Observations in Astrophysics-2

Statistical description of processes

convolution of signal with transfer function, sampling etc

requires the concept of Fourier transforms via the Convolution Theorem & Cross Correlations

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Additional reading

Numerical Recipes

Press et al. 1992

Chapters 12-0,1, 13, 14

check : www.nr.com

Observational Astrophysics

Lena, P., Lebrun, F., Mignard, F.

Data reduction and error analysis

Bevington & Robinson 1992

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Chapters 12-0,1, 13, 14

check : www.nr.com

Observational Astrophysics

Lena, P., Lebrun, F., Mignard, F.

Data reduction and error analysis

Bevington & Robinson 1992

Chapter 1.1, 1.2, 1.3 & 2.1, 2.2 (not 2.2.2), 2.3 OAF-2 & Chapters 3 & 5 of OAF-1

Useful (observational) Astrophysics websites

http://xxx.soton.ac.uk/list/astro-ph/new

http://cdsads.u-strasbg.fr/abstract_service.html

http://simbad.u-strasbg.fr/simbad/sim-fid

http://www.astronomerstelegram.org/

arXiv.org > astro-ph

Search or Article-id (Help | Advanced search)

All papers Go!

Astrophysics (since Apr 1992)

For a specific paper, enter the identifier into the top right search box.

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  - recent (last 5 mailings)
  - current month's astro-ph listings
  - specific year/month:
    2008 all months Go
- **Catch-up:**
  Changes since: 01 09 (Sep) 2008 , view results without abstracts Go
- **Search** within the astro-ph archive
- Submission statistics:
  2008 2007 2006 2005 2004 2003 2002 2001 2000 1999 1998 1997 1996 1995 1994 1993 1992

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HTTP://XXX.SOTON.AC.UK/LIST/ASTRO-PH/NEW

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Databases to query: ☑ Astronomy ☐ Physics ☑ arXiv e-prints

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☐ Exact name matching | Object name/position search

☐ Require author for selection | ☐ Require object for selection

(◉ OR ○ AND ○ simple logic) | (Combine with: ◉ OR ○ AND)

Publication Date between (MM) (YYYY) and (MM) (YYYY)

Enter Title Words ☐ Require title for selection

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Full Text Search: Search OCRd text of scanned articles

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Measurements in general

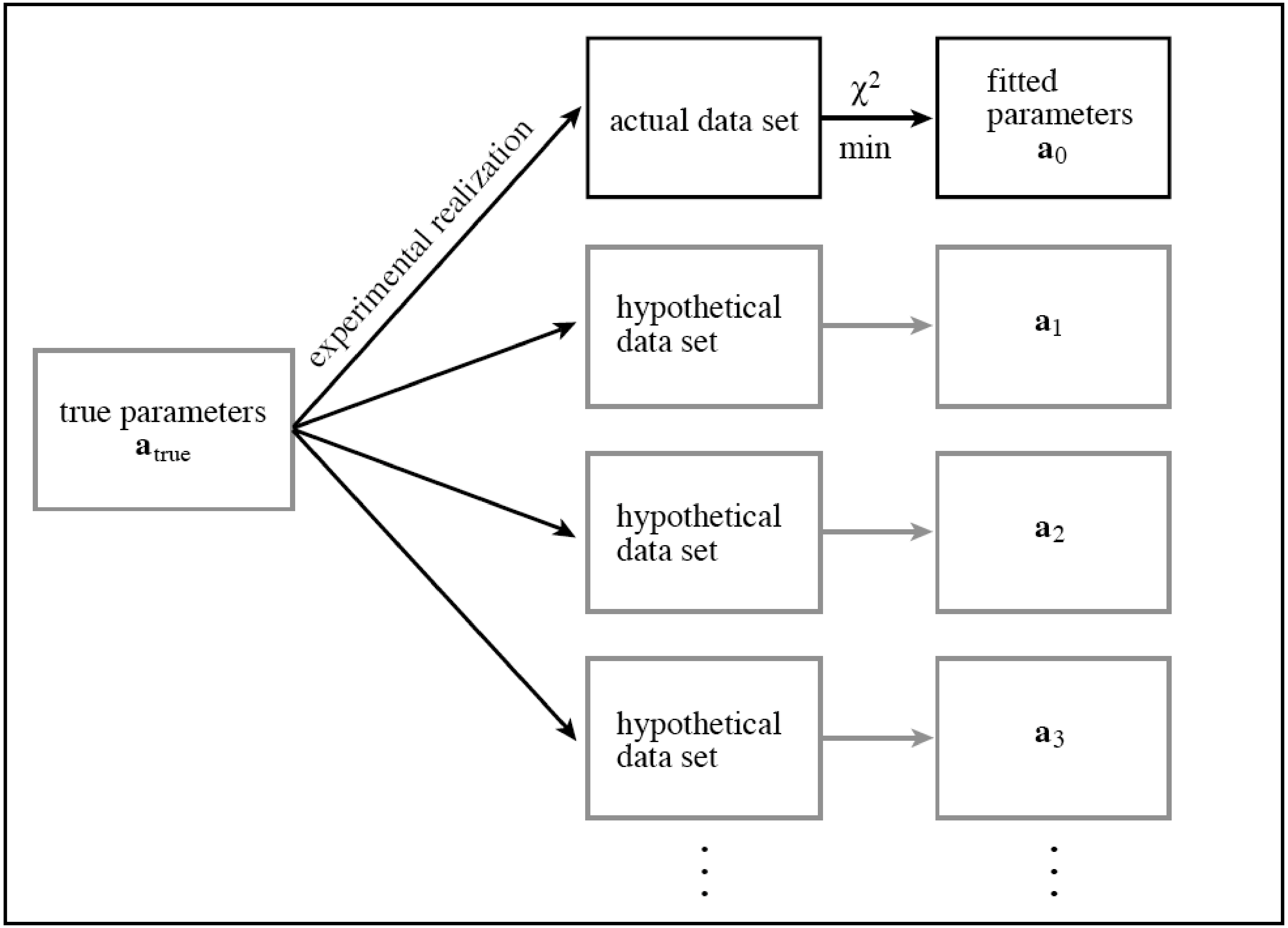
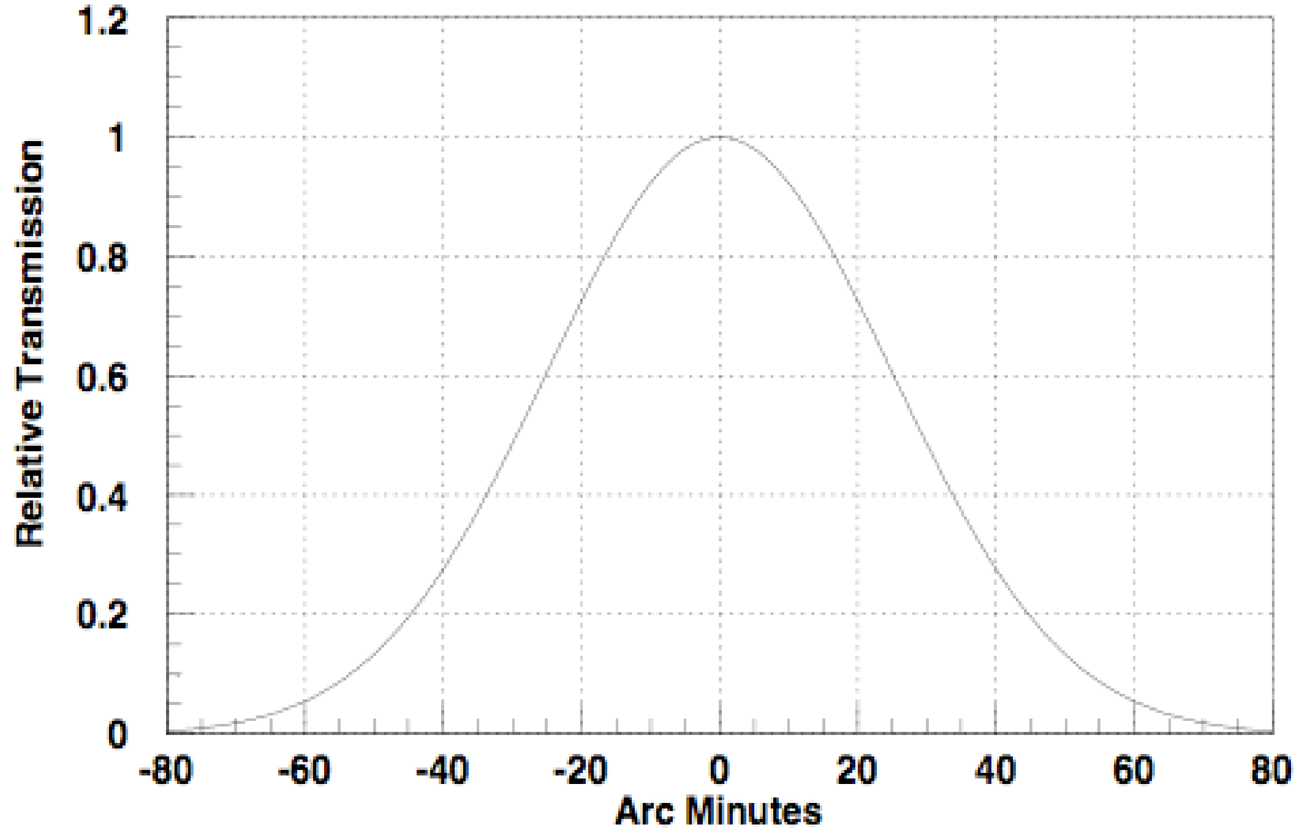


Fig from Numerical Recipes

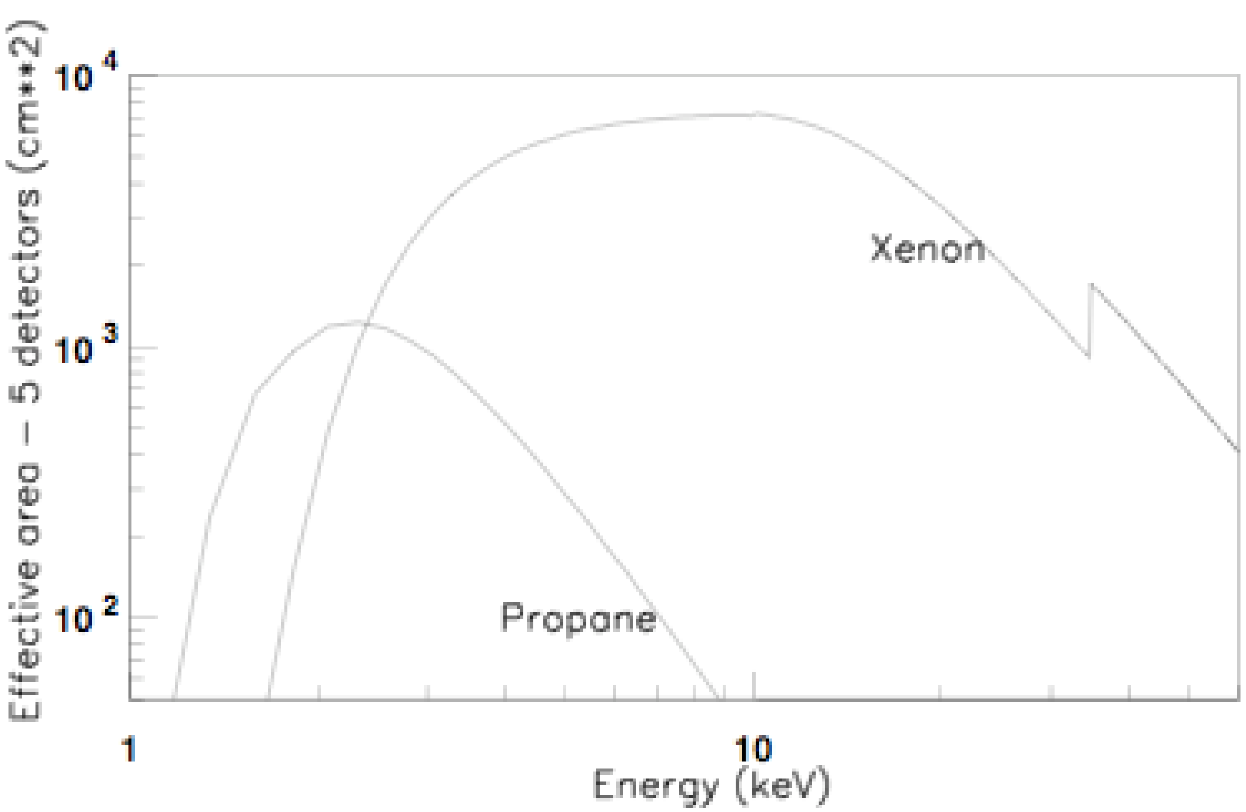
Detection of X-rays with the Rossi X-ray timing Explorer

