Outline

1. Polarized Light in the Universe
2. Fundamentals of Polarized Light
3. Descriptions of Polarized Light

Polarized Light in the Universe

Polarization indicates *anisotropy* $\Rightarrow$ not all directions are equal

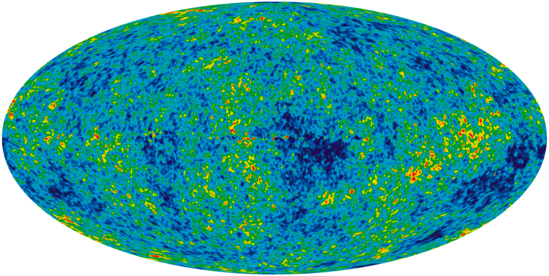
Typical anisotropies introduced by

- geometry (not everything is spherically symmetric)
- temperature gradients
- magnetic fields
- electrical fields

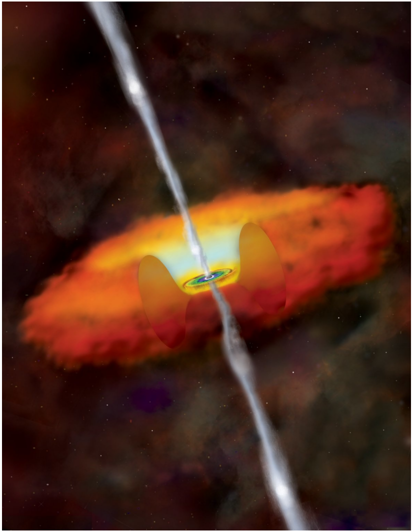
Polarized Light from the Big Bang

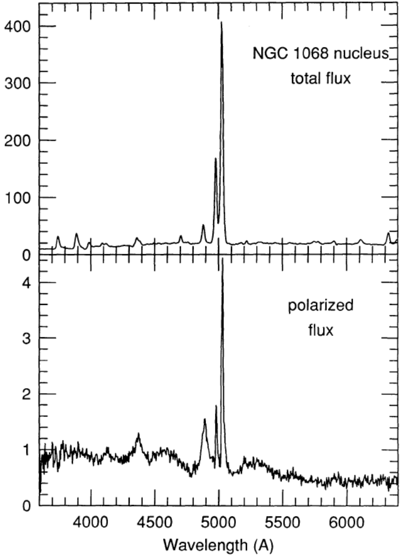
- Cosmic Microwave Background (CMB) is red-shifted radiation from Big Bang 14×10^9 years ago
- age, geometry, density, of universe from CMB intensity pattern
- first 0.1 seconds from polarization pattern of CMB
- inflation $\Rightarrow$ gravitational waves $\Rightarrow$ polarization signals
- polarization expected at (or below) 10^{-6} of intensity

