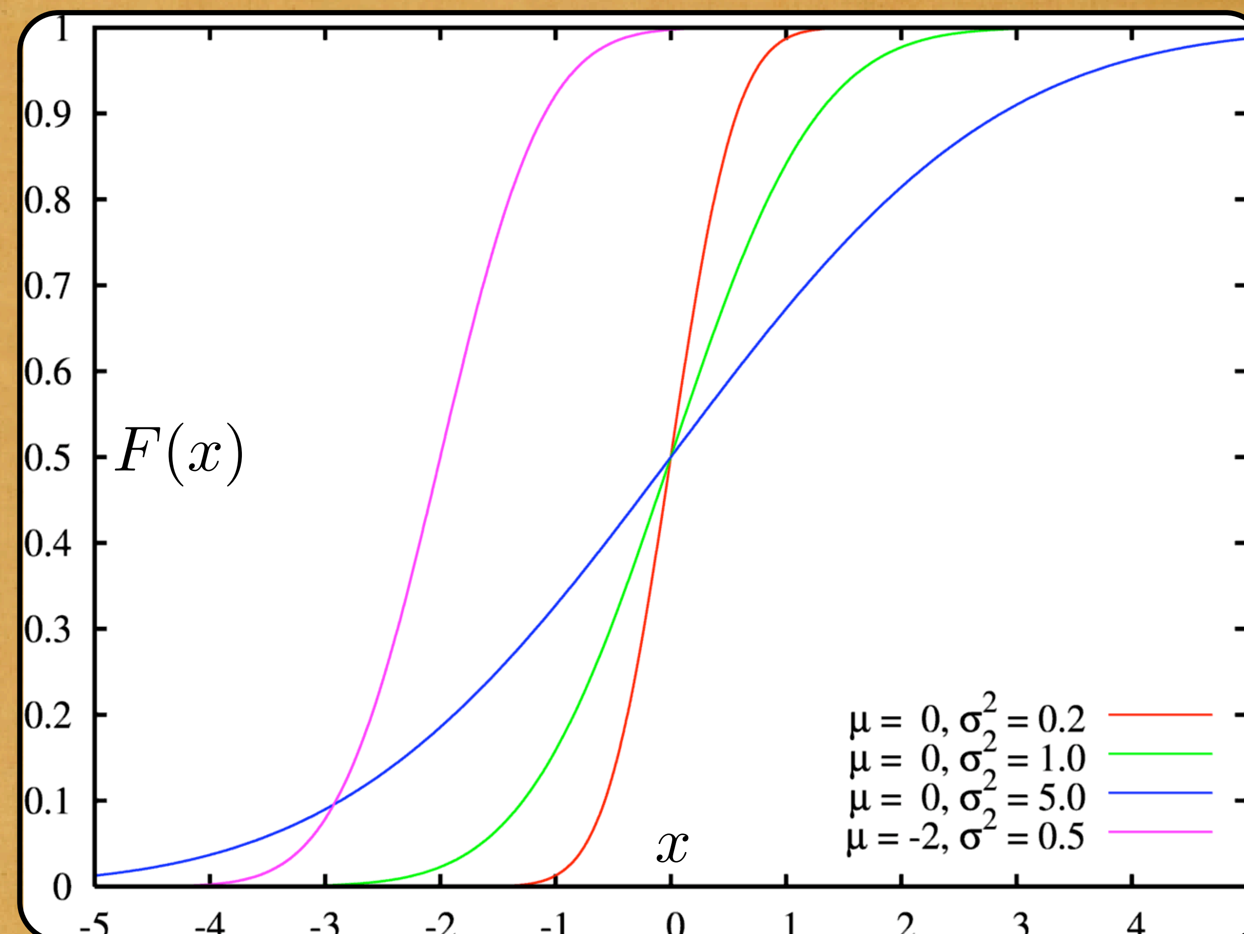
$$F(-\infty) = 0$$

$$F(\infty) = 1$$

Gaussian Cumulative Distribution Function

$$F(x, \eta, \sigma) = 0.5 + erf\frac{x-\eta}{\sigma^2}$$

(see chapter 6 Num Res for special functions such as erf)



Probability density function

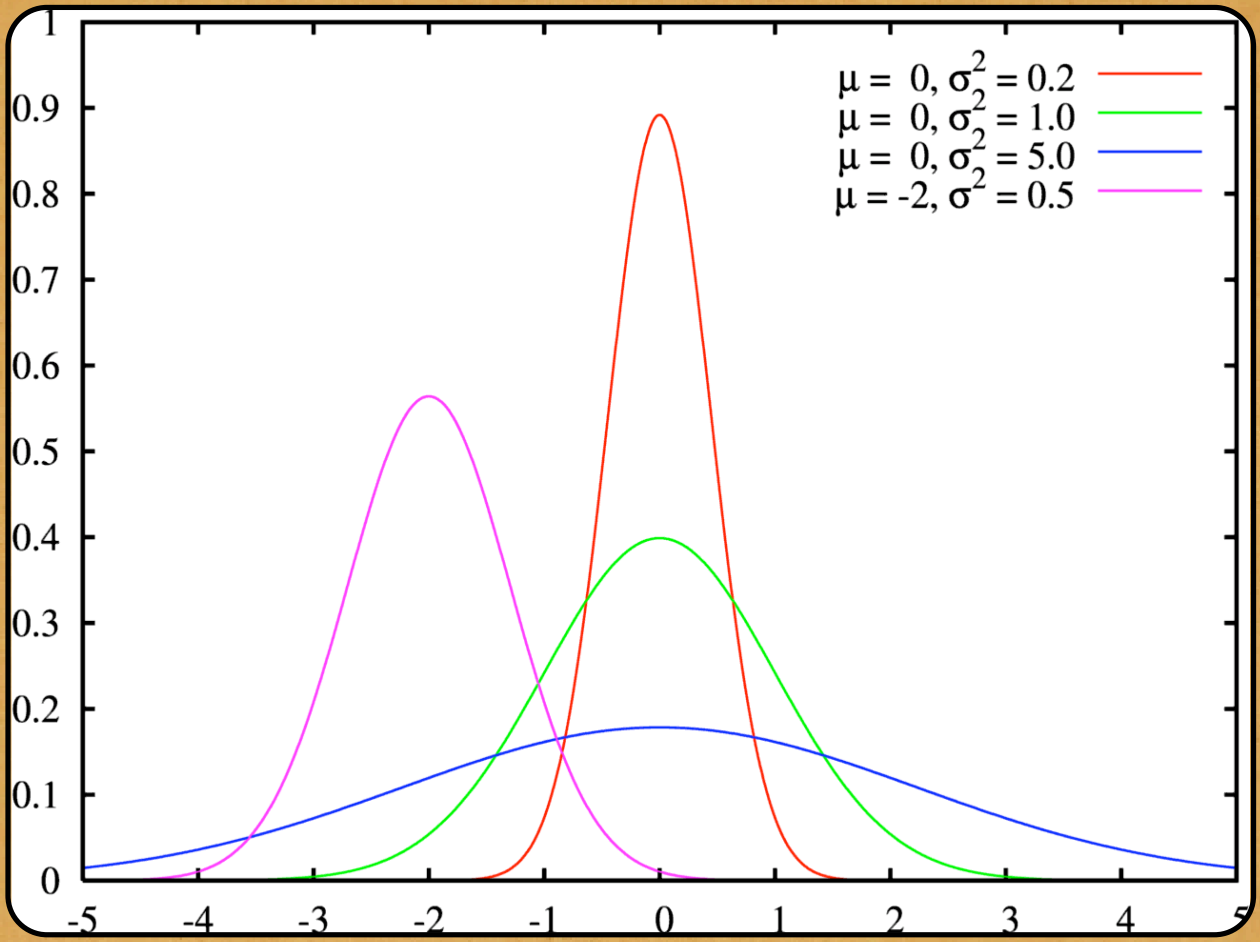
$$\frac{dF(x)}{dx} = f(x)$$

$\hookrightarrow$ Gauss, Poisson, χ^2 , binomial

Aka normal distribution

A Gaussian Distribution

Gaussian or Normal Probability Density Distribution



$$f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} exp(-\frac{1}{2}\frac{(x-\eta)^2}{\sigma^2})$$

Two parameters completely describe the distribution

The mean η and variance σ^2

$$\eta = \lim_{N \to \infty} \left(\frac{1}{N} \sum_i x_i \right)$$

$$\sigma^2 \equiv \lim_{N \to \infty} \left(\frac{1}{N} \sum_i (x_i - \eta)^2 \right)$$

Average of squares of deviations from the mean

Median value $\eta_{\frac{1}{2}}$: $P(x_i > \eta_{\frac{1}{2}}) \equiv P(x_i < \eta_{\frac{1}{2}}) \equiv 0.5$