Three instruments: an ASM, the PCA, and Hexte

Collimator response

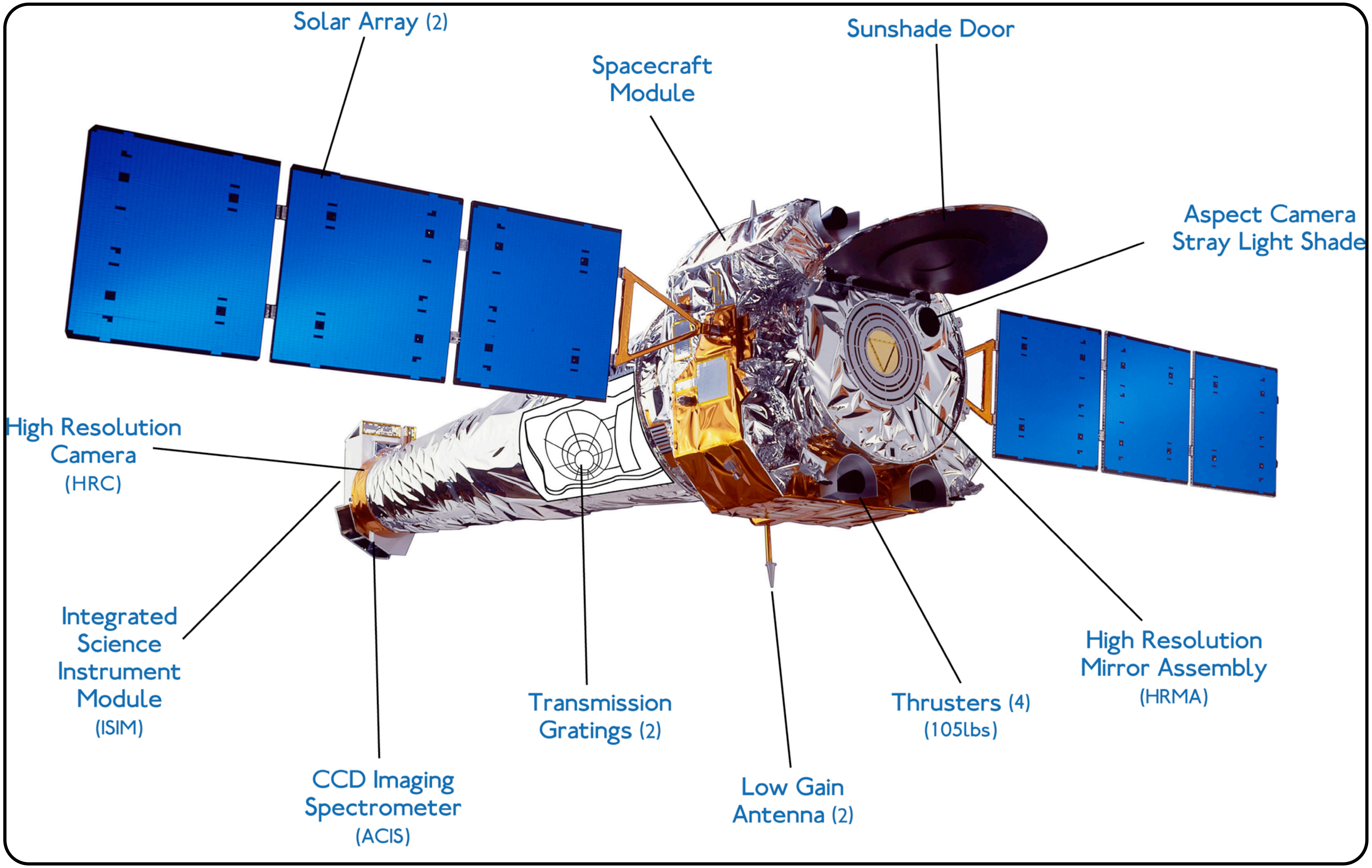


Effective Area

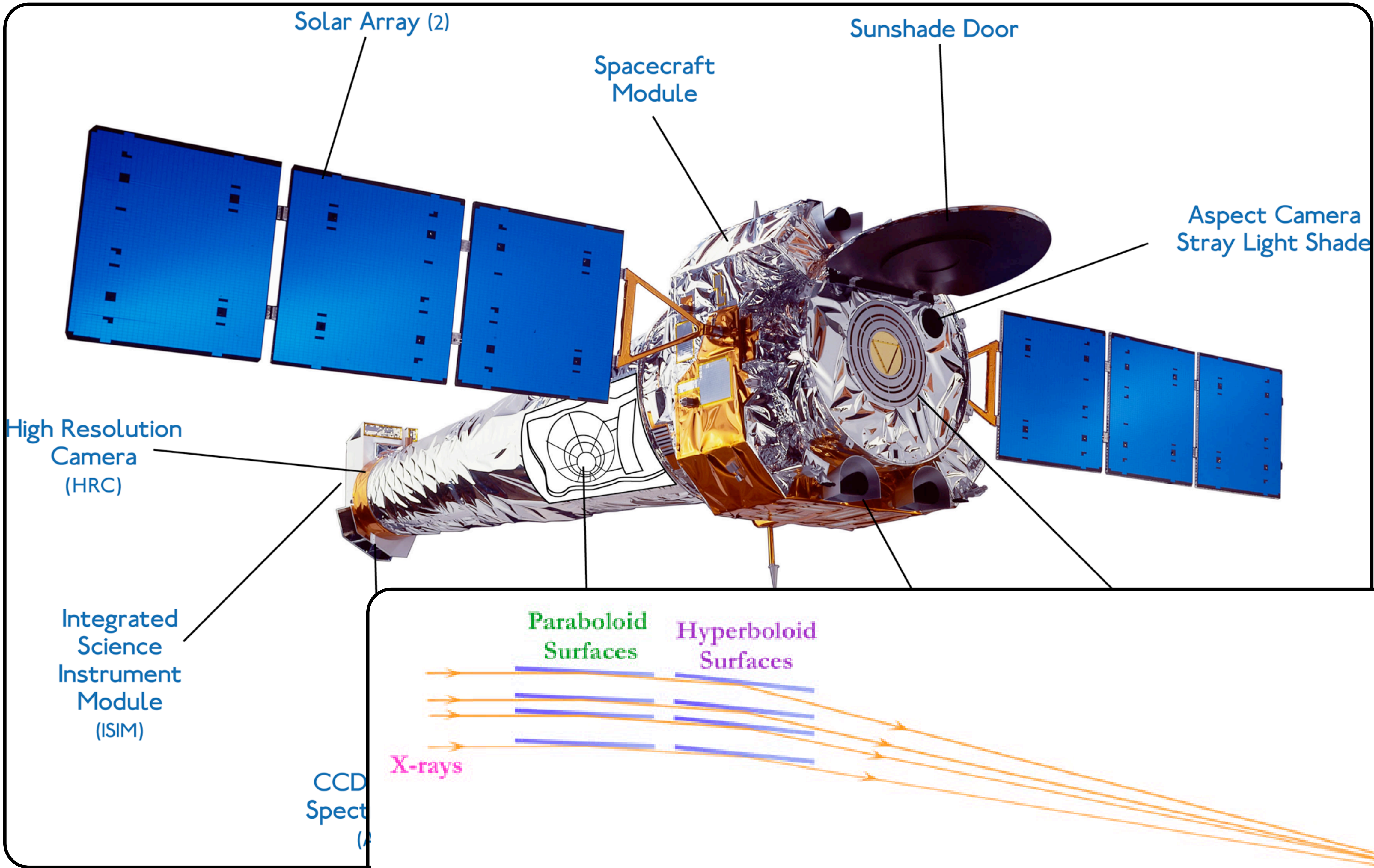


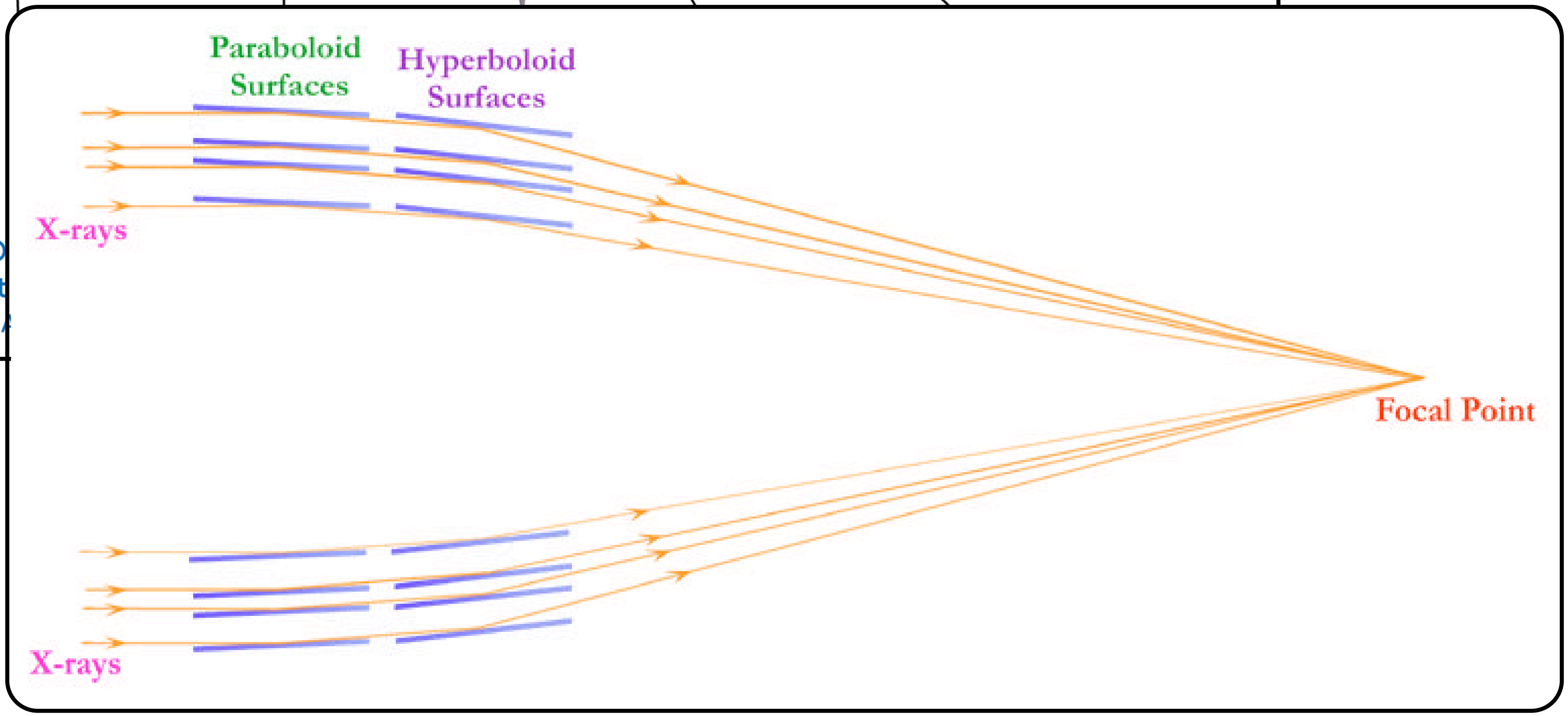
Data sampling & Data binning

Chandra satellite

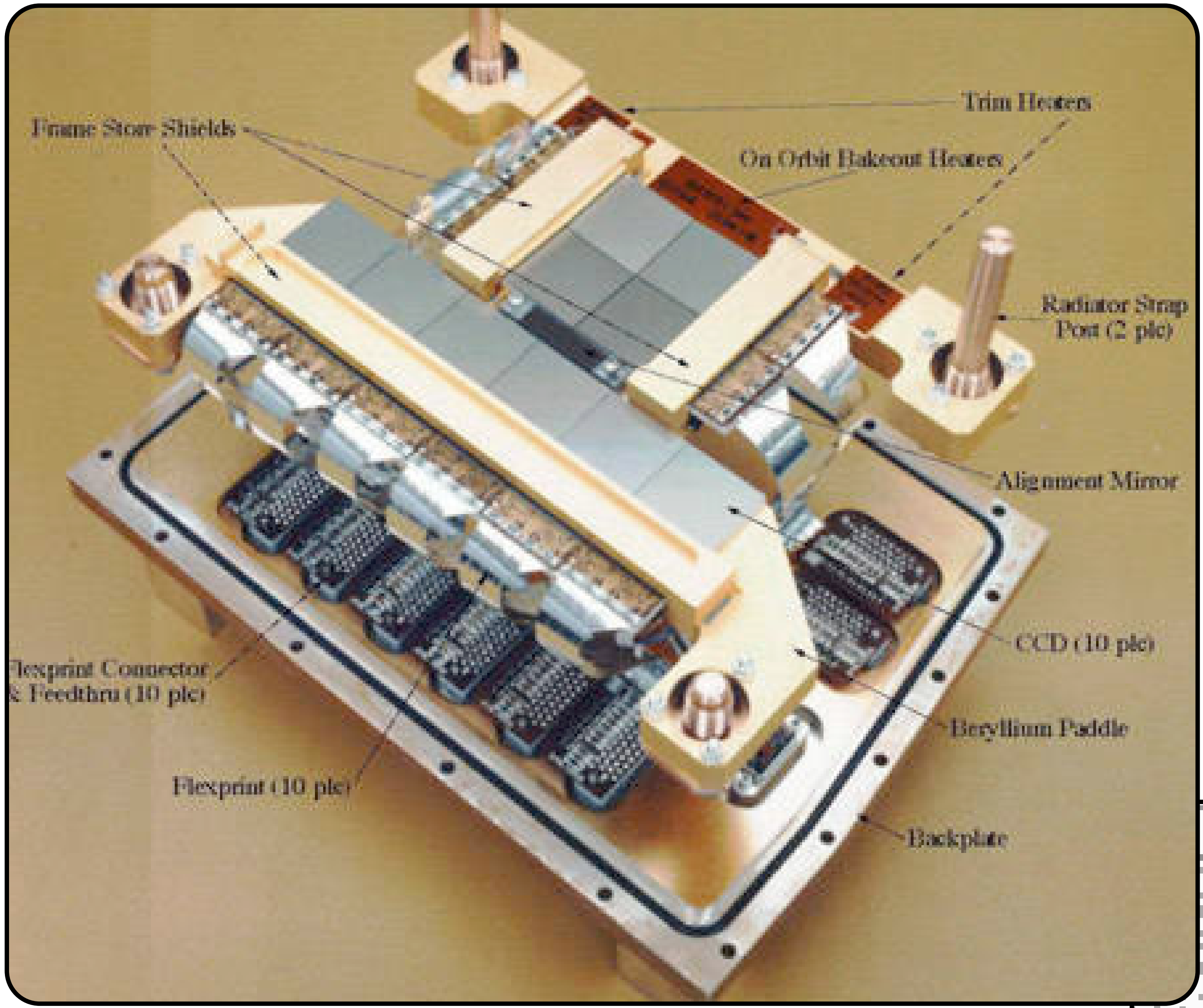


CHANDRA SATELLITE

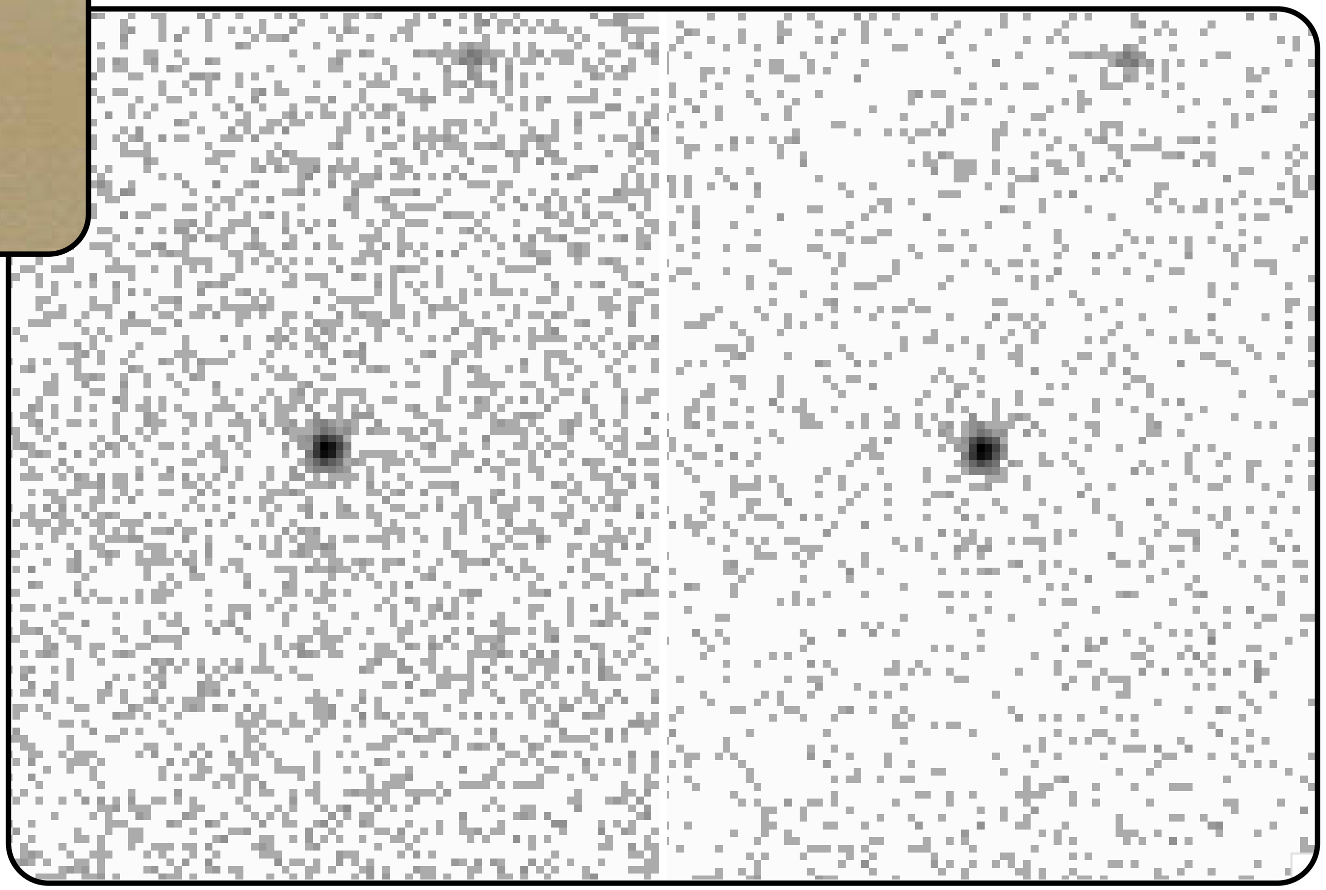




The ACIS CCDs



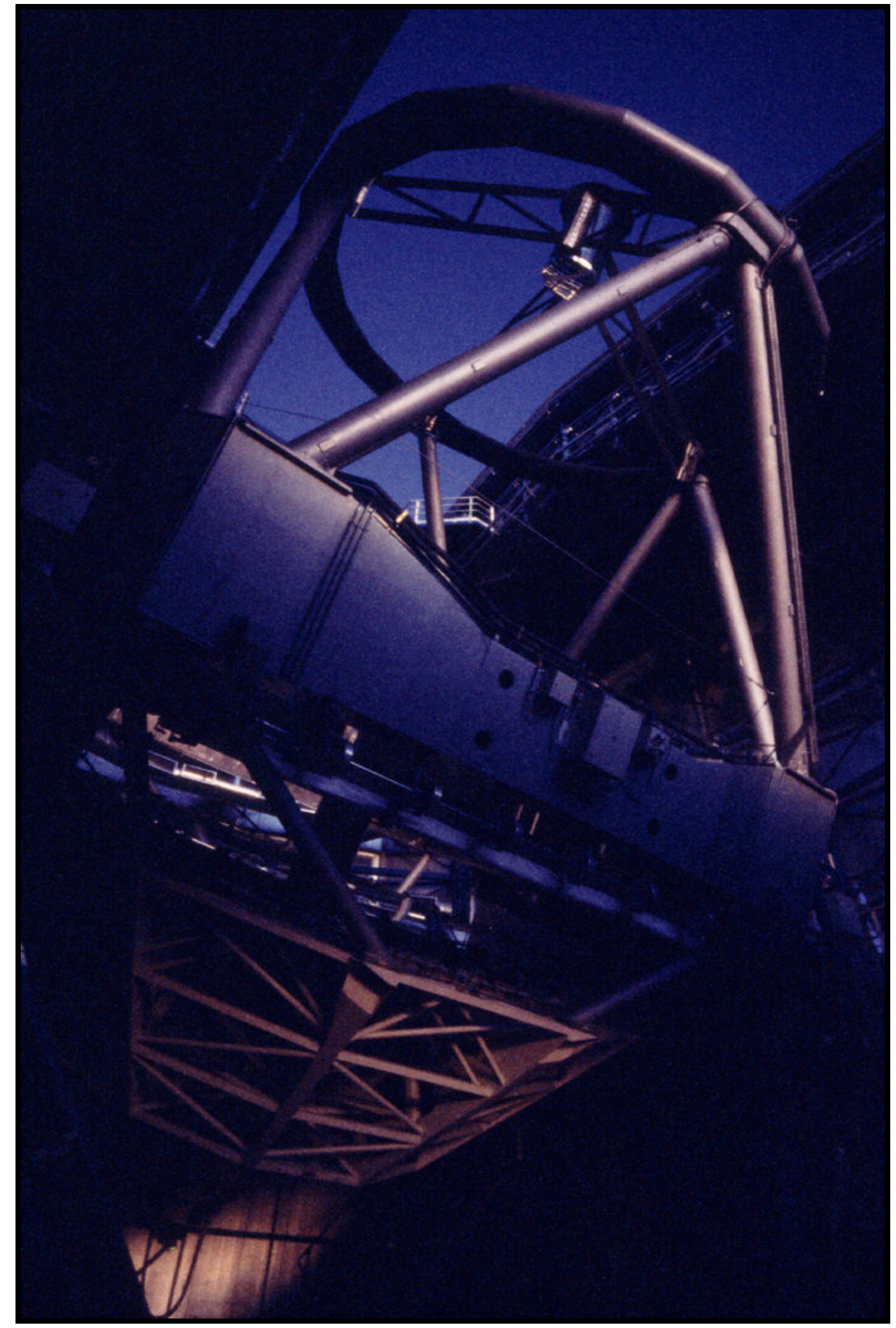
close-up of an observation



Discrete pixels & filtering

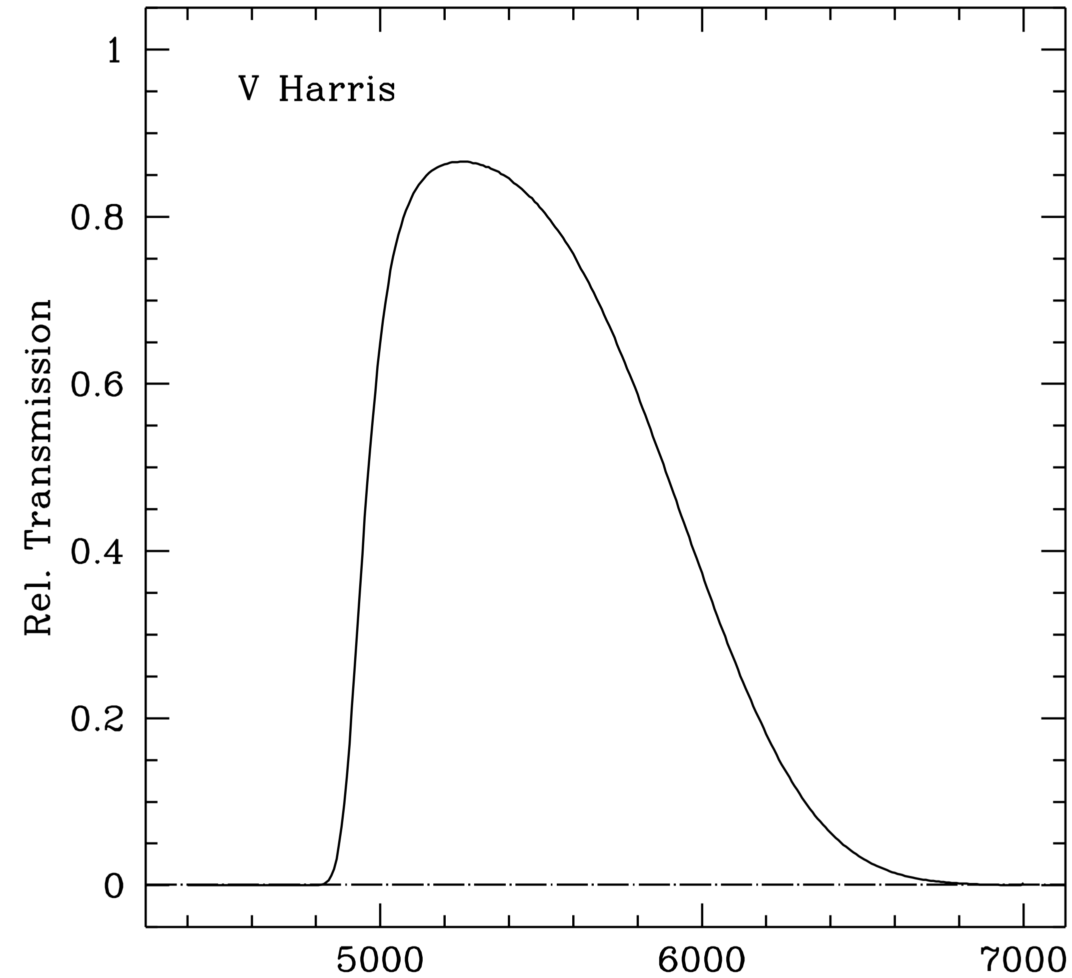
Examples: detection of optical light via a telescope

