Today's course

Chapter 1.5-1.7, 2.2.2 OAF-2

Topics:

- recap: aliasing & nyquist theorem
- (Optimal) Filtering
- measuring moments of a s.p.
- Stochastic nature of radiation processes

SIGNAL DETECTION INVOLVES:

- LIMITED TIME INTERVAL → WINDOWING
- NOT CONTINUOUS → SAMPLING
- SAMPLES NOT INSTANTANEOUS → AVERAGING
- DEALING WITH NOISE → FILTERING
- RESPONSE OF THE DETECTION SYSTEM

THUS THE DETECTED SIGNAL WILL ONLY APPROXIMATE THE SOURCE SIGNAL

Power Spectral Density

(∝ amplitude of individual sinusoids)

(will return in more depth in Chapter 6)

Continuous FT: $$F(f) = \int_{-\infty}^{\infty} f(t)\, e^{-2\pi i f t} dt$$

Continuous PSD: $$P(f) = \tilde{F(f)} \tilde{F(f)}^*$$

for WSS signals: $$P(f) = \int_{-\infty}^{\infty} R(\tau) e^{-2\pi i f \tau} d\tau$$

hence:

$$\widetilde{F(f)} \widetilde{F(f)} = |F(f)|^2 = \int_{-\infty}^{\infty} R(\tau) e^{-2\pi i f \tau} d\tau$$

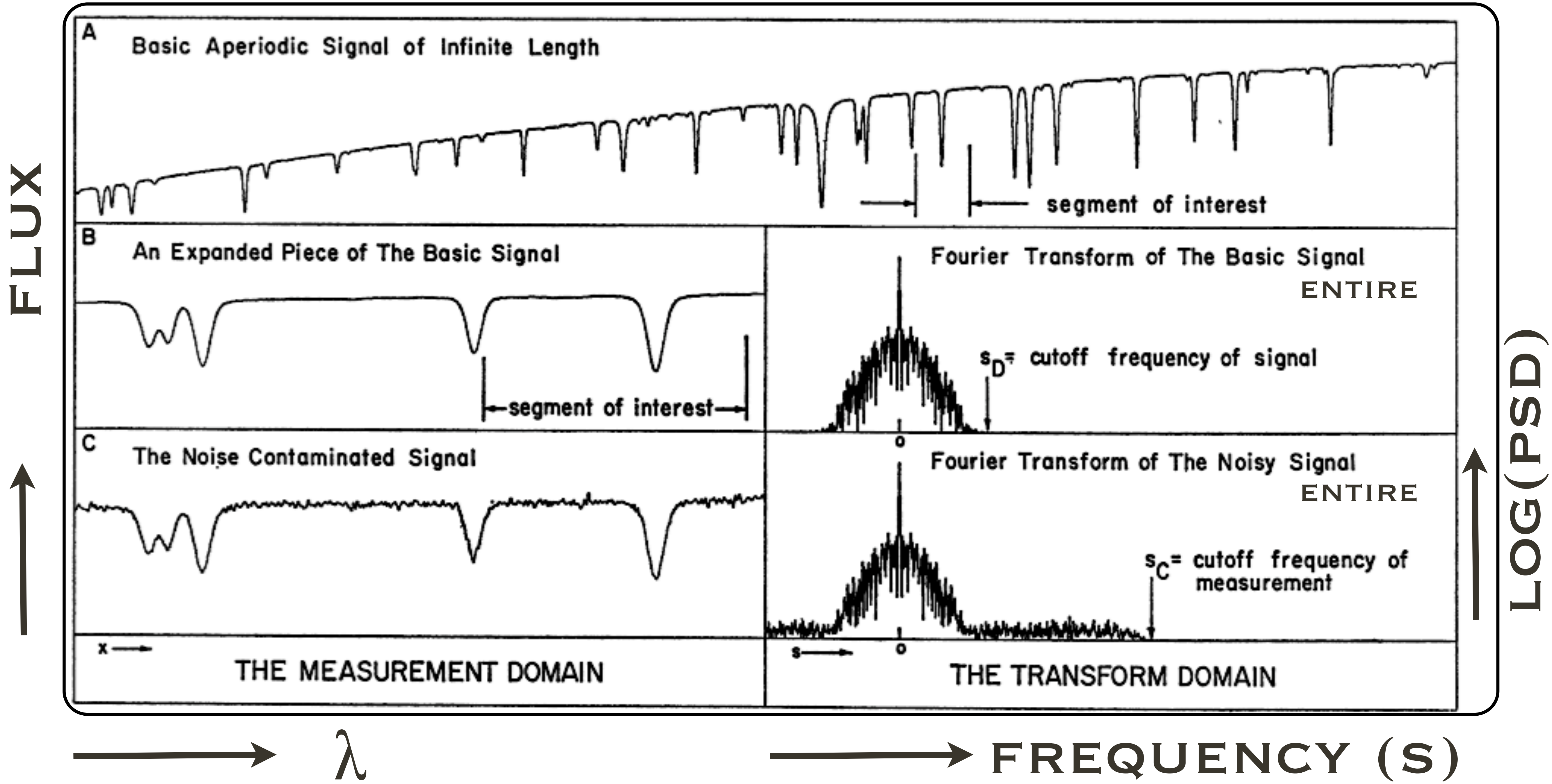
Wiener-Khinchine Theorem

Fourier transform of the autocorrelation of f(x) is equal to the power density spectrum $|F(s)|^2$

$$S(s) = \int_{-\infty}^{\infty} R(\tau) e^{-2\pi i s \tau} d\tau$$

$$R(t) = \int_{-\infty}^{\infty} S(s) e^{2\pi i t s} ds$$

THE NYQUIST THEOREM



WINDOWING & NOISE, BRAULT & WHITE 1971, A&A

Nyquist theorem: cont'd
Data sampling

Data is discrete not continuous

time domain multiply s.p. with shah function

$$m_{samp,n} = m_s(x) = m(x)\frac{1}{\tau}\text{III}(\frac{x}{\tau}) = \sum_n m(n\tau)\delta(x - n\tau)$$

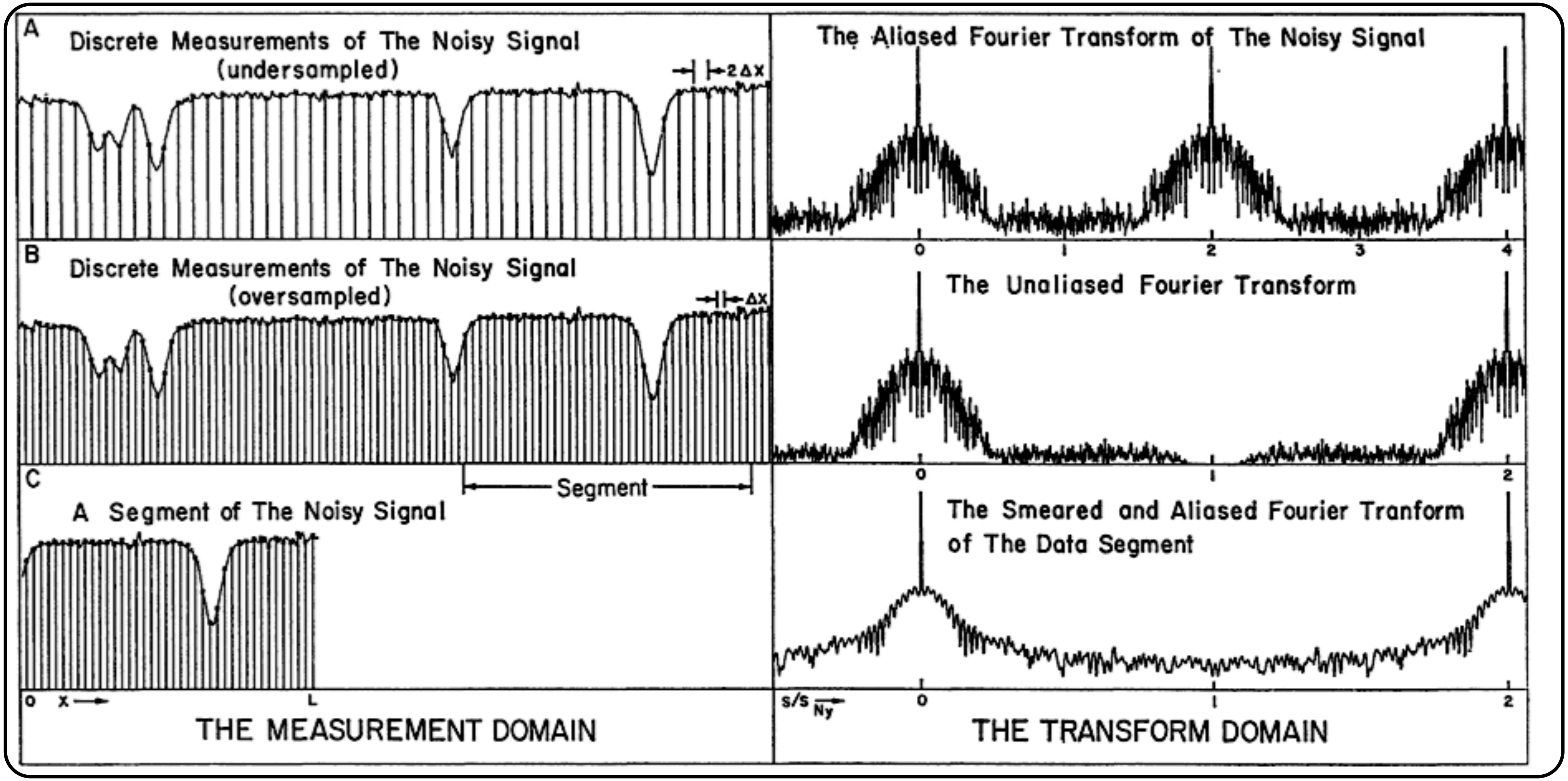
Discrete FT: $$M_{samp,k} = \sum_{n=0}^{N-1} m_{samp,n}\ e^{2\pi i n k/N}$$