For symmetric distributions $\eta = \eta_{\frac{1}{2}} = \eta_{\max}$

Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

λ expectation or expected value

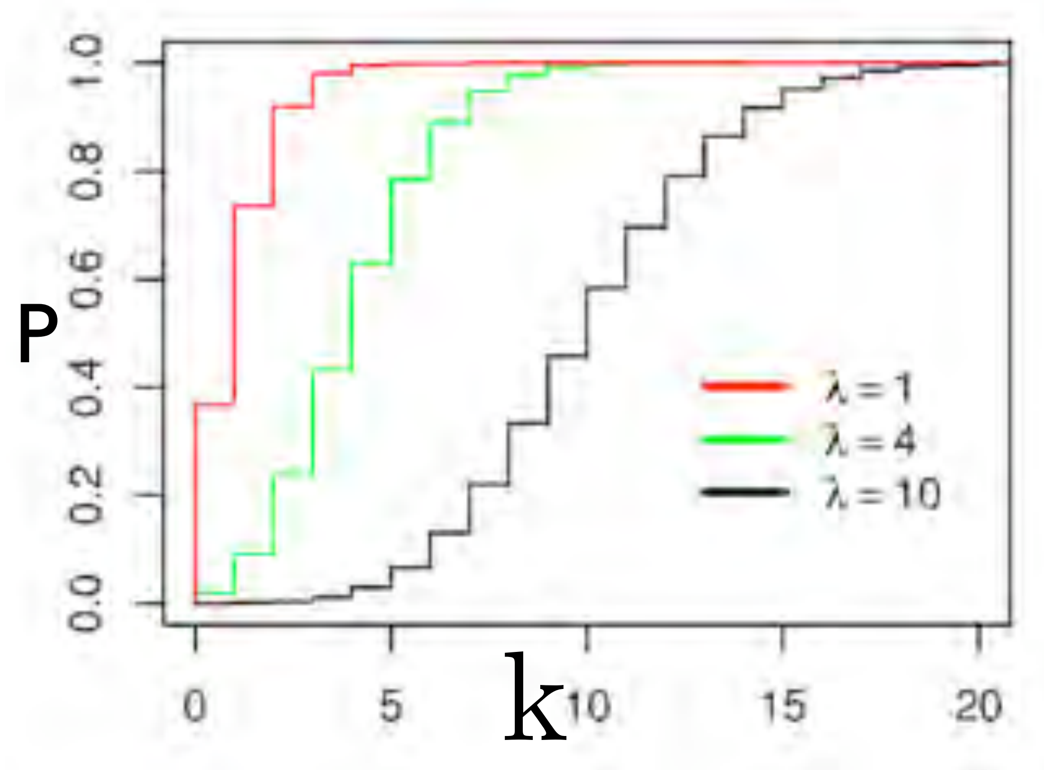
k observed number of events (integer: fie not defined elsewhere!)

$f(k;\lambda)$ probability of observing k number of events when expectation value is λ

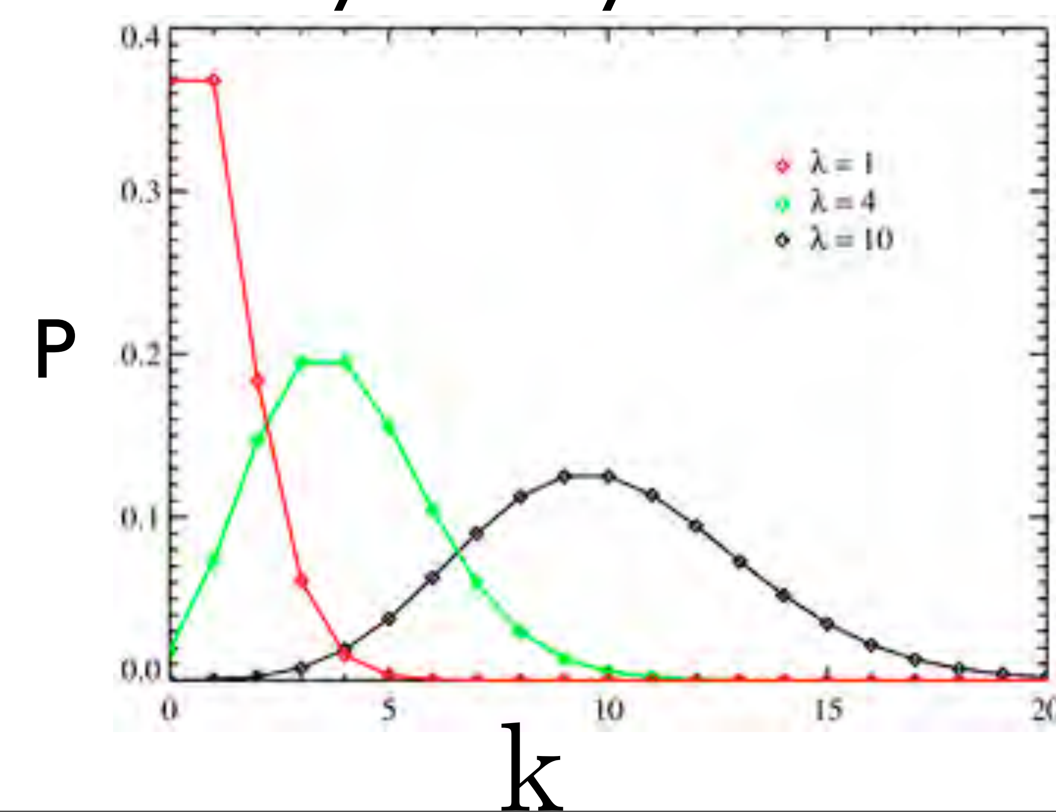
Mean= λ

Variance= λ

Cumulative distribution



Probability density distribution



Expectation values

$$E\{\phi(x)\} = \int_{-\infty}^{\infty} \phi(x) f(x) dx$$

discrete version $$E\{\phi(x)\} = \sum_{n=-\infty}^{\infty} \phi(x_n) P_n$$

Moments of a distribution

moment $\mu'_k = E\{(x)^k\}$

central moment $\mu_k = E\{(x - E\{x\})^k\}$

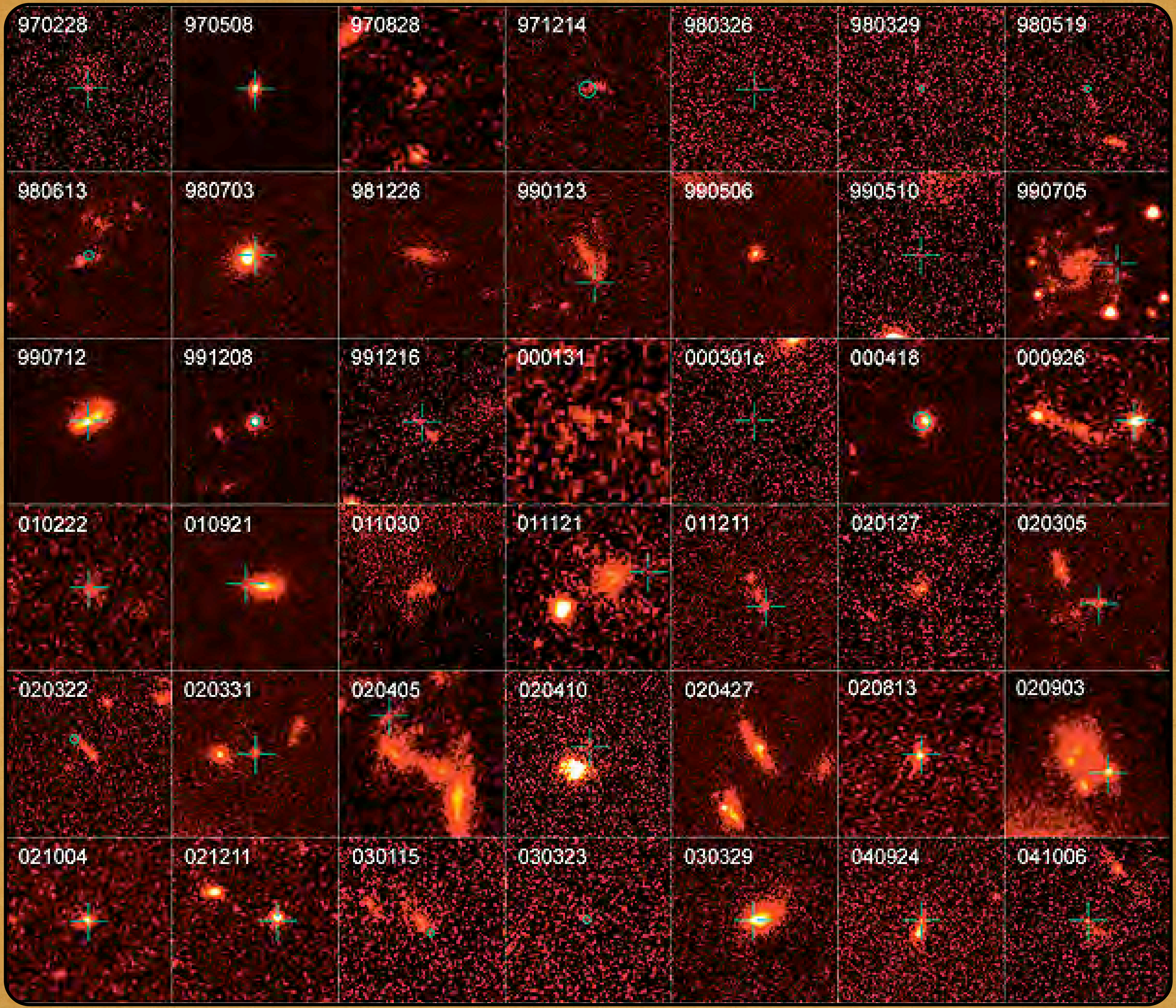
Mean $\eta = E\{x\} = \int_{-\infty}^{\infty} x f(x) dx$

Variance = central moment of 2nd order

$$\mu_2 = E\{(x-\eta)^2\} = \int_{-\infty}^{\infty} (x-\eta)^2 f(x) dx \equiv \sigma^2$$