ESO's 4 Very large telescopes

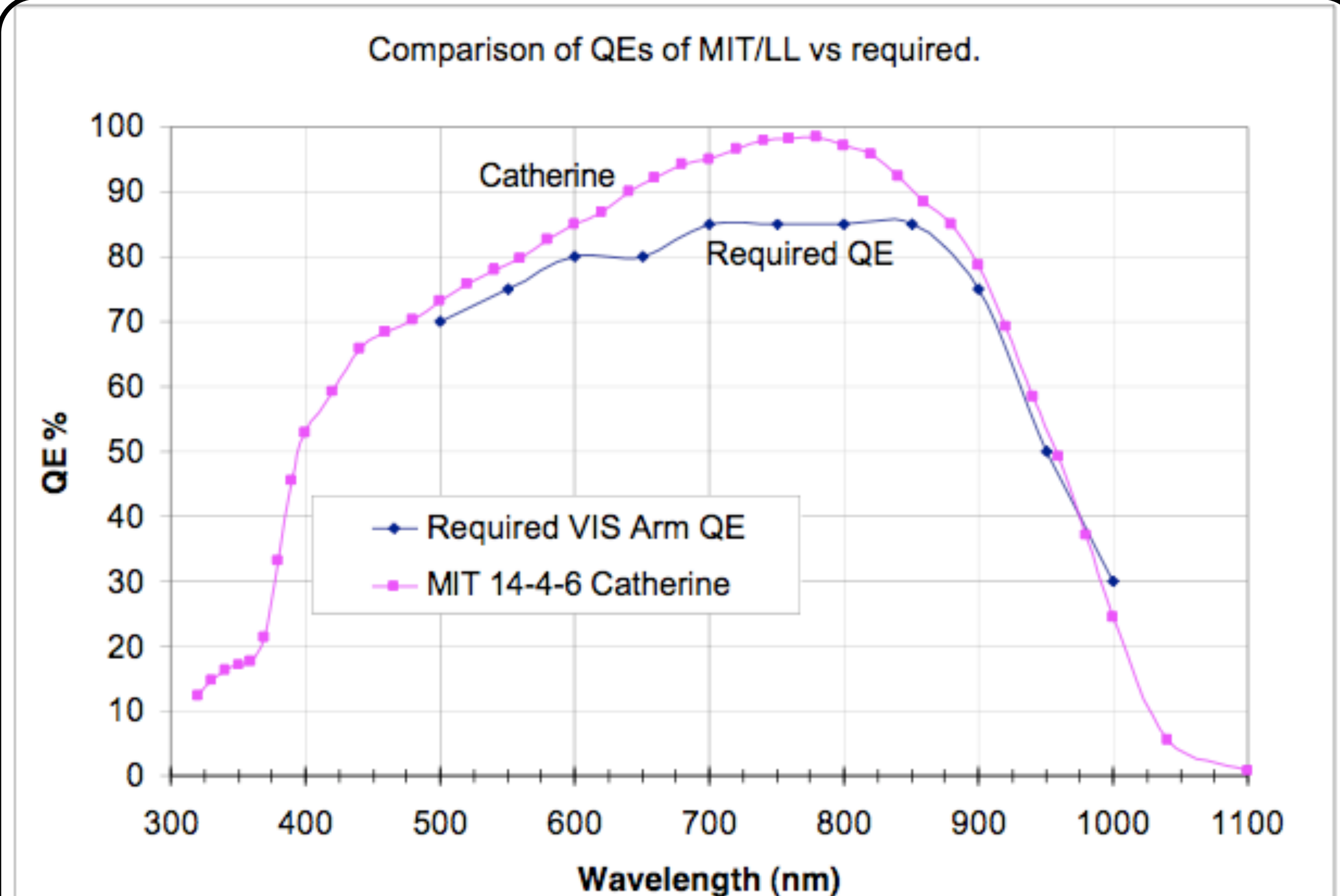


Inner workings

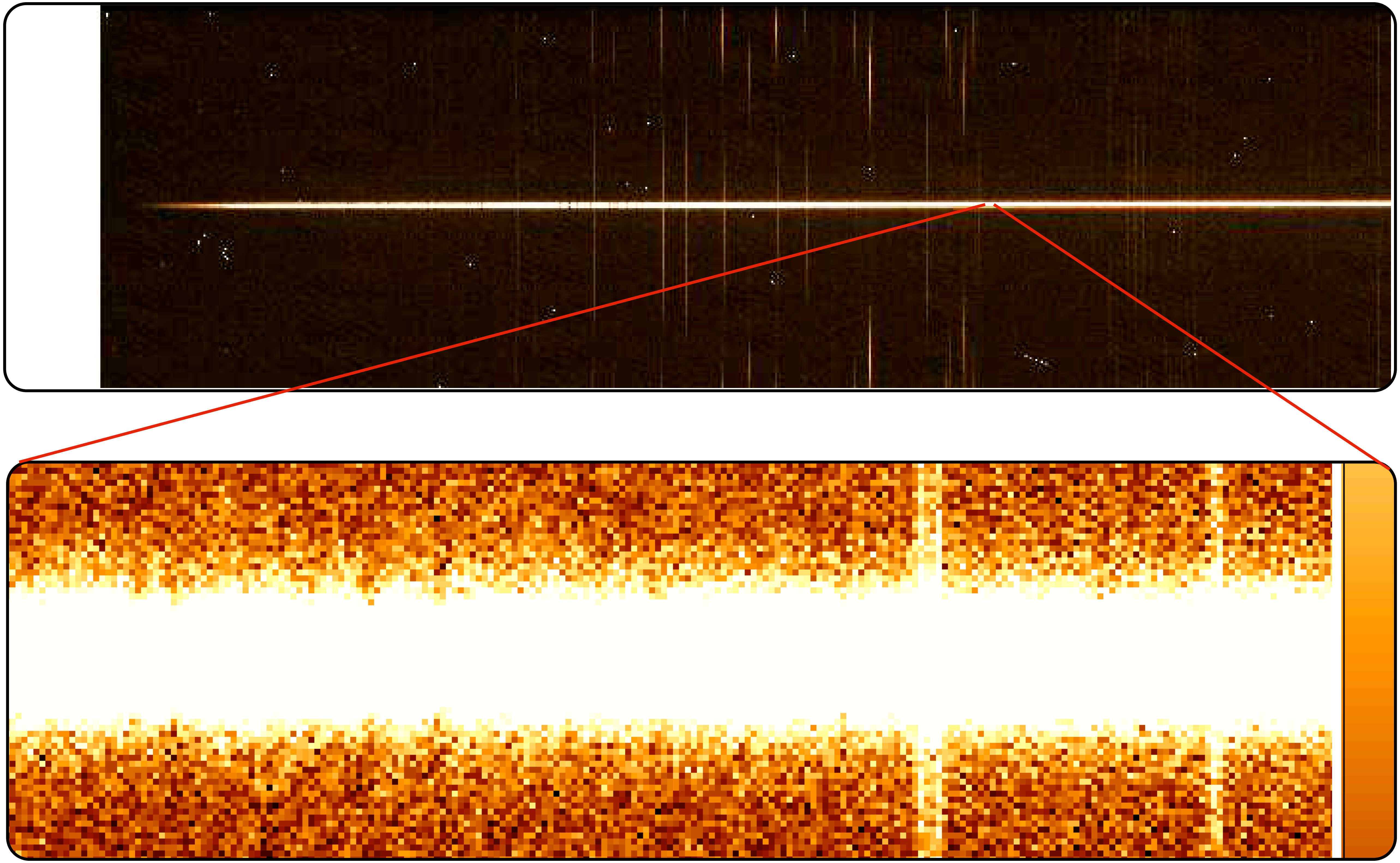
Broadband filter



efficiency detector



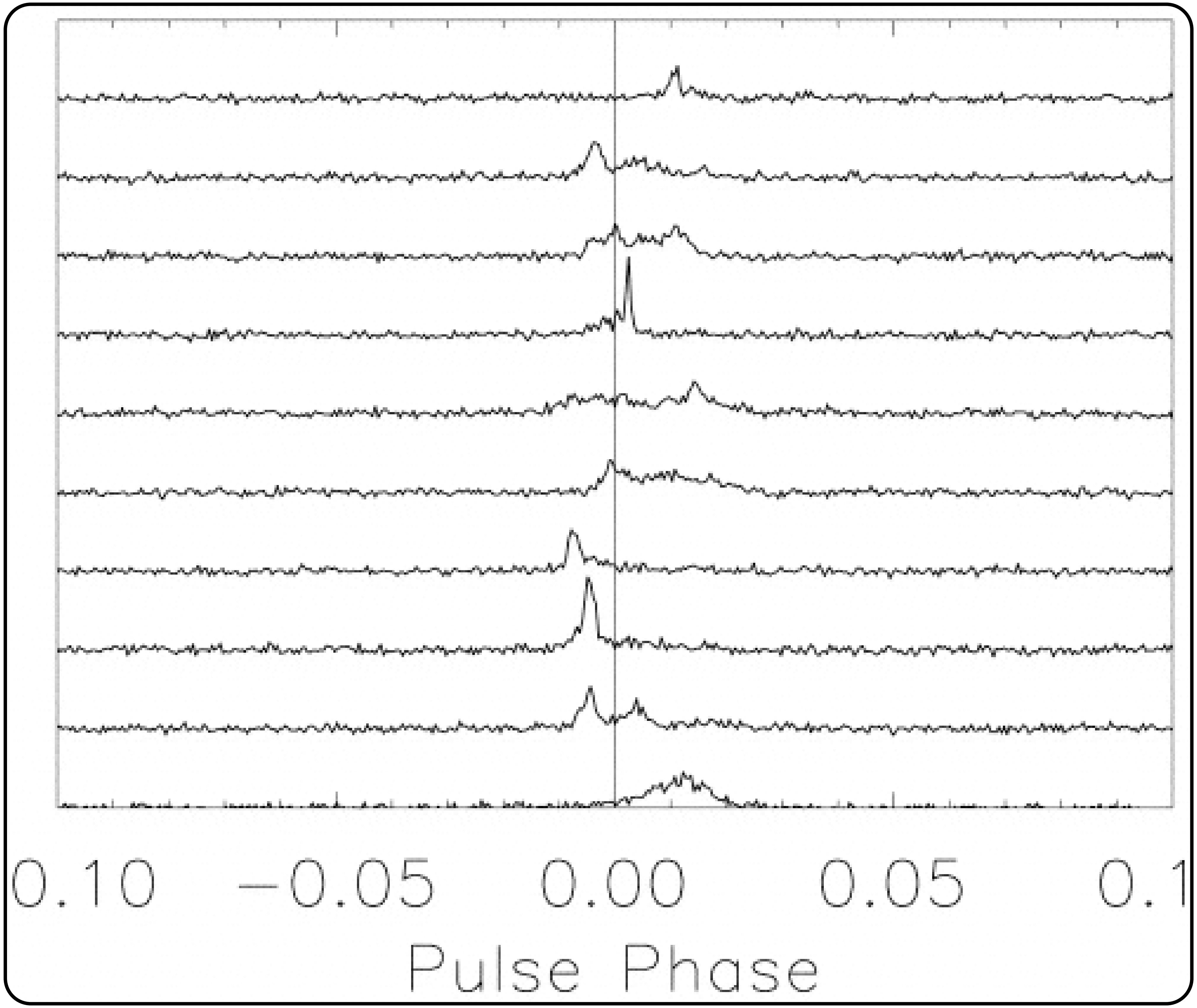
Data is discretely sampled



Optical spectrum recorded with a CCD camera

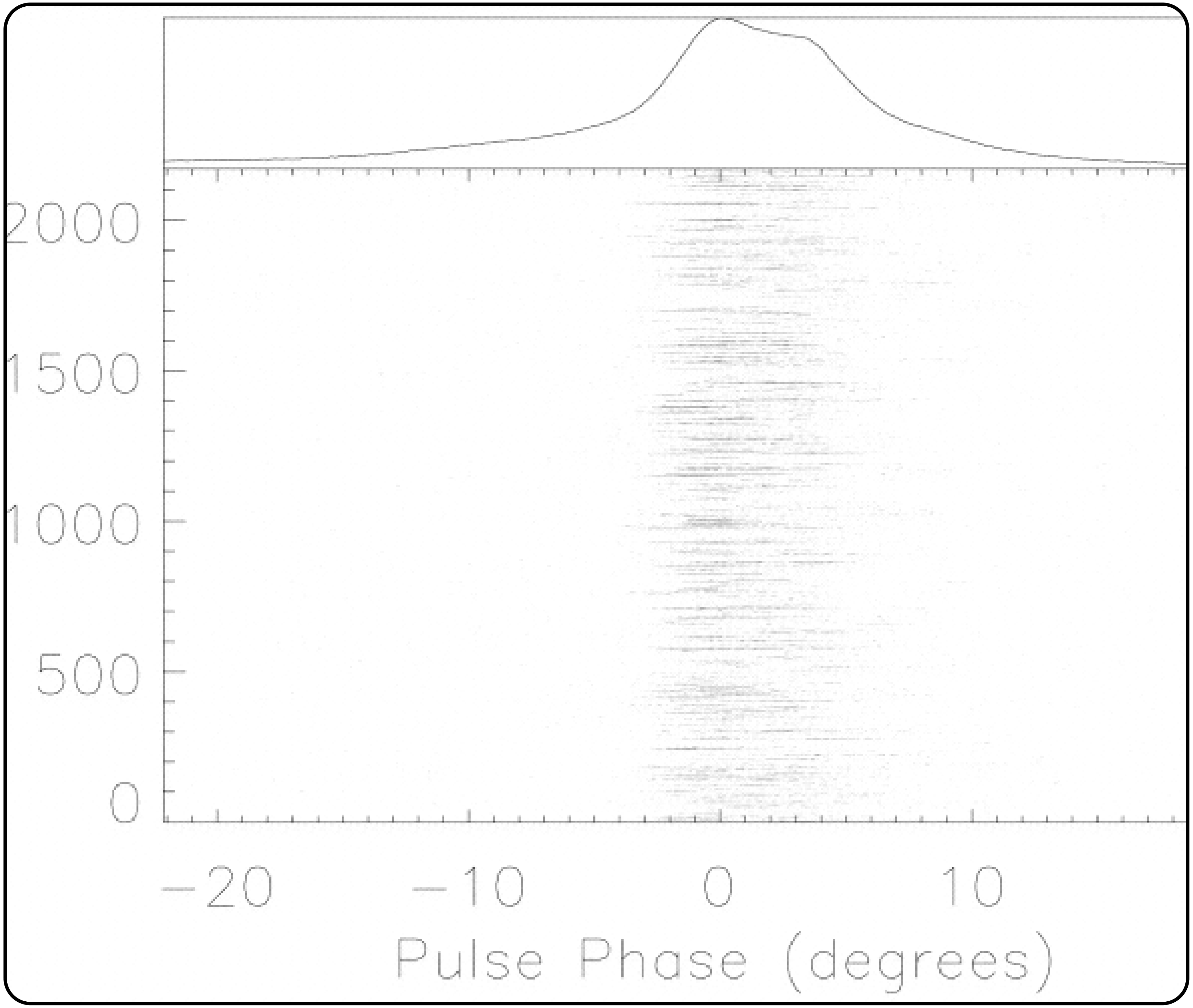
cf. Horne and Harris papers for presentation

Average pulse profile & individual pulses



PSR J0437-4715 Janet et al. 1998

Average pulse profile & individual pulses

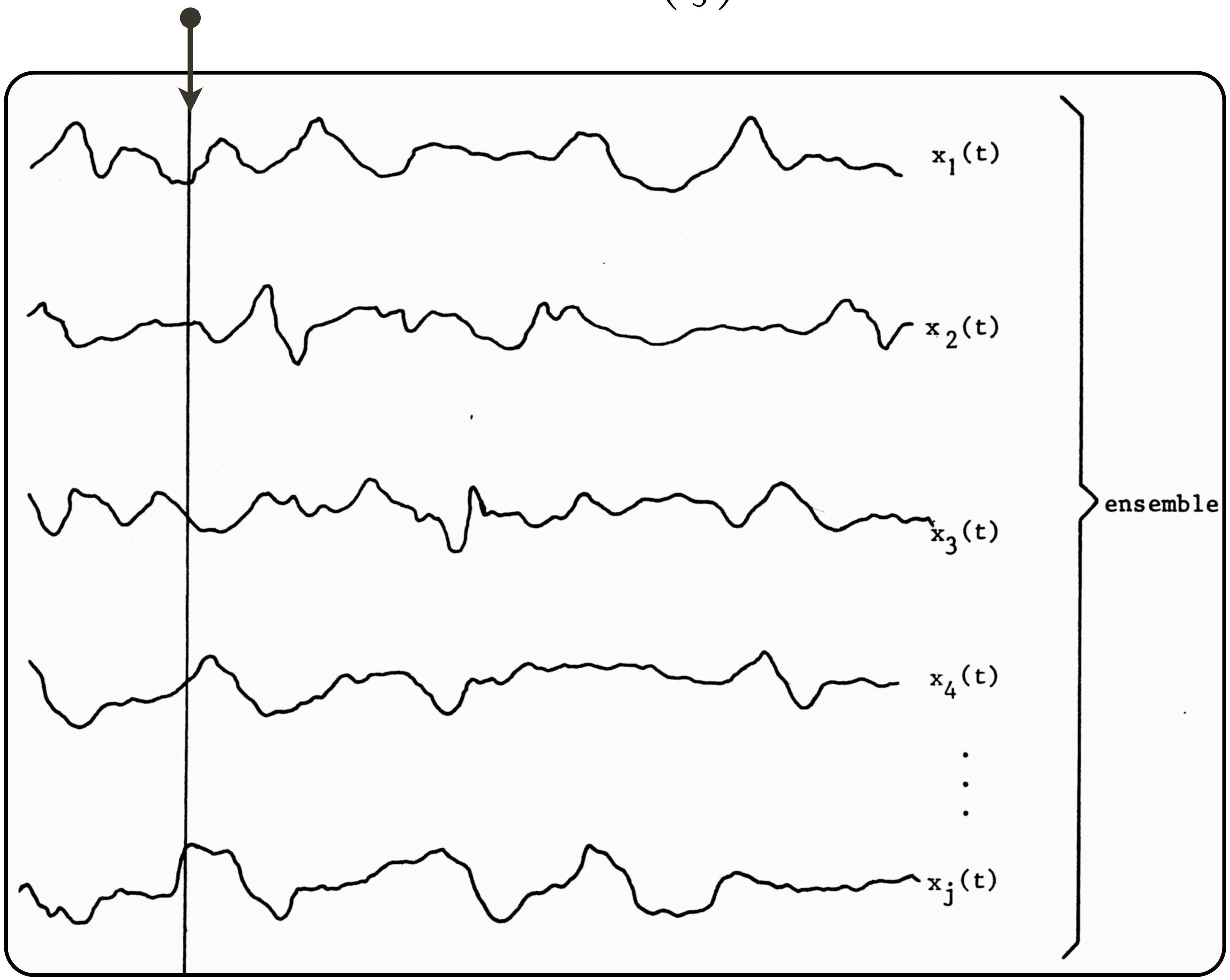


PSR J0437-4715 Janet et al. 1998

- Data is filtered before detection
- Data is filtered during detection
- Data is filtered/processed after detection
- Data is discretely sampled
- Data is stochastic in nature

Stochastic Processes

Random variable $x(\zeta)$



$x(\zeta, t)$

TIME

Stochastic Processes

4 different aspects of a s.p.