13.7 billion year old temperature fluctuations from WMAP

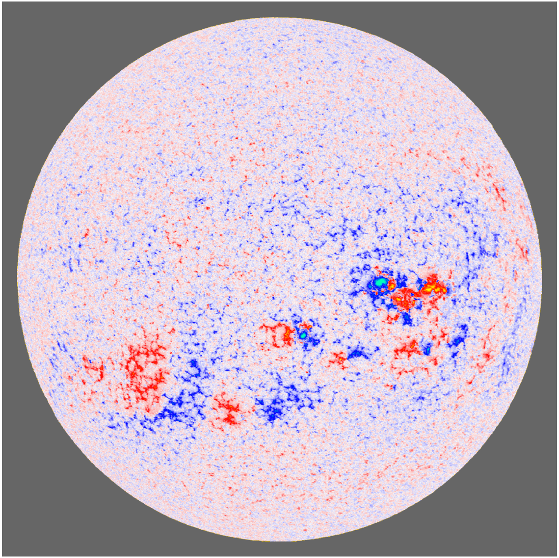


Unified Model of Active Galactic Nuclei

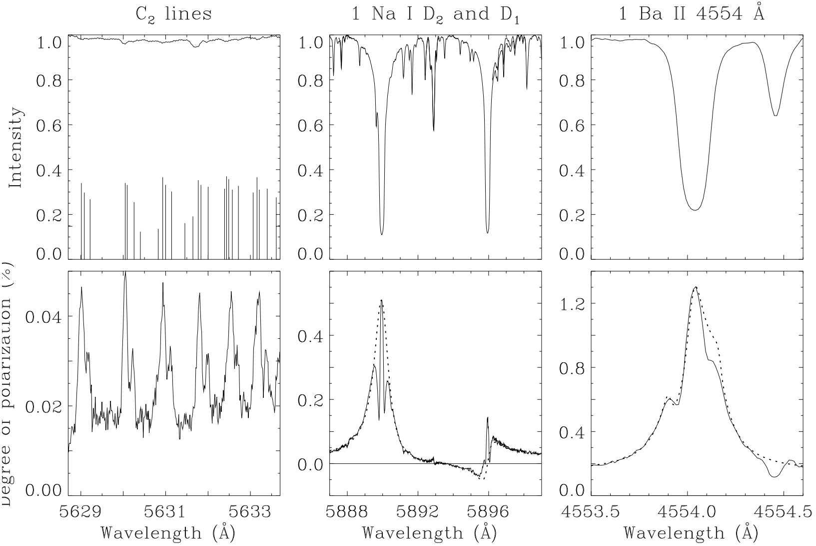




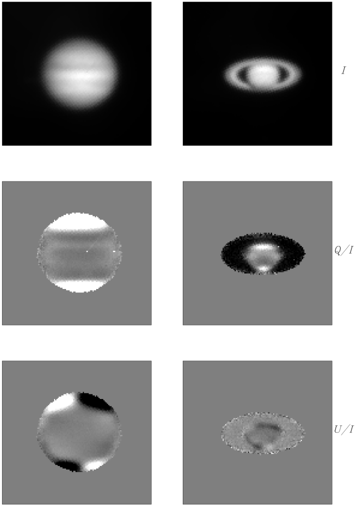
Solar Magnetic Field Maps from Longitudinal Zeeman Effect



Second Solar Spectrum from Scattering Polarization



Jupiter and Saturn



(courtesy H.M.Schmid and D.Gisler)

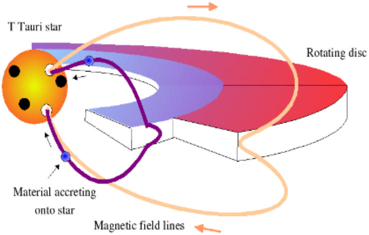
Planetary Scattered Light

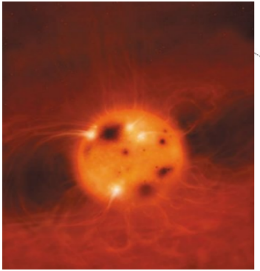
- Jupiter, Saturn show scattering polarization
- much depends on cloud height
- can be used to study exoplanets
- ExPo development at UU

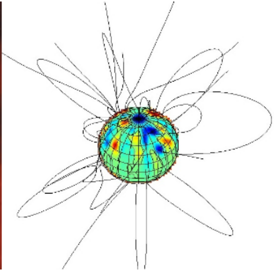
Other astrophysical applications

- interstellar magnetic field from polarized starlight
- supernova asymmetries
- stellar magnetic fields from Zeeman effect
- galactic magnetic field from Faraday rotation

Magnetic Fields of TTauri Stars by Sandra V.Jeffers







Fundamentals of Polarized Light

Electromagnetic Waves in Matter

- *Maxwell's equations* $\Rightarrow$ electromagnetic waves
- optics: interaction of electromagnetic waves with matter as described by *material equations*
- polarization of electromagnetic waves are integral part of optics

Maxwell's Equations in Matter

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{j}$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \cdot \vec{B} = 0$$

Symbols

- $\vec{D}$ *electric displacement*
- ρ *electric charge density*
- $\vec{H}$ *magnetic field*
- c *speed of light in vacuum*
- $\vec{j}$ *electric current density*
- $\vec{E}$ *electric field*
- $\vec{B}$ *magnetic induction*
- t time

Linear Material Equations

$$\vec{D} = \epsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$
$$\vec{j} = \sigma \vec{E}$$

Symbols

- ϵ *dielectric constant*
- μ *magnetic permeability*
- σ *electrical conductivity*