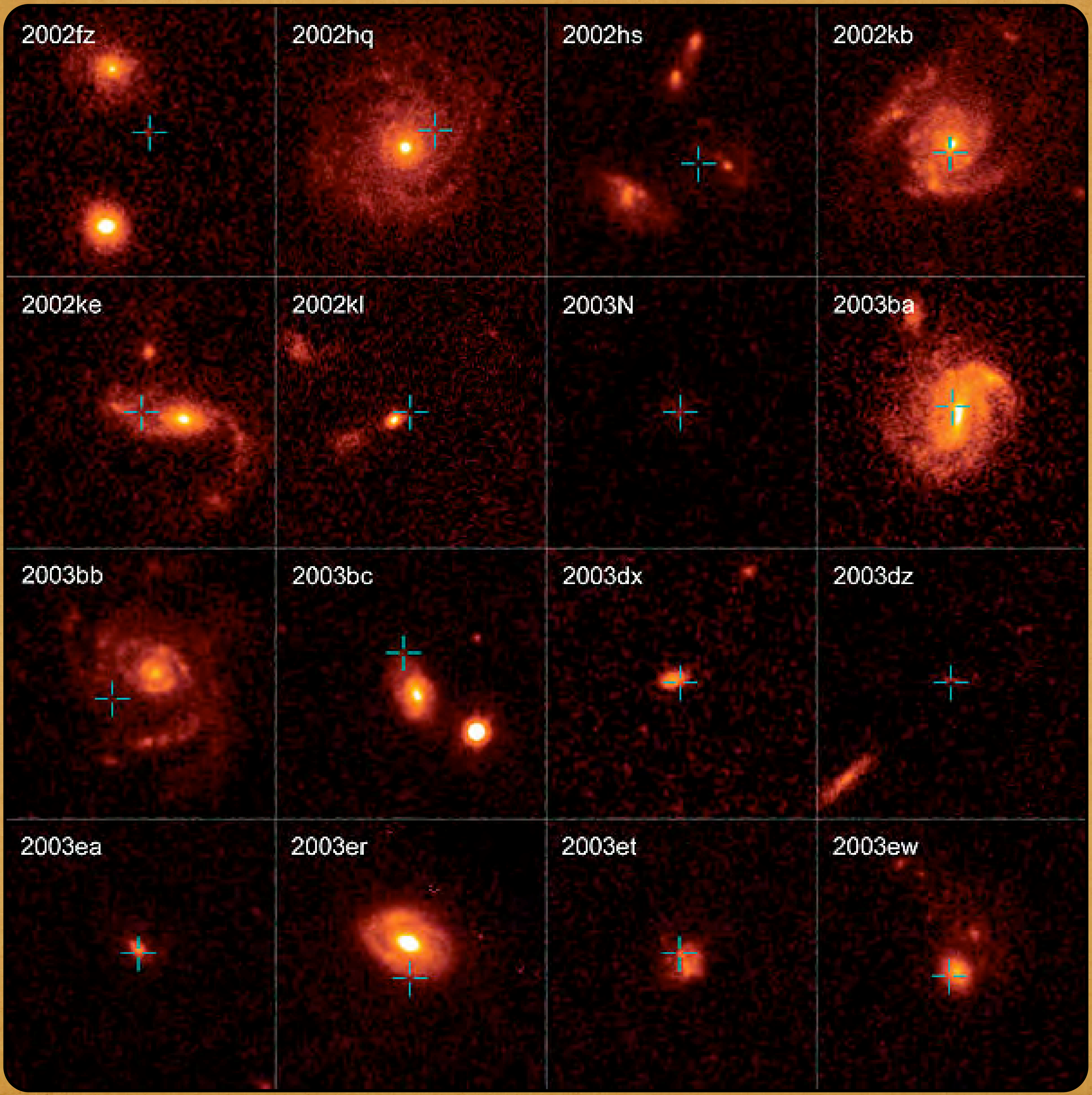
$$\sigma^2 = E\{x^2\} - \eta^2 = E\{x^2\} - (E\{x\})^2$$

Example of use of moments: GRB distribution

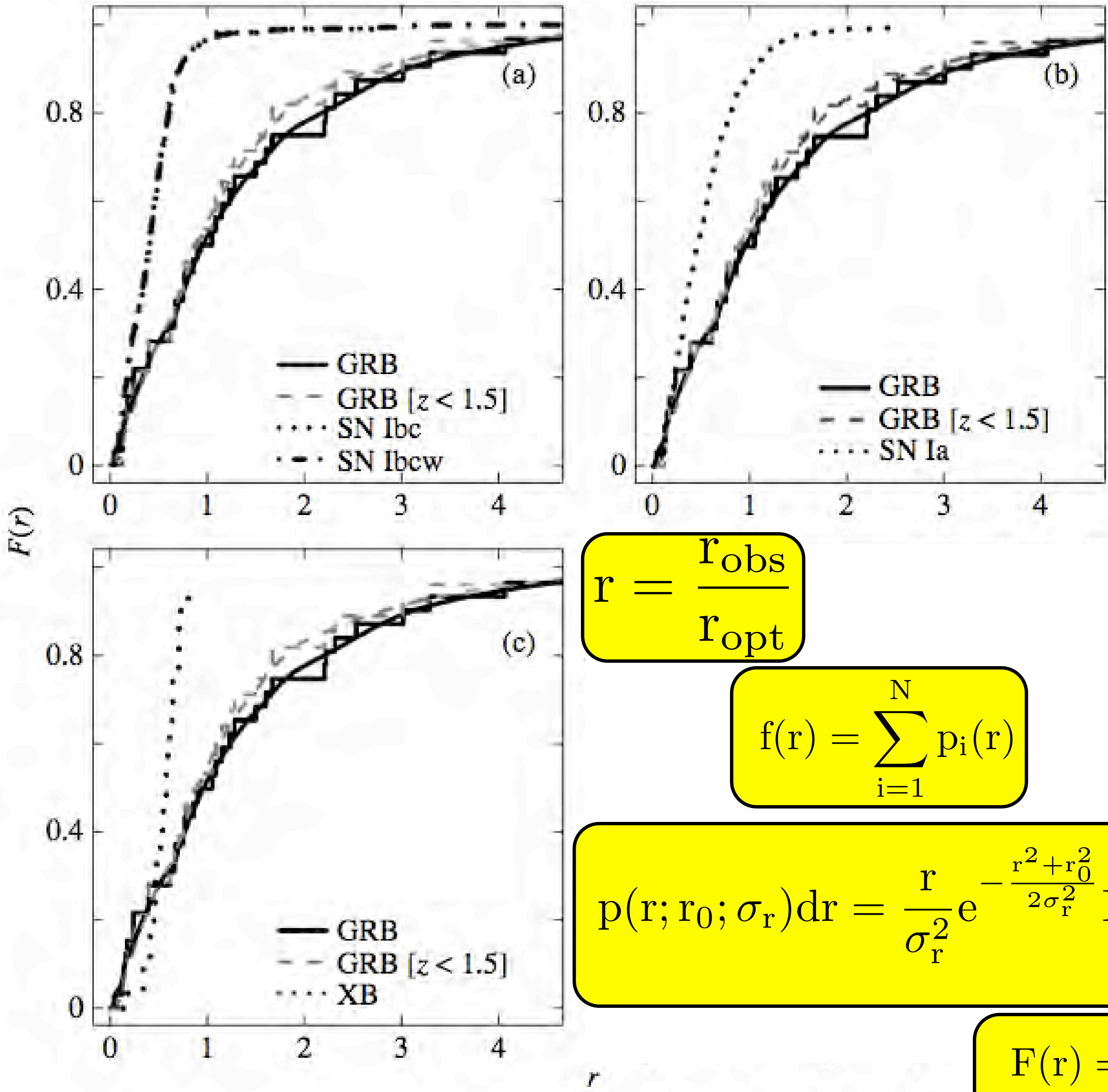


Bloom et al. 2002, Blinnikov et al. 2004, Fruchter et al. 2006

Core-collapse SN distribution



CUMULATIVE DISTRIBUTIONS BLINNIKOV ET AL.2004



$$r = \frac{r_{obs}}{r_{opt}}$$

$$f(r) = \sum_{i=1}^{N} p_i(r)$$

$$p(r; r_0; \sigma_r)dr = \frac{r}{\sigma_r^2} e^{-\frac{r^2+r_0^2}{2\sigma_r^2}} I_o(\frac{r\, r_0}{\sigma_r^2})dr$$

$$F(r) = \int_0^r f(r)dr$$

Less often used moments are the

skewness

how asymmetric is the distribution?

$$Skew(x_1....X_N) = \frac{1}{N}\sum_{j=1}^{N}\left[\frac{x_j - \bar{x}}{\sigma}\right]^3$$