Discrete PSD: $$P_j = \frac{2}{a_0}|a_j|^2$$ power ∝ amplitude squared:

j=k label $$a_0 = M_{samp,k=0} = \sum_{n=0}^{N-1} m_{samp,n} \equiv N_0$$

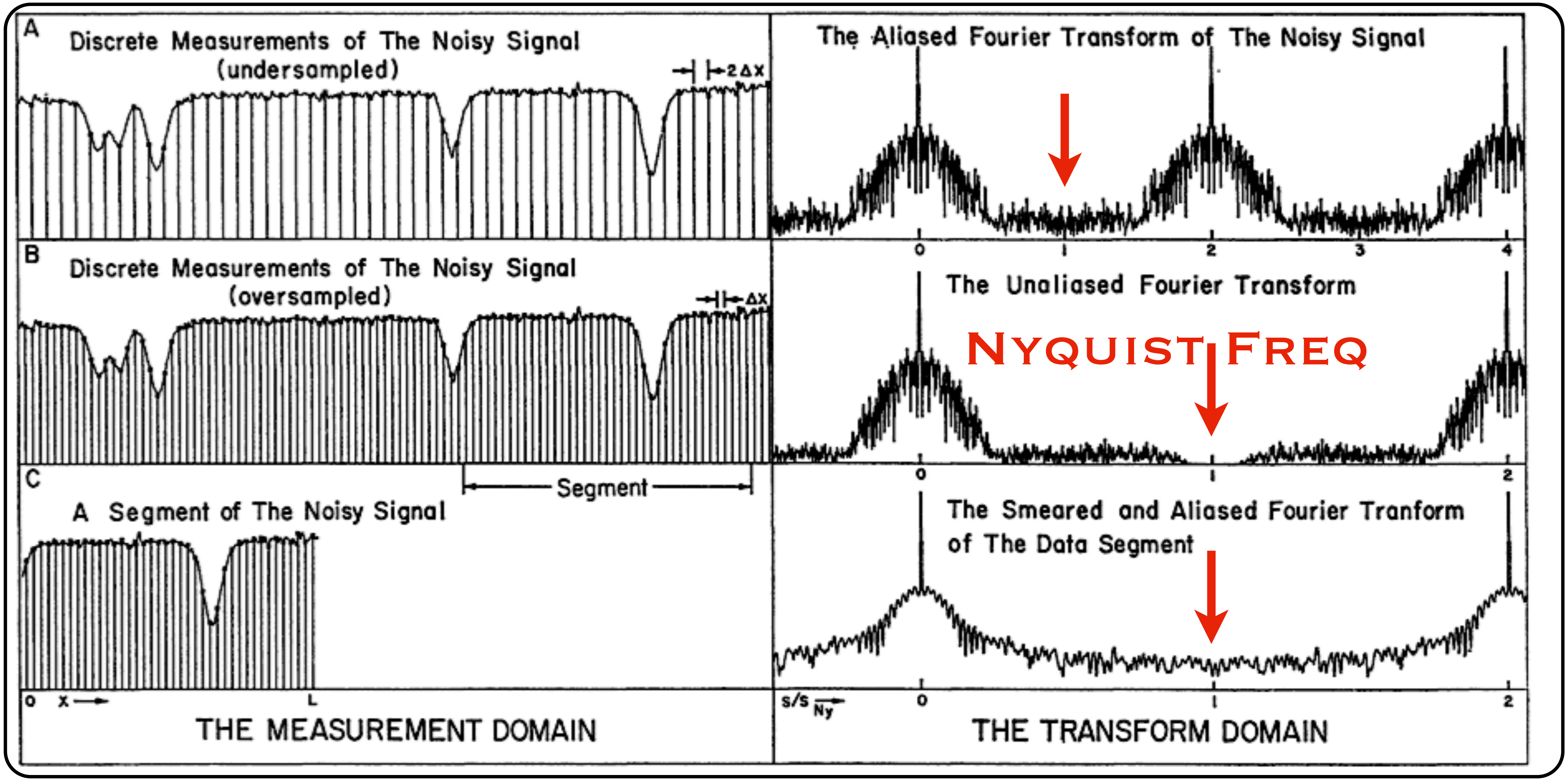
$$a_k = M_{samp,k} = \sum_{n=0}^{N-1} m_{samp,n}\ e^{2\pi i n k/N}$$

Nyquist theorem: cont'd



Sampling; Brault & White 1971, A&A, 13, 169 (in list of presentation papers!)

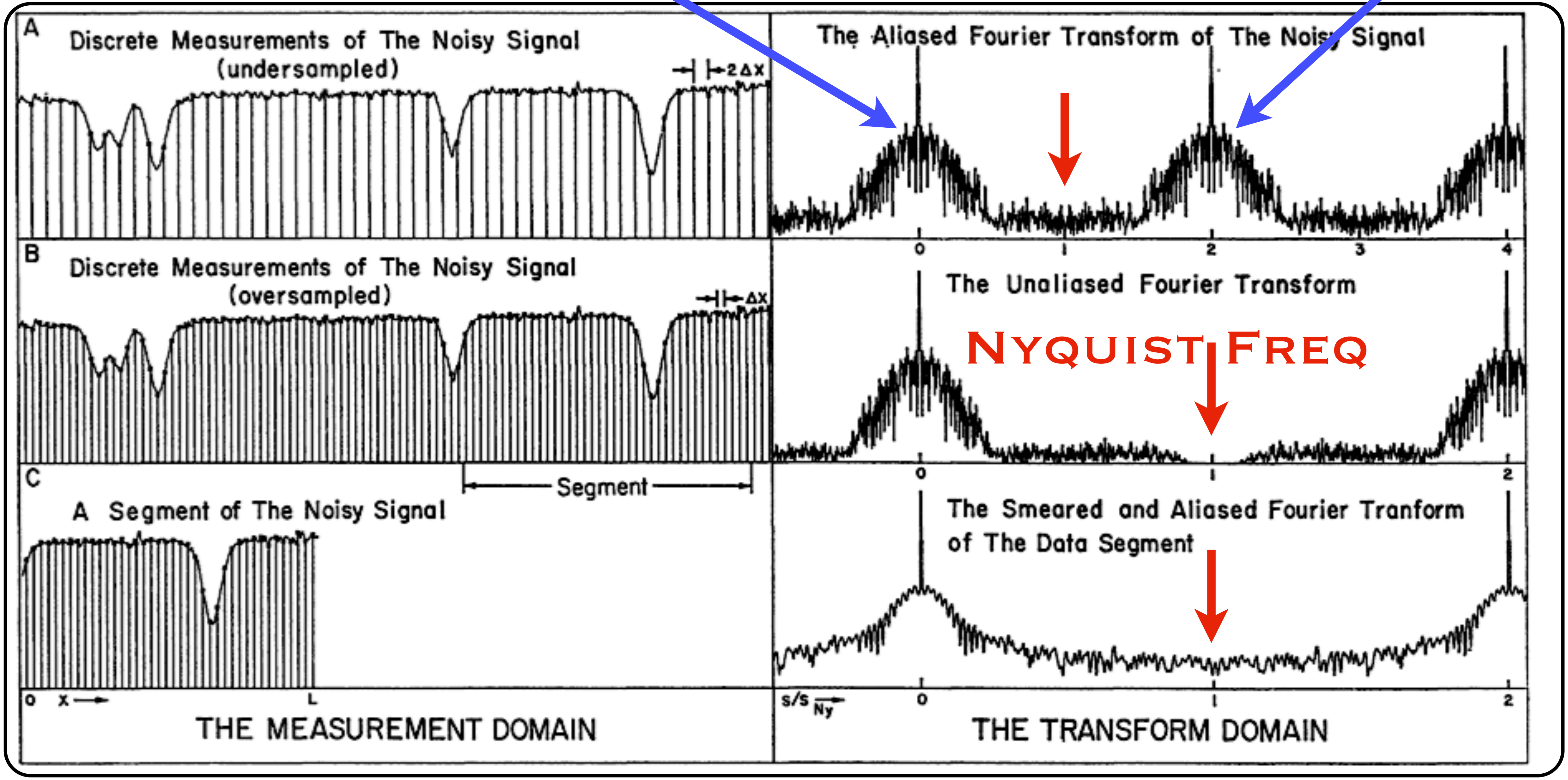
Nyquist theorem: cont'd



Sampling; Brault & White 1971, A&A, 13, 169 (in list of presentation papers!)

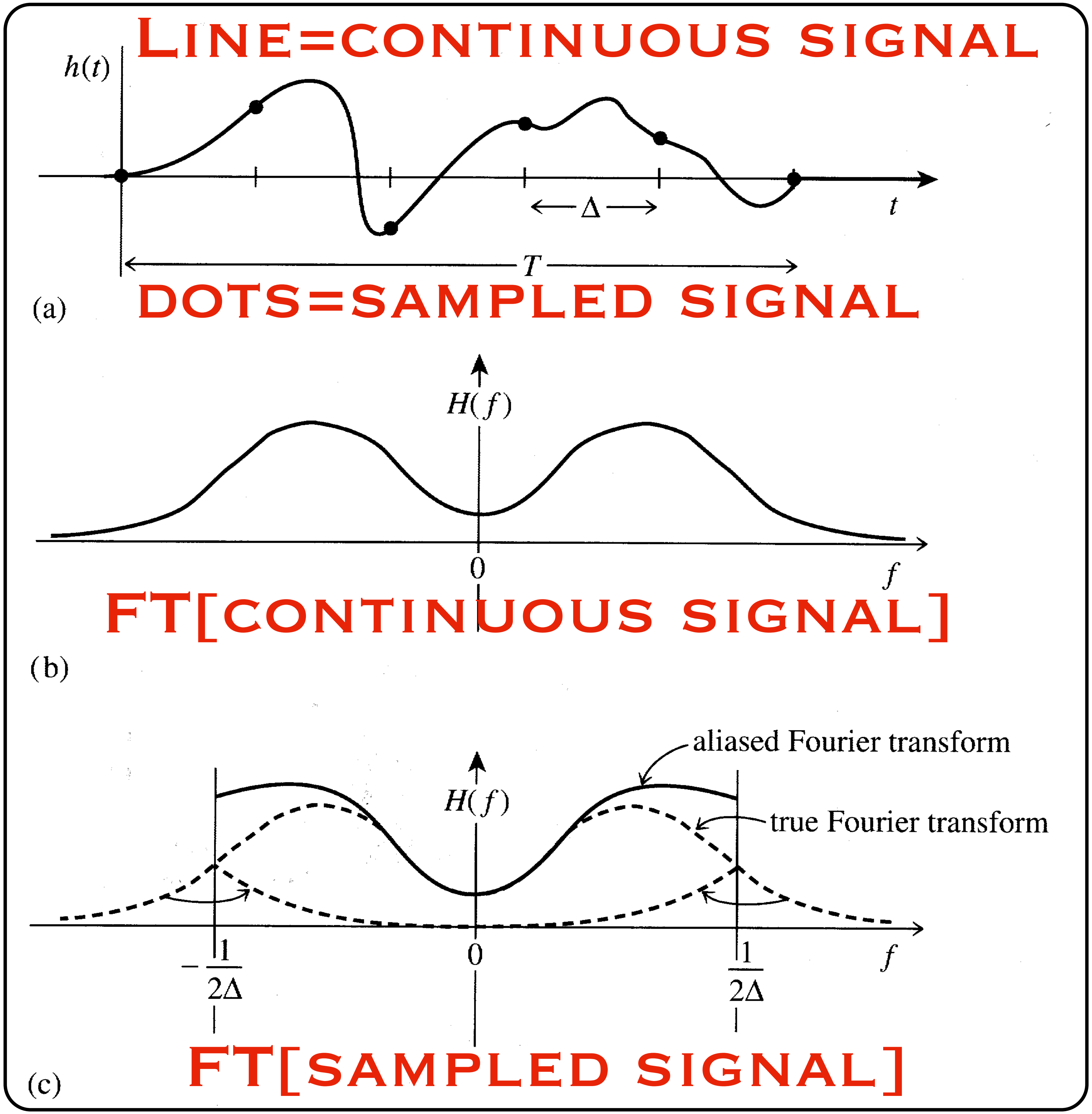
Nyquist theorem: cont'd

Sampling causes replication of signal



Sampling; Brault & White 1971, A&A, 13, 169 (in list of presentation papers!)