Isotropic Media

- isotropic media: ϵ and μ are scalars
- for most materials: $\mu = 1$

Wave Equation in Matter

- static, homogeneous medium with no net charges: $\rho = 0$
- combine Maxwell, material equations $\Rightarrow$ differential equations for damped (vector) wave

$$\nabla^2 \vec{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{E}}{\partial t} = 0$$

$$\nabla^2 \vec{H} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial \vec{H}}{\partial t} = 0$$

- damping controlled by conductivity σ
- $\vec{E}$ and $\vec{H}$ are equivalent $\Rightarrow$ sufficient to consider $\vec{E}$
- interaction with matter almost always through $\vec{E}$
- but: at interfaces, boundary conditions for $\vec{H}$ are crucial

Plane-Wave Solutions

- Plane Vector Wave ansatz

$$\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$\vec{k}$ spatially and temporally constant *wave vector*
$\vec{k}$ normal to surfaces of constant phase
$|\vec{k}|$ *wave number*
$\vec{x}$ spatial location
ω *angular frequency* ($2\pi\times$ frequency)
t time
$\vec{E}_0$ (generally complex) vector independent of time and space

- could also use $\vec{E} = \vec{E}_0 e^{-i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- damping if $\vec{k}$ is complex
- real electric field vector given by real part of $\vec{E}$

Complex Index of Refraction

- temporal derivatives $\Rightarrow$ Helmholtz equation

$$\nabla^2 \vec{E} + \frac{\omega^2 \mu}{c^2} \left(\epsilon + i \frac{4\pi\sigma}{\omega} \right) \vec{E} = 0$$

- *dispersion relation* between $\vec{k}$ and ω

$$\vec{k} \cdot \vec{k} = \frac{\omega^2 \mu}{c^2} \left(\epsilon + i \frac{4\pi\sigma}{\omega} \right)$$

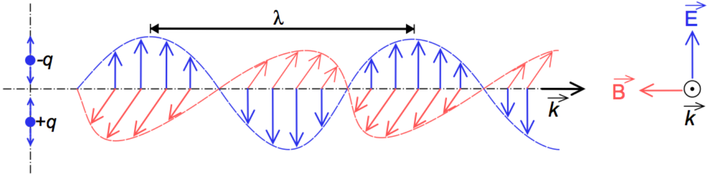
- *complex index of refraction*

$$\tilde{n}^2 = \mu \left(\epsilon + i \frac{4\pi\sigma}{\omega} \right), \quad \vec{k} \cdot \vec{k} = \frac{\omega^2}{c^2} \tilde{n}^2$$

- split into real (n: *index of refraction*) and imaginary parts (k: *extinction coefficient*)

$$\tilde{n} = n + ik$$

Transverse Waves



- plane-wave solution must also fulfill Maxwell's equations

$$\vec{E}_0 \cdot \vec{k} = 0, \quad \vec{H}_0 \cdot \vec{k} = 0, \quad \vec{H}_0 = \frac{\tilde{n}}{\mu} \frac{\vec{k}}{|\vec{k}|} \times \vec{E}_0$$

- isotropic media: electric, magnetic field vectors normal to wave vector $\Rightarrow$ transverse waves
- $\vec{E}_0$, $\vec{H}_0$, and $\vec{k}$ orthogonal to each other, right-handed vector-triple
- conductive medium $\Rightarrow$ complex $\tilde{n}$, $\vec{E}_0$ and $\vec{H}_0$ out of phase
- $\vec{E}_0$ and $\vec{H}_0$ have constant relationship $\Rightarrow$ consider only $\vec{E}$

Energy Propagation in Isotropic Media

- *Poynting vector*

$$\vec{S} = \frac{c}{4\pi}\left(\vec{E} \times \vec{H}\right)$$

- $|\vec{S}|$: energy through unit area perpendicular to $\vec{S}$ per unit time
- direction of $\vec{S}$ is direction of energy flow
- time-averaged Poynting vector given by

$$\left\langle \vec{S} \right\rangle = \frac{c}{8\pi}\mathrm{Re}\left(\vec{E}_0 \times \vec{H}_0^*\right)$$

Re	real part of complex expression
$*$	complex conjugate
$\langle . \rangle$	time average

- energy flow parallel to wave vector (in isotropic media)

$$\left\langle \vec{S} \right\rangle = \frac{c}{8\pi}\frac{|\tilde{n}|}{\mu}\left|E_0\right|^2\frac{\vec{k}}{|\vec{k}|}$$