&

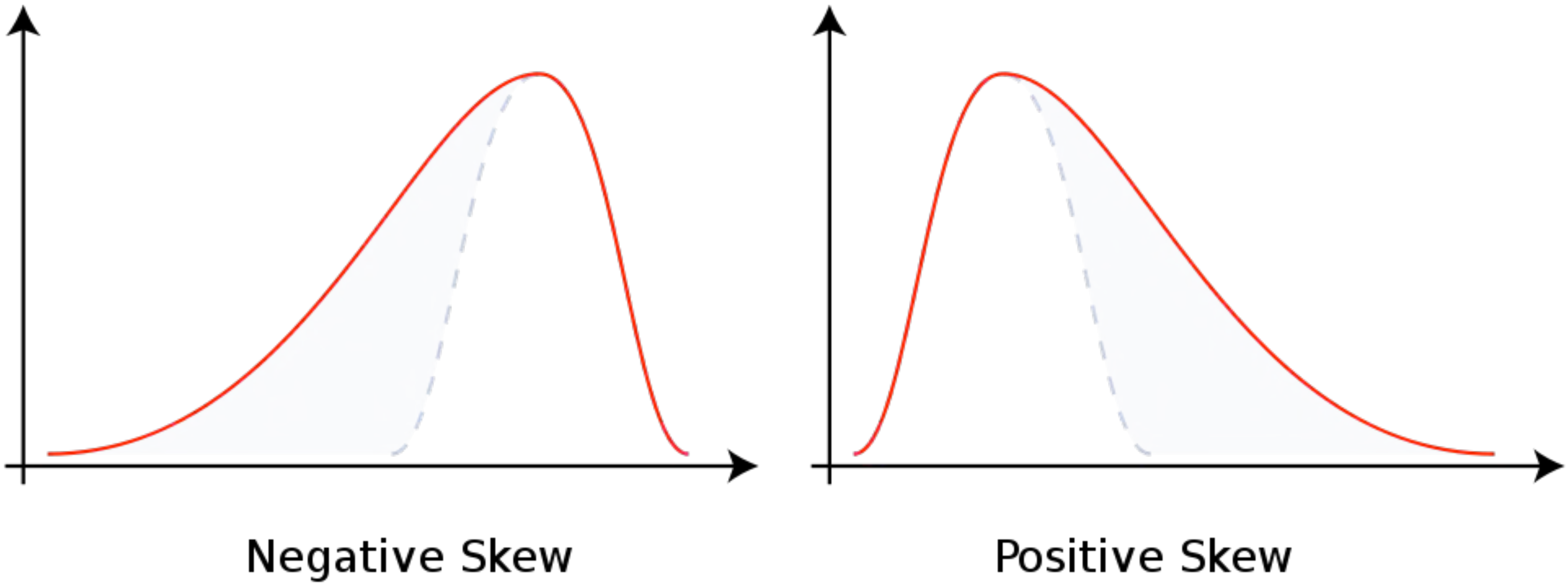
Kurtosis

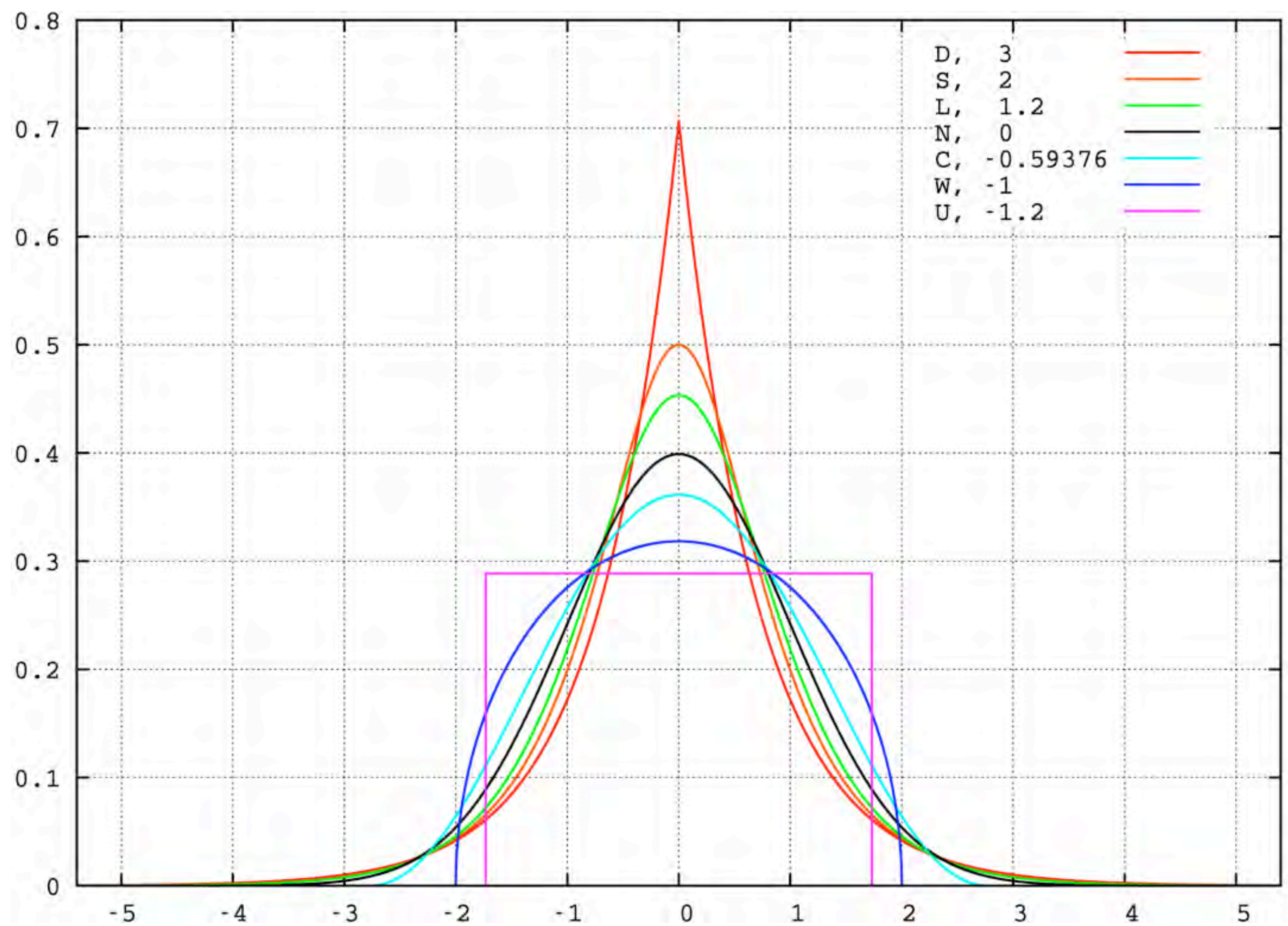
how peaked is the distribution?

$$Kurt(x_1....X_N) = \left\{\frac{1}{N}\sum_{j=1}^{N}\left[\frac{x_j - \bar{x}}{\sigma}\right]^4 - 3\right\}$$

Both measured wrt a normal=Gaussian distribution

Skewness:





Kurtosis of Gaussian distribution =0

Examples from Wikipedia

Correlation, auto-correlation

Correlation

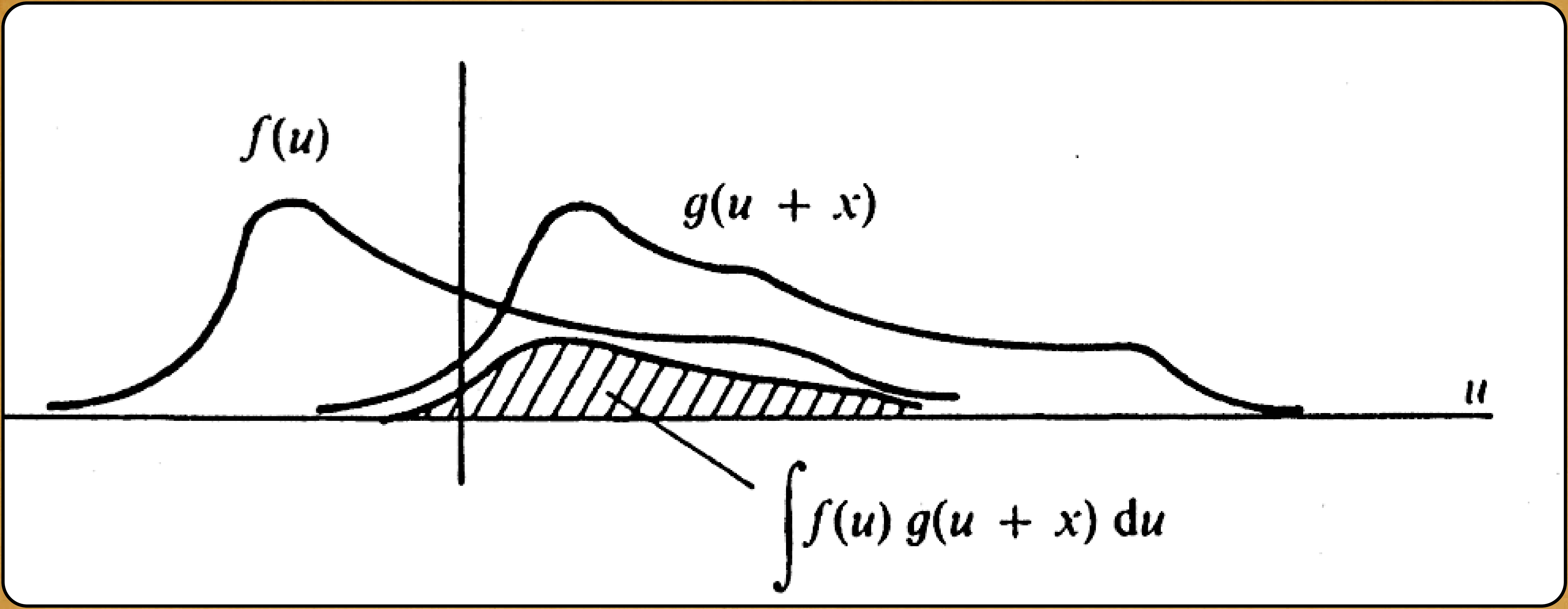
$$k(x) = f(x) \otimes g(x) \qquad k(x) = \int_{-\infty}^{\infty} f(u) g(u+x) du$$

If X and Y are two independent random variables with probability distributions f and g, respectively, then the probability distribution of the difference Y - X is given by the cross-correlation f⊗g.