A: a family of functions depending on time (indexed by ζ)

B: a particular function of time (ζ fixed)

C: a random variable (at fixed t, for a set of trial outcomes ζ

D: a number (at fixed t and for fixed ζ)

$x(\zeta)$ describes the relation between the possible outcomes ζ and the random variable x

E.g.

die throwing: ζ_1 outcome is face 1 of die

$$x(\zeta_1)$$

is for instance the gain in a game of dice

$$x(\zeta_1) = 0€$$

$$x(\zeta_2) = x(\zeta_3) = 10€$$

$$x(\zeta_4) = x(\zeta_5) = 100€$$

$$x(\zeta_6) = 1000€$$

example from book of Lena, appendix b

Another example: the number of ADUs* measured by a CCD camera behind 5 telescopes for a source of magnitude $M_V=15$

$$x(\zeta_1) = 1001$$
$$x(\zeta_2) = 1045$$
$$x(\zeta_3) = 1099$$
$$x(\zeta_4) = 953$$
$$x(\zeta_5) = 988$$

*what are these?

Cumulative distribution function

$$F(x) = P\{x \le y\}$$

The probability that a set of outcomes of the r.v. has a value $\le$ y

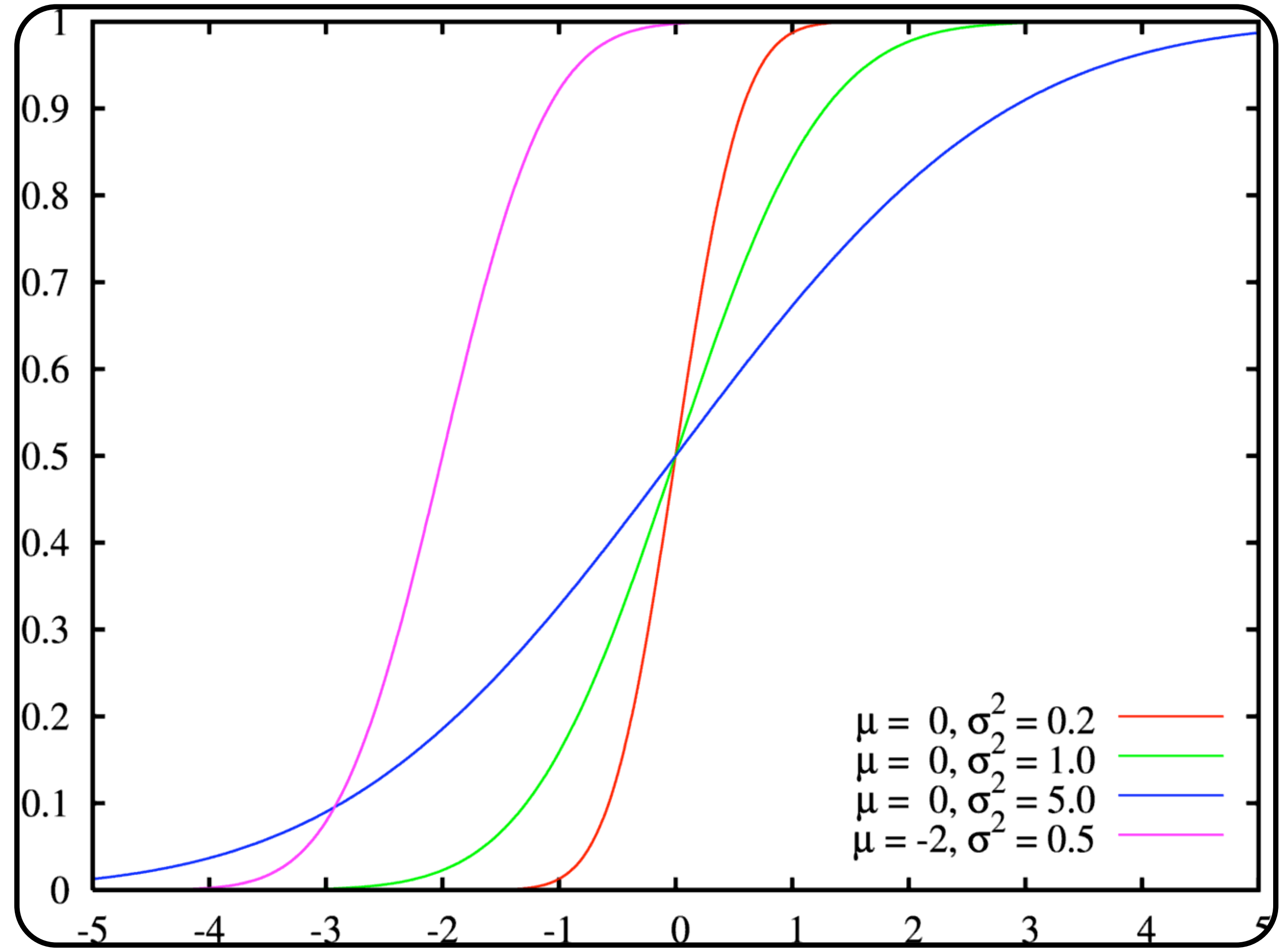
$$F(-\infty) = 0$$

$$F(\infty) = 1$$

Gaussian Cumulative Distribution function

$$F(x, \eta, \sigma) = 0.5 + erf\,\frac{x-\eta}{\sigma^2}$$

(see chapter 6 Num Res for special functions such as erf)



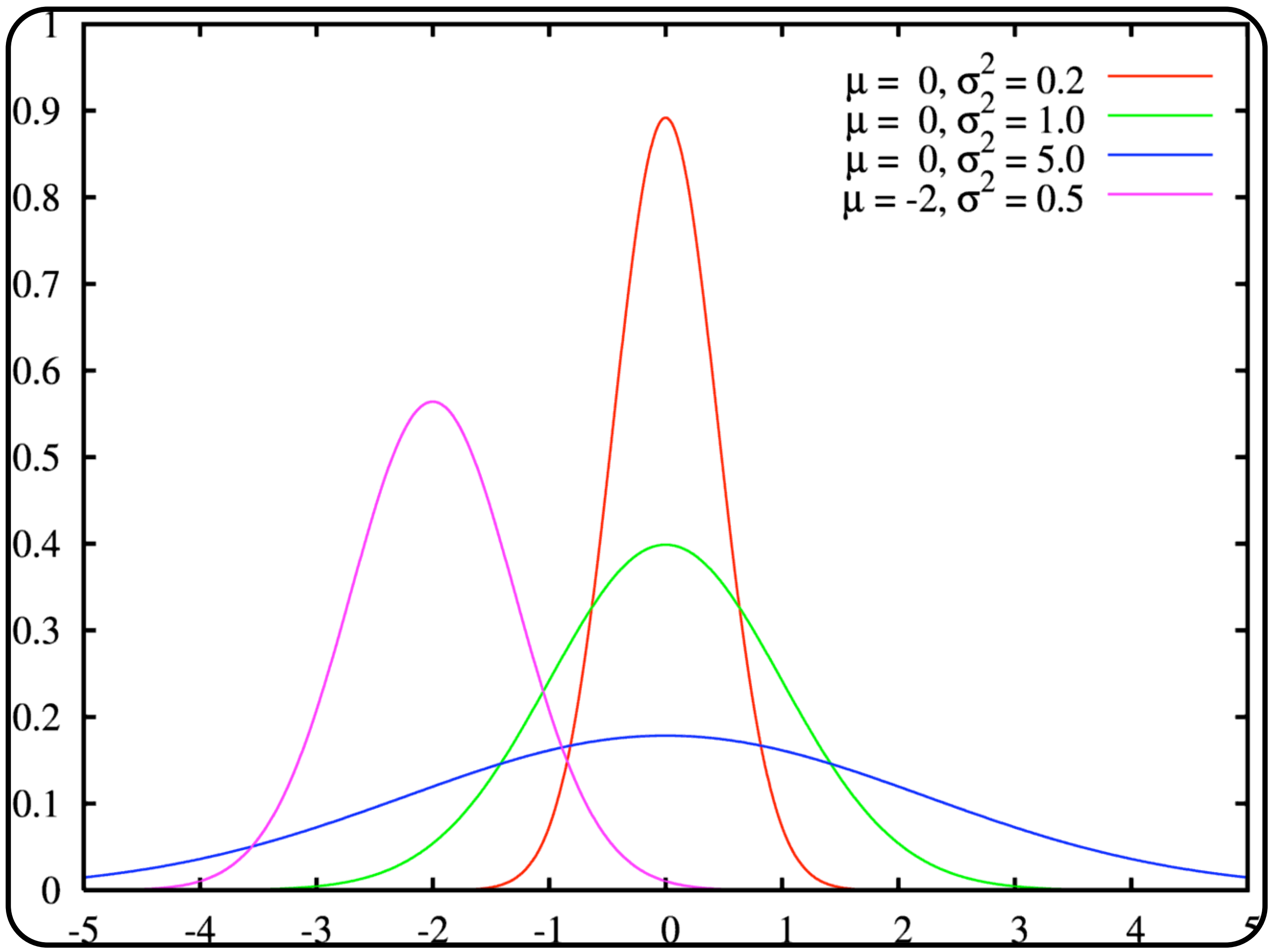
Probability density function

$$\frac{dF(x)}{dx} = f(x)$$

$\hookrightarrow$ Gauss, Poisson, χ^2 etc

A Gaussian Distribution

Gaussian or Normal Probability Density Distribution



$$f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} exp(-\frac{1}{2}\frac{(x-\eta)^2}{\sigma^2})$$

Two parameters completely describe the distribution

Expectation values

$$E\{\phi(x)\} = \int_{-\infty}^{\infty} \phi(x) f(x) dx$$

discrete version $$E\{\phi(x)\} = \sum_{n=-\infty}^{\infty} \phi(x_n) P_n$$

Moments of a distribution

moment $\mu'_k = E\{(x)^k\}$

central moment $\mu_k = E\{(x - E\{x\})^k\}$

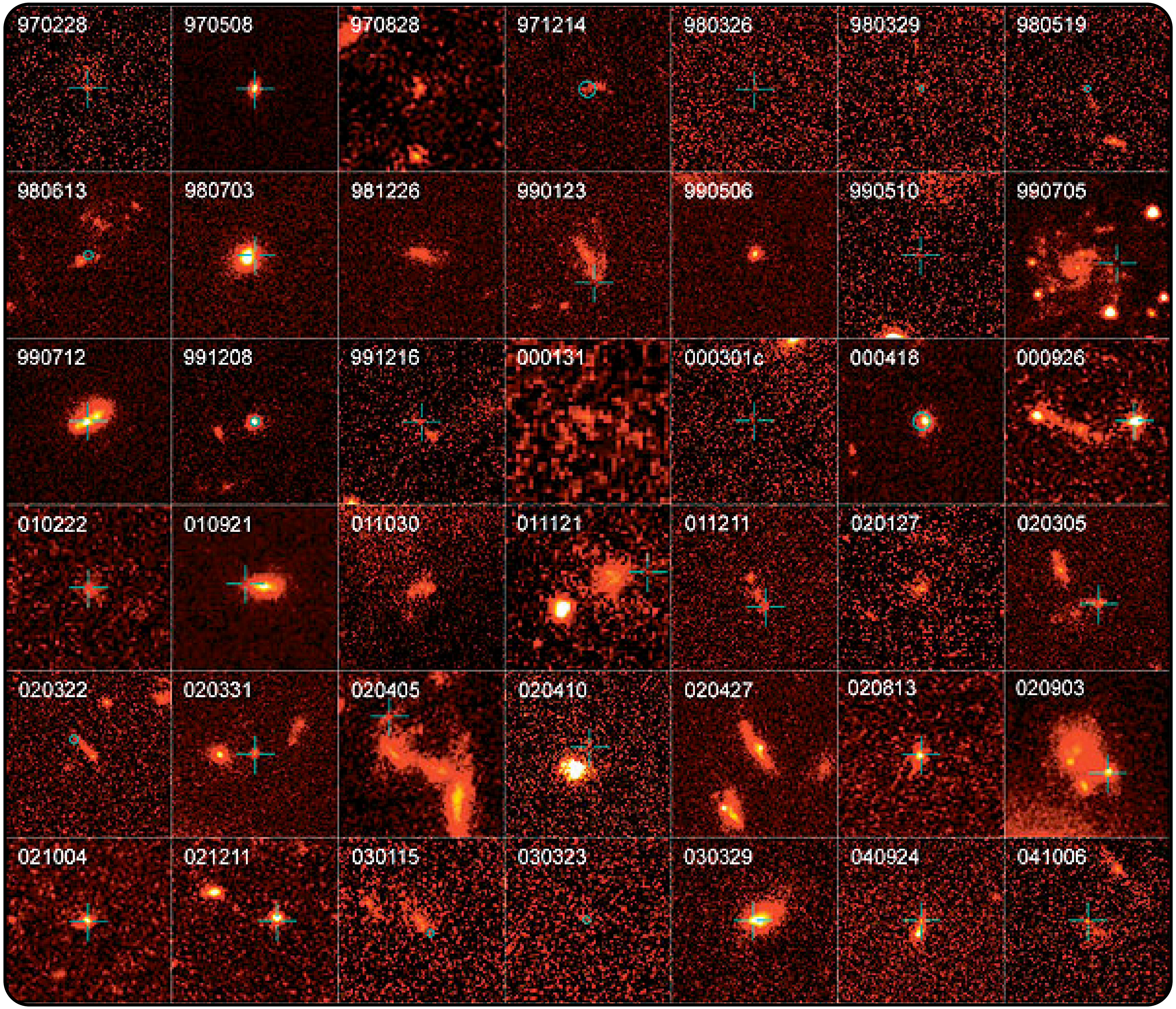
Mean $\eta = E\{x\} = \int_{-\infty}^{\infty} x f(x) dx$

Variance = central moment of 2nd order

$$\mu_2 = E\{(x-\eta)^2\} = \int_{-\infty}^{\infty} (x-\eta)^2 f(x) dx \equiv \sigma^2$$

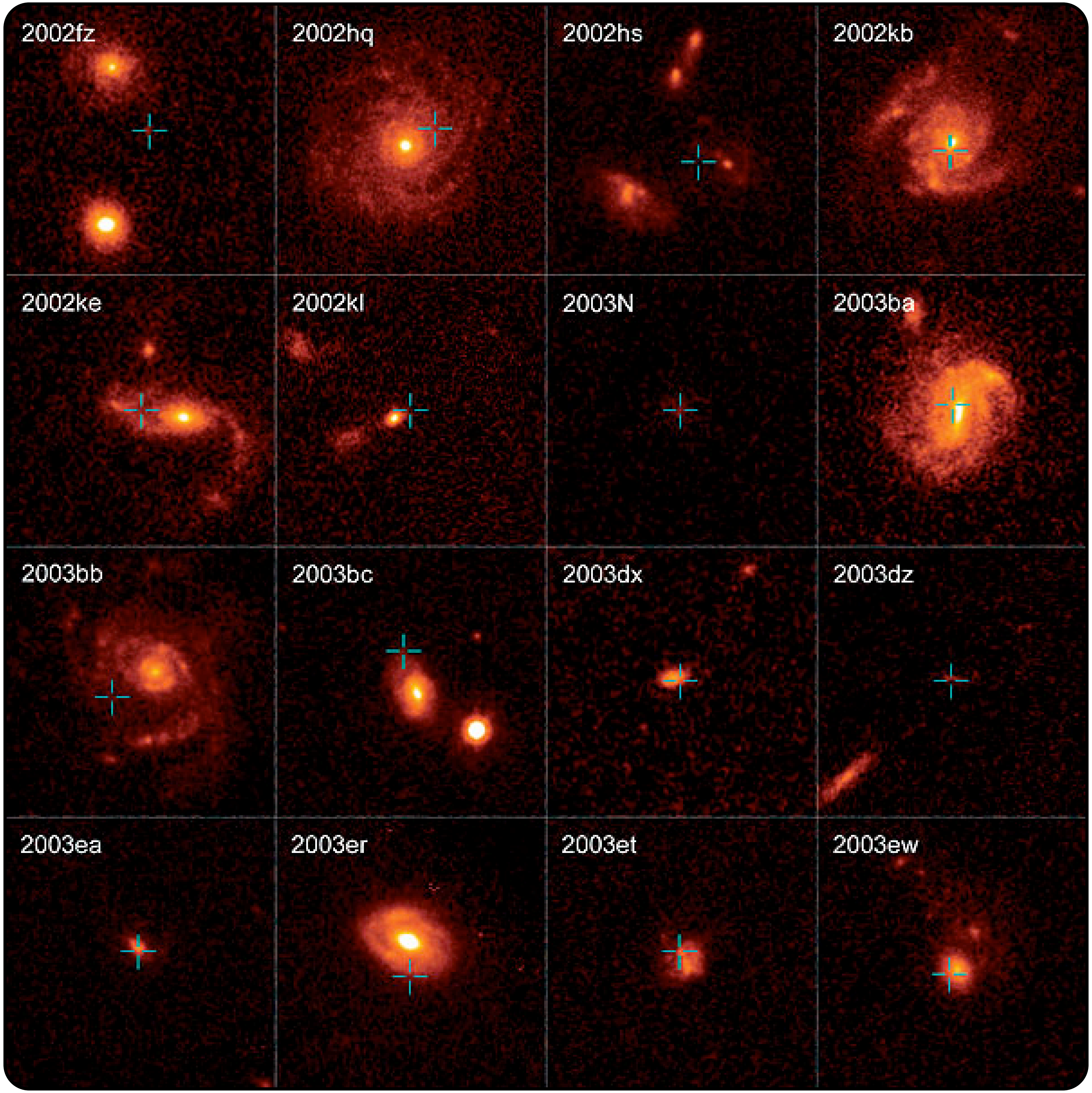
$$\sigma^2 = E\{x^2\} - \eta^2 = E\{x^2\} - (E\{x\})^2$$