Polarization

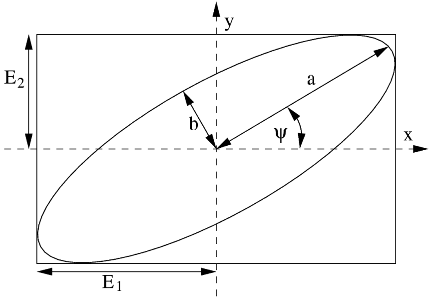
- Plane Vector Wave ansatz $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_1$ and $\vec{e}_2$, both perpendicular to $\vec{k}$

  $$\vec{E}_0 = E_1\vec{e}_1 + E_2\vec{e}_2.$$

  E_1, E_2: arbitrary complex scalars
- damped plane-wave solution with given ω, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called *polarization*
- many ways to represent these four quantities
- if E_1 and E_2 have identical phases, $\vec{E}$ oscillates in fixed plane

Polarization Ellipse

Polarization Ellipse



Polarization

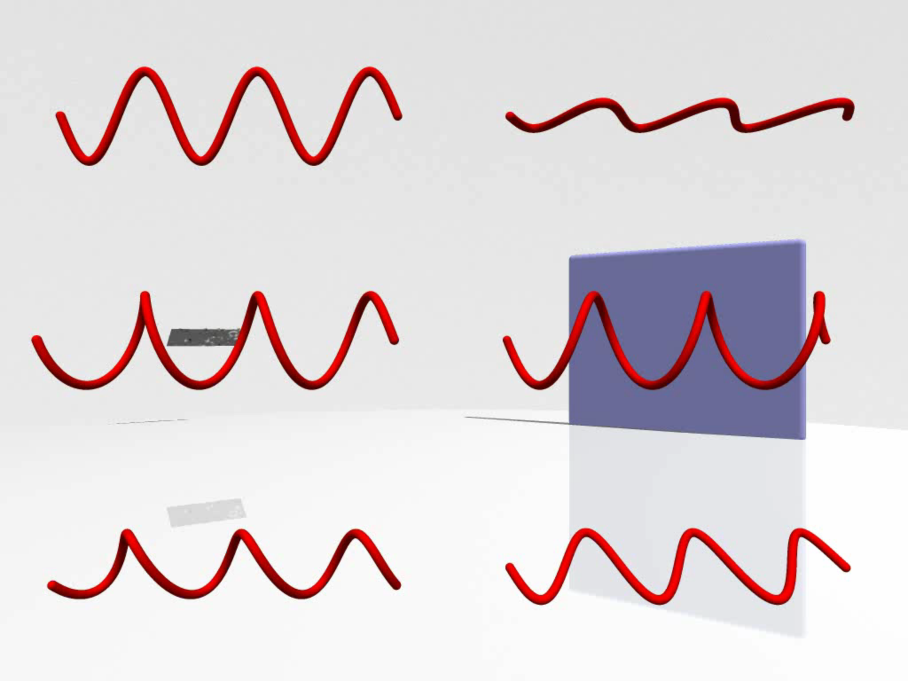
$$\vec{E}(t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$$\vec{E}_0 = E_1 e^{i\delta_1}\vec{e}_x + E_2 e^{i\delta_2}\vec{e}_y$$

- wave vector in z-direction
- $\vec{e}_x$, $\vec{e}_y$: unit vectors in x, y
- E_1, E_2: (real) amplitudes
- $\delta_{1,2}$: (real) phases

Polarization Description

- 2 complex scalars not the most useful description
- at given $\vec{x}$, time evolution of $\vec{E}$ described by *polarization ellipse*
- ellipse described by axes a, b, orientation ψ



Jones Formalism

Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

- beam in z-direction
- $\vec{e}_x$, $\vec{e}_y$ unit vectors in x, y-direction
- complex scalars $E_{x,y}$
- Jones vector
$$\vec{e} = \left(\begin{array}{c} E_x \\ E_y \end{array} \right)$$
- phase difference between E_x, E_y multiple of π, electric field vector oscillates in a fixed plane $\Rightarrow$ *linear polarization*
- phase difference $\pm\frac{\pi}{2}$ $\Rightarrow$ *circular polarization*