ALIASING


LINE=CONTINUOUS SIGNAL
h(t)
Δ
t
T
(a)
DOTS=SAMPLED SIGNAL
H(f)
0
f
FT[CONTINUOUS SIGNAL]
(b)
aliased Fourier transform
H(f)
true Fourier transform
−1/2Δ
0
1/2Δ
f
FT[SAMPLED SIGNAL]
(c)

Recap Nyquist & aliasing

Data sampling: if the Nyquist criterium is fulfilled (i.e. sample at a rate higher than two times the highest frequency in the signal) then sampling does not lead to loss of information

Conditions:

- ➔ band-limited response of the detector removes highest noise powers and the sampling is fast enough to cover the band limit of the detector
- ➔ Signal is band-limited also and

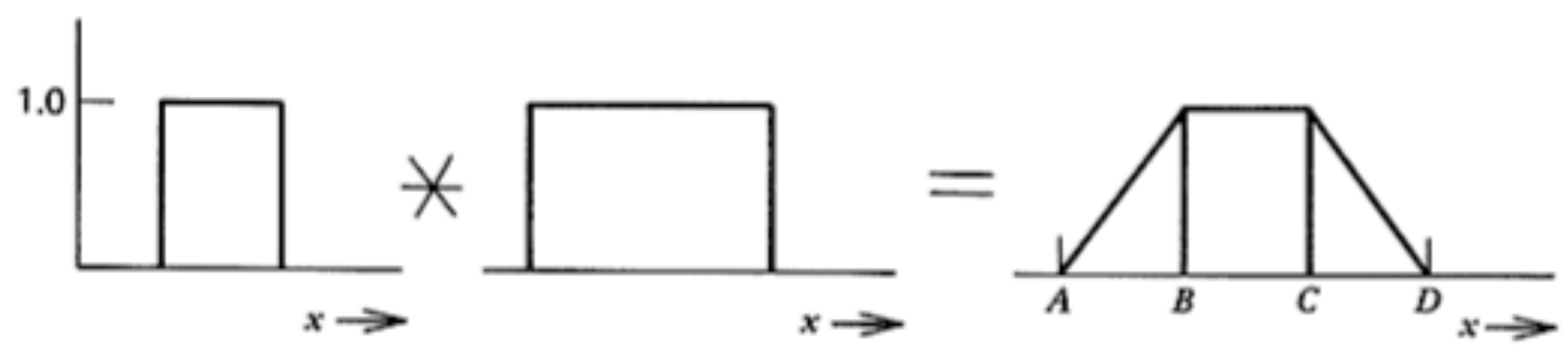
$$\nu_{\text{sampling}} > \nu_{\text{max,detector}} > \nu_{\text{max,signal}}$$

(Optimal) Filtering

Num Res Chapter 13.0-13.3

Deconvolve measured signal and response function of sampled data

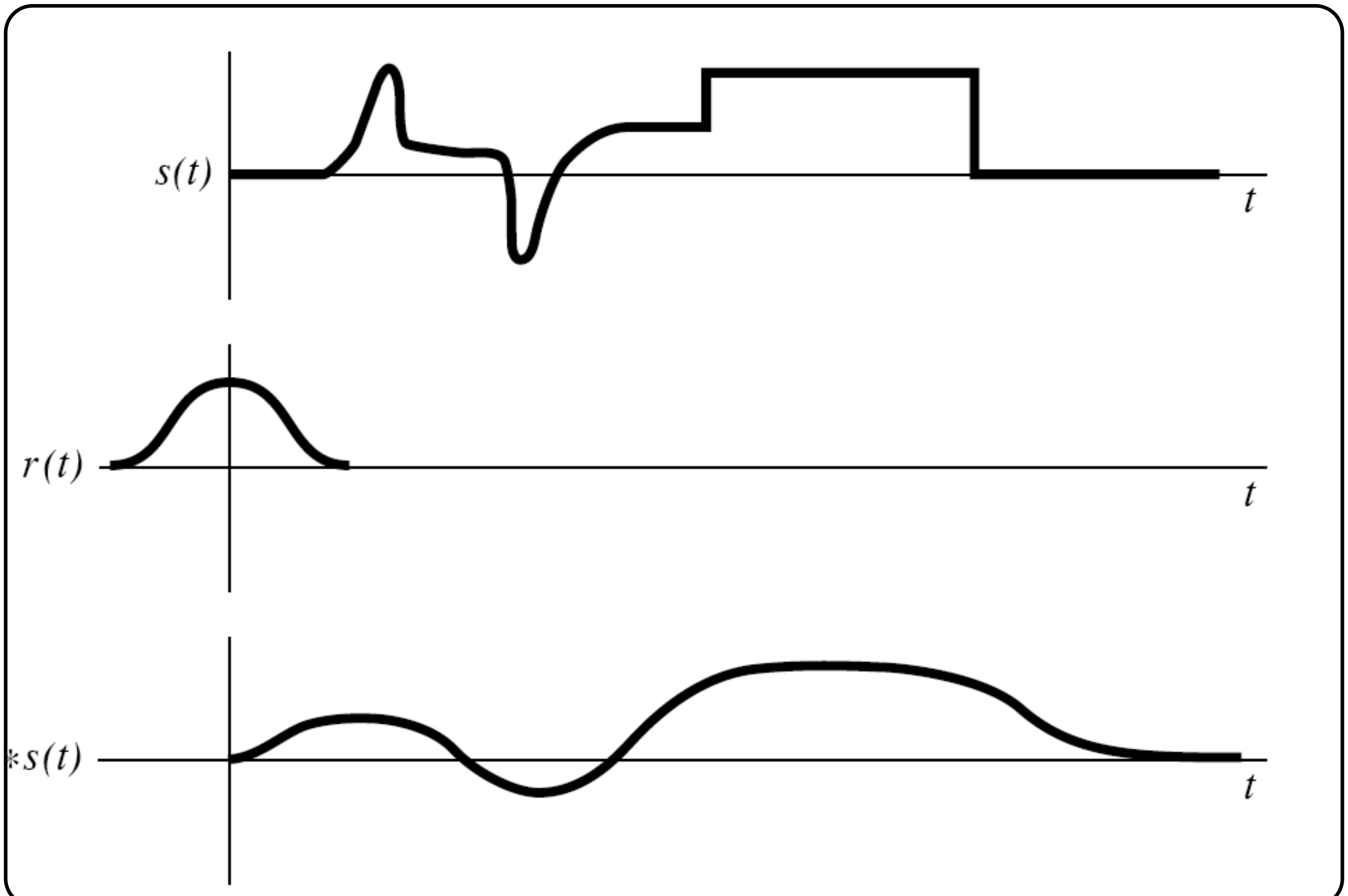
Reminder convolution



(Optimal) Filtering

Num Res Chapter 13.0-13.3

deconvolve measured signal and response function of sampled data



Num Res Chapter 13.1

(Optimal) Filtering

Num Res Chapter 13.0-13.3

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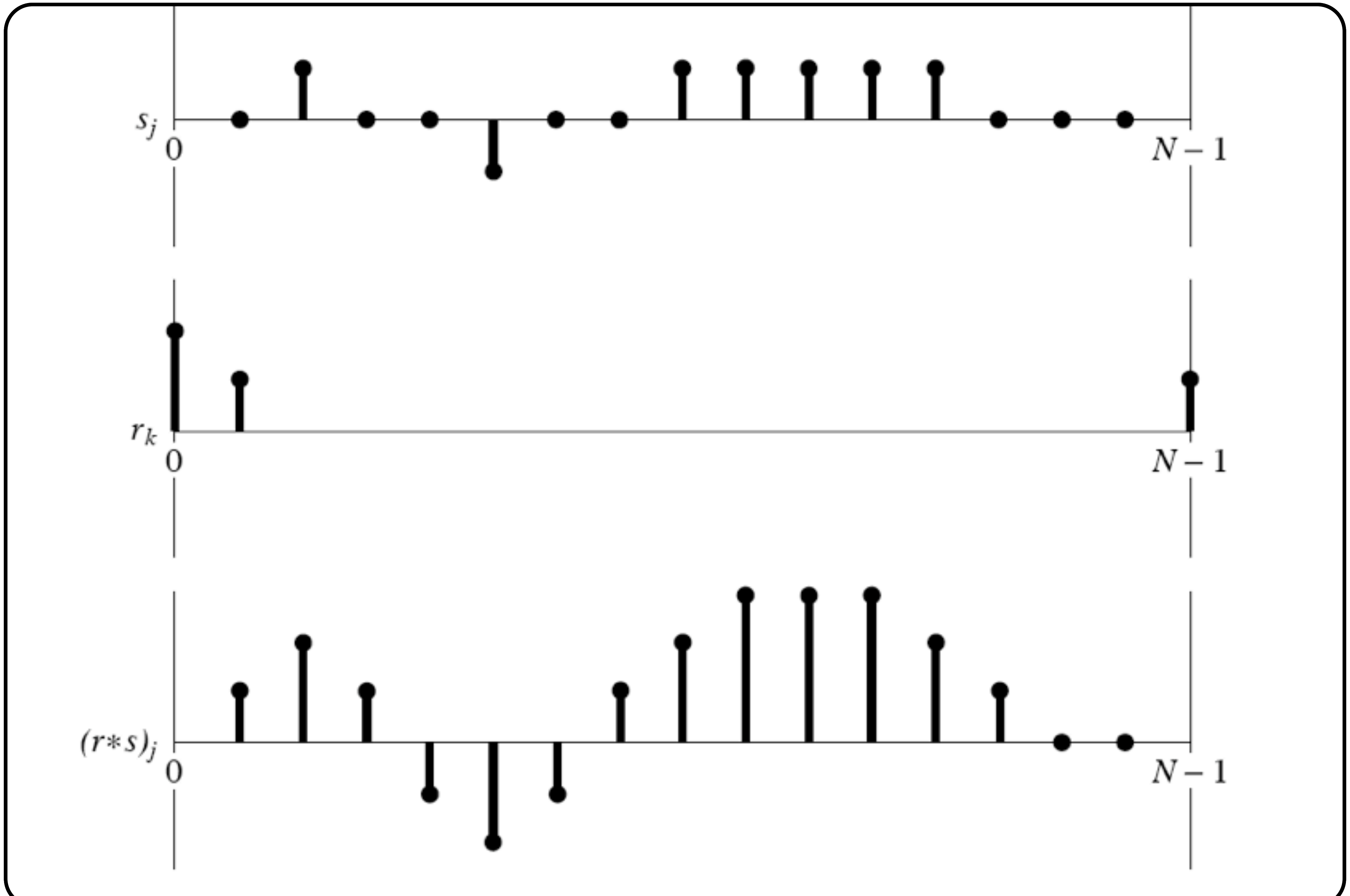