The convolution f * g gives the probability distribution of the sum X + Y

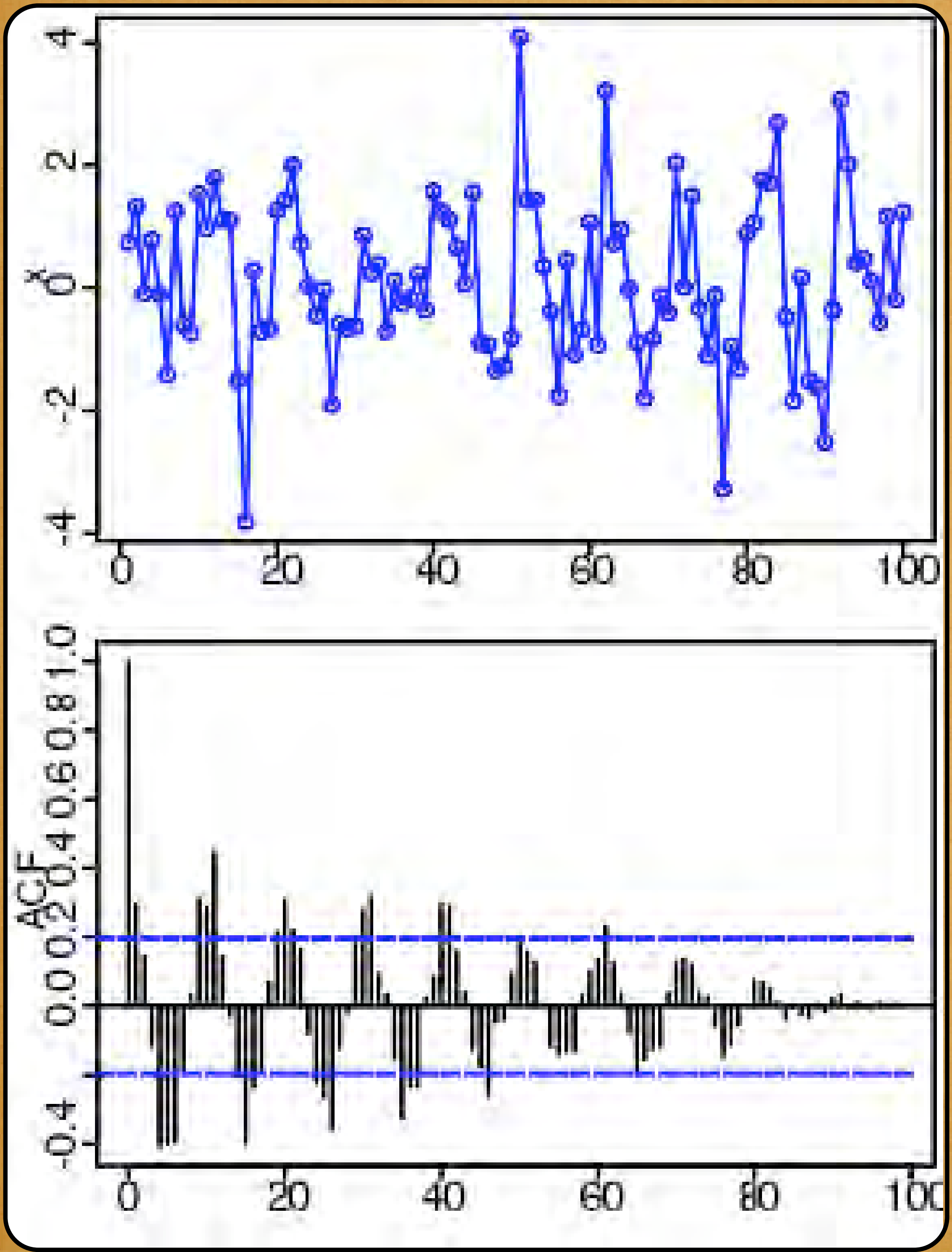
Correlation, auto-correlation

Correlation



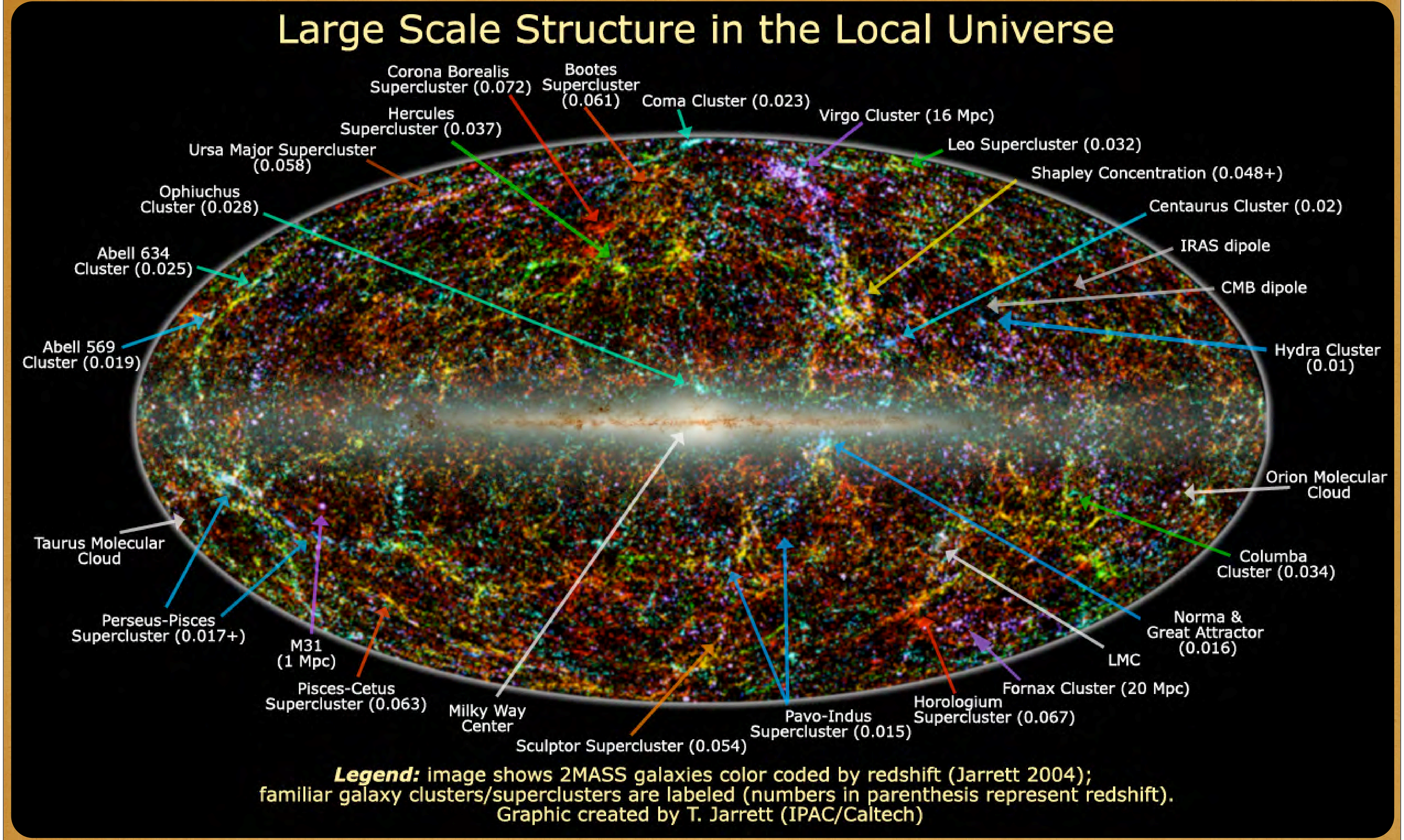
Auto-correlation

$$R(x) = f(x) \otimes f(x) = \int_{-\infty}^{\infty} f(u) f(u+x) du$$



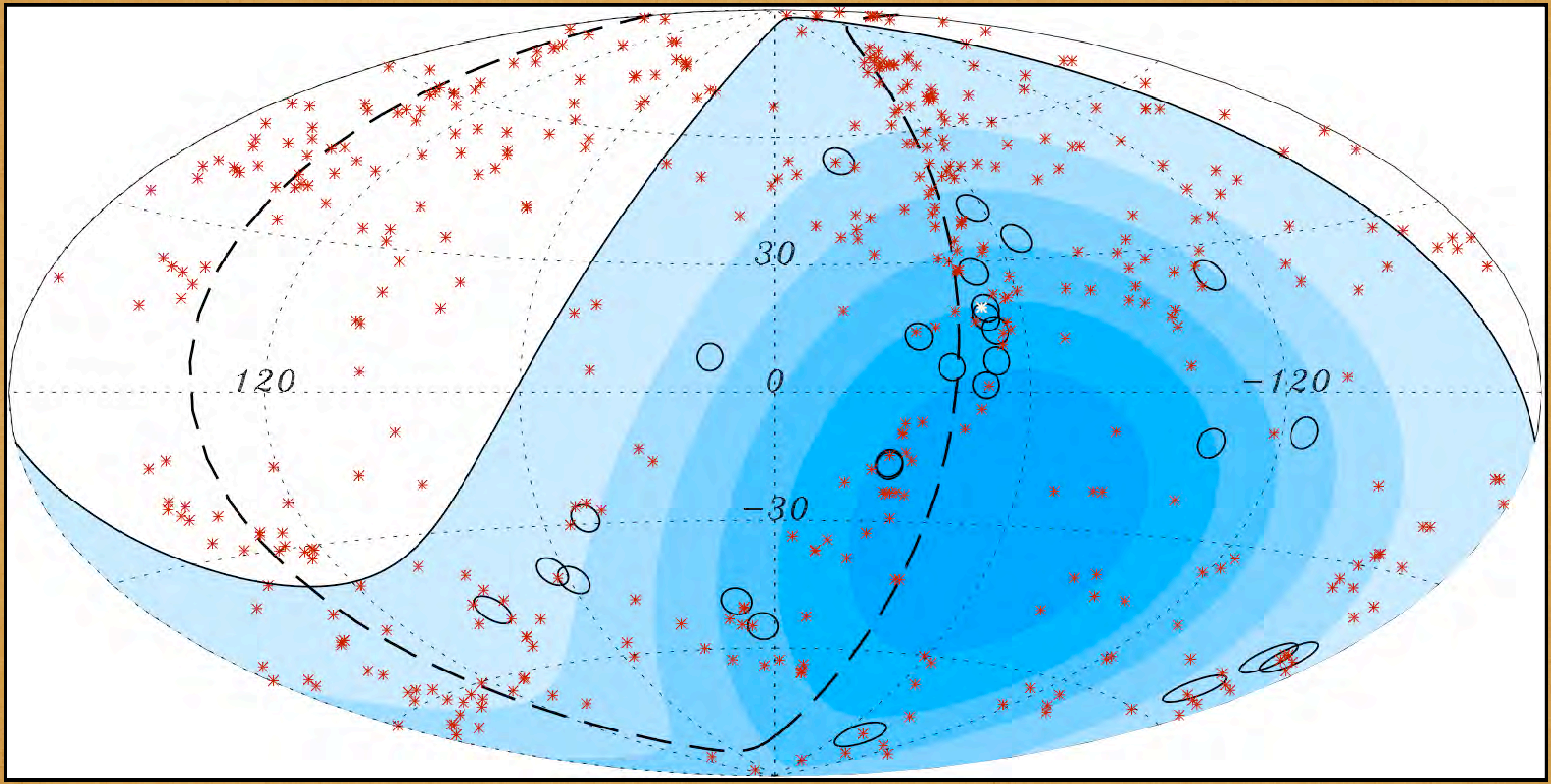
$$R(x) = E\{\underbrace{f(x)}_{f(x_1)}\underbrace{f(x+t)}_{f(x_2)}\}$$