Summing and Measuring Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

$$\vec{e} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- Maxwell's equations linear $\Rightarrow$ sum of two solutions again a solution
- Jones vector of sum of two waves $=$ sum of Jones vectors of individual waves if wave vectors $\vec{k}$ the same
- addition of Jones vectors: *coherent* superposition of waves
- elements of Jones vectors are not observed directly
- observables always depend on products of elements of Jones vectors, i.e. intensity

$$I = \vec{e} \cdot \vec{e}^* = e_x e_x^* + e_y e_y^*$$

Jones matrices

- influence of medium on polarization described by 2×2 complex *Jones matrix* J

$$\vec{e}' = \mathsf{J}\vec{e} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \vec{e}$$

- assumes that medium not affected by polarization state
- different media 1 to N in order of wave direction $\Rightarrow$ combined influence described by

$$\mathsf{J} = \mathsf{J}_N \mathsf{J}_{N-1} \cdots \mathsf{J}_2 \mathsf{J}_1$$

- order of matrices in product is crucial
- Jones calculus describes coherent superposition of polarized light

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- 45°: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Circular Polarization

- left: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$
- right: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

Notes on Jones Formalism

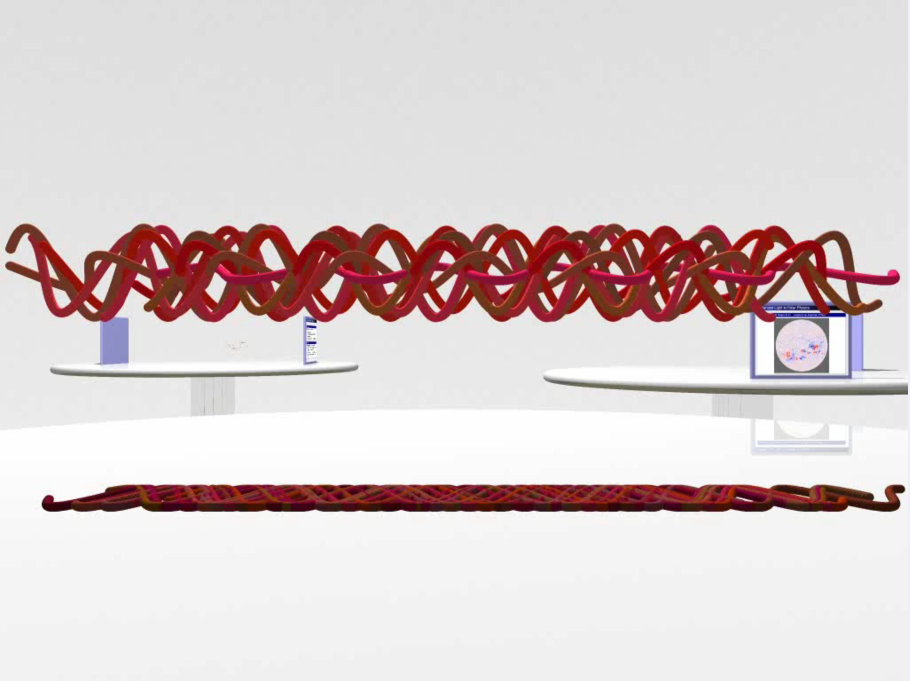
- Jones formalism operates on amplitudes, not intensities
- coherent superposition important for coherent light (lasers, interference effects)
- Jones formalism describes 100% polarized light

Quasi-Monochromatic Light

- monochromatic light: purely theoretical concept
- monochromatic light wave always fully polarized
- real life: light includes range of wavelengths $\Rightarrow$ *quasi-monochromatic light*
- quasi-monochromatic: superposition of mutually incoherent monochromatic light beams whose wavelengths vary in narrow range $\delta\lambda$ around central wavelength λ_0

$$\frac{\delta\lambda}{\lambda} \ll 1$$

- measurement of quasi-monochromatic light: integral over measurement time t_m
- amplitude, phase (slow) functions of time for given spatial location
- *slow*: variations occur on time scales much longer than the mean period of the wave



Polarization of Quasi-Monochromatic Light

- electric field vector for quasi-monochromatic plane wave is sum of electric field vectors of all monochromatic beams

$$\vec{E}\left(t\right) = \vec{E}_0\left(t\right) e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