Num Res Chapter 13.1

(Optimal) Filtering

Num Res Chapter 13.0-13.3

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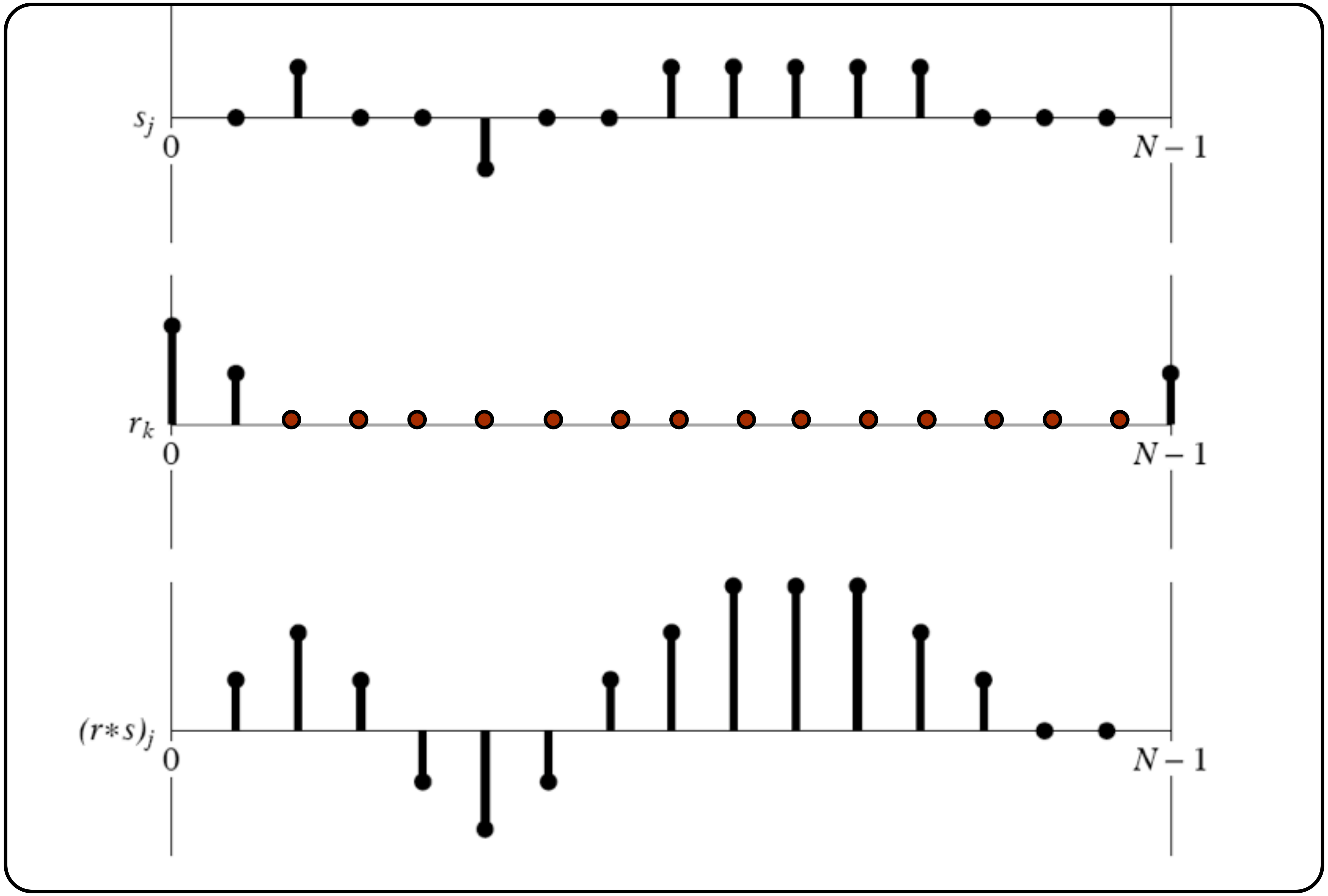
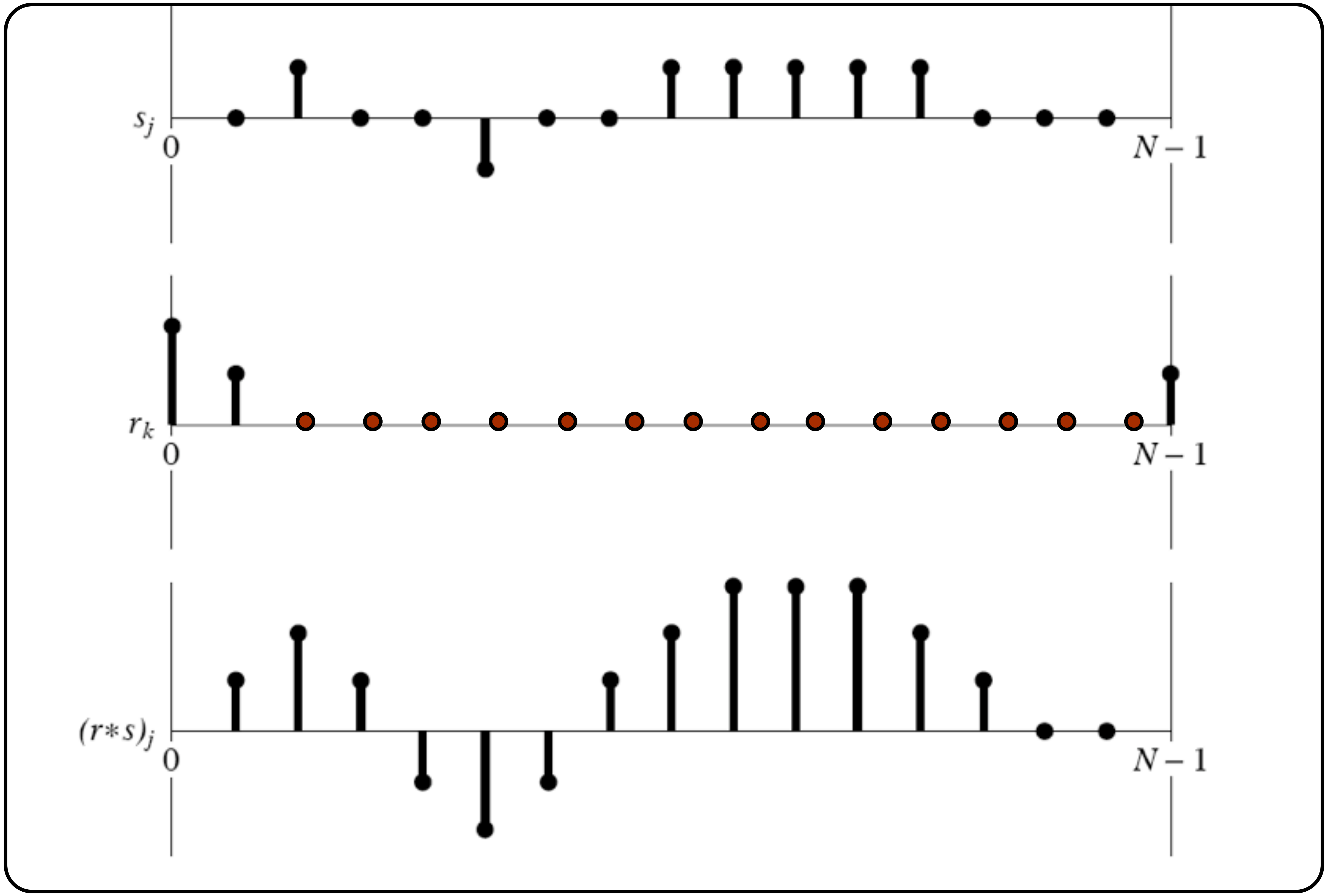