Example from Wikipedia



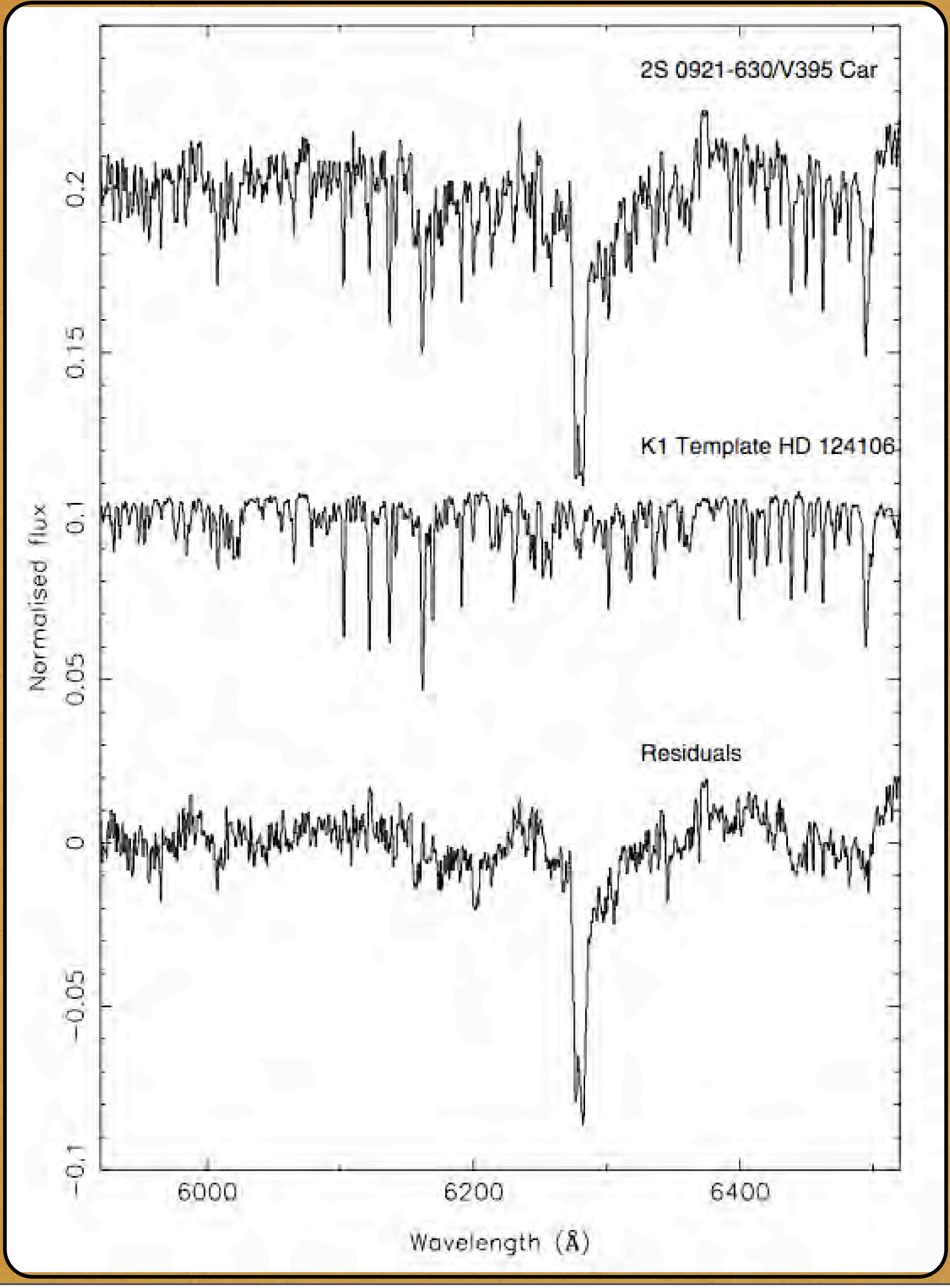
GIVEN A RANDOM GALAXY IN A LOCATION THE CORRELATION FUNCTION DESCRIBES THE PROBABILITY THAT ANOTHER GALAXY WILL BE FOUND WITHIN A GIVEN DISTANCE (PEEBLES 1980)