Example of use of moments: GRB distribution

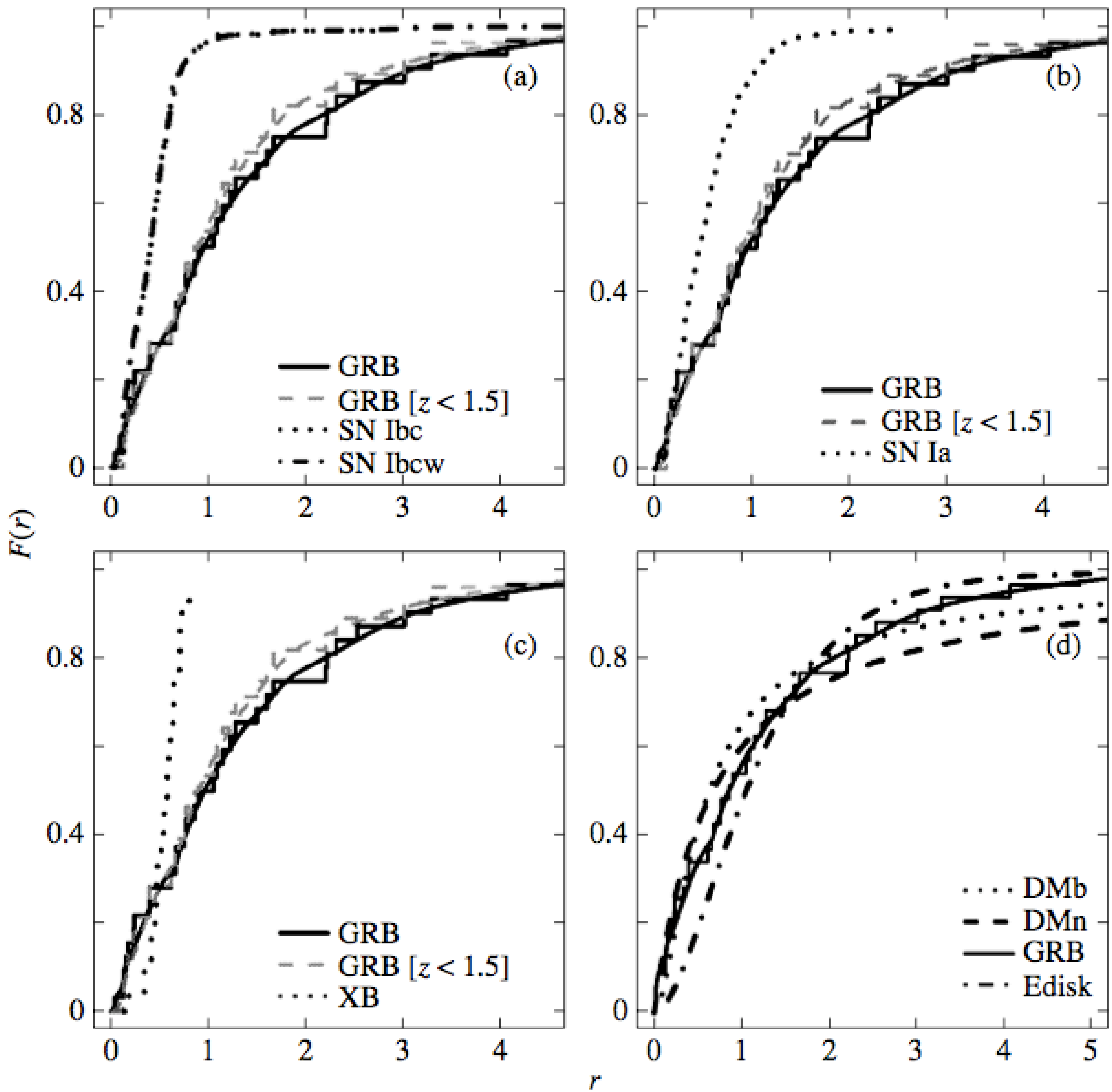


Bloom et al. 2002, Blinnikov et al. 2004, Fruchter et al. 2006

Core-collapse SN distribution



Cumulative distributions Blinnikov et al.2004



Less often used moments are the

skewness

how asymmetric is the distribution?

$$Skew(x_1....X_N) = \frac{1}{N}\sum_{j=1}^{N}\left[\frac{x_j - \bar{x}}{\sigma}\right]^3$$

&

Kurtosis

how peaked is the distribution?

$$Kurt(x_1....X_N) = \left\{\frac{1}{N}\sum_{j=1}^{N}\left[\frac{x_j - \bar{x}}{\sigma}\right]^4 - 3\right\}$$

Both measured wrt a normal=Gaussian distribution

Correlation, auto-correlation

Correlation

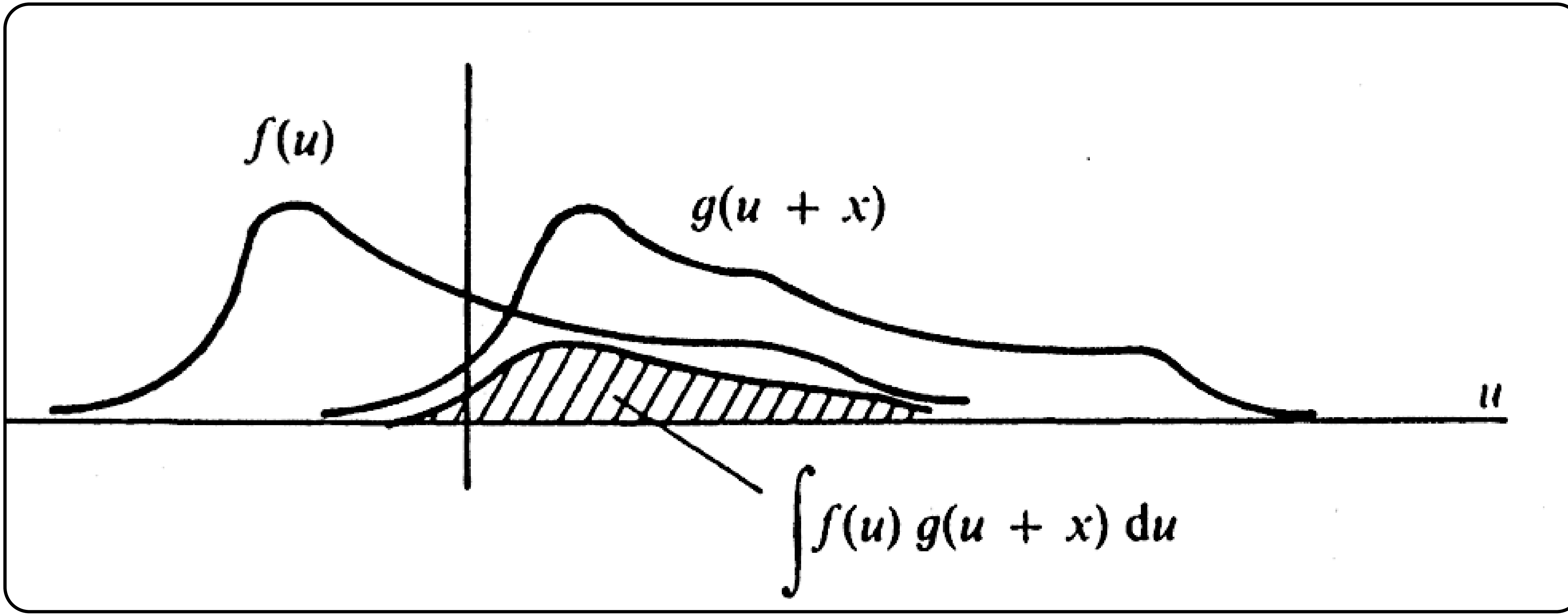
$$k(x) = f(x) \otimes g(x) \qquad k(x) = \int_{-\infty}^{\infty} f(u)g(u+x)du$$

If X and Y are two independent random variables with probability distributions f and g, respectively, then the probability distribution of the difference Y - X is given by the cross-correlation f$\otimes$g.