Num Res Chapter 13.1

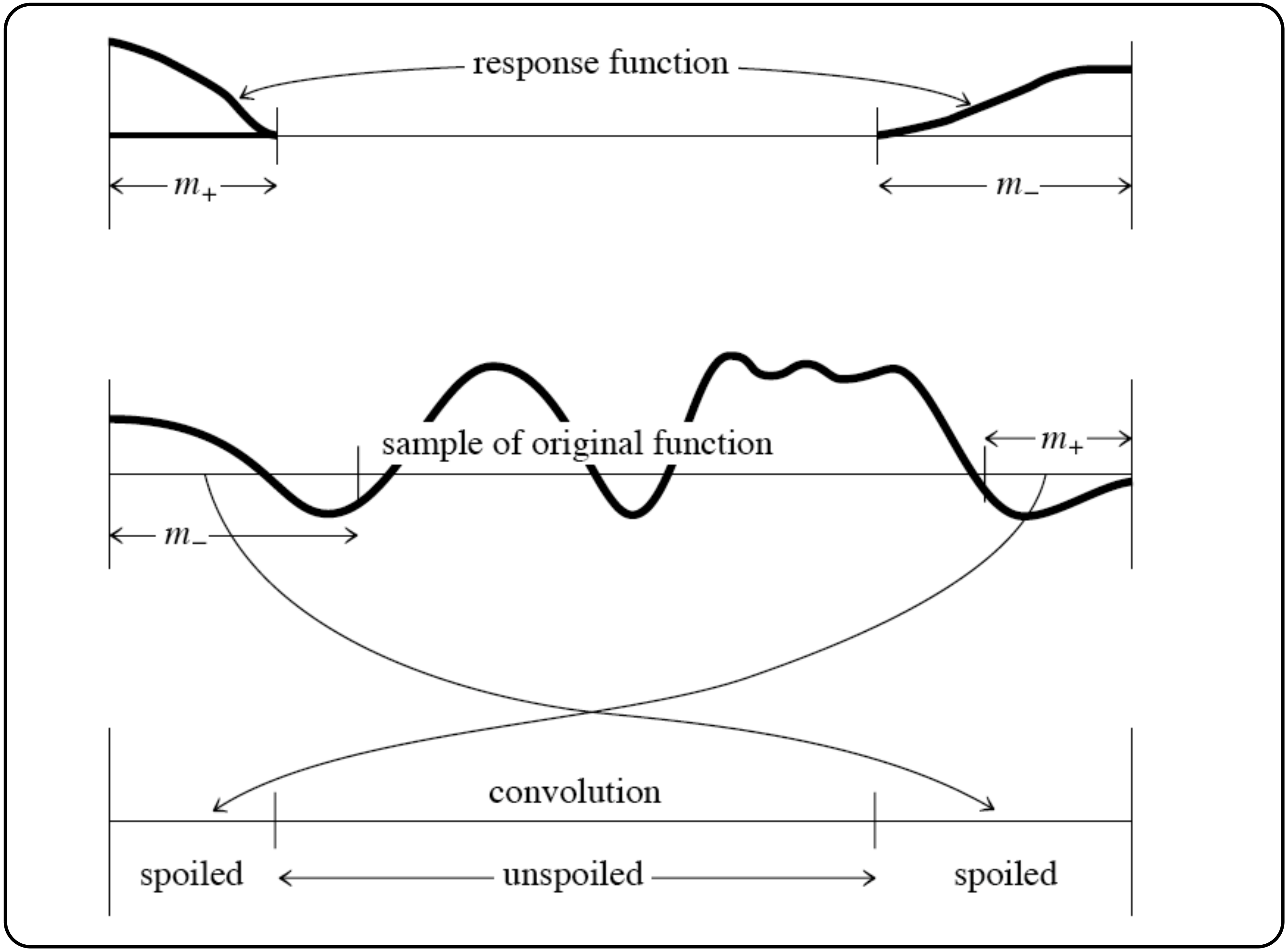
Discrete convolution theorem

$$(r * s)_j \equiv \sum_{k=-M/2+1}^{M/2} s_{j-k} r_k \Leftrightarrow S_n R_n$$

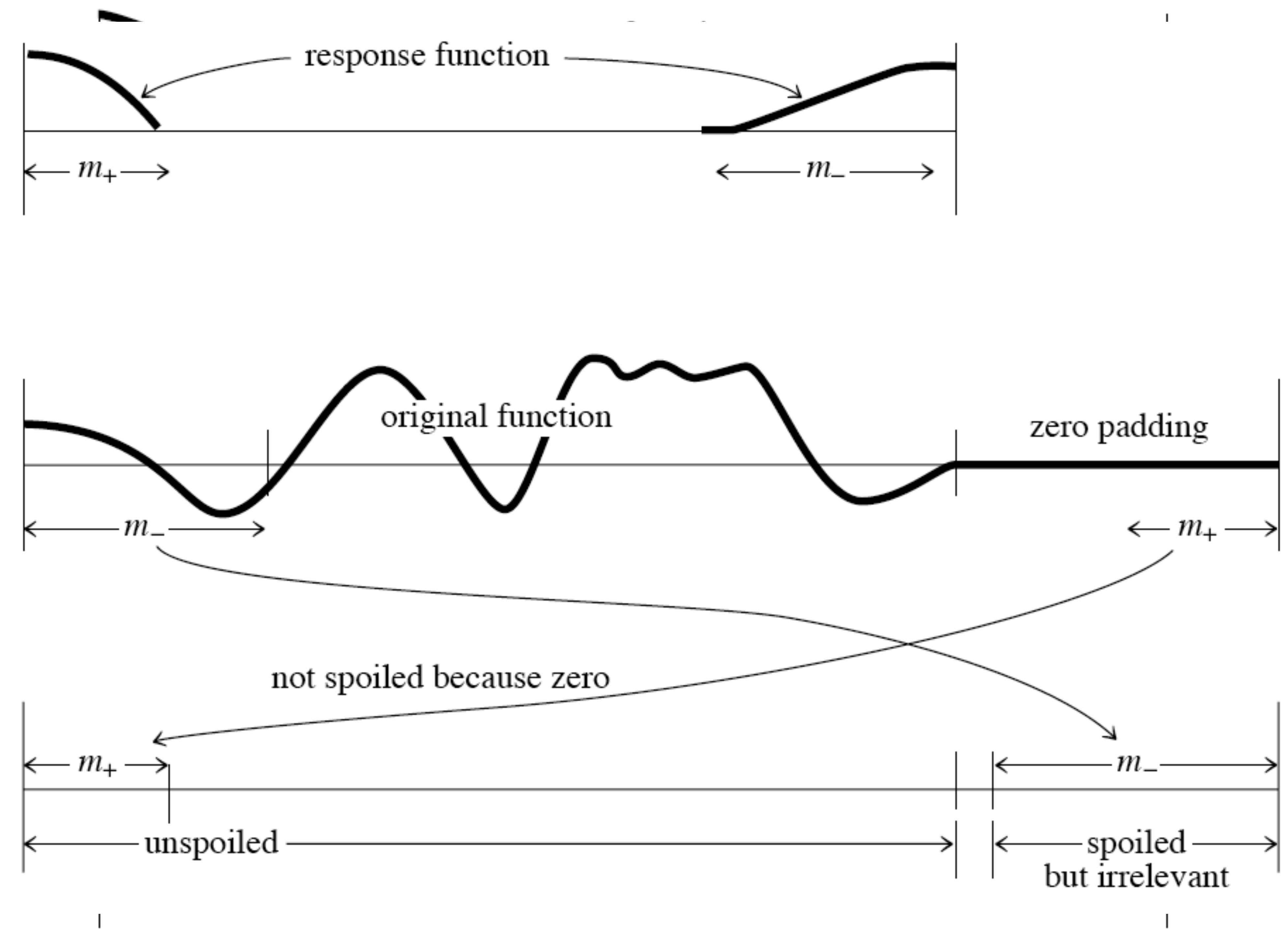
M only non-zero values of r_k



Discrete convolution theorem



Discrete convolution theorem



Discrete convolution theorem

Discrete deconvolution

$$\frac{\tilde{F}(r * s)_j}{R_n} = S_n$$

However noise and uncertainties in response can make this process unreliable

Discrete convolution theorem

Discrete deconvolution

$$\frac{\tilde{F}(r * s)_j}{R_n} = S_n$$

However noise and uncertainties in response can make this process unreliable

Similar to the continuous case!

Noise removal by optimal filtering

$$cs(t) = s(t) + n(t)$$

$s(t)$ is the smeared signal i.e. true $\ast$ response

Design an optimal filter ϕ(t) that produces a signal $\tilde{u}(t)$ as close as possible to u(t)

$$\widetilde{U(f)} = \frac{C(f)\phi(f)}{R(f)}$$

Close in least square sense

$\int_{-\infty}^{\infty} |\widetilde{U(f)} - U(f)|^2 df$ is minimised

Noise removal by optimal filtering

$$\int_{-\infty}^{\infty} \left| \frac{[S(f)+N(f)]\phi(f)}{R(f)} - \frac{S(f)}{R(f)} \right|^2 df$$

$\int S(f)N(f)df$ terms are zero since noise and signal are uncorrelated

$$\int_{-\infty}^{\infty} |R(f)|^{-2} \underbrace{\{|S(f)|^2|1-\phi(f)|^2 + |N(f)|^2|\phi(f)|^2\}}_{\Theta} df$$

Θ minimised with respect to ϕ

Noise removal by optimal filtering

$$\frac{d\theta}{d\phi} = 0$$

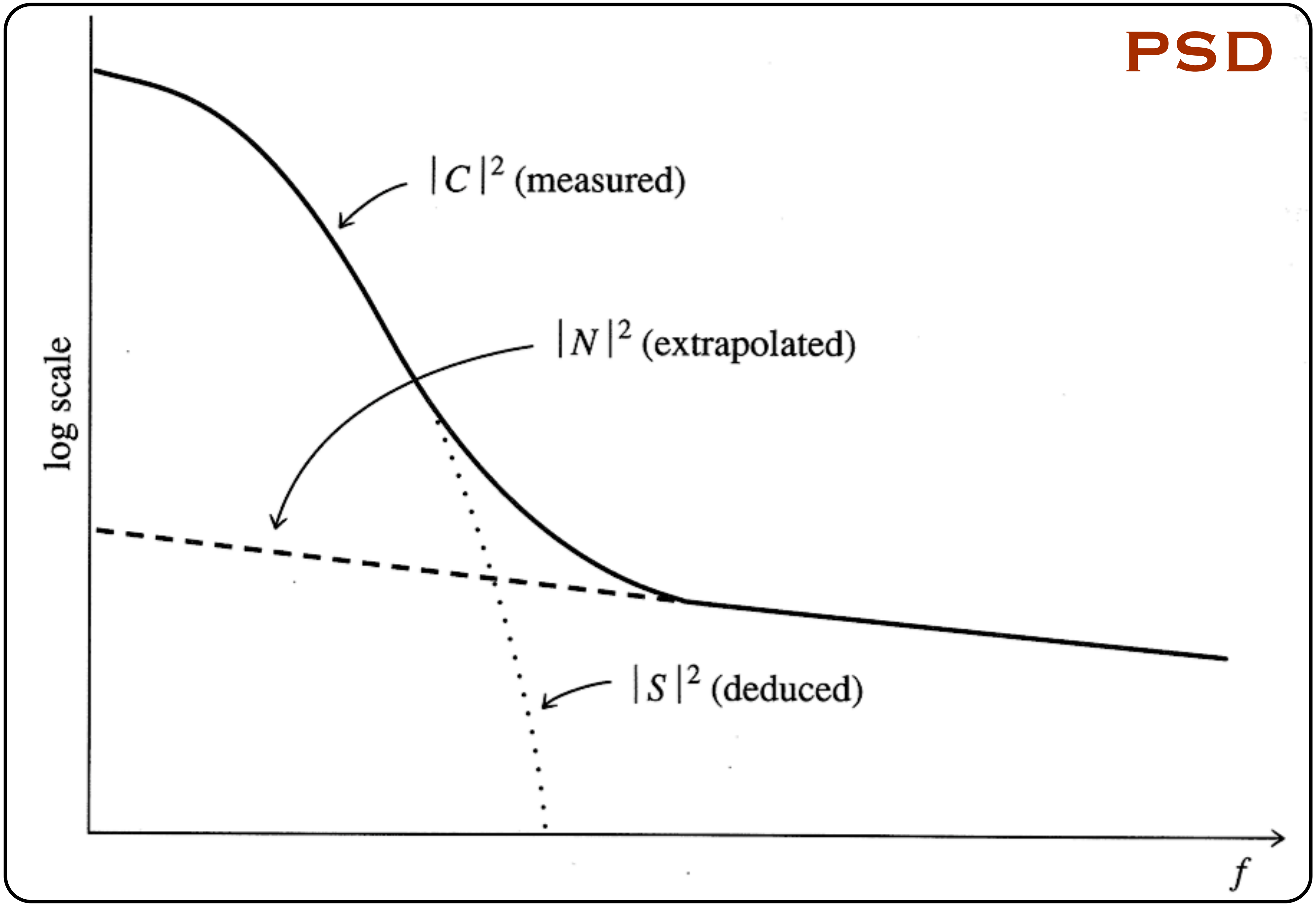
$$-2S^2(1-\phi) + 2N^2\phi = 0$$

Optimal filter $$\phi = \frac{S^2}{S^2 + N^2}$$

Does not contain true signal directly!

$$|S(f)|^2 + |N(f)|^2 = PDS(f) = |CS(f)|^2$$

Noise removal by optimal filtering



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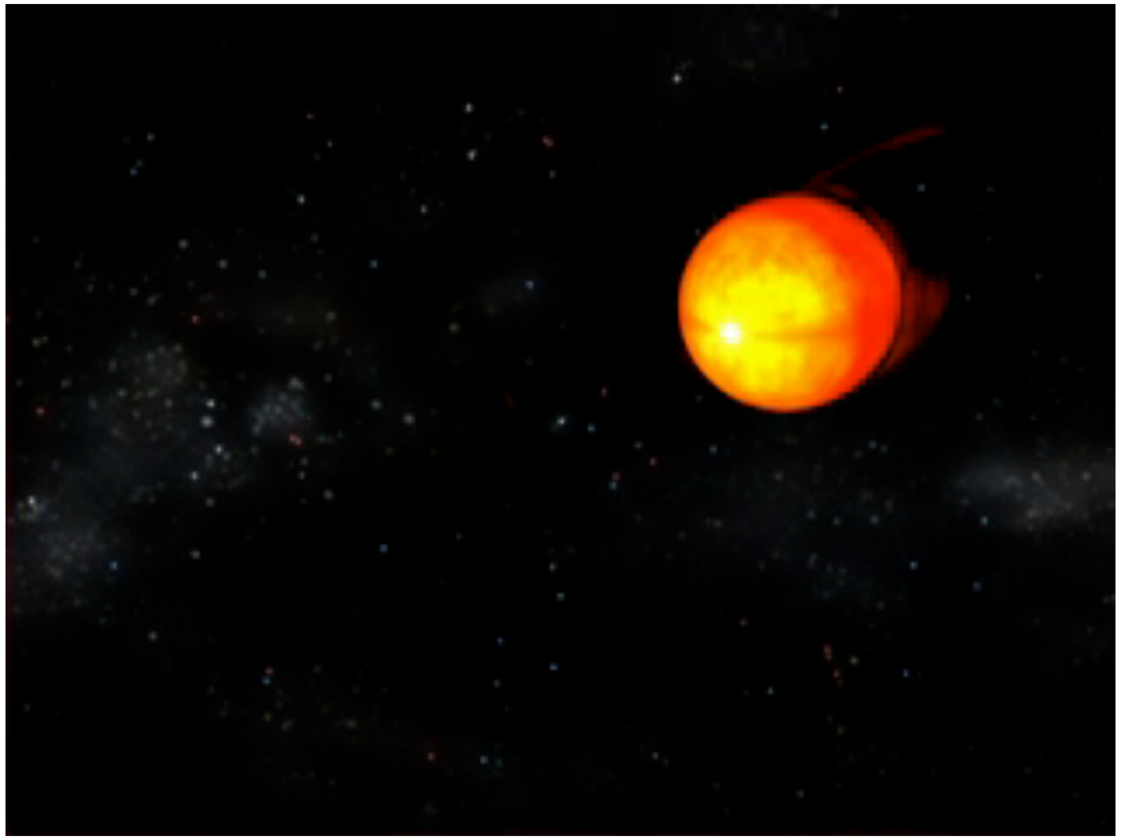
Some Applications of filtering