Cross-correlating cosmic-ray events with the position of nearby AGN

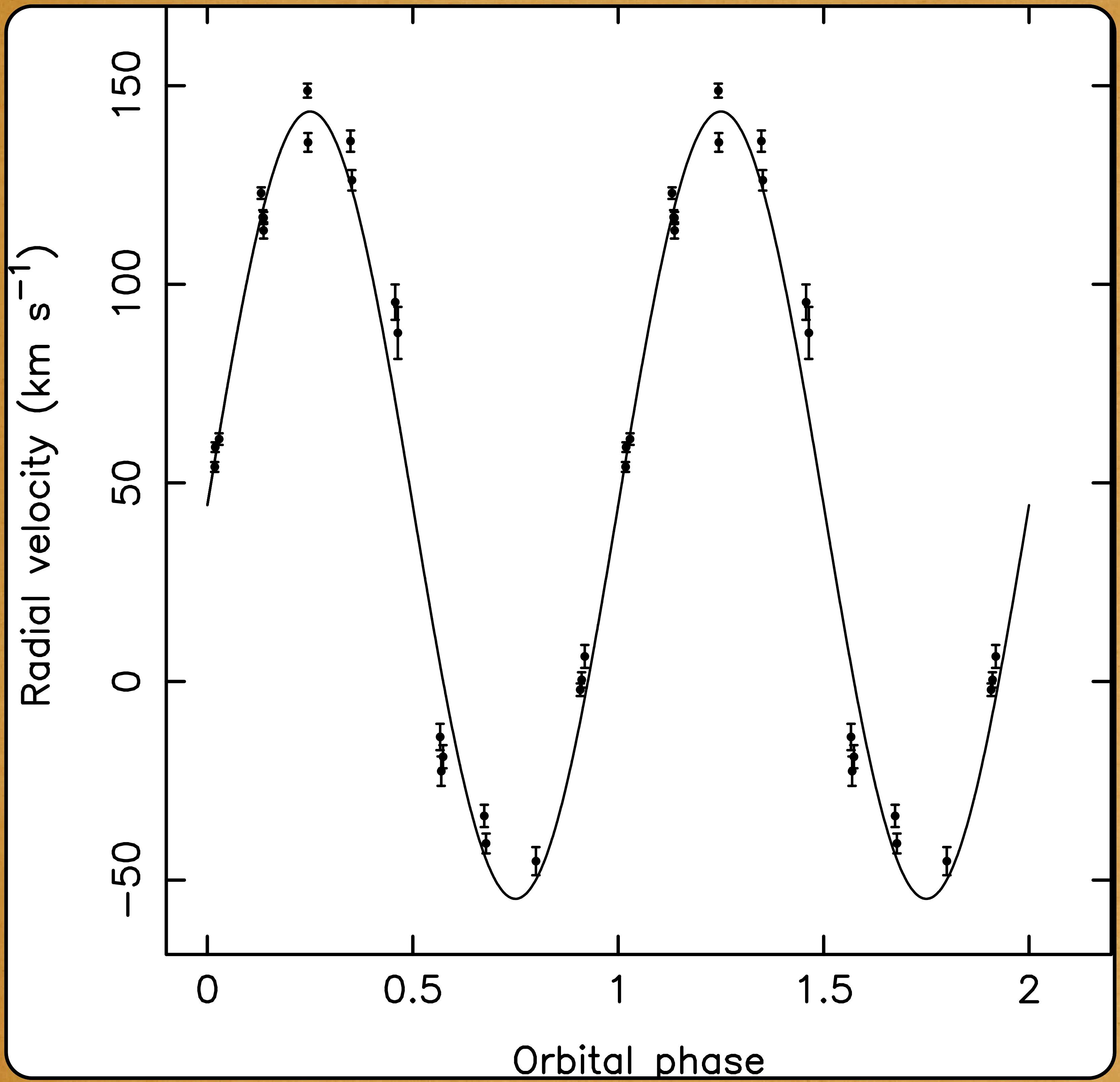


Data from the Pierre Auger collaboration
see astro-ph 0711.2256

Cross-correlating spectra to determine velocities



Cross-correlating spectra to determine velocities and from that a mass-function



More on the auto-correlation

$$R(x) = E\{\underbrace{f(x)}_{f(x_1)}\underbrace{f(x+t)}_{f(x_2)}\}$$

if $x_1 = x_2$

$$R(x) = R(x,x) = \mathbf{E}\{f^2(x)\} = \mathbf{E}\{|f(x)|^2\}$$

Average generally not zero

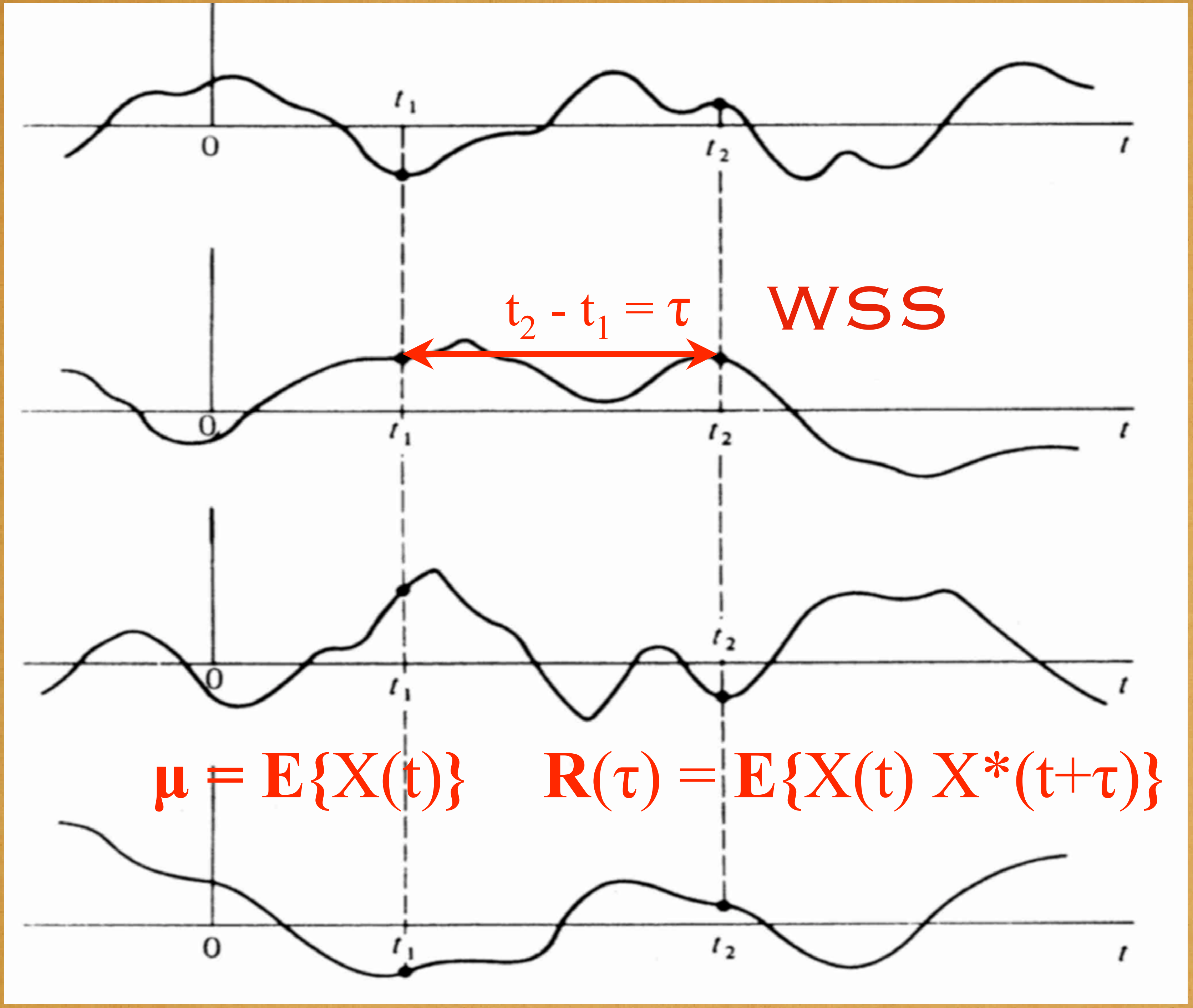
→ autocovariance

$$C(x_1, x_2) = \mathbf{E}\{(f(x_1) - \eta(x_1))(f(x_2) - \eta(x_2))^*\}$$

$$C(x) = R(x) - |\eta(t)|^2 = \sigma^2(x)$$

C(x) → average power in the fluctuations around the mean

Wide-sense stationary S.P.



WSS: mean time independent
& autocorrelation depends on
time difference

Not all signals are WSS:

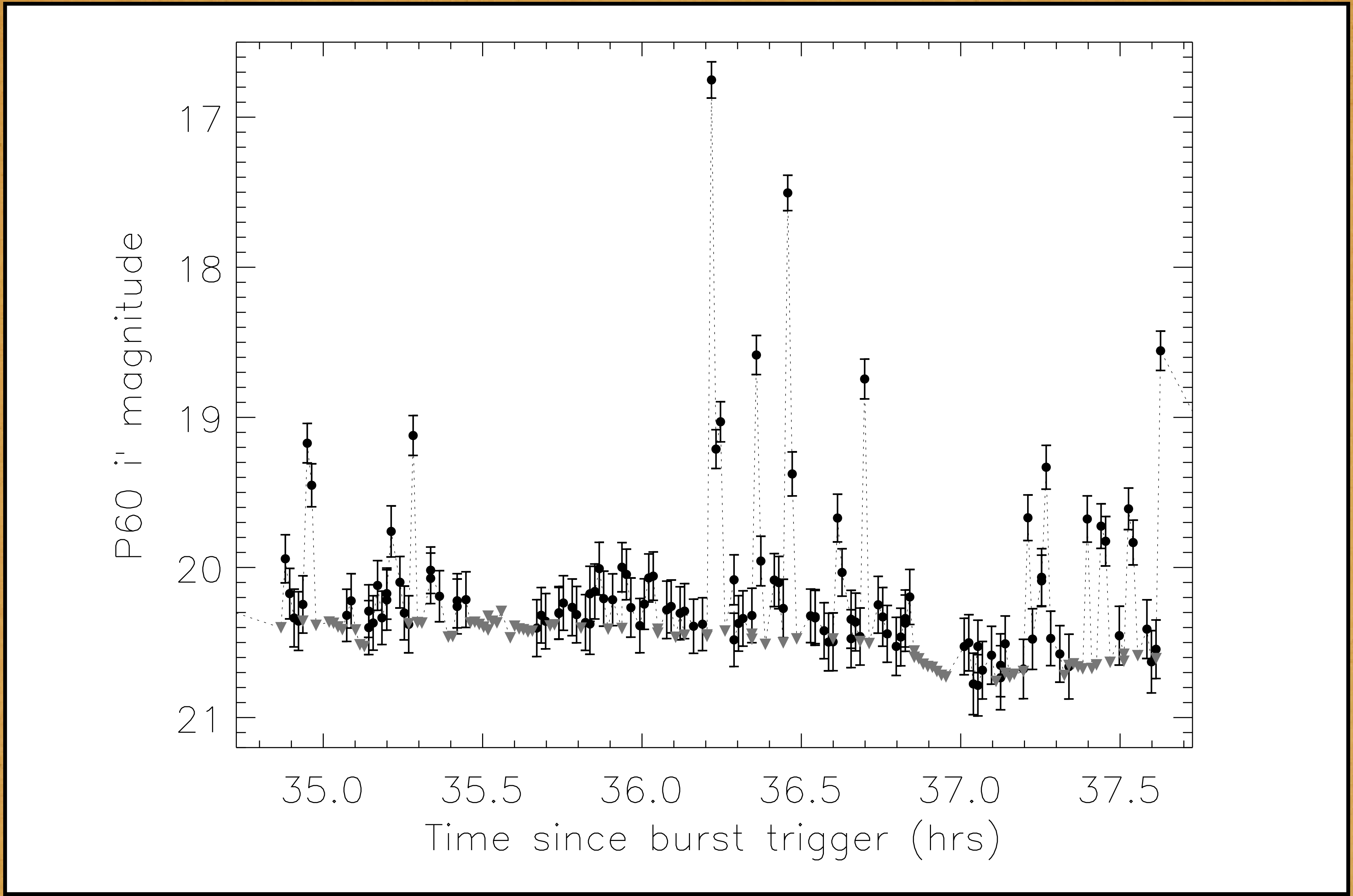
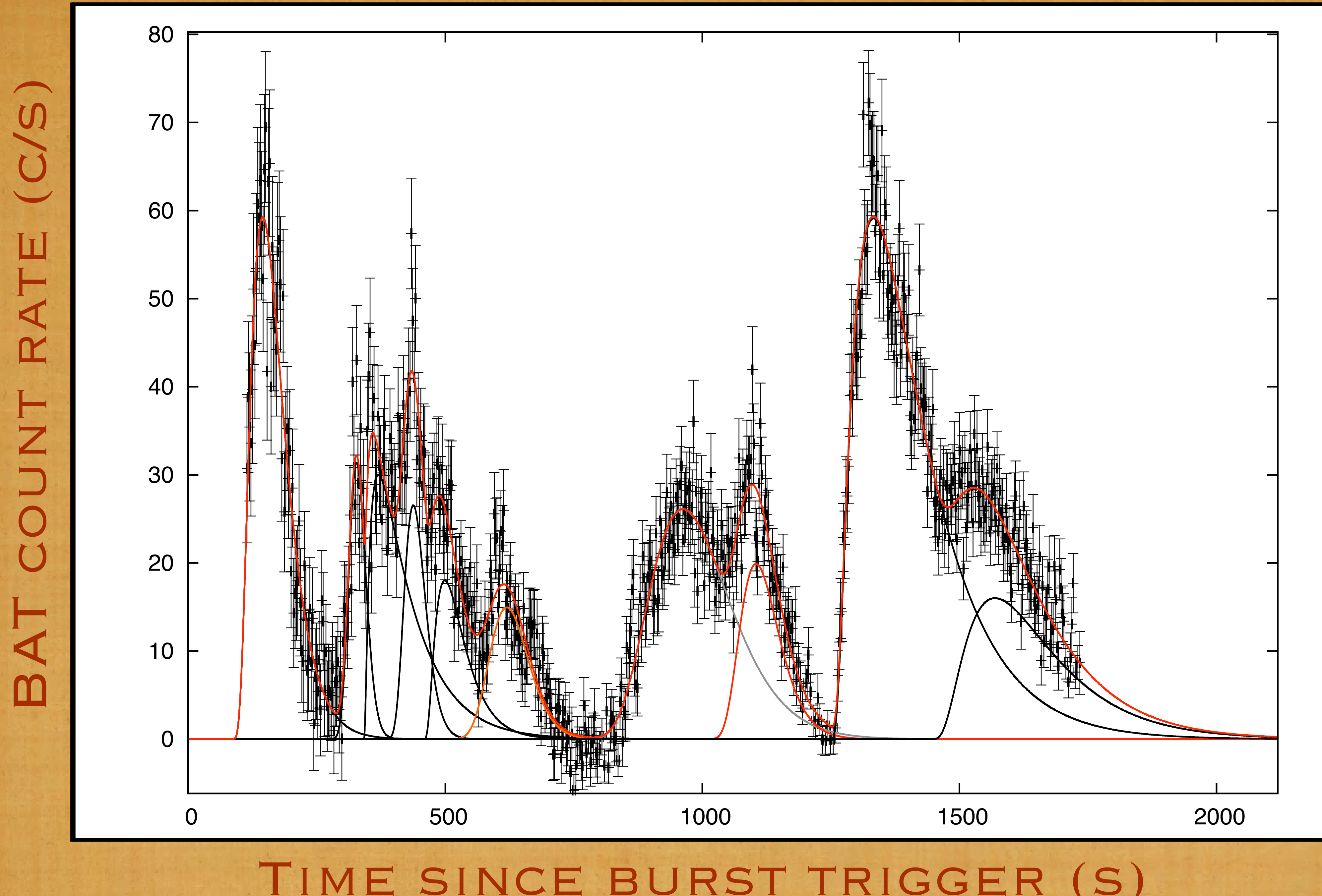