the convolution f * g gives the probability distribution of the sum X + Y

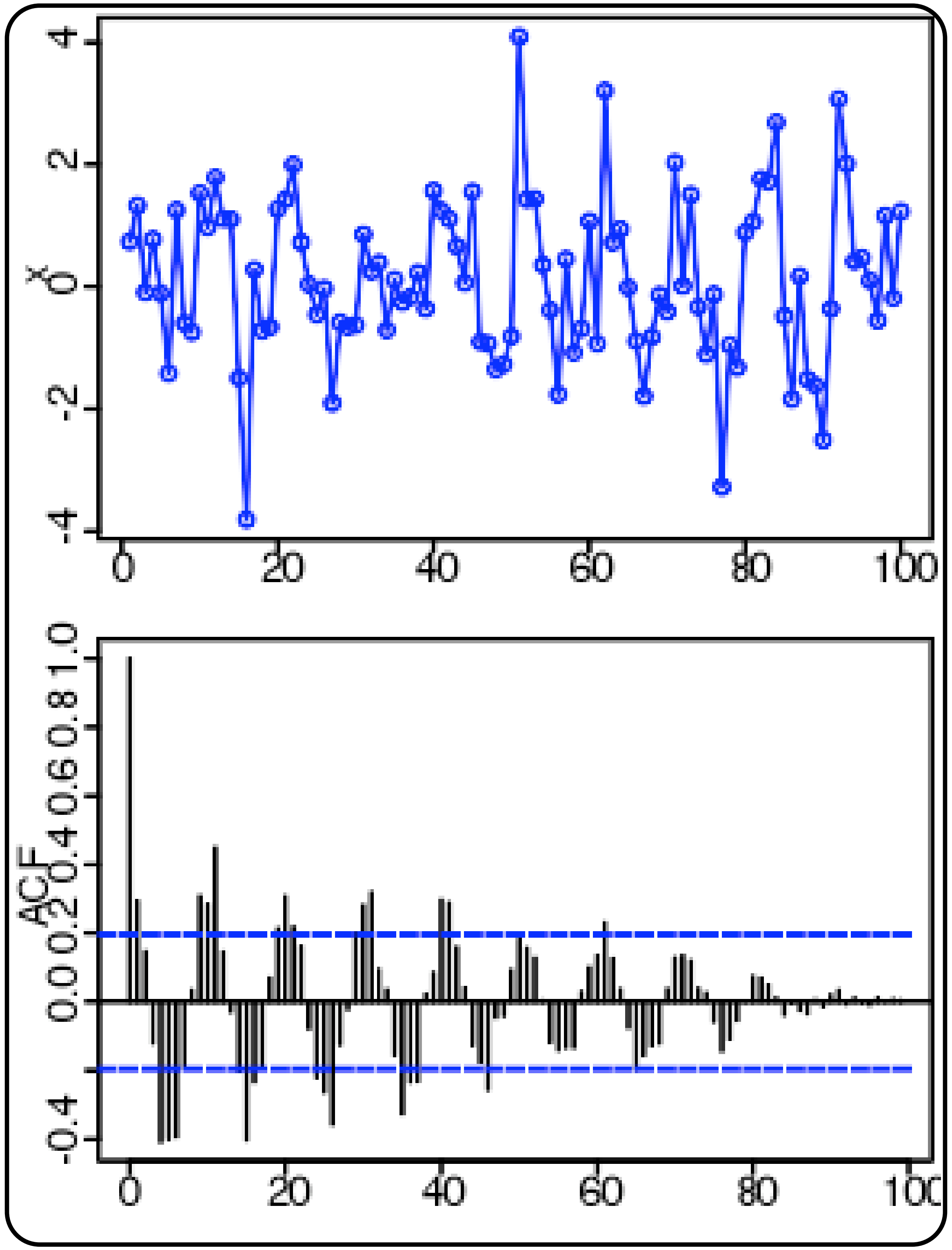
Correlation, auto-correlation

Correlation

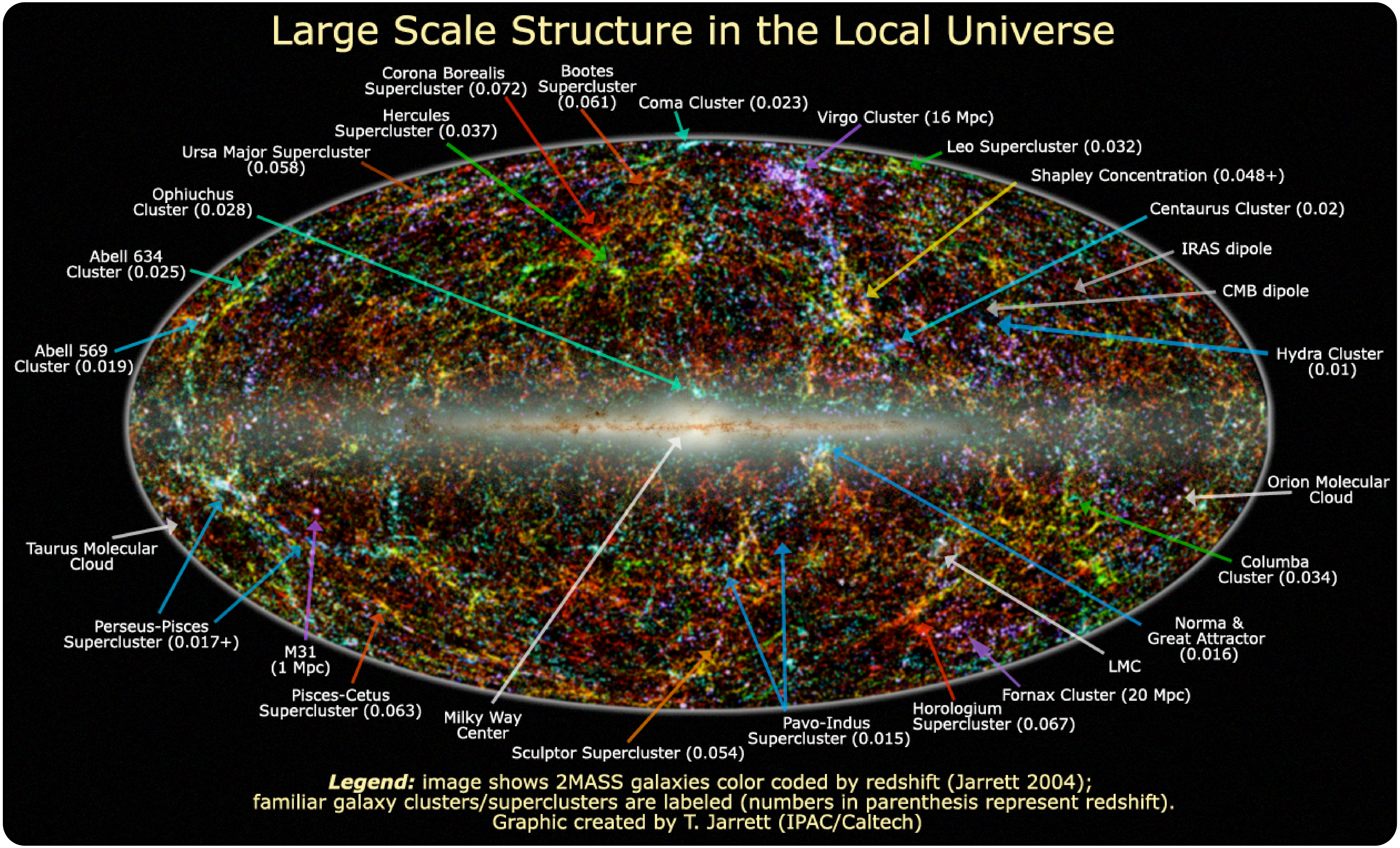


Auto-correlation

$$R(x) = f(x) \otimes f(x) = \int_{-\infty}^{\infty} f(u) f(u+x) du$$

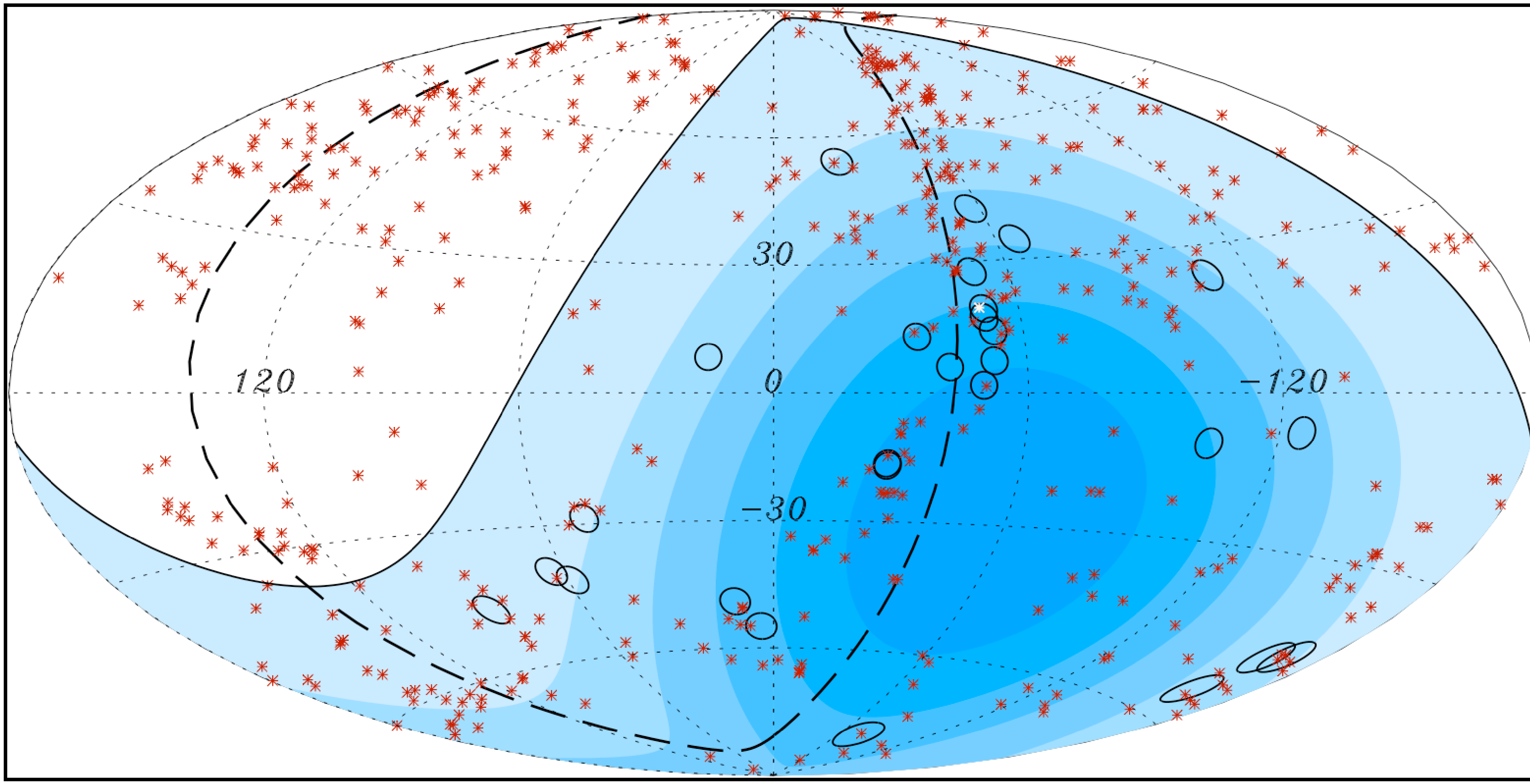


$$R(x) = E\{\underbrace{f(x)}_{f(x_1)} \underbrace{f(x+t)}_{f(x_2)}\}$$



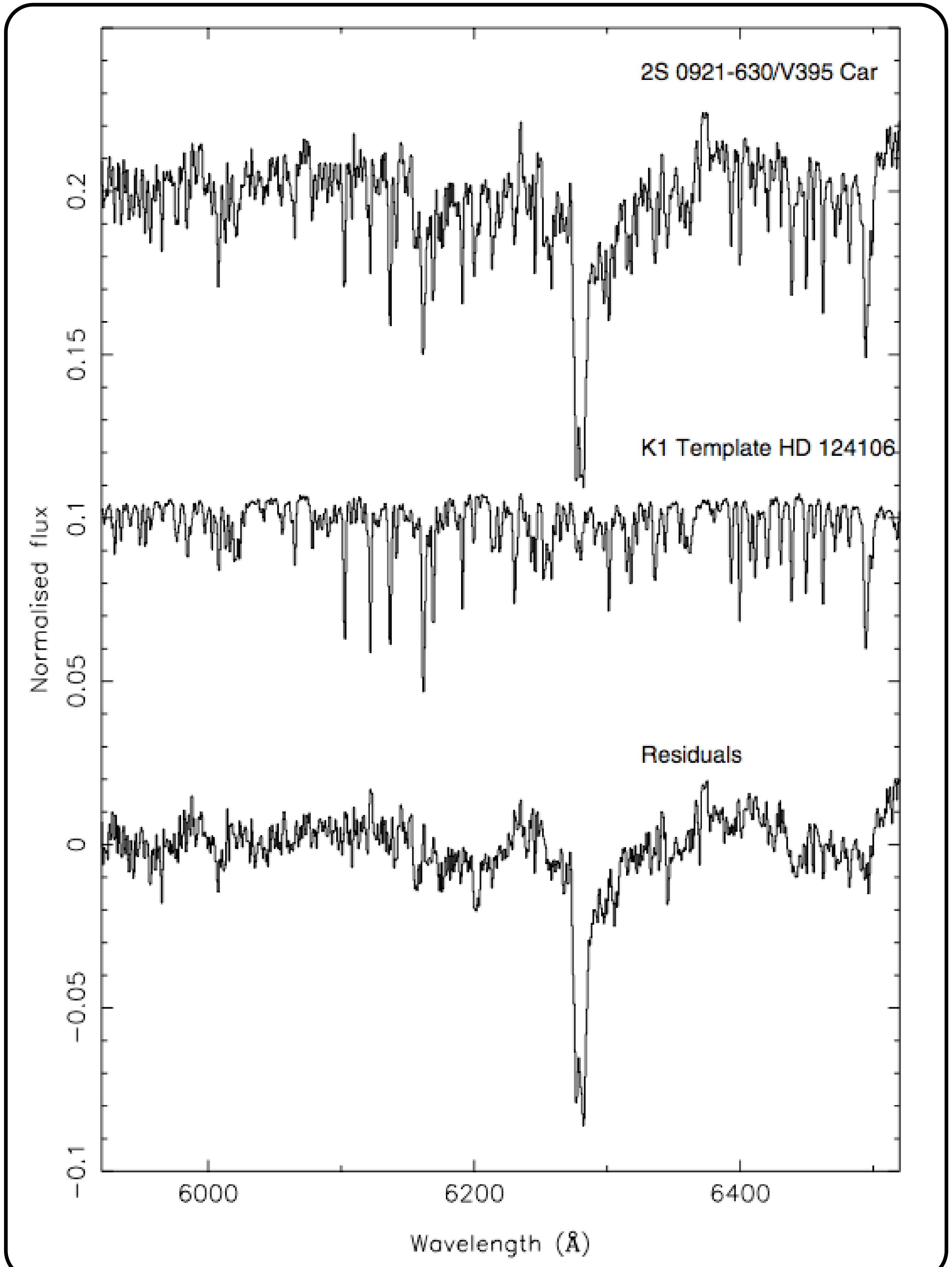
GIVEN A RANDOM GALAXY IN A LOCATION THE CORRELATION FUNCTION DESCRIBES THE PROBABILITY THAT ANOTHER GALAXY WILL BE FOUND WITHIN A GIVEN DISTANCE (PEEBLES 1980)

Cross-correlating cosmic-ray events with the position of nearby AGN

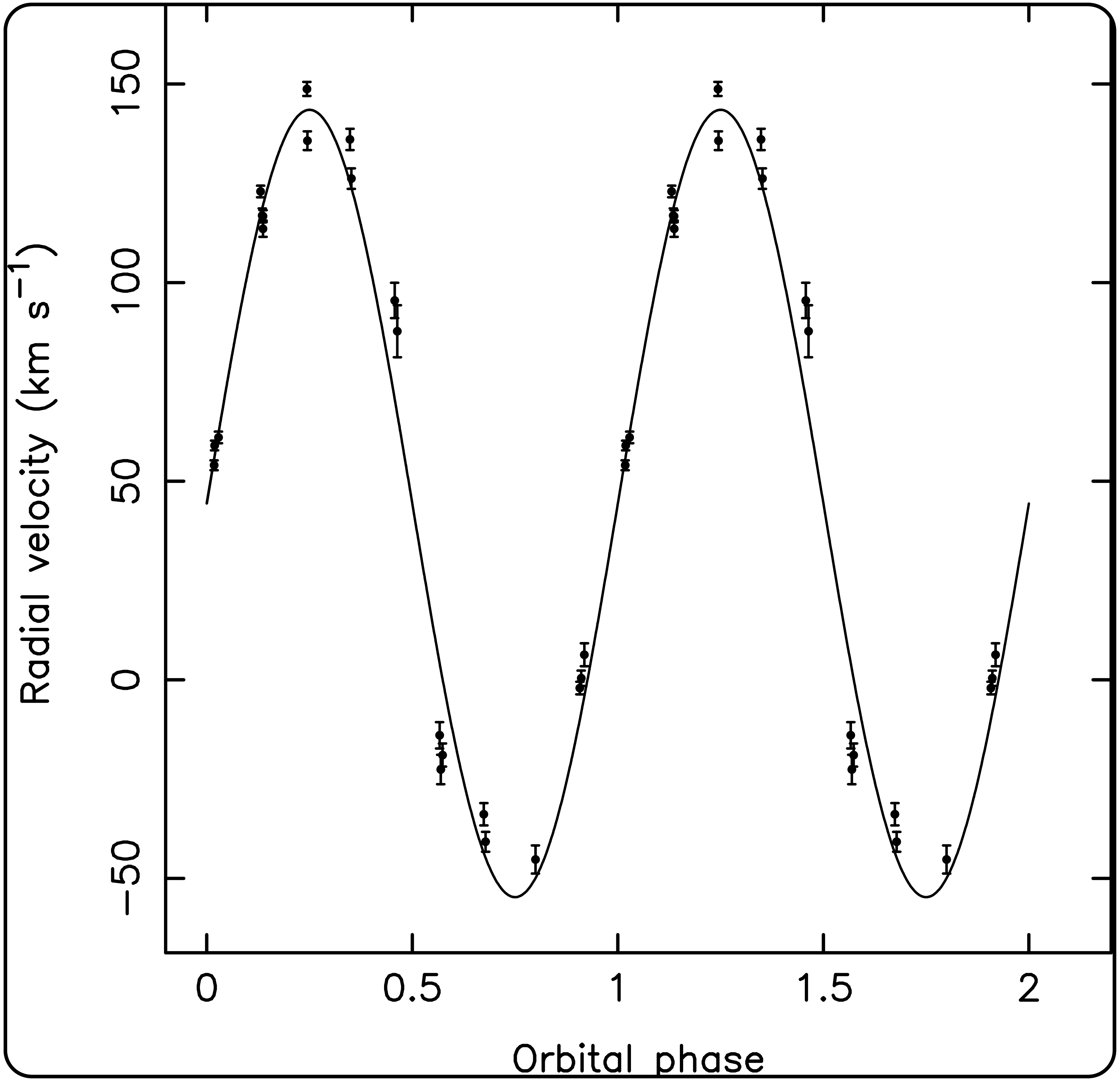


Data from the Pierre Auger collaboration
see astro-ph 0711.2256

Cross-correlating spectra to determine velocities



Cross-correlating spectra



to determine velocities

and from that a mass-function

More on the auto-correlation

$$R(x) = E\{\underbrace{f(x)}_{f(x_1)}\underbrace{f(x+t)}_{f(x_2)}\}$$

$$\text{if} \quad x_1 = x_2$$

$$R(x) = R(x,x) = \mathbf{E}\{f^2(x)\} = \mathbf{E}\{|f(x)|^2\}$$

Average generally not zero → autocovariance

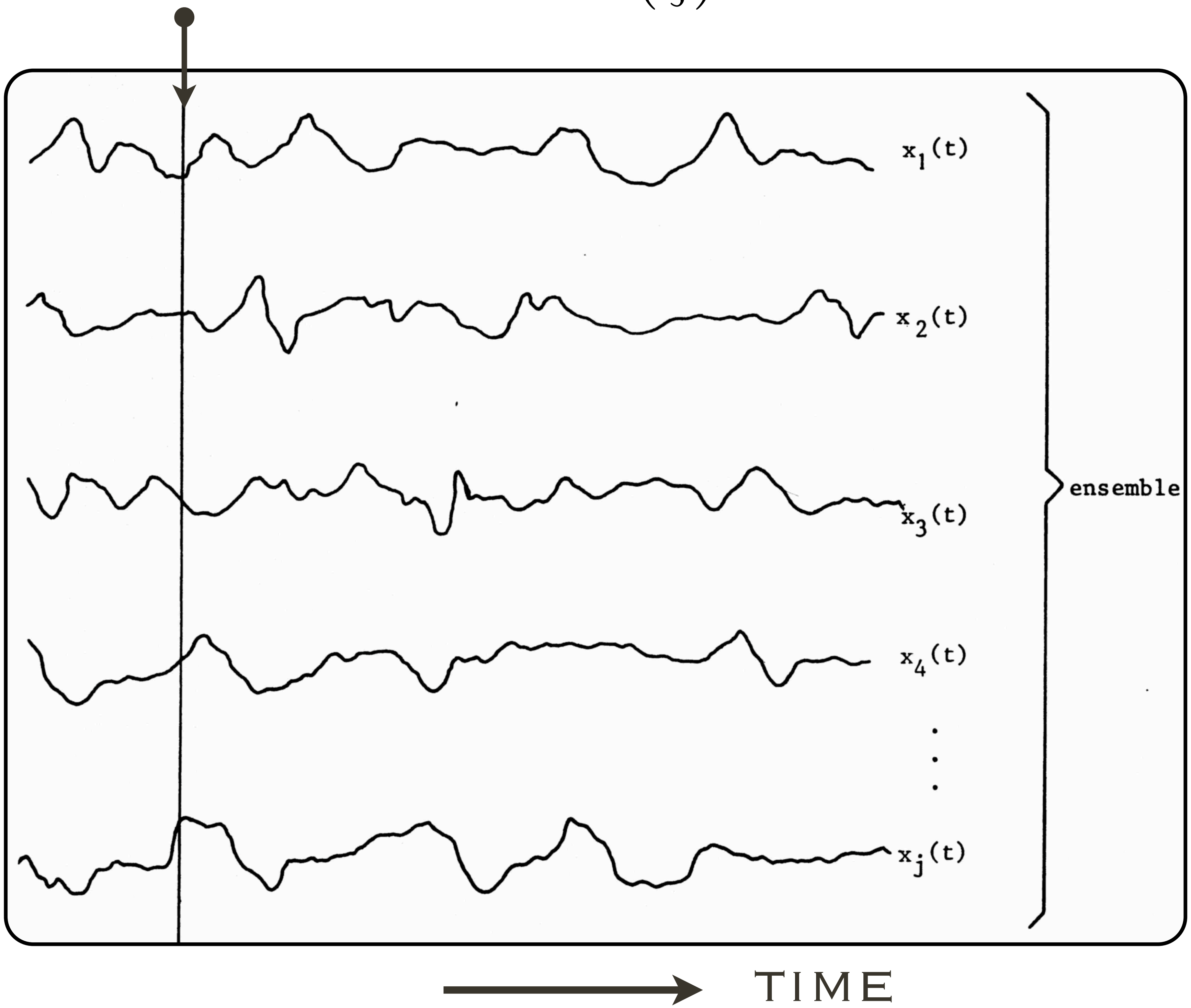
$$C(x_1, x_2) = \mathbf{E}\{(f(x_1) - \eta(x_1))(f(x_2) - \eta(x_2))^*\}$$

$$C(x) = R(x) - |\eta(t)|^2 = \sigma^2(x)$$

C(x) → average power in the fluctuations around the mean

Stochastic Processes

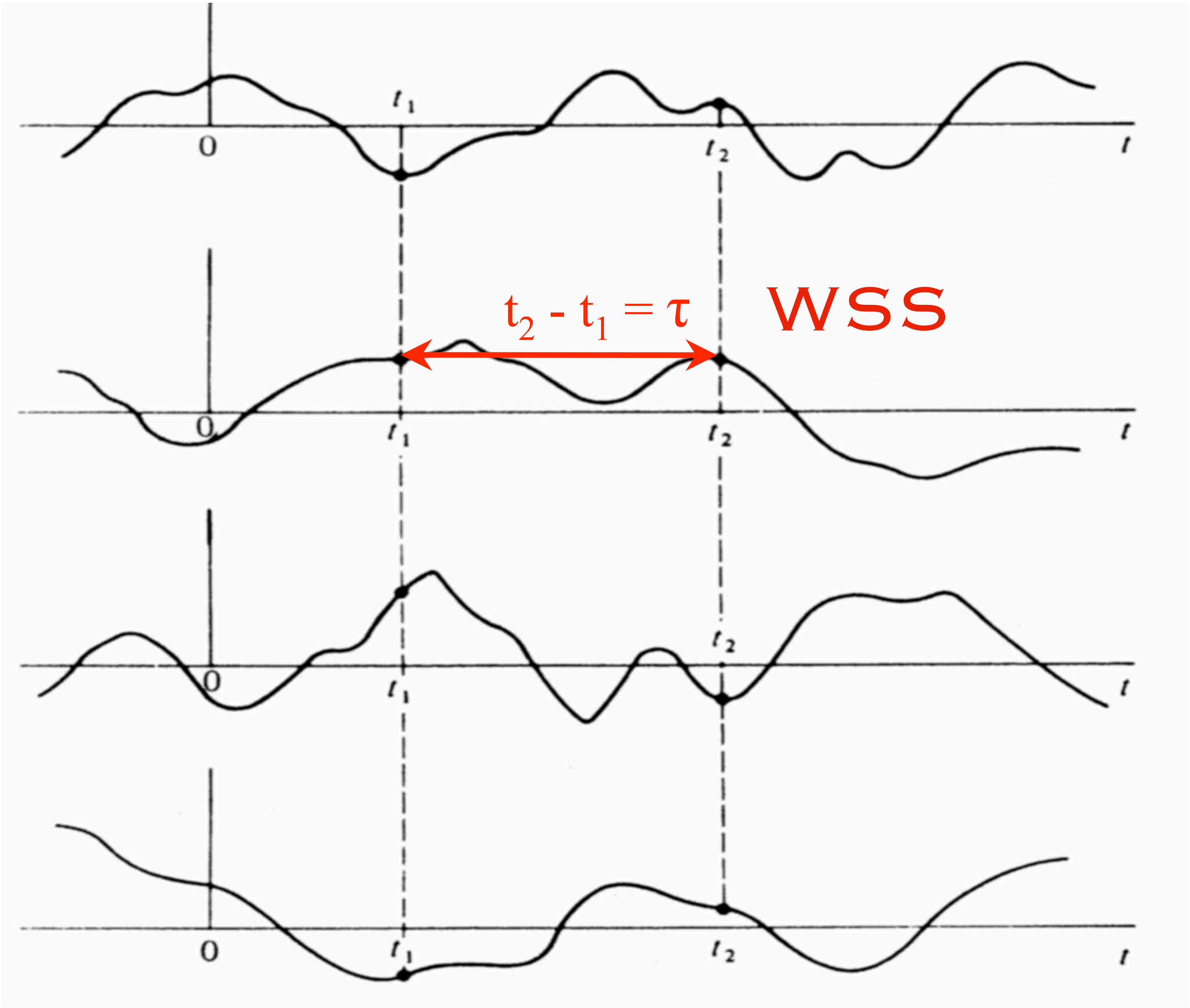
Random variable $x(\zeta)$



$x(\zeta, t)$

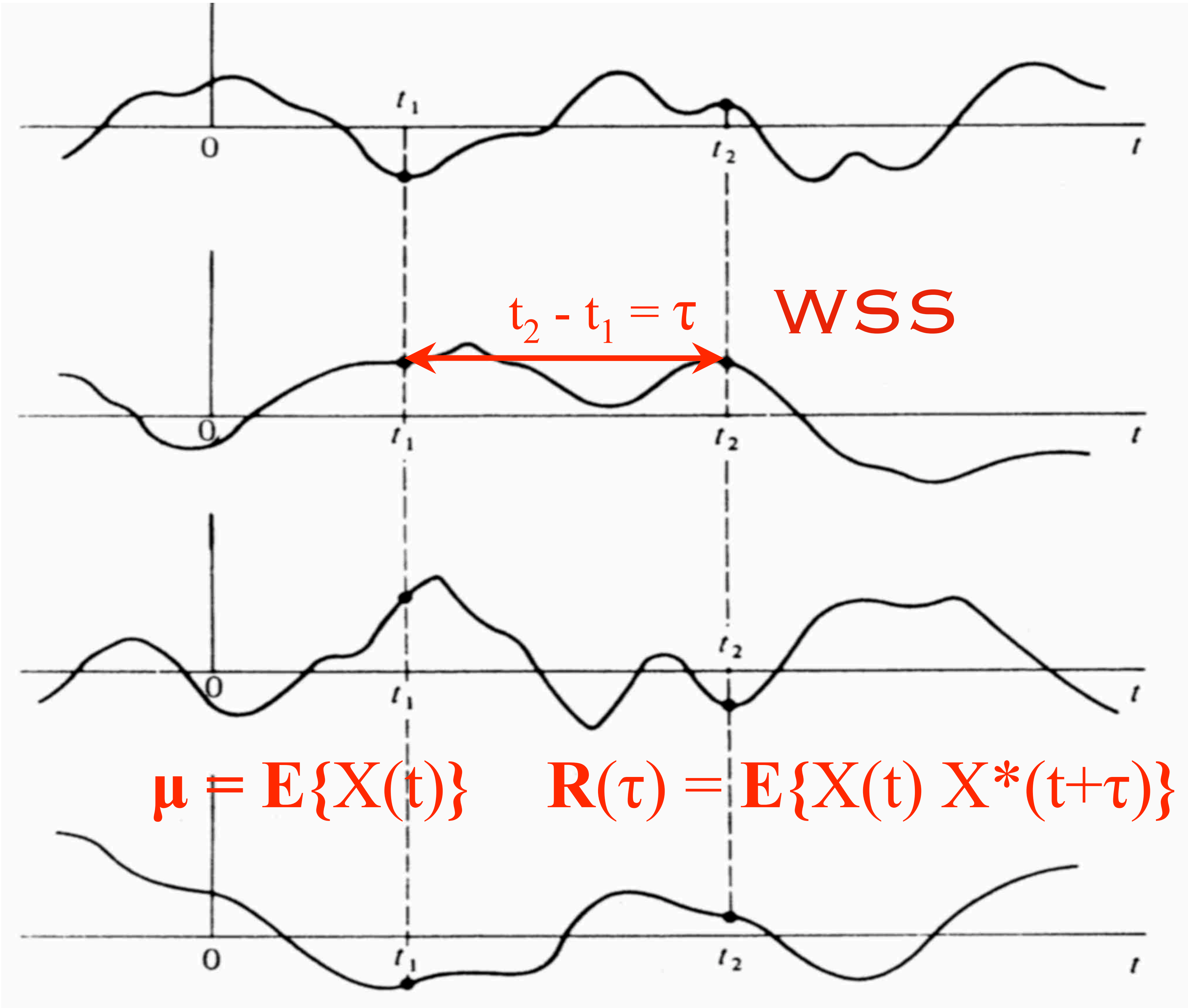
TIME

Wide-sense stationary S.P.