- can write this way because $\delta\lambda \ll \lambda_0$
- measured intensity of quasi-monochromatic beam

$$\left\langle \vec{E}_x \vec{E}_x^* \right\rangle + \left\langle \vec{E}_y \vec{E}_y^* \right\rangle = \lim_{t_m -> \infty} \frac{1}{t_m} \int_{-t_m/2}^{t_m/2} \vec{E}_x(t)\vec{E}_x^*(t) + \vec{E}_y(t)\vec{E}_y^*(t) dt$$

$\langle \cdots \rangle$: averaging over measurement time t_m

- measured intensity independent of time
- quasi-monochromatic: frequency-dependent material properties (e.g. index of refraction) are constant within $\Delta\lambda$

Polychromatic Light or White Light

- wavelength range comparable to wavelength ($\frac{\delta\lambda}{\lambda} \sim 1$)
- incoherent sum of quasi-monochromatic beams that have large variations in wavelength
- cannot write electric field vector in a plane-wave form
- must take into account frequency-dependent material characteristics
- intensity of polychromatic light is given by sum of intensities of constituting quasi-monochromatic beams

Stokes and Mueller Formalisms

Stokes Vector

- formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i\left(E_x E_y^* - E_y E_x^*\right) \end{pmatrix} = \begin{pmatrix} E_1^2 + E_2^2 \\ E_1^2 - E_2^2 \\ 2E_1 E_2 \cos\delta \\ 2E_1 E_2 \sin\delta \end{pmatrix}$$

  Jones vector elements $E_{x,y}$, real amplitudes $E_{1,2}$, phase difference $\delta = \delta_2 - \delta_1$

- $$I^2 \geq Q^2 + U^2 + V^2$$

Stokes Vector Interpretation

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

- *degree of polarization*

$$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

  1 for fully polarized light, 0 for unpolarized light
- summing of Stokes vectors $=$ *incoherent* adding of quasi-monochromatic light waves

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
- 45°: $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Circular Polarization

- left: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
- right: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Mueller Matrices

- 4×4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

$$\vec{I}' = \mathrm{M}\vec{I}\,,$$

$$\mathrm{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

- N optical elements, combined Mueller matrix is

$$\mathrm{M}' = \mathrm{M}_N \mathrm{M}_{N-1} \cdots \mathrm{M}_2 \mathrm{M}_1$$

Vertical Linear Polarizer

$$M_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

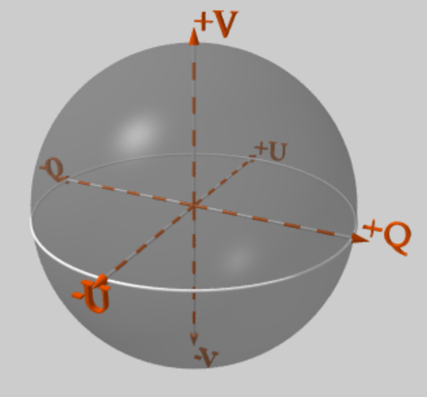
Horizontal Linear Polarizer

$$M_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Mueller Matrix for Ideal Linear Polarizer at Angle θ

$$M_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

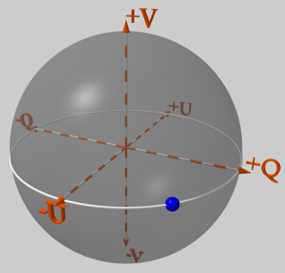
Poincaré Sphere

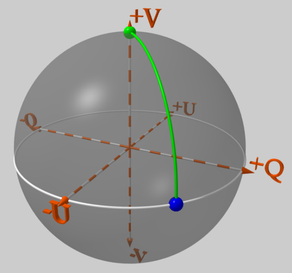


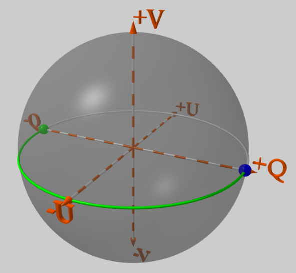
Relation to Stokes Vector

- fully polarized light:
  $I^2 = Q^2 + U^2 + V^2$
- for $I^2 = 1$: sphere in Q, U, V coordinate system
- point on Poincaré sphere represents particular state of polarization
- graphical representation of fully polarized light

Poincaré Sphere Interpretation







- polarizer is a point on the Poincaré sphere
- transmitted intensity: $\cos^2(l/2)$, l is arch length of great circle between incoming polarization and polarizer on Poincaré sphere