On optimal detection of point sources in CMB maps
Vio et al, 2002, A&A, 391, 789

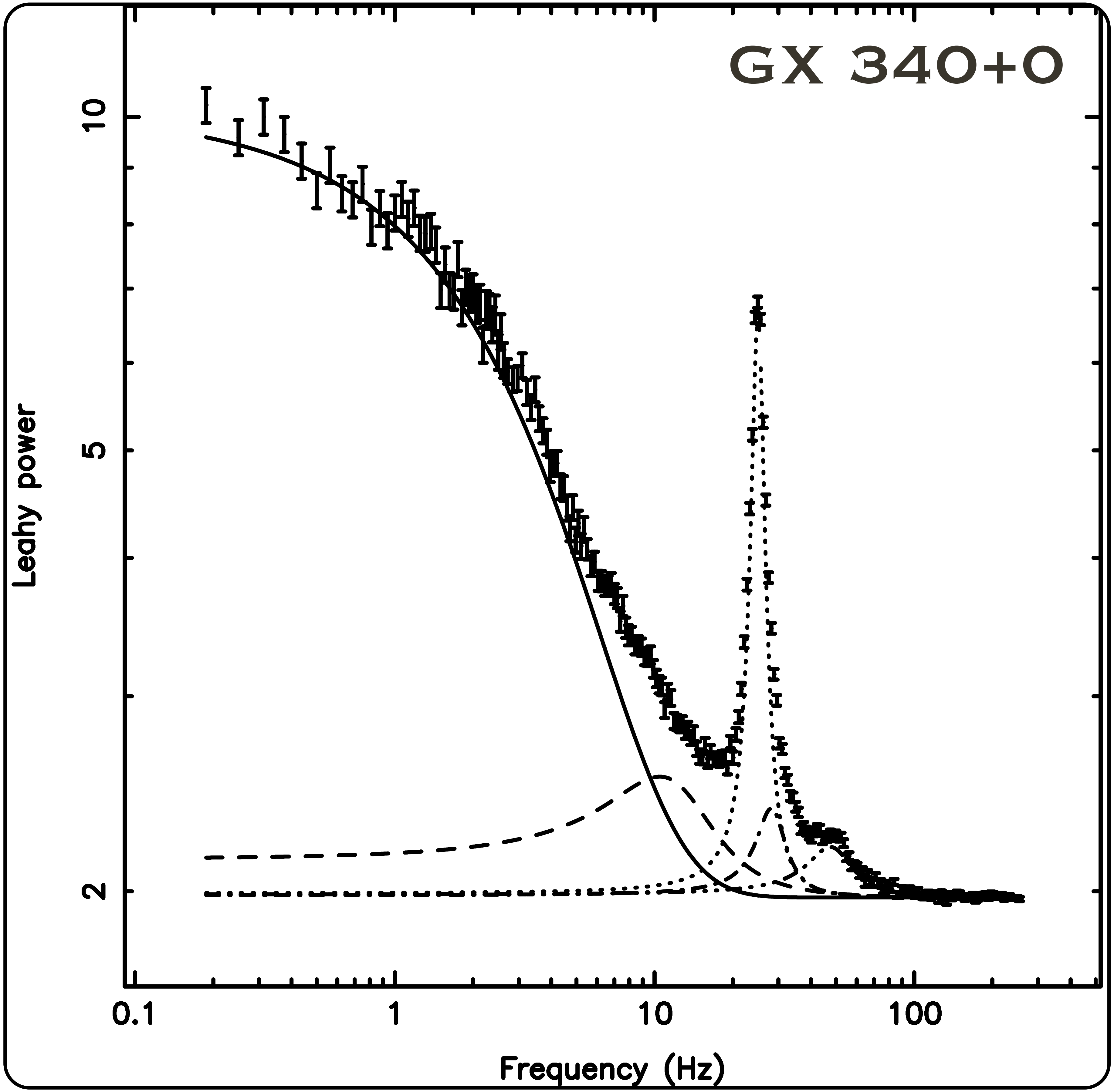
An optimal filter for the detection of galaxy clusters through weak lensing
Maturi, et al. 2005, A&A, 442, 851

The largest scale perturbations: A window on the physics of the beginning
Wandelt, New Astronomy Review, 2006, 11, 900

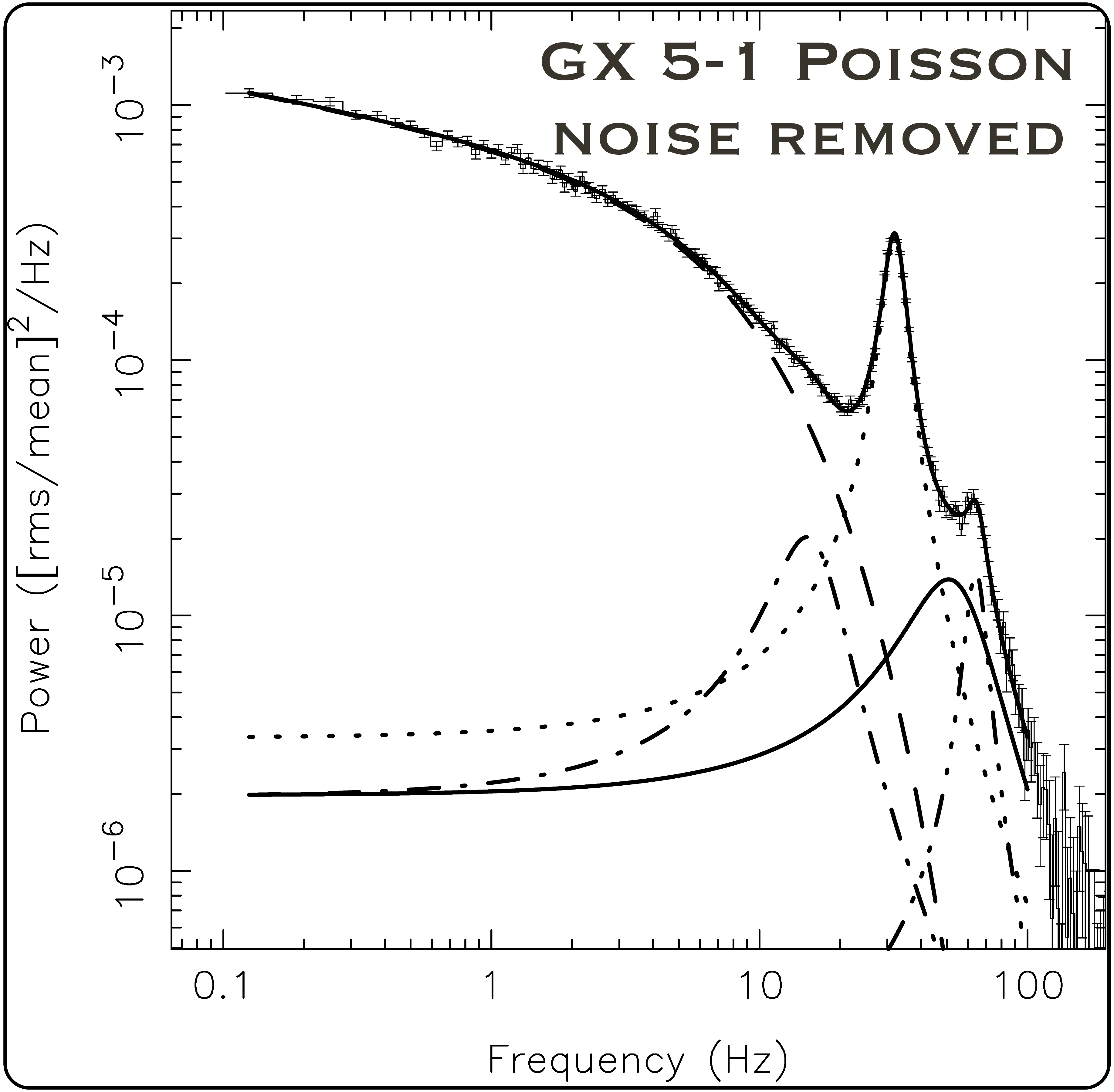
Low-mass X-ray binaries



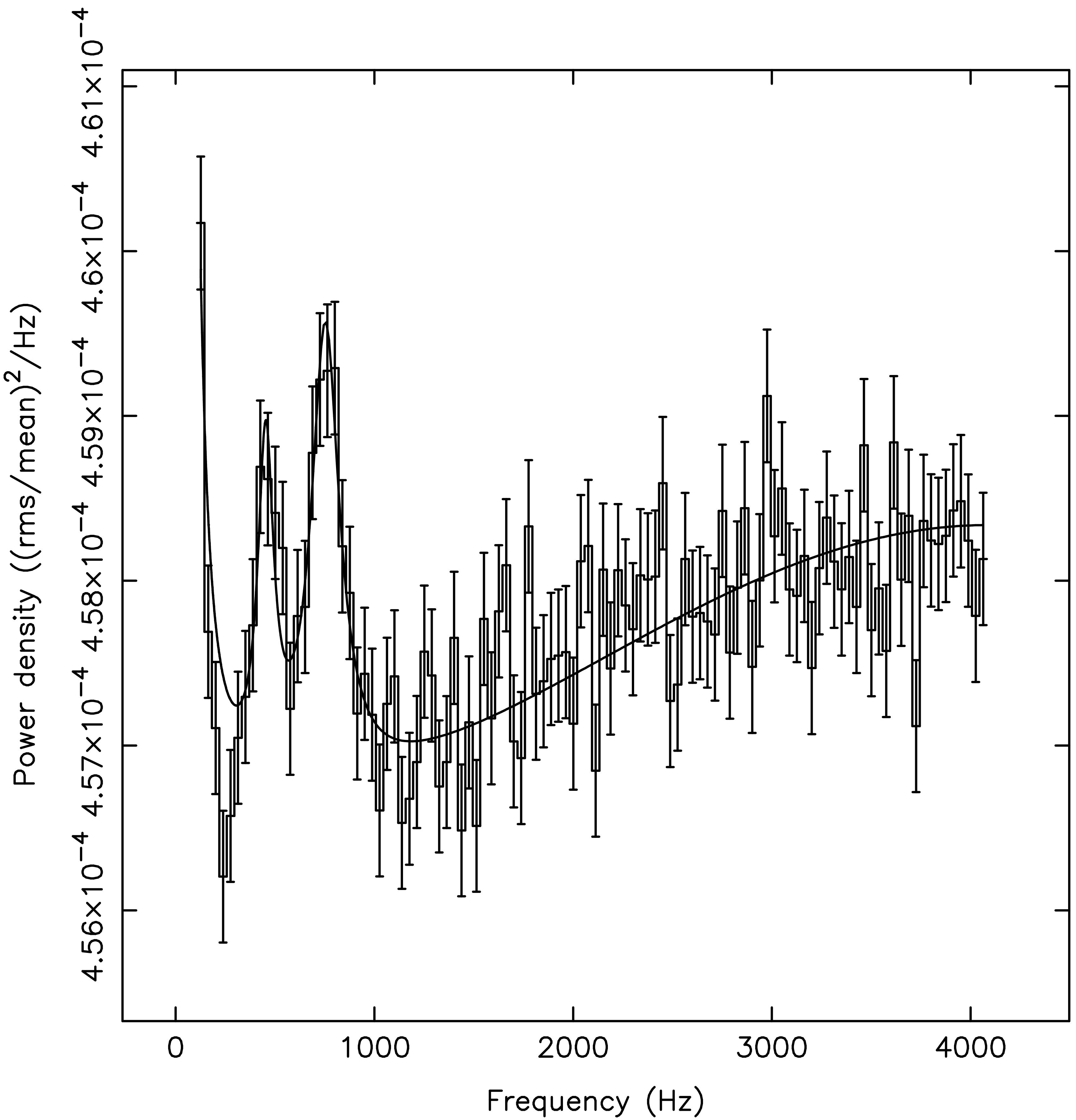
C.F. X-RAY TIMING EXPERIMENTS



X-ray Timing experiments



X-ray Timing experiments



Recap Filtering:

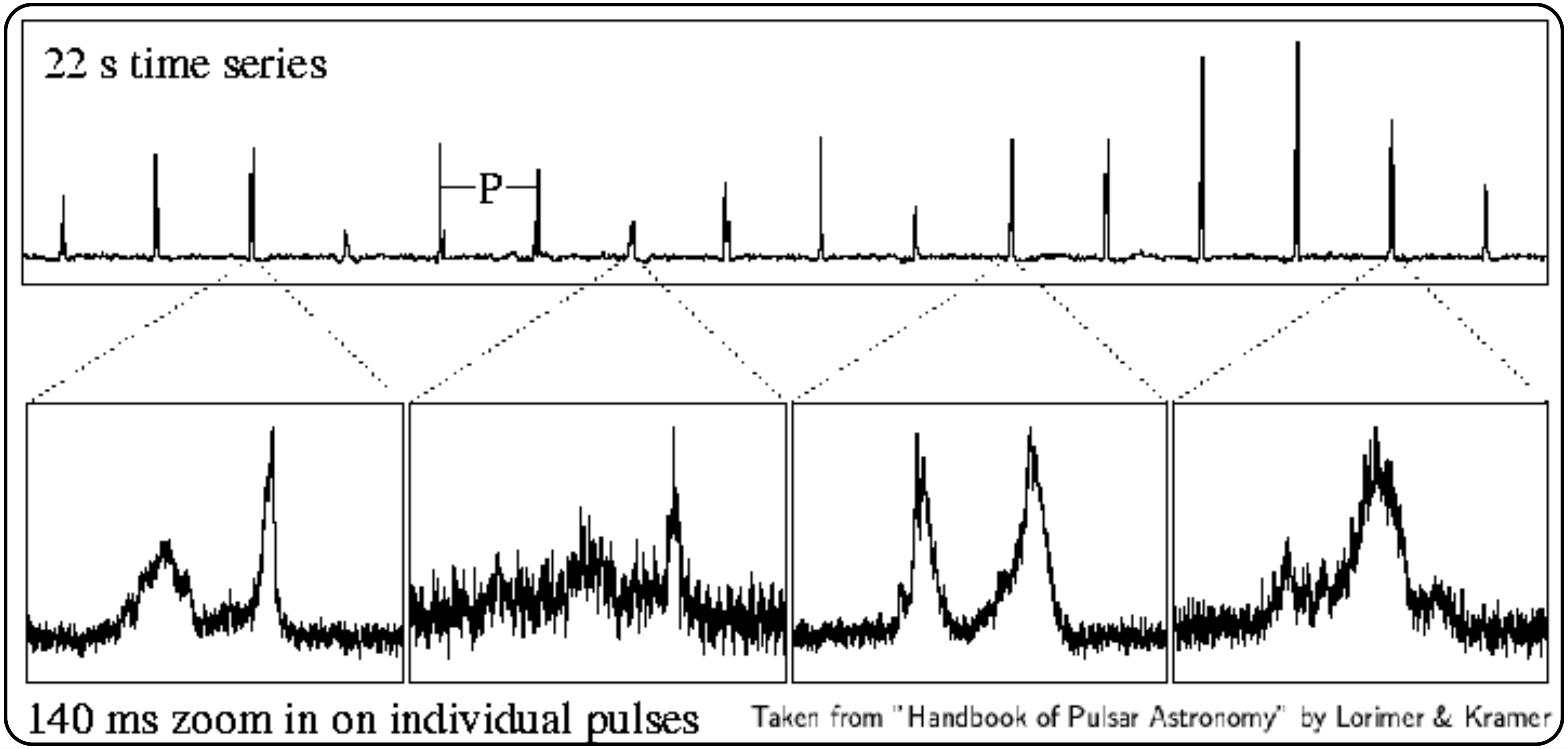
one can design an optimal filter such that the filtered measured data-set is as close as possible (in least-square sense) to the uncorrupted signal

ESTIMATING THE MOMENTS OF A STOCHASTIC PROCESS

Chapter 2.2.2

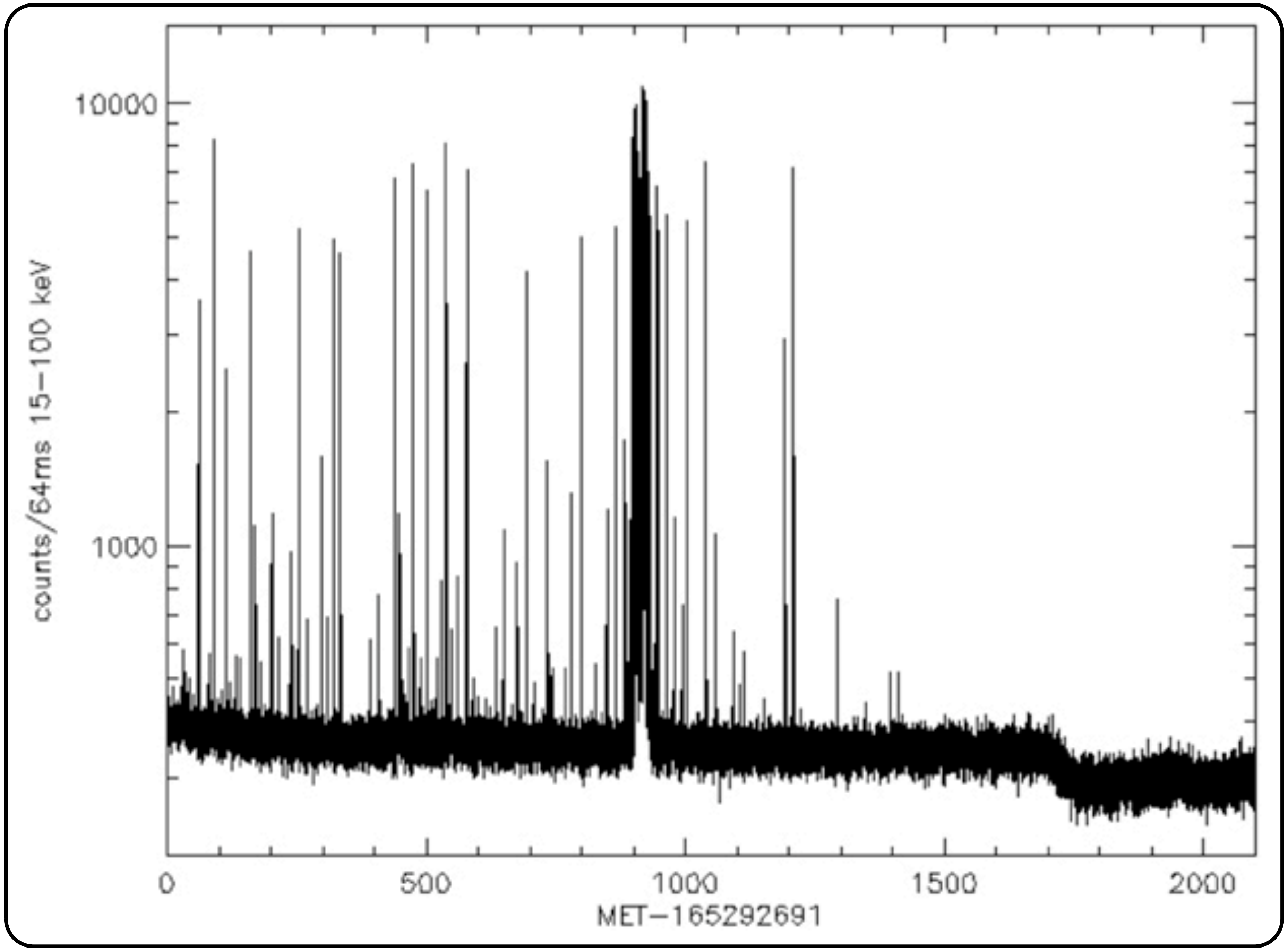
See also Appendix B3.2, Lena ea.

How representative is a measurement of a S.P.?



How representative is a measurement?