WSS: mean time independent & autocorrelation depends on time difference

Wide-sense stationary S.P.



WSS: mean time independent & autocorrelation depends on time difference

Not all signals are WSS:

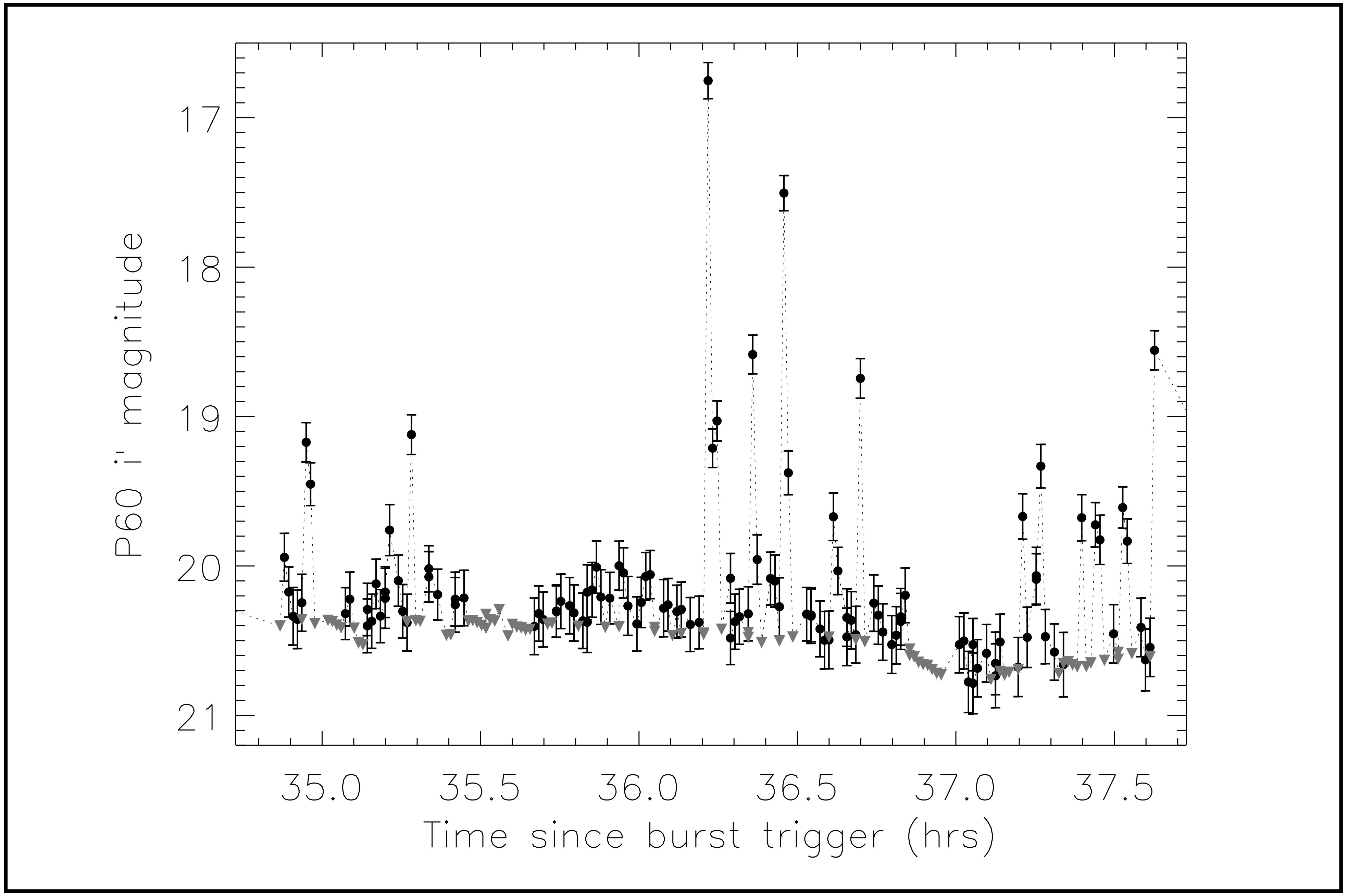
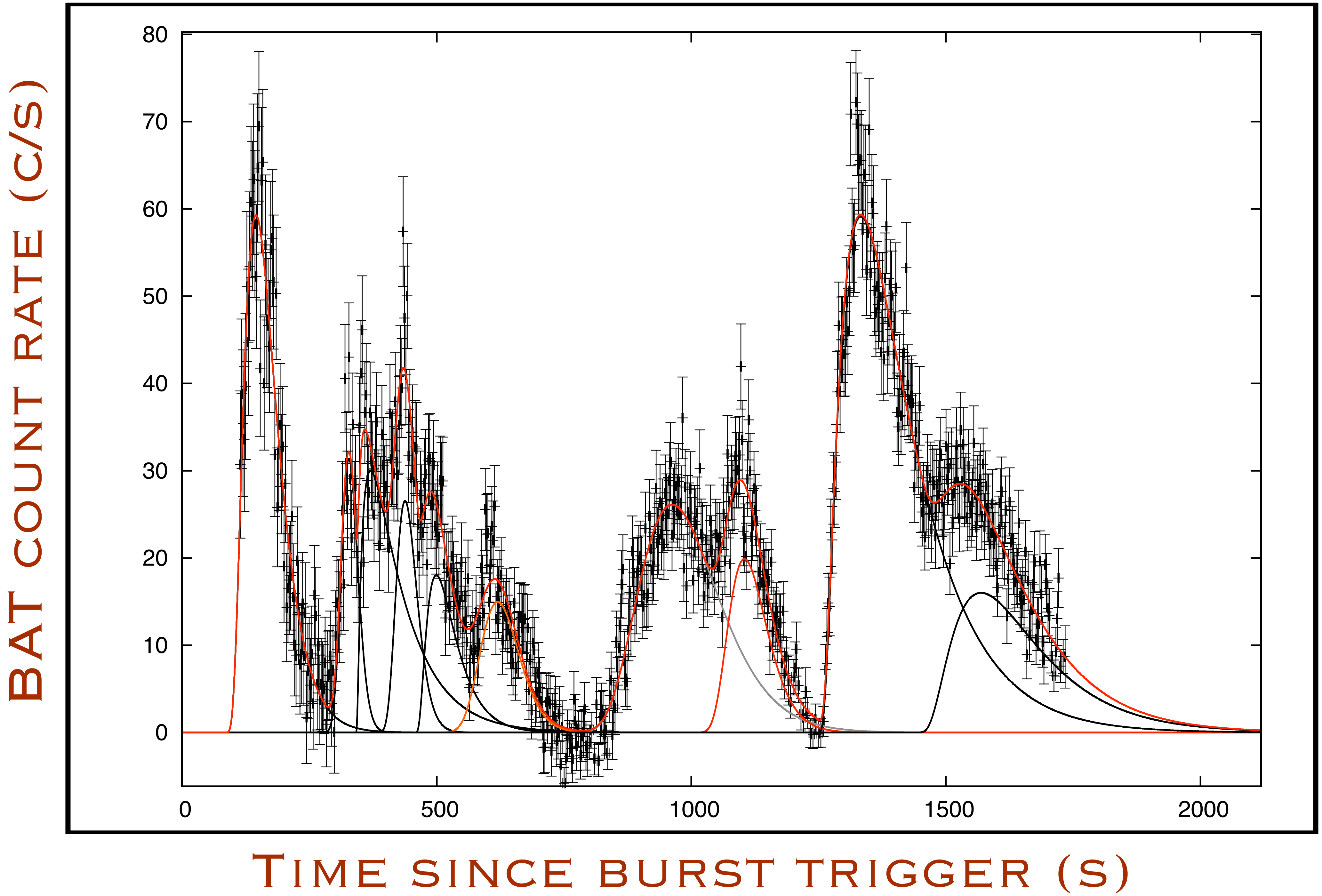


Figure from Kasliwal et al. 2007

Nature of the source uncertain:
GRB, SGR, BH-X-ray binary?

Not all signals are WSS:

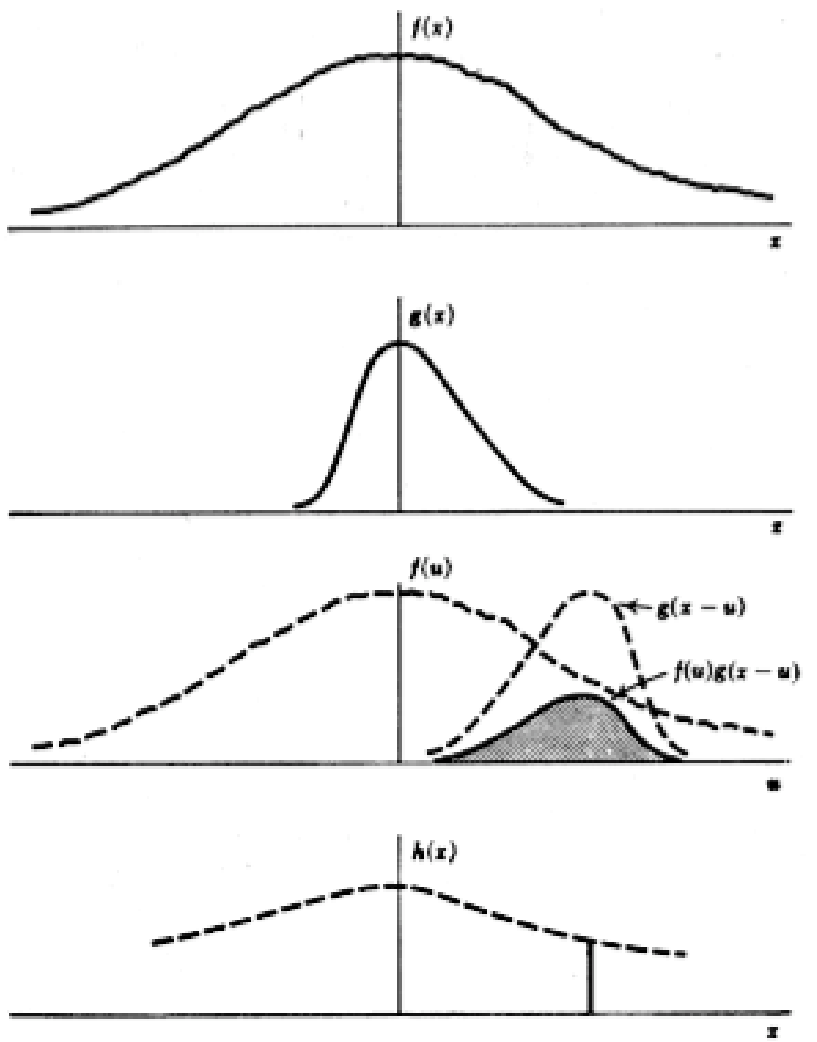


Swift BAT detector light curve

Figure from Chincarini et al. 2008

CONVOLUTION:

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(x)g(x_1 - x)dx$$



Convolution Theorem

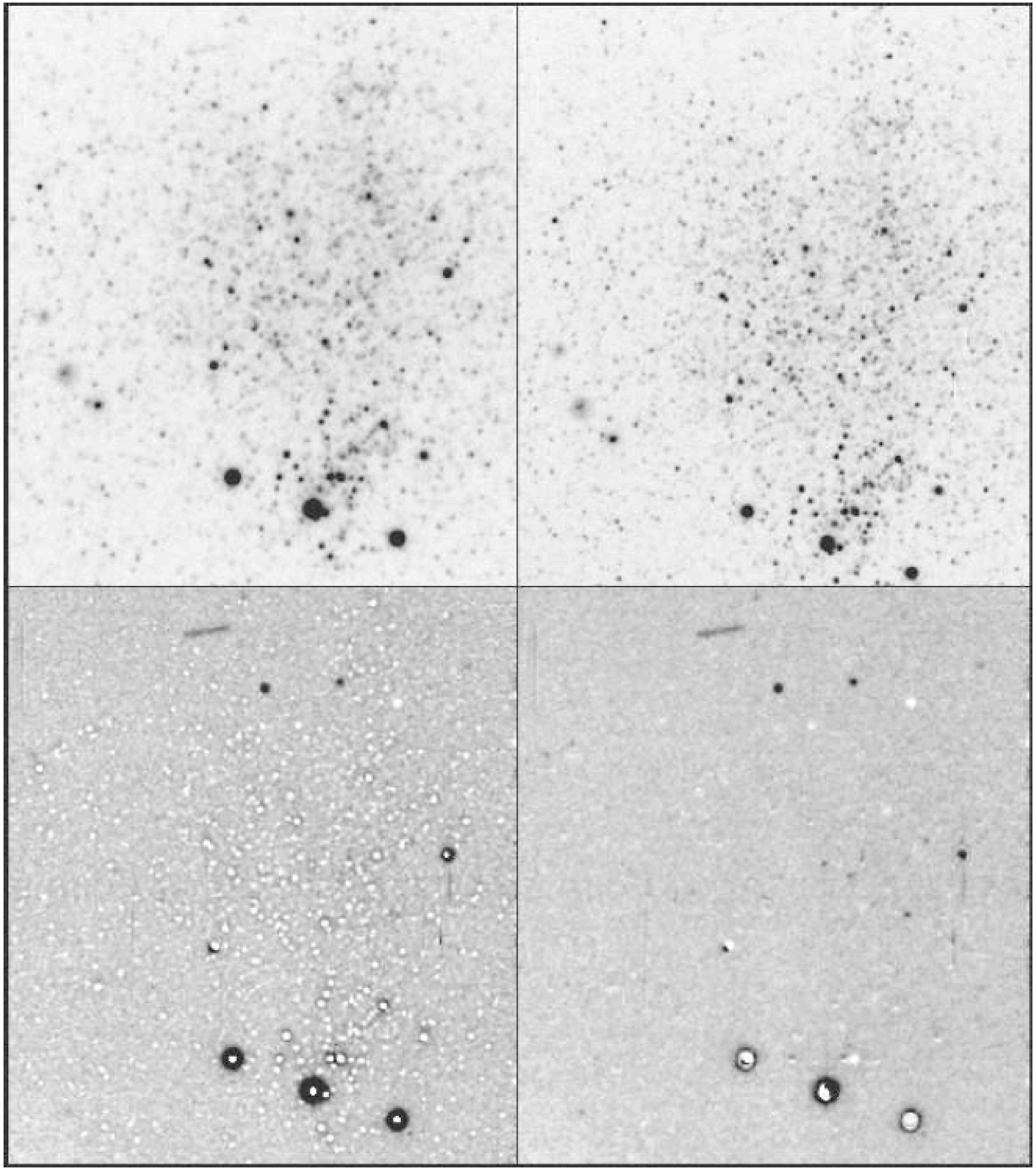
$$F(f(x) * g(x) = F(f(x))F(g(x))$$

$$f(x) * g(x) \Leftrightarrow F(s)G(s)$$

SIMILARLY FOR CROSS CORRELATIONS

$$F(f \otimes g) = F(f)F(g)$$

CONVOLUTE IMAGES



2 B-band images of the Phoenix dwarf galaxy

Difference image after convoluting the better-seeing image with a smoothing kernel and scaling the fluxes