Figure from Kasliwal et al. 2007

Nature of the source uncertain:
GRB, SGR, BH-X-ray binary?

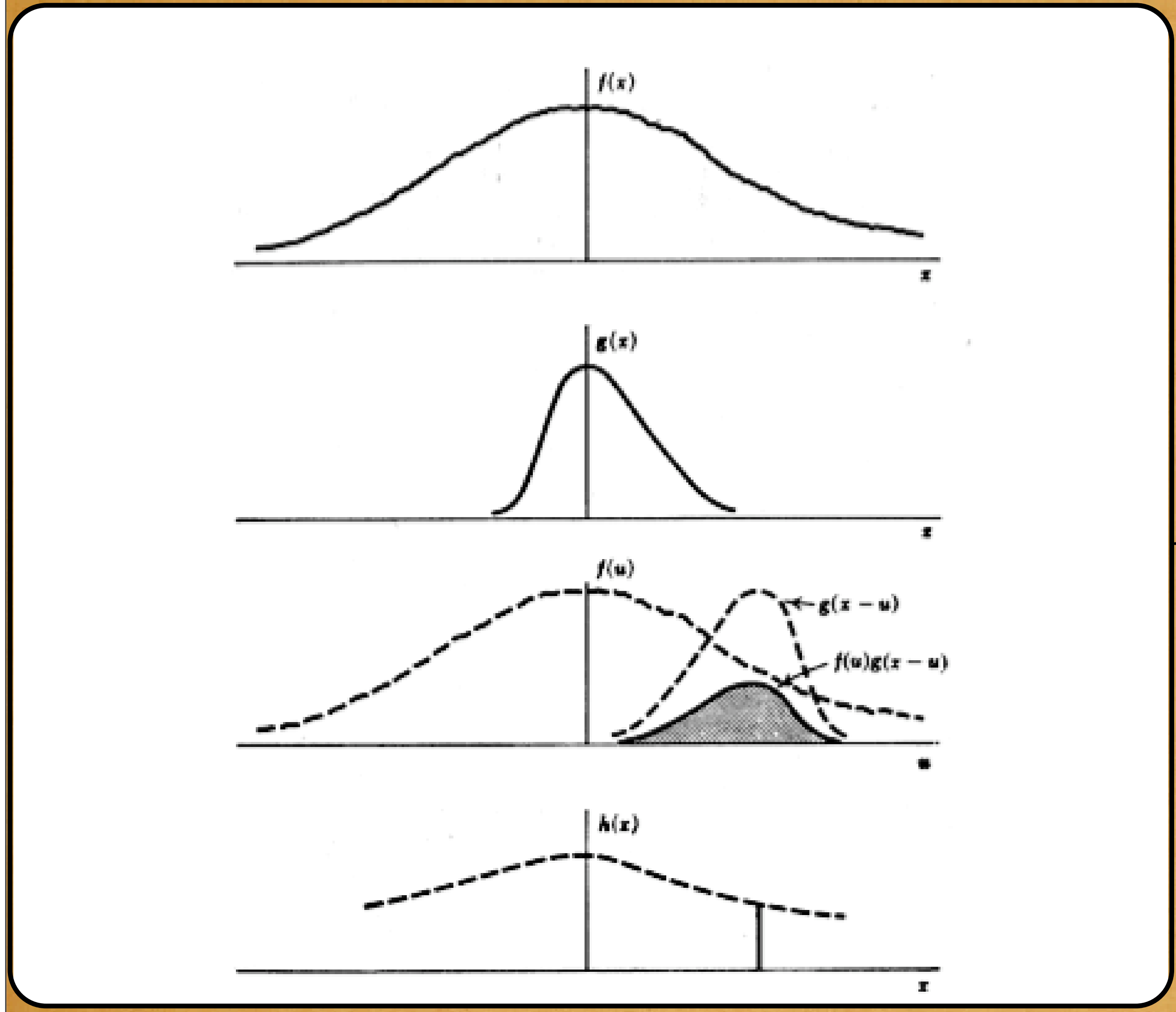
Not all signals are WSS:



Swift BAT detector light curve
of a Gamma Ray Burst
Figure from Chincarini et al. 2008

CONVOLUTION:

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(x) g(x_1 - x) dx$$



CONVOLUTION THEOREM

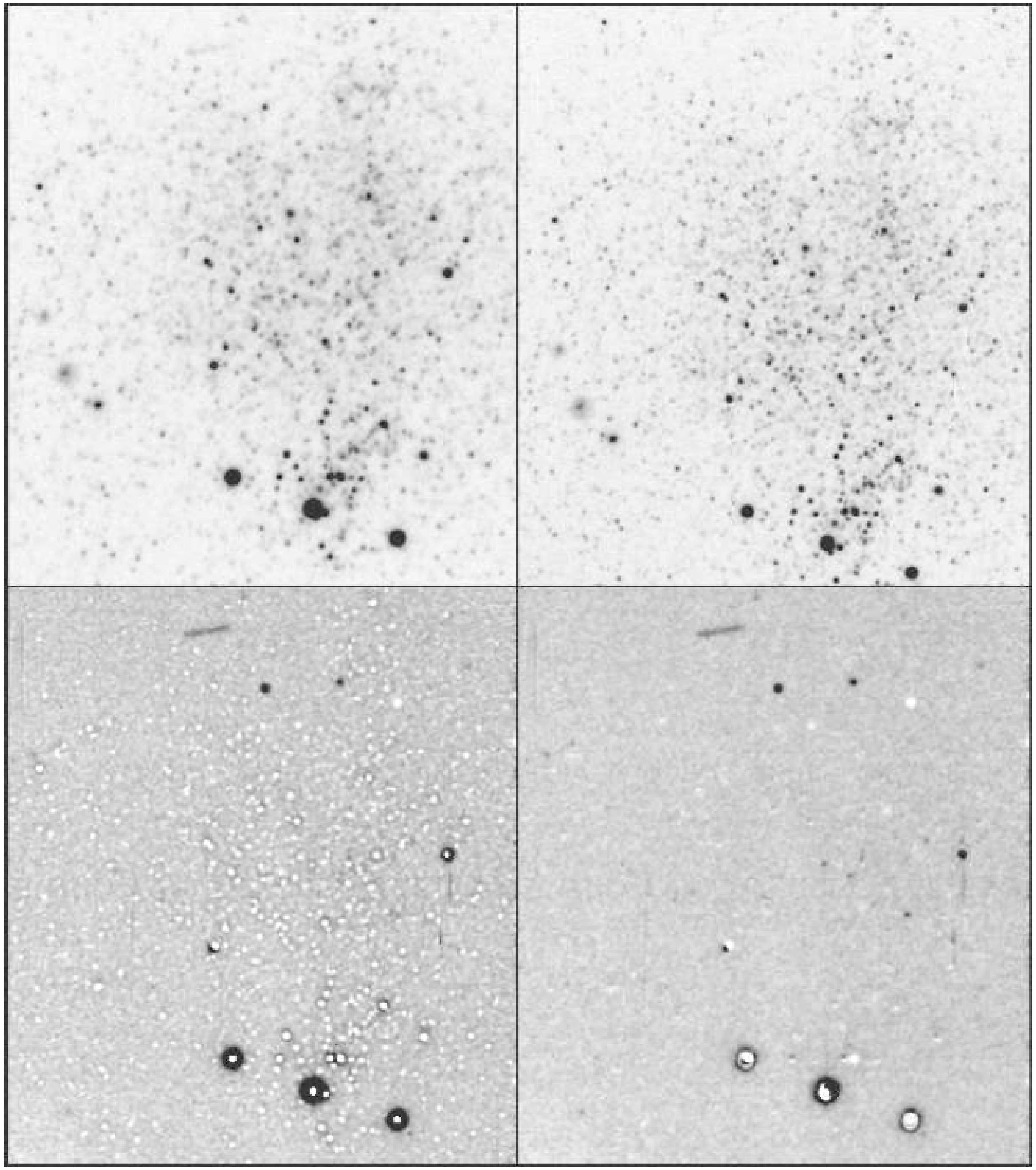
$$F(f(x) * g(x) = F(f(x))F(g(x))$$

$$f(x) * g(x) \Leftrightarrow F(s)G(s)$$

SIMILARLY FOR CROSS CORRELATIONS

$$F(f \otimes g) = F(f)F(g)$$

CONVOLUTE IMAGES



2 B-band images of the Phoenix dwarf galaxy

Difference image after convoluting the better-seeing image with a smoothing kernel and scaling the fluxes

From Phillips & Davis 1995