Exercises OAF2

Bose Einstein statistics

Exercise 3

a) Show $X(t)$ that the probability density function for the intensity of an unpolarised signal from a thermal source is given by:

$$p(I) \;=\; \bar{I}^{-1}\, e^{-I/\bar{I}}, \tag{1}$$

where $\bar{I} \;=\; 2\sigma^2$. The variance σ^2 here relates to the (uncorrelated) gaussian stochastic variables $\mathrm{Re}\big(\tilde{E}_0(t)\big)$ and $\mathrm{Im}\big(\tilde{E}_0(t)\big)$, that give a scalar description of the fluctuating behaviour of the *phasor* $\tilde{E}_0(t)$.

b) Proof that for the variance of the intensity I of an unpolarised signal:

$$\overline{\Delta I^2} = \left(\bar{I}\right)^2 \tag{2}$$

Exercise 4

a) The equation for the stationary state of a free particle follows from the solution of the time-independent Schrödinger equation. The wave amplitude Ψ is then given by:

$$\Psi(\vec{r}) \;=\; e^{\frac{2\pi i}{h}\vec{p}\cdot\vec{r}} \tag{3}$$

When considering a finite volume, the allowed values for the momentum $\vec{p}$ must meet the specific boundary conditions, which causes quantization of the allowed momentum and energy levels.

Show that the number of occupation levels dZ_k for values of the momentum between p_k and $p_k + dp_k$ *per unit of volume* is given by:

$$dZ_k \;=\; \frac{4\pi p_k^2}{h^3} dp_k \tag{4}$$

b) Consider a black body cavity with temperature T. The radiation field can be described as a *boson gas*. Calculate the number of occupation levels per cm^3 for the *average* photon frequency that corresponds to a temperature of $T = 5000K$, integrated over the frequency interval of the natural line width. Assume a coherence time of $\Delta t_c = 10^{-8}$ s.

c) Derive the *energy density per Hz* of the black body radiation field, using the result you found above and the mean occupation number for a photon gas.