From Phillips & Davis 1995

Fourier Transformations

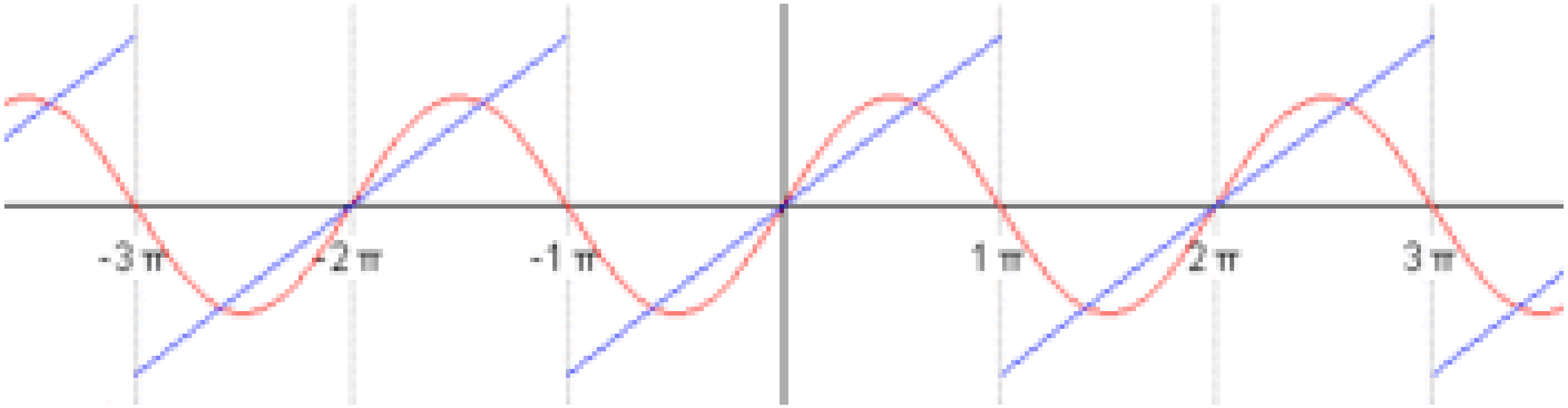


Figure from Wikipedia

$$F(t) \Leftrightarrow f(x)$$

$$F(t) = \int_{-\infty}^{\infty} f(x) e^{-2\pi ixt} dx$$

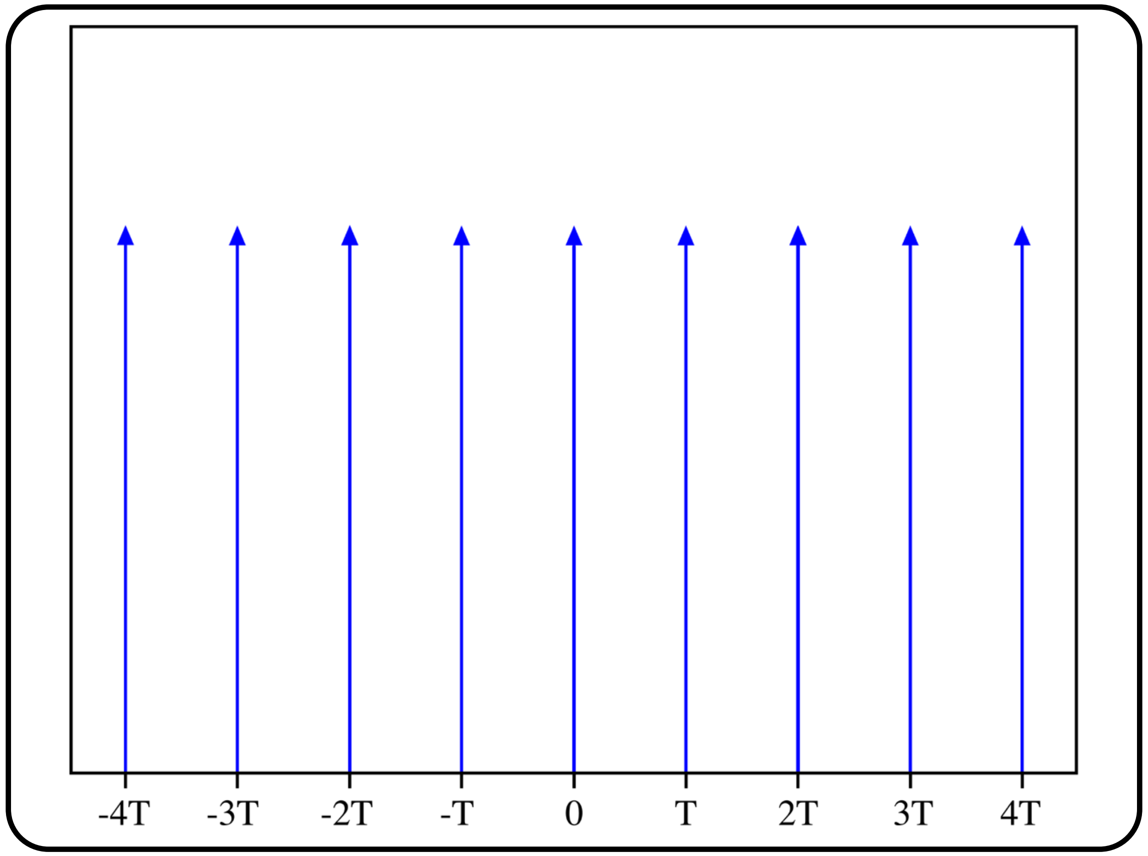
$$f(x) = \int_{-\infty}^{\infty} F(t) e^{2\pi ixt} dt$$

$$Euler's\ relation: \quad e^{ix} = cosx + isinx$$

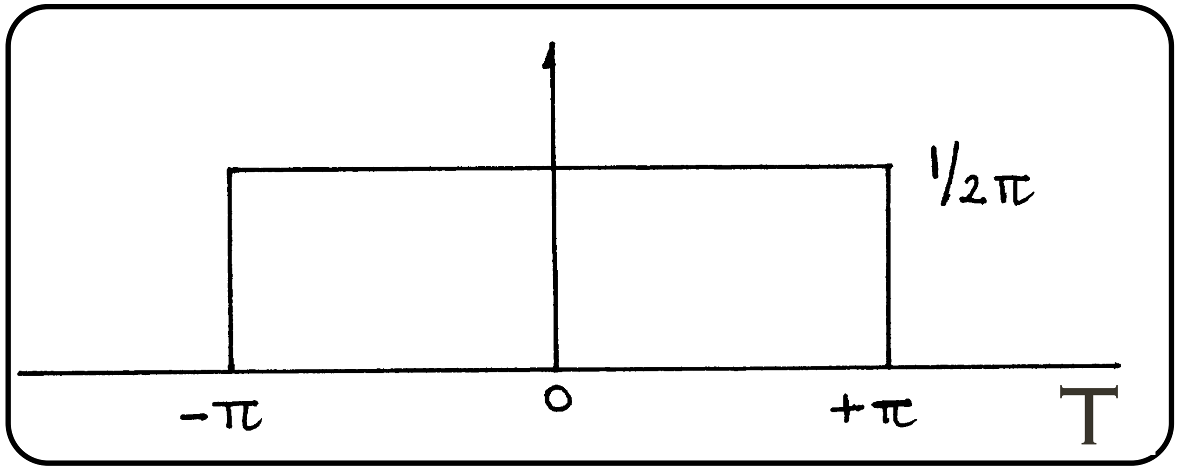
Some special functions:

Shah's function/Dirac comb

$$III(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



Box/window function

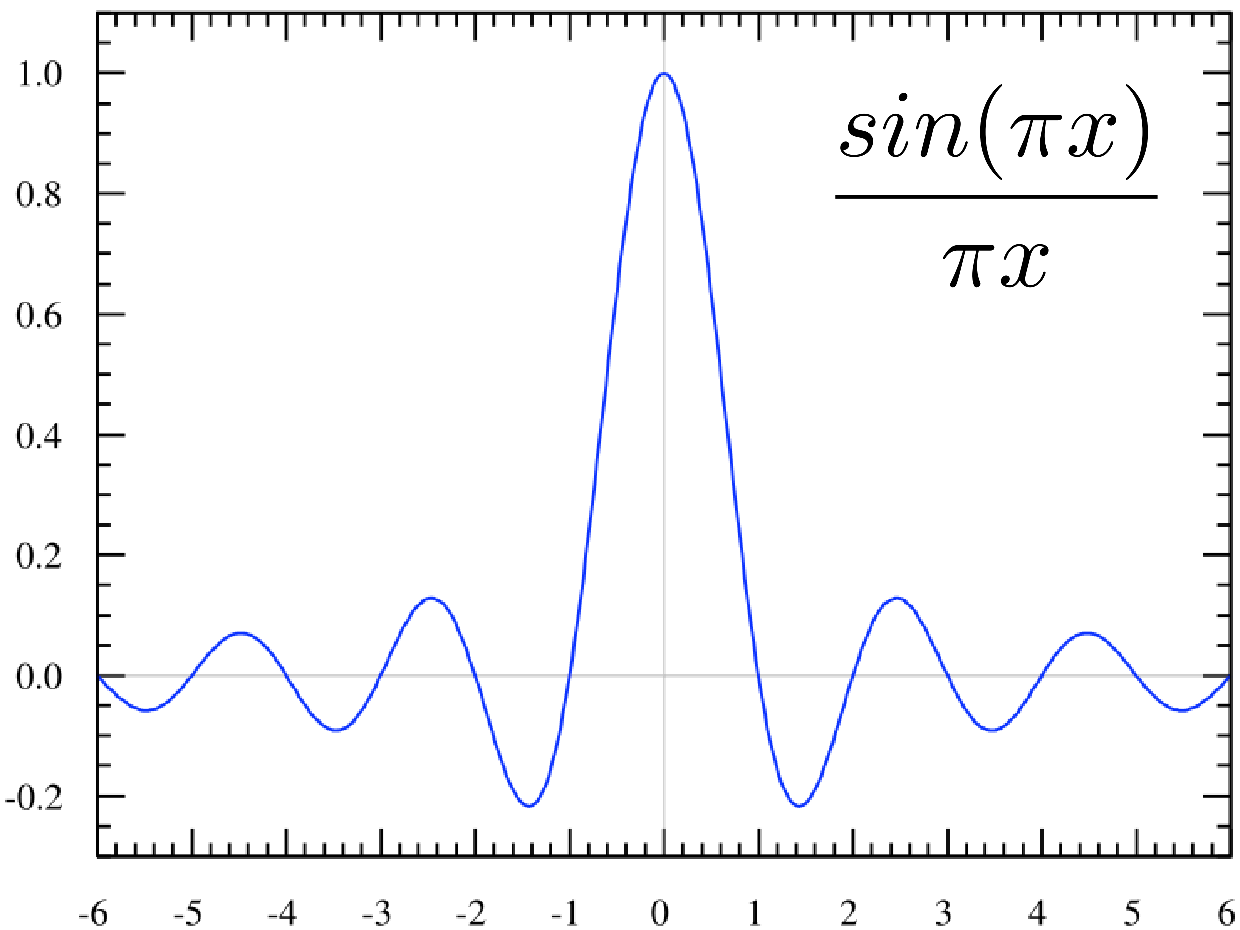


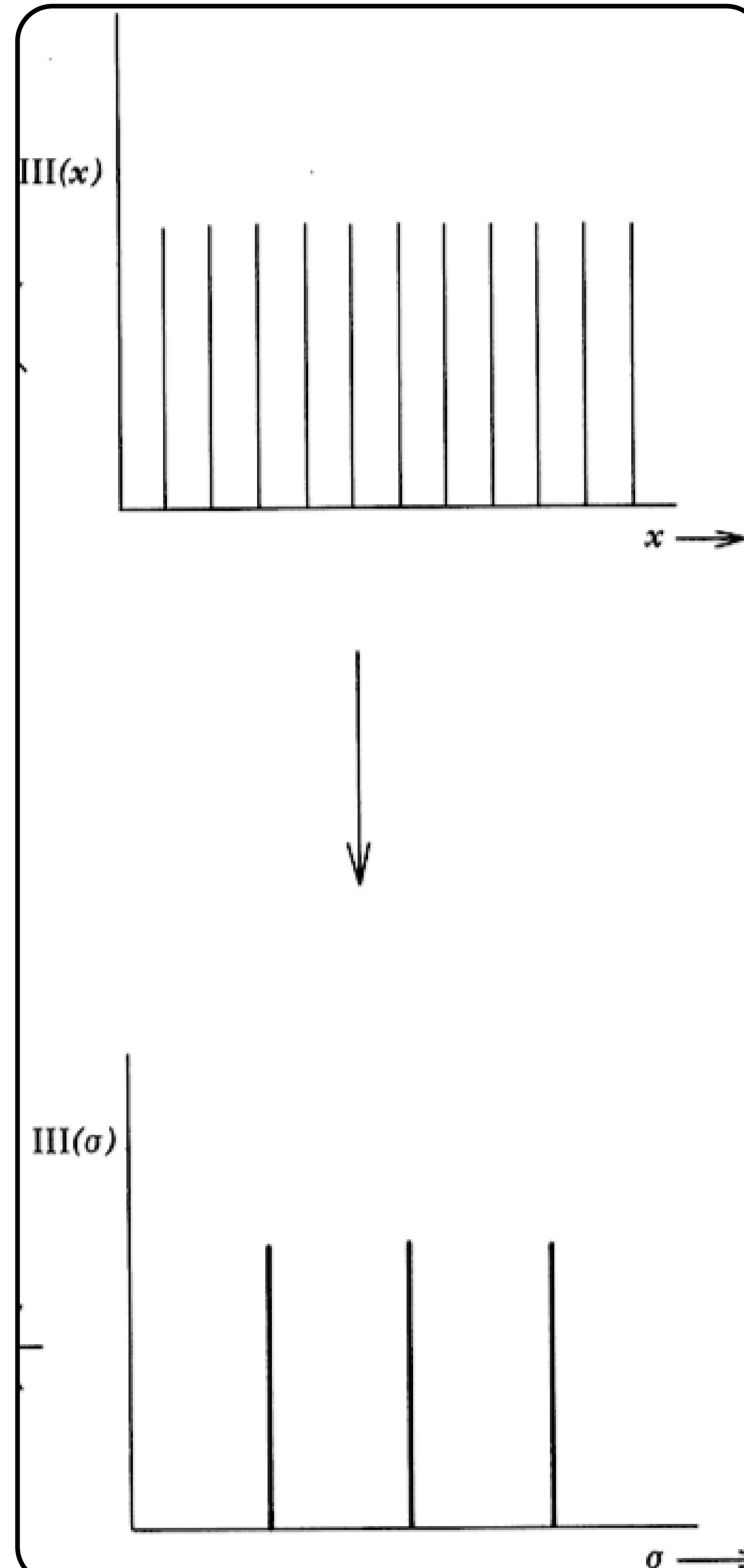
$$B(t) = 0 \ for \ -\frac{W}{2} > t > \frac{W}{2}$$

$$B(t) = 1 \ for \ -\frac{W}{2} < t < \frac{W}{2}$$

Fourier transforms of these special FIE's

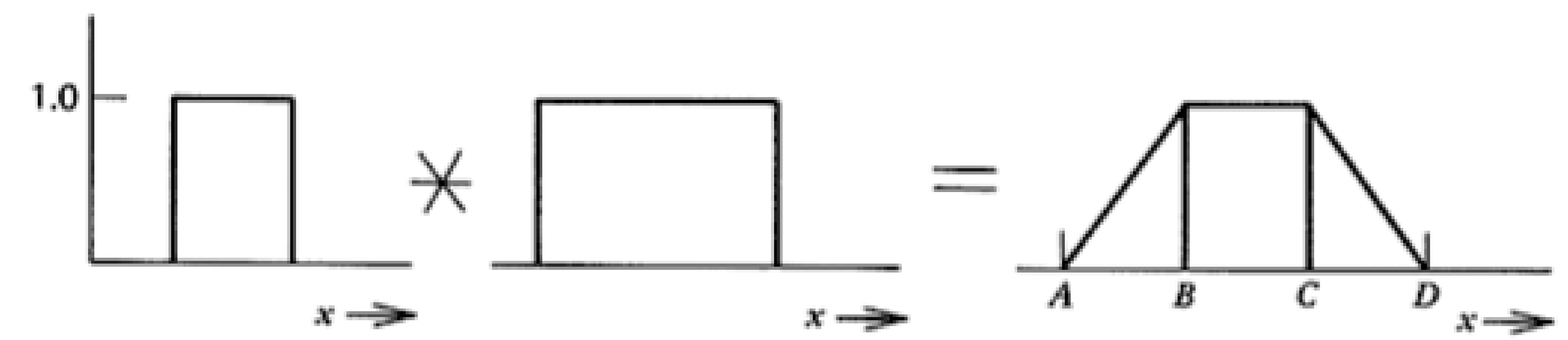
Sinc function



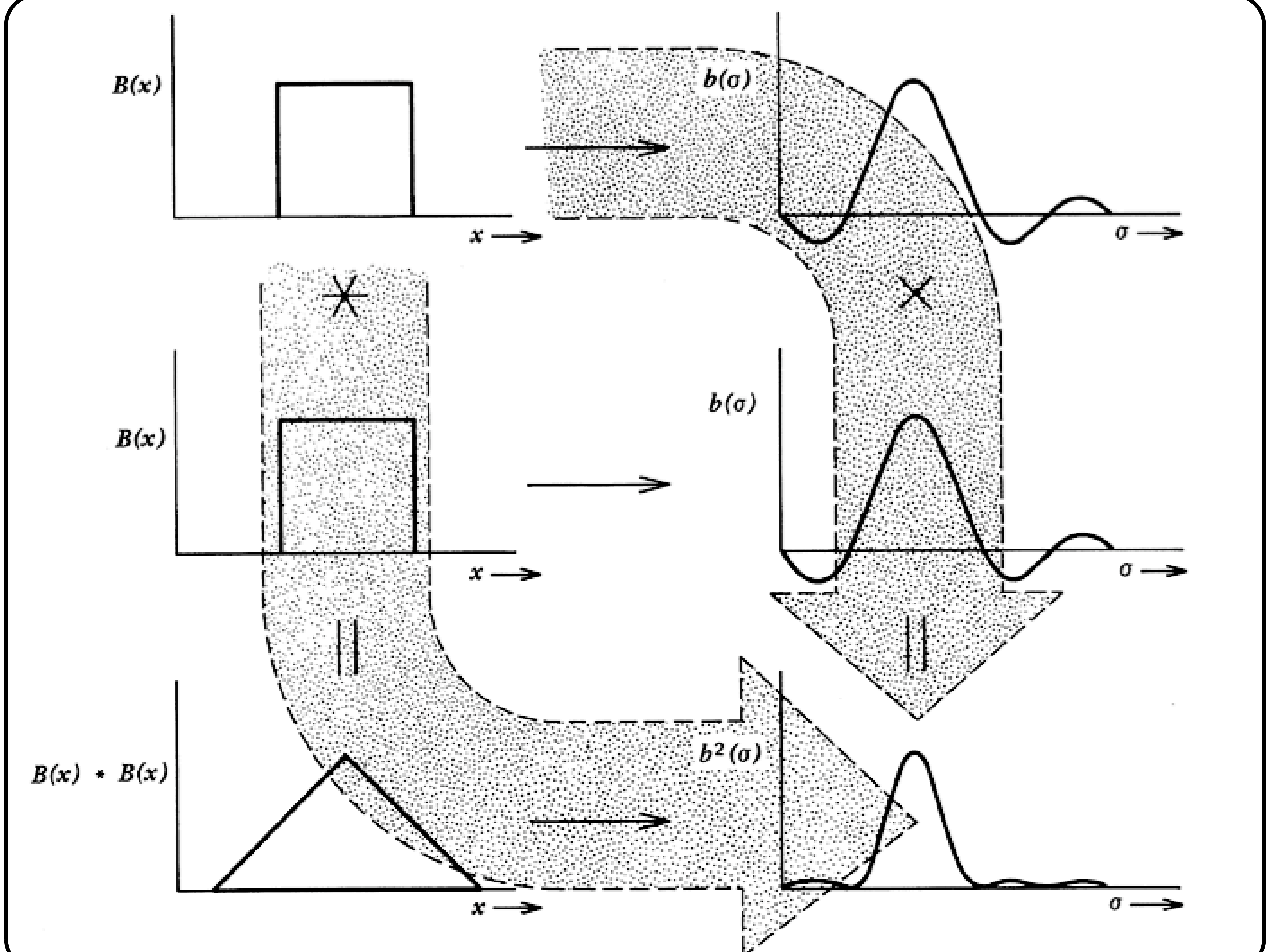


$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \Leftrightarrow \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T})$$

Convolution in practice



Always broadens the input function



Two figures from Gray page 28 & 29

In ideal case find input spectrum back

$$M(\lambda) = \int_{-\infty}^{\infty} S(\lambda')R(\lambda - \lambda')d\lambda'$$

$$S(s) \Leftrightarrow S(\lambda)$$

$$S(s) = \frac{M(s)}{R(s)}$$

$$S(\lambda) = F^{-1}[\frac{M(s)}{R(s)}]$$