SGR 1900+14



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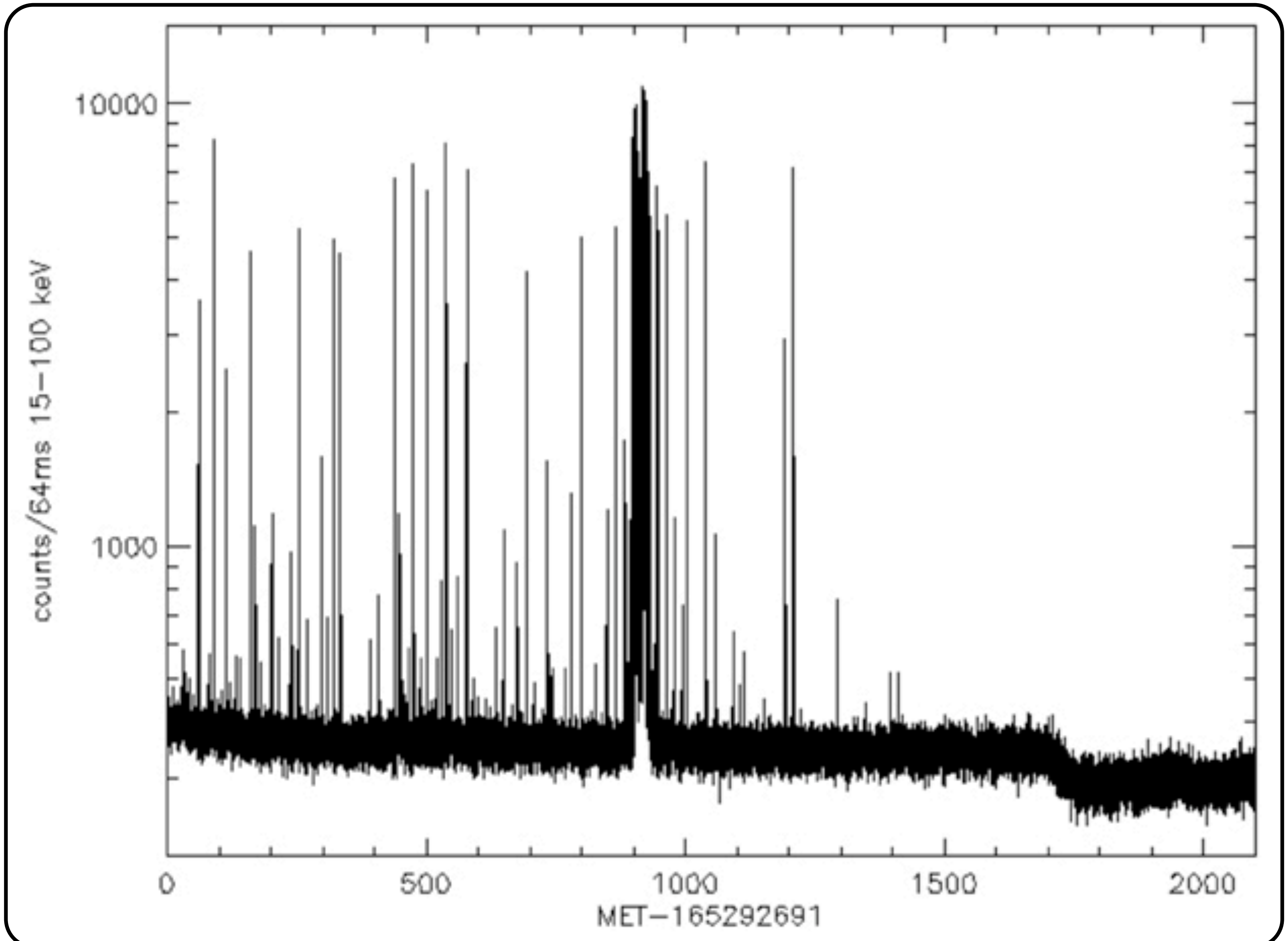
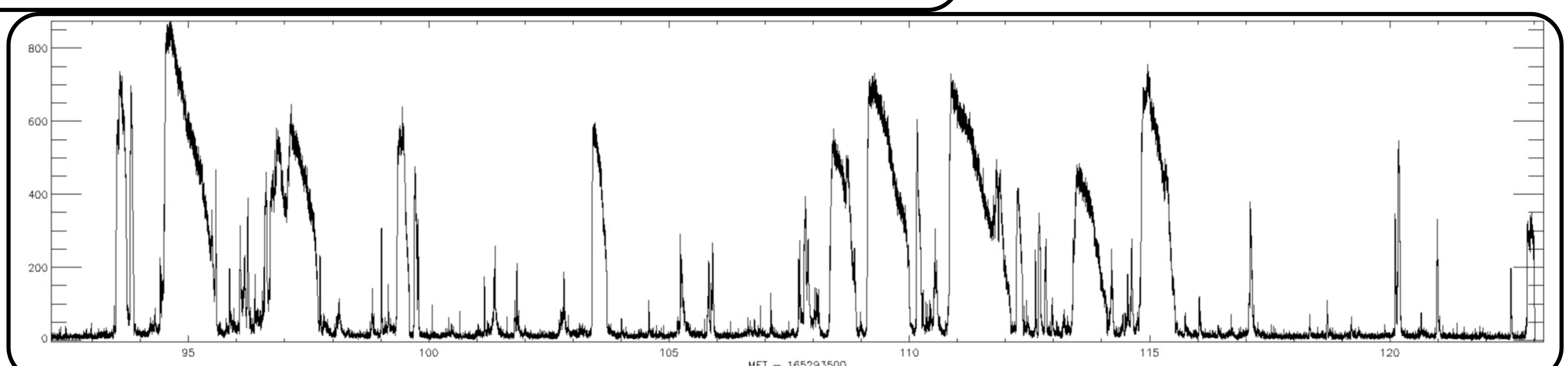
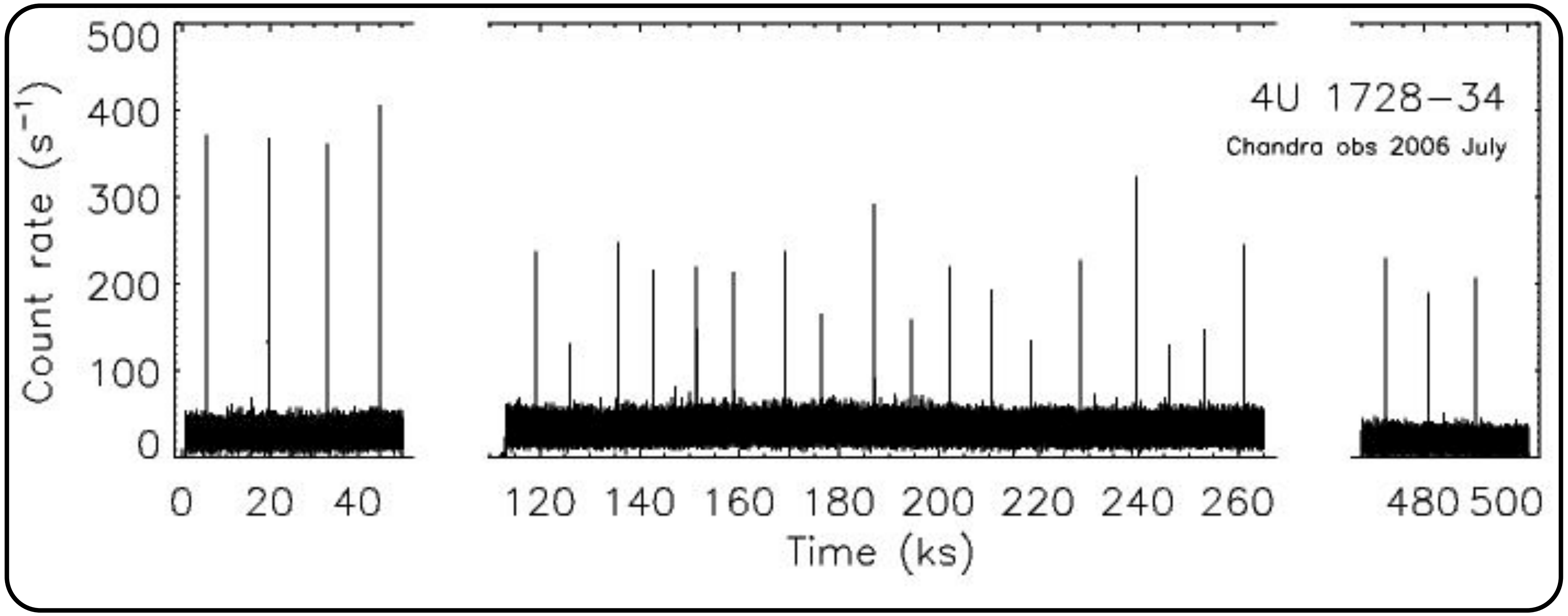


ZOOM



THE CASE OF TYPE I X-RAY BURSTS



GALLOWAY PRIVATE COMMUNICATION

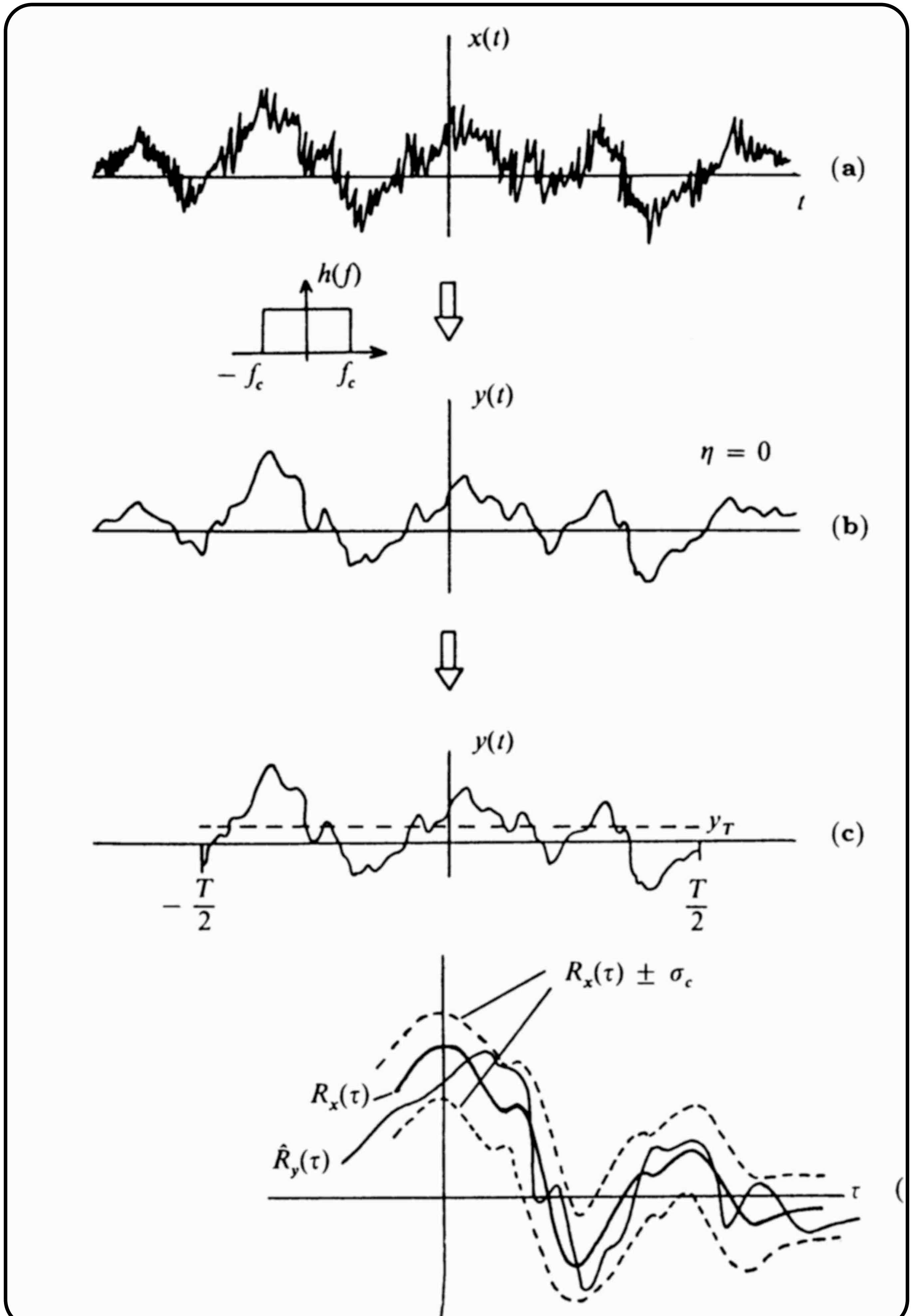
1 MEASUREMENT OF X(T) IN A TIME T => WINDOWING AND AVERAGING OVER TIME ΔT

WINDOWING: $$y(t) = \Pi(\frac{t}{T})x(t)$$

AVERAGING:

$$z(t) \equiv y_{\Delta T}(t) = \frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} y(t')dt' = \frac{1}{\Delta T}\int_{-\infty}^{\infty}\Pi(\frac{t-t'}{\Delta T})y(t')dt'$$

= LOW-PASS FILTER, REMEMBER NYQUIST THEOREM



RESPONSE →

REMOVE HIGH FREQUENCIES DUE TO FINITE RESPONSE OF INSTRUMENT AND DETECTOR

FINITE DURATION →

TIME DOMAIN, MULTIPLICATION WITH BOX

1 SAMPLE → AVERAGING →

TIME -> CONVOLVING, HENCE SMOOTHING, WITH SINC IN FREQ DOMAIN

Derivation on black board page 25,26,27 lecture notes OAF2

$$\sigma^2_{x_T} = \sigma^2/N \text{ for T} >> \tau_0$$

τ_0 depends on transfer function
which in order to avoid aliasing should be suited for the system under study

Estimating σ if the probability density function of the s.p. is known

→

Propagation of Errors chapter 5.2

Estimating σ if the probability density function of the s.p. is not known

→

e.g.

bootstrap method

Jackknife method

More on this later

Stochastic description of radiation fields

What is the size of the fluctuations in the radiation field?